Valuation of Energy Investments as Real Options: The case of an Integrated Gasification Combined Cycle Power Plant

Luis M. Abadie
Bilbao Bizkaia Kutxa
Gran Vía, 30
48009 Bilbao, Spain
Tel +34-607408748
Fax +34-944017996
E-mail: imabadie@euskalnet.net

José M. Chamorro
University of the Basque Country, Spain
Dpto. Fundamentos del Análisis Económico I
Av. Lehendakari Aguirre, 83
48015 Bilbao, Spain
Tel. +34-946013769
Fax +34-946013891
E-mail: jepchgom@bs.ehu.es

February 3, 2005

Abstract

In this paper we analyze the valuation of options stemming from the flexibility in an Integrated Gasification Combined Cycle (IGCC) Power Plant.

First we use as a base case the opportunity to invest in a Natural Gas Combined Cycle (NGCC) Power Plant, deriving the optimal investment rule as a function of fuel prices and the remaining life of the right to invest. Additionally, the analytical solution for a perpetual option is obtained.

Second, the valuation of an operating IGCC Power Plant is studied, with switching costs between states and a choice of the best operation mode. The valuation of this plant serves as a base to obtain the value of the option to delay an investment of this type; it deals with an American option on a series of European nested options.

Finally, we derive the value of an opportunity to invest either in a NGCC or IGCC Power Plant, that is, to choose between an inflexible and a flexible technology, respectively.

Numerical computations involve the use of one- and two-dimensional binomial lattices that support a mean-reverting process for the fuel prices. Basic parameter values refer to an actual IGCC power plant currently in operation.

Keywords: Real options, Power plants, Flexibility, Stochastic Costs.

Contents

1 Introduction

It is broadly accepted in the financial literature that the traditional techniques based on discounted cash flows are not the most appropriate tool to evaluate uncertain investments, especially in presence of irreversibility considerations, or a chance to defer investment, or when there is scope for flexible management. In these cases, it is usually preferable to employ the methods for pricing options, such as Contingent Claims Analysis or Dynamic Programming.

On the other hand, the energy sector is of paramount importance for the development of any society. Besides, its especific weight both in the real and financial sectors of the economy cannot be neglected. Therefore the use of inadecuate instruments in decision making may be particularly onerous. In addition, the high sums involved in the energy industry, its operating flexibility and environmental impact, the progressive liberalization of the markets for its inputs and outputs along with many types of uncertainties, all of them render this kind of investments a suitable candidate to be valued as real options.

The aim of this paper is to use the real options methodolgy to assess decisions of investment in power plants. Especifically, we compare an inflexible technology (Natural Gas Combined Cycle, or NGCC henceforth) with a flexible one (Integrated Gasification Combined Cycle, or IGCC). We derive the best mode of operation, the value of the investments and the optimal investment rule when there exists an option to wait. We consider two stochastic processes with mean reversion for coal and gas prices, and also switching costs between modes of operation. The output (electricity) price, though, follows a deterministic path.

From a growing strand of related literature we would mention we would mention Herbelot [7], who studied the fulfillment of restrictions on SO_2 emissions, be it either through the purchase of emission permits, or changing the fuel or deleting polluting agents in the factory itself.[1] On the other han, Kulatilaka [10] developes a dynamic model that allows to value the flexibility in a flexible manufacturing system with several modes of operation, using a matrix of transition probabilities between states. In this paper, the importance of flexibility in the design of systems is emphasized from the viewpoint of both the engineers and the competitors. Later on, Kulatilaka [12] analyzes the choice between a flexible technology (fired by oil or gas) and two inflexible technologies to generate electricity. Some numerical results appear in Kulatilaka [11], where he normalizes the gas price in terms of the oil price. A similar problem with two stochastic processes is dealt with in Brekke and Schieldrop[3], but in this case there are no switching costs while prices follow standard geometric brownian motions. Finally, Robel [14] and

[1] A summary of this work appears in Dixit and Pindyck [6].

Insley [8] develop some aspects of the stochastic process that we use in this study, namely an Inhomogeneus Geometric Brownian Motion (IGBM).

The paper is organized as follows. In Section 2 we briefly introduce the IGCC and NGCC technologies; we also report some basic parameter values from an actual power plant that we use in our computations. Section 3 presents the stochastic model for fuel price (IGBM), its main features and the binomial lattices for one and two variables that we use later in the numerical applications. The choice of the binomial lattice as a resolution method is due both to its suitability and acceptance by the industry.[2] Then Section 4 focuses on a perpetual option to invest in an asset whose value depends on fuel price. We obtain a solution in terms of Kummer's confluent hypergeometric function. This section also includes a practical application and a sensitivity analysis. The results serve later on to verify how the binomial lattices behave. In Section 5 first we value the option to invest in a NGCC plant, second we value an operating IGCC plant with switching costs. Then we obtain the value of the right to invest in the flexible technology (IGCC) and finally we derive the optimal investment rule when it is possible to choose between the NGCC and IGCC alternatives. A final section with our main findings concludes.

2 The NGCC and IGCC technologies

2.1 Some features of electric power plants

The production of electricity can be viewed as the exercise of a series of nested real options to transform a type of energy (gas, oil, coal, or other) in electric energy. There are two sets of outstanding information. The first one has to do with the characteristics of the energy inputs used in the production process. The second one comprises the operation features of the electric power plants, among them: the net output, the rate of efficiency ("operating heat rate"), the costs to start and stop, the fixed costs, the starting and stopping periods, and the physical restrictions that prevent instantaneous changes between states. These factors determine the wedge between the prices of the energy consumed and produced. On this basis, at each instant it is necessary to decide how much electricity to produce and how, i.e. the possible state changes to realize.

2.2 Natural Gas Combined Cycle (NGCC) Technology

It is based on the employment of two turbines, one of natural gas and another one of steam. The exhaust gases from the first one are used to generate the steam that is used in the second turbine. Thus it consists of a Gas-Air cycle

[2] See Mun [13].

and a Water-Steam cycle. This system allows for a higher net efficiency, [3] close to 55%; future trends aim at reaching net efficiencies of 60% in NGCC plants of 500 Mw.

The advantages of a NGCC Power Plant are:[4]

- a) Lower emissions of CO_2, estimated about 350 g/Kwh, which allow an easier fulfillment of the Kyoto protocol;
- b) Higher net efficiency, between 50% and 60%;
- c) Low cost of the investment, about 500 €/Kw installed;
- d) Less consumption of water and space requirements, which allow to build in a shorter period of time and closer to consumer sites;
- e) Lower operation costs, with typical values of 0.35 cents €/KWh.

In addition, a NGCC power plant can be designed as a base plant or as a peak plant; in the latter case, it only operates when electricity prices are high enough, what usually happens during periods of strong growth in demand.

On the other hand, the disadvantages of a NGCC Power Plant are:

- a) The higher cost of the natural gas fired in relation to coal's;
- b) The insecurity concerning gas supplies, since reserves are more unevenly distributed over the world;
- c) The strong rise in the demand for natural gas, which can cause a consolidation of prices at higher than historical levels.

2.3 Integrated Gasification Combined Cycle (IGCC) Technology

It is based on the transformation of coal into synthesis gas, which is composed principally of carbon monoxide (CO) and hydrogen (H_2).[5] This gas keeps approximately 90% of the heating value of the original coal. However, the appearance of CO as a by-product demands a special care with this technology, given the harmful impact of small quantities of this gas on human health. Besides, the process of gasification increases the costs of the original investment and those of operating a plant of this type.

The synthesis gas obtained has several applications:

[3] The net efficiency refers to the percentage of the heating value of the fuel that is transformed into electric energy.

[4] Our reference is ELCOGAS (2003): "Integrated gasification combined cycle technology: IGCC" and its actual application in the power plant of Puertollano (Spain).

[5] According to the following reactions: $C + CO_2 \rightarrow 2\ CO$ and $C + H_2O \rightarrow CO + H_2$.

- a) Generation of electric energy in IGCC power plants;
- b) Production of hydrogen for diverse uses, like fuel cells; note the promising future of hydrogen as a fuel in general;
- c) As an input to chemical products, like ammonia, for manufacturing fertilizers;
- d) As an input to produce sulphur and sulphuric acid.

After the stage of gasification, it is time to clean the gas by means of washing processes with water and absorption with dissolvents. This makes it possible that an IGCC plant has significantly lower emissions of SO_2, NO_x and particles, in relation to those generated in standard coal power plants. The emissions of CO_2 per Kwh are also lower, but in this case the improvement derives mainly from the higher net efficiency of the cycle (currently, typical values fluctuate around 42%, whith a trend towards approaching 50%, provided this methodology enhances its design, which is in its first steps). Right now, the emissions of CO_2 from an IGCC power plant (about 725 g/Kwh) are estimated at 20% lower than those from a classic coal power plant [6]. However, they are clearly higher than those from NGCC (around 350 g/Kwh).

In addition to the gasification plant, the operation of an IGCC power plant requires another costly element, namely the air separation unit, for producing oxygen as an oxidizing agent. This unit may represent between 10% and 15% of total investment. Needless to say, the units for gasification, air separation, and other auxiliary systems raise the initial outlay and imply higher operation costs.

From the viewpoint of real options, it is necessary to highlight the following aspects of this technology:

- a) The investment in an IGCC plant can be considered as a strategic investment in a new technology, whose ultimate results will depend on its final success or failure;
- b) The IGCC power plant is a flexible technology concerning the possible fuels to use; apart from the synthesis gas, it may fire oil coke, heavy refinery liquid fuels, natural gas, biomasss, and urban solid waste, among others. In this way, at each time it is possible to choose the best input combination according to relative prices. The valuation of this operating flexibility between synthesis gas and natural gas constitutes one of the tasks of this paper.

[6] Coal power plants are classified in subcritical, supercritical and ultrasupercritical, depending on the pressure and the temperature of the steam. The net efficiency for coal power plants rounds about 35% in the supercriticals, and increases with higher investment costs but also entails an increased hazard due to working at higher temperatures.

2.4 Representative values used in the numerical applications

We have used the following standard values:

Table 1. Basic parameters

Concepts	PC Subcritical	IGCC	NGCC Base
Output Mw (P)	500	500	500
Production Factor (%) (FP)	80%	80%	80%
Net Efficiency (%) (RDTO)	35%	41%	50.5%
Investment Cost €/Kw (i)	1.186	1.300	496
O&M cts. €/Kwh (CVAR)	0.68	0.71	0.32

The Production Factor is the percentage of the total capacity used on average over the year. Using these data, the heat rate, the plant's consumption of energy, and the total production of electricity can be computed:

Heat Rate: $HR = 3600/RDTO/1000000$, in GJ/Kwh.[7]

Investment Cost $I = 1000 * i * P$, in Euros.[8]

Total annual production: $A = 1000P * 365 * 24 * FP$, in Kwh.

Fuel energy needs: $B = 1000P * 365 * 24 * FP * HR$, in GJ/year.

Now with these formulae we estimate:

Table 2. Resulting values

Concepts	PC Subcritical	IGCC	NGCC Base
Heat Rate GJ/Kwh	0.0103	0.0088	0.0071
Total Investment (million €)	593	650	248
Annual production (million Kwh)	3,504	3,504	3,504
Fuel Energy (GJ/year)	36,041,143	30,766,829	24,979,010

3 The stochastic model for the fuel price

3.1 The Inhomogeneous Geometric Brownian Motion (IGBM)

In a model for long-term valuation of energy assets, it is convenient to keep in mind that prices tend to revert towards levels of equilibrium after an incidental change. Among the models with mean reversion, we have chosen the Inhomogeneous Geometric Brownian Motion or IGBM process (Bhattacharya [2]):

$$dS_t = k(S_m - S_t)dt + \sigma S_t dZ_t, \tag{1}$$

where:

S_t: the price of fuel at time t.

S_m: the level fuel price tends to in the long run.

[7]One Kwh amounts to 3.600 KJ, and a GJ (Gigajoules) is a million KJ (Kilojoules).

[8]Since power is measured in Mw.

k: the speed of reversion towards the "normal" level. It can be computed as $k = \log 2/t_{1/2}$, where $t_{1/2}$ is the expected half-life, that is the time for the gap between S_t and S_m to halve.

σ: the instantaneous volatility of fuel price, which determines the variance of S_t at t.

dZ_t: the increment of a standard Wiener process. It is normally distributed with mean zero and variance dt.

Some of the reasons for our choice are:

- a) this model satifies the following condition (which seems reasonable): if the price of one unit of fuel reverts to some mean value, then the price of two units reverts to twice that same mean value.
- b) The term $\sigma S_t dZ_t$ in the differential equation precludes, almost surely, the possibility of negative values.
- c) This model admits as a particular solution $dS_t = \alpha S_t dt + \sigma S_t dZ_t$ when $S_m = 0$ and $\alpha = -k$; therefore, it includes Geometric Brownian Motion (GBM) as a particular case.
- d) The expected value in the long run is: $E(S_\infty) = S_m$; this is not true in Schwartz's model [15], where $E(S_\infty) = S_m(e^{-\frac{\sigma^2}{4k}})$.

3.1.1 First and second moments of the actual process

Now we compute the first two moments of an actual path described by an IGBM process.[9]. Following Kloeden and Platen [9],[10] for linear stochastic differential equations, and denoting $m(t) \equiv E(S_t)$ and $P(t) \equiv E(S_t^2)$, we find that:

$$\frac{dm(t)}{dt} = a_1(t)m(t) + a_2(t), \tag{2}$$

$$\frac{dP(t)}{dt} = (2a_1(t) + b_1^2(t))P(t) + 2m(t)(a_2(t) + b_1(t)b_2(t)) + b_2^2(t), \tag{3}$$

where $a_1 = -k$, $a_2 = kS_m$, $b_1 = \sigma$ and $b_2 = 0$.

A) Expected value.

In this case, the expected value satisfies the following differential equation:

$$\frac{dm(t)}{dt} = -km(t) + kS_m = k(S_m - m(t)). \tag{4}$$

This equation can be easily integrated:

$$E(S_t) \equiv m(t) = S_m + (S_0 - S_m)e^{-kt}, \tag{5}$$

[9] The actual path obeys the real trend. Consequently it cannot be used for risk-neutral valuation.

[10] Equations (2.10) and (2.11) in page 113.

where S_0 stands for current price. As mentioned above, when $t \to \infty$ the expected value is $E(S_\infty) = S_m$.

It can be verified that when $k = -\alpha$ and $S_m = 0$, the expected value of the GBM model results: $E(S_t) = S_0^{\alpha t}$.

B) Second non-central moment.

For the second non-central moment of an IGBM process, the ordinary differential equation is:

$$\frac{dP(t)}{dt} = (-2k + \sigma^2)P(t) + 2m(t)kS_m. \tag{6}$$

After substituting and rearranging, this can be rewritten as:

$$\frac{dP(t)}{dt} + (2k - \sigma^2)P(t) = 2kS_m \left(S_m + (S_0 - S_m)e^{-kt}\right). \tag{7}$$

Using an integration factor $\mu = e^{(2k-\sigma^2)t}$:

$$\mu P(t) = 2kS_m \int_0^t e^{(2k-\sigma^2)t} \left(S_m + (S_0 - S_m)e^{-kt}\right) dt + c. \tag{8}$$

After some algebra:

$$P(t) \equiv E(S_t^2) = \frac{2kS_m^2}{2k - \sigma^2}(1 - e^{(\sigma^2-2k)t}) + \frac{2kS_m(S_0 - S_m)}{k - \sigma^2}(e^{-kt} - e^{(\sigma^2-2k)t}) +$$

$$+ S_0^2 e^{(\sigma^2-2k)t}, \tag{9}$$

where we have substituted S_0^2 for the constant c so that in $t = 0$ the moment takes on the value S_0^2.

There are two especial situations, both highly unlikely in practice. They involve cases in which denominators are zero or indeterminacies like $\frac{0}{0}$ appear:

- a) $k = \frac{\sigma^2}{2}$. In this case, applying L'Hospital rule, $P(t)$ simplifies to:

$$P(t) = 2kS_m^2 t + 2S_m(S_m - S_0)(e^{-kt} - 1) + S_0^2. \tag{10}$$

- b) $k = \sigma^2$. In this case, the solution reduces to:

$$P(t) = 2S_m^2(1 - e^{-kt}) - 2kS_m(S_m - S_0)te^{-kt} + S_o^2 e^{-kt}. \tag{11}$$

Now, from the formula for the second non-central moment, we can derive the explicit solution for the variance:

$$Var(S_t) = E\left((S_t - E(S_t))^2\right) =$$

$$= e^{(\sigma^2-2k)t}\left(S_0^2 + \frac{2kS_m^2}{\sigma^2-2k} + \frac{2kS_m(S_0-S_m)}{\sigma^2-k}\right) +$$
$$+e^{-kt}\left(\frac{2kS_m(S_0-S_m)}{k-\sigma^2} + 2S_m(S_m-S_0)\right) - e^{-2kt}(S_0-S_m)^2 + \frac{2kS_m^2}{2k-\sigma^2} - S_m^2. \quad (12)$$

In particular, when $2k > \sigma^2$ and $t \to \infty$, the variance tends to:

$$\frac{2kS_m^2}{2k-\sigma^2} - S_m^2. \quad (13)$$

Only when the speed of reversion k is sufficiently high in relation to σ^2, the variance converges towards a finite value. Otherwise the variance grows without limit with the passage of time.

In subsequent sections it will be convenient to use simpler expressions for the numerical computations. It is known that, when the increment Δt is very small, given: $e^{at} = 1 + at + \frac{(at)^2}{2} + \ldots$, then $e^{a\Delta t} \approx 1 + a\Delta t$. Substituting in expressions (5) and (12), the usual results of Euler-Maruyama's approximation arise:

$$E(S_t) \approx S_{t-1} + k(S_m - S_{t-1})\Delta t, \quad (14)$$

$$Var(S_t) \approx S_{t-1}^2 \sigma^2 \Delta t. \quad (15)$$

Finally, when $S_m = 0$ and $k = -\alpha$ we get:

- the expected value: $S_0 e^{\alpha t}$;
- the second non-central moment: $S_0^2 e^{(\sigma^2+2\alpha)t}$;
- the variance: $S_0^2 e^{(\sigma^2+2\alpha)t} - S_0^2 e^{2\alpha t} = S_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$.

That is, we get the formulas corresponding to a standard GBM process.

3.2 Risk-Neutral IGBM Process

When using binomial lattices below, we will make use of the risk-neutral valuation principle and assume that the world is risk neutral.

3.2.1 The fundamental pricing equation

The change from an actual process to a risk-neutral one is accomplished by replacing the drift in the price process(in the GBM case, α) with the growth rate in a risk-neutral world ($r-\delta$, where δ denotes the net convenience yield). Note, though, that the convenience yield is not constant in a mean-reverting process .[11]

[11] If we equate $(r-\delta)S$ to the difference between the coefficient of dt in (1) and the risk premium, the resulting expression for δ is a function of S_t.

In order to obtain the risk-neutral version of the IGBM process, we simply discount a risk premium (which, according to the CAPM, is)

$$\rho\sigma\phi S \tag{16}$$

to its actual rate of growth. In this expression, ρ is the correlation between the returns on the market portfolio and the fuel asset, and σ is the asset's volatility. Finally, ϕ denotes the market price of risk, which is defined as:

$$\phi \equiv \frac{\alpha_M - r}{\sigma_M}, \tag{17}$$

where α_M is the expected return on the market portfolio, r is the riskless interest rate, and σ_M denotes the volatility of market returns.

If certain "complete market" assumptions hold, it can be shown that the value of an investment $V(S,t)$, which is a function of fuel price and calendar time, follows the differential equation:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + [k(S_m - S) - \rho\sigma\phi S]\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV + C(S,t) = 0, \tag{18}$$

where $C(S,t)$ is the instantaneous cash flow generated by the investment.

When the problem is to price an option whose value F depends on S and t, there is no cash flow and the above equation reduces to:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} + [k(S_m - S) - \rho\sigma\phi S]\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} - rF = 0. \tag{19}$$

3.2.2 First and second moments of the risk-neutral process

Let $\hat{S}_t$ denote the risk-neutral version of S_t:

$$d\hat{S}_t = [k(S_m - \hat{S}_t) - \rho\sigma\phi\hat{S}_t]dt + \sigma\hat{S}_t dZ_t. \tag{20}$$

Then, it can be shown that it has an expected value:

$$E(\hat{S}_t) = \frac{kS_m}{k+\rho\sigma\phi}[1 - e^{-(k+\rho\sigma\phi)t}] + S_0 e^{-(k+\rho\sigma\phi)t}. \tag{21}$$

Using the first two elements of a Taylor's expansion, this can be approximated by:

$$S_{t-1} + (kS_m - S_{t-1}(k+\rho\sigma\phi))\Delta t. \tag{22}$$

Concerning the second non-central moment, it is:

$$\begin{aligned} E(\hat{S_t^2}) = {} & \frac{2k^2 S_m^2}{(k+\rho\sigma\phi)} \frac{(1 - e^{-(2k+2\rho\sigma\phi-\sigma^2)t})}{(2k+2\rho\sigma\phi-\sigma^2)} - \\ & - \frac{2k^2 S_m^2}{(k+\rho\sigma\phi)} \frac{(e^{-(k+\rho\sigma\phi)t} - e^{-(2k+2\rho\sigma\phi-\sigma^2)t})}{(k+\rho\sigma\phi-\sigma^2)} + \\ & + 2kS_m S_0 \frac{(e^{-(k+\rho\sigma\phi)t} - e^{-(2k+2\rho\sigma\phi-\sigma^2)t})}{(k+\rho\sigma\phi-\sigma^2)} + S_0^2 e^{-(2k+2\rho\sigma\phi-\sigma^2)t}. \end{aligned} \tag{23}$$

After discretization, there results: $Var(\hat{S}_t) \approx \sigma^2 S_{t-1}^2 \Delta t$.

3.3 Binomial Lattice for a risk-neutral IGBM variable

The time horizon T is subdivided in n steps, each of size $\Delta t = T/n$. Starting from an initial value S_0, at time i, after j positive increments, the value of the fuel asset is given by $S_0 u^j d^{i-j}$, where $d = 1/u$.

Consider an asset whose risk-neutral behaviour follows the differential equation:

$$d\hat{S} = (k(S_m - \hat{S}) - \rho\sigma\phi\hat{S})dt + \sigma\hat{S}d\hat{Z}. \tag{24}$$

This can also be written as:

$$d\hat{S} = \left(\frac{k(S_m - \hat{S})}{\hat{S}} - \rho\sigma\phi\right)\hat{S}dt + \sigma\hat{S}d\hat{Z} = \mu^*\hat{S}dt + \sigma S d\hat{Z}. \tag{25}$$

Since it is usually easier to work with the processes for the natural logarithms of asset prices, we carry out the following transformation: $X = ln\hat{S}$. Thus $X_s = 1/\hat{S}$, $X_{ss} = -1/\hat{S}^2$, $X_t = 0$, and by Ito's Lemma:

$$dX = (\frac{k(S_m - \hat{S})}{\hat{S}} - \rho\sigma\phi - \frac{1}{2}\sigma^2)dt + \sigma d\hat{Z} = \mu^{**}dt + \sigma d\hat{Z}, \tag{26}$$

where $\mu^{**} \equiv k(S_m - \hat{S})/\hat{S} - \rho\sigma\phi - \frac{1}{2}\sigma^2$ depends at each moment on the asset value $\hat{S}$.

Following Euler-Maruyama's discretization, the probabilities of upward and downward movements must satisfy three conditions:

- a) $p_u + p_d = 1$.
- b) $E(\Delta X) = p_u\Delta X - p_d\Delta X = \left(\frac{k(S_m - \hat{S})}{\hat{S}} - \rho\sigma\phi - \frac{1}{2}\sigma^2\right)\Delta t = \mu^{**}\Delta t$. The aim is to equate the first moment of the binomial lattice ($p_u\Delta X - p_d\Delta X$) to the first moment of the risk-neutral underlying variable ($\mu^{**}\Delta t$).
- c) $E(\Delta X^2) = p_u\Delta X^2 + p_d\Delta X^2 = \sigma^2\Delta t + (\mu^{**})^2(\Delta t)^2$. In this case the equality refers to the second moments. For small values of Δt, we have $E(\Delta X^2) \approx \sigma^2\Delta t$.

From a) and b) we obtain the probabilities, which can be different at each point of the lattice (because μ^{**} depends on $\hat{S}$, which may vary from node to node):

$$p_u = \frac{1}{2} + \frac{\mu^{**}\Delta t}{2\Delta X}. \tag{27}$$

From c) there results $\Delta X = \sigma\sqrt{\Delta t}$; therefore, $u = e^{\sigma\sqrt{\Delta t}}$. The probability of an upward movement at point (i, j) is:

$$p_u(i,j) = \frac{1}{2} + \frac{\mu^{**}\sqrt{\Delta t}}{2\sigma}, \tag{28}$$

where:

$$\mu^{**}(i,j) \equiv (\frac{k(S_m - \hat{S}(i,j))}{\hat{S}(i,j)} - \rho\sigma\phi - \frac{1}{2}\sigma^2). \quad (29)$$

3.4 Binomial Lattice for two risk-neutral IGBM variables

Consider two assets whose prices are governed by the following risk-neutral processes:

$$d\hat{S}_1 = (k_1(S_{m_1} - \hat{S}_1) - \rho_1\sigma_1\phi\hat{S}_1)dt + \sigma_1\hat{S}d\hat{Z}_1, \quad (30)$$

$$d\hat{S}_2 = (k_2(S_{m_2} - \hat{S}_2) - \rho_2\sigma_2\phi\hat{S}_2)dt + \sigma_2\hat{S}d\hat{Z}_2, \quad (31)$$

$$d\hat{Z}_1 d\hat{Z}_2 = \rho dt, \quad (32)$$

where ρ_1 and ρ_2 denote the correlations of their respective returns with those of the market portfolio. Adopting the transformation $X_1 = ln\hat{S}_1$, $X_2 = ln\hat{S}_2$, and applying Ito's Lemma:

$$dX_1 = \left(\frac{k_1(S_{m_1} - \hat{S}_1)}{\hat{S}_1} - \rho_1\sigma_1\phi - \frac{1}{2}\sigma_1^2\right)dt + \sigma_1 d\hat{Z}_1 = \hat{\mu_1}dt + \sigma_1 d\hat{Z}_1, \quad (33)$$

$$dX_2 = \left(\frac{k_2(S_{m_2} - \hat{S}_2)}{\hat{S}_2} - \rho_2\sigma_2\phi - \frac{1}{2}\sigma_2^2\right)dt + \sigma_2 d\hat{Z}_2 = \hat{\mu_2}dt + \sigma_2 d\hat{Z}_2. \quad (34)$$

Now it is necessary to solve a system of six equations:

- a) $p_{uu} + p_{ud} + p_{du} + p_{dd} = 1$. The probabilities must sum to one.
- b) $E(\Delta X_1) = (p_{uu} + p_{ud})\Delta X_1 - (p_{du} + p_{dd})\Delta X_1 = \hat{\mu_1}\Delta t$. This is the expected value of the increment in X_1.
- c) $E(\Delta X_1^2) = (p_{uu} + p_{ud})\Delta X_1^2 + (p_{du} + p_{dd})\Delta X_1^2 = \sigma_1^2\Delta t + \hat{\mu_1}^2\Delta t^2$. This is the expected value of the second non-central moment of the increment in X_1.
- d) $E(\Delta X_2) = (p_{uu} + p_{du})\Delta X_2 - (p_{ud} + p_{dd})\Delta X_2 = \hat{\mu_2}\Delta t$. This is the expected value of the increment in X_2.
- e) $E(\Delta X_2^2) = (p_{uu} + p_{du})\Delta X_2^2 + (p_{ud} + p_{dd})\Delta X_2^2 = \sigma_2^2\Delta t + \hat{\mu_2}^2\Delta t^2$. This is the expected value of the second non-central moment of the increment in X_2.
- f) $E(\Delta X_1 \Delta X_2) = (p_{uu} - p_{ud} - p_{du} + p_{dd})\Delta X_1 \Delta X_2 = \rho\sigma_1\sigma_2\Delta t + \hat{\mu_1}\hat{\mu_2}\Delta t^2$. This is the expected value of the product $\Delta X_1 \Delta X_2$, which amounts to satisfying the correlation condition.

The solution to this system of equations, ignoring the elements in Δt^2, is:[12]

$$\Delta X_1 = \sigma_1\sqrt{\Delta t}, \tag{35}$$

$$\Delta X_2 = \sigma_2\sqrt{\Delta t}, \tag{36}$$

$$p_{uu} = \frac{\Delta X_1\Delta X_2 + \Delta X_2\hat{\mu}_1\Delta t + \Delta X_1\hat{\mu}_2\Delta t + \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{37}$$

$$p_{ud} = \frac{\Delta X_1\Delta X_2 + \Delta X_2\hat{\mu}_1\Delta t - \Delta X_1\hat{\mu}_2\Delta t - \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{38}$$

$$p_{du} = \frac{\Delta X_1\Delta X_2 - \Delta X_2\hat{\mu}_1\Delta t + \Delta X_1\hat{\mu}_2\Delta t - \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{39}$$

$$p_{dd} = \frac{\Delta X_1\Delta X_2 - \Delta X_2\hat{\mu}_1\Delta t - \Delta X_1\hat{\mu}_2\Delta t + \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}. \tag{40}$$

The branches of the lattice have been forced to recombine by taking constant increments ΔX_1 and ΔX_2 once the step size Δt has been chosen. Thus it is easier to implement the model in a computer program. However, the probabilities change from one node to another by depending on $\hat{\mu}_1$ and $\hat{\mu}_2$. Besides, it is necessary that at any time the four probabilities take on values between zero and one.

4 Perpetual Investment Option

4.1 Analytical Solution

Next we obtain the analytical solution for the value F of a perpetual call option on an investment V which in turn depends on the value of an asset whose price S follows an IGBM process.

In this case, the term F_τ disappears in (19):

$$\frac{1}{2}F_{ss}\sigma^2S^2 + (k(S_m - S) - \rho\sigma\phi S)F_s - rF = 0. \tag{41}$$

This equation may be rewritten as:

$$F_{ss}S^2 + (\alpha S + \beta)F_s - \gamma F = 0, \tag{42}$$

where the following notation has been adopted:

$$\alpha = -\frac{2(k + \rho\sigma\phi)}{\sigma^2},$$

$$\beta = \frac{2kS_m}{\sigma^2},$$

[12] Clewlow and Strickland [5] show a multidimensional lattice with two assets which follow correlated GBM's.

$$\gamma = \frac{2r}{\sigma^2}.$$

The solution to this equation can be written as:[13]

$$F(S) = A_0(\beta S^{-1})^\theta h(\beta S^{-1}). \tag{43}$$

The first and second derivatives, divided by $A_0\beta^\theta$, are:

$$\frac{F_S(S)}{A_0\beta^\theta} = (-\theta)S^{-\theta-1}h(\beta S^{-1}) + S^{-\theta-2}h^{'}(\beta S^{-1})(-\beta),$$

$$\begin{aligned}\frac{F_{SS}(S)}{A_0\beta^\theta} &= \theta(\theta+1)S^{-\theta-2}h(\beta S^{-1}) + (-\theta)S^{-\theta-3}h^{'}(\beta S^{-1})(-\beta) + \\ &\quad + S^{-\theta-3}(-\theta-2)(-\beta)h^{'}(\beta S^{-1}) + S^{-\theta-4}h^{''}(\beta S^{-1})\beta^2.\end{aligned}$$

Substituting these expressions in (42) and simplifying we get:

$$\begin{aligned}S^{-\theta}h(\beta S^{-1})\left(\theta(\theta+1) - \alpha\theta - \gamma\right) + S^{-\theta-1}\Big(S^{-1}\beta^2 h^{''}(\beta S^{-1}) + \\ + h'(\beta S^{-1})(\beta\theta + (\theta+2)\beta - \alpha\beta - S^{-1}\beta^2) - h(\beta S^{-1})\theta\beta\Big) &= 0\end{aligned}$$

For this equality to hold, first it must be:

$$\theta(\theta+1) - \theta\alpha - \gamma = \theta^2 + \theta(1-\alpha) - \gamma = 0. \tag{44}$$

This equation allows to determine the value of θ, since the remaining terms are known constants.

Once the value of θ has been obtained, the remainder of the equation is:

$$(\beta S^{-1})h^{''}(\beta S^{-1}) + h^{'}(\beta S^{-1})(2\theta + 2 - \alpha - \beta S^{-1}) - \theta h(\beta S^{-1}) = 0.$$

This is Kummer's Differential Equation, [14] where: $a = \theta$, $b = 2\theta + 2 - \alpha$ and $z = (\beta S^{-1})$. The general solution to this equation has the form:

$$A_1U(a,b,z) + A_2M(a,b,z), \tag{45}$$

where $U(a,b,z)$ is Tricomi's or second-order hypergeometric function, and $M(a,b,z)$ is Kummer's or first-order hypergeometric function.

Therefore, the general solution to (43) will be:

$$A_0(\beta S^{-1})^\theta(A_1U(a,b,z) + A_2M(a,b,z)). \tag{46}$$

The second-order hypergeometric function has the following representation:

[13] In Dixit and Pindyck [6], $F(S) = A_0S^\theta h(S)$ is used as the solution to the Logistic or Pearl-Verhulst equation.

[14] See Abramowitz and Stegun [1], equation 13.1.1.

$$U(a,b,z) = \frac{\Gamma(1-b)}{\Gamma(1+a-b)} M(a,b,z) + \frac{\Gamma(b-1)}{\Gamma(a) z^{(b-1)}} M(1+a-b, 2-b, z), \quad (47)$$

where Γ(.) is the gamma function and M(a,b,z) is Kummer's function, whose value is given by:

$$M(a,b,z) = 1 + \frac{a}{b} z + \frac{a(a+1)}{b(b+1)} \frac{z^2}{2!} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} \frac{z^3}{3!} + \dots \quad (48)$$

The derivatives of Kummer's function have the following properties:

$$\frac{\partial M(a,b,z)}{\partial z} = \frac{a}{b} M(a+1, b+1, z),$$

$$\frac{\partial^2 M(a,b,z)}{\partial z^2} = \frac{a(a+1)}{b(b+1)} M(a+2, b+2, z).$$

The derivatives of Tricomi's function satisfy:

$$\frac{\partial U(a,b,z)}{\partial z} = -aU(a+1, b+1, z),$$

$$\frac{\partial^2 U(a,b,z)}{\partial z^2} = a(a+1) U(a+2, b+2, z).$$

The boundary conditions will determine whether A_1 or A_2 in (46) are zero. In our case, S refers to a fuel, so we face stochastic costs. An upward evolution in S entails a reduction in profits, so that $F(\infty) = 0$ and $z = 0$, then $A_1 = 0$ and the term in Kummer's function remains: $M(a,b,z)$.[15] The solution has the form $A_m(\beta S^{-1})^{\theta} M(a,b,z)$, with $A_m \equiv A_0 A_2$.[16]

The constant A_m and the critical value S^* below which it is optimal to invest, must be jointly determined by using the remaining two boundary conditions:

- Value-Matching: $F(S^*) = V(S^*) - I(S^*)$,
- Smooth-Pasting: $F_S(S^*) = V_S(S^*) - I_S(S^*)$.

[15]Due to the fact that $U(a,b,0) = \infty$ if $b > 1$, which below will be shown to hold with our parameter values.

[16]When a downward evolution of S entails a reduction in profits, so that $F(0) = 0$ and $z = \infty$, $A_2 = 0$ and the term in Tricomi's function remains: $U(a,b,z)$. The solution is $A_u(\beta S^{-1})^{\theta} U(a,b,z)$ with $A_u \equiv A_0 A_1$, since $M(a,b,\infty) = \infty$ and then it must be $A_m = 0$. This would be the case of stochastic revenues.

4.2 Numerical Application

Consider the case of a NGCC plant whith the following features from Table 1:

- Plant size or Output: 500,000 Kwh;
- Production Factor: 80%;
- Net Efficiency: 50.5%.

Table 2 shows that the plant consumes 0.007129 GJ per KWh. Therefore, annual consumption will be 24,979,010 GJ, and 3,504 million Kwh will be generated. With an investment cost of 496 euros per Kilowatt installed, the total cost of the plant would amount to 248 million euros. The useful life of the investment would be 25 years, and we set the riskless interest rate at 5%. We assume a fixed price of 3.5 cents €/Kwh for the electricity, and a variable cost of 0.32 cents €/Kwh.

Under these conditions, the net present value of revenues, net of the investment outlay and the present value of variable costs, is:

$$\int_0^{25} 3,504,000,000\frac{(3.5-0.32)}{100}e^{-0.05t}dt - 248,000,000 = 1,342,000 \text{ €}.$$

The present value of gas costs may be computed by means of the formula for the value of an annuity in presence of an IGBM process (developed in the Appendix):

$$V = \frac{kS_m(1-e^{-r\tau})}{r(k+\rho\sigma\phi)} + \frac{S - \frac{kS_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}(1-e^{-(r+k+\rho\sigma\phi)\tau}), \tag{49}$$

where S stands for the gas price.

Assuming the additional values: $S = 5.45$ €/GJ, $S_m = 3.25$ €/GJ, $\rho = 0$, $\phi = 0.40$, $k = 0.25$, $\sigma = 0.20$ and substituting them in the above formula we obtain the average cost per unit consumed yearly during the facility's life: $35.5498 + 3.3315S$. Now multiplying the annual unit cost by the annual consumption we get $887,999,971 + 83,217,315S$.

From the present value of revenues, we subtract the investment outlay, the present value of variable costs, and the value of fuel to obtain the value of the investment if realized now: $V(S) - I = 454,055,483 - 83,217,315S$.

In this case, the boundary conditions would be:

- Value-Matching: $F(S^*) = A_m(\beta(S^*)^{-1})^\theta M(a,b,\beta(S^*)^{-1})$ =454,055,483-83,217,315$S^* = V(S^*) - I(S^*)$
- Smooth-Pasting: $F_S(S^*) = A_m(\beta(S^*)^{-1})^\theta(-\theta(S^*)^{-1}M(a,b,\beta(S^*)^{-1}) - \frac{a\beta}{b(S^*)^2}M(a+1,b+1,\beta(S^*)^{-1})) = -83,217,315 = V_S(S^*) - I_S(S^*)$

By computing $\alpha = -12.5$, $\beta = 40.625$, $\gamma = 2.5$, and $\theta = 0.1827$, we derive a and b: $a = 0.1827$, $b = 14.8654$.

Thus we have a system of two equations that will allow us to determine A_m and S^*. Substituting the value $A_m(\beta(S^*)^{-1})^{\theta}$, from the Value-Matching condition, in the Smooth-Pasting condition, there results an equation in S^* which has as its solution $S^* = 2.7448$.

Next it is easy to determine that $A_m = 94.394.000$ and, finally, that the value of the option for a gas price S is:

$$94,394,000 * (\frac{40.625}{S})^{0.1827} M(0.1827, 14.8654, \frac{40.625}{S})$$

In our case, with $S = 5.45$, initially the option value is $153,870,000$ €, against a $NPV = 521,120$ €; thus it is optimal to wait.

If there were no other option but to invest now or never, the equilibrium point would be $V(S^{**}) = I(S^{**})$, whence we get $S^{**} = 5.4563$. However, with a perpetual option it is preferable to wait, since in principle and in the long run the gas cost is going to decrease and fluctuate around a price $S_m = 3.25$, and then to go on waiting until it reaches $S^* = 2.7448$ or below this value, so that the option price be equal to the net value of the investment. When the option may be exercised only during a finite period, the threshold S^* will take on a value between 5.4563 and 2.7448.

4.3 The deterministic case

In this case $\sigma = 0$ and hence the gas cost follows a deterministic path: $S_t = S_m + [S_0 - S_m]e^{-kt}$, which corresponds exactly to the mathematical expectation of the stochastic case. If an investment is made at time T, it will be worth: $454,055,483 - 83,217,315 S_T$, which is the same value as in the stochastic case when $\rho = 0$. The only difference is that this is a future value and that $S_T = S_m + [S_0 - S_m]e^{-kT} = 3.25 + 2.2e^{-0.25T}$. Therefore the present value at initial time of an investment to be made at time T is derived by discounting at the riskless interest rate:

$$[454,055,483 - 83,217,315[3.25 + 2.2e^{-0.25T}]]e^{-0.05T}.$$

Differentiating with respect to T, the optimal investment time is $T^* = 7.1557$ years. In that moment the benefits from waiting for a sure drop in gas prices fail to balance the effect of discounting. Therefore investment would take place when $S^* = S_T = 3.6177$. This is the value S^* will approach to as $\sigma \to 0$.

4.4 Sensitivity analysis

Next we analyze the sensitivity of the results with respect to changes in σ, k and S_m.

- We assume several values for σ while the others remain the same.

Table 3. Critical value S^* as a function of σ										
σ	0.00	0.02	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
S^*	3.62	3.59	3.49	3.25	2.99	2.74	2.52	2.30	2.11	1.94

As could be expected, the value of the option to wait increases with volatility, which is reflected in a lower threshold S^*.

- We consider several values for k while $\sigma = 0.20$. First the special case in which $k = 0$ is computed;[17] then the differential equation has the form: $\frac{1}{2}F_{ss}\sigma^2 S^2 - rF = 0$, whose solution is: $F(S) = A_1 S^{\gamma_1} + A_2 S^{\gamma_2}$, where $\gamma_1 > 0$ and $\gamma_2 < 0$ are the roots of a quadratic equation. Since $F(\infty) = 0$, it must be: $A_1 = 0$. In our case, $\gamma_2 = -1.1583$.

  The boundary conditions are:

  Value-Matching: $A_2(S^*)^{\gamma_2} = 1,342,055,454 - 356,448,075S^*$,

  Smooth-Pasting: $A_2\gamma_2(S^*)^{\gamma_2-1} = -356,448,075$.

  They allow to determine the value of S^* for $k = 0$, which is 2.0206.

  Computing the remaining parameters by the usual procedure, the following results apply:

Table 4. Critical value S^* as a function of k		
k	$V(S) - I(S)$	S^*
$k = 0.00$	$1,342,055,454 - 356,448,075S$	2.0206
$k = 0.05$	$928,778,968 - 229,286,079S$	2.2594
$k = 0.10$	$712,083,008 - 162,610,399S$	2.4276
$k = 0.20$	$507,699,469 - 99,723,156S$	2.6608
$k = 0.25$	$454,055,483 - 83,217,315S$	2.7448
$k = 0.30$	$415,510,405 - 71,357,291S$	2.8143
$k = 0.40$	$364,000,824 - 55,508,189S$	2.9225

  According to (49), gas costs depend on k. Therefore, $V(S) - I(S)$ also depends on k. An increase in k pushes the critical value S^* upwards, since prices will drop faster. In the limit, with an infinite reversion speed, prices reach the equilibrium value instantaneously offsetting the opportunity to wait until they fall. When k tends to zero, the result converges towards the values of the case in which $k = 0$, obtained from the analytic resolution of a simpler option.

[17] The same results can be obtained by using equation (46) noting that $\alpha = 0$, $\beta = 0$, $\gamma = 2.5$, $\theta = 1.1583$, $A_1 = 0$ and $M(a, b, 0) = 1$. In this case $A_m(\beta)^\theta = A_2$. It is shown that the solution using Kummer's hypergeometric function includes other simpler kinds of options as particular solutions.

- Finally, assume several values of S_m while $\sigma = 0.20$ and $k = 0.25$. First the special case in which $S_m = 0$ is computed; although it is not very realistic, it allows to analyze the convergence towards another type of option. In this case, the differential equation has the form: $\frac{1}{2}F_{ss}\sigma^2 S^2 - kS - rF = 0$, whose solution is: $F(S) = A_1 S^{\gamma_1} + A_2 S^{\gamma_2}$. Given that $F(\infty) = 0$, it must be: $A_1 = 0$. In our case $\gamma_2 = -0.182712$.

  In this case the conditions:

  Value-Matching: $A_2(S^*)^{\gamma_2} = 1.342.055.454 - 83.217.315S^*$,

  Smooth-Pasting: $A_2\gamma_2(S^*)^{\gamma_2 - 1} = -83.217.315$,

  allow to determine the value of S^* for $S_m = 0$, which is 2.4914.

  Obtaining the remaining parameters by the usual procedure, the following results apply:

Table 5. Critical value S^* as a function of S_m

S_m	$V(S) - I(S)$	S^*
$S_m = 0.00$	$1,342,055,454 - 83,217,315S$	2.4914
$S_m = 0.05$	$1,328,393,916 - 83,217,315S$	2.5016
$S_m = 0.50$	$1,205,440,074 - 83,217,315S$	2.5884
$S_m = 1.00$	$1,068,824,964 - 83,217,315S$	2.6737
$S_m = 2.00$	$795,593,934 - 83,217,315S$	2.7925
$S_m = 3.00$	$522,363,173 - 83,217,315S$	2.7848
$S_m = 3.25$	$454,055,483 - 83,217,315S$	2.7448
$S_m = 3.50$	$385,747,793 - 83,217,315S$	2.6769
$S_m = 3.75$	$317,440,103 - 83,217,315S$	2.5645
$S_m = 4.00$	$249,132,413 - 83,217,315S$	2.3687

  It can be observed that changes in the critical value S^* are relatively small in the range between $S_m = 1.00$ and $S_m = 3.50$.

  Given $S = 5.45$, for very low values of S_m the option value increases more than the immediate investment value, with the equilibrium being reached for lower values of S^*. For high values of S_m the option value is low, while the investment value reaches negative values which are offset if the critical value S^* falls more.

5 Valuation of alternative technologies

5.1 Valuation of the opportunity to invest in a NGCC plant

Our purpose is to determine the optimal investment rule in a NGCC facility, when the opportunity is available between time 0 and T.

In the final moment $t = T$, there is no other option but to invest right then or not to invest. The decision to go ahead with the investment is taken if:

$$V = NPV(Revenues) - NPV(Expenditures) > 0.$$

A binomial lattice is arranged with the following values in the final nodes:

$$W = max(V, 0).$$

The revenues are the receipts from the electricity generated, whereas the costs refer to the initial investment, the average operation costs and those of the fuel consumed, in this case natural gas. We consider a deterministic evolution of electricity prices and variable costs, both of which would grow at a constant rate a; the initial investment would grow at a rate b.

The present value of revenues would be computed according to the following formula:

$$A.E.e^{aT}\frac{(1-e^{-dur(r-a)})}{r-a}, \tag{50}$$

where:

A = Annual production in Kwh = 500,000 Kwh.

E = Current electricity price = 0.035 €/Kwh.

a = rate of growth of electricity price and variable costs = 0%.

b = rate of growth of the initial investment: 0%, 2.5% and 5%.

T = Total time interval for the investment opportunity: though a representative period of 5 years is considered, the optimal exercise curve is computed for 1 to 30 years.

r = riskless interest rate: 5%.

dur = Estimated useful life of the electric facility upon investment: up to 25 years.

In the particular case in which $a = r$, the present value of revenues would be:

$$A.E.e^{aT}dur. \tag{51}$$

The costs of the initial investment are Ie^{bT}.

Variable expenditures have a present value:

$$A.C_{var}.e^{aT}\frac{(1-e^{-dur(r-a)})}{r-a}. \tag{52}$$

For each GJ consumed over the year, and using the reversion formula (49), assuming $\rho\sigma\phi = 0$, the value of an annuity consumed per year multiplied by the annual consumption is:

$$B[\frac{S_m(1-e^{-r\tau})}{r} + \frac{S-S_m}{r+k}(1-e^{-(r+k)\tau})], \tag{53}$$

where B is the annual fuel energy needed in GJ, which has been multiplied by the cost of one unit consumed per year.

In previous moments, that is, when $0 \leq t < T$, depending on the current gas price we compute the present value of investing (V) and compare it with the value of waiting, choosing the maximum between them:

$$W = Max(V, e^{-r\Delta t}(p_u W^+ + p_d W^-)).$$

The lattice is solved backwards, which provides the time 0 value. If we compare this value with that of an investment made at the outset, the difference will be the value of the option to wait. Logically, this option's value will always be nonnegative. By changing the initial value of the fuel unit cost it is possible to determine the fuel price at which the option value changes from positive to zero. This will be the optimal exercise price at $t = 0$.

Similarly, arranging a binomial lattice for the investment opportunity with maturity $t < T$ and changing the gas price, the optimal exercise price for intermediate moments is determined.

At the final date, investment is realized only if $V > 0$. The optimal point to invest will be found by computing the gas price for which $V = 0$; in our case, this value is $S_0^* = 5.4563$.(see Section 4.2)

As could be expected, when $a = 0$ and $b = 0$, it must be $S_\infty^* = 2.7448$, which come from the analytical solution for the perpetual option.

Our results, which show a convergence towards those of the perpetual option when the maturity of the investment opportunity increases, are the following:

Table 6. Critical value S^* with finite time

Term	$a = 0, b = 0$	$a = 0, b = 0.025$	$a = 0, b = 0.05$
0	5.4563	5.4563	5.4563
$\frac{1}{2}$	3.3268	3.5587	3.7823
1	3.2200	3.4417	3.6503
2	3.0864	3.3035	3.5107
3	3.0040	3.2250	3.4394
4	2.9480	3.1751	3.3982
5	2.9079	3.1413	3.3731
6	2.8782	3.1179	3.3575
7	2.8557	3.1012	3.3481
8	2.8386	3.0893	3.3423
9	2.8253	3.0808	3.3392
10	2.8151	3.0746	3.3378
∞	2.7448	-	-

It can be observed that an increase of investment costs over time, keeping the remaining variables constant, quickens the time to invest, as can be seen from the higher S^*.

When the time to maturity is zero, there is no influence from the rate of growth of the initial investment.

Next we analyze the optimal choice between investing or waiting, when the investment opportunity is available for five years and depending on the current fuel cost, as a function of NPV and the option value:

Table 7. NPV and option values with $dur = 5$ years

S_0	NPV	Option value	Max(NPV,Option)	Optimal decision
5.45	521,120	119,170,000	119,170,000	Wait
5.00	37,969,000	129,040,000	129,040,000	Wait
4.50	79,578,000	141,640,000	141,640,000	Wait
4.00	121,190,000	156,820,000	156,820,000	Wait
3.50	162,790,000	176,420,000	176,420,000	Wait
3.00	204,400,000	205,010,000	205,010,000	Wait
2.9079	212,070,000	212,070,000	212,070,000	Indifferent
2.50	246,010,000	245,900,000	246,010,000	Invest
2.00	287,620,000	287,450,000	287,620,000	Invest

When $S = 5.45$, the NPV is very low and the option to wait is worth more than 119 million euros. As the value of S_0 decreases, the option to wait increases in value but less than the NPV, with equality being reached when $S = 2.9079$. For lower values, it is preferable to invest immediately.

5.2 Valuation of an operating IGCC plant

Now we must compute the value of an operating IGCC power station, both right upon the initial outlay and at any moment along its useful life to derive its remaining value. We accomplish this by means of two two-dimensional binomial lattices which refer to initial states consuming either coal or gas, respectively.

At the last moment of the plant's useful life, its value is zero, whether that instant has been reached consuming coal or gas. In that final date:

$$Wc = 0 \text{ if useful life is finished consuming coal,}$$
$$Wg = 0 \text{ if useful life is finished consuming natural gas,}$$

where W_c stands for the values of the lattice nodes for the coal and W_g denotes those of the lattice nodes for the gas. In earlier times t we compute, for a time interval Δt:

a) Revenues from electricity:

$$A.E.e^{at}\frac{(1-e^{-\Delta t(r-a)})}{r-a}.$$

b) Variable costs:

$$A.C_{var}.e^{at}\frac{(1-e^{-\Delta t(r-a)})}{r-a}.$$

We assume a variable cost of $C_{var_c} = 0.0071$ €/Kwh in coal-mode and $C_{var_g} = 0.0032$ €/Kwh in gas-mode.[18]

c) Costs of coal:

$$B_c \Delta t P_c;$$

d) Costs of natural gas:

$$B_g \Delta t P_g,$$

where:

B_c: the coal energy needed per year in GJ,
B_g: the natural gas energy needed per year in GJ,
P_c: current coal price,
P_g: current gas price.

With this data, the profits by mode of operation are determined as the difference between electricity revenues and the sum of variable plus fuel costs.

Further notation is as follows:

π_c : Net profits from operating with coal.
π_g : Net profits from operating with natural gas.
$I(c \to g)$: Switching cost from coal to gas (in euros)= $20,000$ €.
$I(g \to c)$: Switching cost from gas to coal (in euros)= $20,000$ €.

The expressions for the profits are:

$$\pi_c = A.E.e^{at} \frac{(1 - e^{-\Delta t(r-a)})}{r - a} - B_c \Delta t P_c - A.C_{var_c}.e^{at} \frac{(1 - e^{-\Delta t(r-a)})}{r - a}, \quad (54)$$

$$\pi_g = A.E.e^{at} \frac{(1 - e^{-\Delta t(r-a)})}{r - a} - B_g \Delta t P_g - A.C_{var_g}.e^{at} \frac{(1 - e^{-\Delta t(r-a)})}{r - a}. \quad (55)$$

If initially the IGCC plant was consuming coal, the best of two options is chosen:[19]

- continue: the present value of the coal lattice is obtained, plus the profits from operating in coal-mode at that instant.
- switch: the present value of the gas lattice is obtained, plus the profits from operating in gas-mode at that instant, minus the costs to switching from coal to gas: $I(c \to g)$.

[18] Again, these values are taken from ELCOGAS (2003) "Integrated gasification combined cycle technology: IGCC".

[19] We have not considered the option not to operate, though it could be taken into account easily. It could be included by a third lattice, corresponding to an idle initial sate. At every time we should maximize over three possible values, taking into account the switching costs between states. If we denote the idle state by p, in this case there could be a stopping cost: $I(c \to p)$ or $I(g \to p)$, and a starting cost: $I(p \to c)$ or $I(p \to g)$. If the starting costs were very high, we would be in the case in which stopping amounts to abandone.

The binomial lattices will take on the following values:[20]

$$Wc = Max(\pi_c + e^{-r\Delta t}(p_{uu}Wc^{++} + p_{ud}Wc^{+-}p_{du}Wc^{-+} + p_{dd}Wc^{--}), \pi_g - I(c \to g) + \\ + e^{-r\Delta t}(p_{uu}Wg^{++} + p_{ud}Wg^{+-}p_{du}Wg^{-+} + p_{dd}Wg^{--})). \quad (56)$$

When the initial state corresponds to operating with natural gas, we would compute:

$$Wg = Max(\pi_c - I(g \to c) + e^{-r\Delta t}(p_{uu}Wc^{++} + p_{ud}Wc^{+-}p_{du}Wc^{-+} + p_{dd}Wc^{--}), \\ \pi_g + e^{-r\Delta t}(p_{uu}Wg^{++} + p_{ud}Wg^{+-}p_{du}Wg^{-+} + p_{dd}Wg^{--})). \quad (57)$$

Finally, at time zero the optimal initial mode of operation is chosen by:

$$Max(Wc, Wg).$$

In this way, we have derived the value of a flexible plant in operation.

In this computation, for a plant operating for a while, the cost of the initial investment plays no role, but it can be included at the outset in order to compare the outlay with the present value of expected profits.

The following values are adopted in the base case:

Table 8. An IGCC Plant

Concepts	Coal Mode	Gas Mode
Plant Size Mw (P)	500	500
Production Factor(FP)	80%	80%
Net Efficiency(%) (RDTO)	41.0%	50.5%
Investment cost €/Kw (i)	1,300	1,300
Operation cost (€cents/Kwh) (CVAR)	0.71	0.32
Fuel price (€/GJ)	1.90	5.45
Reversion value S_m(€/GJ)	1.40	3.25
Reversion speed (k)	0.125	0.25
Plant's useful life (years)	25	25
Risk-free interest rate (r)	0.05	0.05
Market price of risk	0.40	0.40
Volatility	0.05	0.20
Correlation with the market	0	0
Correlation between fuels	0.15	0.15
Cost rate of growth	0	0
Electricity price rate of growth	0	0
Switching costs (€)	20,000	20,000

Making the computations with a lattice of 300 steps (one for each month of useful life), the following results ensue, as a function of switching costs:

[20] We follow a similar procedure to that used by Trigeorgis [17] in pages 177-184 (the case with switching costs).

Table 9. Value of an IGCC Plant		
Switching costs	Plant's value	Plant's value - Initial investment
0 €	702,662,000	52,662,000
10,000 €	702,598,000	52,598,000
20,000 €	702,534,000	52,534,000
50,000 €	702,345,000	52,345,000
100,000 €	702,129,000	52,129,000
1,000,000 €	700,049,000	50,049,000
∞ €	691,987,000	41,987,000

When there are no switching costs, the value of the plant exceeds the initial investment (650,000,000 €) by 52,662,000 €. It is worth an 8.10% more than the amount disbursed.

Comparing the value in each case with that of infinite switching costs, the difference shows the value of flexibility, which amounts to 10,675,000 € in the absence of switching costs. Flexibility in the IGCC plant may arise because of reasons different from harnessing at each instant the best fuel option. For instance, it may be due to failures in elements necessary to operate in coal-mode, but that do not prevent the plant from operating in gas-mode and avert the total stopping of the facility. The same could happen in the case of problems concerning supplies of a certain kind of fuel.

In the case of infinite switching costs, in which operation is only in coal-mode, the value of the plant changes as the lattices include more steps. With 3,000 steps we reach a value (net of investment cost) of 43,353,000 €, a figure which is very close to the 43,594,000 € obtained analytically. The plant's value as a function of coal and gas costs appears in Figure 1.
Next we show the value of the plant, as a function of the remaining life, for the base case with switching costs of 20,000 €, without taking into account the initial investment of 650,000,000 €. For the sake of consistence we have used a step per month.

Table 10. Value of an operating IGCC plant as a function of the growth rate of electricity price and variable cost (a)		
Remaining life	Plant value $a = 0.00$	Plant value $a = 0.03$
25 years	702,534,000	1,245,700,000
20 years	613,835,000	999,120,000
15 years	501,490,000	742,186,000
10 years	361,090,000	479,990,000
5 years	191,020,000	223,980,000
4 years	153,950,000	175,460,000
3 years	116,150,000	128,500,000
2 years	77,806,000	83,409,000
1 year	39,044,000	40,477,000
0 year	0	0

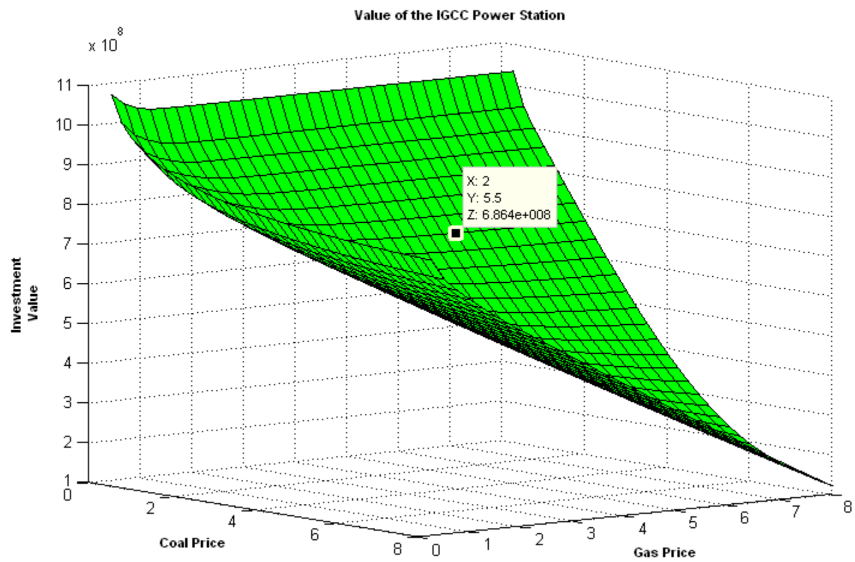

Figure 1: Value of an IGCC Plant € (a=0.00).

The shape of the curve for $a = 0$ has to do with the discount rate $r = 0.05$, whereas the electricity price remains unchanged. If $a = 0.03$, the value with two years to operate exceeds twice that with one year since, because of the reversion effect, a bigger drop in fuel cost would be expected over two remaining years.

5.3 Valuation of the opportunity to invest in an IGCC plant

In this case, the procedure is very similar to that for the optimal timing to invest in a NGCC plant. However, the value of the plant in operation must be determined at each node of the lattice. In other words, each point in the binomial lattice results from another two-dimensional binomial lattice which takes as a starting point the prices at that node.

At the final date,[21] the choice must be made between investing then if the plant's value is positive or not to invest:

$$W = max(V_{igcc}, 0).$$

With switching costs of 20,000 € as in the base case, Table 11 shows combinations of coal and gas prices imply a zero value (that is, the plant value matches initial outlay) at maturity.

[21] It is the moment at which the opportunity to invest in an IGCC plant expires.

Table 11. IGCC Critical curve at $t = T$	
Gas price (€/GJ)	Coal price (€/GJ)
∞	2.1410
5.4563	2.2325
5.45	2.2327
5.00	2.2446
4.50	2.2642
4.00	2.2974
3.50	2.3684
3.25	2.4485
3.00	2.5802
2.50	3.0682
2.00	4.0610
1.50	6.2646
1.00	18.9226
0.9897	∞

If, with a very high price for coal, the IGCC plant is to be profitable, it is necessary that gas prices remain below 0.9897. This is due to the fact that the reversion process pushes gas prices towards 3.25 €/GJ.

At previous instants, the best of the two options (to invest or to continue) has to be chosen:

$$W = Max(V_{igcc}, e^{-r\Delta t}(p_{uu}W^{++} + p_{ud}W^{+-}p_{du}W^{-+} + p_{dd}W^{--})). \quad (58)$$

If, at $t = 0$, the value obtained from the lattice is compared with that of an investment realized right then, the difference is the value of the option to wait.

By using a lattice with quarterly steps for the option to wait, and one with monthly steps to value the plant, we get the following valuations, in the base case, as a function of the time to maturity of the investment option:

Table 12. Value of the option to wait (IGCC plant)			
Term	Plant value	Option to wait	Option to wait - Plant value
5 years	52,534,000	76,398,000	23,864,000
4 years	52,534,000	74,179,000	21,645,000
3 years	52,534,000	71,048,000	18,514,000
2 years	52,534,000	66,651,000	14,117,000
1 year	52,534,000	60,592,000	8,058,000
0,5 year	52,534,000	56,835,000	4,301,000
0 year	52,534,000	52,534,000	0

The second column refers to the plant value under the assumption of a "now or never" investment in the flexible technolgy. It includes the value of the plant's flexibility.

The value of the option to wait, given the starting point of the prices, increases with the maturity of the opportunity to invest. The highest yearly increase takes place in the first period (8, 058, 000 €); henceforth, that increase is much lower, since the effect of reversion is stronger in the initial periods.[22]
The optimal investment rule in an IGCC technology can be derived following a procedure akin to that used for the NGCC technology in the base case. Nonetheless, in this case and at each time, we will have to compute the combinations of coal price and gas price for which the option to wait is worthless.

At every t, a range of initial prices for coal is chosen; then, for each one of them, we compute the gas price for which the option to wait turns from positive to zero. For an option to wait up to two years, we get:

Table 13. Optimal values with option to wait 2 years (IGCC)

Gas price	Coal price
∞	1.57
5.45	1.56
5.00	1.56
4.50	1.56
4.00	1.56
3.50	1.56
3.25	1.57
3.00	1.57
2.50	1.59
2.25	1.85
2.00	2.98
1.50	5.15
0.96	∞

These results appear in Figure 2.

[22]With $t_{\frac{1}{2}} = \frac{ln(2)}{k}$ one would guess that gas price will change from 5.45 €/GJ to 4.35 €/GJ in 2.77 years time. On the other hand, the expected price for coal 5.55 years from now would be 1.65 €/GJ. The value of $t_{\frac{1}{2}}$ results from solving: $S_0 - (\frac{S_0 - S_m}{2}) = S_m + (S_0 - S_m)e^{-kt_{\frac{1}{2}}}$.

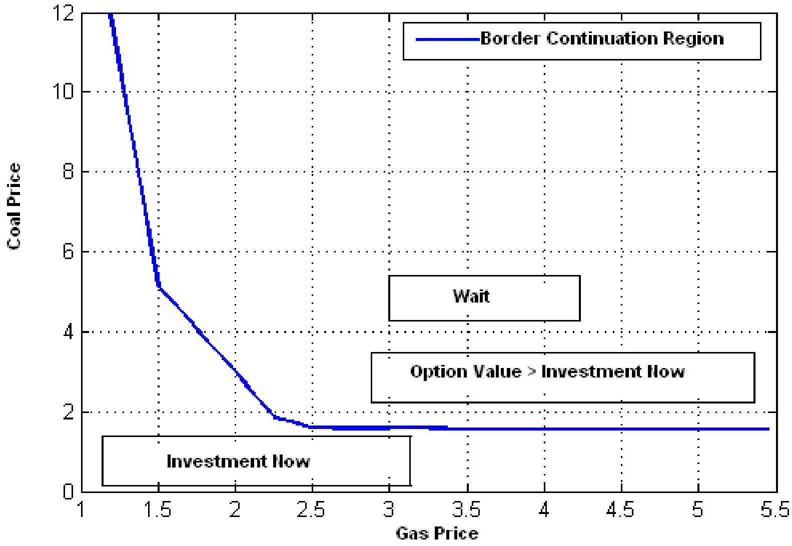

Figure 2: Border of Continuation area for investment in an IGCC Power Plant with option to wait for two years.

In Table 13, it can be observed that the curve has shifted downwards and to the left, in relation to the case in which there is no option to wait (Table 11). For a gas price of 5.00€/GJ, the resulting values for the option to invest in the IGCC plant and for the option to wait appear in Figure 3.

5.4 Valuation of the opportunity to invest in NGCC or IGCC

When there is an opportunity to invest in either one of the two technologies, at each moment we face the choice:

- to invest in the inflexible technology (NGCC),
- to invest in the inflexible technology (IGCC),
- to wait and at maturity give up the investment.

At the final date, since there is no remaining option, the best alternative among the three avaliable is chosen:

$$W = max(V_{igcc}, V_{ngcc}, 0),$$

where the value V_{igcc}, at each point in the binomial lattice, is computed by means of a two-dimensional binomial lattice with the values at that node.

If, at time T, the only possibility is to invest in the NGCC technology, the investment is realized whenever the gas price is lower than 5.4563 €/GJ.

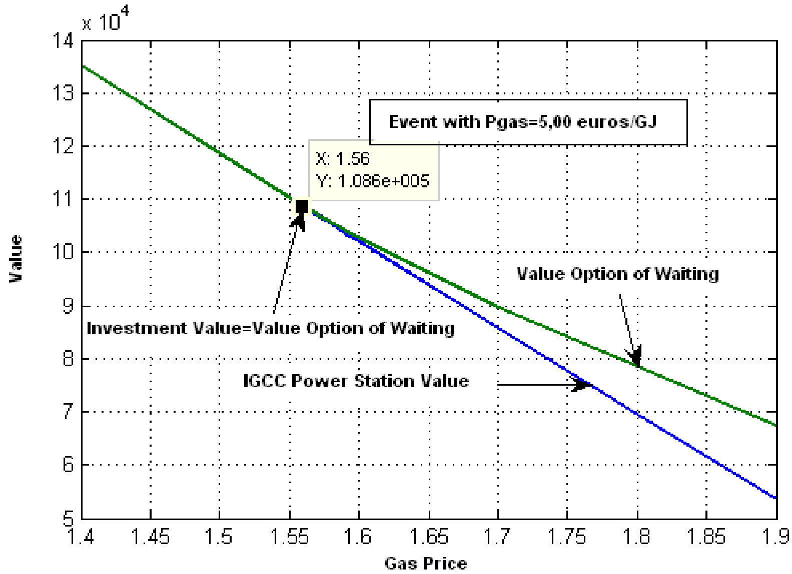

Figure 3: Values of the investment and the option to wait.

Similarly, if, at time T, the only alternative is to invest in the IGCC technology, the plant is built when the combinations of gas price and coal price lay below the curve resulting from Table 11 and shown in Figure 2.

Figure 4 shows both decision rules, but it must be noted that, in this case, it is not possible to choose the best possibility.

When, at time T, it is possible to choose between the two alternatives, there is a set of combinations gas price-coal price in which the value of the investment is positive for the two plants; consequently the investment with the highest value will be chosen. This area will be divided in two by a line starting at the point ($P_{gas} = 5.4563; P_{coal} = 2.2325$), along which there is indifference among investing in IGCC, investing in NGCC, and not investing. Figure 5 shows the three areas in which the price space divides when there is also the option not to invest.

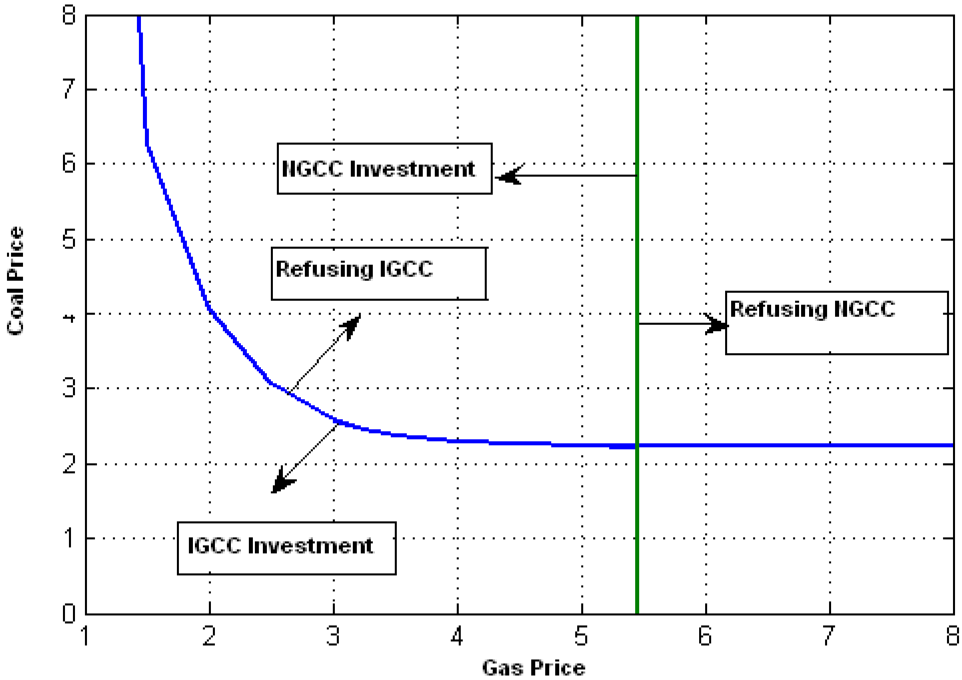

Figure 4: Border of Continuation areas without choosing between alternatives at time T.

At previous moments, the choice is:

$$W = max(V_{igcc}, V_{ngcc}, e^{-r\Delta t}(p_{uu}W^{++} + p_{ud}W^{+-}p_{du}W^{-+} + p_{dd}W^{--})).$$

This computing procedure is iteratively followed until the initial value is obtained. At that instant, the NGCC technology will be adopted if $W = V_{ngcc}$; similarly, the IGCC technology is chosen if $W = V_{igcc}$. If there is no investment at $t = 0$, this means that the best option is to wait.

The points along the optimal exercise curve are those for which the value of the option to wait changes from positive to zero. In principle, there may be two curves, one for the IGCC plant and another one for the NGCC plant. See Figure 6. It can be observed that:

- In order to invest, prices must be rather lower than those when there is no option to wait. The reversion effect, given the initial prices, promotes this behaviour.
- For a very high gas price, there will be investment in an IGCC plant if coal price is below 1.57 €/GJ, which is the same that we computed for the waiting option in the IGCC investment and this is the only technology available.

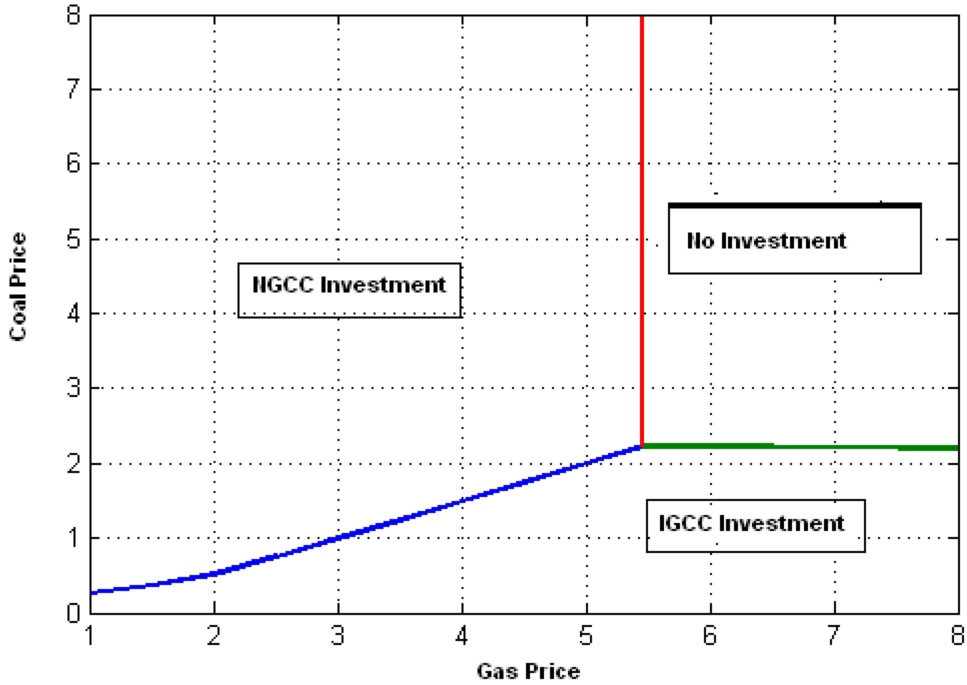

Figure 5: Border of Continuation areas choosing between alternatives at time T.

- Similarly, for a very high coal price, there will be investment in a GNCC plant if gas price is below 3.17 €/GJ.[23]
- For values close to the iso-value line between immediate investment in NGCC technolgy and IGCC, the best choice is to wait in order to see how uncertainty unfolds. The waiting zone expands into the regions of immediate investment like a wedge.

The shape of the three zones in Figure 6 somehow resembles that in Brekke and Schieldrop [3] as the analytic result of a perpetual option to wait. In this case the solution is a numerical one because the option to wait is finite.

6 Concluding remarks

Many investments in the energy sector can be conveniently valued as real options. Frequently appearing features are the operating flexibility and the

[23]This result would apply as an option to wait for two years in a NGCC plant with a lattice of 8 steps, the same step size used in the option to wait with the two technologies on offer.

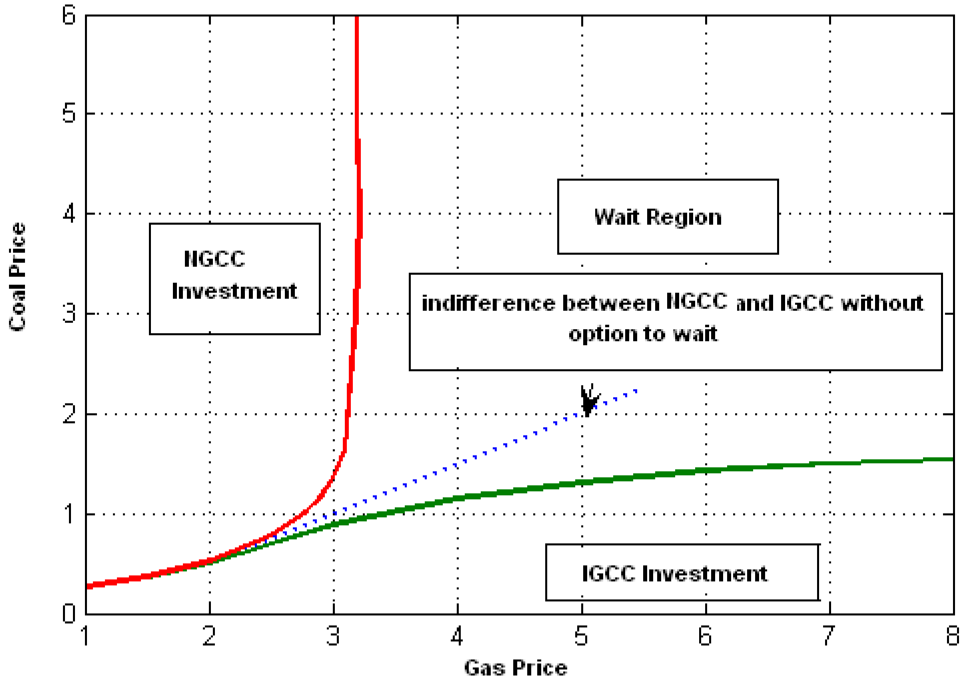

Figure 6: Border of Continuation areas without selecting between alternatives with two years of option of waiting.

possibility to delay the investment or even not to invest at the final moment. Special attention in the valuation must be paid to the nature of the stochastic processes that govern the underlying variables.

In this paper we have analyzed the valuation of the oportunity to invest in an IGCC power plant as opposed to the alternative of an NGCC power plant according to the current prices of gas and coal when they follow a mean-reverting process, namely an IGBM. As a reference point we have computed the value of a perpetual option to wait. The IGCC power plant has been valued with switching costs and choice of the optimal mode of operation.

Our results show the influence of the mean-reverting process on the critical values when current fuel prices are above their level expected in the long run. In this case the optimal price to invest is lower than without reversion, and gets even lower when the time to maturity of the right of invest is longer. The value of the operating flexibility in the IGCC power plant is low, because it is designed to function mainly with coal.
We show that there is a small region in the prices space in which it is optimal

to wait instead of investing, since the values of both technologies are very close. Outside it, the decisions are clear-cut.

A Valuation of an annuity with mean reversion

A.1 An IGBM process

Our aim is to value an asset V wich pays Xdt continuously over τ periods, with X following a mean-reverting process of the type:

$$dX = k(X_m - X)dt + \sigma X dZ_t.$$

It can be shown that the asset V satisfies the differential equation:

$$\frac{1}{2}V_{xx}\sigma^2 X^2 + (k(X_m - X) - \rho\sigma\phi X)V_x - rV - V_\tau = -X, \tag{59}$$

where it is assumed that the existing traded assets dynamically span the price X. Let ρ denote the correlation with the market portfolio, and ϕ the market price of risk:

$$\phi = \frac{(r_M - r)}{\sigma_M},$$

where r_M stands for the expected return on the market portfolio and σ_M its standard deviation. The solution $V(X, \tau)$ to the differential equation must satisfy the following boundary conditions:

- At $t = 0$ the value must be zero: $V(X, 0) = 0$,
- Bounded derivative as $X \to \infty$: $V_x(\infty, \tau) < \infty$,
- Bounded derivative as $X \to 0$: $V_x(0, \tau) < \infty$.

Using Laplace transforms we get:

$$\frac{1}{2}h_{xx}\sigma^2 X^2 + (k(X_m - X) - \rho\sigma\phi X)h_x - h(r + s) = -\frac{X}{s}.$$

Rearranging:

$$\frac{1}{2}h_{xx}\sigma^2 X^2 - (k + \rho\sigma\phi)Xh_x - h(r + s) = -kX_m h_x - \frac{X}{s}.$$

The general solution has the form:

$$h(x) = A_1 X^{\beta_1} + A_2 X^{\beta_2} + \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{s(s + r + k + \rho\sigma\phi)} + \frac{kX_m}{(k + \rho\sigma\phi)s(s + r)}.$$

The derivative is bounded; thus $A_1 = 0$. Besides, $h(0) = 0$; therefore $A_2 = 0$.

The solution simplifies to:

$$h(x) = \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{s(s+r+k+\rho\sigma\phi)} + \frac{kX_m}{(k+\rho\sigma\phi)s(s+r)}.$$

With the first and second derivatives: $h_x = \frac{1}{s(s+r+k+\rho\sigma\phi)}$; $h_{xx} = 0$, it is possible to show, by substitution, that the differential equation applies.

At this moment, the inverse Laplace transforms are taken. To do so we employ formula 29.3.12 in Abramowitz and Stegun [1], the final result being:

$$V = \frac{kX_m(1-e^{-r\tau})}{r(k+\rho\sigma\phi)} + \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}(1 - e^{-(r+k+\rho\sigma\phi)\tau}). \quad (60)$$

This formula may be useful to compute the present value of fuel costs over the whole life of a plant with inflexible technology, like an NGCC or coal plant.

A series of particular, frequently used, cases may be derived from the above general solution:

- a) If $\phi = 0$ or $\rho = 0$, then the formula reduces to

$$V = \frac{X_m(1-e^{-r\tau})}{r} + \frac{X - X_m}{r+k}(1 - e^{-(r+k)\tau}).$$

- b) If $\tau \to \infty$:

$$V = \frac{kX_m}{r(k+\rho\sigma\phi)} + \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}. \quad (61)$$

  In this case, it can be observed that the project value is the sum of two components: one related to the reversion value and another one which is a function of the initial difference between the observed value and the "normal" level of X.

- c) When it is perpetutity and also $X_m = 0$ and $k + \rho\sigma\phi = -\alpha$, then:

$$V = \frac{X}{r-\alpha}. \quad (62)$$

- d) When it is perpetutity and also $X_m = 0$ and $k + \rho\sigma\phi = -r + \delta$, then:

$$V = \frac{X}{\delta}. \quad (63)$$

References

[1] M. Abramowitz and I. Stegun. *Handbook of Mathematical Functions.* Dover Publications, 1972.

[2] S. Bhattacharya. Project valuation with mean-reverting cash flow streams. *Journal of Finance*, XXXIII(5):1317–1331, 1978.

[3] J.A. Brekke and B. Schieldrop. *Investment in Flexible Technologies under Uncertainty*, volume in Project Flexibility, Agency and Competition, Brennan M. J. and L. Trigeorgis (eds.), chapter 3, pages 34–49. Oxford University Press, 2000.

[4] M.J. Brennan and L. Trigeorgis. *Project Flexibility, Agency, and Competition.* Oxford University Press, 2000.

[5] L. Clewlow and C. Strickland. *Implementing Derivatives Models.* John Wiley and Sons, Inc., 1998.

[6] A. K. Dixit and R. S. Pindyck. *Investment under Uncertainty.* Princeton University Press, 1994.

[7] O. Herbelot. *Option Valuation of Flexible Investments: The Case of Environmental Investments in the Electric Power Industry.* PhD thesis, Massachusetts Institute of Technology, 1992.

[8] M. Insley. A real option approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management*, 44:471–492, 2002.

[9] P. E. Kloeden and E. Platen. *Numerical Solution of Stochastic Differential Equations.* Springer, 1992.

[10] N. Kulatilaka. Valuing the flexibility of flexible manufacturing systems. *IEEE Transactions on Engineering Management*, 35(4):250–257, 1988.

[11] N. Kulatilaka. The value of flexibility: The case of a dual-fuel steam boiler. *Financial Management*, 22(3):271–279, 1993.

[12] N. Kulatilaka. *The Value of Flexibility: A General Model of Real Options*, volume in Real Options in Capital Investment, Lenos Trigeorgis (ed.). Praeger, 1995.

[13] J. Mun. *Real Option Analysis.* John Wiley and Sons, Inc., 2002.

[14] G. F. Robel. Real options and mean-reverting prices. the 5th annual **R**eal **O**ptions conference. 2001.

[15] E. S. Schwartz. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance*, 52(3):923–973, 1997.

[16] E. S. Schwartz and L. Trigeorgis. *Real Options and Investment under Uncertainty*. MIT, 2001.

[17] L. Trigeorgis. *Real Options - Managerial Flexibility and Strategy in Resource Allocation*. The MIT Press, Cambridge, Massachusetts, 1996.