# The value of market switching options for combination carriers

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**Abstract:** The paper derives a real options model of flexibility and applies it to shipping, valuing the option to switch between the dry bulk market and wet bulk market for a combination carrier which is able to operate in both markets. The model is a mean-reverting (Ornstein-Uhlenbeck) version of a standard entry-exit model under stochastic prices. A fixed cost is incurred every time switching takes place. The estimated value of flexibility is related to historical price differentials between combination carriers and exclusive oil tankers of comparable size. It is concluded that new combination carriers are likely to enter the market in a few years unless a shift in current market trends takes place.

**Keywords:** Investment, uncertainty, flexibility, real options, bulk shipping.

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## 1. Introduction

In maritime economics, the fleet of combination carriers, which are hybrid vessels designed to carry either wet or dry cargo, has traditionally been the factor that linked the freight markets for tankers and dry bulk vessels. In times of strong transportation demand for oil relative to other dry bulk commodities, the combined fleet would switch to the tanker market and increase supply in the short run until increased newbuilding activity and subsequent deliveries restored the market balance. Thereby it acted as a mechanism to integrate the two markets. In the absence of extreme supply and demand imbalances, as in the tanker market in the early 1980s, such swing tonnage ensured that the freight rate differential between the tanker and dry bulk markets did not diverge very far from some long-term average.

Consequently, combination carriers do not have a market of their own – they take advantage of events in the dry and liquid bulk markets. In theory this leaves a ship owner with two types of real options: an option to switch between the tanker market and the dry bulk market when there is a rate advantage to be gained, and an option to reduce ballast time by carrying dry and liquid cargoes on alternate voyage legs (triangulation). The latter ability to lift 'backhaul' cargoes out of areas where competing conventional vessels depart unladen represents a competitive advantage. However, following the introduction of the oil/bulk/ore (OBO) design in 1966, the capacity of the combination carrier fleet outgrew the availability of combined-carrier voyages and so this competitive advantage was lost (Stopford, 1997). Moreover, the presence of a large combination carrier fleet ensured that surplus capacity was transmitted between the tanker and dry bulk markets, making the two markets highly integrated and thereby reducing the value of the option to switch between them. As so often happens, therefore, the concept became a victim of its own success. Having reached a peak of around 48.3 million deadweight tons (DWT) in 1979, comprising about 9.4% of the total bulk ship fleet, the fleet of combination carriers shrank gradually to 117 vessels of 11.7 million DWT in December 2003, or 1.9% of the total bulk ship fleet (Clarksons Research, 2004). Figure 1 shows the development of the combo fleet since 1970.

<Figure 1 inserted here>

There are currently no combination carriers on order in the world, illustrating how the vessel type has fallen out of favor with the shipping community. However, recent market trends make it natural to ask whether the fleet of combination carriers is now down to a level where the value of flexibility for such vessels once again is about to exceed the cost. There is a tendency of increasing freight differentials between the dry bulk and wet bulk markets. Quoted newbuilding prices also indicate shrinking average price differentials between combination carriers and comparable specialized vessels. These trends which are described in more detail in the empirical part of the paper, work in favor of combination carriers.

Motivated by the development just described, the objective of this paper is to assess the value of the flexibility of combination carriers in a formal real options framework under stochastic prices. We will only be considering the flexibility to switch between these markets for the longer run, rather than switching between oil and dry bulk freights during the same (triangular) voyage. This is consistent with what is observed.

The required technical tool is a (real options) entry-exit model as in Dixit (1989), but one based on mean-reverting, Ornstein-Uhlenbeck prices as opposed to geometric Brownian prices used by Dixit. The model is developed in Sect. 2, and represents a separate theoretical contribution of the paper. Section 3 contains the empirical analysis, estimating the value of flexibility under various assumptions that seem reasonable from historical data. Section 4 concludes that combination carriers are likely to enter the market within a few years if current market trends are sustained.

## 2. The model

Value of flexibility characterizing the combination carrier fleet arises from freight rate differentials between the two markets. As long as the freight rate processes satisfy certain charactertistics, an optimal policy will consist of switching to the other market as soon as the freight rate there exceeds the freight rate in the current market of operation by a certain amount. The objective of this section is to determine the expected and discounted benefit from such a policy, relative to staying in one market all the time. If the value of flexibility exceeds the extra price one will have to pay, it is natural to conclude that the combination carrier is a better buy than a single-role ship.

Suppose a new combination carrier is acquired at time $t$, and starts to operate in the oil market because freight rates in this market are relatively high. The freight rate, interpreted as the flow of net revenue, equals $p_w(t)$. A different revenue, $p_d(t)$, would have been obtained if the ship instead had been operating in the dry bulk market at the same time. The freight rate differential, $p(t)=p_d(t)-p_w(t)$, is assumed to follow a continuous and autonomous Itô process

$$(1) \qquad dp = \mu_p dt + \sigma_p dz$$

Here $dt$ is the time increment and $dz$ is a standard Wiener process. The drift and volatility parameters, $\mu_p$ and $\sigma_p$, may depend on $p$ but not on explicit time, $t$. For a market with free entry it is reasonable to assume that the freight rates do not drift very far apart in the long run, so $\mu_p$ can be expected to be more negative the higher $p$, and more positive the lower $p$.

Future cash flows are discounted at a constant rate $\rho$. The discount rate can be seen as the sum of a real interest rate, $r$, a rate of depreciation, $\lambda$, and a possible adjustment for risk. The depreciation rate embodies all lifetime considerations for the ship. The interpretation could be deterministic or stochastic. The deterministic interpretation assumes an infinite lifetime but net earnings from operation decreases at rate $\lambda$. By a stochastic (Poisson) interpretation there could be a constant probability $\lambda$ per unit of time that the ship sinks or exits for another reason, thereby cutting off the cash flow; see Dixit and Pindyck (1994).

Following the methodology of Sødal (2002), the expected net present gain from switching once to the dry bulk market and remaining there, becomes

(2) $$P(t) = E\left[\int_t^{\infty} p(s)e^{-\rho(s-t)}ds\right]$$

This is an important variable in the further description of the model, showing the gain from a switch as well as the opportunity cost of a return to the oil market.

A fixed cost, $F^+$, applies when switching from the oil market to the dry bulk market. A similar fixed cost, $F^-$, applies when returning. There may also be an additional fixed cost flow, *c*, arising from possible more expensive operations in the dry bulk market. (This would be negative if dry bulk operations are more costly.)

The autonomous character of this model implies that the optimal policy consists of switching to the dry bulk market whenever the freight rate differential $p(t)$ reaches some fixed value, $p_H$, and switching back whenever some lower value, $p_L$, is reached. Unless fixed operating costs differ highly, $p_L$ will be negative and $p_H$ will be positive.

The development over time for a ship starting out in the oil market is illustrated in Fig. 2, where switching occurs twice: from wet to dry bulk at time $t_1$ and back again at time $t_2$.

<Figure 2 inserted here>

Assume for a moment that the current freight rate differential equals $p_L$. The ship owner, knowing all the above but not yet the exact optimal values for $p_L$ and $p_H$, decides to switch according to the given policy. The expected net present value of flexibility arising from switching, denoted $W_L$ (as the starting point is $p_L$), becomes

(3) $$W_L = Q(p_L, p_H)(P_H - F^+ - c/\rho + Q(p_H, p_L)(c/\rho - F^- - P_L + W_L))$$

$Q(p_L,p_H) \equiv E[e^{-\rho T}]$ is the *expected discount factor* when moving from $p_L$ to $p_H$, and $T$ is the (stochastic) first-hitting time. $Q(p_H,p_L)$ is the similar expected discount factor in the opposite case, while $P_L$ and $P_H$ are expected and discounted values for the differential as given by the integral in (2). The discount factor function takes on values between zero and one, and it depends on the process characteristics; see Dixit et al. (1999) for a discussion.

Eq. (3) can be explained as follows: No additional earnings are obtained as long as the ship remains in the oil market. This continues until $p_H$ is hit. The starting point is $p_L$, so the discount factor $Q(p_L,p_H)$ reduces the value of future revenues and costs by the appropriate amount. The net present gain from remaining in the dry bulk market equals $P_H$. The switching cost, $F^+$, and the additional fixed cost of operating in this market, $c/\rho$, are subtracted. There is also an option to return to the oil market. The ship returns as soon as the freight rate differential equals $p_L$. The discount factor, $Q(p_H,p_L)$, reduces the value of future gains. One gain consists of (possibly) lower fixed costs ($c/\rho$). The switching cost $F^-$ must be paid, and the net present revenue $P_L$ is subtracted as the ship returns to its default cash flow. (Remember that $P_L$ is typically negative as the ship should not return to the oil market unless there is a gain from doing so.) Right afterwards, the status of the ship is exactly as at the initial point, so the value of further options, $W_L$, is simply added.

Suppose now that the initial freight rate differential no longer happens to be equal to $p_L$, but rather a lower, *fixed* value $p_0$. Then the value of flexibility, $W_0$, becomes

(4) $$W_0 = Q(p_0, p_L)W_L$$

as more discounting applies when $p_H$ is farther away. The following obviously also holds for $p_0<p_L<p_H$:

(5) $$Q(p_0, p_H) = Q(p_0, p_L)Q(p_L, p_H)$$

Re-arranging (3) and using (4) and (5), $W_0$ can be written

(6) $$W_0 = \frac{Q(p_0, p_H)\left(P_H - F^+ - c/\rho + Q(p_H, p_L)(c/\rho - F^- - P_L)\right)}{1 - Q(p_L, p_H)Q(p_H, p_L)}$$

Eq. (6) states the value of flexibility contingent on the specific switching policy ($p_L$,$p_H$). The maximum value of the option is given by the maximum of $W_0$ as a function of $p_L$ and $p_H$. The discount factor function must be identified in order to determine this optimum for specific processes.

For an autonomous Itô process (1), the expected discount factor $Q(p_1,p_2)$ – i.e., when moving from $p_1$ to $p_2$ – is found by solving the equation

(7) $$\tfrac{1}{2}\sigma_p^2 Q''(p_1, p_2) + \mu_p Q'(p_1, p_2) - \rho Q(p_1, p_2) = 0$$

where primes denote derivatives with respect to the first argument; see Dixit et al. (1999). Let the freight rate differential be given by an Ornstein-Uhlenbeck process

(8) $$dp = \mu(m - p)dt + \sigma dz$$

where $m$ is the long-run mean, $\mu$ is a mean-reverting speed parameter, and $\sigma$ is a measure of volatility. Setting $\mu_p=\mu(m-p)$ and $\sigma_p=\sigma$, the Appendix finds the following general solution to (7):

(9) $$Q(p_L, p_H) = K_1 M(p_L) + K_2 U(p_L),$$

where $$M(p_L) = KummerM\left(\frac{\rho}{2\mu}, \frac{1}{2}, \frac{\mu}{\sigma^2}(m - p_L)^2\right);$$

$$U(p_L) = (p_L - m) KummerM\left(\frac{1}{2}\left(1 + \frac{\rho}{\mu}\right), \frac{3}{2}, \frac{\mu}{\sigma^2}(m - p_L)^2\right)$$

and $K_1$ and $K_2$ are arbitrary constants. *KummerM*(·) is the confluent hypergeometric function, which has the following series representation (Slater 1960):

(10) $$KummerM(\theta, b, z) = 1 + \frac{\theta}{b} z + \frac{\theta(\theta+1)z^2}{b(b+1)2!} + \frac{\theta(\theta+1)(\theta+2)z^3}{b(b+1)(b+2)3!} + \ldots$$

The two constants are determined by two boundary conditions: $Q(p_1,p_2)=1$ if $p_1=p_2$, and $Q(p_1,p_2)\to 0$ as $|p_1-p_2|\to\infty$. As shown in the Appendix, the expression for the discount factor depends on whether the motion is upward or downward. Assuming $p_H \geq p_L$, the two parts can be written as follows:

(11a) $$Q(p_L, p_H) = \frac{M(p_L) - R^- U(p_L)}{M(p_H) - R^- U(p_H)}, \qquad R^- = \lim_{p\to-\infty} \frac{M(p)}{U(p)}$$

(11b) $$Q(p_H, p_L) = \frac{M(p_H) - R^+ U(p_H)}{M(p_L) - R^+ U(p_L)}, \qquad R^+ = \lim_{p\to+\infty} \frac{M(p)}{U(p)}$$

The discount factor function is too complicated for deep analytical investigations, but it is clearly symmetric around the mean – i.e., $Q(m-x,m+x)=Q(m+x,m-x)$ for any real $x$. This is intuitive as the Ornstein-Uhlenbeck process is symmetric around $m$. Some other properties are discussed in the Appendix.

The expected price differential at time $s$ ($>t$), starting from $p_t$ at time $t$, equals

(12) $$E(p_s) = m + (p_t - m)e^{-\mu(s-t)};$$

see Dixit and Pindyck (1994, p. 74). Inserted into (2) this implies

(13) $$P_i = E\left[\int_t^\infty p_s e^{-\rho(s-t)} ds\right]_{p_t = p_i} = \frac{m}{\rho} + \frac{p_i - m}{\rho + \mu}, \quad i = L, H$$

The value of flexibility for a switching policy $(p_L,p_H)$ is found by inserting (11) and (13) into (6). The *optimal* policy follows from maximizing $W_0$ with respect to $p_L$ and $p_H$. The final result, $W_0^{opt}=W_0(p_L^{opt}, p_H^{opt})$, will depend on the current freight rate differential, $p_0$. If $W_0^{opt}$ exceeds the difference in vessel prices (including all irreversible costs, and discounting over the life-cycle), the combination carrier is expectedly more profitable than the oil tanker.

A comparison between combination carriers and dry bulk carriers can be carried out similarly, imagining a new combination carrier starting out in the dry bulk market. The symmetry of the model ensures that no new computations will be needed.

## 3. Empirical analysis

### Fleet and freight rate data

As mentioned in the introduction, the combination carrier fleet has shrunk so much that it has little influence on the relative level of freight rates in the dry bulk and wet bulk markets. This is illustrated in the graph below, plotting the average spot earnings (or timecharter equivalent spot freight rates) of a 1990/91-built Capesize bulk vessel and a Suezmax tanker. These vessels have similar cargo carrying capacity in practice (around 140,000 metric tons).

<Figure 3 inserted here>

Figure 4 plots the freight rate differential based on the same data; i.e., the difference between the two curves in Fig. 3.

<Figure 4 inserted here>

A casual look at the graphs suggests that since about 1994, freight rates in the two markets for bulk vessels around 150,000 DWT have not been as closely correlated as in the first part of the sample. This latter part of the sample coincides with an accelerated decline in the combo fleet size, as shown in Fig. 1. The hypothesis that the fleet of combination carriers is no longer sufficient to integrate the two bulk markets is strengthened by the observation that average tanker freight rates have been significantly higher than bulker rates since 1998 even though 80% of the combination carriers, on average, have traded in oil during the same time period; see Fig. 5. Accordingly, even a large shift of combination carriers into the tanker markets has not prevented the divergence of freight rates in the two markets.

<Figure 5 inserted here>

Ironically, the decreasing number and corresponding inability of combination carriers to maintain a highly integrated bulk market increases the value of their flexibility to switch between markets. The lower the correlation of freight rates in the tanker and dry bulk freight markets, the higher is the value of the switching option embedded in the design of the combination carrier.

Constructing a vessel that can carry both crude oil and dry bulk cargoes such as coal and ore is a more complex undertaking than building a standard bulk vessel. This results in a higher initial investment, as illustrated in Fig. 6, which compares the newbuilding price of a combination carrier, a Suezmax tanker, and a Capesize bulk carrier during the period January 1993 to December 2003.

<Figure 6 inserted here>

The cost differential has been shrinking over the last decade, and in December 2003 the quoted price for a combination carrier was $7 million higher compared to a tanker of the same size, and $10 million higher compared to a standard bulker design. This represents premia of 14% and 21%, respectively. It should be noted that since no combination carriers have been ordered over the past few years, the quoted prices are the best estimate of shipbrokers. They may not reflect the actual cost of ordering.

In addition to the higher initial investment, there are costs related to switching between the wet and dry bulk freight markets. These primarily relate to cleaning of the holds and cargo restrictions in an initial time period after the switch. The switching costs are varied parameterically in the empirical analysis.

Assuming the price differential can be described by the stochastic process in (8), the discrete time equivalent is an AR(1) process given as

(14) $$p_t = A + Cp_{t-1} + \varepsilon_t$$

where $C$>1 implies mean reversion and $\varepsilon$ is a normally distributed error term. Estimated parameters from this regression are given in the table below.

**Table 1: Estimated parameters from the autoregression in (14)**

| *Parameter* | *Coefficients* | *Standard Error* | *t Stat* | *p-value* |
|---|---|---|---|---|
| *A* | 243.1 | 127.40 | 1.9 | 0.056 |
| *C* | 0.955 | 0.011 | 87.3 | 0 |

The standard deviation of the residuals is 3070.58. The constant term in the regression is positive, but barely significantly different from zero. This means that the data reveals huge uncertainty with respect to differences in freight rates between the two markets in a normal situation. It also suggests varying this parameter over a wide parameter space in numerical experiments aiming to quantify the switching flexibility.

The relationships between the parameters in the discrete time model in (14) and the continuous time version in (8) are given by

(15a) $$\mu = \frac{\ln A}{\Delta}$$

(15b) $$m = C\frac{\mu}{1-e^{-\mu\Delta}}$$

(15c) $\sigma = \sqrt{S^2 \dfrac{2\mu}{1-e^{-2\mu\Delta}}}$

where $S$ is the standard deviation of $\varepsilon$, and $\Delta$ is the time between observations. Setting $\Delta$=1/52, using the parameter estimates in Tab. 1, and assuming 330 sailing days per year, the annualized parameters in equation (8) are given by estimated parameters $\hat{\mu} = 2.38782335$, $\hat{m} = 4{,}268{,}085.62$ and $\hat{\sigma} = 7{,}475{,}335.98$.

## Ship valuation

The analysis will show that the model is sensitive to some variables, including those estimated above. In order to set the results into perspective, it is therefore convenient to start the discussion with a simple argument that does not apply real options tools.

Suppose that the combination carrier could move freely between the two markets at any time. Based on historical data, what would be the average net gain of doing so instead of operating an exclusive oil tanker? (This means to compute the sum of the areas bounded by the zero-line and the curve segments above this line in Fig. 5.) The average annual gain when measured over the entire period 1990-2004 turns out to be \$515.000, or \$5.15 mill. in net present value if the discount rate is 10 percent. The average price quote for the combination carrier was \$55.5 mill. in 2003; the price of the Suezmax tanker was \$47.5 mill. The price differential \$8.0 mill. is higher than the estimated net present value difference just estimated. However, the annual gain from free switching increases to \$554.000 if only considering the last 5 years (1999-2003), and to \$1.283 mill. if only considering the last 2 years (2002-2003).

These calculations indicate that a closer look at combination carriers is worthwhile. The numerical analysis below will support this conclusion, in addition to shedding light on what parameters are the most critical. Table 2 contains the base case data set.

**Table 2: Base case data**

| *Parameter* | *Base case value* | *Range of variation* |
|---|---|---|
| *m (\$ per year)* | $4{,}268{,}085.62(=\frac{1}{2}\hat{m})$ | $0-\hat{m}$ |
| *$\sigma$ (\$ per year)* | $7{,}475{,}335.98(=\hat{\sigma})$ | – |
| *$\mu$ (\$ per year)* | $2.38782335(=\hat{\mu})$ | – |
| *$\rho$ (annual rate)* | 0.10 | 0.05 – 0.15 |
| *$F^+$ & $F^-$ (\$)* | 40,000 | 0 – 200,000 |
| *c* | 0 | – |
| $p_0$ | 0 | $-6\hat{m}-0$ |

The estimate $\hat{m}$ was unreliable and we argued that this be varied within a broad spectrum. The simple mean freight rate differential in the raw data set is \$5,259 per day, which on a yearly basis corresponds to somewhat less than $\frac{1}{2}\hat{m}$. Therefore the base case mean value is set at $m = \frac{1}{2}\hat{m}$ (\$6,467 per day with 330 sailing days per year), but the spectrum from 0 to $\hat{m}$ is studied. This assumption can also be supported by equilibrium arguments. For competitive markets it is reasonable to expect that average prices get fairly close to average costs in the long run. The average newbuilding price differential between the Suezmax oil tanker and the Capesize bulk carrier was approximately \$11.2 mill. (51.4 versus 40.2) over the period 1990-2003, moving down from \$13.0 to \$9.4 mill. between the first and the second half. On average, this corresponds to just three years of extra earnings in tanker trades when $m = \frac{1}{2}\hat{m}$. Value of flexibility is decreasing in *m*, so the base case assumption is probably conservative with respect to expected profitability of combination carriers.

The volatility and drift parameters estimated above, $\hat{\sigma}$ and $\hat{\mu}$ are kept unaltered. The base case discount rate is set at 10 percent, but discount rates ranging from 5 percent to 15 percent have been studied. The base case can be seen as the sum of 5 percent expected annual return and 5 percent expected depreciation. The latter corresponds to roughly 20 year expected life-time for the ship.

The default value of both switching costs is \$40,000. This corresponds to 3-4 days with cleaning etc. More extensive switching operations might be needed in case of long transit, so switching costs up to \$200,000 in both directions have been studied. Differences in operating costs between the markets are included in the spot earnings data, so we set *c*=0 throughout the entire analysis. Effects of short-term market conditions are considered by varying the initial freight rate differential between $-6\hat{m}$ and zero.

Table 3 sums up the value of flexibility for several switching cost assumptions. The impact of switching costs is significant in the base case scenario ($F^{+}=F^{-}=\$40{,}000$), but more so for the optimal policy than for the overall value of flexibility. The value of flexibility is close to \$4.6 mill., or 23 percent less than what would be obtained with free switching ($F^{+}=F^{-}=0$). Both switching costs must be increased to \$200,000 to cut in half the value of flexibility when comparing with the free switching option.

**Table 3: Option value and switching policy for various switching costs**

| $F^{+}$ & $F^{-}$ (\$) | *Value of flexibility (\$)* | $p_L$ (\$ per day) | $p_H$ (\$ per day) |
|---|---|---|---|
| 0 | 5,658,945 | 0 | 0 |
| 20,000 | 4,970,878 | −3,366 | 3,906 |
| 40,000 | 4,585,911 | −4,169 | 5,047 |
| 60,000 | 4,274,272 | −4,716 | 5,889 |
| 80,000 | 4,004,949 | −5,140 | 6,587 |
| 100,000 | 3,764,894 | −5,490 | 7,198 |
| 200,000 | 2,827,114 | −6,694 | 9,608 |

Even low switching costs have significant impact on the optimal policy, imposing a drag in both directions. In the base case scenario, a combination carrier currently transporting oil should shift to dry bulk when the freight rate differential exceeds $5,047 per day. If already being in the dry bulk market, the ship should not leave that market until the rates there are $4,169 lower. The reason for the asymmetry lies in the (long-run) mean differential, favoring the oil market. This makes the ship owner more inclined to switching to tanker trades than the other way around. The asymmetry gets increasingly more significant as the switching costs increase.

Table 4 shows how the mean freight rate differential affects the value of flexibility. The empirical analysis did not provide credible estimates here. By splitting up the 14-year data period in shorter sub-periods, it is evident that there is a downward trend in this variable. (This could also be shown by expanding to a two-factor model in which the long-term differential was also made stochastic.)

**Table 4: Option value and switching policy for various freight rate differentials**

| *Mean freight rate differential ($ per day)* | *Value of flexibility ($)* | $p_L$ *($ per day)* | $p_H$ *($ per day)* |
|---:|---:|---:|---:|
| 0 | 12,162,437 | –4,600 | 4,600 |
| 3,233 ($=\frac{1}{4}\hat{m}$) | 7,727,759 | –4,382 | 4,822 |
| 6,467 ($=\frac{1}{2}\hat{m}$) | 4,585,911 | –4,169 | 5,047 |
| 9,700 ($=\frac{3}{4}\hat{m}$) | 2,534,522 | –3,962 | 5,276 |
| 12,934 ($=\hat{m}$) | 1,304,789 | –3,761 | 5,507 |

The mean freight differential has minor impact on the switching policy. The value of flexibility increases almost ten-fold by reducing the mean from $12,934 to zero. The switching points, or the freight rate "band of in-action", move to the left with only $8-900 per day, from (–3,761;5,507) to (–4,600; 4,600).

The average quoted price differential was $8 mill. when comparing a combination carrier and a Suezmax tanker in 2003. The value of flexibility in Tab. 4 is close to this level for $m=\frac{1}{4}\hat{m}$, which from the equilibrium arguments mentioned above seems more reasonable as a future estimate than the base case assumption $(m=\frac{1}{2}\hat{m})$.

Table 5 shows the influence of the discount rate. The results here can be summed up easily because frequent switching eliminates most second-order effects. First, the optimal policy is hardly influenced at all by discounting. Second, the value of flexibility decreases in almost exact proportion to the discount rate. For example, increasing it from 10 to 15 percent decreases the option value with one third, from $4.586 mill. to $3.076 mill.

**Table 5: Option value and switching policy for various discount rates**

| *Discount rate (%)* | *Value of flexibility ($)* | $p_L$ *($ per day)* | $p_H$ *($ per day)* |
|---|---|---|---|
| 15.0 | 3,076,245 | –4,172 | 5,050 |
| 12.5 | 3,680,398 | –4,171 | 5,049 |
| 10.0 | 4,585,911 | –4,169 | 5,047 |
| 7.5 | 6,094,106 | –4,168 | 5,046 |
| 5.0 | 9,108,954 | –4,167 | 5,045 |

Table 6 demonstrates that the initial freight rate differential has little impact on the value of flexibility. The optimal policy is obviously not affected, so all thresholds in the two rightmost columns are identical. A spectrum of initial values, ranging from 0 to $77,602 dollars per day in favor of oil freights have been studied to show the effect of mean reversion.

**Table 6: Option value and switching policy for various initial freight rates**

| $p_0$ *($ per day)* | $Q(p_0,0)$ | *Value of flexibility ($)* | $p_L$ *($ per day)* | $p_H$ *($ per day)* |
|---|---|---|---|---|
| $-77{,}602\ (=-6\hat{m})$ | 0.86 | 3,952,670 | –4,169 | 5,047 |
| $-51{,}734\ (=-4\hat{m})$ | 0.88 | 4,025,723 | –4,169 | 5,047 |
| $-25{,}867\ (=-2\hat{m})$ | 0.91 | 4,156,109 | –4,169 | 5,047 |
| 0 | 1.00 | 4,585,911 | –4,169 | 5,047 |

For the first row of results, the discount factor is 0.86, implying 14 percent value reduction. The discount rate is 10 percent, so this corresponds to 16-18 months of waiting. The discount factor difference between the two first rows is only 2 percent (0.86 versus 0.88), which translates into a couple of months. Thus it would usually not take long for the freight rate differential to decrease from $77,000 to $52,000.[1]

The value of flexibility decreases in direct proportion to the initial discount factor. For example, the result in the third row ($4.156 mill.) is 9 percent lower than that of the last row ($4.586 mill.). This corresponds to 10-11 months of delay. The difference could be decisive for whether one should buy a combination carrier or an oil tanker.

Similar comparisons as all of those above can be made between combination carriers and dry bulk carriers. Since the Ornstein-Uhlenbeck process is symmetric, this requires no further complex analysis. Nor does it add much to the results above, as long as the two shipping markets are linked by a fairly competitive newbuilding

[1] Measuring expected times to move by the discount factor in this way is a shortcut that neglects second-order effects. More complex methods are needed for exact estimates; see Dixit (1993).

market. Just as low freight rates in the oil market relative to dry bulk encourages newbuilding of oil tankers relative to combinations carrier, it tends to make make an oil tanker a better choice than a bulk carrier (and likewise the other way around).

## 4. Conclusions

The main conclusion can be summed up as follows: *Combination carriers, with the capability of transporting dry bulk as well as wet bulk commodities, are about to become profitable once again.* Hardly any such ships have been built for several years, but current market trends are working in their favor. This mainly includes decreasing correlation of freight rates and decreasing vessel price differentials. Unless a change in these trends occurs, and unless the real prices of new combination carriers turn out to be significantly higher than the quoted prices, such ships are likely to enter the market within a few years.

The possibility of triangulation, which has not been studied here, may represent another option for combination carriers in the future even if it is not a very valuable one with the current trade patterns for bulk commodities.

The empirical analysis indicated that profitability of combination carriers is greatly influenced by several factors among which some are more transparent than others. The expected long run mean differential ($m$) seems equally as decisive as volatility and correlation of freight rates. Within reasonable limits (see Tab. 3), the switching costs could be somewhat less important.

The observed increase in freight rate volatility over time implies that the empirical analysis may well have underestimated overall values of flexibility. In order to check the robustness of the results, similar numerical experiments were undertaken based on shorter sub-periods. Focusing on the 5-year period 1999-2003, which is obviously the most interesting one, two effects work in opposite directions. Volatility is higher but the mean freight rate differential is also higher. This motivates a follow-up study more than a revision of the conclusions just made. Such a follow-up study ought to be based on a two-factor model where the mean freight rate differential is made endogenous in a long-term stochastic equilibrium setting.

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## Appendix. The Ornstein-Uhlenbeck discount factor

This appendix derives the general solution to the differential equation (7) for an Ornstein-Uhlenbeck process (8), using the result to determine the discount factor (11a,b). To simplify notation, set $p_1=x$ and $Q(p_1,p_2)=y(x)$, and define the variable

(A1) $$z = a(x-m)^{\varphi}$$

where $a$ and $\varphi$ are arbitrary constants. This implies

(A2) $$\frac{dz}{dx} = a\varphi(x-m)^{\varphi-1} = \varphi a^{-1/\varphi} z^{(\varphi-1)/\varphi}$$

and

(A3) $$\frac{d^2z}{dx^2} = a\varphi(\varphi-1)(x-m)^{\varphi-2} = \varphi(\varphi-1)a^{2/\varphi}z^{1-2/\varphi}$$

Then

(A4) $$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \varphi a^{-1/\varphi} z^{(\varphi-1)/\varphi} y'(z)$$

and

(A5) $$\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2}\left(\frac{dz}{dx}\right)^2 + \frac{dy}{dz}\frac{d^2z}{dx^2} = \varphi^2 a^{2/\varphi} z^{(2\varphi-2)/\varphi} y''(z) + \varphi(\varphi-1)a^{2/\varphi}z^{1-2/\varphi}y'(z)$$

Replace $Q'(p_1,p_2)$ with (A4) and $Q'(p_1,p_2)$ with (A5) in eq. (7). By collecting terms, (A5) simplifies to the following when $\varphi=2$:

(A6) $$2a\sigma^2 zy''(z) + (a\sigma^2 - 2\mu z)y'(z) - \rho y(z) = 0$$

Then set $a=\mu/\sigma^2$, divide with $2\mu$, and (A6) simplifies to a standard form,

(A7) $$zy''(z) + (b-z)y'(z) - \theta y(z) = 0\,,$$

where $b=½$ and $\theta=\rho/2\mu$. This is the Kummer equation, whose simplest solution is the Kummer function or the hypergeometric function of the first kind, $KummerM(\theta,b,z)$, as characterized by (10); see Slater (1960). This fixes the first part, $M(p_L)$, of the general solution (9).

The argument $z=(x-m)^2$ in the Kummer function suggests a linearly independent solution of the form $(x-m)KummerM(\gamma,c,z)$, where $\gamma$ and $c$ are constants. By differentiating this function and picking appropriate values for $\gamma$ and $c$, using the same procedure as above, the second part of the solution is obtained – i.e., $U(p_L)$ in (9).

The discount factor for upward motions, Eq. (11a), is determined by imposing appropriate boundary conditions. This requires a two-step procedure. First, a *conditional* discount factor function is derived, describing the discount factor when moving from $p_L$ to $p_H$ ($>p_L$) without hitting a certain value $p<p_L$ first. Then let $p\to-\infty$, which implies $\mu(m-p)\to\infty$. Since the process is continuous, any finite value $p_H$ will be hit before $p$ with probability one, so $Q(p_L,p_H)$ is reached in the limit.

The appropriate boundary conditions are $Q(p_H,p_H)=1$ and $Q(p,p_H)=0$, reflecting that no discounting takes place if $p_L$ has already hit its destination $p_H$, whereas the destination is never hit before $p$ if starting out at (or sufficiently close to) $p$. Hence,

(A8) $$K_1M(p_H)+K_2U(p_H)=1\,; \qquad K_1M(p)+K_2U(p)=0$$

Solving (A8) for $K_1$ and $K_2$, inserting into (9) and letting $p\to-\infty$ gives the desired result (11a). The solution with downward motion, Eq. (11b), is derived similarly.

As mentioned in the text, the discount factor is symmetric around $m$. Numerical analysis confirms that the mean-reverting properties of the process are reflected in the discount factor. For example, $Q(m-x_1,m+y_1)> Q(m-x_2,m+y_2)$ when $x_1$, $x_2$, $y_1$ and $y_2$ are positive numbers satisfying $x_1+y_1=x_2+y_2$, $x_1>x_2$ and $y_1>y_2$. Thus mean reversion reduces the amount of discounting for a given distance the farther away from the mean the point of departure is located.

Figure 7 plots the discount factor $Q(p_L,p_H)$ for the base case data in the text. The starting point is $p_L=-6\hat{m}$ as in the first row of Tab. 6. The destination, $p_H$, varies from $p_L$ and upwards. The discount factor $Q(-6\hat{m},0)=0.86$ from the first row of Tab. 6 is indicated with the horisontal and vertical lines. The figure illustrates clearly the effect of mean reversion. The curve is flat and the discount factor close to one for low values of $p_H$ because of the drift towards the mean. The curve gets steeper for increasingly higher $p_H$ as mean reversion makes it less likely that $p_H$ will be hit soon. Eventually, it approaches the horizontal axis as the probability of ever reaching the destination point vanishes for high enough $p_H$.

Figure 1. Combo fleet trends. Source: Clarksons Research (2004).

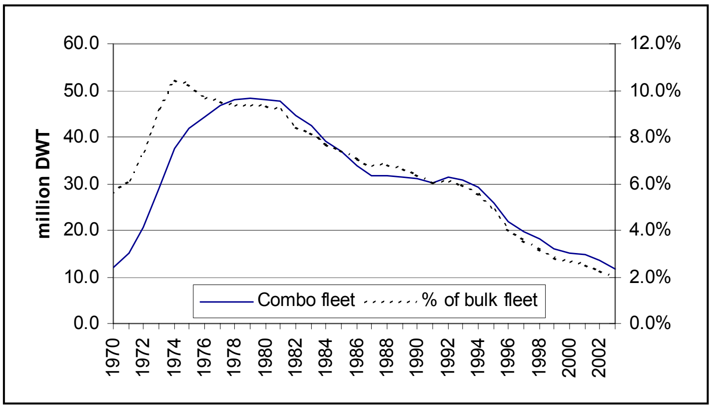


Figure 2. Market switching.

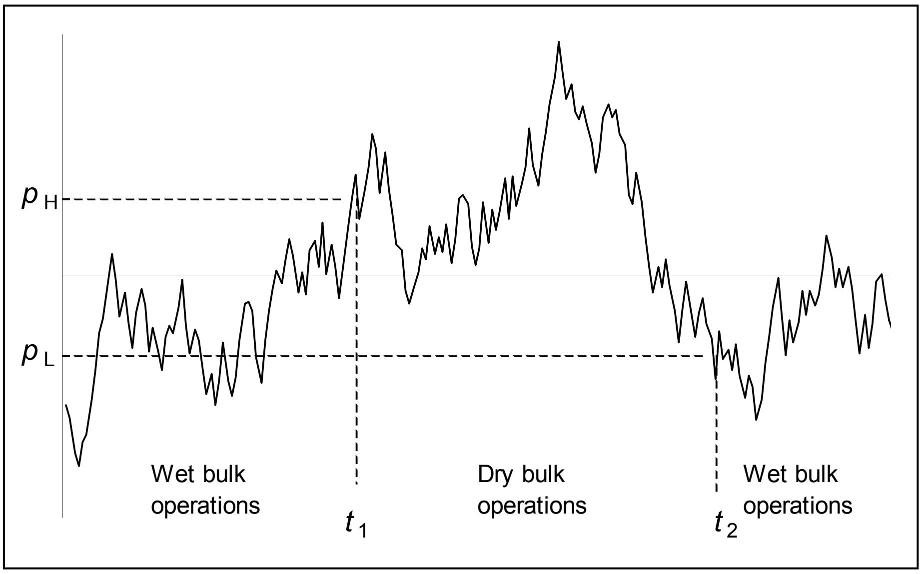


Figure 3. Daily spot earnings *(135,000-150,000 DWT)*. Source: Clarksons Research (2004).

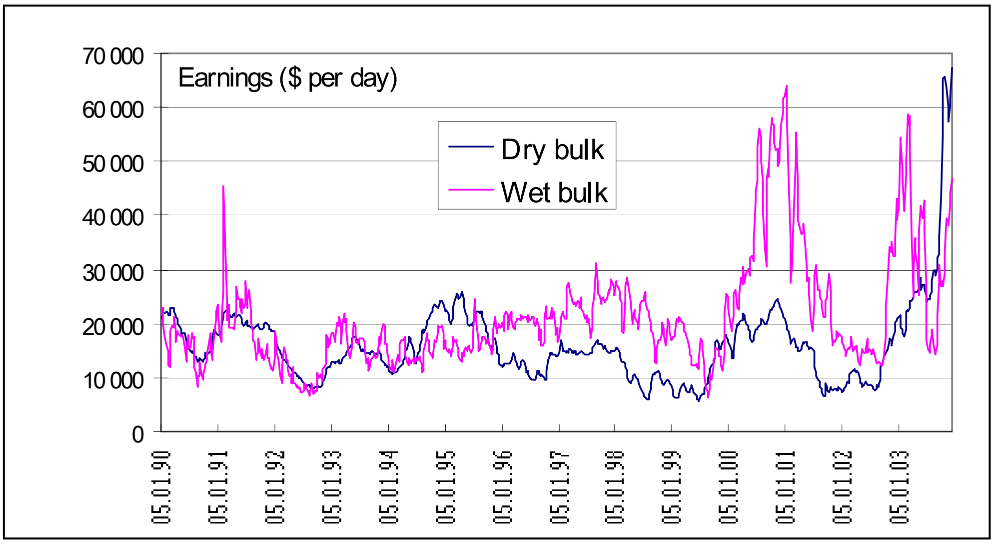

Figure 4. Daily spot earnings differential. Source: Clarksons Research (2004).

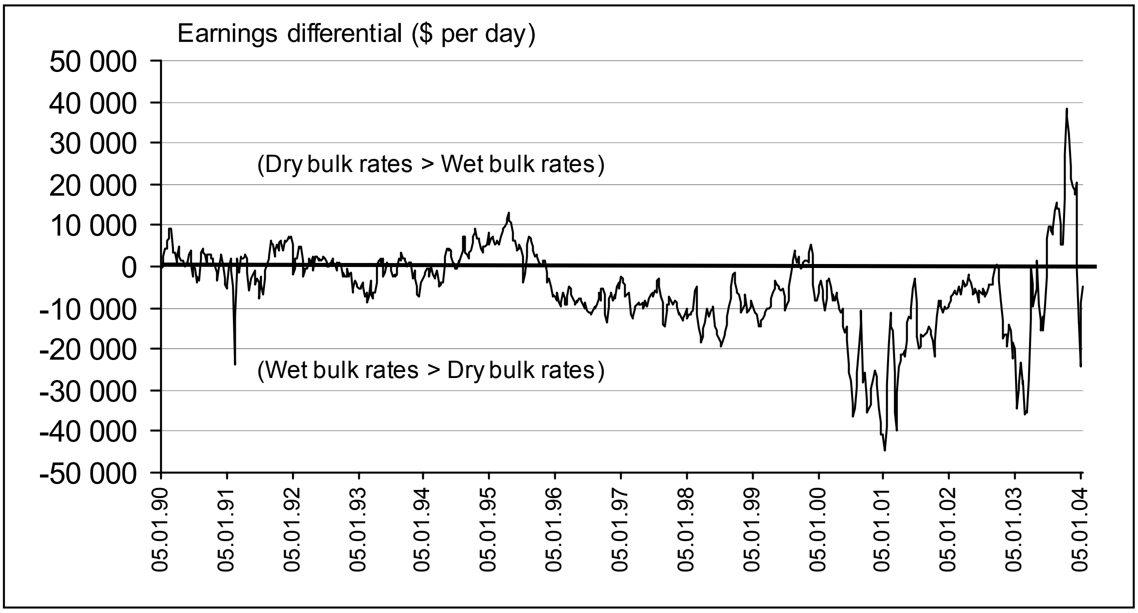


Figure 5. Share of combination carriers trading in oil. Source: Clarksons Research (2004).

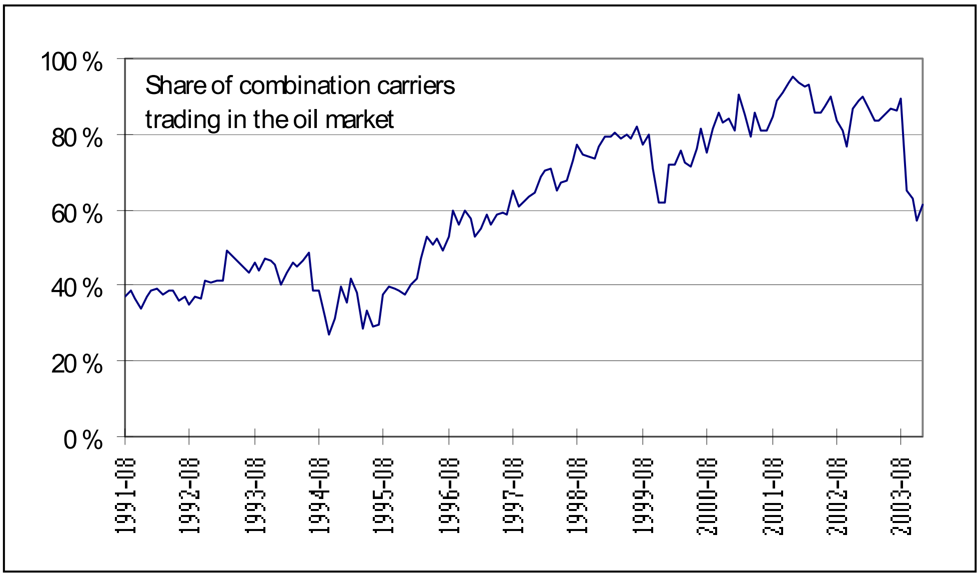


Figure 6. Comparison of newbuilding prices. Source: Clarksons Research (2004).

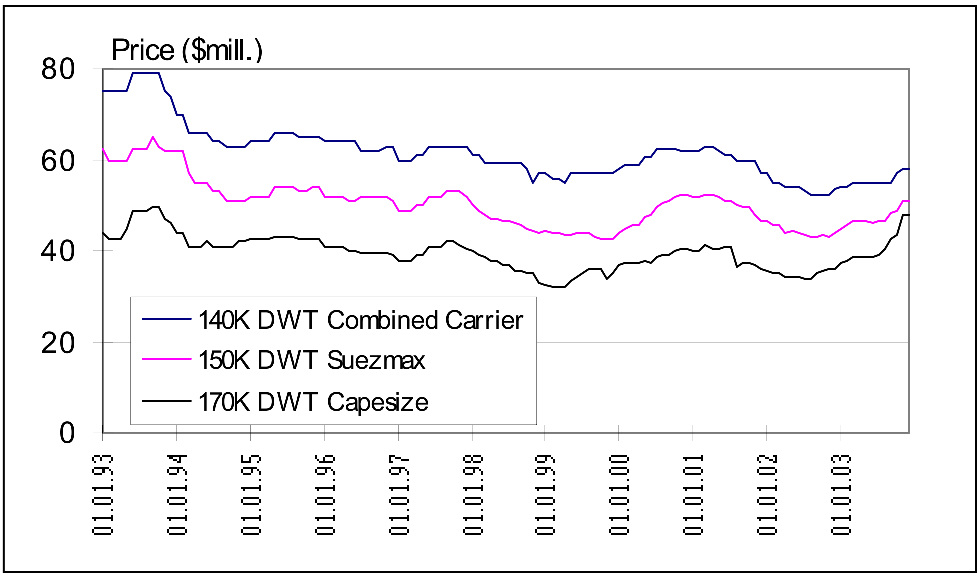

Figure 7. The discount factor function.