Network Investment and Competition with Access-to-Bypass*

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Abstract

This paper examines firms' incentives to make irreversible investments under an open access policy with stochastically growing demand. Using a simple model, we derive an access-to-bypass equilibrium. Analysis of the equilibrium confirms that the introduction of competition in network industries makes a firm's incentive to make investments greater than those of a monopolist. We then show that a change in access charges induces a trade-off in social welfare. That is, a decrease in the access charge expands the social benefit flow in the access duopoly, and deters not only the introduction of a new network facility, but also a positive network externality generated by the construction of an additional bypass network. The feasibility of socially optimal investment timing is then discussed.

Keywords: Open access policy; Investment; Real options; Network facility; Access charge

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1 Introduction

Since the early 1980s, competition has been introduced in public utility industries (such as telecommunications, natural gas and electricity) in OECD countries to increase efficiency and innovation. An important example of competition policy is the *open access policy*, which grants entrants that do not have a network facility access to an incumbent's network.[1] Nevertheless, these industries remain characterized by large sunk costs of investment, and increasing uncertainty in business environments. In addition, competition lowers the expected profit flow from investment, so that it tends to delay investment. Hence, the open access policy may reduce incentives for a *facility-based entry*. Given the potentially adverse effects on incentives, the open access policy has been reconsidered in some countries. For example, in 2003, the Federal Communications Commission (FCC) adopted new rules concerning the network unbundling obligations of incumbent local phone carriers, with the aim of providing incentives for carriers to invest in broadband.[2] That is, policy makers who have recommended the introduction of competition or open access policies are uncertain about how effective competition in public utility or network industries is in enhancing social welfare.

Note that an access charge in an open access policy is a crucial factor that affects both the profit of firms and social benefits in network industries. However, we should not ignore its effect on the timing of investment in infrastructure in these industries, especially when demand is expanding. In telecommunications, in addition to the traditional telephone, several kinds of communication devices, such as mobile telephones and Internet telephony, expand demand in the industry. For example, the annual growth rate of information services and telecommunications industries in Japan has been around 4% since 1997,[3] which suggests that more broadband networks are required. Similarly, demand for natural gas has been increased by environmental protection, which suggests greater demand for broader gas pipeline networks in the future. With growing demand, infrastructure

[1] See OECD (2001) for details.

[2] The new rules do not require unbundling of hybrid loops and fiber-to-the-home (FTTH) loops for both broadband and narrowband services. Michael K. Powell, the chairman of FCC, states: "Today's decision makes significant strides to promote investment in advanced architecture and fiber by removing impeding unbundling obligations."

[3] See InfoCom Research, Inc.(2003).

investment can be stimulated by access charges or other policy instruments in an open access environment.

This paper analyzes the competitive environment in public utility or network industries by focusing on the effects of access charges on firms' incentives to invest when there is stochastically growing demand. We employ *a real options approach* to examine issues related to investment because the industries are characterized by large sunk costs of investment and increasing demand (or cost) uncertainty. The real options approach features irreversibility of investment under uncertainty. It highlights the option value of delaying an investment decision. In fact, a decision on the timing of irreversible investment under uncertainty is crucial for firms in public utility or network industries.

In particular, the real options approach is useful when a player has a sequential opportunity of investment timing. As is well-known, public utility industries comprise a production facility and a network facility. (For example, in the electricity industry, a plant for generating electricity is the production facility, whereas transmission and local distribution wires are network facilities.) In the industries, then, an entrant or follower has a sequential opportunity of investment timing; that of a construction of a *bypass* or another network facility. This is because an important characteristic of network industries is approval for a common use of network facilities. Since network (or essential) facilities are characterized by large sunk costs, their common use is recommended from a social point of view, as long as congestion problems do not occur. An entrant's decision to construct a bypass may be controversial with respect to improving welfare. In that case, the real options approach is suitable for examining the properties of an entrant's sequential investment decision (i.e., *from access to bypass*) because the application of a simple net present value (NPV) approach cannot provide adequate understanding of an entrant's incentives to construct a bypass when there is uncertainty and investment is irreversible. With an NPV approach, one would characterize the entrant's decision about whether (or when) to construct a bypass by comparing the net present value of profit under access with that under use of the bypass. However, such an approach would be inappropriate because it ignores the option value of delaying additional investment in the bypass. This is the main reason for adopting the real options approach to examine the incentives for investment in network industries. To sum up, this approach is useful for studying network industries and, in particular, for studying the effect of regulatory policies on the performance of these industries.

Using a simple model of an option-exercise game, we examine two specific issues. The first issue is the effect of open access policy on the timing of investment or entry, while the second is to confirm its welfare implication. We then ask: can an appropriate level of access charge achieve the socially optimal investment timing? To examine these two issues, we first derive an *access-to-bypass equilibrium* by allowing an entrant to access an incumbent's network facility. In particular, we characterize the entrant's sequential investment timing for the construction of an additional network facility, having accessed the incumbent's network, in terms of an access charge and the level of network investment cost. Analysis of the equilibrium confirms that the introduction of competition in network industries makes a firm's incentive to invest greater than that of a monopolist. That is, in an access-to-bypass equilibrium, an open access policy leads a firm to enter earlier than if there were no competition. This implies that an open access policy provides a strong pre-emptive incentive to a firm, irrespective of the level of the access charge, as long as the access-to-bypass equilibrium holds. We then show that a change in the access charge induces a trade-off in social welfare. That is, a decrease in the access charge expands social benefit flow in the access duopoly, and deters not only the introduction of a new network facility, but also a positive network externality generated by the construction of an additional bypass network. This trade-off occurs even when there is a usage access charge, since the trade-off is due to its effect on profit flows in the access duopoly. Then, we examine the feasibility of socially optimal investment timing. We show that the use of lump-sum subsidies or taxes in conjunction with a usage access charge not only leads to the achievement of the desired social benefit flow in the access oligopoly, but it also induces socially optimal investment timing for infrastructure construction. These policies have recently been pursued by governments in Japan and Korea, in the form, for example, of direct funding and tax exemptions for the construction of telecommunications broadband infrastructure.

Many studies have addressed the access pricing problem in public utility industries in static economic environments. (See Armstrong (2002) for an elegant survey.) To the best of our knowledge, only a few papers have examined the effect of access pricing on the incentive to invest in network facilities. Examples are Sidak and Spulber (1997), Gans and Williams (1999) and Gans (2001), who considered incentives to invest in infrastructure when there is no uncertainty. The effect of uncertainty on irreversible investment has been formally examined by Biglaiser and Riordan (2000). However, they neither analyzed a game between an incumbent and an entrant nor allowed an entrant to construct a bypass. Our study is the first to analyze the investment game in public utility or network industries by focusing on an entrant's decision to make an additional investment in bypass construction when there is stochastically growing demand.

In the next section, we describe the framework of a real options model for an imperfectly competitive network industry. In section 3, we derive the access-to-bypass equilibrium in which the entrant first adopts an access strategy and then converts to a bypass strategy. In section 4, we examine the properties of the equilibrium and achievement of the social optimum. Section 5 concludes the paper.

2 The Model

There are two risk-neutral firms, $i = 1, 2$, which plan to enter a network industry, such as electricity, telecommunications or natural gas. The network industry needs two types of facility to serve their customers: a production facility and a network facility. Each firm has the opportunity to invest in both types of facility, and the investment decisions in each type are assumed to be irreversible. The investment cost for the production facility is $I^e > 0$, and that for the network facility is $I^m > 0$. Both I^e and I^m are sunk costs.

Investments in the two types of facility may be undertaken simultaneously or sequentially. A firm constructs the production facility at cost I^e, and at the same time or in the next stage, the network facility is built at an additional cost of I^m. However, not all firms need to invest in the network facility, provided that at least one firm maintains the facility. That is, the firm without a network facility may utilize the existing network facility to distribute products. The firm that initially enters the market with both production and network facilities is a *leader*, whereas the other firm, which may or may not have a network facility, is a *follower*. We assume that the follower can access the existing network facility through a usage access charge, $v > 0$, which is given for each firm and determined by a policy maker.[4] When the follower uses the leader's existing network facility, the leader incurs an access (or usage) cost for the network facility, c, which is the same as the cost of its own production. Moreover, the follower, having access to the leader's network facility, may invest in the construction of its own network facility in the future. For simplicity, production costs other than access costs are assumed to be zero.

We assume that the two firms compete in a market for a homogeneous good produced in the network industry. The profit flows of the firms are uncertain because the firms face an aggregate exogenous industry shock. The profit flow of a firm is represented by $\pi = Y\Pi(N)$, where Y is the aggregate exogenous shock, $N = 0, 1, 2$ is the number of active firms and $\Pi(N)$ is interpreted as the non-stochastic part of the firm's profit flow at the industry equilibrium.

Y evolves exogenously and stochastically according to a geometric Brownian motion, with drift given by the following expression:

$$dY_t = \alpha Y_t dt + \sigma Y_t dW,$$

where $\alpha \in (0, r)$ is the drift parameter measuring the expected growth rate of Y, r is the risk-free interest rate, $\sigma > 0$ is a volatility parameter and dW is the increment of a standard Wiener process, where $dW \sim N(0, dt)$. Note that $\alpha > 0$ implies that the firm's profit flow $\pi = Y\Pi(N)$ is enhanced stochastically.

A firm's profit flow in the monopolistic equilibrium is represented by $Y\Pi(1)$. When the two firms are active in the market, we must distinguish between two duopolistic market structures: 'access duopoly', in which the follower has access to the leader's network facility; and 'bypass duopoly', in which the follower maintains its own network facility. Let $Y\Pi^L(2; v)$ represent the profit flow of the leader in the access duopoly equilibrium,

[4] Regulated access pricing in some public utility industries is based on a two-part pricing formula (i.e., a usage charge and a fixed charge). For analytical simplicity, we analyze only a usage charge in this paper. See Hori and Mizuno (2003) for an analysis of a lump-sum or fixed access charge.

and let $Y\Pi^F(2;v)$ represent the profit flow of the follower. Similarly, $Y\Pi(2)$, which is the same for the leader and follower, represents the profit flow in the bypass duopoly equilibrium.

The following relationship is assumed to hold.

Assumption 1 (i) $\Pi(1) > \Pi(2)$ and $\Pi(1) > \Pi^i(2;v)$ $(i = L, F)$, (ii) $\Pi(2) \geq \Pi^F(2;v)$, (ii) $\Pi^L(2;v) \underset{(<)}{\geq} \Pi^F(2;v)$ if $v \underset{(<)}{\geq} c$, (iii) $\frac{\partial \Pi^L(2;v)}{\partial v} > 0$, (iv) $\frac{\partial \Pi^F(2;v)}{\partial v} < 0$.

In (i), it is reasonable to assume that the equilibrium profit $\Pi(N)$ is a decreasing function of N. The assumption of (ii) is based on the notion that additional supply of the network facility improves the quality of goods or generates a positive network externality. For example, a decrease in the probability of blackout may be generated by the construction of another transmission wire in a local electricity market, or the capability to provide high calorie gas may be due to the construction of additional gas pipelines in a gas market. In telecommunications, the construction of another broadband cable can benefit the population of Internet users, which in turn increases the firms' profits. Note that we do not exclude the case in which $\Pi^L(2;v)$ is greater than $\Pi(2)$. This occurs when the access charge v is so high that it generates more profit for a leader than is generated by a positive network externality. As shown below, there exists a unique equilibrium (on which we focus) not only in that case, but also when the leader's profit in the access duopoly is less than that in the bypass duopoly.

We validate Assumption 1 by using a numerical example. Suppose the inverse demand function is linear and represented by $p = a - bQ$ $(a,\, b > 0)$. Suppose also that the inverse demand function is converted to $p = (a + \theta) - bQ$ when the market is a bypass duopoly, where the parameter $\theta\,(> 0)$ represents a positive network externality. In a monopoly, a firm chooses Q to maximize $(p - c)\,Q$, so we have $\Pi(1) = \frac{(a-c)^2}{4b}$. Similarly, a firm's profit in a bypass duopoly under Cournot competition is given by $\Pi(2) = \frac{(a+\theta-c)^2}{9b}$. In an access duopoly, the leader's profit is given by $(p - c)\,q^L + (v - c)\,q^F$, while the follower's profit is $(p - v)\,q^F$. Under Cournot competition, equilibrium profits in the access duopoly are given by $\Pi^L(2;v) = \frac{1}{9b}\left[(a + v - 2c)^2 + 3(v - c)(a - 2v + c)\right]$ and $\Pi^F(2;v) = \frac{1}{9b}(a - 2v + c)^2$. Hence, the condition, $\Pi(2) \geq \Pi^F(2;v)$, requires $v \geq c - \frac{\theta}{2}$, which means that there exists a positive network externality that causes the *virtual* unit cost in the bypass duopoly to be lower than that in the access duopoly by a factor of $\frac{\theta}{2}$.

We restrict our attention to Markov strategies for a firm's decision about when to enter (or when to invest): each firm's decision depends noly on the state variable Y. As an equilibrium concept fro the game, we use a subgame perfect equilibrium.

The follower has three possible strategies. First, the follower may want access to the network facility constructed by the leader forever to save on investment costs, I^m. Second, the follower may want to construct its own network facility to save on an access payment. Another possibility is that the follower initially has access to the leader's network facility, but then decides to construct its own network facility.[5] We refer to these three alternatives as the '*access strategy*', the '*bypass strategy*' and the '*access-to-bypass strategy*', respectively. The preference of the follower may depend on the conditions relating to the level of investment costs, the equilibrium profit under product market competition, the level of the access charge, and so on. In the next section, we examine the follower's choice of strategy before deriving the equilibrium of the game.

3 The Access-to-Bypass Equilibrium

We derive the equilibria of the game described in section 2.

3.1 The follower's choice of strategy

First, we must consider the follower's strategy choice when the follower is allowed to not only have access to the leader's network facility, but also to construct its own network facility. As mentioned in the previous section, in this environment, the follower has three alternative strategies: the access strategy, the bypass strategy and the access-to-bypass strategy. We ask the question, under what conditions does the follower choose one strategy over the others? We note that the follower's choice can only be appropriately determined by the real options approach under irreversible investment and uncertainty. The stan-

[5] Yet another possibility is that the follower first constructs its own network plant and then uses the leader's plant by paying an access charge. However, we can ignore this possibility because network investment and the access payment are irreversible.

dard net present value approach ignores the option value of waiting generated by the irreversibility of the two types of investment that the follower can make.

To answer the question, we first derive the value of each project before obtaining the values of the three strategies.

When the access project is undertaken, its value is:

$$V^A(Y) = \frac{Y\Pi^F(2;v)}{r-\alpha}. \tag{1}$$

When the bypass project is undertaken, the value of the project is:

$$V^B(Y) = \frac{Y\Pi(2)}{r-\alpha}. \tag{2}$$

Using (1) and (2), we can define the value of *the transition project*, $\Delta V(Y)$, which is the difference between the values of the bypass project and the access project:

$$\Delta V(Y) \equiv V^B(Y) - V^A(Y) = \frac{Y\Delta\Pi(2;v)}{r-\alpha}, \tag{3}$$

where $\Delta\Pi(2;v) \equiv \Pi(2) - \Pi^F(2;v)$ is referred to as the *incremental profit flow from access to bypass.*

Suppose that the bypass project is undertaken. Then, there must be a trigger point Y^{B*}, at which the bypass project begins. Defining the option value of the transition project and using the value-matching and smooth-pasting conditions, we derive the trigger point Y^{B*}:[6]

$$Y^{B*} = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Delta\Pi(2;v)}I^m. \tag{4}$$

Next, we derive the trigger point Y^{A*} at which the access project begins. Note that, when the bypass project is allowed, the *effective* value of the access project includes not only its own project value, but also the option value of the transition project. Hence, defining the option value of the access project and using the value-matching and smooth-

[6] The derivation follows a standard technique in the real options literature. See Dixit and Pindyck (1994).

pasting conditions, we determine the trigger point Y^{A*}.[7]

$$Y^{A*} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{\Pi^F (2; v)} I^e. \tag{5}$$

From (4) and (5), an increase in uncertainty deters not only the follower's entry by access, but also its construction of a bypass facility. That is, $\partial Y^{A*}/\partial\sigma > 0$ and $\partial Y^{B*}/\partial\sigma > 0$, since β_1 is a decreasing function of the volatility parameter σ.[8]

The derivation clarifies which strategy is adopted by the follower. When $Y^{B*} < +\infty$ and $(0 <) Y^{A*} \leq Y^{B*}$, the follower adopts the access-to-bypass strategy. When $Y^{B*} = +\infty$ and $Y^{A*} (> 0)$, the follower adopts the access strategy. When $Y^{B*} < Y^{A*}$, the follower adopts the bypass strategy. The following lemma states the conditions under which each strategy is adopted by the follower.

Lemma 1 *Under* $\Delta\Pi (2; v) > 0$ of *Assumption 1(i), the follower adopts the access-to-bypass strategy (the bypass strategy) if and only if:*

$$\Pi (2) \leq (>) \left(1 + \frac{I^m}{I^e}\right) \Pi^F (2; v) . \tag{6}$$

Proof. See Appendix. ■

When an incremental profit flow from access to bypass is positive, i.e., when $\Delta\Pi (2; v) > 0$, the access strategy is not adopted by the follower. This is because the aggregate shock Y evolves according to a geometric Brownian motion such that it has the expected growth rate αThe condition (6) defines a hyper-plane that separates the access-to-bypass strategy and the bypass strategy.

[Insert Figure 1]

[7] We can ensure that the trigger point Y^{A*} is the same when the bypass construction is not allowed, which means that the option value of the transition project does not affect the trigger point for the access project Y^{A*}. This is because the option to enter the market by access includes the option value of the transition project. In fact, the option value of the transition project is canceled out in the process of deriving Y^{A*}:

[8] See pp.143-144 of Dixit and Pindyck (1994) for the effect of σ on β_1.

Figure 1 illustrates the follower's choice of strategy in terms of the access charge v and the investment cost for the network facility I^m. In Figure 1, the follower adopts the access-to-bypass strategy in the region above the hyper-plane $I^m = \Psi(v)$, which is an explicit representation of the function $\Pi(2) = \left(1 + \frac{I^m}{I^e}\right) \Pi^F(2; v)$.[9] Otherwise, the follower adopts the bypass strategy. The division of the regions is intuitive. When the investment cost for the network facility is higher than the access payment, the follower initially accesses the incumbent's network and then constructs its own network facility.

3.2 The equilibrium

In this subsection, we focus on the case in which the follower adopts the access-to-bypass strategy. We do so because this is a general case in the sense that it includes two actions of the follower and shows some peculiar characteristics of network industries.

When the follower chooses the access-to-bypass strategy, the value function is as follows.

$$V_F^{AB}(Y) = \begin{cases} \left(\frac{Y}{Y^{A*}}\right)^{\beta_1} \left\{ \frac{Y^{A*}\Pi^F(2;v)}{r-\alpha} - I^e + \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1} \left[\frac{Y^{B*}\Delta\Pi(2;v)}{r-\alpha} - I^m\right] \right\} & if \quad Y < Y^{A*} \\ \frac{Y\Pi^F(2;v)}{r-\alpha} - I^e + \left(\frac{Y}{Y^{B*}}\right)^{\beta_1} \left[\frac{Y^{B*}\Delta\Pi(2;v)}{r-\alpha} - I^m\right] & if \quad Y^{A*} \leq Y < Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha} - (I^e + I^m) & if \quad Y^{B*} \leq Y \end{cases} \tag{7}$$

The trigger points $Y^{A*} > 0$ and $Y^{B*} > 0$ are (5) and (4), respectively.

Next, we consider the leader's value function when the follower adopts the access-to-

[9] It is easy to verify that $\Psi'(v) > 0$. A sufficient condition for $\Psi"(v) > 0$ is $\frac{\partial^2 \Pi^F(2;v)}{\partial v^2} \leq 0$.

bypass strategy. In that case, the value function of the leader is derived as follows.

$$V_L^{AB}(Y) = \begin{cases} \frac{Y\Pi(1)}{r-\alpha}\left[1-\left(\frac{Y}{Y^{A*}}\right)^{\beta_1-1}\right]+\left(\frac{Y}{Y^{A*}}\right)^{\beta_1}\left\{\frac{Y^{A*}\Pi^L(2;v)}{r-\alpha}\left[1-\left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1-1}\right]\right. \\ \quad \left.+\left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1}\frac{Y^{B*}\Pi(2)}{r-\alpha}\right\}-(I^e+I^m) \qquad if \;\; Y<Y^{A*} \\ \frac{Y\Pi^L(2;v)}{r-\alpha}\left[1-\left(\frac{Y}{Y^{B*}}\right)^{\beta_1-1}\right]+\left(\frac{Y}{Y^{B*}}\right)^{\beta_1}\frac{Y^{B*}\Pi(2)}{r-\alpha}-(I^e+I^m) \\ \qquad if \;\; Y^{A*}\leq Y<Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha}-(I^e+I^m) \qquad if \;\; Y^{B*}\leq Y \end{cases} \tag{8}$$

[Insert Figure 2]

Let us focus on the asymmetric leader-follower equilibrium, which we refer to as the '*access-to-bypass equilibrium*'. To guarantee the existence and uniqueness of the equilibrium, it is sufficient to ensure that $V_L^{AB}\left(Y^{A*}\right) > V_F^{AB}\left(Y^{A*}\right)$ and that the difference between the leader's value and the follower's value decreases monotonically. The sufficient condition for the existence and uniqueness of the access-to-bypass equilibrium are derived in the proof of Proposition 1 in the Appendix. Although the condition is complicated, sets of numerical values that satisfy the condition are easily found. Figure 2 shows the region in which the sufficient condition holds when $\beta_1 = 2$. The horizontal axis represents $x \equiv \frac{\Pi^L(2;v)-\Pi^F(2;v)}{\Pi(2)-\Pi^F(2;v)}$, and the vertical axis represents $y \equiv \left[\frac{\Pi^L(2;v)}{\Pi^F(2;v)}-1\right]\left(\frac{I^m}{I^e}\right)$. When $\beta_1 = 2$, the sufficient condition holds in the shaded region defined by $\left\{(x,y) \mid y \leq x \text{ and } y \geq \frac{x^2}{2x-1}\right\}$. For example, when $\frac{1}{2}\left[\frac{\Pi^L(2;v)}{\Pi^F(2;v)}-1\right] = \frac{I^m}{I^e}$ and $\Pi^L(2;v) - 2\Pi^F(2;v) = 3\Pi(2)$, the access-to-bypass equilibrium is unique. This set of numerical examples indicates that, when the investment cost for the network facility is small relative to the cost for the production facility and when the access charge is sufficiently high that it offsets the benefit generated by a positive network externality for the leader, there exists a unique equilibrium in which the follower enters the market by access and then builds its own bypass facility in the future. We summarize this result in the following proposition.

Proposition 1 *There exists a unique access-to-bypass equilibrium in which the leader's* *trigger point Y_L^{AB} is characterized by*

$$\begin{array}{rclll} V_L^{AB}(Y) & < & V_F^{AB}(Y) & if & Y < Y_L^* \\ V_L^{AB}(Y) & = & V_F^{AB}(Y) & if & Y = Y_L^* \\ V_L^{AB}(Y) & > & V_F^{AB}(Y) & if & Y \in \left(Y_L^*, Y^{B*}\right) \\ V_L^{AB}(Y) & = & V_F^{AB}(Y) & if & Y \geq Y^{B*}, \end{array}$$

under the condition that

$$\begin{aligned} I^m - \frac{\beta_1}{\beta_1 - 1}\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)}I^e &< (\beta_1 - 1)\left(\frac{\Delta\Pi(2;v)}{\Pi^F(2;v)}\frac{I^e}{I^m}\right)^{\beta_1}\left[1 - \frac{\beta_1}{\beta_1 - 1}\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)}\right]I^m \\ &< -\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)}\frac{I^e}{I^m}. \end{aligned} \tag{9}$$

where $\Pi^{LF}(2;v) \equiv \Pi^L(2;v) - \Pi^F(2;v)$.

Proof. See Appendix. ■

[Insert Figure 3]

Figure 3 shows the access-to-bypass equilibrium. For $Y \in [0, Y_L^*)$ where Y_L^* is the trigger point at which the leader enters the market, the two firms do not enter the market. For $Y \in \left[Y_L^*, Y^{A*}\right)$, the leader earns monopoly profits. For $Y \in \left[Y^{A*}, Y^{B*}\right)$, the follower has access to the leader's network facility. For $Y \in \left[Y^{B*}, +\infty\right)$, the follower constructs its own network facility.

4 Properties of the Equilibrium

4.1 The effect of competition

Using a real options approach, and comparing the optimal strategy of a monopolist with the optimal strategy of a leader in duopoly, Nielsen (2002) showed that, even with irreversible investment and uncertainty, competition induces firms to invest earlier. We can extend this result to the access-to-bypass equilibrium derived in the previous section.

As a benchmark, we present the investment trigger point of a monopolist, which is given by

$$Y^* = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{\Pi(1)} (I^e + I^m). \tag{10}$$

We then derive the following proposition.

Proposition 2 *The leader in an access-to-bypass equilibrium enters the market earlier than a monopolist.*

Proof. See appendix. ■

To prove Proposition 2, we show in the Appendix that $V_L^{AB}(Y^*) > V_F^{AB}(Y^*)$ for $Y \in (0, Y^{A*})$. The condition that $V_L^{AB}(Y^{A*}) > V_F^{AB}(Y^{A*})$, which guarantees the existence and uniqueness of the access-to-bypass equilibrium, plays a crucial role in the proof of the proposition. For the condition to be satisfied, as we explained before stating Proposition 1, the access charge v must exceed the access cost c, or the investment cost for the network facility I^m must be small. Then, the leader's value is higher than that of the follower at $Y \in (Y_L^*, Y^{B*})$, which means that both firms have strong pre-emptive incentives under competition.

The meaning of this result warrants a detailed explanation. Note that the introduction of competition reduces a firm's profit flow, i.e., from $\Pi(1)$ to $\Pi^i(2; v)$ $(i = L, F)$, which lowers a firm's incentive to enter. In a real options approach, the effect of the decrease in profit flow on the timing of entry is more severe than in an NPV approach. This is because the option value of waiting is due to irreversible investment and uncertainty, which should be added to the net present value of profit.[10] However, each firm has a strategic motive to extract a monopoly rent, i.e., it has a pre-emptive incentive. Hence, the result implies that the pre-emptive incentive for being a leader dominates the effect of a decrease in the profit flow, even if the option value of waiting is realized. This *pre-emptive-incentive-domination effect* was also found by Nielsen (2002). We extend his result to an open access competitive environment.[11]

[10] This point is made by the formula for the trigger point (e.g., Y^*) by the multiplication of $\frac{\beta_1}{\beta_1 - 1}$ (> 1).

[11] Grenadier (2002) also emphasized the impact of competition on an exercise strategy of investment in an N-player Cournot-Nash competition.

Furthermore, the follower in the access-to-bypass equilibrium might also enter the market earlier than a monopolist.

Corollary 1 *When* $\Pi(1) < \left(1 + \frac{I^m}{I^e}\right) \Pi^F(2; v)$*, the follower in the access-to-bypass equilibrium enters the market earlier than a monopolist.*

Proof. Comparing Y^{A*} with Y^* proves this point. ■

Note that, even when the follower's profit flow under access duopoly is less than that of a monopolist (i.e., $\Pi^F(2; v) < \Pi(1)$) as stated in Assumption 1(i), the follower may enter the market earlier than a monopolist. This is because the follower is allowed to access the leader's network facility by paying an access charge v. In fact, this can be the case when the investment cost for a network facility is small relative to the cost of a production facility, and when the follower's profit is not too small. For example, for $\beta_1 = 2$, under a set of $\left\{I^m = 0.5I^e,\ \Pi^F(2; v) = 0.7\Pi(1),\ \Pi(2) = \frac{2.8}{3}\Pi(1)\right\}$, which guarantees the existence and uniqueness of the equilibrium, the follower enters the market earlier than a monopolist.

4.2 The effect of the access charge

In the previous subsection, we showed that the introduction of competition makes the leader enter earlier than a monopolist even when there is irreversible investment under uncertainty. However, the access charge also affects the firm's incentive to enter an open-access competitive environment. We examine the effect of the access charge on the trigger points in the access-to-bypass equilibrium.

Proposition 3 *(i)* $\partial Y_L^* / \partial v < 0$*, (ii)* $\partial Y^{A*} / \partial v > 0$*, (iii)* $\partial Y^{B*} / \partial v < 0$*.*

Proof. See Appendix. ■

From Proposition 3, a unit access charge can affect the investment timing of firms. In particular, a decrease in the unit access charge can induce the follower to enter the market early through access and construct its bypass facility late, and induces the leader to enter late. That is, in the access-to-bypass equilibrium, a change in the access charge has a positive effect on the follower's entry with access, but has a negative effect on the leader's entry and the introduction of bypass construction.

The result and its welfare implications are intuitive. When the access charge decreases, consumers cannot be served early through the construction of a new network facility by a leader (such as a new broadband cable in a rural area) and neither can they enjoy positive network externalities early. However, they can enjoy a longer access duopoly equilibrium in which social welfare is higher than in a monopoly equilibrium. In addition, a decrease in the unit access charge increases the social benefit flow *itself* in the access duopoly equilibrium. Therefore, there is a trade-off in the policy of changing the access charge, which gives rise to a dilemma for a policy maker.

4.3 The feasibility of optimal investment timing

In this subsection, we compare investment timing in the access-to-bypass equilibrium with socially optimal investment timing.

To examine this issue, we need to define the social optimum in our model. Bearing in mind the open access policy, we ignore the case in which a regulator can control the retail price. We represent the consumer surplus flow (excluding the random term) by $S(.)$ and the social benefit flow (excluding the random term) by $SB(.)$. Recall that the unit access charge v affects not only firms' investment timing, but also the social benefit flow under the access duopoly in the access-to-bypass equilibrium. In addition, several types of market structure sequences may be socially optimal in an expanding economy. This depends on the level of social benefit flow in each market equilibrium, the parameters of the geometric Brownian motion and levels of investment costs.

However, in an expanding economy, it is reasonable and useful to focus on the case in which the social optimum is the environment that has the same sequence of market structures (i.e., the monopoly, the access duopoly and the bypass duopoly) as has the access-to-bypass equilibrium. This is justified by the following argument. When demand is low and a firm's profit flow is small, the (natural) monopoly may be desirable since the duplication of sunk investment for both production and network facilities would be wasteful, even if the social benefit flow in a monopoly equilibrium is small. When demand increases, the duopoly is desirable. Then, which type of duopoly is desirable depends on the magnitude of the positive network externality generated by construction of the bypass.

Suppose the social benefit flow induced by the positive network externality from the bypass construction exceeds that under an access duopoly in which the unit access charge is set so low that it induces the socially optimal retail price (as long as the firm's feasibility (or non-negative profit) condition is met). Then, it is desirable that the access duopoly is followed by the bypass duopoly from a welfare point of view in an expanding economy. Hence, we limit our attention to the case in which the social optimum in an open-access competitive environment is the one that has the same sequence of market structures as has the access-to-bypass strategy equilibrium. The following assumption guarantees the existence of the social optimum.

Assumption 2 (i) $SB(1) < SB_v^{**}(2) < SB(2)$, (ii) $\frac{\Delta SB(2)}{SB_v^{**}(2) - SB(1)} < \frac{I^m}{I^e} < \frac{2SB(1) - SB_v^{**}(2)}{SB_v^{**}(2) - SB(1)}$,

where $SB(1) \equiv S(1) + \Pi(1)$, $SB_v^{**}(2) \equiv S_v^{**}(2) + \Pi_v^{L**}(2) + \Pi_v^{F**}(2)$, $SB(2) \equiv S(2) + 2\Pi(2)$ and $\Delta SB(2) \equiv SB(2) - SB_v^{**}(2)$. Here, $SB_v^{**}(2)$ is defined as the maximized social benefit flow achieved by a regulator who controls access pricing under the access duopoly, i.e., the social benefit flow in the access duopoly in which the unit access charge is so low that it induces the socially optimal retail price as long as the firm's feasibility condition is met.[12] ($S_v^{**}(2)$, $\Pi_v^{L**}(2)$ and $\Pi_v^{F**}(2)$ are the associated consumer surplus and the firms' profit flows, respectively.) Assumption 2(i) states that the social benefit flow in the monopoly market is less than that in the access duopoly, which in turn is less than that in the bypass duopoly. Assumption 2(ii) guarantees the existence and uniqueness of the social optimum in which the same sequence of market structures as those in the access-to-bypass equilibrium prevails according to the development of Y. In particular, this assumption guarantees that the trigger point of the monopoly project is below that of the access project, which in turn is below that of the bypass project.

Let us derive the socially optimal investment timing for the construction of each facility. The procedure used is the same as that used to derive the firm's optimal investment timing except that the social benefit flow $SB(.)$ is used instead of the profit flow. Hence, we report only the value of the social benefit at the social optimum, which is given by:

[12] See Armstrong and Vickers (1998) and Lewis and Sappington (1999) for an optimal access charge with an unregulated retail price.

$$V^{**}(Y)=\begin{cases} H_1 Y^{\beta_1} & if \quad Y<Y_L^{**} \\ \frac{YSB(1)}{r-\alpha}+GY^{\beta_1}-(I^e+I^m) & if \quad Y_L^{**}\leq Y<Y^{A**} \\ \frac{YSB_v^{**}(2)}{r-\alpha}+F_1Y^{\beta_1}-(2I^e+I^m) & if \quad Y^{A**}\leq Y<Y^{B**} \\ \frac{YSB(2)}{r-\alpha}-2(I^e+I^m) & if \quad Y^{B**}\leq Y \end{cases} \tag{11}$$

where $F_1=\frac{\Delta SB(2)\left(Y^{B**}\right)^{1-\beta_1}}{(r-\alpha)\beta_1}$, $G_1=\frac{(SB_v^{**}(2)-SB(1))\left(Y^{A**}\right)^{1-\beta_1}+\Delta SB(2)\left(Y^{B**}\right)^{1-\beta_1}}{(r-\alpha)\beta_1}$, and $H_1=\frac{SB(1)\left(Y^{L**}\right)^{1-\beta_1}+(SB_v^{**}(2)-SB(1))\left(Y^{A**}\right)^{1-\beta_1}+2\Delta SB(2)\left(Y^{B**}\right)^{1-\beta_1}}{(r-\alpha)\beta_1}$.

The trigger points, representing socially optimal investment timing, are given by:

$$Y_L^{**} = \frac{\beta_1}{\beta_1-1}\frac{r-\alpha}{SB(1)}(I^e+I^m) \tag{12}$$

$$Y^{A**} = \frac{\beta_1}{\beta_1-1}\frac{r-\alpha}{SB_v^{**}(2)-SB(1)}I^e \tag{13}$$

$$Y^{B**} = \frac{\beta_1}{\beta_1-1}\frac{r-\alpha}{\Delta SB(2)}I^m \tag{14}$$

Comparing the trigger points in the access-to-bypass equilibrium with those in the social optimum yields the welfare implications of the equilibrium investment timing. First, $Y^{B**}<Y^{B*}$ as long as $\Delta SB(2)>\Delta\Pi(2;v)$. This is because the follower is not concerned about a change in the consumer surplus or the leader's profit flow. If a positive network externality induces a higher increase in social benefit under bypass than does an increase in follower's profit flow, the timing of the social bypass is earlier than that of the equilibrium.

Similarly, we can compare Y^{A*} with Y^{A**}, or Y_L^* with Y_L^{**}. However, since the results depend not only on the level of the access charge, but also on the parameters representing the economic environment (such as demand and costs), the derivation of interesting welfare implications is too complex. Hence, rather than compare the timing of investment, we examine the feasibility of the socially optimal investment timing in the access-to-bypass equilibrium.

By comparing with the trigger points in the access-to-bypass equilibrium, it is easy to verify that only the usage access charge v as a policy variable cannot generically induce socially optimal investment timing, Y_L^{**}, Y^{A**}, and Y^{B**}. This result contrasts with that of Gans (2001). In his paper, a two-part (i.e., lump-sum plus usage) access charge can achieve the optimal timing of infrastructure investment in the absence of uncertainty, as long as the firm's feasibility requirement is not violated. Optimal investment timing cannot be determined in our model, even if the two-part structure of access pricing is assumed, for the following reasons. First, we assume growing demand in addition to the positive network effect generated by construction of the bypass, whereas Gans (2001) assumed a stationary economic environment, which explains why one of the two regimes (access or bypass) is socially preferred under a duopoly in his model. Second, since in his paper the follower is not required to invest in a production facility to enter a market through access, the follower must gain access as soon as possible from a welfare point of view. This second point is crucial in deriving the social optimum by controlling the access charge in his paper. By contrast, in our model, the follower must undertake irreversible investment in its own production facility. Furthermore, the trigger point for construction of the bypass is also affected by the access charge. Therefore, only the usage access charge v as a policy variable cannot generically induce socially optimal investment timing, Y_L^{**}, Y^{A**}, and Y^{B**}.

However, as stated in the following proposition, socially optimal investment timing can be achieved if lump-sum subsidies (and taxes), as well as the usage access charge, are introduced.

Proposition 4 *The social optimum can be achieved by the following regulatory policy.*

For the follower: (i) the usage access charge v^{**} *is such that* $SB\,(2; v^{**}) = SB_v^{**}\,(2)$ $\left(\equiv S\,(2; v^{**}) + \Pi^L\,(2; v^{**}) + \Pi^F\,(2; v^{**})\right)$*, and the lump-sum subsidy* T^{A**} *in the access duopoly, and the lump-sum subsidy* T^{B**} *in the bypass duopoly are such that*[13]

$T^{A**} = \left[SB_v^{**}\,(2) - \Pi_v^{F**}\,(2)\right] - SB\,(1),$

$T^{B**} = \left[SB\,(2) - \Pi\,(2)\right] - SB\,(1).$

For the leader: (i) the lump-sum subsidy T^{B**} *in the bypass duopoly, and the lump-sum tax* T_L^{**} *in the monopoly are such that:*[14]

[13]Note that T^{A**} or T^{B**} can be negative, in which case, a lump-sum tax is applied.
[14]The remark in footnote 13 applies here.

$$
\begin{aligned}
T_L^{**} \;=\;& \frac{Y_L^{**}\Pi(1)}{r-\alpha}\left[1-\left(\frac{Y_L^{**}}{Y^{A**}}\right)^{\beta_1-1}\right] \\
&+\left(\frac{Y_L^{**}}{Y^{A**}}\right)^{\beta_1}\left\{\frac{Y^{A**}\left[\Pi_v^{L**}(2)-\Pi_v^{F**}(2)\right]}{r-\alpha}\right. \\
&-\frac{Y^{A**}\left[\Pi_v^{L**}(2)+T^{A**}\right]}{r-\alpha}\left(\frac{Y^{A**}}{Y^{B**}}\right)^{\beta_1-1}+I^e \\
&\left.+\left(\frac{Y^{A**}}{Y^{B**}}\right)^{\beta_1}\left[\frac{Y^{B**}\Pi_v^{F**}(2)}{r-\alpha}+I^m\right]\right\}-\left(I^e+I^m\right) \qquad (15)
\end{aligned}
$$

Proof. See Appendix. ■

The premise for achievement of the social optimum is as follows. The optimal social benefit flow under the access duopoly can be manipulated only by the usage access charge, which means that v^{**} has a role in achieving the optimal social benefit flow under the access duopoly. Hence, setting v^{**} disrupts a firm's investment timing. To adjust the timing, disrupted by v^{**}, to the socially optimal timing requires lump-sum subsidies and taxes.

Note that, while the role of the usage access charge v^{**} is familiar, the role of lump-sum subsidies or taxes differs from its role in the literature that focuses on static analysis. Lump-sum subsidies and taxes are usually used to meet firms' feasibility requirements. In our model, they are needed to correct investment timing, which is affected by the usage access charge v^{**}. In particular, the magnitudes of T^{A**} and T^{B**} are intuitive. They represent shortages of social benefits that are ignored by the follower when the market structure changes from monopoly to duopoly. Note that both T^{A**} and T^{B**} are expressed in terms of flow variables, whereas T_L^{**} is expressed as a stock variable. Note also that T^{A**}, T^{B**} and T_L^{**} can be negative, and so policy makers need detailed information on the environment to determine them.

In practice, similar policies that perform the role of these subsidies and tax benefits are pursued. In telecommunications in Japan, under the *e-Japan* plan, Japanese governments have provided funds of up to 50% (25% from central government and 25% from local government) for infrastructure cost (cable, equipment and installation) to NTT/cable TV operators using FTTH or Hybrid networks. Similarly, in South Korea, the Korean

Information Infrastructure (KII) strategy included the construction of new high capacity backbone infrastructure with more than US$1.5 billion of direct government funding.[15] These policies should be accompanied by an appropriate usage access charge that induces the optimal social benefit flow in access duopoly.

It should be noted also that, instead of the lump-sum subsidy T^{A**}, we could introduce a lump-sum access charge, in the form of a two-part tariff. However, the introduction of a lump-sum access charge generates a suspension value in a stochastic environment, which complicates the analysis. (See Hori and Mizuno (2003) for an analysis of lump-sum access charges.) To avoid unnecessary complexity, we introduced T^{A**}. If we had used the two-part tariff structure for access pricing, an adjustment for suspension value would have been required to the subsidy T^{B**} and the tax T_L^{**}.

5 Concluding Remarks

In this paper, we have investigated the effects of access charges on firms' incentives to invest in network public-utility industries when investment is irreversible and there is uncertainty. Since the industries are characterized by large sunk costs for investment with stochastically growing demand, we employed a real options approach to examine some policy issues in an open-access competitive environment.

Using a simple model, we derived an *access-to-bypass equilibrium* by allowing an entrant the opportunity to access an incumbent's network facility. In particular, we characterized an entrant's sequential investment timing for the construction of an additional network facility, having accessed the incumbent's network, in terms of an access charge and the level of network investment costs. Analysis of the equilibrium confirmed that the introduction of competition in network industries makes a firm's incentive to invest greater than that of a monopolist. That is, in an access-to-bypass equilibrium, a firm enters earlier in an open access policy if there is competition. We then showed that a change in the access charge induces a trade-off in social welfare. That is, a decrease in the access charge expands social benefit flow in the access duopoly equilibrium, and deters the introduction

[15] See Broadband Stakeholder Group (2003) for international experience of funding investment in next-generation broadband.

of a new network facility and a positive network externality generated by the construction of an additional bypass network. This trade-off occurs even when there is only a usage access charge through its effect on profit flows in the access duopoly equilibrium. We also examined the feasibility of socially optimal investment timing. We showed that, if lump-sum subsidies or taxes can be used with a usage access charge, the desired social benefit flow and socially optimal investment timing for infrastructure construction can be achieved in access oligopoly. These policies have recently been applied in practice. For example, the governments of Japan and Korea have introduced direct government funding and tax exemptions for the construction of telecommunications broadband infrastructure.

One may argue that the regulatory policy that we introduced in this paper is unrealistic because it uses a usage access charge and lump-sum subsidies and taxes. However, other regulatory policies could be used to achieve the optimal investment timing. For example, if the access charge depends on the state, such as $v(Y)$, it may be possible to achieve the optimum. However, in that case, firms' profits would be non-linear functions of Y, which would complicate the analysis. In addition, voluntary agreements on access charges between network providers and access seekers may induce an approximate social optimum. The search for policy tools that will achieve the social optimum is an important issue for future research.

Appendix

Proof of Lemma 1

First, it is easily shown that the follower does not adopt the access strategy. This is because the access project is not converted to the bypass project if and only if the net value of the transition project is non-positive, i.e., $\Delta V(Y) - I^m \leq 0$ for $\forall Y$. However, when $\Delta \Pi(2; v) > 0$, there exists $\overline{Y}$ such that the condition for $\Delta V(Y) - I^m = \frac{Y \Delta \Pi(2;v)}{r-\alpha} - I^m > 0$ holds for $Y \geq \overline{Y}$.

Next, observe that, when $Y^{B*} < +\infty$ and $(0 <) Y^{A*} \leq Y^{B*}$, the follower adopts the access-to-bypass strategy. When $Y^{B*} < Y^{A*}$, the follower adopts the bypass strategy. In fact, using (4) and (5), the condition that $Y^{A*} \leq (>) Y^{B*}$ is rewritten as (6). ■

Proof of Proposition 1

Since we have already derived the value function of the leader and that of the follower, the characteristics of which are standard in the real options literature, it is enough to show a sufficient condition for the existence and uniqueness of the access-to-bypass equilibrium. As stated in the text, to guarantee the existence and uniqueness of the equilibrium, it is sufficient to ensure that $V_L^{AB}\left(Y^{A*}\right) > V_F^{AB}\left(Y^{A*}\right)$ and that the difference between the leader's value and the follower's value decreases monotonically.

Let us define $Q^{AB}(Y) \equiv V_L^{AB}(Y) - V_F^{AB}(Y)$ at $Y \in \left[Y^{A*}, Y^{B*}\right)$. Substituting (7) and (8) into $V_L^{AB}(Y)$ and $V_F^{AB}(Y)$, we have

$$Q^{AB}(Y) = \left(\frac{Y}{Y^{B*}}\right)^{\beta_1}\left[I^m - \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}\right] - \left[I^m - \frac{Y\Pi^{LF}(2;v)}{r-\alpha}\right]. \tag{16}$$

where $\Pi^{LF}(2;v) \equiv \Pi^L(2;v) - \Pi^F(2;v)$.

So, we have

$$Q^{AB\prime}(Y) = \beta_1\frac{1}{Y^{B*}}\left(\frac{Y}{Y^{B*}}\right)^{\beta_1-1}\left[I^m - \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}\right] + \frac{\Pi^{LF}(2;v)}{r-\alpha} \tag{17}$$

$$Q^{AB"}(Y) = \beta_1(\beta_1-1)\frac{1}{(Y^{B*})^2}\left(\frac{Y}{Y^{B*}}\right)^{\beta_1-2}\left[I^m - \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}\right]. \tag{18}$$

For $Q^{AB\prime}(Y) < 0$ for $Y \in \left[Y^{A*}, Y^{B*}\right)$, it is necessary that $I^m < \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}$, which implies that $Q^{AB"}(Y) < 0$ (i.e., $Q^{AB}(Y)$ is not smooth at $Y = Y^{B*}$). For $Q^{AB}(Y)$ to be monotonically decreasing in Y, we require that $Q^{AB\prime}\left(Y^{A*}\right) < 0$. Inserting Y^{A*} and Y^{B*} into $Q^{AB\prime}\left(Y^{A*}\right) < 0$, we have

$$\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)} + (\beta_1-1)\left(\frac{\Delta\Pi(2;v)}{\Pi^F(2;v)}\frac{I^e}{I^m}\right)^{\beta_1-1}\left[1 - \frac{\beta_1}{\beta_1-1}\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)}\right] < 0, \tag{19}$$

where $1 < \frac{\beta_1}{\beta_1-1}\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)}$ by the condition that $I^m < \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}$.

In addition, the condition that $Q^{AB}\left(Y^{A*}\right) > 0$ is rewritten by inserting Y^{A*} and Y^{B*} as follows:

$$(\beta_1-1)\left(\frac{\Delta\Pi(2;v)}{\Pi^F(2;v)}\frac{I^e}{I^m}\right)^{\beta_1}\left[1 - \frac{\beta_1}{\beta_1-1}\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)}\right]I^m > I^m - \frac{\beta_1}{\beta_1-1}\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)}I^e. \tag{20}$$

Note that the right-hand side of (20) must be negative. Combining (19) and (20) and rearranging, we can summarize a sufficient condition for the existence and uniqueness of the access-to-bypass equilibrium:

$$\begin{aligned} I^m - \frac{\beta_1}{\beta_1 - 1} \frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)} I^e &< (\beta_1 - 1) \left(\frac{\Delta\Pi(2;v)}{\Pi^F(2;v)} \frac{I^e}{I^m} \right)^{\beta_1} \left[1 - \frac{\beta_1}{\beta_1 - 1} \frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)} \right] I^m \\ &< -\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)} \frac{I^e}{I^m}. \end{aligned} \tag{21}$$

■

Proof of Proposition 2

We define $P^{AB}(Y) \equiv V_L^{AB}(Y) - V_F^{AB}(Y)$ at $Y\left(< Y^{A*}\right)$. To prove the proposition, we need to show that $P^{AB}(Y^*) > 0$, since $P^{AB}(Y) > 0$ for $Y \in \left(Y_L^*,\ Y^{B*}\right)$. Substituting (7) and (8) into $P^{AB}(Y)$ and arranging, we have

$$\begin{aligned} P^{AB}(Y) &= \frac{Y\Pi(1)}{r-\alpha} - (I^e + I^m) \\ &\quad + \left(\frac{Y}{Y^{A*}}\right)^{\beta_1} \left\{ I^e + \frac{Y^{A*}\Pi^{LF}(2;v)}{r-\alpha} - \frac{Y^{A*}\Pi(1)}{r-\alpha} \right\} \\ &\quad + \left(\frac{Y}{Y^{B*}}\right)^{\beta_1} \left\{ I^m - \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha} \right\}, \end{aligned} \tag{22}$$

where $\Pi^{LF}(2;v) \equiv \Pi^L(2;v) - \Pi^F(2;v)$.

Let us evaluate (22) at Y^*. Inserting Y^*, Y^{A*}, and Y^{B*} in $P^{AB}(Y^*)$, we have

$$\begin{aligned} P^{AB}(Y^*) &= \frac{I^e + I^m}{\beta_1 - 1} \\ &\quad + \left(\frac{\Pi^F(2;v)}{\Pi(1)} \frac{(I^e + I^m)}{I^e} \right)^{\beta_1} \left\{ 1 + \frac{\beta_1}{\beta_1 - 1} \frac{\Pi^{LF}(2;v) - \Pi(1)}{\Pi^F(2;v)} \right\} I^e \\ &\quad + \left(\frac{\Delta\Pi(2;v)}{\Pi(1)} \frac{(I^e + I^m)}{I^m} \right)^{\beta_1} \left\{ 1 - \frac{\beta_1}{\beta_1 - 1} \frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)} \right\} I^m. \end{aligned} \tag{23}$$

We define $a \equiv \frac{I^m}{I^e}$, $b \equiv \frac{\Pi^F(2;v)}{\Pi(1)}$, $d \equiv \frac{\Pi^L(2;v)}{\Pi^F(2;v)}$, and $e \equiv \frac{\Pi(2)}{\Pi(1)}$, where $b \in (0,\ 1)$ and $e \in (0,\ 1)$ from Assumption 1. Then, applying this notation to (23) and dividing by I^e

yields

$$\begin{aligned}\frac{P^{AB}(Y^*)}{I^e} &= \frac{1}{\beta_1 - 1}(1+a) \\ &+ (b(1+a))^{\beta_1}\left[1 + \frac{\beta_1}{\beta_1 - 1}\left((d-1) - \frac{1}{b}\right)\right] \\ &+ \left((e-b)\frac{1+a}{a}\right)^{\beta_1}\left[a - \frac{\beta_1}{\beta_1 - 1}\frac{ab(d-1)}{e-b}\right]. \end{aligned} \tag{24}$$

Observe that the existence and uniqueness of the access-to-bypass equilibrium is guaranteed by the condition that $V_L^{AB}(Y^{A*}) > V_F^{AB}(Y^{A*})$. Using the above notation, the condition that $V_L^{AB}(Y^{A*}) \geq V_F^{AB}(Y^{A*})$ is rewritten as:

$$\left(\frac{e-b}{ab}\right)^{\beta_1}\left[a - \frac{\beta_1}{\beta_1 - 1}\frac{ab(d-1)}{e-b}\right] > a - \frac{\beta_1}{\beta_1 - 1}(d-1). \tag{25}$$

Multiplying both sides of (25) by $(b(1+a))^{\beta_1}$, we have

$$\left((e-b)\frac{1+a}{a}\right)^{\beta_1}\left[a - \frac{\beta_1}{\beta_1 - 1}\frac{ab(d-1)}{e-b}\right] > (b(1+a))^{\beta_1}\left[a - \frac{\beta_1}{\beta_1 - 1}(d-1)\right]. \tag{26}$$

Comparing (24) with (26) yields

$$\frac{P^{AB}(Y^*)}{I^e} > \frac{1}{\beta_1 - 1}(1+a) + (b(1+a))^{\beta_1}\left[(1+a) - \frac{\beta_1}{\beta_1 - 1}\frac{1}{b}\right] \equiv \chi(a, b, \beta_1) \tag{27}$$

To ensure that $P^{AB}(Y^*) > 0$, it is sufficient to show that $\chi(a, b, \beta_1) \geq 0$. To show that $\chi(a, b, \beta_1) \geq 0$, we examine two cases according to the value of $b(1+a)$.

Case 1:

When $b(1+a) \geq \frac{\beta_1}{\beta_1 - 1}$, it is obvious that $\chi(a, b, \beta_1) > 0$.

Case 2:

Let us examine the case in which $b(1+a) < \frac{\beta_1}{\beta_1 - 1}$. First, we rewrite $\chi(a, b, \beta_1)$.

$$\begin{aligned}\chi(a, b, \beta_1) &= \frac{1+a}{\beta_1 - 1}\left\{1 + (\beta_1 - 1)\left[(b(1+a))^{\beta_1} - \frac{\beta_1}{\beta_1 - 1}(b(1+a))^{\beta_1 - 1}\right]\right\} \\ &\equiv \frac{1+a}{\beta_1 - 1}\widetilde{\chi}, \end{aligned} \tag{28}$$

where $\widetilde{\chi} \equiv 1+(\beta_1 - 1)\left[(b(1+a))^{\beta_1} - \frac{\beta_1}{\beta_1 - 1}(b(1+a))^{\beta_1 - 1}\right]$. Hence, the sign of $\chi(a, b, \beta_1)$ is equal to that of $\widetilde{\chi}$. Let us define $x \equiv \beta_1 - 1$ and $\gamma \equiv b(1+a)$. Then, we have

$$\widetilde{\chi}(\gamma, x) = x\gamma^{x+1} - (x+1)\gamma^x + 1. \tag{29}$$

Here, $\frac{\partial \widetilde{\chi}}{\partial \gamma} = x(x+1)\gamma^{x-1}(\gamma - 1)$, so that $\frac{\partial \widetilde{\chi}}{\partial \gamma} \geq (<) \, 0$ if and only if $\gamma \geq (<) \, 1$. This means that $\widetilde{\chi}(x, \gamma)$ takes its minimum value at $\gamma = 1$, given any $x > 0$. In fact,

$$\widetilde{\chi}(1, x) = x - (x+1) + 1 = 0. \tag{30}$$

Note that this does not depend on the level of x. That is, for all $x > 0$, $\widetilde{\chi}(\gamma, x) \geq 0$. Therefore, $\chi(a, b, \beta_1) \geq 0$, which in turn implies $P^{AB}(Y^*) > 0$. ■

Proof of Proposition 3

First, let us prove (i). Remember that Y_L^* is defined by $V_F^{AB}(Y_L^*) = V_L^{AB}(Y_L^*)$, or

$$B_F (Y_L^*)^{\beta_1} = \frac{\Pi(1)}{r - \alpha} Y_L^* - B_L (Y_L^*)^{\beta_1} - (I^e + I^m), \tag{31}$$

where $B_F \equiv \frac{\Pi^F(2;v)}{\beta_1(r-\alpha)}\left(Y^{A*}\right)^{1-\beta_1} + \frac{\Delta\Pi(2;v)}{\beta_1(r-\alpha)}\left(Y^{B*}\right)^{1-\beta_1}$ and $B_L \equiv \frac{\Pi(1)-\Pi^F(2;v)}{r-\alpha}\left(Y^{A*}\right)^{1-\beta_1} - \frac{\Delta\Pi(2;v)}{r-\alpha}\left(Y^{B*}\right)^{\beta_1}$. Differentiating this, we have

$$C dY_L^* + \frac{\partial (B_F + B_L)}{\partial v} (Y_L^*)^{\beta_1} dv = 0, \tag{32}$$

where $C \equiv V_F^{AB\prime}(Y_L^*) - V_L^{AB\prime}(Y_L^*) < 0$. Substituting (5) and (4) into Y^{A*} in B_F and Y^{B*} in B_L, respectively, and differentiating, we have

$$\begin{aligned} & \frac{\partial (B_F + B_L)}{\partial v} \\ = \; & \beta_1 (\beta_1 - 1)\left[\left(\Pi^F(2;v)\right)^{\beta_1 - 2}\left(\Pi(1) - \Pi^F(2;v)\right) K_1 + (\Delta\Pi(2;v))^{\beta_1 - 1} K_2\right] \\ & \times \frac{\partial \Pi^F(2;v)}{\partial v} \\ < \; & 0, \end{aligned} \tag{33}$$

where $K_1 \equiv (\beta_1 (r-\alpha))^{\beta_1} \left(\frac{\beta_1 - 1}{I^e}\right)^{\beta_1 - 1}$ and $K_2 \equiv (\beta_1 (r-\alpha))^{\beta_1} \left(\frac{\beta_1 - 1}{I^m}\right)^{\beta_1 - 1}$. Hence, we have $\partial Y_L^* / \partial v < 0$.

The proofs of (ii) and (iii) are easily derived from the assumption that $\frac{\partial \Pi^F (2;v)}{\partial v} < 0$. ■

Proof of Proposition 4

First, let us denote the optimal regulatory policy for the follower by $\{v^{**}, T^{A**}, T^{B**}\}$, as in the proposition. The optimal social benefit flow under the access duopoly can be manipulated only by a usage access charge v, so that v^{**} should be characterized by $SB_v^{**}(2) = SB(2; v^{**})$. Substituting these into the project values of (1) to (3), we derive the trigger points for the access and the bypass, respectively.

$$Y^{A*} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{\Pi^F (2; v^{**}) + T^{A**}} I^e \tag{34}$$

$$Y^{B*} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{\Delta\Pi (2; v^{**}) + [T^{B**} - T^{A**}]} I^m. \tag{35}$$

Comparing (13) with (34) reveals that $T^{A**} = \left[SB_v^{**}(2) - \Pi_v^{F**}(2)\right] - SB(1)$, which makes Y^{A*} equal to Y^{A**}. Similarly, we get $T^{B**} = [SB(2) - \Pi(2)] - SB(1)$ by substituting T^{A**} into (35) and comparing it with (14).

Since the leader's trigger point Y_L^* is characterized by $V_F^{AB}\left(Y_L^*; Y^{A*}, Y^{B*}\right) = V_L^{AB}\left(Y_L^*; Y^{A*}, Y^{B*}\right)$, T_L^{**} is obtained by solving the equation after the substitution of Y^{A**}, Y^{B**}, v^{**}, T^{A**} and T^{B**}. In addition, for $V_F^{AB} \leq V_L^{AB}$ for all Y, T^{B**} is required for the leader in the bypass duopoly. ■

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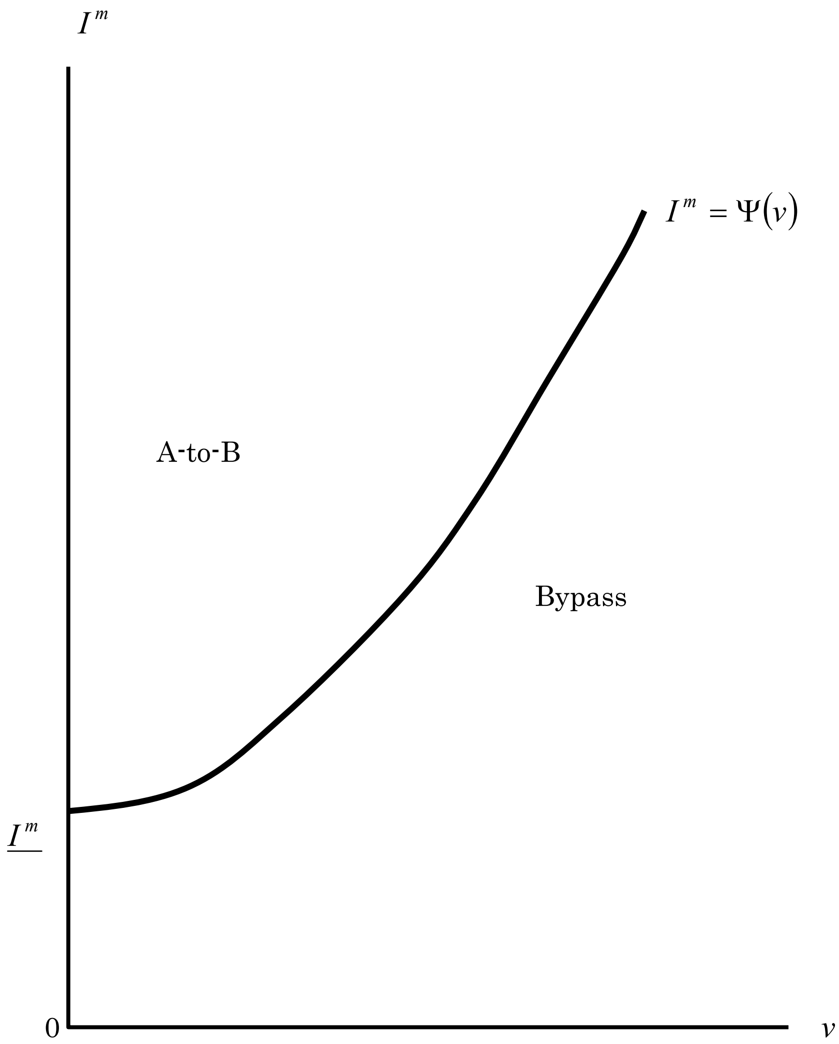

Figure 1

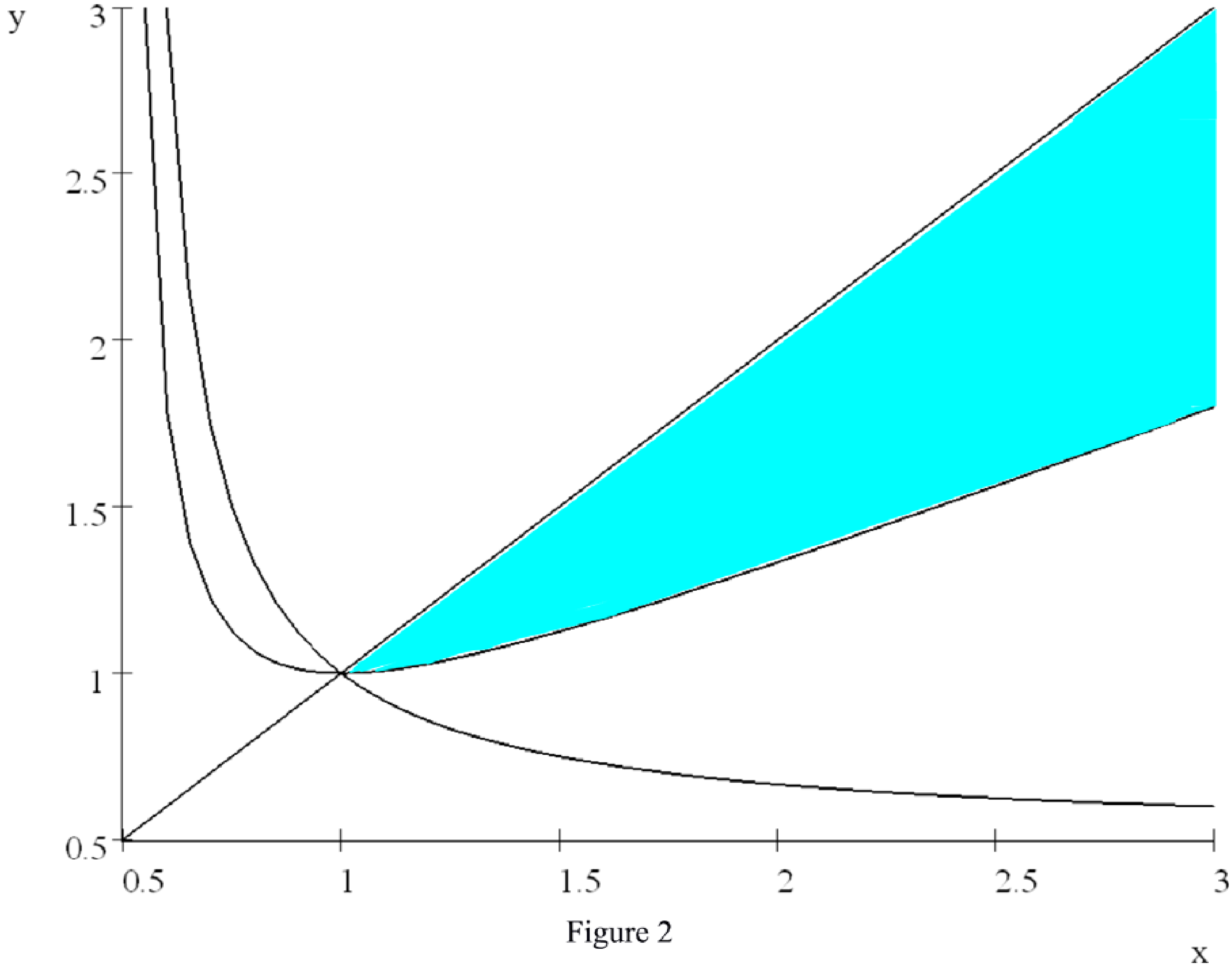

Figure 2

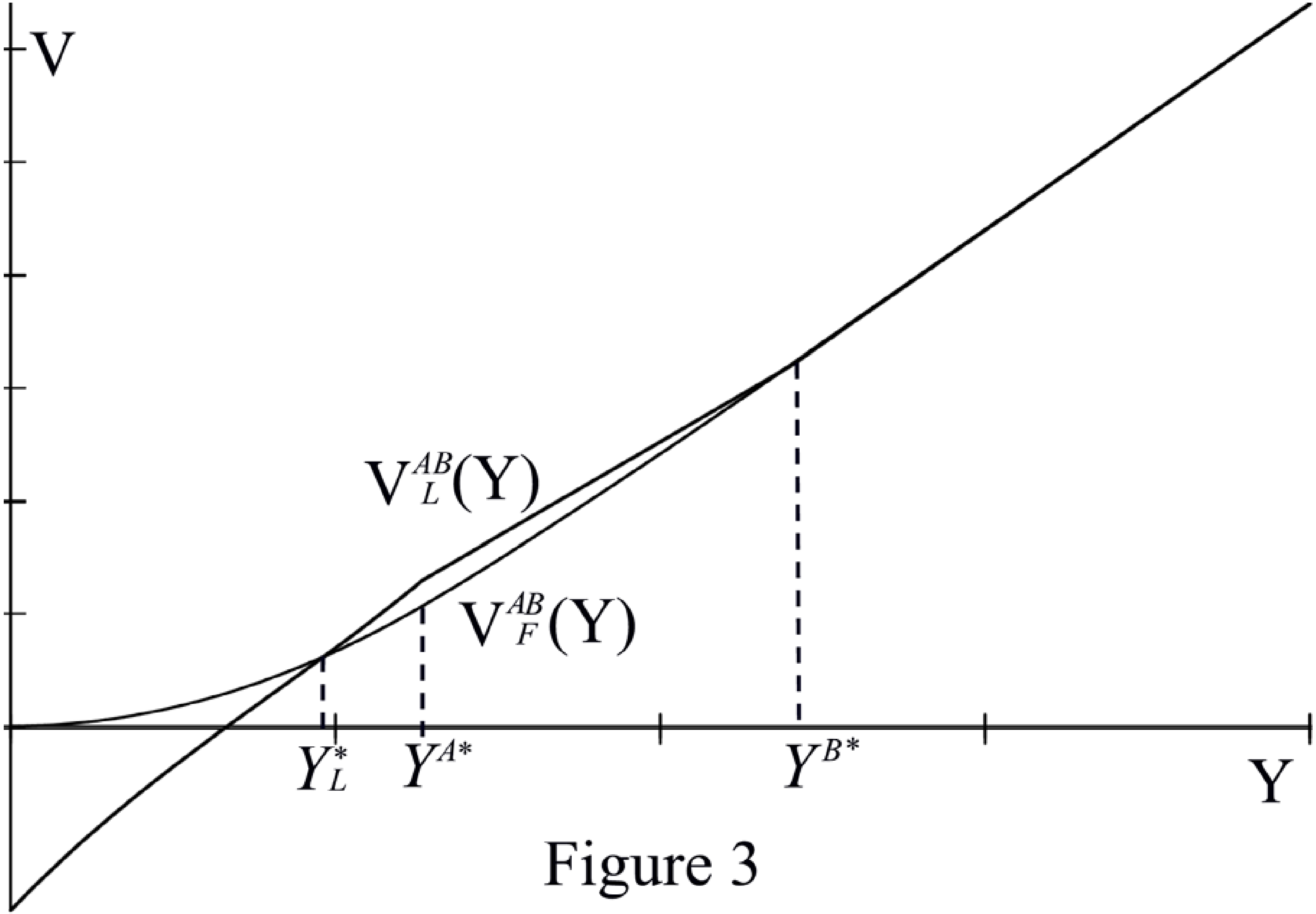

Figure 3