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A Real OptionsApproach to Technological Portfolio Diversification†

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1 Introduction

The value of a project follows not only from its discounted cash flow but should include new options and future opportunities. Thus, a project might be thought of as a compound real option. It is not uncommon that a decision-maker may have to decide between competing projects and, moreover, most investment decisions embody operating flexibilities which must be endogenized when assessing projects. These operating options confer possibilities of future adjustments in response to changes in economic conditions. The firm is a combination of assets and options, and may be viewed as a portfolio of projects and current real assets, offering managers the ability to dynamically optimize this portfolio by exploiting interactions between options (flexible) and existing assets (irreversibly held).

Many papers in the literature on real options consider portfolios of real options in an implicit manner. Trigeorgis (1993) treats valuation of investments with multiple options and shows the necessity of simultaneous evaluation due to options interactions and non-additivity of their values. Kulatilaka (1995) investigates the effect of interdependencies between options on the valuation and operating rules for a multiple options project. Childs, Ott and Triantis (1998) analyzes interaction between two projects and the choice of a development strategy (parallel or sequential) knowing that only one project can be implemented. He and Pindyck (1992) examines investments in flexible production capacity. The problem considered is whether to buy flexible or non-flexible equipment and how much capacity to invest in. Epaulard and Gallon (2001) analyze the decision to invest in an *European Pressurised-water Reactor* prototype that would enable the decision maker to have the choice between nuclear power plants and gas-fired plants when comes the time of replacing the existing nuclear power stations in France.

Brosch (2001) deals with portfolio aspects of real options. He points out three relevant aspects: first, the direct qualitative interactions between existing projects and potential ones (non-stochastic relationships resulting from physical properties: substitute, complement); second, options interactions resulting from the fact that they affect the underlying assets; and third, correlation, which is of importance only if projects exhibit interaction and have to be priced simultaneously. As Brosch (2001), we emphasize that diversification of a portfolio of real options must not be viewed in a classical way: one fails to identify the additional value created by a real options portfolio if one only considers the stochastic relationship (correlation) between the market values of separate projects, while disregarding operating options.

In this paper, we adopt a portfolio analysis in a real options framework; more precisely, we study an investment problem when competing technologies are able to achieve a specific production objective while having the potential to generate operating options.

We present a model where a firm has an exclusive opportunity to acquire production units of a given capacity. Two technologies are available. These technologies have different risk characteristics because (say) they rely on different inputs with different prices volatilities. Besides the choice between the two competing technologies, the investor has to decide how many capacity units to install (growth options) with no constraints on technological uniformity or timing. Thus the firm may decide to hold zero units, one unit of either type, two units of one type, two units of the alternative type, one unit of each type, etc. We limit the analysis to two units, but the methodology can be extended to many units.

Changes in the production scale of any unit of capacity are costless; this gives a technologically diversified firm a certain operating flexibility to adjust to fluctuations in input prices. This operating flexibility is similar to a switch option but differs in two important ways: first any combination of production between two units is possible, subject to the capacity of each unit; second the operating flexibility option may be acquired as a one shot purchase of two capacity units, or as a decision to extend capacity by choosing a different technology at the second acquisition. This apparently benign feature in the acquisition process of a technology that, *ex post*, may be considered a flexible unit, turns out to make may take place in two steps

In the model described above, with its multiple underlying options, we analyze the operational and investment decision processes and show how and under which conditions portfolio optimization leads to a technologically diversified, or a specialized, portfolio. The analysis emphasizes that project evaluation is firm and history specific in such a context.

2 The model:

A firm faces a maximum demand of $Q = 1$ which is constant over time. It may satisfy only part, or the totality, of that demand if it wishes to and if it holds the required capacity. To strip the problem of any non crucial complication, we further assume that the unit price P is exogenous to the firm. This assumption is not crucial but allows us to abstract from such issues as canibalism or strategic behavior, and to compare technologies on their own sake. The firm holds the opportunity of investing in discrete units of production of fixed capacity λQ with $\frac{1}{2} < \lambda < 1$; thus it will have to build at least two plants and no more than two plants if it finds it optimal to produce and sell a quantity $Q = 1$. It may also abstain from building any capacity or acquire only one unit and satisfy only part of the demand.

Two different technologies are available; they do not depreciate and last forever once in place. Operating a plant with the first one, referred to as Technology A, entails a constant cost C per unit of output. We assume that C is always lower than $P(t)$. One can think of Technology A as certain in that neither the price of its inputs nor the production process itself are subject to fluctuations. The second available production process, referred to as Technology B, relies on a specific input whose price varies stochastically, or is subject to technological uncertainty, so that its unit production cost S follows a geometric Brownian motion with zero drift.

$$\frac{dS}{S} = \sigma dz \tag{1}$$

The firm observes the current value $S(t)$, and knows that S evolves over time with a volatility σ measured by σ.

The cost I of acquiring and installing a production unit (of capacity λ) is the same whatever technology is chosen and is irrecoverable. The firm has the possibility of investing in a maximum of two production units[1] with no constraints on the timing or the sequence of investment, nor on the choice of technology. At any time, the value of the firm is equal to the value of its installed capacity plus the value of its options still alive.

We assume that

$$\frac{P - C}{r}(1 - \lambda) > I \tag{2}$$

This assures that buying Technology A is a profit making alternative, not only as a first but also as a second, investment, where the second unit of production would be used it at a rate $\frac{1-\lambda}{\lambda}$ of its capacity, in such a way as to supply the entire demand while using the first unit at full capacity. As will be seen, this does not rule out waiting to invest, or choosing Technology B, as rational alternatives; this assumption only ensures that we do not study a degenerate problem where investing in a second unit of Technology A would be ruled out by the choice of parameters.

By choosing between Technology A and Technology B, the firm chooses the technological composition of its assets, and thus, the degree of risk at which characterizing its future cash flows.

If the firm holds two units of production, its total capacity of 2λ exceeds demand. Assuming that varying the rate of output production of a plant is costless, this means that being technologically diversified is equivalent to possessing a flexible production system with a potential for dynamic management. Indeed, such a firm is able to organize production in such a way that the lowest cost technology produces at full capacity, acounting for a larger proportion λQ of total output while the second plant provides the remaining quantity $(1 - \lambda)$. In that case, production decisions are modeled as a

[1] This assumption reduces the size of the problem without removing any interesting characteristics. Without that assumption it would be conceivable, and optimal in some parameter configurations and levels of S, that the firm acquire a maximum of two capacity units of each technology. The methodology presented here can easily be adapted to cover these possibilities, as it could also be adapted to constant rate depreciation.

continuous sequence of binary choices based on the evolution of the stochastic cost S; this is described in detail in the next section.

Before analysing the various decisions faced by a firm in such a context, it is useful to examine the special case where S is known. The firm acquires two identical capacity units simultanously at the beginning of the period. There is no need for technological diversification nor for spreading the investments. Indeed, perfect certainty implies that the current situation will not change. Thus, if any action is to be taken, it must be taken immediately in order not to postpone profits. Since Technology A is profitable by assumption (2) both as a first and as a second unit, the only consideration is whether Technology A is superior to B or vice versa. Thus if $S > C$, two units of A are acquired immediately; in the apposite case, two units of Technology B are acquired immediately. Diversification may be considered in the limiting case where costs are identical; then, it is of no consequence as it does not affect the value of the firm.

The decision process is much more complex under uncertainty. In particular any decision concerning the certain Technology A is contaminated by the uncertainty surrounding Technology B; also, both the choices between technologies and the timing of investments differ according to the firm's existing assets. This is the portfolio dimension of the analysis. We find:

- Although production is profitable, waiting may be optimal
- A trivial, certain, project requires a different evaluation if it is in competition with another, uncertain, project;
- Project evaluation depends on the circumstances of the firm.
- In contrast with financial portfolios whose composition must meet restrictions stemming from current information only, real portfolios (Technical units held by a firm) are determined and constrained by the past.

3 Evaluating final “technological portfolios”

Let $V_{ij}(S)$ denote the value of the when it holds one unit of Technology i and one unit of Technology j, where $i, j = A, B$. When the firm holds its maximum total capacity of two units, its real portfolio can be technologicaly specialized; its value is then $V_{AA}(S)$ or $V_{BB}(S)$. Or the real portfolio may be diversified; its value is then $V_{AB}(S)$.

3.1 Values of technologically specialized portfolios

The firm's value maximizing choice of technology and capacity can eventually lead to Portfolio AA composed of two units of production, both using Technology A, or to Portfolio BB consisting of two plants operating with Technology B. Since Technology A is always profitable, the units will always be in operation and will never be shut down once the firm has acquired them. Consequently, the value of Portfolio AA is

$$V_{AA}(S) = \frac{P - C}{r} \tag{3}$$

If the firm holds two units of Technology B, it is possible for S to exceed the price at some dates, in which case it is optimal not to operate the units until they becomes profitable again, when $S(t) \leq P$. In the first instance the instant profit is zero, while it is $P - S$ in the second instance. This is a version of the well known problem of a firm with the option to shut down first solved by McDonald and Siegel (1985). The value of the firm holding Portfolio BB is

$$V_{BB}(S(t)) = \begin{cases} (P - S(t))\,dt + e^{-rdt} E_t V_{BB}(S(t+dt)),\ S(t) \leq P \\ e^{-rdt} E_t V_{BB}(S(t+dt)),\ S(t) > P \end{cases}$$

As shown in the Appendix (xxnot available in this version), the Bellman equation that must be satisfied by the function $V_{BB}(.)$ is

$$\begin{aligned} \frac{\sigma^2}{2} c_B^2 V_{BB}''(S) + \alpha S V_{BB}'(S) - r V_{BB}(S) + P - S &= 0 \text{ if } P \geq S \\ \frac{\sigma^2}{2} c_B^2 V_{BB}''(S) + \alpha S V_{BB}'(S) - r V_{BB}(S) &= 0 \text{ if } P < S \end{aligned}$$

According the current state S that equation admits the following solutions:

$$V_{BB}(S(t)) \equiv V_{BB}(S(t);\ \text{on}) = AS^{\beta_1} + \frac{P - S(t)}{r},\ \text{if } P \geq S \tag{5}$$

$$V_{BB}(S(t)) \equiv V_{BB}(S(t);\ \text{off}) = BS^{\beta_2},\ \text{if } P < S \tag{6}$$

where A and B are positive constants to be determined, and

$$\beta_1 = 0.5 - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{\alpha}{\sigma^2} - 0.5)^2 + \frac{2r}{\sigma^2}} > 1 \tag{7}$$

$$\beta_2 = 0.5 - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{\alpha}{\sigma^2} - 0.5)^2 + \frac{2r}{\sigma^2}} < 0 \tag{8}$$

The first terms in (5) and in (6) represent the value of the flexibility provided by the opportunity to switch back and forth from operating to being inactive as many times as necessary; the first switch, if any, is from operating to being inactive, in the case of Equation (5), and the other way around in the case of Equation (6). The second term in (5) reflect the expected net present value of sticking to the current mode of operation forever, never using the option to switch; its counterpart in (6) is zero as the current mode does not yield any profit.

Where the two regions meet, the two forms must connect in a continuous and smooth way, i.e. they must have the same value and equal derivatives at $S = P$[2].

$$V_{BB}(P;\ \text{on}) = V_{BB}(P;\ \text{off}) \tag{9}$$

$$V'_{BB}(P;\ \text{on}) = V'_{BB}(P;\ \text{off}) \tag{10}$$

Conditions (9) and (10) xx determine the constants A and B; both are strictly positive.

$$A = \frac{1}{r(\beta_1 - \beta_2)} P^{(1-\beta_1)}$$

$$B = \frac{1}{r(\beta_1 - \beta_2)} P^{(1-\beta_2)}$$

[2]See Dixit and Pindyck (1994), p 188.

3.2 Value of the technologically diversified portfolio

Owning both technologies gives the firm an operating option enabling it to combine its two production units in such a way as to minimize its unit cost given the cost $S(t)$. Two modes of operation may be optimal according to the value of S:

- mode Ab: unit A runs at full capacity, producing λ; unit B supplies $(1-\lambda)$. The instant profit is

$$\pi_{Ab}(S(t)) = [P - (\lambda C + (1-\lambda)S)] \tag{11}$$

- mode Ba: unit B runs at full capacity, producing λ; unit A supplies $(1-\lambda)$. The instant profit is

$$\pi_{Ba}(S(t)) = [P - ((1-\lambda)C + \lambda S)] \tag{12}$$

At any time, the firm's instant profit is $\pi(S(t)) = \max\{\pi_{Ab}(S(t)), \pi_{Ba}(S(t))\}$, i.e.:

$$\pi(S(t)) = \begin{cases} \pi_{Ab}(S(t)) & \text{if} \quad S(t) > C \\ \pi_{Ba}(S(t)) & \text{if} \quad S(t) < C \end{cases} \tag{13}$$

To determine the value of Portfolio AB, we proceed as above with Portfolio BB. The value of a firm running in mode m and holding the option to switch to mode m^{-1} (e.g. running in mode Ab with the option of switching to mode Ba) is:

$$\begin{aligned} V_{AB}(S(t);m) \quad = \quad & \max\big\{V_{AB}(S(t);m^{-1}) \ ; \\ & \pi(S(t))dt + e^{-rdt}E_t V_{AB}(S(t+dt);m) \text{ subject to } (1)\big\} \end{aligned} \tag{14}$$

The Bellman equation that must be satisfied by the function $V_{AB}(.)$ is

$$\frac{\sigma^2}{2}c_B^2 V_{AB}''(S;m) + \alpha S V_{AB}'(S;m) - rV_{AB}(S;m) + \pi(S;m) = 0 \tag{15}$$

According to the current state S and the corresponding mode of operation, the value function takes two alternative forms, each corresponding to the two alternative solutions to equation 15:

$$V_{AB}(S) \equiv V_{AB}(S(t);Ab) = KS^{\beta_2} + \Pi_{Ab}(S(t)) \text{ if } S(t) > C \tag{16}$$

$$V_{AB}(S) \equiv V_{AB}(S(t);Ba) = HS^{\beta_1} + \Pi_{Ba}(S(t)) \text{ if } S(t) < C \tag{17}$$

with

$$\Pi_{Ab}(S(t)) = \left[\frac{P-\lambda C}{r} - \frac{(1-\lambda)S}{r}\right] \tag{18}$$

$$\Pi_{Ba}(S(t)) = \left[\frac{P-(1-\lambda)C}{r} - \frac{\lambda S}{r}\right] \tag{19}$$

The first terms in (16) and in (17) represent the value of the flexibility provided by technological diversification, i.e. the value of the option to switch as many times as required from mode Ab to mode Ba if S falls below C (16) and from mode Ba to mode Ab if S increases beyond C. The form (16) covers situations where the first switch, if any, is from mode Ab to mode Ba, and vice versa for (17). The second terms in (16) and (17) reflect the expected net present value of keeping to the current mode of operation forever, never using the option to switch.

At the frontier between the two operating regions, the two forms of the value function must connect in a continuous and smooth way, i.e. they must have the same value and equal derivatives at $S = C$ (Dixit and Pindyck, 1994, p 188):

$$V_{AB}(C; Ab) = V_{AB}(C); Ba) \tag{20}$$

$$V'_{AB}(C; Ab) = V'_{AB}(C); Ba) \tag{21}$$

Conditions (20) and (21) determine the constants K and H; it shown in the Appendix that they are strictly positive.

$$K = \frac{2\lambda - 1}{r(\beta_1 - \beta_2)} C^{(1-\beta_2)}$$

$$H = \frac{2\lambda - 1}{r(\beta_1 - \beta_2)} C^{(1-\beta_1)}$$

Figure 1 shows the value of *Portfolio AB* with and without the option to actively manage production in order to adjust to changes in relative production costs C and S. The value of the flexible portfolio is always higher than if either mode of operation was imposed. More uncertainty over future values of S increases the value the operating option and thus the value of the technologically diversified *Portfolio AB*. The values of

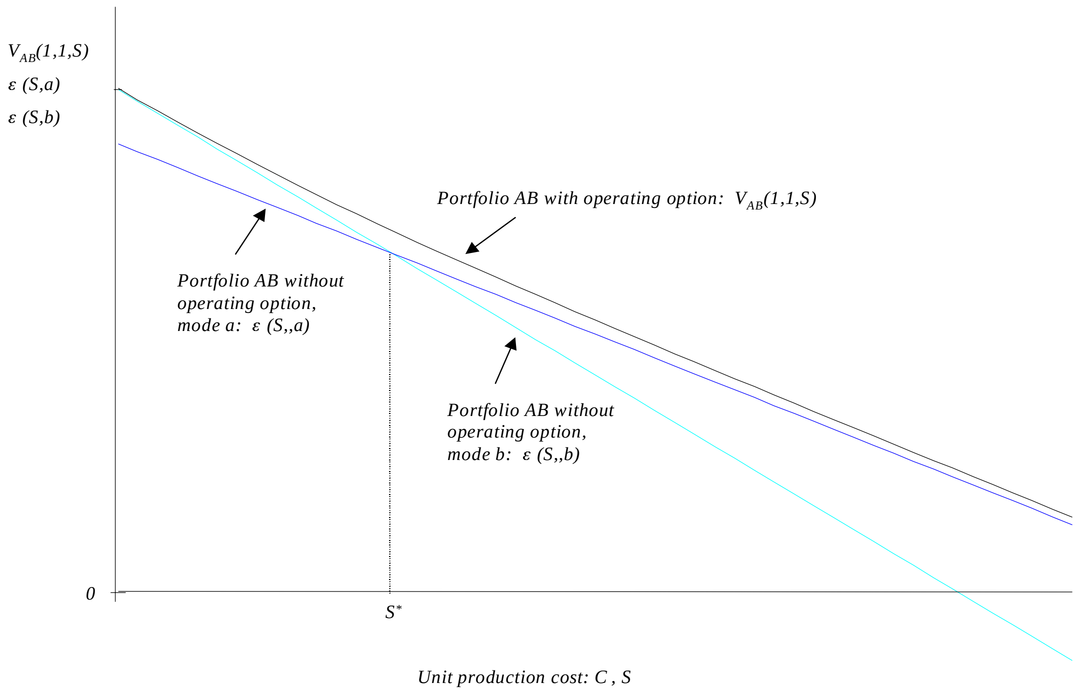

Figure 1: The Value of the flexibility of a technologically diversified portfolio

the three final portfolios AA, BB, and AB are illustrated in Figure 2. **xxcorriger la notation du 2ème graphe**

xx**corriger les noms des portefeuilles: AA et BB, V**$_{AA}$, V$_{BB}$. Each of the three possible final portfolios dominate the other two over some range of S values. Which one should be held by the firm? The answer does not depend as much on the current value of S as on its past trajectory. While the firm starts with a set of three possible final portfolios, the possible outcomes of the investment process will be reduced to only two possibilities once the first investment is undertaken and one technology is chosen.

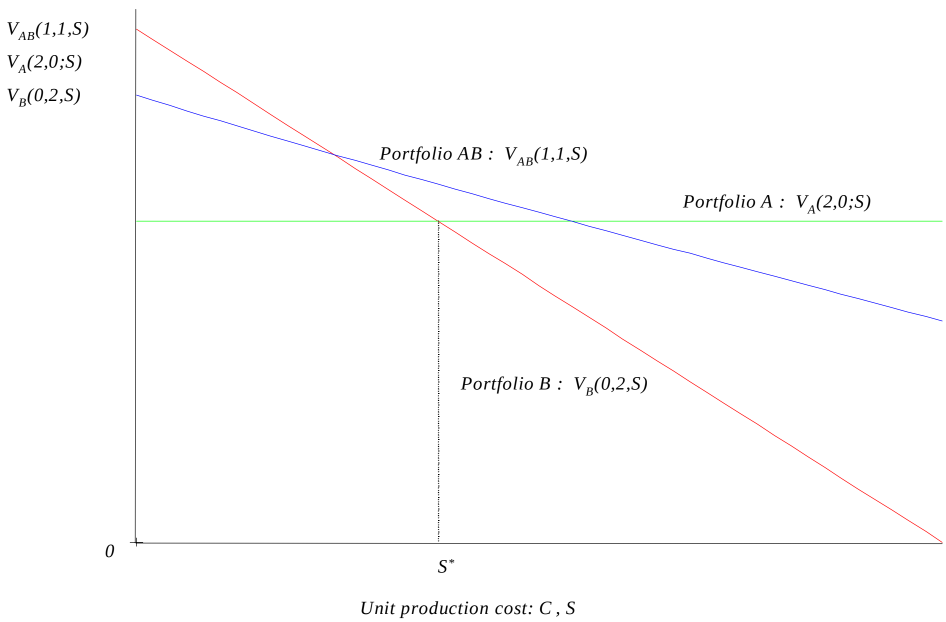

Figure 2: Values of the three possible final portfolios

4 Evaluation of intermediate portfolios

By installing a first unit of production the firm trades its initial portfolio, consisting exclusively of options for an intermediary portfolio composed of a real asset (*unit A* or *unit B*) and a growth option compounded with a technology choice option and, possibly, an operating option. According to the technology chosen for the first unit of capacity the firm may hold one of the following intermediate portfolios:

- *Portfolio A*: one *unit of Technology A* in operation, and the option to add a second unit of either technology. At the end of investment process the firm may end up with *Portfolio AA* or *Portfolio AB*.

- *Portfolio B*: one *unit of Technology B* in operation, and the option to add a second unit of either technology. At the end of investment process the firm may end up with *Portfolio BB* or *Portfolio AB*.

4.1 Valuing intermediate portfolio A

Let $V_A(S)$ be the value of a firm which has already acquired a *unit A* and produces a quantity λ at a unit cost C while holding the option to **increase its** capacity in the future by investing an amount I in a second unit of production. The choice of technology for the latter fixes the technological composition of the final portfolio. The last investment step may result in a specialized *Portfolio AA*, worth $V_{AA}(S)$, or in a diversified *Portfolio AB*, **worth** $V_{AB}(S)$.

Maximizing the firm's value $V_A(S)$, requires finding the decision rule for the last investment. This rule involves two threshold values noted S_{ij} where i refers to the technology currently installed while j indicates the technology in which it is optimal to invest when S lies in the appropriate region, delimited by S_{ij}. Precisely:

- S_{AA} : trigger cost S above which it is optimal for the firm owning a unit of Technology A to opt for technological specialization by investing in a second unit of Technology A;

- S_{AB} : trigger cost S below which it is optimal for the firm owning a unit of Technology A to choose diversification by investing in a unit of Technology B.

The value of Portfolio A is

$$\begin{aligned} V_A(S(t)) &= \max\{\max\{V_{AA}(S(t)) - I\ ,\ V_{AB}(S(t)) - I\}\ ; \\ &\quad (P-C)\lambda dt + e^{-rdt}E_t\left[V_A(S(t+dt))\right] \text{ subject to } (1)\Big\} \end{aligned} \tag{22}$$

For this optimal control problem and for each technology (asset), we can show that a particular value S separating a continuation region (waiting, status quo) from a stopping region (investing), exits. We also show that S_{AA} and S_{AB} are respectively an upper boundary and a lower boundary of a range of values of S.

The first term on the right-hand side of (22) is the termination payoff that the firm gets if the optimum decision is to invest in a second unit immediately. The second term is the sum of the instantaneous profit earned from operating the existing capacity unit[3] and the expected value of the current portfolio at the end of a time interval dt, conditional on the value of S at t.

The Bellman equation corresponding to the continuation region for the problem (22) is

$$\frac{\sigma^2}{2}S^2V_A'' + \alpha SV_A' - rV_A + (P-C)\lambda = 0 \tag{23}$$

and admits a complete solution

$$V_A(S) = D_1S^{\beta_1} + D_2S^{\beta_2} + \frac{P-C}{r}\lambda \tag{24}$$

where D_1 and D_2 are two positive constants to be determined.

The first two terms of the solution (24) reflect the value of the option to add capacity. As S increases, Technology A gains ground over Technology B and it becomes more likely that the former would be chosen if an expansion was decided. Consequently, the value of the option to invest in a second unit A rises; the term $D_1S^{\beta_1}$corresponds to that option. A decrease in S would produce the reverse effect, in favour of Technology B; the

[3]Since *Technology A* is always profitable it is certain that the firm operates the first unit.

term $D_2 S^{\beta_2}$) corresponds to that option. The last term is the present expected value of maintaining the current status quo forever.

The boundaries S_{AA} and S_{AB} define an interval over which equation (23) must be satisfied and over which its solution (24) applies.

4.1.1 Exercising the option of specialization or diversification

For values of S exceeding the threshold S_{AA}, it is optimal to carry out the second investment by acquiring a second of Technology A, thereby endowing the firm with a specialized portfolio. At such high levels of S, Technology A has such an advantage that a reversal in favour of Technology B is too unlikely to justify waiting for such an occurence while foregoing profits. When S overtakes S_{AA} from below, the usual value matching and smooth-pasting conditions must hold:

$$V_A(S_{AA}) = V_{AA}(S_{AA}) - I \tag{25}$$

$$V_A^{'}(S_{AA}) = V_{AA}^{'}(S_{AA}) \tag{26}$$

where $V_{AA}(S_{AA})$ is defined by (3) .

For values of S smaller than the threshold S_{AB}, Technology B has such an advantage that exercising the growth option by diversifying is optimal. At the threshold point S_{AB}, the value matching and smooth-pasting conditions are:

$$V_A(S_{AB}) = V_{AB}(S_{AB}) - I \tag{27}$$

$$V_A^{'}(S_{AB}) = V_{AB}^{'}(S_{AB}) \tag{28}$$

where $V_{AB}(S)$ may be defined by (16) or by (17) depending on the relative values of S_{AB} and C, the frontier between the two possible alternative forms taken by V_{AB}. Intuitively, one would expect $S_{AB} < C$, i.e. Technology B to dominate Technology A in current operation at the time of the investment. However, if the advantage of diversification is high enough, it defines a premium in favour of Technology B as a second investment that may exceed the cost of loosing flexibility; in that case $S_{AB} > C$.

Equations (25), (26), (27) and (27) give the constants D_1 and D_2 and the critical values S_{AA} and S_{AB}, given the model's parameters. We show that $D_1 > 0$ and $D_2 > 0$ and $S_{AB} < S_{AA}$ in the Appendixxxmake appendix. At the investment's threshold, whatever the choice of technology, the value of the received portfolio is a flow of expected revenues; in case of diversification the expectation takes account of any future operating option. This expected revenue flow must offset the direct cost I, plus the value of the option to postpone the investment, plus the value of the foregone option to acquire the competing technology.

4.2 Valuing intermediate portfolio B

We call $V_B(S)$ the value of a firm owning a production unit B and holding the opportunity to expand capacity by choosing either Technology A or Technology B. The decision to invest is ruled by two critical values S_{BB} and S_{BA}; respectively the lower and the upper boundary of a region where the status quo is optimal:

- S_{BB} : trigger cost S below which it is optimal for a firm owning a unit B to opt for technological specialization by investing in a second unit B.
- S_{BA} : trigger cost S above which it is optimal to choose diversification by investing in a unit A.

An important difference between holding a unit of Technology B and a unit of A is that, in the former instance, the firm has the option to abstain from producing if $S \geq P$. However it cannot be optimum to invest in any production unit if the latter is not to be put in operation immediately: in the opposite case, the same revenue flow could be achieved by not investing, while postponing the purchase expenditure. Thus in a region where the firm is considering investing in a second unit of Technology B, it is certain that the first unit is in operation. Over such a region, the value of the firm is then

$$\begin{aligned} V_B(S(t)) &= \max\{\max\{V_{BB}(S(t)) - I\ ,\ V_{AB}(S(t)) - I\}\ ; \\ &\quad (P-S)\lambda Q dt + e^{-rdt} E_t\left[V_B(S(t+dt))\ \mid\ S(t)\right]\} \end{aligned} \tag{29}$$

subject to process (1)

Similar steps as for problem (22) lead to a solution for $V_B(S)$ which holds as long as no investment is made.

$$V_B(S) = G_1 S^{\beta_1} + G_2 S^{\beta_2} + \frac{P - S}{r}\lambda \tag{30}$$

G_1 and G_2 are positive constants to be determined with the thresholds S_{BB} and S_{BA}.

4.2.1 Exercising the option of specialization

For all S smaller than S_{BB} investing in a production *unit B* is optimal. S_{BB} is such as value-matching and smooth pasting conditions hold

$$V_B(S_{BB}) = V_{BB}(S_{BB}) - I \tag{31}$$

$$V_B^{'}(S_{BB}) = V_{BB}^{'} S_{BB}) \tag{32}$$

4.2.2 Exercising the option of diversification

When values of S go up and overtake S_{BA} from below, the optimal decision is to opt for a technologically diversified final portfolio immediately. At S_{BA} value-matching and smooth pasting conditions are

$$V_B(S_{BA}) = V_{AB}(S_{BA}) - I \tag{33}$$

$$V_B^{'}(S_{BA}) = V_{AB}^{'}(S_{BA}) \tag{34}$$

The same discussion, as for the critical value S_{AB}, is valid for the comparative position of S_{BA} and S^* on one hand, and S_{BA} and C in the other hand.

(if the firm requires that Technology A offers a smaller expected present cost ($\frac{C}{r} < \frac{S_{BA}}{r}$) we will have $S_{BA} > S^*$, else if no such advantage is required to choose Technology

A we can have $S_{BA} < S^*$)

In appendix, we verify that G_1 and G_2 are positive and that $S_{BB} < S_{BA}$.

5 Evaluation of the initial portfolio

The zero capacity firm is worth $V(S)$ the value of its set of investment options. Managing this portfolio consists in deciding whether an option has come to maturity, at each time t. These options could be exercised simultaneously or in a strict sequential fashion, immediately or after a waiting period.

If at the very beginning, the firm observe that S has already gone beyond the high level S_{AA} or under the low level S_{BB}, the best action is to install two identical units of production (total capacity $2\lambda Q$) immediately. Conversely if S lays between these two critical values, the firm faces a set of four possible actions: wait, invest in one *unit* A, invest in one *unit* B or, acquire Portfolio AB immediately.

According to the model parameters we can meet with two situations when choosing a diversified portfolio:

- value of the flexibility created by the operating option is sufficiently high to justify a technological choice not necessarily involving the cheapest production system in terms of expected present cost per unit of production: $S_{BA} < S^* < S_{AB}$.
- the acquired technology must offer the lowest expected present cost to produce a unit of output: $S_{AB} < S^* < S_{BA}$.

The investment process and the initial set of available actions will depend on the prevailing situation.

5.1 Case where $S_{AB} < S^* < S_{BA}$

In the present case, a Portfolio AB will never be acquired in one invesment of $2I$: to install a first *unit* A, S must be higher than S^*. Since $S_{AB} < S^*$, a non-zero amount of time is necessary to allow cost S to fall and reach S_{AB}. A similar argument is valid when we consider a first investment in a *unit B* ($S_B < S^* < S_{BA}$).

Therefore, we can determine two critical values S_A and S_B such as, the firm acquires **one** *unit B* for $S_B - \varepsilon \leq S \leq S_B$, waits for $S_B < S < S_A$ and, acquires a *unit A* for $S_A \leq S \leq S_A + \varepsilon'$ **where** ε and ε are some positive numbers.

In **the A**ppendix, **we prove that a waiting region exists:** $S_B < S_A$; assume that S and recall that we are interested in the interval $]S_{BB}, S_{AA}[$, the problem of the firm is to decide whether to maintain the initial portfolio $V(S)$ or, to spend I in exchangefor an intermediate portfolio **containing one unit of either** *Technology A* or *Technology B*

$$\begin{aligned} V(S(t)) &= \max\left\{\max\left\{V_A(S(t)) - I \ , \ V_B(S(t)) - I\right\} \ ; \right. \\ & \left. e^{-rdt}E_t\left[V(S(t+dt))\right] \text{ subject to } (1)\right\} \end{aligned} \tag{35}$$

A complete solution takes the form

$$V(S) = L_1 S^{\beta_1} + L_2 S^{\beta_2} \tag{36}$$

constants L_1 and L_2 and thresholds S_B and S_A are determined by the boundary conditions[4]

$$V(S_A) = V_A(S_A) - I \tag{37}$$

$$V'(S_A) = V_A'(S_A) \tag{38}$$

$$V(S_B) = V_B(S_B) - I \tag{39}$$

$$V'(S_B) = V_B'(S_B) \tag{40}$$

[4]Proof that L_1 and L_2 are positive in appendix ?

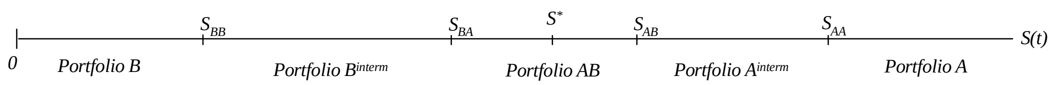

Figure 3: Investment decision when inactivity is never optimal

5.2 Case where $S_{BA} < S^* < S_{AB}$

If a waiting region before proceeding to first investment exists: $S_B < S < S_A$, necessarily we have $S_B < S^* < S_A$. Therefore, we would have: $S_B < S_{AB}$ and $S_A > S_{BA}$, meaning that the criterion to chose a technology is more severe when dealing with a first unit of production than a second unit and, that for each technology.

However, we know that a *unit A* (or B) generates more profit as a first unit than as a second unit and also that a first *unit A* (or B) creates two options (specialization; diversification) whereas a second *unit A* (or B) consumes an option of specialization. Consequently, the firm must be more "prudent" with a decision to acquire a second unit of a given technology. If that technology is B, it requires a lower trigger for the second unit than for the first one: $S_{AB} < S_B$. If that technology is A, it requires a higher trigger for the second unit than for the first one: $S_{BA} > S_A$. This is in contradiction with the assumption of the existence of a waiting region.

Thus, we can conclude that the optimal decision is an immediate acquisition of Portfolio AB for all costs S, $S_{BA} < S < S_{AB}$.

Portfolio A for $S_{AB} < S < S_{AA}$, Portfolio B for $S_{BB} < S < S_{BA}$ and a specialized portfolio for S below S_{BB} or above S_{AA}.

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