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Rules of Thumb in Real Options Analysis

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Abstract

In this normative paper, we derive payback period (PBP) and internal rate of return (IRR) in the presence of real options. In a Kulatilaka - Trigeorgis General Real Option Pricing Model, we derive the expected value of these two decision rules that corresponds to the expected NPV Bellman dynamic programming maximizing strategy in the presence of the options to wait, to mothball and to abandon. A number of original results are derived for an all equity financed firm. Expected PBP and IRR at time 0 are derived together with their distribution. These new methods are applied to a case study in shipping finance. Real options are shown to be value enhancing and shortfall decreasing also with respect to thumb rules: expected IRR is increased while expected PBP is decreased. Probabilities of earning negative returns are reduced together with those of not recovering initially invested capital.

Our model gives a more intuitive insight into the dynamic optimal behavior which is endogenous to real options valuation models showing plainly how the representative agent would probably manage optimally her project. This would help to compare optimally dynamic behavior with current practice and to conclude whether real options are a simple pure academic abstraction or a realistic model.

JEL classification code: G13, G31.

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Introduction

Although real options were proposed more than twenty years ago,[1] they are still a side product or an application of financial options models. As the latters, real options literature focuses on project evaluation, i.e. expanded NPV computation, neglecting all the other parameters widely used in capital budgeting practice, namely IRR and PBP.[2] In this normative paper, we compute these decision rules in the presence of real options providing practitioners with figures which, most of the times, they are more familiar with. After twenty years real options have been proposed for the first time, one of the factors that prevents their becoming widely used in practice is that they are perceived as a valuation criterion on its own. Instead, in this paper we show how even other capital budgeting parameters, such as those previously mentioned, can be computed in a real options framework. In other words, it is possible to translate the active dynamic management of an industrial investment project even in thumb rules.

The model we have devised is simple and it can be easily adapted by practitioners to the various contingencies of the business life. We have derived NPV_e, the usual expanded NPV, in a version of the Kulatilaka - Trigeorgis General Real Option Pricing Model (GROPM). Together with this expanded NPV we have derived optimal exercise thresholds of the real options to wait, to mothball, to restart and to abandon for the whole life of the project. Having established a Bellman's Dynamic Programming (DP) optimal policy, we were able to compute forward the same expanded NPV that was previously derived in the usual backward induction process running an Euler Scheme Monte Carlo simulation. The average of these Monte Carlo experiments corresponds to the expected expanded NPV computed in the original backward induction. This fact assures that both backward and forward computations are modeling the same DP optimal policy.

Using the DP optimally managed time series of cash flows and the corresponding NPV_es, a number of original results can be derived. Value at risk of the project at time $t = 0$, VaR_{NPV_e}, and the Cash Flow at Risk for each epoch of the investment project horizon, $CFaR_{CF_{DP}}$ can be easily derived, see (Alesii, 2003). Moreover, investment criteria which are intrinsically path dependent like IRR and PBP at time $t = 0$ can be computed taking into account the optimal exercise of real options.

This paper is organized as follows. In section 1 the use of thumb rules is briefly examined both from an empirical and a theoretical point of view. As a matter of fact a short review in positive capital budgeting literature is provided in order to show that, after all, NPV and *a fortiori* real options, are not very used in practice while IRR and PBP are. Moreover, theoretical justifications for the use of these thumb rules is

[1] The original intuition of a (real) growth option is already in (Myers, 1977) but the expression "real option" was coined by (Myers, 1984).

[2] We should be wary in defining criteria some decision rules that simply do not have a representative agent criterion function to maximize or which do not respect the Value Additivity Principle, see for instance (Rao, 1992) page 240.

provided. In section 2 it is described the method adopted to compute forward the same $E(NPV_e)$ usually computed backward in a GROPM framework. This same method is used to compute the two thumb rules object of this paper. In section 3 the method previously devised is applied to a shipping finance stylized case study. In section 4, conclusions are drawn and several extensions are proposed. The appendix reports an extensive numerical proof of convergence of expected forward computed NPV to the values obtained in the usual backward induction process.

1 Practice and Theory of Thumb Rules in Capital Budgeting

The main justification of this paper is that thumb rules, such as PBP and IRR, are widely used in practice and their use can be upheld from a theoretical point of view. The widespread use of thumb rules in the practice of capital budgeting is described taking some evidence from the positive capital budgeting literature. Instead, theoretical justifications for the use of these non-NPV decision rules have been devised in both a behavioral and a rational framework, respectively in organizational behavior literature and in agency theory and real options analysis.

To begin with, the most recent surveys in positive capital budgeting have shown how rules of thumb are still widely used in practice, see for instance (Graham and Harvey, 2001) and (Ryan and Ryan, 2002). Although there is an increasing number of firms that use DCF methods and expecially NPV,[3] the use of thumb rules is still important: according to (Graham and Harvey, 2001) page 197, more than 75% of US firms still use IRR as an individual criterion, inasmuch as NPV, and more than 50% use PBP. Those figures are more or less the same as those reported by (Ryan and Ryan, 2002). The use of less sophisticated capital budgeting rules is more widespread among small and medium sized firms with an older CEO (Graham and Harvey, 2001). Although that is true, even for large firms, such as the Fortune 1000, thumb rules are important for small sized capital budgets, see Exhibit 2 in (Ryan and Ryan, 2002). The US evidence is confirmed by a stack of studies in other countries. [4]

Another pattern in the practice of thumb rules in capital budgeting that emerges from these studies is that they are used in combination with NPV to explore the many faceted aspects of investment performance (Pike, 1996), or because some Non-NPV investment parameters, such as IRR, are a more cognitively efficient measure of comparison, (Binder and Chaput, 1996), or simply because they are provided in any spreadsheet

[3] See for instance (Binder and Chaput, 1996) for a description of trends from the '50s till the '80s in capital budgeting practices in the US.

[4] See for instance references in (Boyle and Guthrie, 1997) for survey results about Australia, Canada and New Zealand. See (Scapens and Sale, 1981) for two parallel surveys about UK and US firms, (Runyon, 1983) and (Coulthurst and McIntyre, 1987) for a survey about small-medium sized firms respectively in the US and the UK. See (Segelod, 2000) and (Sandahl and Sjögren, 2003) for evidence about Swedish professional service groups and large companies.

package (Pike, 1996). PBP and IRR are often found as one of the most popular combinations, see (Mills and Herbert, 1987) and (Cullinane and Panayides, 2000) page 323.

The use of thumb rules in combination with NPV can be explained from two points of view. From an agency theory, organizational behavior point of view, in small firms or in large ones but for small capital budgets, quite often investments are mostly defensive and sometimes they are not even evaluated (Runyon, 1983). In divisionalized companies, managers mostly implement headquarters decisions although they exert a certain influence and interact informally with them. In this bargaining process, there is limited role for the rigorous textbook like NPV only evaluation technique, (Scapens and Sale, 1981). In conclusion, formal capital budgeting is only one of the performance control tools and, sometimes, it is not the most important, (Segelod, 2000). As a consequence, informal bargaining takes place better on a whole string of investment project parameters instead of only one, i.e. NPV.

This has been given a theoretical rational justification by (Berkovitch and Israel, 1998) who show how in a bargaining process between headquarters and divisions managers, IRR and PI are useful in curbing empire building because, when selecting between mutually exclusive projects, they tend to bias against large scale projects, expecially when allocation according to these criteria is in conflict with the NPV criterion. Hence, thumb rules can be rational on their own when taking into account agency considerations. For instance, a manager may maximize her utility increasing early cash flows at the expense of ultimate overall profitability of the firm, *visibility bias*, see (Hirshleifer, 1993). In pursuing this kind of optimization, she may find more convenient to use PBP instead of NPV favoring the former investment projects that produce early cash flows (Narayanan, 1985).

On the other hand, leaving aside agency theory and organizational behavior considerations, the use of thumb rules alone or in combination with passive NPV has been shown to produce the same allocation as real options models, see (McDonald, 1998) in (Brennan and Trigeorgis, 1998) for a general view about these *hybrid rules*. This has been proved for models with time homogeneous cash flows with individual irreversible real options for IRR, see (Dixit, 1992), PBP, see (Boyle and Guthrie, 1997) and PI, see (McDonald, 1998). Basically, an hybrid rule is stated deriving endogenously to the real option model the level of the threshold for the thumb rule which corresponds to the real option exercise, using it, in the case of PBP, in combination with a passive NPV.

Our model differs from those just mentioned in a number of ways. To begin with, our model is numerical while the previous models are based on elegant but difficult to adapt to reality stochastic algebra. Moreover, since the method chosen is numerical, we were able to apply it to a general model with reversible, switching, and irreversible options. Finally, our aim in this paper has been to provide practitioners with the same string of investment parameters they crave for in the bargaining process to get funds to invest. The crucial difference is that in this model IRR and PBP are the translation in a different metrics of NPV maximizing Bellman Dynamic optimal strategy with the exercise of real options to wait, mothball, restart and abandon.

This paper and (Alesii, 2003) strive toward establishing a solid link between real options analysis and capital budgeting methods that are commonly considered as an alternative to the former. As a matter of fact, being perceived as a criterion on its own, real options analysis is still used by very few companies and in very few cases, 25% in (Graham and Harvey, 2001) but much less, 1.6% in (Ryan and Ryan, 2002), 0.5% in (Leliveld and Jeffery, 2003), nil in (Sandahl and Sjögren, 2003), and with a decreasing trend, see (Rigby, 2002). Proposing to the practitioners' audience the Kulatilaka-Trigeorgis GROPM extended for its risk dimensions and translated in other capital budgeting thumb rules mostly used in practice would probably help real options to take root in corporate culture.

2 The Computation of Thumb Rules in the Presence of Real Options

The procedure we have followed is numerical and it can be easily adapted by practitioners to any kind of investment project. The model of (Kulatilaka and Trigeorgis, 1994), (hence after KT), has been used to derive both the expanded NPVs of the investment project at time zero and the real options optimal exercise thresholds throughout the whole life of the project. Then, an Euler Scheme Monte Carlo simulation of the same state variable is performed with the same discretization. On each simulated path observation the exposure mapping equation of the firm is computed taking into account real options exercise thresholds. Hence, a simulated CF history, which has been managed exercising optimally real options, corresponds to each path of the state variable.

On these CFs histories it is possible not only to compute expected expanded net present values and its distribution but also to assess CF variability for each epoch, see (Alesii, 2003).[5] Moreover, on the same CF time series it is possible to compute any path dependent investment parameter such as PBP and IRR at time $t = 0$.

In the remaining part of this section, a minor extension of the KT real option pricing model is proposed motivating its choice among the vast variety of real options models, see section 2.1. Moreover, the scenario construction method is described giving a graphic portrayal of the CF computation according to optimal

[5] In essence, we generate scenarios that can be used for both risk management and pricing purposes. In this case, the natural probability measure used for pricing is shown to be also an EMM, see appendix of (Kulatilaka, 1993). Although that is true, it is considered correct to use in general an EMM for risk management purposes, while it is not right to use a natural measure of probability for pricing purposes, see (Tavella, 2002) page 77.

exercise real options thresholds, see section 2.2. Methods adopted to compute PBP and IRR are reported in section 2.3.

2.1 An Extension of the Kulatilaka Trigeorgis Model

KT general real options pricing model (GROPM) has been chosen for a variety of reasons.[6] To begin with, it is a general model for pricing simultaneously, or individually, a variety of real options while most of the other models are *ad hoc* individual options pricing models. Moreover, it is one of the few not based on a pseudo asset approach but on a running present value computation of expanded NPV. For these reasons GROPM accommodates general specifications of the exposure mapping and it enables to assess the variability of CF in each epoch of the investment project.

The version of the KT model we have used here is univariate, with a stochastic state variable specified as an arithmetic Ornstein-Uhlenbeck process, see equation (1), discretized in a grid (Kulatilaka, 1993). The choice of this specification is instrumental to the numerical example we have developed in section 3 where the state variable we have chosen evolves like a mean reverting process.

$$d\,\theta_t = \eta \cdot \left(\overline{\theta} - \theta_t\right)\, d\,t + \sigma_\theta\, d\,Z \tag{1}$$

where,

η	:	the speed of reversion, e.g. for $\eta = 0$ the process becomes a geometric Brownian motion while for $0 < \eta < 1$ the process tends to be mean reverting, negative levels are excluded to avoid mean aversion, one is excluded to avoid overshooting ;
$\overline{\theta}$	:	the normal level of θ, i.e. the level at which θ tends to revert;
σ_θ^2	:	instantaneous variance rate;
$d\,t$	:	time differential;
$d\,Z$	:	standard Wiener process, normally distributed with $E\left(d\,Z\right) = 0$ and $Var\left(d\,Z\right) = E\left(\left(d\,Z\right)^2\right) = d\,t$.

In essence, GROPM is based on a Bellman Dynamic Programming (hence after DP) method on a finite horizon solving an impulse control problem. Controls available to the firm are its various operating modes, namely, in this case, being idle before investing, operating, being mothballed, abandoned. Real options, then, are the capabilites to pass from one operating mode to the other, respectively option to wait, option to mothball, to restart and option to abandon. Because of this, GROPM accommodates several degrees of irreversibility of investment decisions being possible to specify a transition cost for each passage between operating modes. [7]

The solution of a dynamic optimization problem has two faces: the *max argument* derived from recursions of the Bellman equation, see expression (2); and the *argmax argument* or *optimal policy* , see expression (3). We claim that the derivation of the latter for the whole life of the project is an original contribution of

[6] The definition of a general model of real option is in (Kulatilaka, 1995) in (Trigeorgis, 1995). The acronym is ours.

[7] Bellman's Optimum Principle has been applied to the pricing or real options also by (Dixit and Pindyck, 1994) page 95.

this paper. These thresholds have been constructed simply recording the *argmax* function during each backward induction recursion, hence in a numerical procedure that can be easily generalized to any number of operating modes. [8]

$$F(\theta_t, \ell', \mathrm{t}) = \underset{\ell'}{max} \left\{ \Pi(\theta_t, \ell', t) - c_{\ell,\ell'} + \rho \cdot E_t^{\theta^*_{t+1}} \left[F(\theta_{t+1},, \ell', \mathrm{t}+1) \right] \right\} \tag{2}$$

$$\left.\begin{array}{l} \theta_{j,t} \Longrightarrow \\ \theta_{j,t} \Longleftrightarrow \end{array}\right\} \widehat{\ell'}_{j,t} = \underset{\ell'}{\mathrm{argmax}} \left\{ F(\theta_t, \ell', \mathrm{t}) \right\} \tag{3}$$

where,

$F(\theta_t, \ell, \mathrm{t})$	:=	value of the plant for the level of the state variable θ_t, for an optimizing operating mode ℓ at time t;
$E_t^{\theta^*_{t+1}}[\,]$	:=	expectation operator on equivalent martingale measure, hence starred, of the process θ_t;
$\rho = 1/\left(1 + i_{1/m}\right)$	:=	present value factor, in which $i_{1/m} = (1 + r_f)^{1/m} - 1$.
$c_{\ell,\ell'}$	:=	operating mode transition cost, being l the beginning mode and l' the ending mode;
$\Pi(\theta_t, \ell', t)$	:=	individual period operating cash flow.

(Kulatilaka, 1993) shows that under the restrictive condition of the investment project having a $\beta = 0$, lemma 4 of (Cox et al., 1985) is applicable and the drift of the arithmetic Ornstein Uhlenbeck can be considered a certainty equivalent drift rate. Hence, EMM and natural probability measure coincide under this restrictive hypothesis. Without loss of generality we consider an individual risk free rate. However, the model could easily accommodate a whole term structure of interest rates.[9]

2.2 Scenario Construction

The optimal exercise thresholds partition the discretized space of the state variable in regions in which different operating modes are optimal according to a Bellman DP procedure, see upper graph in figure 1. We run an Euler Scheme Monte Carlo simulation of the solution of the SDE in expression (1) and we obtain recursively an approximation to the path of the levels of θ_t $\forall t = 0, \ldots, T$, see equation (4).[10] This path meanders on the grid going through the thresholds, passing from an hysteresis, $\theta_{j,t} \Longrightarrow \widehat{\ell'}_{j,t}$ in (3), to a *one mode* region, $\theta_{j,t} \Longleftrightarrow \widehat{\ell'}_{j,t}$ in (3), and the other way around, see upper graph in figure 1.

$$\theta_t = \theta_{t-1} \cdot e^{-\eta \Delta\, t} + \overline{\theta} \cdot \left(1 - e^{-\eta \Delta\, t}\right) + \epsilon_t \tag{4}$$

[8]For details of the numerical procedure followed see (Alesii, 2000).

[9]We wish to thank Professor Richard Stapleton for pointing out this issue.

[10]See page 87 in (Tavella, 2002). In practice we simulate the Markov Chain that has a one step transition probability matrix given for each level θ_t by the discretization of the normal distribution of the arithmetic Ornstein Uhlenbeck additive shocks. It is worth noting that although a solution to the OU SDE does exist, see equation 4, the Euler scheme discretization has been adopted to maintain the same metrics on which results were derived in the backward induction procedure. We wish to thank Christian Schlag for pointing out this issue.

where, in addition to the previous notation:

$\epsilon_t \sim N\left(0, \frac{\sigma_\theta^2}{2\cdot\eta}\cdot\left(1-e^{-\eta\Delta\, t}\right)\right)$: noise term distributed normally with mean zero and variance as a fraction of σ_θ^2.

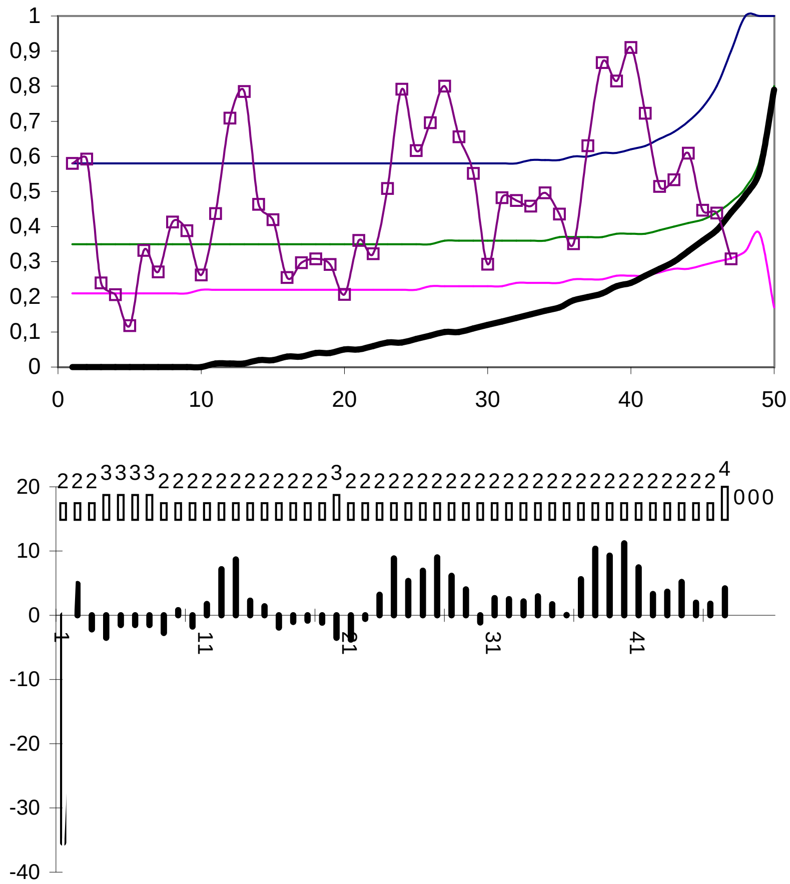

Figure 1: Path of θ and CF computation

Legend:

The upper graph in the figure represents the real options optimal exercise thresholds in a time - state variable Cartesian space, time unit is the dynamic system resetting period. To be specific, the highest represents the investment trigger threshold, the lowest, in bold, represents the abandonment threshold. In the middle, the higher is the restarting threshold while the lower is the mothballing threshold. An indicative path of θ_t is represented on the same graph. In the lower graph in the figure the corresponding time series of CF_t is represented in bold, together with an indicator function, in bars, representing the operating mode in which CFs at that epoch were generated. 2:= operating mode; 3:= mothballed mode; 4:= abandoned mode.

For each θ_t in each path we were able to compute $\Pi(\theta_t, \ell, t)$ netting these cash flows of the transition costs when and if they are due. Therefore, a θ_t path corresponds to a time series of CFs optimally managed according to Bellman Optimum Principle, see lower graph in figure 1 in which also a mode indicator function is reported between the two graphs.

In the example represented in figure 1, the investment project, e.g. a ship, is implemented at time $t = 0$ since the θ_t series starts at a level higher than the investment threshold, (operating mode=2). The very low CF at the beginning is the result of the lump sum initially invested and the first operating CF. After three epochs the time charter rate goes below the mothballing threshold. Hence the ship is laid up for four periods (operating mode=3) until the time charter rate reaches the restarting threshold. Even if in the following epoch it goes below it, the project is kept in operating mode, hysteresis situation. The same kind of situation takes place just before epoch 20 and 30. At period 47 the time charter rate goes below the abandonment threshold and the ship is scrapped for its salvage value (operating mode=4).

On this CF_t time series it is possible to compute NPV expanded for real options using the same risk free rate used in the backward induction process. Averaging these results across different Montecarlo experiments, expected values converge to those found using expression (2). This has been thoroughly shown in the Appendix. The extension for a passively managed project is trivial implying simply the computation of CF_t without taking into account real options exercise thresholds.

2.3 Forward Computation of IRR and PBP

On these cash flows it is possible to compute several thumb rules widely used in the practice of capital budgeting. Payback period (PBP) and Internal Rate of Return (IRR) can be computed in a path wise way only going forward on each optimally managed cash flow time series.

PBP computation is straightforward, see for instance (Lefley, 1996), (Kruschwitz and Loffler, 1999), (Yard, 2000). As a matter of fact, it implies simply the cumulation of CF_t until the initial investment is completely recovered, see expression (5).

$$PB = pb \Leftrightarrow I = \sum_{t=1}^{int(pb)} CF_t + [pb - int\,(pb)] \cdot CF_{int(pb+1)} \tag{5}$$

where:

PB	:=	payback period, expressed in the same frequency units as CF_t;
$int\,(pb)$	:=	integer number of periods before the last;
$CF_{int(pb+1)}$	:=	cash flow of the last period. Under the hypothesis of equally distributed cash flows within each period, a fraction of the last cash flow covers the initial capital which is still to be recovered at time $int\,(pb)$.

In CF_t histories in which investment does not pay back, i.e. inflows do not recover completely initial investment, we have computed a fractionary *recovery ratio* to show how much of the initial capital is actually recovered.

IRR computation, instead, can be computationally burdensome.[11] A textbook like version of this investment parameter is simply not applicable to the generality of the CF_t histories both with and without real options. As a matter of fact, CF_t series change sign more than once. We have discarded the truncation

[11] For some analytical proofs about existence of internal rates of returns and computational feasibility of NPV see (Saak and Hennessy, 2001) and (Oehmke, 2000).

theorem solution and we have adopted a generalized version of the internal rate of return according to Teichroew, (Teichroew et al., 1965b), (Teichroew et al., 1965a), (Teichroew, 1964). In essence, IRR according to Teichroew IRR_T is the internal rate of return which equals the running compound value (RCV) to the last cash flow produced by the investment project, taken with the negative sign, compounding positive RCV with the opportunity cost of capital, in our case r_f, and negative ones with IRR_T, see expressions (6)-(8). In other words, IRR according to Teichroew sets to zero the compound value of cash flows produced by an investment at the end of its life. This is equivalent to setting up to zero its present value. Because of this, the simple textbook like IRR is a particular case of IRR_T. Results have been found using a simple grid search combined with a secants algorithm. IRR_T does not exist in the cases in which cash flows have all negative signs, or are not "well behaved", see last column in table 1. The probability of these occurrences is very low.

at $t = 0$:

$$RCV_{0,\,1} = \begin{cases} CF_0 \cdot (1+r) \\ CF_0 \cdot (1+IRR) \end{cases} \text{for } CF_0 \begin{cases} > 0 \\ < 0 \end{cases} \tag{6}$$

$\forall\ t = 1,\ \ldots\ N-2$:

$$RCV_{t,t+1} = \begin{cases} [CF_t + RCV_{t-1,t}] \cdot (1+r) \\ [CF_t + RCV_{t-1,t}]\,(1+IRR) \end{cases} \text{for } [CF_t + RCV_{t-1,t}] \begin{cases} > 0 \\ < 0 \end{cases} \tag{7}$$

at $t = N$:

$$RCV_{N-1,N} = -CF_N \tag{8}$$

In order to get the annualized interest rate for IRR, linear compounding was used, i.e. $IRR = IRR_{1/h} \cdot h$ for a subperiod $\Delta\, t = 1/h$. As a matter of fact, subannual compounding does not allow to compute equivalent annual rates for $IRR < -100\%$. Both PBP and IRR computed on each of the Montecarlo experiments were averaged to get expected payback period and internal rate of return of the project. Moreover, we could provide the whole distribution of these investment parameters.

3 Numerical example in shipping finance

In this section we apply to a stylized case study in shipping finance the methods previously devised for computing PBP and IRR in the presence of real options. The choice of the shipping industry has been motivated both on positive and normative capital budgeting grounds. A short comparison with other case studies in recent literature is drawn to show that our approach is definetly a new contribution to the field. Setup and motivation of the numerical example and results obtained conclude the section. The main conclusion is that real options are effective in reducing downside risk in IRR and in increasing its expected value. Moreover, real options accelerate recovery of capitals invested, reducing both payback period and the expected life of the project and increasing the expected recovery rate.

Investments in the shipping industry has been often studied using real options, see for instance (Dixit and Pindyck, 1994) on page 237, (Goncalves de Oliveira, 1999) page 185. This is due to the fact that in tramp shipping services, strategic interactions among competitors are really meaningless. This allows us to apply a reduced model in which a representative agent is faced by a whimsical nature generating the most important profitability driver of the industry, time charter rates. The data generating process of this series has been specified as a driftless GBM by (Dixit and Pindyck, 1994) and a GBM with drift by (Goncalves de Oliveira, 1999). Instead, it is possible to show, see appendix A in (Alesii, 2003), that tramp shipping time charter rates are well described by an Arithmetic Ornstein Uhlenbeck.

Moreover, both these models are derived in a infinite time horizon, being based on stationary dynamic programming. Being a ship a finite lived asset, we have preferred to study the opportunity to invest in a ship over a finite time horizon, T=10 years. Because of these specific features, we were able to derive exercise thresholds for the whole life of the project while this was not the case for the two papers previously mentioned in which only individual levels of the state variable are given as thresholds. Finally, while (Dixit and Pindyck, 1994) on page 237 evaluates the vessel with all the real options we have considered here, namely option to wait, to mothball and to restart, and option to abandon, (Goncalves de Oliveira, 1999) page 185 studies only the switching options to mothball and to restart. Is it worth emphasizing the fact that both the previous models are based on expanded NPV only and that, to our knowledge, there is not in the current literature any extension of the elegant symbolic stochastic algebra that derives the implied values of PBP and IRR.

Although any textbook in financial management shows that the use of these thumb rules does not lead to shareholders' value maximization, practitioners in the shipping industry use them thoroughly. For instance, leading shipping management consultants propose together with second hand ship evaluations also PBP and IRR, see (Drewry and Jupe, 2001) and (Drewry and Kellock & Co, 1999). A recent survey by (Cullinane and Panayides, 2000) reports IRR as the mostly used decision rule, followed by NPV and PBP. These analyses are usually performed on an expected scenario discounted using a risk adjusted rate. Risk is taken into account through sensitivity analysis. Optimal dynamic decisions are simply ignored.

The widespread use of IRR in the evaluation of shipping investment is mirrored in some normative literature that applies risk analysis à la Hertz to IRR distribution, see (Haralambides, 1993) in (Gwilliam, 1993). There a distribution of returns is derived under a probability assigned by the representative agent, more or less subjectively. In conclusion, the derivation of the whole distribution of PBP and IRR in the presence of real options for a shipping industry investment can be of some interest to ship owners and financiers being these thumb rules more easily understood by both the industrial and the banking practice.

The numerical example has been set up as follows. The initial investment is $c_{1,2} = 40$, or cost to move the dynamic system from mode 1, wait, to mode 2, operate. Costs to mothball the project are $c_{2,3} = 2$, cost to move from mode 2, to mode 3, laid up ship, e.g. Fujairah anchorage off Oman coast. Costs to restart the project are $c_{3,2} = 4$. If the project is abandoned it yields $c_{3,4} = 5$, cost to move from mode 3 to mode 4, abandoned, e.g. Bangladesh wrecking yard. [12]

The project has an expected technical life of 10 years and its operating mode can be revised every six months exercising the options to start the project, to mothball, to restart or to abandon it. In operating mode the profit is $\pi_O = 20 \cdot \theta_s - 7$ while in mothballing mode it is $\pi_M = -1.5$. In both waiting mode and abandonment mode cash flows are nil. [13]

The state variable has been specified as an arithmetic Ornstein Uhlenbeck with the following parameters: $\eta = .125$, $\overline{\theta} = .5$, $\sigma_\theta = .125$, in a grid with $\theta_{min} = 0$ and $\theta_{max} = 1$ with $\Delta\,\theta_t = 1\%$. This process has been chosen after estimating the process parameters on 53 years of monthly time series reported in appendix A of (Alesii, 2003). The proportions between the normal value and volatility are equivalent to those of dry bulk time charter computed in appendix B of (Alesii, 2003). Reversion speed resembles that of the same time series.

We have derived the value of the investment project at time $t = 0$ in a backward induction procedure applied to equation (2). Results are represented by the smoothed lines without markers in figure 2. The same procedure has been run both for dynamic active management and passive management. From the same procedure we have derived the real options exercise thresholds, see equation (3), for the whole life of the project as represented in figure 1.

We have performed 80,000 Euler Scheme Monte Carlo simulations of the state variable θ_t approximating equation (4) with the same discretization used in the backward induction. Then, on each of these time series

[12] We consider the wreckage profit instead of the second hand ship price because the former can be considered deterministic while the latter varies considerably in the horizon chosen for the model. Hence, a better specification to take into due account second hand ship price would be as a second state variable. We save this extension for another paper.

[13] It is worth noting that this specification of the model does not consider that different real options have different lags between their exercise and their effect. For instance, the option to wait is effective at least after two or three years the ship has been ordered to the shipyard. This would require a new specification of the model. We save this extension for another paper.

we have computed CF_t net of transition costs taking into due account the optimal operating modes indicated by real options exercise thresholds. Results are reported in figure 1 as smoothed lines with markers. As a matter of fact, while the RPVs have been derived for 100 different initial values simultaneously, Monte Carlo simulations have been performed for 10 initial values $\theta_0 = 0.0, 0.1, 0.2, \ldots, 0.8, 0.9, 1.0$.

At a first glance on figure 2 forward and backward computed expected values seem to be the same both with and without real options. This is confirmed by convergence tests performed in Appendix A in which expected values for actively managed projects differ on the average at most 1.5% for $\theta_0 = 0$ and absolute differences are really meaningless with respect to the sum initially invested. This assures that both backward induction and forward computation are modeling the same optimal dynamic behavior. Convergence for passively managed projects expected values is, instead, complete. Results on NPV distributions are thoroughly commented in (Alesii, 2003) where it is shown that real options are effective not only in enhancing value but also in taming risk, reducing the so called project at risk.

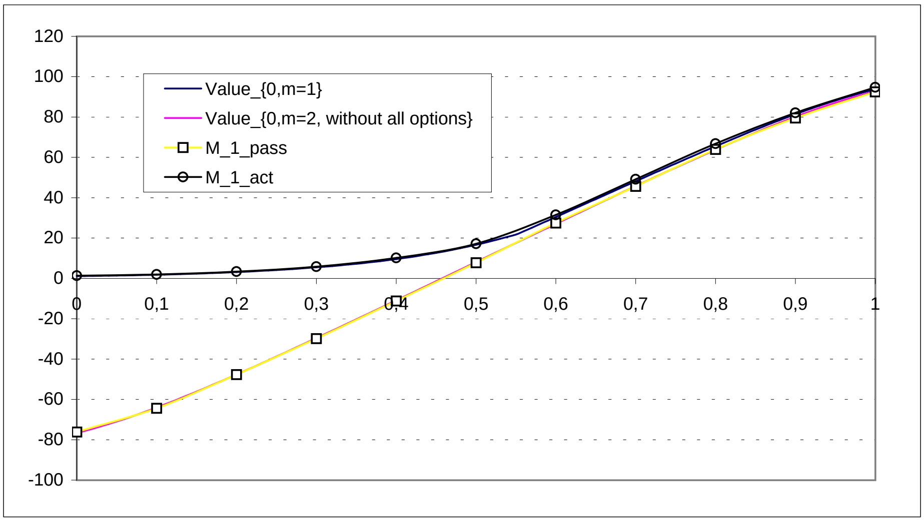

Figure 2: RPV and Monte Carlo Markov Chain Expected Expanded NPVs

Legend: The graph reports RPVs for both the active and passive management of the project, with and without real options, smoothed line without markers. Together with these values, the corresponding averages from the Monte Carlo Markov Chain for selected levels of $\theta_{t=0}$ are reported, namely $\theta_{t=0} = 0, .1, .2, \ldots, .9, 1$. These are represented with markers, circles and squares respectively.

In this paper we focus on IRR and PBP which were computed on the same CF_t series both passively and actively managed with the optimal exercise of real options. Table 1 reports results about internal rate of return computed according to Teichroew, while tables 2 and 3, to be read together with table 4, report results about the payback period, the recovery rate when the investment is not completely repaid and the life of the project in the case in which it is actively managed. As a matter of fact, passively managed projects last just as much as the technical life of the project.

A. Passive management:

θ_0	$Pr(IRR < 0)$	$Pr(IRR < r_f)$	$q_{.99}$	$q_{.95}$	$q_{.50}$	$M(IRR)$	$std(IRR)$	$max(IRR)$	Sk_{co}	Ku_{co}	$Pr(CF < 0\ \forall t)$	$Pr(IRR = n.a.)$
0.0	92.3%	96.7%	-493%	-430%	-30%	-100.6%	143.5%	28.8%	-1.53	3.88	18.6%	0.026%
0.1	86.8%	92.9%	-486%	-417%	-25%	-92.4%	139.2%	35.4%	-1.59	4.12	14.9%	0.025%
0.2	77.4%	85.4%	-481%	-401%	-18%	-81.2%	134.7%	40.5%	-1.69	4.54	10.5%	0.013%
0.3	65.2%	73.8%	-482%	-386%	-10%	-69.5%	131.3%	47.3%	-1.82	5.10	5.1%	0.015%
0.4	51.4%	59.6%	-481%	-361%	-1%	-52.4%	123.5%	52.3%	-2.12	6.50	1.5%	0.015%
0.5	36.4%	42.9%	-453%	-304%	10%	-28.8%	106.6%	62.2%	-2.65	9.61	0.2%	0.015%
0.6	22.9%	27.0%	-392%	-217%	21%	-4.7%	85.5%	68.5%	-3.37	15.03	0.0%	0.000%
0.7	12.7%	14.8%	-318%	-84%	33%	17.6%	65.5%	76.0%	-4.31	24.54	0.0%	0.003%
0.8	6.3%	7.2%	-223%	-11%	47%	38.0%	47.8%	83.1%	-5.38	40.41	0.0%	0.001%
0.9	2.6%	2.9%	-82%	22%	63%	57.1%	34.2%	89.4%	-6.77	71.90	0.0%	0.001%
1.0	1.0%	1.1%	0%	46%	79%	74.1%	22.9%	96.1%	-7.02	89.46	0.0%	0.003%

B. Active management:

θ_0	$Pr(IRR < 0)$	$Pr(IRR < r_f)$	$q_{.99}$	$q_{.95}$	$q_{.50}$	$M(IRR)$	$std(IRR)$	$max(IRR)$	Sk_{co}	Ku_{co}	Pr(Not taken)	$Pr(IRR = n.a.)$
0.0	25.1%	30.7%	-79%	-43%	18%	13.4%	28.7%	75.9%	-1.19	5.19	88.9%	0.000%
0.1	24.8%	30.3%	-75%	-42%	18%	13.3%	28.2%	84.6%	-1.13	4.80	84.5%	0.000%
0.2	24.5%	29.9%	-74%	-45%	19%	13.8%	28.5%	76.5%	-1.11	4.56	77.1%	0.000%
0.3	23.7%	28.8%	-71%	-46%	20%	14.1%	28.4%	79.9%	-1.10	4.35	65.3%	0.000%
0.4	23.7%	28.4%	-71%	-47%	20%	14.1%	28.4%	83.1%	-1.13	4.36	49.7%	0.000%
0.5	22.0%	26.6%	-72%	-47%	21%	14.9%	28.4%	80.8%	-1.20	4.51	29.4%	0.000%
0.6	20.5%	25.0%	-70%	-49%	21%	14.8%	27.5%	68.5%	-1.31	4.55	0.0%	0.000%
0.7	10.0%	12.3%	-56%	-21%	34%	29.1%	24.2%	76.0%	-1.46	5.81	0.0%	0.000%
0.8	3.9%	4.9%	-34%	6%	47%	44.1%	21.3%	83.1%	-1.46	6.61	0.0%	0.000%
0.9	1.1%	1.4%	-2%	27%	63%	60.1%	18.3%	89.4%	-1.50	7.17	0.0%	0.000%
1.0	0.2%	0.3%	27%	48%	79%	75.5%	14.8%	96.1%	-1.69	8.38	0.0%	0.003%

Table 1: IRR Variability

Legend: $q_{.99}$:= 99th quantile; $q_{.95}$:= 95th quantile; $q_{.5}$:= median; $M(IRR)$:= average of the simulated generalized Teichroew IRRs; $std(IRR)$:= standard deviation of the simulated generalized Teichroew IRRs; $Sk_{co} = \frac{\mu^3}{\sigma^3} \frac{E\left[(x-\mu)^3\right]}{\sigma^3}$: standardized skewness coefficient according to Irving Fisher; $Ku_{co} = \frac{\mu^4}{\sigma^4}$:= standardized kurtosis coefficient, a value of 3 indicates a normal, less than 3 a platicurtic, more than 3 a leptocurtic; $Pr(CF < 0\ \forall t)$:= probability of all negative cash flows for passively managed projects or of a never taken investment for actively managed ones; $Pr(IRR = n.a.)$:= probability of non existence of Teichroew IRR.

From table 1 we can conclude that real options [14] are effective not only in increasing expected return from investment but also in reducing downside risk. As a matter of fact, both expected values and medians for the actively managed project are definetly higher than those of the passively managed ones. This is true for all the levels of θ_0 but those above the initial investment threshold $\theta_{1\to 2} = .59$.

Hence, we can conclude that the option most effective in increasing expected value is the option to wait. Instead, the options to mothball and to restart are the least effective. No conclusion we can reach for the option to abandon being this thoroughly used for all the initial θ_0.

Moreover, downside risk in actively managed projects is definetly reduced. Not only the probability of having a negative return is decreased many times, see first two columns in table 1, but also, $VaR_{.99}$ and $VaR_{.95}$ are definetly reduced. In essence, real options reduce negative skewness of IRR distributions, decreasing occurrences in which IRR is below 100%, i.e. occurrences in which investment projects not only burn all the initially invested capital, namely $c_{1,2}$, but also require additional capital to be maintained over the project horizon. This dovetails with the results found by (Alesii, 2003) in which expanded NPV is rarely found to be lower than minus the initially invested sum. The fact that we have used here a version of IRR which ex ante does not always exist does not prevent us to draw this parallelism since it was not possible to get an IRR in a negligible number of cases, see last column in table 1.

It is worth noting that these results are obtained following a very specific dynamically optimal behavior. To limit ourselves to the exercise of the option to wait, higher IRRs are obtained because investment is implemented in slightly more than 10% of the cases for $\theta_0 = 0$ or more than 70% for $\theta_0 = .50$. Obviously, investment project is always implemented for levels above the investment threshold. [15] In this way, representative agent discards also all the occurrences in which every cash flow from investment is negative, see last but one column in panel B in table 1.

Payback Period is definetly reduced by real options, see mean and median columns in table 2. Although that is true, the most effective option in reducing PBP seems to be the option to wait. As a matter of fact, for levels of θ_0 above the investment threshold, mean and median are the same or not significantly different.

From results reported in the same table, real options appear even more effective in reducing downside risk for PBP, see $q_{.99}$ and $q_{.95}$ columns. Probabilities of not finding a PBP are drastically reduced because investment is not taken in a high percentage of cases as already noticed above for IRR results. As a matter of fact, probabilities that investment does not pay back are very low in the neighborhood of the investment

[14] In this case the option to wait, to mothball, to restart and to abandon, typically downside risk real options.

[15] To give a thorough understanding of this optimal behavior, the probability of exercising the option to mothball once, twice etc should be included. This would help to understand whether the optimal behavior endogenous to the expanded net present values is actually feasible or it is a simple abstraction. We save this extension for another paper.

threshold while are nil for the other initial levels of θ_0. Hence, probabilities that investment pays back when and if it is taken are very high.

Even when investment project does not pay back completely, real options allow to recover a higher percentage than in the case of a passively managed project, see table 3. For instance for $\theta_0 = .6$ in almost 20% of the cases investment does pay back initially invested capital only fractionally, see table 2. Recovery rates that can be read in table 3 are much higher expecially in the lower tail.

The beneficial effects of real options on both IRR and PBP, both in terms of improved expected values and reduced downside risk, are due to an investment behavior that delays investing until the state variable reaches the threshold level for exercising the option to wait and abandons the investment as soon as the abandonment threshold is reached. This shortens the expected life of the project when it is implemented, see table 4. For instance, for levels lower than the investment threshold $\theta_{1\to 2} = .59$, investment projects have a very short expected life being implemented late and abandoned early.

Considered all together, tables 1 - 4 give a whole string of parameters on which negotiation can take place between headquarters and division managers or between the shipowner and her banker. For instance, for a level of $\theta = .6$, the expected level of net present value is $E(NPV_p) = 27.18$ for the passively managed project while it is $E(NPV_e) = 30.51$ for the actively managed one, being the difference the value of the switching options to mothball, to restart and to abandon.[16] This present value has been translated into PBP and IRR. The levels of expected IRR are respectively $E(IRR_T)_p = -5\%$ and $E(IRR_T)_e = 15\%$. It is worth noting that rates of return are more effective in underlining the difference between an active and a passive management. Instead, the corresponding expected levels of PBP are not significantly different since the option to wait has been exercised for $\theta_0 > .59$. Although that is true, the option to abandon together with the switching options, to mothball and to restart, are effective in increasing the recovery rates. As a matter of fact, not only occurrences in which capital is not recovered are drastically reduced, from 7.8% to 0.5%, but also cases in which only a fractional recovery takes place are increased, from 12.0% to 19.8%. Even in those cases, capital recovered in the worst 5% occurrences is 11.8% if real options are used to manage the project, while it is 5.5% in cases in which it is passively managed.

In conclusion, in this paragraph we apply the methods previously devised to a sketched case study in shipping finance deriving the distributions of IRR, PBP together with recovery rates and life of the project. This allows us to reach conclusions not only about the value enhancing properties of real options but also about the risk reducing ones. As a matter of fact, IRR is not only increased in its expected values but also its downside risk is definetly trimmed down by real options. To the same token, PBP is reduced in its

[16] Results about NPV are provided in (Alesii, 2003).

expected values and its upside risk is definetly decreased.

Computing these thumb rules has allowed us to give a more intuitive insight into the optimal behavior endogenous to real options valuation. As a matter of fact we have shown what is the probability of exercising the option to wait and how the life of the project is affected by optimal investment delaying and abandoning strategies.

A. Passive management:

θ_0	$q_{.01}$	$q_{.05}$	$q_{.50}$	$M(PBP)$	std	Sk_{co}	Ku_{co}	Pr(Not taken)	Pr(No PB)	$Pr(PB)$	Pr(Frac Rec)
0.0	10.0	9.9	8.6	8.4	1.2	-0.69	2.84	0.00%	79.6%	6.3%	14.1%
0.1	10.0	9.8	8.2	8.0	1.3	-0.50	2.43	0.00%	69.8%	11.3%	18.8%
0.2	10.0	9.8	7.6	7.5	1.5	-0.27	2.16	0.00%	56.2%	20.5%	23.3%
0.3	9.9	9.6	6.9	6.9	1.7	0.00	2.01	0.00%	40.8%	33.7%	25.5%
0.4	9.9	9.4	6.0	6.2	1.8	0.31	2.11	0.00%	26.6%	49.2%	24.2%
0.5	9.8	9.0	4.9	5.3	1.8	0.71	2.62	0.00%	15.2%	65.7%	19.1%
0.6	9.6	8.2	3.8	4.4	1.7	1.25	4.00	0.00%	7.8%	80.2%	12.0%
0.7	9.0	6.6	3.0	3.5	1.4	2.02	7.43	0.00%	3.4%	90.7%	5.9%
0.8	7.6	4.5	2.4	2.7	1.0	3.24	16.81	0.00%	1.2%	96.6%	2.2%
0.9	4.6	3.0	1.9	2.1	0.6	5.25	45.73	0.00%	0.3%	99.1%	0.6%
1.0	2.7	2.1	1.7	1.7	0.3	8.86	159.02	0.00%	0.0%	99.9%	0.1%

B. Active management:

θ_0	$q_{.01}$	$q_{.05}$	$q_{.50}$	$M(PBP)$	std	Sk_{co}	Ku_{co}	Pr(Not taken)	Pr(No PB)	$Pr(PB)$	Pr(Frac Rec)
0.0	6.3	5.1	3.0	3.2	1.0	1.17	4.44	88.86%	0.0%	8.3%	2.8%
0.1	6.8	5.5	3.1	3.3	1.1	1.27	4.85	84.44%	0.0%	11.7%	3.9%
0.2	7.2	5.8	3.2	3.5	1.1	1.26	4.72	77.00%	0.0%	17.3%	5.7%
0.3	7.6	6.1	3.3	3.6	1.2	1.26	4.60	65.25%	0.0%	26.5%	8.2%
0.4	8.1	6.6	3.4	3.8	1.4	1.26	4.43	49.62%	0.1%	38.5%	11.8%
0.5	8.6	7.1	3.6	4.0	1.5	1.23	4.21	29.36%	0.2%	55.2%	15.2%
0.6	9.5	8.0	3.8	4.3	1.7	1.25	4.04	0.00%	0.5%	79.7%	19.8%
0.7	8.8	6.5	3.0	3.4	1.4	1.99	7.39	0.00%	0.0%	90.3%	9.6%
0.8	7.3	4.5	2.4	2.7	1.0	3.14	16.25	0.00%	0.0%	96.4%	3.6%
0.9	4.5	3.0	1.9	2.1	0.6	4.93	41.52	0.00%	0.0%	99.1%	0.9%
1.0	2.7	2.1	1.7	1.7	0.3	7.64	122.96	0.00%	0.0%	99.8%	0.2%

Table 2: PBP Variability

Legend: $q_{.01}$: first centile; $q_{.05}$:= first ventile; $q_{.5}$: median; $M(PBP)$: mean of computed payback periods; std: standard deviation of computed payback periods; $Sk_{co} = \frac{\mu^3}{\sigma^3}\frac{E[(x-\mu)^3]}{\sigma^3}$: standardized skewness coefficient according to Irving Fisher; $Ku_{co} = \frac{\mu^4}{\sigma^4}$:= standardized kurtosis coefficient, a value of 3 indicates a normal, less than 3 a platicurtic, more than 3 a leptocurtic; Pr(Not taken):= probability of not taking the investment project; Pr(No PB):= probability that the investment project does not pay back at all; $Pr(PB)$:= probability that the investment pays back completely and more; Pr(Frac Rec):= probability that the investment project yields back only a fraction of the initially invested lump sum, see table 3 for the distribution of these fractional recovery.

A. Passive management:

θ_0	Min	$q_{.99}$	$q_{.95}$	$q_{.5}$	M(ReRa)	std	max	Sk_{co}	Ku_{co}
0.0	00.0%	00.5%	3.0%	39.0%	42.2%	28.0%	99.5%	0.30	1.95
0.1	00.0%	00.5%	3.5%	41.5%	44.2%	28.4%	99.5%	0.23	1.89
0.2	00.0%	00.5%	4.0%	45.5%	46.5%	28.3%	99.5%	0.13	1.84
0.3	00.0%	01.0%	4.5%	47.5%	48.1%	28.4%	99.5%	0.06	1.83
0.4	00.0%	01.0%	5.0%	50.0%	49.7%	28.3%	99.5%	-0.02	1.84
0.5	00.0%	01.0%	5.5%	51.5%	50.8%	28.0%	99.5%	-0.05	1.86
0.6	00.0%	01.0%	5.5%	52.0%	51.1%	28.0%	99.5%	-0.07	1.86
0.7	00.0%	01.0%	6.0%	52.5%	51.5%	27.8%	99.5%	-0.09	1.88
0.8	00.0%	00.5%	5.0%	53.0%	51.6%	27.6%	99.5%	-0.13	1.92
0.9	01.0%	02.3%	7.3%	54.5%	52.2%	27.3%	99.5%	-0.12	1.91
1.0	00.0%	00.0%	6.5%	51.8%	51.6%	26.3%	96.5%	-0.11	2.05

B. Active management:

θ_0	Min	$q_{.99}$	$q_{.95}$	$q_{.5}$	M(ReRa)	std	max	Sk_{co}	Ku_{co}
0.0	01.5%	15.6%	28.5%	72.3%	68.3%	22.3%	99.8%	-0.55	2.34
0.1	00.3%	13.8%	25.5%	70.3%	66.9%	23.1%	99.8%	-0.50	2.27
0.2	01.3%	10.3%	22.8%	69.3%	65.5%	23.8%	99.8%	-0.48	2.23
0.3	00.3%	08.5%	19.3%	66.0%	62.9%	24.5%	99.8%	-0.39	2.13
0.4	00.3%	07.1%	17.8%	63.8%	61.2%	25.3%	99.8%	-0.32	2.03
0.5	00.0%	04.8%	14.8%	60.8%	58.5%	25.9%	99.8%	-0.24	1.98
0.6	00.0%	03.4%	11.8%	55.1%	54.5%	26.5%	99.8%	-0.09	1.89
0.7	00.0%	10.8%	22.8%	65.8%	63.4%	22.9%	99.8%	-0.38	2.25
0.8	01.3%	21.3%	36.8%	75.3%	72.0%	19.0%	99.8%	-0.71	3.01
0.9	24.8%	35.3%	48.0%	79.8%	77.4%	15.2%	99.8%	-0.77	3.16
1.0	52.8%	53.3%	63.3%	81.8%	81.1%	11.6%	99.3%	-0.26	2.16

Table 3: Recovery Rate Variability

Legend: The table reports descriptive statistics for recovery rates of fractionary recovering investments. The probability of these occurrences are those reported in the last column of table 2. $q_{.99}$:= 99th quantile; $q_{.95}$:= 95th quantile; $M(ReRa)$= average of the recovery rates; *std*: standard deviation computed on recovery rates; $Sk_{co} = \frac{\mu^3}{\sigma^3} \frac{E\left[(x-\mu)^3\right]}{\sigma^3}$: standardized skewness coefficient according to Irving Fisher; $Ku_{co} = \frac{\mu^4}{\sigma^4}$:= standardized kurtosis coefficient, a value of 3 indicates a normal, less than 3 a platicurtic, more than 3 a leptocurtic. Values of -1 in both coefficients indicate degenerate distributions on which it was not possible to compute them.

θ_0	Min	$q_{.99}$	$q_{.95}$	$q_{.5}$	M(Project Life)	std	max	Sk_{co}	Ku_{co}
0.0	0.5	1.0	3.0	5.0	5.1	1.4	9.0	-0.24	3.05
0.1	0.5	2.0	3.0	5.5	5.4	1.5	9.0	-0.24	2.80
0.2	0.5	2.5	3.0	6.0	5.9	1.6	9.5	-0.30	2.65
0.3	0.5	2.5	3.5	6.5	6.4	1.7	9.5	-0.40	2.53
0.4	0.5	3.0	4.0	7.5	7.1	1.8	9.5	-0.63	2.62
0.5	0.5	3.0	4.5	8.5	7.9	1.8	9.5	-1.08	3.29
0.6	2.0	3.5	5.5	10.0	9.3	1.5	10.0	-2.20	7.00
0.7	2.0	4.5	6.5	10.0	9.5	1.2	10.0	-2.72	10.00
0.8	2.5	5.0	7.5	10.0	9.7	1.0	10.0	-3.29	14.04
0.9	3.0	6.0	8.0	10.0	9.8	0.8	10.0	-3.89	19.33
1.0	3.0	6.5	8.5	10.0	9.8	0.6	10.0	-4.36	23.87

Table 4: Life of the Actively Managed Project Variability

Legend: The table reports descriptive statistics for actively managed projects lives. The probability of these occurrences are the complement to one of Pr(Not taken) reported in table 2. $q_{.99}$:= 99th quantile; $q_{.95}$:= 95th quantile; M(Project Life)= expected life of the actually taken projects; *std*: standard deviation computed on recovery rates; $Sk_{co} = \frac{\mu^3}{\sigma^3} \frac{E\left[(x-\mu)^3\right]}{\sigma^3}$: standardized skewness coefficient according to Irving Fisher; $Ku_{co} = \frac{\mu^4}{\sigma^4}$:= standardized kurtosis coefficient, a value of 3 indicates a normal, less than 3 a platicurtic, more than 3 a leptocurtic. Values of -1 in both coefficients indicate degenerate distributions on which it was not possible to compute them.

4 Conclusions

In this paper it is devised a new method to translate into two of the most used thumb rules, namely PBP and IRR, the effect of the optimal exercise of real options. Using a numerical method already used by (Alesii, 2003) to derive project at risk in the presence of real options, we were able to derive the whole distribution of the Teichroew version of the internal rate of return and of simple payback period together with fractional recovery rate and life of the project in the presence of the option to wait, to mothball, to restart and to abandon.

The effect of these downside risk options on the computed thumb rules is not only in improving their expected value but also in reducing their shortfall. Moreover, this paper explores when real options are actually exercised giving a more intuitive insight into the endogeneity of real options valuations models with respect to actual optimally dynamic behavior. This in turn could help to consider real options models not as a black box but simply as a way of quantifying optimal management of an industrial project.

A possible extension of this paper would be in delving into the description of this dynamic optimal behavior giving practitioners a probable sequence of optimal actions they should conform to in order to create value with the optimal exercise of real options, e.g. in this case how many time the options to mothball and to restart are exercised. This would allow to compare current practice with optimal project management according with real options and to conclude whether these are a simple pure abstraction or a realistic model.

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A Convergence Tests

The gist of the present paper stays in the convergence of the expected Running Present Value in the DP procedure and the average of the Markov chain Monte Carlo simulations. This fact guarantees that these two procedures model the same dynamic optimal behavior although the former is based on backward induction while the latter is based on a forward computation of the expanded net present value.

Therefore, this appendix reports some results about convergence of expected value of Markov chain Monte Carlo simulations towards the values produced by the Dynamic Programming procedure. We have checked both the results based on active and passive management. Moreover, we have checked whether the initial level has some influence on convergence choosing a level in the middle of the discretized state variable space $\theta_0 = .5$ together with the minimum and the maximum, $\theta_0 = 0$ and $\theta_0 = 1$.

To test convergence we have computed for 500 experiments expected values of the Markov chain Monte Carlo simulations for several number of trials n, namely 1000, 5000, 10000, 20000, 40000 and 80000. Changing the step allowed us to test the variability of the results in different neighborhoods of n. Because of this, experiments have a higher degree of granularity in the hundreds while this decreases as the number of experiments increases. This experiment design was chosen to show that results variability decreases as the number of experiments increases. We have considered the relative difference between the averages as computed above and the corresponding RPV, see expression (9), where the exponents ^{f} and ^{b} indicate respectively forward and backward computation.

$$\left\{ \begin{array}{c} Relative \\ Difference \end{array} \right\} = \frac{M_1\left(NPV^f_{\theta_0}\right) - E^*\left(RPV^b_{\theta_0}\right)}{E^*\left(RPV^b_{\theta_0}\right)} \tag{9}$$

Results are reported in table 5. Observing means and medians together with skewness and kurtosis coefficients, we can conclude that results distributions fairly approximate normal distributions expecially for high number of trials. Because of this we can reasonably detect convergence from mean and standard deviations of (9).

Passive management results show a complete convergence in the mean and a fast convergence in the standard deviation of the results. This is true for all the three levels of θ_0 that have been chosen. Instead, active management results depend on the initial level of θ_0. While for low levels relative difference converges to around 1.5%, for middle and high levels this is definitely good, between 0.06% and 0.003%. However it should be stressed that absolute differences are really meaningless when compared to the initial investment.

It is impressive that in both active and passive management results, convergence in standard deviation follows the same proportions for the same increases in the number of trials. Convergence in standard deviation does follow the usual square of the number of trials rule. As a matter of fact, to double the accuracy of a simulation we must quadruple the number of trials.

In conclusion, passive management results convergence is not dependent on the initial value of θ_0, while active management results depend, to a certain extent, on the initial value. However, averages of forward computed NPV from the Markov chain Monte Carlo simulations converge to the Running present values computed in the backward induction process with a relative difference that at most, on the average, reaches 1.5% and is never lower than 0.003%. A possible explanation of this difference, which would be hardly acceptable for financial derivatives models, stays in the short life of the project and in the coarse time discretization chosen.

A) Passive Management:								**B) Active Management:**						
θ		1000	5000	10000	20000	40000	80000		1000	5000	10000	20000	40000	80000
0	M_1	0.064%	-0.027%	0.006%	0.007%	0.017%	0.002%	M_1	-3.041%	-1.577%	-1.464%	-1.547%	-1.982%	-1.582%
	median	0.045%	0.001%	0.008%	0.010%	0.014%	0.004%	median	-2.249%	-1.675%	-1.525%	-1.685%	-2.162%	-1.487%
	std	1.423%	0.650%	0.447%	0.325%	0.243%	0.163%	std	18.939%	8.344%	6.323%	4.407%	3.292%	2.192%
	skew	0.205	-0.057	0.017	0.209	-0.173	0.023	skew	0.241	0.013	0.111	0.135	0.056	0.076
	kurt	-0.209	-0.016	-0.023	0.518	0.147	-0.431	kurt	-0.109	-0.192	-0.078	0.062	-0.029	0.073
	min	-3.444%	-1.856%	-1.407%	-0.838%	-0.812%	-0.433%	min	-45.645%	-25.913%	-20.502%	-13.689%	-13.106%	-7.567%
	max	4.671%	2.042%	1.411%	1.367%	0.653%	0.441%	max	66.389%	24.944%	16.498%	12.580%	6.553%	5.193%
0.05	M_1	1.777%	0.117%	0.146%	-0.381%	0.108%	-0.018%	M_1	0.202%	-0.144%	-0.121%	-0.175%	-0.052%	-0.063%
	median	1.816%	0.174%	0.040%	-0.516%	0.126%	-0.010%	median	0.231%	-0.149%	-0.109%	-0.215%	-0.050%	-0.067%
	std	16.699%	7.404%	5.576%	3.793%	2.814%	2.075%	std	5.380%	2.487%	1.825%	1.199%	0.932%	0.684%
	skew	0.134	-0.023	0.014	0.132	0.044	-0.074	skew	0.138	-0.047	0.000	0.198	-0.106	0.098
	kurt	0.157	0.074	-0.088	0.270	0.466	0.041	kurt	0.110	-0.071	-0.341	0.215	0.203	-0.047
	min	-44.995%	-24.301%	-17.876%	-14.204%	-7.972%	-5.848%	min	-13.925%	-9.673%	-5.252%	-4.340%	-2.878%	-1.949%
	max	64.374%	21.155%	13.731%	11.593%	9.494%	6.260%	max	17.990%	7.064%	5.370%	3.978%	2.850%	1.977%
1	M_1	0.040%	-0.037%	-0.038%	-0.005%	0.005%	0.001%	M_1	0.033%	-0.036%	-0.036%	-0.008%	0.002%	-0.003%
	median	0.025%	-0.048%	-0.048%	-0.011%	-0.002%	-0.005%	median	0.026%	-0.034%	-0.043%	-0.015%	-0.005%	-0.009%
	std	1.271%	0.540%	0.381%	0.279%	0.194%	0.141%	std	1.199%	0.512%	0.362%	0.262%	0.182%	0.133%
	skew	-0.100	0.055	-0.046	0.039	-0.059	0.005	skew	-0.093	0.107	-0.026	0.059	-0.058	0.006
	kurt	0.238	0.275	-0.038	-0.144	0.010	0.263	kurt	0.281	0.302	-0.014	-0.106	0.024	0.267
	min	-4.607%	-1.810%	-1.202%	-0.860%	-0.527%	-0.434%	min	-4.410%	-1.754%	-1.168%	-0.822%	-0.480%	-0.419%
	max	3.993%	1.601%	1.009%	0.861%	0.579%	0.469%	max	3.613%	1.558%	0.947%	0.843%	0.556%	0.449%

Table 5: Convergence Tests Results

Legend:Monte Carlo simulations have been replicated 500 times for different number of trials, namely 100, 500, 1000, 2000, 4000 and 8000 for three different initial levels of the state variable $\theta = 0.0, 0.5, 1.0$. Statistic have been computed on the difference between the average of the trials and the the Running Present Value at time $t = 0$ in the DP procedure divided by the latter. Skewness and kurtosis have been computed according to the following expressions:

$$\hat{\gamma} = \frac{n}{(n-1)\cdot(n-2)} \cdot \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s}\right)^3$$

$$\hat{\delta} = \left\{ \frac{n\cdot(n+1)}{(n-1)\cdot(n-2)\cdot(n-3)} \cdot \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s}\right)^4 \right\} - 3 \cdot \frac{(n-1)^2}{(n-2)\cdot(n-3)}$$