Investing in Urban Transportation Infrastructure Under Uncertainty

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Abstract

We analyze the impact of uncertainty and irreversibility on the timing to invest in urban transportation infrastructure using real options. We consider a monocentric city whose population varies stochastically, which impacts land rents, land prices, and transportation costs through congestion; a transportation agency can decide when to buy land and build transportation infrastructure to reduce congestion. Our numerical results, using realistic parameters values, show that some of the basic insights of simple investment models do not hold because of barriers on the city population. Our results also underline the importance of accounting for uncertainty when investing to relieve urban externalities.

Keywords: transportation infrastructure; congestion; uncertainty; irreversibility; real options.

JEL codes: D61, D81, R42.

I. INTRODUCTION

When to invest in transportation infrastructure is an essential question for transportation agencies, elected officials, and private transportation entrepreneurs alike. Leaving demand unmet for too long can entail large social costs as congestion and pollution build-up. On the other hand, building too early risks incurring large social and/or private costs as underused capacity creates revenue shortfalls. Recent examples of private highway projects that faced smaller than anticipated demand in their early years include the Dulles Toll Road extension in suburban Virginia and the San Joaquin Hills Transportation Corridor in Orange County, California. Similar timing problems can also plague public projects, although underused public facilities are not as commonly publicized in the press; more generally, timing is a problem for any type of congestible infrastructure.

Surprisingly, however, very few papers deal with the timing of infrastructure investment. One exception is Szymanski (1991), who examines this problem for both a welfare maximizing public agency and a profit maximizing private firm in a deterministic framework, where he assumes a fixed rate of growth for infrastructure demand. To the best of our knowledge, no paper in the literature analyzes the optimal timing of investing in urban transportation infrastructure under uncertainty in spite of the importance of this topic for public policy. This paper aims at filling that gap. We show that the interplay between population volatility and upper (limit on population density) and lower (the population is not allowed to crash) barriers on city population is richer than for simpler investment problems. A complicating factor here is that both the benefits and the costs (through the cost of land) of exercising the option to build infrastructure vary with the city population and with its volatility. For realistic parameter values, we find that the investment threshold, x^*, first increases and then decreases with uncertainty. If x^* is close to its upper barrier, increasing uncertainty reduces the value of the project because the upper barrier prevents higher population levels while allowing more frequent and larger population drops; conversely, if x^* is close to its lower barrier, it becomes optimum to invest immediately because the lower barrier increases the likelihood of higher population levels. While we focus on population uncertainty, our approach can be generalized to other types of uncertainty.

Finding the right timing for investing in transportation infrastructure is inherently difficult, however, for two key reasons. First, the net social benefits and, in the case of private projects, the net revenues from transportation infrastructure are stochastic because of regional and local economic booms and busts, the competition from other transportation projects, or technological changes, but the most commonly acknowledged uncertainties are in project costs and user demand. These difficulties are compounded by the lack of a reliable methodology to quantify user costs as well as external costs and benefits from large transportation projects.

Here we focus on uncertainty in user demand linked to population growth. Our reasoning is twofold – (1) errors in population forecasts are typically a crucial source of uncertainty in infrastructure project analysis, and (2) other types of uncertainty can often be proxied by uncertainty in population forecasts. Indeed, population projections of the time-scale required for major infrastructure projects are subject to considerable uncertainty. In a recent summary, Johnson (1999) remarks that reputable population projections for the State of California in the year 2025 vary from a low of 41.5 million (Census Bureau alternative estimate) to a high of 52 million persons (Center for the Continuing Study of the California Economy high estimate). Johnson notes that this wide range results largely from different assumptions about net U.S. to California migration in the future – a trend that is difficult to forecast accurately given fluctuations in net domestic migration between California and other states since the mid-1980s.[1]

Second, the decision to invest in infrastructure is largely irreversible: if it turns out that a new transportation project was not needed, only a small fraction of the initial investment can typically be recovered and environmental impacts are likely to be felt for years. The importance of irreversibility is amplified by the large amounts of capital usually required by infrastructure projects. Uncertainty and irreversibility also characterize most private investment decisions, but infrastructure investments, from highways to rail transit to airports, are often complicated by the availability of public funds and by political considerations.

The transportation literature has long been concerned with benefit-cost assessments of transportation projects (e.g., see Mohring 1961 or Mohring and Harwitz 1962 on transportation investment benefits), and especially with the impacts of public infrastructure on private sector productivity (e.g., see Aschauer 1989, Gramlich 1994, or Boarnet 1997a-b), but it does not seem to have incorporated insights from the theory of investment under uncertainty (see Dixit and Pindyck 1994 and the references therein). Indeed, it is now well known that when an investment is uncertain and irreversible, a standard cost-benefit analysis (which is static and deterministic) can be seriously biased as it ignores the value of waiting and of acquiring new information.

As a result, real options have become quite popular to analyze private investment decisions. This approach has been fruitfully applied to the understanding of land conversion (e.g., see Capozza and Helsley 1990, Capozza and Sick 1994, or Clarke and Reed 1998), but it has not received much attention for public investments. Although we analyze urban transportation infrastructure, our framework is applicable to most large infrastructure investments undertaken by the public sector.

In this paper, we propose a continuous time framework based on real options (Dixit and Pindyck 1994) to analyze the timing of the decision to build urban transportation infrastructure to relieve congestion in a monocentric city. People commute daily to a business district located at its center. Transportation costs and land rents increase stochastically with the city population. A transportation agency can decide if and when to buy land and build transportation infrastructure that relieves congestion. We formulate the corresponding optimal stopping problem and propose a simple numerical solution. An illustration shows that uncertainty plays an important role in the timing of building transportation infrastructure to relieve congestion.

This paper is organized as follows. In the next section, we develop a model for analyzing the optimal timing of urban transportation infrastructure investments. In Section III, we introduce some simplifying assumptions based on economic considerations. Section IV presents a numerical illustration of the impact of uncertainty, irreversibility, and time of completing the project on the decision to build transportation infrastructure. Section V summarizes our conclusions and presents suggestions for future research. A summary of our notation can be found in Table 1.

II. THE MODEL

A. City Geography, Population, and Congestion

We consider a monocentric, circular city located on a homogeneous plain. We suppose that the city radius, denoted by Z, is fixed to abstract from the problem of land conversion (e.g., see Capozza and Helsley 1990, Capozza and Sick 1994, or Clarke and Reed 1998). Employment and production are located at the central business district (CBD), at the center of the city. City residents are identical and commute daily to the CBD. We denote by $z \in [0,Z]$ the distance between a point in the city and the CBD.

The population of this city, denoted by X, is growing stochastically according to the autonomous diffusion process:

$$dX = f(X)dt + s(X)dw. \tag{1}$$

In (1), $f(.)$ and $s(.)$ are continuously differentiable; $s(.)$ is strictly positive on $(0,+\infty)$; dt is an infinitesimal time increment; and dw is an increment of a standard Wiener process (Karlin and Taylor 1981). Throughout this paper, X stands for the random variable describing the evolution of the city population and x is one of its realizations. We also assume that the city population evolves between two reflecting barriers, $\underline{x}$ and $\overline{x} > \underline{x}$, so each time X reaches $\underline{x}$ ($\overline{x}$), it is reflected upwards (downwards). The upper barrier results from density constraints, since the city size is fixed. The lower barrier $\underline{x}$ simplifies the analysis (see below) by precluding very low population levels, which would require considering the decision to abandon infrastructure.

Because of congestion, population growth increases the flow of transportation costs, which equals $\gamma x^{\delta} z$ per unit of time for a person leaving at distance z from the CBD; $\gamma>0$ is the congestion coefficient and $\delta \geq 1$ is the congestion exponent. Initially, $\gamma = \gamma_0$.

B. Demand for Land and Land rents

City residents derive utility from land, L, and from a composite numeraire good. Their preferences can be described by the Cobb-Douglas utility function

$$U(g,L) = a.\ln(g) + [1-a].\ln(L), \tag{2}$$

where $a \in (0,1)$. The budget constraint of a city resident living at a distance z from the CBD when the city population totals x is

$$g + R_0(x,z).L + \gamma_0 x^{\delta} z = m. \tag{3}$$

In the above:

- g is income spent on the composite numeraire good per person and per unit of time.
- $R_0(x,z)$ is the land rent per unit area at a distance z from the CBD;
- L is the corresponding amount of land rented per person;
- $\gamma_0 x^{\delta} z$ represents transportation costs per person per unit of time.
- m is total income per person per unit of time. For simplicity, m is assumed constant.

We then know (e.g., see Varian 1992) that the demand functions (for land and for the numeraire) of a city resident living at a distance z from the CBD are respectively

$$L(R_0(x,z),m) = \frac{m - \gamma_0 x^{\delta} z}{(1+\nu)R(x,z)}, \text{ and} \tag{4}$$

$$g(R_0(x,z),m) = \frac{\nu}{1+\nu}[m - \gamma_0 x^{\delta} z], \tag{5}$$

where ν is the ratio of the numeraire good elasticity of utility to the land elasticity of utility:

$$\nu = \frac{a}{1-a}. \tag{6}$$

At equilibrium, all city residents have the same level of utility. Equating the utility of a resident of the CBD and the utility of a resident located at a distance z from the CBD tells us how land rent changes as we move away from the city center:

$$R_0(x,z) = R_0(x,0)\left[\frac{m-\gamma_0 x^{\delta} z}{m}\right]^{\nu+1}. \tag{7}$$

Now let $N(x,z)$ designate the number of city dwellers in the disk of radius z centered on the CBD when total city population is *x*. In a ring of radius *z* and thickness *dz*, there are $N(x,z+dz)-N(x,z)=N'(x,z)dz$ city residents. To find the total demand for land in this ring, we multiply $N'(x,z)dz$ by the demand for land (Equation (4)). The total supply of land in this ring is $2\pi z dz$ so equating demand and supply for land leads to the equilibrium relationship:

$$N'(x,z) = \frac{2\pi(1+\nu)z}{m-\gamma_0 x^{\delta} z} R_0(x,z). \tag{8}$$

When we introduce (7) into (8) and integrate $N'(x,z)$ over *z* between 0 and the city radius, *Z*, we find that the land rent per unit area per unit of time at the CBD equals

$$R_0(x,0) = \frac{x}{1+\nu}\frac{m^{1+\nu}}{2\pi I_0(x)}, \tag{9}$$

where $2\pi I_0(x)$ is the sum over all city residents of their income, raised to the power *ν*, available for spending on land and on the composite numeraire good:[2]

$$2\pi I_0(x) = 2\pi \int_0^Z \xi[m - \gamma_0 x^\delta \xi]^\nu \, d\xi. \tag{10}$$

Since all residents have the same indirect utility per unit of time $V_0(x)$, it equals the indirect utility per unit of time at the CBD:

$$V_0(x) = \frac{\nu}{1+\nu} \ln\left(\frac{\nu}{1+\nu}\right) + \frac{1}{1+\nu} \ln\left(\frac{2\pi I_0(x)}{x}\right). \tag{11}$$

C. Transportation Infrastructure and Congestion

To relieve congestion, we suppose that a transportation agency can buy a slice of urban land and build a project that lowers γ from γ_0 to γ_2 where $0<\gamma_2<\gamma_0$; during project construction, $\gamma=\gamma_1\geq\gamma_0$. This land slice, measured by angle θ, extends from the CBD to the edge of the city. Let us denote by x^* the population level at which it is optimal to start the project.[3] Assuming that the population living on the land purchased by the agency relocates elsewhere in the city, the land available to city residents decreases from πZ^2 to $(\pi\text{-}0.5\theta)Z^2$, and the supply-demand equilibrium for land is restored at higher land prices and densities.

Project costs include the cost of land, $C_L(x^*)$, construction costs, and all future maintenance and operation costs, C_P, which we lump together. The cost of land depends on x^* because total city population has an impact on land rents, and therefore on land prices. C_P is assumed fixed and known. If $C(x^*)\equiv C_L(x^*) + C_P$ is annualized over an infinite time horizon and shared equally among city residents, the cost of the project per city resident per unit of time equals $\rho \frac{C(x^*)}{x}$, where ρ is the discount rate. Once the project is under way, the budget constraint of a resident located at a distance z from the CBD becomes

$$g + R_i(x,z).L + \gamma_i x^{\delta} z = m - \rho \frac{C(x^*)}{x}, \tag{12}$$

with i=1 when the project is under construction and i=2 when it is complete. To derive the new demand functions from (4) and (5), we just replace m by $m - \rho \frac{C(x^*)}{x}$. Following the same logic as above, the rent per unit area and per unit of time after the project has started is

$$R_i(x,z) = \frac{x}{(2\pi - \theta)(1+\nu)} \frac{\left[m - \rho \frac{C(x^*)}{x} - \gamma_i x^{\delta} z \right]^{1+\nu}}{I_i(x,x^*;0)}, \tag{13}$$

where, for $k\geq$ integer,

$$I_i(x,x^*;k) = \int_0^Z \xi [m - \rho \frac{C(x^*)}{x} - \gamma_i x^{\delta} \xi]^{\nu+k} \, d\xi. \tag{14}$$

In $I_i(x,x^*;k)$, i=1,2, k is an index introduced for convenience. $I_i(x,x^*;k)$ is obtained from $I_0(x)$ by replacing m with $m - \frac{C(x^*)}{x}$, ν with ν+k, and γ_0 with γ_i. From its value at the CBD, the indirect utility per unit of time of a city resident during (i=1) and after (i=2) construction is

$$V_i(x,x^*) = \frac{\nu}{1+\nu} \ln\left(\frac{\nu}{1+\nu} \right) + \frac{1}{1+\nu} \ln\left(\frac{(2\pi - \theta) I_i(x,x^*;0)}{x} \right). \tag{15}$$

Let us now derive an expression for $C_L(x^*)$.

D. The Cost of Project Land

If the land market is competitive, the price of land equals the expected present value of future land rents. Let $W_i(x,x^*)$ denote the flow of land rents for a slice of land of angle θ during (i=1) and after (i=2) project construction. Integrating $R_i(x,z)$ over this area gives the flow of rents

$$W_i(x,x^*) = \frac{\theta}{2\pi-\theta}\frac{x}{1+\nu}\frac{I_i(x,x^*;1)}{I_i(x,x^*;0)}. \tag{16}$$

To find the price of the land slice, we integrate $W_i(x;x^*)$ over time after the start of the project:

$$C_L(x^*) = E_{x^*}\left(\int_0^{\Delta} W_1(X,x^*)e^{-\rho t}dt + \int_{\Delta}^{+\infty} W_2(X,x^*)e^{-\rho t}dt\right). \tag{17}$$

In (17), E_{x^*} is the expectation operator with respect to X conditional on $X(0)=x^*$, and Δ is the time it takes to buy land and build the project. The first term on the right side of (17) represents the discounted flow of rents from the moment the land is purchased until the end of construction. The second term is the discounted flow of rents thereafter. From (14)- (17), it is important to note that Equation (17) only defines $C_L(x^*)$ implicitly.

E. The Transportation Agency's Problem

We suppose that the objective of the transportation agency is to maximize the expected present value of the utility of city residents by deciding when, if ever, to build additional transportation infrastructure to relieve congestion. This is a standard stopping problem: when the city population is relatively small, congestion is low and investing in transportation infrastructure is not attractive; this defines the waiting region. Conversely, when the city population is large, so is congestion and it is optimum to invest immediately to improve transportation; this defines the so-called "stopping region." The agency wants to find x^*, the value of the city population above which it is optimum to invest in additional transportation infrastructure; x^* separates the waiting from the stopping region. This transportation agency's problem can therefore be written

$$\underset{\underline{x} \le x^*}{Max}\, E\left\{ \int_0^{T_{x^*|x_0}} X.V_0(X).e^{-\rho t}dt + \int_{T_{x^*|x_0}}^{T_{x^*|x_0}+\Delta} X.V_1\left(X,x^*\right)e^{-\rho t}dt + \int_{T_{x^*|x_0}+\Delta}^{+\infty} X.V_2\left(X,x^*\right)e^{-\rho t}dt \right\}, \quad (18)$$

where X follows (1) and the cost of land (which appears in $V_i(X,x^*)$) is defined by (17). Moreover:

- E is the expectation operator with respect to X;
- $T_{x^*|x_0}$ is the minimum time it takes X to hit x^* for the first time, starting from x_0; and
- $V_0(X)$ and $V_i(X,x^*)$, i=1,2, are indirect utilities per unit of time (see (11) and (15)).

In (18), the first integral represents the aggregate utility of city residents before the project; the second one is their aggregate utility during construction; and the last one is their aggregate utility after project completion. It is convenient, however, to express (18) as (see the appendix)

$$\underset{\underline{x} \le x^*}{Max}\, D_{x^*|x_0}\left\{ E_{x^*}\left[\int_0^{+\infty} X.[V_1\left(X,x^*\right) - V_0(X)]e^{-\rho t}dt\right] + e^{-\rho\Delta}\left[\int_0^{+\infty} p(\Delta,x^*,\xi)E_{\xi}\left[\int_0^{+\infty} X.[V_2(X,x^*) - V_1(X,x^*)]e^{-\rho t}dt\right]d\xi\right]\right\}, \quad (19)$$

again subject to (1) for X and to (17). In the above:

- $p(\Delta,x,y)$ is the density of X at time Δ given that $X(0)=x$; and

- $D_{x^*|x_0} \equiv E(e^{-\rho T_{x^*|x_0}})$ is the expected discount factor.

The first integral in (19) is the present value of the loss in expected utility from the project, assuming it lasts forever: during construction, congestion is higher and a flow of payments is required for the project. Conversely, terms in factor of $e^{-\rho\Delta}$ are the net expected utility gains from congestion relief. Alternatively, the objective function maximizes the net utility gains from the option (i.e., the possibility but not the obligation) of relieving congestion.

Equation (19) offers two advantages over (18). First, it shows that the solutions of the first order necessary condition associated to (19) are independent of x_0: indeed, for $x_0 \leq x_1 \leq x^*$, $T_{x^*|x_0} = T_{x^*|x_1} + T_{x_1|x_0}$ and therefore $D_{x^*|x_0} = D_{x^*|x_1} D_{x_1|x_0}$; as x_0 intervenes only in $D_{x^*|x_0}$, changing x_0 is just akin to multiplying (19) by a constant. Second, it is easier to calculate the expected value of integrals defined over [0;∞) (Karlin and Taylor, 1981). In (19), however, the cost of land is still implicit, even without uncertainty. Let us now address this problem.

III. AN EXPLICIT APPROXIMATION OF THE OBJECTIVE FUNCTION

Our formulation so far does not account for the fact that $y_d \equiv \frac{\rho C(x^*)}{mx}$ and $y_i \equiv \frac{\gamma_i x^\delta Z}{m}$ are typically much smaller than 1 (see next section). The former represents the ratio of the flow of individual congestion costs divided by the flow of personal income; the latter is the ratio of the flow of project costs divided by the flow of total income. For convenience, we define

$$\varpi_i \equiv \frac{\gamma_i \bar{x}^\delta Z}{m}, \text{ and} \qquad (20)$$

$$\Omega_P \equiv \frac{\rho C_P}{m\bar{x}}, \tag{21}$$

where $0 \le \varpi_i \ll 1$ and $0 \le \Omega_P \ll 1$. In addition, the percentage of land dedicated to a new transportation project is typically small compared to the city area, so $0 < \theta \ll 2\pi$. We take advantage of these properties to derive approximate expressions of the objective function (19) and of the cost of land (Equation (17)). We present first order approximations here because we found them to be adequate for our numerical illustration; second order approximations are provided in the appendix.

A. Cost of Land

After expanding $I_i(x, x^*; k)$, i=1,2, k=0,1, we form and simplify the ratio $\dfrac{I_i(x, x^*;1)}{I_i(x, x^*;0)}$, introduce it in the expression of $C_L(x^*)$, and derive the first order approximation

$$C_L(x^*) \approx \frac{\theta(1-a)}{2\pi} m\bar{x} \left\{ M_1(x^*) - \frac{\Omega_P}{\rho} - \frac{2}{3}\varpi_1 M_{\delta+1}(x^*) + e^{-\rho\Delta} \frac{2}{3}(\varpi_1 - \varpi_2) N_{\delta+1}(x^*) \right\}, \tag{22}$$

where, for any real number α,

$$M_\alpha(x^*) \equiv \frac{1}{\bar{x}^\alpha} E_{x^*} \left\{ \int_0^{+\infty} X_t^\alpha e^{-\rho t} dt \right\}, \text{ and} \tag{23}$$

$$N_\alpha(x^*) \equiv \int_{\underline{x}}^{\bar{x}} p(\Delta, x^*, \xi) M_\alpha(\xi) d\xi. \tag{24}$$

In (22), $m\bar{x}\left\{M_1(x^*) - \rho^{-1}\Omega_P\right\} = m\bar{x}M_1(x^*) - C_P$ is the expected present value of the income of present and future city residents net of the costs of building the project. Terms in ϖ_1 and ϖ_2 represent the contribution of congestion costs during and after project construction. Finally, we note that $C_L(x^*)$ is approximately proportional to the utility elasticity of land, 1-*a* (see again (6)).

B. Objective Function

To approximate the objective function, we plug expansions of $\ln(I_i(x,x^*;k))$, *i*=1,2, and $\ln(I_0(x))$ into the differences $V_1(X,x^*)-V_0(X)$ and $V_2(x,x^*)-V_1(x,x^*)$, insert these results into the objective function (19), and simplify to get

$$\underset{\underline{x}\le x^*}{Max}\, D_{x^*|x_0}\, a\overline{x}\left\{\frac{\theta}{2\pi}\frac{a-1}{a}M_1(x^*)-\frac{C(x^*)}{m\overline{x}}+\frac{2}{3}\left(\varpi_0-\varpi_1\right)M_{\delta+1}(x^*)+ e^{-\rho\Delta}\frac{2}{3}\left(\varpi_1-\varpi_2\right)N_{\delta+1}(x^*)\right\}. \tag{25}$$

The first three terms of (25) represent the expected discounted utility loss from the cost of the project (first two terms) and the extra congestion ($\gamma_1>\gamma_0$) it creates (third term), assuming construction lasts forever. The last terms are the expected discounted utility gains from the project. Inserting (22) into $C(x^*)$ in (25) gives the first order expansion of the objective function

$$\underset{\underline{x}\le x^*}{Max}\, D_{x^*|x_0}\, a\overline{x}\left\{\frac{\theta}{2\pi}\frac{a^2-1}{a}M_1(x^*)+\left[\frac{\theta(1-a)}{2\pi}-1\right]\frac{\Omega_P}{\rho}+ \frac{2}{3}\left(\left[\frac{\theta(1-a)}{2\pi}-1\right]\varpi_1+\varpi_0\right)M_{\delta+1}(x^*)+e^{-\rho\Delta}\frac{2}{3}\left(\varpi_1-\varpi_2\right)\left[1-\frac{\theta(1-a)}{2\pi}\right]N_{\delta+1}(x^*)\right\}. \tag{26}$$

Finally, we need to show how to calculate the discount factor $D_{x^*|x_0}$ as well as

$M_\alpha(x^*)$ and $N_\alpha(x^*)$. This is done in the appendix: see (A.10) to (A.12) for $D_{x^*|x_0}$, and (A.13) to (A.14) for $M_\alpha(x^*)$. Once $M_\alpha(x^*)$ and $p(\Delta, x^*, \xi)$ are known, $N_\alpha(x^*)$ follows from (24). This framework is valid for a wide range of processes for the city population. To obtain numerical results, however, the population process (Equation (1)) needs to be specified. This is done below.

IV. ILLUSTRATION

A. Assumptions and Derivations

To explore the properties of x^* for a very simple case, let us now assume that the city population follows the trendless Brownian motion (a continuous random walk)

$$dX = \sigma dw, \tag{27}$$

where dw is an increment of a standard Wiener process (Karlin and Taylor, 1981). This formulation is probably better suited to an older city where there is no clear population trend; we retain it as a starting point for its simplicity. To calibrate σ but also for management purposes, it is useful to derive $E\left(T_{x_1|x_0}\right)$, the expected time it takes for the city population to increase from x_0 to $x_1 > x_0$. From Chapter 15 in Karlin and Taylor (1981),

$$E\left(T_{x_1|x_0}\right) = \frac{(x_1 - x_0)(x_1 + x_0 - 2\underline{x})}{\sigma^2}, \tag{28}$$

so X grows more slowly away from $\underline{x}$ and $E\left(T_{x_1|x_0}\right)$ is inversely proportional to σ^2.

From (A.10)- (A.12), the discount factor for $0<y<x$, is

$$D_{x|y} = \frac{e^{\lambda(y-\underline{x})} + e^{-\lambda(y-\underline{x})}}{e^{\lambda(x-\underline{x})} + e^{-\lambda(x-\underline{x})}}, \quad (29)$$

where

$$\lambda = \frac{\sqrt{2\rho}}{\sigma}. \quad (30)$$

We can also obtain an explicit expression of $M_\alpha(x^*)$ (see (A.16) in the appendix). In addition, we know from Saphores (2003) that

$$p(t,x,y) = \frac{1}{\overline{x}-\underline{x}}\left\{1 + 2\sum_{n=1}^{+\infty} \exp\left(-\left(\frac{n\pi\sigma}{\overline{x}-\underline{x}}\right)^2 \frac{t}{2}\right) \cos\left(n\pi \frac{x-\underline{x}}{\overline{x}-\underline{x}}\right) \cos\left(n\pi \frac{y-\underline{x}}{\overline{x}-\underline{x}}\right)\right\}. \quad (31)$$

Combining (A.16) and (31) allows us to calculate $N_\alpha(x^*) \equiv \int_{\underline{x}}^{\overline{x}} p(\Delta, x^*, \xi) M_\alpha(\xi) d\xi$.

B. Data

Let us first assess urban congestion costs, transportation project costs, and the reduction in congestion that can reasonably be expected from transportation infrastructure projects.

Congestion Costs as a Fraction of Income

According to the Texas Transportation Institute (TTI, 2002) congestion delay costs per driver in 2000 averaged \$1,160 for the 75 metropolitan statistical areas studied, and \$1,590 in the largest metropolitan statistical areas. From the U.S. census data, the year 2000 median household income and per capita income were \$42,228 and \$21,587 respectively. This gives estimates of congestion as a fraction of income that range from 2.7% (=$1,160/$42,228) to 7.4% (=$1,590/$21,587).

An alternative approach yields more conservative estimates. DeLucchi (1998) finds that external time costs from traffic congestion per person-mile range from 0.91 cents to 4.01 cents (in 1990 $). The most recent National Household Transportation Survey estimates that Americans travel 26.9 miles per day per person (Pucher and Renne, 2003), which translates into a delay cost comprised between $112.97 and $497.79 per person per year in 2000 $.[4] Dividing by per capita income for 2000 gives delay costs as a fraction of income ranging from 0.5% to 2.3%.

Magnitude of Congestion Reduction

Weisbrod, Vary, and Treyz (2001) use simulation to analyze the impacts of congestion reduction for the National Cooperative Highway Research Program. They assume that congestion reduction ranges from 10% of travel time (which implies a 2.5% total travel cost reduction) to 25% of travel time (a 6.3% reduction in total travel cost). More evidence is provided by efforts to track regional congestion delays. The recession of the early 1990s contributed to reducing traffic congestion in Southern California. Surveys of travelers reveal that average two-way commute times (to and from work) in Los Angeles County decreased from 79 minutes in 1992 to 64 minutes in 1994 – a 19% reduction in travel time. This is a regional congestion reduction due to changes in travel (commute) demand. Targeted infrastructure investments might lead to larger travel time reductions along specific corridors. For that reason, we include estimates of total travel time reduction as large as 40%, which based on the values in Weisbrod, Vary, and Treyz (2001) corresponds to approximately a 10% reduction in total travel costs. This is likely an upper bound of what could be feasibly achieved through infrastructure investments.[5]

Transportation Project Costs

Since it is most reasonable to compare our hypothetical investment to major highway or rail transit corridor investments in metropolitan areas, we assess recent costs for building highway or rail transit projects in the United States. Table 2 shows some construction costs data for six major transportation infrastructure projects. Excluding maintenance, we see that $\frac{\rho C_P}{mx}$ is in the interval [0.000056, 0.00089]. Assuming that maintenance adds 50% to annualized construction costs changes this range to [0.000084, 0.00134]. For our base case, we set Ω_P to 0.0002.

Time Unit and Population Variability

Annual changes in urban populations are typically estimated between censuses, so we adopt a time unit of 10 years. We consider a city whose population varies between $\underline{x} = 1$ million and $\overline{x} = 2$ million. To assess population variability over 10 years, we examine population changes between consecutive censuses in a number of U.S. metropolitan areas with approximately 1 to 2 million residents between 1970 and 2000. Most cities considered grew, although some older cities lost population: Buffalo-Niagara lost 0.106 million residents between 1970 and 1980. Population changes between consecutive censuses are frequently less than 0.1 million (e.g., Indianapolis for 1970-1980 and 1980-1990; Milwaukee; or New Orleans for 1980-1990 and 1990-2000) but a number of cities saw drastic changes (+0.346 million for Oakland between 1980 and 1990; +0.486 million for the Denver metropolitan area between 1990 and 2000). To cover a wide range of possible situations, we vary σ over [0.03; 0.3]; σ is in millions of people divided by the square root of time. For future reference, the expected time it would take for the city population to increase from 1.4 million to 1.6 million is 90 years for σ=0.1, 22 years for σ=0.2, and 10 years for σ=0.3 (see Equation (28)).

Base Case and Sensitivity Analysis

For a city whose population varies between 1 and 2 million, a transportation project would reduce ϖ (the maximum ratio of the flow of individual congestion costs divided by the flow of personal income) from $\varpi_0 = 0.05$ to $\varpi_2 = 0.0465$ (a 7% reduction). For simplicity, congestion does not increase during project construction (so $\varpi_1 = 0.05$); we explore the impact of this hypothesis in our sensitivity analysis. The congestion exponent is δ=2. The transportation project requires an angle $\theta=0.001*2\pi$ of the city area (0.1% of urban land) and it takes Δ=3 years to complete. The fraction of people's income flow needed to pay for project construction (excluding land purchases) is Ω_P=0.0001. The utility elasticity of the numeraire good divided by the utility elasticity of land is ν=2. Finally, our annual discount rate is 7%, the value used by the federal government for transportation projects cost benefit analyses (Circular No. A-94 Revised from the OMB). The corresponding 10 year effective rate is therefore $\rho=1.07^{10}-1=0.967$.

For our sensitivity analysis, we vary the congestion coefficient during construction (ϖ_1 changes by 0%, 1.5% and 3%) and for each pre-construction congestion level (ϖ_0), we choose ϖ_2 to reflect congestion reductions of 4%, 7%, and 10%, so $\varpi_2 \in \{0.048, 0.0465, 0.045\}$. We also change the time to complete the project ($\Delta \in \{1, 3, 5, 7\}$ years) and the % of people's income needed annually to pay for project construction ($\Omega_P \in \{0.00005, 0.0001, 0.00015\}$. We then repeat our calculations for a yearly discount rate of 4%, a value used in the past for transportation projects. Finally, we explore systematically the impact of σ by varying its value over [0.03; 0.3].

C. Results

Results were generated using MathCad on a PC. They are partially summarized on Figures 1 to 4. From Figure 1, we observe that the discount rate has a steady impact on x^* (a 4% to 6% change on x^* on the range of σ considered). Decreasing the annual effective discount rate (denoted by ρ_y) from 7% to 4% causes the investment in congestion reduction to take place at a lower value of the city population (so the project land can be purchased for approximately 40% less) because it increases the present value of the project net benefits. This decrease in ρ_y also substantially increases the individual utility from the project: its relative impact is largest when σ is low, but the ratio of individual utilities still reaches a factor 8 when σ=0.3. The effect of a drop in the discount rate on the timing of the infrastructure investment is largest when σ is small because changes in the city population are driven by σ.

We also see that when ρ_y=4%, x^* first increases with σ and then starts tapering off (for ρ_y=7%, x^* behaves similarly for higher values of σ). This behavior results from the presence of barriers ($\underline{x}$ and $\overline{x}$) on the city population. Starting from small values of σ, increasing the volatility first increases the congestion costs incurred during construction more than it decreases the per capita cost of project land, so x^* increases and the project is postponed. For higher values of σ, however, X is constrained by $\overline{x}$ so the increase in congestion costs during construction is limited while project benefits increase, so x^* diminishes.

The role of $\underline{x}$ and $\overline{x}$ is partly confirmed by the evolution with σ of $\Omega_L(x_0) \equiv \frac{\rho C_L(x_0)}{m x_0}$, the fraction of the flow of total income devoted to the flow of payments on project land. From Figure 2, $\Omega_L(\overline{x})$ decreases with σ whereas the reverse is true for $\Omega_L(\underline{x})$. For the former, an increase in σ decreases the expected population and therefore the cost of land by allowing X to move faster away from $\overline{x}$; an increase in σ when X=$\underline{x}$ has the opposite effect. For x_0 between $\underline{x}$ and $\overline{x}$, the behavior of $\Omega_L(x_0)$ depends on how close x_0 is from either barrier.

From Figure 3, x^* increases as the congestion relief brought about by the project decreases (i.e., increasing ϖ_2 makes the project less attractive), as expected. We observe again that $\underline{x}$ and $\overline{x}$ play important roles. When ϖ_2 is large, it takes higher population levels to justify the project; when σ is large enough, the population varies more and is more frequently below the population level that makes the project worthwhile (because of the upper barrier) so the project becomes uneconomical (the case ϖ_2=0.048). Conversely, when ϖ_2 is small (and the project is attractive at low population levels), increasing σ makes the project more attractive at increasing low population levels because the population is bounded from below and tends faster towards high values; at some point the project should be built immediately (the case ϖ_2=0.045).

A similar logic is at work for Figure 4, which shows how project duration (Δ, the time to buy land and build the project) influences its timing: increasing Δ augments the present value of congestion costs during construction and decreases the present value of project benefits. The project becomes unattractive when σ is high enough for a long project duration (Δ=7 years) because the upper population barrier ($\bar{x}$) truncates the expected benefits from investing to relieve congestion. For projects that are sufficiently attractive (a low value of ϖ_2) and of short duration, a project could also become increasingly attractive as σ increases. These results point to the detrimental effects of congestion relief projects that drag on; if project cost/duration tradeoffs were known, our approach could be used to estimate optimum project costs and duration.

Other results from our sensitivity analysis show, as expected, that x^* increases with $\Omega_P \equiv \dfrac{\rho C_P}{m\bar{x}}$: it is optimal to wait for a larger population to share larger project costs. Likewise, increasing congestion during project construction (ϖ_1) makes the project less attractive and therefore increases x^*. As for the discount rate, increasing ϖ_1 just shifts x^* upwards over the range of σ considered by approximately 3%-3.5% for each 1.5% increase in ϖ_1.

V. CONCLUSIONS

To the best of our knowledge, this paper is the first one to analyze the impacts of uncertainty and irreversibility on the timing of building urban transportation infrastructure. Our results show that some of the basic insights of simple investment models (i.e., see Dixit and Pindyck, 1994) do not carry over to our problem; more uncertainty, for example, does not necessarily delay the decision to invest; it may instead call for early investment. Our numerical illustration based on plausible parameter values show that the investment threshold, denoted by x^*, first increases and then decreases with population uncertainty. When x^* is close to the upper population barrier, increasing uncertainty may make the project unattractive; the reverse is true when x^* is close to the lower population barrier (it may be optimal to invest immediately). Several reasons explain these differences: the stochastic process considered for our population variable is of course important, but even more important probably are the upper and lower population barriers and the dependence of the project costs and benefits on the level of the city population.[6]

Our approach is applicable to other externality problems linked to population increases in general, and to congestion problem in particular. It also illustrates the importance of barriers in stochastic investment problems, a point already made elsewhere in different contexts.

Future work will explore the value of buying land in anticipation of future infrastructure needs (land banking) and allow for the possibility of abandoning transportation infrastructure if the city population decreases too much. In addition, our framework could be used to study the impact of transportation projects on population dynamics and on future city growth. Dealing with increasing income and technological progress is possible but substantially more complex.

APPENDIX

Derivation of the objective function (Equation (19)).

Let J_i, $i \in \{1,2,3\}$, denote the i^{th} integral term of (18). Let us first add to and subtract

$$E_{x_0}\left\{\int_{T_{x^*|x_0}}^{+\infty} X.V_0(X).e^{-\rho t}dt\right\} = D_{x^*|x_0} E_{x^*}\left\{\int_0^{+\infty} X.V_0(X).e^{-\rho t}dt\right\}$$

from J_0; the equality results from the Markov property and from the definition of the discount factor $D_{x^*|x_0} \equiv E_{x_0}\left(e^{-\rho T_{x^*|x_0}}\right)$. After simplifying,

$$J_0 \equiv E_{x_0}\left\{\int_0^{+\infty} X.V_0(X).e^{-\rho t}dt\right\} - D_{x^*|x_0} E_{x^*}\left\{\int_0^{+\infty} X.V_0(X).e^{-\rho t}dt\right\};$$

we note that the first term of J_0 is independent of x^*; it represents the discounted utility of city residents if the project is never built.

Using the same approach, we can transform J_1 and J_2 to

$$J_1 \equiv D_{x^*|x_0}\left[E_{x^*}\left\{\int_0^{+\infty} X.V_1\left(X,x^*\right)e^{-\rho t}dt\right\} - e^{-\rho\Delta}\int_0^{+\infty} p(\Delta,x^*,\xi)E_\xi\left[\int_0^{+\infty} X.V_1(X,x^*)e^{-\rho t}dt\right]d\xi\right]$$

and

$$J_2 \equiv e^{-\rho\Delta}D_{x^*|x_0}\left[\int_0^{+\infty} p(\Delta,x^*,\xi)E_\xi\left[\int_0^{+\infty} X.V_2(X,x^*)e^{-\rho t}dt\right]d\xi\right].$$

When we omit terms independent of x^*, the transportation agency's objective becomes (19).

Approximate expression of the cost of land.

As noted above, y_i, the ratio of the flow of congestion costs divided by income flow, and y_d, the ratio of the flow of project debt divided by income flow, are both typically small, i.e.

$\forall x \in [\underline{x}, \overline{x}]$, $\forall i \in \{0,1,2\}$, $0 \le y_i \equiv \dfrac{\gamma_i x^\delta Z}{m} \ll 1$ and $0 \le y_d \equiv \dfrac{\rho C(x^*)}{mx} \ll 1$.

Neglecting terms in $y_i^{n_1} y_d^{n_2}$ with $n_1 \geq 0$, $n_2 \geq 0$, and $n_1 + n_2 > 2$, a second order expansion of $I_i(x,x^*;0)$ for $i \in \{1,2\}$ leads to

$$\frac{I_i(x,x^*;0)}{m^\nu Z^2} \approx \frac{1}{2} - \frac{\nu}{3} y_i - \frac{\nu}{2} y_d + \frac{\nu(\nu-1)}{8} y_i^2 + \frac{\nu(\nu-1)}{3} y_i y_d + \frac{\nu(\nu-1)}{4} y_d^2. \tag{A.1}$$

To derive $I_0(x)$ from (A.1), simply set y_d to 0 and y_i to y_{0c}. Moreover, replacing ν with $\nu+1$ in (A.1) gives $I_i(x,x^*;1)$. With these results, we can calculate the second order expansion:

$$\frac{I_i(x,x^*;1)}{I_i(x,x^*;0)} \approx m\left\{1 - \frac{2}{3} y_i - y_d + \frac{\nu}{18} y_i^2\right\}. \tag{A.2}$$

When we combine (A.2) and (16), we find

$$W_i(x,x^*) \approx \frac{x\theta m}{(2\pi - \theta)(1+\nu)}\left\{1 - \frac{2}{3} y_i - y_d + \frac{\nu}{18} y_i^2\right\}. \tag{A.3}$$

Inserting (A.3) into (17) and isolating $C_L(x^*)$ leads to

$$\begin{aligned} C_L(x^*) \approx \frac{\theta m \overline{x}}{(1+\nu)(2\pi-\theta)+\theta} \Bigg\{ & M_1(x^*) - \frac{\Omega_P}{\rho} - \frac{2}{3}\varpi_1 M_{\delta+1}(x^*) + e^{-\rho\Delta}\frac{2}{3}(\varpi_1 - \varpi_2) N_{\delta+1}(x^*) \\ & + \frac{\nu}{18}\left[\varpi_1^2 M_{2\delta+1}(x^*) - e^{-\rho\Delta}(\varpi_1^2 - \varpi_2^2) N_{2\delta+1}(x^*)\right]\Bigg\}, \end{aligned} \tag{A.4}$$

where ϖ_i is defined by (20). Terms in factor of $\dfrac{\nu}{18}$ are a second order congestion correction. To get (22), note that a 1st order approximation of $\dfrac{\theta m}{(1+\nu)(2\pi-\theta)+\theta}$ is simply $\dfrac{\theta}{(1+\nu)2\pi} = \dfrac{\theta(1-a)}{2\pi}$.

Objective function

We start by looking for a second order expansion of $V_1(X,x^*)-V_0(X)$. From (A.1),

$$V_1(x,x^*)-V_0(x)\approx\frac{1}{1+\nu}\left\{\ln\left(\frac{2\pi-\theta}{2\pi}\right)+\right.$$
$$\left.\frac{2\nu}{3}\left(y_0-y_1\right)+\frac{\nu(9-\nu)}{36}(y_0^2-y_1^2)-\frac{2\nu}{3}y_1y_d-\nu y_d-\frac{\nu}{2}y_d^2\right\}. \tag{A.5}$$

The first integral of (19) can thus be approximated by

$$E_{x^*}\left[\int_0^{+\infty}X.[V_1\left(X,x^*\right)-V_0(X)]e^{-\rho t}dt\right]\approx a\bar{x}\left\{\frac{1}{\nu}\ln\left(\frac{2\pi-\theta}{2\pi}\right)M_1(x^*)\right.$$
$$+\frac{2}{3}\left(\varpi_0-\varpi_1\right)M_{\delta+1}(x^*)+\frac{9-\nu}{36}(\varpi_0^2-\varpi_1^2)M_{2\delta+1}(x^*) \tag{A.6}$$
$$\left.-\frac{2}{3}\frac{\rho C(x^*)}{m\bar{x}}\varpi_1M_\delta(x^*)-\frac{1}{\rho}\frac{\rho C(x^*)}{m\bar{x}}-\frac{1}{2}\left(\frac{\rho C(x^*)}{m\bar{x}}\right)^2M_{-1}(x^*)\right\},$$

where $M_\alpha(x^*)$ is defined by (23).

Likewise, a second order expansion of $V_2(x,x^*)-V_1(x,x^*)$ is

$$V_2(x,x^*)-V_1(x,x^*)\approx\frac{\nu}{1+\nu}\left\{\frac{2}{3}(1+y_d)\left(y_1-y_2\right)+\frac{9-\nu}{36}(y_1^2-y_2^2)\right\}, \tag{A.7}$$

so the second integral of (19) can be approximated by

$$\int_0^{+\infty}p(\Delta,x^*,\xi)E_\xi\left[\int_0^{+\infty}X.[V_2(X,x^*)-V_1(X,x^*)]e^{-\rho t}dt\right]d\xi\approx$$
$$a\bar{x}\left\{\frac{2}{3}\left(\varpi_1-\varpi_2\right)\left[N_{\delta+1}(x^*)+\frac{\rho C(x^*)}{m\bar{x}}N_\delta(x^*)\right]+\frac{9-\nu}{36}(\varpi_1^2-\varpi_2^2)N_{2\delta+1}(x^*)\right\}, \tag{A.8}$$

where $N_\alpha(x^*)$ is defined by (24).

From (A.6) and (A.8), the objective function becomes

$$\underset{\underline{x}\le x^*}{Max}\, D_{x^*|x_0}\, a\overline{x}\left\{\frac{1}{\nu}\ln\left(\frac{2\pi-\theta}{2\pi}\right)M_1(x^*)-\frac{1}{\rho}\frac{\rho C(x^*)}{m\overline{x}}+\frac{2}{3}(\varpi_0-\varpi_1)M_{\delta+1}(x^*)\right.$$
$$-\frac{2}{3}\frac{\rho C(x^*)}{m\overline{x}}\varpi_1 M_\delta(x^*)+\frac{9-\nu}{36}(\varpi_0^2-\varpi_1^2)M_{2\delta+1}(x^*)-\frac{1}{2}\left(\frac{\rho C(x^*)}{m\overline{x}}\right)^2 M_{-1}(x^*)+ \qquad \text{(A.9)}$$
$$\left. e^{-\rho\Delta}\left[\frac{2}{3}(\varpi_1-\varpi_2)\left[N_{\delta+1}(x^*)+\frac{\rho C(x^*)}{m\overline{x}}N_\delta(x^*)\right]+\frac{9-\nu}{36}(\varpi_1^2-\varpi_2^2)N_{2\delta+1}(x^*)\right]\right\},$$

where $C(x^*)=C_P+C_L(x^*)$ and $C_L(x^*)$ is defined by (A.4) and C_P is known.

Derivation of $D_{x|y}$, $M_\alpha(x^*)$ *and* $N_\alpha(x^*)$.

The discount factor $D_{x|y}$ with a reflecting barrier at ℓ is easily derived. From Karlin and Taylor (1981) we know that $W(y)\equiv D_{x|y}$ verifies the linear, second-order, ordinary differential equation

$$\frac{s^2(y)}{2}\frac{d^2W(y)}{dy^2}+f(y)\frac{dW(y)}{dy}-\rho W(y)=0. \qquad \text{(A.10)}$$

We therefore need two conditions to fully define $D_{x|y}$. By construction, $D_{x|x}=1$, so

$$W(x)=1. \qquad \text{(A.11)}$$

The other condition comes from a reflecting barrier at ℓ. Saphores (2002) shows that, for $x\ge\ell$,

$$\frac{dW(y)}{dy}\Big|_{y=\ell}=0. \qquad \text{(A.12)}$$

A closed-form expression can be derived for $M_\alpha(x^*)\equiv\overline{x}^{-\alpha}E_{x^*}\left\{\int_0^{+\infty}X_t^\alpha e^{-\rho t}dt\right\}$ for specific choices of (1). From Karlin and Taylor (1981), $M_\alpha(x)$ verifies

$$\frac{s^2(x)}{2}M_\alpha''(x)+f(x)M_\alpha'(x)-\rho M_\alpha(x)+\left(\frac{x}{\overline{x}}\right)^\alpha=0, \tag{A.13}$$

with boundary conditions

$$M_\alpha'(\underline{x})=M_\alpha'(\overline{x})=0. \tag{A.14}$$

To derive $M_\alpha(x)$ for $\alpha\geq 0$ when X follows (27), we first look for two independent solutions of the homogenous equation associated to (A.13). We find

$$F_1(x)=e^{\lambda x},\ F_2(x)=e^{-\lambda x}, \tag{A.15}$$

where $\lambda=\frac{\sqrt{2\rho}}{\sigma}$. We then look for a particular solution of (A.13) as a product of F_1 and an unknown function. After some algebra, the general solution of (A.13)-(A.14) can be written:

$$M_\alpha(x)=\frac{e^{\lambda(x-\underline{x})}+e^{-\lambda(x-\underline{x})}}{e^{\lambda(\overline{x}-\underline{x})}-e^{-\lambda(\overline{x}-\underline{x})}}\frac{H_\alpha'(\overline{x})}{\lambda}-H_\alpha(x), \tag{A.16}$$

$$H_\alpha(x)=\frac{1}{\lambda\sigma^2\overline{x}^\alpha}\int_{\underline{x}}^{x}\xi^\alpha[e^{\lambda(x-\xi)}-e^{-\lambda(x-\xi)}]d\xi. \tag{A.17}$$

Finally, $N_\alpha(x^*)$ can be obtained numerically once $M_\alpha(x)$ and $p(\Delta,x,y)$ are known.

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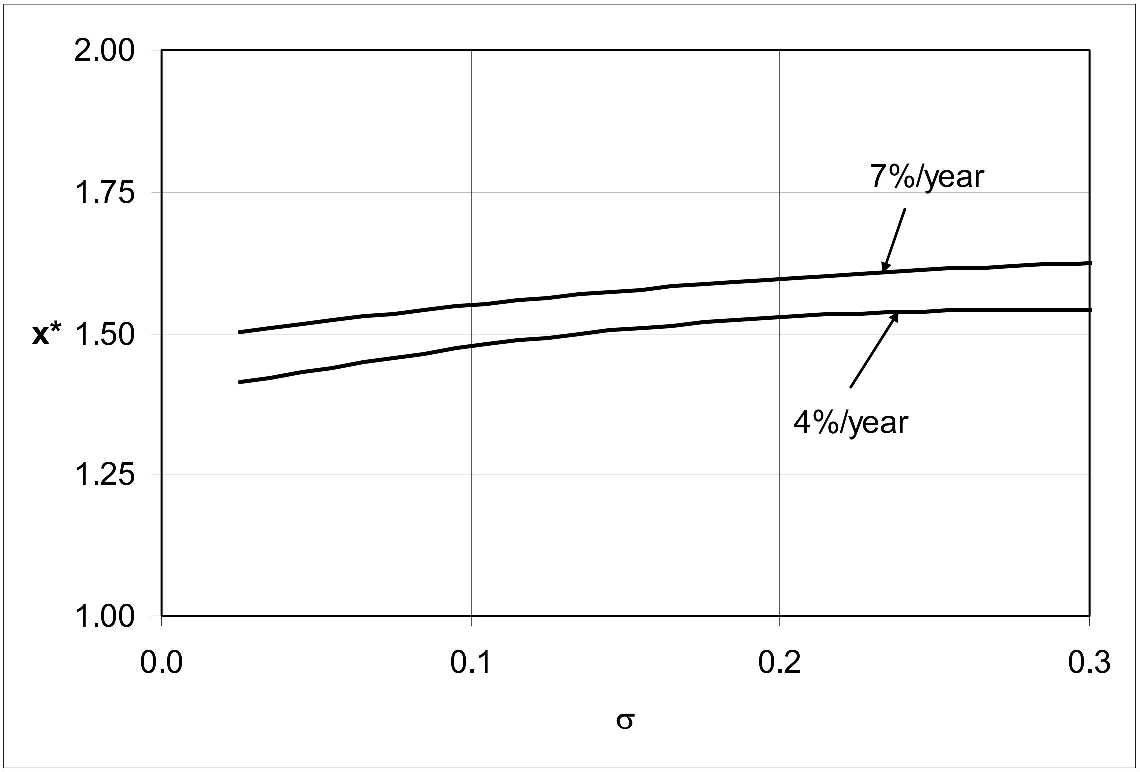

Figure 1: x^* versus σ for different values of the discount rate.

Notes. Model parameters for generating the above results: $\underline{x} = 1.0$ million, $\overline{x} = 2$ million, ν=2, θ=0.001*2π, Δ=3 years, δ=2, Ω_P=0.0002, ϖ_0=0.05, ϖ_1=0.05, and ϖ_2=0.0465; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. See Table 1 for the meaning of the different parameters. x^* is in millions of inhabitants.

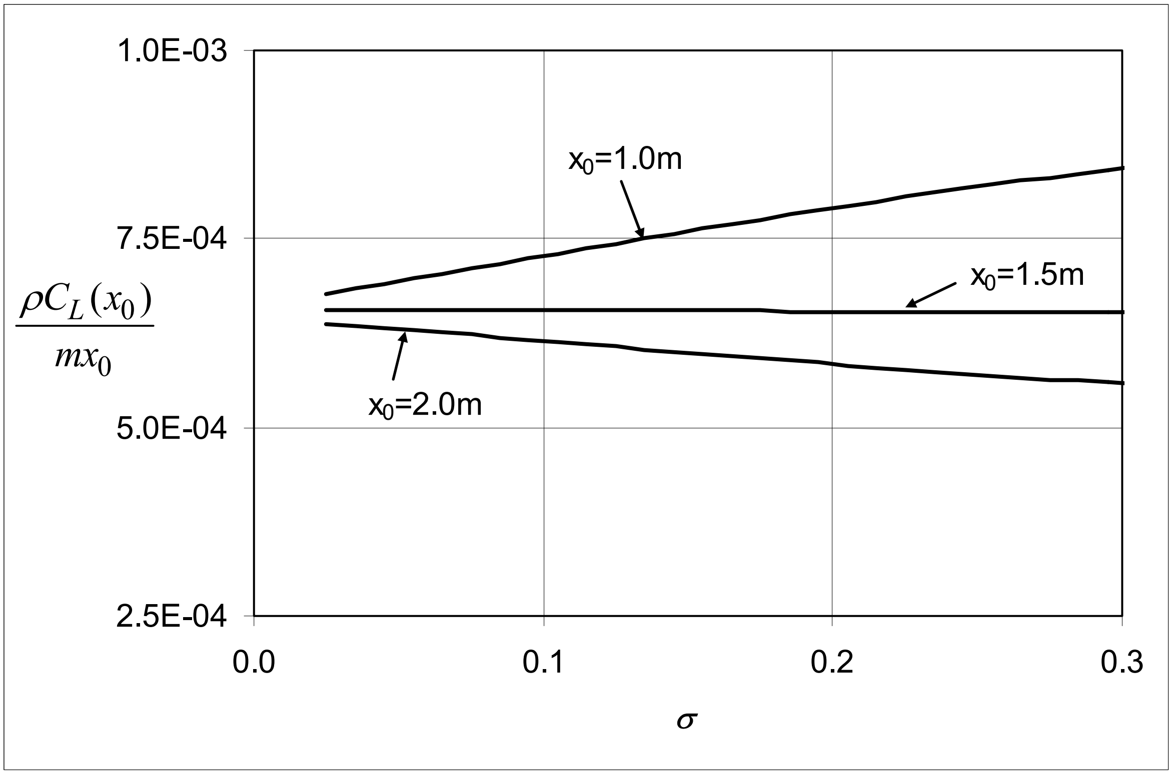

Figure 2: Fraction of total income for project land versus σ for various current population values.

Notes. Model parameters for Figure 2: $\underline{x} = 1.0$ million, $\overline{x} = 2$ million, ν=2, θ=0.001*2π, Δ=3 years, δ=2, Ω_P=0.0002, ϖ_0=0.05, ϖ_1=0.05, ϖ_2=0.0465, and an annual discount rate of 4%; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. x_0 designates the current city population. The numerator of $\frac{\rho C_L(x_0)}{m x_0}$ is the interest payments on project land if the project were started when $X = x_0$; $m x_0$ is the total income flow of the city population. See Table 1 for the meaning of the different parameters. x^* and x_0 are in millions of inhabitants.

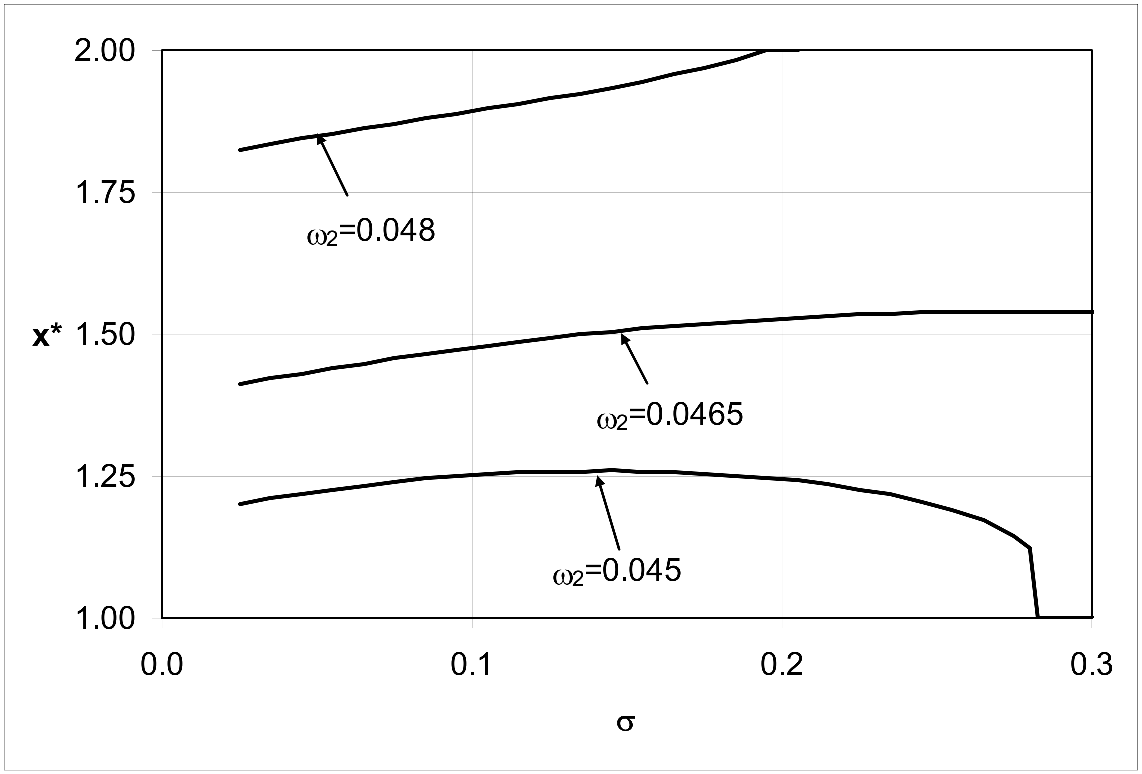

Figure 3: x^* versus σ for different values of ϖ_2.

Notes. Model parameters for Figure 3: $\underline{x} = 1$ million, $\overline{x} = 2$ million, ν=2, θ=0.001*2π, Δ=3 years, Ω_P=0.0002, ϖ_0=0.05, ϖ_1=0.05, δ=2, and an annual discount rate of 7%; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. See Table 1 for the meaning of the different parameters. x^* above is in millions of inhabitants.

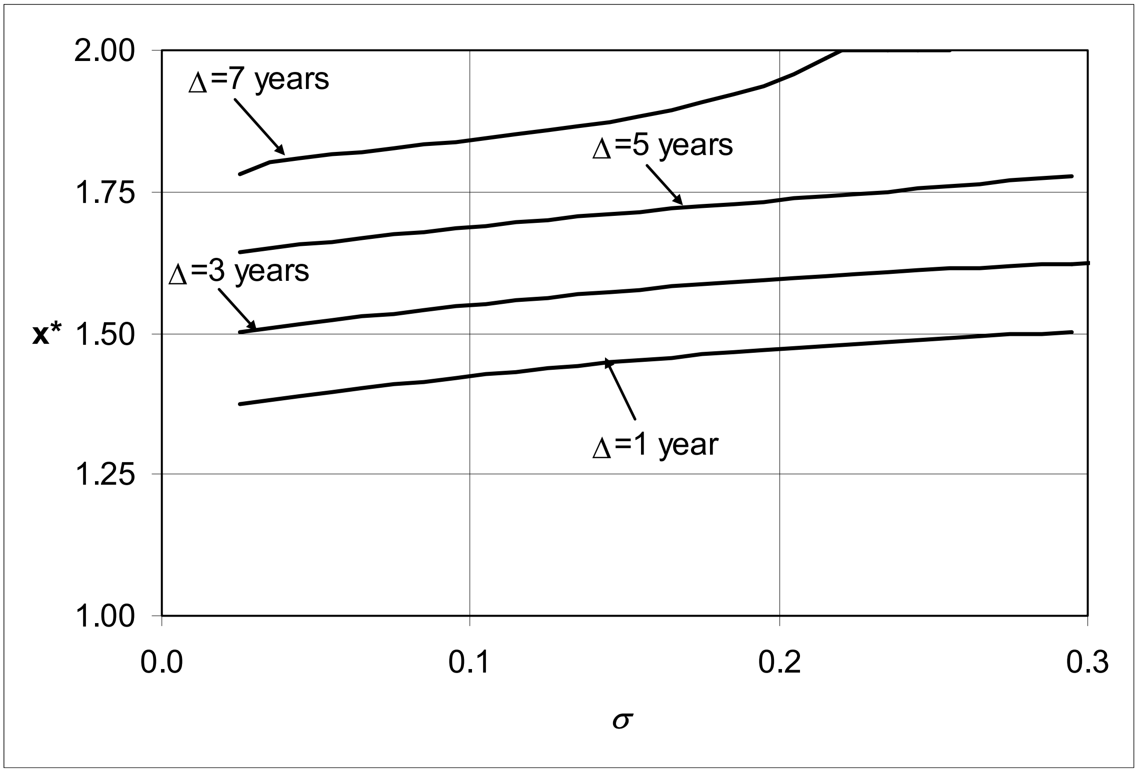

Figure 4: x^* versus σ for different values of Δ.

Notes. Model parameters for Figure 4: $\underline{x} = 1$ million, $\overline{x} = 2$ million, ν=2, θ=0.001*2π, Ω_P=0.0002, ϖ_0=0.05, ϖ_1=0.05, ϖ_2=0.0465, δ=2, and an annual discount rate of 7%; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. Δ is the time necessary to acquire the land for the project and to build the project. x^* is in millions of inhabitants. See Table 1 for the meaning of the different parameters.

Table 1: Key Notation.

Variable	Description	Values
$\underline{x}$, $\overline{x}$	Lower and upper bounds on *x*, the city population (in million)	1 and 2
x^*	Population threshold for building the project	Calculated
σ	Population volatility (in millions per $\sqrt{\text{unit of time}}$)	0.03 to 0.30
m	Individual income per unit of time	See ϖ_i
z, Z	Radial distance from the CBD and city radius	See ϖ_i
θ	Angle of project land slice	$0.001*2\pi$
γ	Congestion coefficient	See ϖ_i
δ	Congestion exponent	2
a, 1-a	Utility elasticity of the numeraire good, utility elasticity of land	See ν
$\nu = \frac{a}{1-a}$	Ratio of elasticities	2
ρ	Social discount rate	4% or 7%
$\varpi_i = \frac{\gamma \overline{x}^{\delta} Z}{m}$	Ratio of flow of congestion costs to income flow, before (i=0)	0.05
	during (i=1)	0.050, 0.0507, 0.0515
	and after (i=2) project completion	0.048, 0.0465, 0.0450
$C_L(x^*)$	Cost of buying project land when X=x^*	Calculated
C_P	Cost of building the project	See Ω_P
$\Omega_P \equiv \frac{\rho C_P}{m\overline{x}}$	Ratio of interest on building costs divided by total income flow	0.0002
C(x^*)	Total direct project cost (land + construction)	$C_L(x^*)+ C_P$
$V(x)$	Project value	Calculated
$N_i(x, z)$	Population within distance *z* of the CBD for population *x*	Calculated
$R(x, z; \gamma)$	Unit land rent at *z* when total city population is x	Calculated

Table 2: Selected Construction Costs for Recent Transportation Infrastructure Projects

Project	Description	Cost (in million of 2001 $)	Cost as a fraction of annual personal income, $\rho C_p/(mx)$ $\rho = 4\%$	$\rho = 7\%$
a): SR-91	10 miles of privately franchised toll lanes in median of pre-existing freeway, Orange County, California; opened 1995.	$146	0.000056	0.000098
b): Dulles Toll Road Extension	14 mile private extension of toll highway into Loudon County, Virginia; opened 1995.	$379	0.00021	0.00036
c): Century Freeway	17 miles of urban freeway, Los Angeles, California; opened 1993.	$2,696	0.00037	0.00065
d): I-210 extension	First phase of 28 mile extension of I-210, Los Angeles, California; opened 2001	$1,100 (estimated for entire project)	0.00015	0.00026
e): Mission Valley East line	5.8 mile extension of San Diego light rail system; construction in progress.	$424 (estimated)	0.00017	0.00031
f): Dallas Area Rapid Transit light rail	First phase of 20 mile light rail line	$971	0.00050	0.00089

Notes. 1) Cost data sources: for a), Boarnet, DiMento, and Macey (2002); for b), Pae (1995); for c), Zamichow (1993); for d), Martin (2001); for e), San Diego Metropolitan Transit Development Board (2002); for f), Howell (1996). Costs include only construction and land acquisition costs, not maintenance. 2) Population data are from 1997 State and Metropolitan Area Data Book and http://www.nv.cc.va.us/oir/va_pop/popxco.html (> 08/24/03). Population data are for surrounding metropolitan statistical area except for b), for which only northern Virginia suburbs of Washington D.C. were used. 3) Per capita income data (2001 dollars) for metropolitan statistical areas is from www.bea.gov/bea/newsreel/MPINewsRelease.htm (> 08/24/03).

[1] Johnson (1999) acknowledges the link between population forecasts and infrastructure project analysis, stating, "Planning and building infrastructure for the wrong population can be costly." Yet Johnson's focus is on understanding the different projections, which differs from our modeling effort to incorporate demand uncertainty into infrastructure project analysis.

[2] $I_0(x) = \frac{m^{\nu} Z^2}{(\nu+1)(\nu+2)} \left(\frac{\gamma_0 x^{\delta} Z}{m} \right)^{-2} \left\{ 1 - \left[1 - \frac{\gamma_0 x^{\delta} Z}{m} \right]^{\nu+2} - (\nu+2) \frac{\gamma_0 x^{\delta} Z}{m} \left[1 - \frac{\gamma_0 x^{\delta} Z}{m} \right]^{\nu+1} \right\}$ is easy to calculate by integration.

[3] The project starts here with a land purchase. For simplicity, we assume that there is no possibility of land banking.

[4] This is based on the estimates of external cost of travel, 0.91 cents per person-mile and 4.01 cents per person-mile, given in DeLucchi (1998) and converted to 2000 dollars using the CPI-U.

[5] An example of a large congestion reduction from a corridor infrastructure investment is the privately owned toll lanes in the median of the pre-existing (and not tolled) of State Route 91 in California. Boarnet (1997) finds that the toll lanes decreased peak period commute times on the adjacent free lanes by up to 20 minutes. The commute pattern for the State Route 91 is from Riverside County into Los Angeles and Orange Counties; it often takes at least an hour (SCAG's State of the Commute Report, 1999), which suggests a 33 percent reduction in travel time.

[6] A mean-reverting process for X may better represent urban population dynamics but it would be more complex; it is also unlikely that it would qualitatively change our results on the joint impact of population volatility and barriers (as volatility increases, a mean reversion becomes more like a random walk), although mean reversion is likely to speed up population changes.