

Equity Carve-outs as Acquisitions of Strategic Real Options

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Abstract

Equity carve-outs, the listing of a stake in a subsidiary on the stock market, are empirically transitory arrangements. This paper studies decisions aimed to gaining access to the choice over a "second stage" event, namely either a sell-off or a buy-back decision. Hence, an equity carve out can be seen as a compound option. The decision is driven by learning over strategic synergies between the parent firm and the subsidiary, which evolves stochastically over time. We find that the decisions to carve-out, to buy-back and to sell off are critically influenced by uncertainty over future synergies. We also find that there is an optimal amount of shares sold, which is positively related to uncertainty over future synergies.

1 Introduction

In an equity carve-out (ECO), a parent company sells a portion of a subsidiary's common stock through an initial public offering. ECOs constitute a significant fraction of IPOs which are still expanding over time: in the '90s, around 10% of the IPOs were ECOs; in 1993, five of the six largest IPOs in the US capital market history were ECOs (Allen and McConnell 1998).¹

The selling firm is usually a large firm and the subsidiary represents a small fraction of the parent activities. The typical ECO candidates are subsidiaries characterized by strong growth prospects, independent borrowing capacity, a unique corporate culture, unique industry characteristics and/or problematic management performance measurement (Miles, Woolridge, and Tocchet 1999). ECOs appear more common in industries with a high degree of value uncertainty, high sales growth and considerable investments in R&D and marketing (Allen and McConnell 1998). This impression is confirmed by the high price/earnings (Schipper and Smith 1986), market-to-book ratios (Powers 2000) and the high R&D expenses of carved-out subsidiaries (relative to the parent).²

Many studies document parent abnormal returns of approximately 2% in the days surrounding the initial ECO announcement. Schill and Zhou (2001) find even higher abnormal returns, 25%, but their sample is focused on Internet stocks and is particularly small. Furthermore, the carved-out firm outperforms control benchmarks significantly after the IPO (Powers (2000), Anslinger, Carey, Fink, and Gagnon (1997), Anslinger, Bonini, and Patsalos-Fox (2000) and Miles, Woolridge, and Tocchet (1999)) or shows an improvement relative to the performance prior the carve-out (Michaely and Shaw 1995). See Table 1 for a review on the main results of these studies.

Notwithstanding the evidence on the value enhancing potential of ECOs, there is no consensus about the sources of these gains. ECO may promote information gathering by investors and so improves access to capital (the 'financing rationale', Holmstrom and Tirole (1993) and Powers (2000)); by increasing the transparency of the performance of the subsidiary, it allows market-based incentives for the subsidiary' management ('restructuring rationale', Schipper and Smith (1986) and Chemmanur and Paeglis (2000)). Table 2 summarizes the motives for performance on ECO related to the re-

¹The most emblematic carve-outer is the Thermo Electron: in 1997, seven of its already publicly traded companies carved-out fifteen further public companies (Allen 1998).

²Despite the important role that growth opportunities play in the corporate finance literature, there is no consensus on how to measure the value of a firm's investment opportunity set. Researchers and investors rely on proxy variables such as market-to-book and P/E ratios (Adam and Goyal 2000).

structuring nature obtained with the aid of questionnaire and interviews by Schipper and Smith (1986).

Nanda (1991) argues that when a parent firm considers its own assets to be undervalued by the market, it prefers to carve out a subsidiary instead of issuing shares to finance a new profitable project. Knowing this, the market perceives the ECO as a positive signal on the value of the parent's assets value and so an increase in the stock price occurs.

An interesting feature of ECO is that within 2-6 years, most of these listings have ceased to exist, as a result of a so-called 'second stage' event. Schipper and Smith (1986) found that 44 of the 73 examined subsidiaries were later reacquired by the parent, completely divested, spun-off, or liquidated (see Table 3). Klein, Rosenfeld, and Beranek (1991) found similar results: 56% of all carve-outs are re-acquired and 38% are followed by a sell-off (see Table 4). Hand and Skantz (1999b) find that 42,7% of the carved-out subsidiaries are subject to a sell-off, 17,4% are reacquired, and 13,2% are spun-off (Table 5). A recent study of the Salomon Brothers also found that between 1983-1995, 29% of the ECOs have been spun off or split-off while 15% they have been bought back. Similar results have been found by Miles, Woolridge, and Tocchet (1999).

The results presented in these tables offer several interesting facts. The decision on the final second event chosen by the parent, is correlated to both the percentage retained by the parent as well as the elapsed time (Klein, Rosenfeld, and Beranek 1991). The shorter the time span, the greater the likelihood that the second event will actually be a sell-off of the remaining interest. This suggests that parents choose to sell-off faster than to re purchase. When a larger stake is retained, the ECO is more often reacquired. Overall, ECOs seem to be a temporary stage in acquainting the market with the value of the subsidiary, which appears to be a relatively fast process.

Perhaps the objectives of an ECO may well be accomplished with only a temporary flotation of the subsidiary shares. For example, the need for external equity financing of growth for the subsidiary will decline once the subsidiary's investments mature to the point where they generate enough profits for internal equity financing. A temporary listing of a subsidiary may also fulfill the objective of informing investors and potential acquirers about the subsidiary's growth potential as potential firm ahead of a sale.

Even the need for an improved management incentive program, may be related to a temporary restructuring stage. At the same time, the autonomy granted to the subsidiary may become costly, if strategic synergies with the parent company call for closer co-ordination of activities. Since, there are fixed costs for a separate listing, once the benefits no longer outweigh these costs, the parent company would either sell the remainder of its stake in the

subsidiary or reacquiring the floated minority share on the market.

Thus, we argue that performing an ECO can be seen as the creation of future real options, either a "call option to reacquire" or "put option to sell". The ECO is then a phase in a dynamic strategy, in which the critical uncertainty concerns the synergies of the subsidiary with the parent. As management obtains new information (in part generated by the ECO itself) on the presence of positive or negative strategic synergies, the subsidiary will be either reacquired by the parent or sold-off completely.

The key ingredient of the strategic option connected to the ECO is therefore that the final strategic decision needs not to be pre planned by the parent, but can be left to a future date. More information will tend to become available, once the subsidiary has traded publicly for some time (Klein, Rosenfeld, and Beranek 1991). In general, upon announcement of the ECO the parent company normally does not commit to either solution, as this would obviously decrease flexibility and destroy strategic options. Once a choice is announced, deviating from it at a later stage would also harm the company's credibility (Eijgenhuijsen 1999). As in a classic real option, the value of an ECO strategy depends on the flexibility gained. This raises the question whether, its value will be higher, the higher is the strategic uncertainty in its business.

In our model we argue that owning a subsidiary creates interactions, which change over time. A close co-ordination between two firms may create operating, marketing and financial synergies, which increase the total value of the group. These synergies, however, may well turn negative. The lack of focus on the core business may create conflicting business interests; the lack of market-based incentives for the subsidiary's management can outweigh the positive synergies.

If synergies become negative, the parent firm can sell off or spin-off the subsidiary. Yet intuitively is not optimal to do an irreversible disinvestment as soon as the synergies turn negative. If changes in technology, regulation or demand cause synergies to turn positive again, the parent firm may not be able to reacquire the subsidiary and would miss some profit opportunities.

The ECO appears to constitute a valid strategy to 'buy time': it gives the opportunity to mitigate the negative synergies, by selling out part of the share, while retaining the flexibility to subsequently reacquire or to sell-off the subsidiary depending on how synergies evolves over time.

In this paper, we use a real option pricing theory in order to determine the optimal timing to perform the carve out, and thereafter to exercise the sell-out or buy back options.³

³For an introduction on real option methodology see Dixit and Pindyck (1994).

A relevant element in the model is played by the fraction of retained shares during the carve-out as it heavily influences the timing of the decisions with the uncertainty. In particular in the moment in which we impose a relation between retained shares and underpricing as indicated by the signaling models in IPOs, an optimal fraction of retained shares can be found. This stake retained varies depending on the uncertainty of the synergies, although it is generally quite high as predicted by Klein, Rosenfeld, and Beranek (1991).

In the next section we present the basic model. Section 3 and 4 study respectively the effect of uncertainty and drift of synergies on the decision process Section 4 presents some extensions. And the final section summarizes the results and concludes.

2 The model

2.1 The Time Structure

To illustrate the strategic value of the ECO option, we present a simple model to describe the optimal decision process of the parent with an ECO option.

We distinguish two components of the subsidiary firm's value: the value as an independent units given by the expected flow of profits, π , and the expected value over time of synergy flow, s . Hence, for the parent the total value of the subsidiary is given by the discounted flow of profits and synergies. For simplicity, we assume the profit flow, π , is non stochastic.

The value of the synergies is uncertain due to rapidly advancing technology, changing regulations or unpredictable demand. Thus, we assume that this synergy flow, s_t , evolves over time following an Arithmetic Brownian Motion:

$$ds_t = \mu dt + \sigma dz \tag{1}$$

where:

μ is the drift term;

dt is the time variation;

σ is the variance parameter;

dz is the increment of a Wiener process;

For the moment, we consider the special case of no drift, $\mu = 0$, as it allows more tractable solutions. In an extension we will study the effect of the drift on the decision process.

The choice of the Arithmetic Brownian Motion rather than an other stochastic processes, is due to the consideration that synergies move continuously over time and that can take on negative values, so that the total value of the subsidiary for the parent, $\frac{\pi+s_t}{r}$ with r the discount factor, can be negative.

The firm is sold at a price based on its value, $\frac{\pi}{r}$, without considering the synergies as they are not captured by other investors. Obviously upon selling off, all internal synergies (positive or negative) disappear, while in a partial sale, they are reduced proportionally.

Furthermore, taking a firm public implies some costs, among which the most distinguished is the underpricing of the initial stake sold.⁴

Figure 1 illustrates the decision tree, the management of the parent firm has to face at each instant. At the beginning, when the management holds full control of the subsidiary, he can at each instant either sell-off the subsidiary immediately, perform an ECO or postpone the decision and retain it as a fully owned subsidiary.

By selling off its subsidiary at t^* , the company will rid itself of all synergies ($\frac{s_t^*}{r}$) and receive the stand-alone value for the subsidiary assets. Yet, a complete sell-off will eliminate the possibility to regain control of the subsidiary if it were profitable to do so; it represents a loss of strategic options. In general, the former subsidiary, now independent, might be restructured in a way that makes a repurchase unattractive. More critically, if positive synergies were to emerge between the sector of the parent company and the former subsidiary, the subsidiary might be acquired by a competitor with similar ambitions. Moreover, the management of the sold-off subsidiary, having realized the value of the potential synergies, might not be willing to return under the control of its former parent. Even if it were possible to re-acquire the former subsidiary, the purchase would require a significant premium since the seller would demand the full added value the subsidiary has for its former parent. Finally, a re-purchase after an initial sell-off would also seriously damage management's reputation. Therefore, we assume that there cannot be a subsequent "second event" in case of a complete sell-off at t_0 .

When instead a firm is carved-out, the parent is retaining a fraction, α , of the total subsidiary shares and incurs in a proportional underpricing,

⁴We do not want to go into the formalization of why there is underpricing and the definition of the information asymmetries. We refer for that to the existing literature. The most common explanation of underpricing phenomenon is the cost of signaling (see Ibbotson and Ritter (1995) and Welch and Ritter (2002)). For a modelization of underpricing and fraction selling during IPO see among the others Leland and Pyle (1977), Grinblatt and Hwang (1989), Welch (1989) and Rossetto (2002).

$(1 - \alpha) \frac{\delta}{r}$.⁵ At that moment outside investors gather new information on the subsidiary value and this information will be incorporated in the market price. It is therefore possible to sell the rest of the shares at a later date without any discount.

The ECO option allows the parent to alter its future decision depending on the actual future value of s_t . If later on, s_t keeps on decreasing, the best solution will be to exercise the option to sell-off the remaining subsidiary shares, in which case the parent would receive $(1 - \alpha) \frac{\pi}{r}$. If, instead, the parent learns that it would be profitable to remain integrated with the subsidiary as s_t increases over time, it will choose to capture all the synergy benefits: it will reacquire the floated shares, paying $\alpha \frac{\pi}{r}$ to the subsidiary's shareholders and regaining full control of its assets $\frac{\pi + s_t}{r}$.⁶ In either case, an ECO through a partial selling of the share reduces the discount, δ/r , needed to place shares of a little-known subsidiary on the market, and reduces the conglomeration synergies s_t in proportion to the amount sold.

An alternative to the sell off and the carve-out is the retention of the subsidiary. This would mean the company would keep on encountering the synergies (positive or negative) in their entirety, s_t . At the same time, however, the parent firm could "wait and see" how the synergies evolve over time. In case they were too low at a later date, the company has still the option to rid itself of the subsidiary by a complete sale or a carve-out.

2.2 The basic model

For the moment we do not consider the case of sell out; later we will study it and compare it with the carve-out.

At the beginning, at each instant, the parent firm has to decide if it is the case to carve out or to hold control without taking any action.

Intuition suggests that when the synergies turn too low and the subsidiary is no more attractive, the parent will perform a carve-out incurring in some underpricing costs. Subsequently depending on the synergies improvement or worsen, the parent firm will decide to sell off or to reacquire the firm.⁷ Hence, we can define three threshold synergy flow levels, for which it is optimal for the parent to exercise the American option to carve out, s_C^* , to sell-out, s_S^* , and to buy back, s_B^* .

⁵We define underpricing as $\frac{\delta}{r}$, instead of the more simple D , as it will be more practical during the calculations.

⁶Note that it is critical for the parent firm to retain control to avoid having to pay a premium reflecting all synergy gains.

⁷We assume that there are no costs related to the loss of control.

Proposition 1 *The optimal thresholds to carve out, to buy-back and sell out are given by the solution of the following system:*

$$\beta s_B^* = \beta s_S^* + \ln \frac{-1 + \alpha - \sqrt{1 - 2\alpha + \alpha^2 \beta^2 s_S^{*2}}}{\alpha (1 + \beta s_S^*)} \quad (2)$$

$$\beta s_S^* = \beta s_B^* + \ln \frac{-\alpha + \sqrt{-1 + 2\alpha + (1 - \alpha)^2 \beta^2 s_B^{*2}}}{(1 - \alpha) (1 + \beta s_B^*)} \quad (3)$$

$$\beta s_C^* = -1 - \beta \delta - \text{ProductLog} \left[\frac{-2 \left((1 - \alpha) \exp^{\beta s_B^*} + \alpha \exp^{\beta s_S^*} \right)}{(1 - \alpha) (\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*}) \exp^{1 + \beta \delta}} \right] \quad (4)$$

where $\text{ProductLog}[z]$ gives the principal solution for w in $z = w \exp^w$. and $\beta = \frac{\sqrt{2r}}{\sigma}$

Proof. See Appendix A ■

These threshold synergy values cannot be derived analytically, but the solutions for βs_B^* and βs_S^* can be computed graphically for any level of share sold during the carve-out. In Fig. 2, we present these thresholds in addition to the threshold value to carve out in case of no underpricing, $\delta = 0$. Furthermore in the following propositions we derive analytically many of their features.

To better understand the economic implications of the results, we use as benchmark the following thresholds: the synergy levels for which it is optimal to buy back and to carve-out when no option to sell out exists, s_{BB}^b and s_{CB}^b , the synergy levels for which it is optimal to sell out and to carve-out when no option to buy back exists, s_{SS}^b and s_{CS}^b , and the synergy level for which it is optimal to carve-out when no option to buy back and to sell out exist, s_{CC}^b . These values are computed in Appendix B.⁸

Proposition 2 *The optimal synergy level for which it is optimal to sell out, s_S^* , always exists, is negative and is smaller than $s_{SS}^b = -\frac{1}{\beta}$.*

Proof. See Appendix C ■

Proposition 3 *The optimal synergy level for which it is optimal to sell out, s_S^* is smaller the higher the amount of shares retained, α . More precisely, when $\alpha = 0$, s_S^* tends to minus infinity and when $\alpha = 1$, $s_S^* = -\frac{1}{\beta}$.*

⁸Note that in the benchmark cases the parent payoffs are influenced by the fraction of shares retained, while the optimal thresholds are constant in respect of the amount of shares retained: the elimination of the interactions between the option to sell out and buy back, makes the optimal thresholds indifferent to the fraction of shares retained.

Proof. See Appendix E ■

These propositions imply that when the subsidiary is already carved out and the synergies are too low, it is optimal for the parent to sell out and that this level of synergies depends heavily on the how much it has been retained during the carve out. The more has been retained during the carve out, the more severe is the influence of the synergies on the payoff of the parent and so the parent is induced to sell out at higher (less negative) synergies giving up the opportunity to buy back the subsidiary and to exploit eventually the positive synergies for higher (less negative) synergies. On the contrary, the lower the fraction retained, the less is the influence of the synergy on the parent payoff and hence the parent is willing to give up the opportunity to buy back only when the synergies are heavily negative.

Proposition 4 *The optimal synergy level for which it is optimal to buy back, s_B^* , always exists, is positive and is always higher than $s_{BB}^b = \frac{1}{\beta}$*

Proof. See Appendix D ■

Proposition 5 *The optimal synergy level for which it is optimal to buy back, s_B^* , is higher the higher the amount of shares retained. More precisely, when $\alpha = 0$, $s_B^* = \frac{1}{\beta}$ and when $\alpha = 1$, s_B^* tends to infinity.*

Proof. See Appendix E ■

The parent firm buys back the subsidiary only when the synergies are positive. The level of the synergies for which it is optimal to buy back depends on the shares retained during the carve-out. The higher the amount of shares retained the higher the synergies for which it is optimal to give up the option to sell out: the synergies that the parent foregoes during the carve out phase are few and so it prefers not to gain fully from the positive synergies and to hold the option to sell out. When instead the parent has sold almost all the shares during the carve out the optimal level of synergies for which it is optimal to buy back is very low: the synergies that the parent is foregoing are relevant and it prefers to give up the option to sell out taking the risk the synergies turn negative again.

Eventually, comparing these results with the cases in which there is only one of the two options (see Appendix B), the optimal levels of synergies are more negative in case of sell out option and more positive in case of the buy back option. Intuitively this derives from the fact that when both options exist the firm prefers not to take a definitive decision and to have a more indications on the future synergies to be sure not to regret the renounce of the options.

In Fig. 3 it is shown the size of the range of the synergies for which it is convenient to be in the carve out state for different levels of share retained without taking actions. The size of the range is a convex function of α , that tends to infinity for α that tends to 0 and 1 and that has a minimum for $\alpha = 0.5$. This implies that the size of the range of the synergies for which it is convenient to remain in the carve out status and, hence, the expected time of being in the carve out status heavily depends from the amount of share sold during the carve out: the more or the less the parent retains the more the subsidiary stays in the carve out status.

This element together with the empirical tendency to retain many shares (around 80%) during ECOs, indicates that parent firm wants to hold the flexibility of the carve out status as much as possible holding the control on the subsidiary, preferring to keep far away the decision to buy back as much as possible while having the option to sell out very close to be exercised.

Proposition 6 *The optimal synergy level for which it is optimal to perform the carve out exists when:*

$$\alpha < \frac{\exp^{2\beta s_B^* - 2\beta s_B^* - \exp^{2\beta s_S^*}}}{\exp^{2\beta s_B^* - 2\beta s_B^* - \exp^{2\beta s_S^*}} + 2 \exp^{\beta s_S^*}} \cong 0.997 \quad (5)$$

When the threshold exists, it is always larger than $-\frac{1}{\beta} - \delta$, it decreases as α increase and tends to $s_{BC}^b = -\frac{1}{\beta} - \delta - \frac{1}{\beta} \text{ProdcutLog}[-2 \exp^{-2-\beta\delta}]$ when α tends to 0 and to $s_{SC}^b = s_{CC}^b \frac{-1-\delta}{\beta}$ when α tends to 1.

Proof. See Appendix F ■

As shown in Fig. 2, the parent will perform the carve out always for negative synergy levels. Furthermore, at the optimal synergy level the losses due to the negative synergies are bigger than the costs due to underpricing and this is why the parent prefers to incur in the underpricing rather than keep on incurring in the negative synergies.

The carve-out option is exercised for more negative synergy values than if no option to sell out exists. In case of no option to sell out, the carve out is performed earlier as the losses have a higher weight: if synergies will turn positive the parent can buy back but there is no possibility to sell off the remaining shares and so it is more convenient to reduce earlier the negative synergies.

On the contrary the optimal threshold to carve out is always higher than the correspondent if not option to buy back existed. If no option to buy back exists, the parent is induced to wait longer to perform the carve-out because with this action it loses forever the possibility to fully benefit fro;

the positive synergies. Hence, the flexibility given by the option to buy back induces the parent to give up earlier the negative synergies that burden its pay off.

Furthermore the higher the amount of shares retained, the more negative are the synergies for which it is optimal to carve out: the higher the shares retained, the smaller the advantage due to the reduction of the negative synergies is smaller and so the later the option is exercised.

The less the shares sold during the IPO the more the parent waits to perform the carve out: there is a sort of trade off between perform an early carve out, but selling many shares and so more underpricing, or wait and sell less shares.

We now consider the sell-off case in order to know when the carve-out is preferred. For this we assume that in case of direct sell-off, there is no underpricing.⁹

Proposition 7 *If no option to carve-out exists, the parent firm sells off at $s_S^*O = -\frac{1}{\beta}$*

Proof. See Appendix G ■

Proposition 8 *When there is no underpricing, $\delta = 0$, the parent firm prefers to carve out rather than sell-off.*

Proof. See Appendix H ■

Studying the derivative of s_C^* given by equation (4), the higher the underpricing the lower are the synergies level for which it is optimal to carve out to the extent that it can be lower than the optimal synergy value of selling out (see Fig. 4). This implies that the higher the underpricing the least is convenient for the parent to carve out because they encounter too many costs and so they prefer to incur in the negative synergies rather than to carve out. Furthermore, a severe underpricing can induce the parent to prefer selling off and loose the option to buy back rather than carving out.

3 Effect of uncertainty on timing of financial decision

Uncertainty is one of the most important factor that affects the optimal synergy level to carve out, buy back and sell out: uncertainty affects the

⁹The acquirer of a sell off is usually an investor in the same industry and faces less severe asymmetry of information.

β parameter that subsequently affects the optimal synergy levels. In particular β decreases as uncertainty increases and it goes from infinity when uncertainty tends to zero and tends to zero as uncertainty tends to infinity.

Proposition 9 *The higher the uncertainty the higher the optimal threshold to buy back, s_S^* and the lower the optimal threshold to sell out, s_B^* , and to carve out, s_C^* .*

Proof. See Appendix I ■

This result is on line on the traditional results of real option theory: the higher the uncertainty the later it is optimal to exercise the option to sell out. Higher uncertainty implies a higher probability that the synergies take higher values on the future and so the parent is less willing to give up the possibility to exercise the option to buy back and exploit the positive synergies.

As in the sell out case, a higher uncertainty increases the option value to buy back, that is, the parent firm prefers to wait rather than buy back in order not give up the sell out option as it is more likely synergies may turn negative.

Again, also in the carve out case, with higher uncertainty extremes values of synergies are more likely and hence it is optimal for the parent to wait and see rather than exercise the option to carve out in order to have higher probability that the synergies will not turn positive soon after having lost the option to carve out.

4 Extensions

4.1 Optimal share retained

As we saw there is a trade off between early carve out and the amount of shares sold. To see which is the optimal level of optimal shares retained we can study the option value.

As it is shown in Fig. 5, when there is no or a low underpricing, the subsidiary value is low relative to the synergies and the synergies are very uncertain, it is optimal for the parent to sell the highest amount possible of shares: when the synergies are very negative relative to the subsidiary profit flow, the gain due to the decrease in the synergies prevails and hence the parent sells the highest amount of share possible.

When instead there is a heavy underpricing and the subsidiary profit flow is high relative to the synergies the loss due to the synergies because

irrelevant relative to the loss of the underpricing and so the parent sells the smallest amount possible (see Fig. 6).

The conclusions indicates the influences of uncertainty on the optimal shares retained, but many and crucial aspects have not been considered: the possible relation between underpricing and shares retained, control issues regulations of the stock markets and tax dispositions.¹⁰

4.2 The effect of the non-zero drift term

So far we have studied the case where the synergies have no drift term, that is $\mu = 0$. Adding the drift term we do not have anymore one β parameter, but two, one for the put options, that is the option to sell out and carve out, $\beta_2 (< 0)$, and one for the call option, namely the buy back option, $\beta_1 (> 0)$.

Whatever is the sign of the drift term, the optimal synergy level to buy back is always positive and greater than $\frac{1}{\beta_1}$ and the optimal synergy level to sell out is always negative and smaller than $\frac{1}{\beta_2}$ (see Appendix J for a demonstration).

Given the complexity of the calculations, it is hard to demonstrate analytically further results. However, we carried out some simulations to derive some features of the ECO when there is the drift term.

Traditional real option theory predicts that an increase in drift increases the threshold values as for higher drift holding the option without exercising it turns less expensive in terms of opportunity costs: that is the higher the drift, the investment in the risk free asset rather than on the investment is relatively less expensive and so holding the option alive rather than exercised it turns less expensive and the investment decision is taken for higher threshold values. This effect prevails always on another drift effect component: the higher the drift the higher the underline investment value and so the more attractive is exercise of the option.

For the carve-out case the situation, this traditional real option forecast does not always apply.

¹⁰The taxation of ECOs is complex depending if it involves primary and/or secondary issue of equity. For US regulation, when the parent holds more than 80% of the subsidiary, the parent can consolidate the subsidiary, deduct 100% of the dividends and spin off the remaining equity tax free in Section 355 spin off. Hence, the dividends are nontaxable. When the parent holds between 50 and 80% the parent can consolidate the subsidiary only for accounting but not taxes purposes. Under the tax point of view, it loses the tax advantage of Section 355 spin off though it can still deduct 80% of the dividends of the subsidiary. When the parent holds between 20 to 50%, the parent uses equity method for accounting purposes. At least, when holding less than 20%, the cost method should be applied. For more details see Willens and Zhu (1999).

In Fig. 7 we present the various optimal threshold values to buy back to sell out and carve out when the retained shares, α , is 0.5. The drift acts differently on the synergy thresholds.

For the buy back option in general, a drift increase determines a higher optimal threshold. The increase of the option value prevails in the increase of the investment value and so the parent waits higher synergy values to buy back; a higher drift makes waiting less expensive and hence increases the weight of the possible downside risk, as higher synergies are more likely.

However when the drift is very negative, a drift increase increases more the investment value than the option value and so the optimal synergy level decreases. When the drift is very low, the parent's firm wants to be guaranteed about the future synergies and so a decrease in the drift induces to wait for higher synergies levels in order to be sure that they will be positive for a enough length of time.

This effect is heavily determined by the amount of shares retained. The amount of shares retained heavily influences the payoff of the option: the higher the share retained the smaller is the payoff of the buy back option and its weight on the decision and the lower the optimal threshold for low values of the drift (see Fig. 8).

In the sell out case, in general an increase in the drift term increases the optimal synergies value for which it is optimal to sell out as the opportunity cost of the put option to sell out increases; a higher drift decrease more the put option value rather than the underlying investment value, that is, the parent firm prefers to exercise the option earlier.

However when the drift term is very high, the decrease in the underlying investment value of the put option value decreases more than the option value itself and so a higher drift induces the parent to wait longer, that is, to exercise the option to sell out later. Again as for the buy back case, the amount of retained shares plays an important role on the effect of the drift term on the optimal threshold: the lower the amount of shares retained the higher the effect of the drift on the payoff relative to the effect on the option so that a higher drift reduces the optimal threshold for very high drift levels (see Fig. 8).

Regarding the optimal threshold to carve out, an increase in the drift term acts univoquely positively: a higher drift decreases the optimal synergy value level this is the put option value increase more than the underlying investment due to the fact that the option is a compound option. The interesting element is this effect is so strong that when the drift is very high, the parent firm when the drift is very high to exercise the carve out option when the synergies are very positive (see Fig. 7). When the drift is very high it can be convenient to give up part of the synergies and invest the payoff in the risk free asset

knowing that likely you will buy it back.

The total effect of the retained shares and of the drift as it can be seen in Fig. 8 is not very relevant for the carve out case as the effect of the buy back and the sell out option compensates each other.

5 Concluding Remarks

Equity carve-outs have become popular in European and American corporate society as a way to refocus their businesses without relinquishing strategic control over the carved-out unit. This provides the company with a high degree of flexibility concerning the future corporate strategy conduct. At the same time, performing a carve-out can generate immediate benefits on operational performance, reduce financing constraints and enable improved corporate governance. For these reasons equity carve-outs have been witnessed in high growth and high uncertainty sectors such as telecommunications and Internet. We show that the benefits of an equity carve-out can be described in terms of real options. An alternative motive to consider an equity carve-out is therefore the creation of real options.

The appeal of strategic real options theory, is due to its ability to describe and value strategic plans which may be executed in the future once more information will be gained on the attractiveness of different alternatives. This new information may concern the evolution of consumer demands, new technological developments, and even the changing regulatory environment. We have applied such an approach to give an explanation for the so-called 'second event' of equity carve-outs. The strategic real option approach describes the potential benefits of equity carve-outs as the acquisition of future strategic opportunities of either capturing positive synergies or avoiding conglomeration costs, while at the same time allowing the market to acquire information on the value of its subsidiary.

We conclude that the choice of an equity carve-out is just the first part of a two-step strategy to either sell-off the subsidiary in a phased manner or re-acquire it if favorable information on its internal strategic value emerges. This so-called second event has been well documented in several studies, although the motivations for this strategy have not been modeled. Our application of strategic real options theory, gives a comprehensible explanation for the empirical results found in these studies. The performance of an equity carve-out actually creates both a put and a call option on the potential synergies of two connected businesses. The possibility to sell-off the remaining shares is equivalent to a put option. The ability to sell the subsidiary when it is more valuable outside the company than inside is obviously valuable. Yet

circumstances might change over time, and it may turn out to be beneficial for the parent to remain connected to the subsidiary. The equity carve-out allows a valuable deferral option.

The possibility to return to the initial situation by reacquiring the floated shares can be seen as a call option. More specifically, it is equivalent to a strategic growth option such as described by Kulatilaka and Perotti (1997,1998). It enables the firm to take advantage of future growth opportunities better and faster than its rivals when conditions change favorably.

An ECO appears to be optimal even when the likely outcome is a final sell off. The parent may choose to sell the subsidiary in a phased manner to signal better insider information allowing the market to recognize its value before selling out at a higher price. The ECO then establishes a good track record for the subsidiary before announcing a 'second stage' exit. The exposure of the carved-out subsidiary to analysts allows to generate new information on its value and to diffuse it in the market, accelerating the learning process.

On the other hand, if by some dates the benefits of a higher autonomy granted to the subsidiary no longer outweigh the costs of a separate listing and the lack of closer coordination, the parent company decides to reacquire the floated minority share on the market and get back full control.

The key ingredient of the compound option connected to the ECO is therefore that the final decision on buying back or selling out need not be pre planned by the parent, but can be left to a future date, when more information has become available, after the subsidiary has traded publicly for some time (Klein, Rosenfeld, and Beranek 1991). In general, upon announcement of the ECO the parent company normally does not commit to either solution: this would obviously decrease flexibility and destroy some embedded options.

These options have considerable value for firms facing uncertainty surrounding these effects. The recent carve-out wave in the biotechnology, telecommunications, and Internet sectors, are natural examples of companies seeking to create opportunities for maximizing value while retaining strategic flexibility. Using a simple option-timing model, we illustrated how these options can be valued.

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A Proof Proposition 1

In order to find the optimal threshold values, we derive the equation that equals the expected returns of the parent firm for each possible status to the expected rate of capital appreciation, that is, we derive the Bellman equation for each situation: full control, carve out, buy back and sell out. This will

allow us to find the general solutions of the option values. Subsequently, we will set all together the boundary conditions and solve them in a unique system.

Full control status. When the firm is in the full control status, the Bellman equation indicates that for an interval of time dt the total expected return on the full control together with the compound option to carve out has to be equal to the expected rate of capital appreciation.

As the expected return of the full control case with the option to carve out depends on the value of the synergies, through the Ito's lemma, the Bellman equation can be expressed as the following differential equation:

$$\frac{1}{2}\sigma^2V''[s] - rV[s] + (\pi + s) = 0 \quad (6)$$

The general solution of this differential equation is given by:

$$V_F = A_1 \exp^{\beta s} + A_2 \exp^{-\beta s} + \frac{\pi + s}{r} \quad (7)$$

where A_1 and A_2 are the differential constant that have to be found and $\pm\beta$ are the two solutions of the quadratic equation derived from the differential equation (6) and they are:

$$\pm\beta = \pm \frac{\sqrt{2r}}{\sigma} \quad (8)$$

Carve-out status. Following the same procedure we can find the general solution for the subsidiary value when the parent has performed a carve-out and has the option to buy back and to sell out. The differential equation is equal to:

$$\frac{1}{2}\sigma^2V''[s] - rV[s] + \alpha(\pi + s) = 0 \quad (9)$$

and the general solution is equal to:

$$V_C = B_1 \exp^{\beta s} + B_2 \exp^{-\beta s} + \alpha \frac{\pi + s}{r} \quad (10)$$

Buy back status and sell-off status. In the buy back and sell off status, no options are embedded and so the subsidiary value corresponds to what traditional NPV theory would predict. They are respectively:

$$V_B = \frac{\pi + s}{r} \quad V_S = 0 \quad (11)$$

Optimal exercises. In order to find each time the optimal exercise points and the three constants that define the options values, we have to impose what is commonly defined, the value matching condition and smooth pasting condition; these two conditions together imply that at the optimal exercise synergies level, the pay-offs of taking and not taking an action meet tangentially (see Appendix C of Chapter 4 of Dixit and Pindyck (1994) for an explanation on this type of boundary condition).

Hence, when the parent holds the full control, the optimal threshold to carve out has to satisfy the following conditions:

$$V_F(s_C^*) = V_C(s_C^*) + (1 - \alpha) \left(\frac{\pi}{r} - \frac{\delta}{r} \right) \quad (12)$$

$$V'_F(s_C^*) = V'_C(s_C^*) \quad (13)$$

On top of it, there is the condition at the limit such that when the synergies are very positive, that is when they tend to infinity, the option to exercise a carve-out is almost worthless as it is impossible that the parent firm will ever exercise it. This implies that $A_1 = 0$.

When the subsidiary has been already carved-out, the optimal threshold to buy back has to meet the following conditions:

$$V_C(s_B^*) = V_B(s_B^*) - (1 - \alpha) \frac{\pi}{r} \quad (14)$$

$$V'_C(s_B^*) = V'_B(s_B^*) \quad (15)$$

In the case of a switch from carve out to sell -out, at the optimal threshold the following condition have to be verified:

$$V_C(s_S^*) = V_S(s_S^*) + \alpha \frac{\pi}{r} \quad (16)$$

$$V'_C(s_S^*) = V'_S(s_S^*) \quad (17)$$

This system of six equations determines the three thresholds and the three remaining constants. The system cannot be solved entirely explicitly. The constant can be solve analytically and are given by:

$$B_1 = \frac{\exp^{\beta s_B^*} (1 - \alpha) + \alpha \exp^{\beta s_S^*}}{(\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*}) \beta r} \quad (18)$$

$$B_2 = \exp^{2\beta s_S^*} B_1 + \frac{\alpha}{r\beta} \exp^{\beta s_S^*} \quad (19)$$

$$A_2 = -\exp^{2\beta s_C^*} B_1 + B_2 + \frac{1 - \alpha}{r\beta} \exp^{\beta s_C^*} \quad (20)$$

The threshold points are given by the solution of the following system of equations:

$$\beta s_B^* = \beta s_S^* + \ln \frac{-1 + \alpha - \sqrt{1 - 2\alpha + \alpha^2 \beta^2 s_S^{*2}}}{\alpha (1 + \beta s_S^*)} \quad (21)$$

$$\beta s_S^* = \beta s_B^* + \ln \frac{-\alpha + \sqrt{-1 + 2\alpha + (1 - \alpha)^2 \beta^2 s_B^{*2}}}{(1 - \alpha) (1 + \beta s_B^*)} \quad (22)$$

$$\beta s_C^* = -1 - \beta \delta - \text{ProductLog} \left[\frac{-2 \left((1 - \alpha) \exp^{\beta s_B^*} + \alpha \exp^{\beta s_S^*} \right)}{(1 - \alpha) (\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*}) \exp^{1 + \beta \delta}} \right] \quad (23)$$

where $\text{ProductLog}[z]$ gives the principal solution for w in $z = w \exp^w$.

B Properties of entry benchmarks

Option to buy back and no option to sell out. The value of the marching condition and then the smooth pasting condition change between the carve-out and the buy back status change. The payoff of the buy back status remains the same as for the general case and it is given by equation 11.

When instead the firm is in the carve out status as there is no option to sell out, the constant B_2 is equal to 0: as when the value of the synergies tends to minus infinity the value of the pay-off as to be equal only to the expected profits because the option to buy back has to be worthless as the probability to exercise it is zero. Hence in this case:

$$V_C = B_1 \exp^{\beta s} + \alpha \frac{\pi + s}{r} \quad (24)$$

It follows that the system of equations that has to be solved constituted by the different value matching smooth pasting conditions are given by equations (12), (13), (14), (15) and the solutions are given by:

$$A_{1S}^b = \frac{(1 - \alpha) \left(\exp^{\beta s_{CB}^b} - \exp^{-1 + 2\beta s_{CB}^b} \right)}{\beta r} \quad (25)$$

$$B_1^b = \frac{(1 - \alpha)}{\beta \exp r} \quad (26)$$

$$s_B^b = \frac{1}{\beta} \quad (27)$$

$$s_{CB}^b = -\frac{1}{\beta} - \delta - \frac{1}{\beta} \text{ProductLog} \left[-2 \exp^{-2 - \beta \delta} \right] \quad (28)$$

where $ProductLog[z]$ gives the principal solution for w in $z = w \exp^w$.

In particular if there is no underpricing:

$$s_{CB}^b = -1.04 \frac{1}{\beta} \quad (29)$$

Option to sell out and no option to buy back. Again, the value matching and the smooth pasting conditions for the switching between the carve out and the sell out change. The expected value in the sell out status remains the same: the solution is constituted by equation (11).

When instead the firm is in the carve-out status and there is no option to buy back, the constant $B_1 = 0$, as when the value of the synergies tends to infinity the value of the pay-off as to be equal only to the expected profits. The option to sell out in fact in this case has to be worthless as the probability to exercise it, is zero. Hence:

$$V_C = B_1 \exp^{\beta s} + B_2 \exp^{-\beta s} + \alpha \frac{\pi + s}{r} \quad (30)$$

It follows that the system of equations that has to be solved constituted by the different value matching smooth pasting conditions are given by equations (12), (13), (16), (17) and the solutions are given by:

$$B_2^b = \frac{\alpha}{\exp \beta r} \quad (31)$$

$$A_{1C}^b = \frac{(1 - \alpha) \exp^{-\beta \delta} + \alpha}{\exp \beta r} \quad (32)$$

$$s_{SS}^b = -\frac{1}{\beta} \quad (33)$$

$$s_{CS}^b = -\frac{1}{\beta} - \delta \quad (34)$$

No option to buy back and no option to sell out. When the firm is in the carve-out status and there are no options to buy back and to sell out later, the value matching and the smooth pasting between the full control and the carve-out status change. The pay-off of the carve out status is equal to the expected profits without any option value added, that is $B_1 = B_2 = 0$.

The system of equations is given by equations (12) and (13) and the solutions are given by:

$$A_{1C}^b = \frac{(1 - \alpha) \exp^{-1 - \beta \delta}}{\beta r} \quad (35)$$

$$s_{CC}^b = -\frac{1}{\beta} - \delta \quad (36)$$

C Proof Proposition 2

At the optimal switching time, s_S^* , equation (16) has to be verified. This implies that the payoff at s_S^* between the carve-out status and the selling out one has to be equal to zero. That is, inserting equations (18), (19) and (20), the following equation:

$$\frac{(1 - \alpha) 2 \exp^{\beta(s_B^* + s_S^*)} + \alpha (1 - \beta s_S^*) \exp^{2\beta s_S^*} + \alpha (1 + \beta s_S^*) \exp^{2\beta s_B^*}}{\beta (\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*}) r} \quad (37)$$

has to be equal to zero.

This function is an increasing monotonic function that goes to minus infinity when s_S^* tends to minus infinity and to infinity when s_S^* tends to infinity. It follows that there is a unique value of s_S^* for which this function equals zero.

For $s_S^* = s_S^b = -\frac{1}{\beta}$, the function (37) is:

$$2 \frac{(1 - \alpha) \exp^{\beta s_B^* + 1} + \alpha}{\beta (\exp^{2(1 + s_B^* \beta)} - 1)} r \quad (38)$$

It is always positive as far as $s_B^* > -\frac{1}{\beta}$. This condition is not relevant as it is shown in Appendix D that $s_B^* > -\frac{1}{\beta}$.

It follows that $\beta s_S^* < -1 < 0$.

D Proof Proposition 4

At the optimal switching time, s_B^* , equation (14) has to be verified. This implies that the payoff function between staying in the carve out status and buying back has to be equal to zero. That is, inserting equations (18), (19) and (20), the following equation:

$$\frac{2\alpha \exp^{\beta(s_B^* + s_S^*)} - (1 - \alpha) \exp^{2\beta s_B^*} (\beta s_B^* - 1) + (1 - \alpha) \exp^{2\beta s_S^*} (\beta s_B^* + 1)}{\beta (\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*}) r} \quad (39)$$

has to be equal to zero.

This function is a increasing monotonic function that goes to minus infinity when s_B^* tends to minus infinity and to infinity when s_B^* tends to infinity. It follows that there is a unique value of s_B^* for which this function equals zero.

For $s_B^* = S_B^b = \frac{1}{\beta}$, the function (39) is:

$$\frac{2 \left((1 - \alpha) \exp^{2\beta s_S^*} + \alpha \exp^{1+\beta s_S^*} \right)}{\beta (\exp^2 - \exp^{2\beta s_S^*}) r} \quad (40)$$

It is always negative as far as $s_S^* > \frac{1}{\beta}$. This condition is not relevant as it is logical that the firm will sell out for negative values of the synergy.

It follows that $0 < 1/\beta_1 < s_B^*$

E Proof Proposition 3 and 5

The derivative of Equation (37), that we indicate as f , in respect of α is given by:

$$\frac{\partial f}{\partial \alpha} + \frac{\partial f}{\partial s_B^*} \frac{\partial s_B^*}{\partial \alpha} \quad (41)$$

where

$$\frac{\partial f}{\partial \alpha} = \frac{\exp^{\beta s_S^*} (\beta s_S^* - 1) - \exp^{\beta s_B^*} (\beta s_S^* + 1)}{\beta (\exp^{\beta s_B^*} + \exp^{\beta s_S^*}) r} > 0 \quad (42)$$

$$\frac{\partial f}{\partial s_B^*} = \frac{-2 \exp^{\beta (s_B^* + s_S^*)} \left((1 - \alpha) (\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}) + 2\alpha \exp^{\beta (s_B^* + s_S^*)} \right)}{(\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*})^2 r} \quad (43)$$

At the same time the derivative of equation (39), that we indicate with g , in respect of α is given by:

$$\frac{\partial g}{\partial \alpha} + \frac{\partial g}{\partial s_S^*} \frac{\partial s_S^*}{\partial \alpha} \quad (44)$$

where

$$\frac{\partial g}{\partial \alpha} = \frac{(\beta s_B^* - 1) \exp^{\beta s_S^*} + (\beta s_B^* + 1) \exp^{\beta s_S^*}}{\beta (\exp^{\beta s_B^*} + \exp^{\beta s_S^*}) r} < 0 \quad (45)$$

$$\frac{\partial g}{\partial s_S^*} = \frac{4(1 - \alpha) \exp^{2(s_B^* + s_S^*)} + 2\alpha \left(\exp^{\beta(3s_B^* + s_S^*)} + \exp^{\beta(s_B^* + 3s_S^*)} \right)}{(\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*})^2 r} < 0 \quad (46)$$

Putting together these elements it results that as the fraction of retained shares increase both s_B^* and s_S^* increases.

Furthermore studying the limit of the behavior of the optimal thresholds at the limit cases of α equal to 0 and 1 we obtain that:

$$\lim_{\alpha \rightarrow 0} s_S^* = -\infty \quad (47)$$

$$\lim_{\alpha \rightarrow 1} s_S^* = -\frac{1}{\beta} \quad (48)$$

$$\lim_{\alpha \rightarrow 0} s_S^* = \frac{1}{\beta} \quad (49)$$

$$\lim_{\alpha \rightarrow 1} s_S^* = \infty \quad (50)$$

F Proof Proposition 6

The optimal switching time, s_C^* is given by equation (4). This expression is real if the part inside the *ProductLog* is greater than $-\frac{1}{\exp}$.

This is always true given as far as:

$$\alpha < \frac{\exp^{2\beta s_B^* - 2\beta s_B^* - \exp^{2\beta s_S^*}}}{\exp^{2\beta s_B^* - 2\beta s_B^* - \exp^{2\beta s_S^*}} + 2 \exp^{\beta s_S^*}} \cong 0.997 \quad (51)$$

As the numerator is positive and the denominator is always slightly greater than the numerator, we have that there is an optimal carve out threshold for almost every possible fraction of retained share.

Computing the limit for α that tends, s_C^* tends to a value that is smaller than $1 - \delta$, while for α that tends to 1, s_C^* tends to $-\frac{1}{\beta}$.

Furthermore as the derivative of equation (4) in respect of α is always negative, s_C^* diminishes as α increases.

G Proof Proposition 7

The sell-off can be considered as a special case of the carve-out without option to buy back and to sell out when $\alpha = 1$ and $\delta = 0$. Hence, adapting the results of Appendix ?? we obtain:

$$s_{SO}^b = -\frac{1}{\beta} \quad (52)$$

H Proof Proposition 8

Inserting equation (3) in equation (4) we obtain:

$$-\frac{1}{\beta} - \frac{1}{\beta} \text{ProductLog} \left[- (1 + \beta s_B^*) \exp^{-1 - \beta s_B^*} \right] \quad (53)$$

This expression is always higher than $-\frac{1}{\beta}$ that is for Proposition 2, that is the superior limit of s_S^*O .

I Proof Proposition 9

Form equations (2) and (3) we know that βs_B^* , βs_S^* , βs_C^* are constant values given the fraction of retained shares. Hence, the higher the uncertainty, the lower β , the higher s_B^* and the lower s_S^* and s_C^* (as they are negative).

J The range of s_B^* and s_S^* in case of $\mu \neq 0$

Inserting the drift terms we obtain the following results:

$$\beta_1 = \frac{-\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} \quad \beta_2 = \frac{-\mu - \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} \quad (54)$$

$$B_1 = \frac{\alpha \exp^{\beta_2 s_B^*} + (1 - \alpha) \exp^{\beta_2 s_S^*}}{\beta_1 (\exp^{\beta_1 s_B^* + \beta_2 s_S^*} - \exp^{\beta_2 s_B^* + \beta_1 s_S^*}) (r - \mu)} \quad (55)$$

$$B_2 = \frac{\alpha \exp^{\beta_1 s_B^*} - (1 - \alpha) \exp^{\beta_1 s_S^*}}{\beta_2 (\exp^{\beta_2 s_B^* + \beta_1 s_S^*} - \exp^{\beta_1 s_B^* + \beta_2 s_S^*}) (r - \mu)} \quad (56)$$

$$A_1 = B_2 + B_1 \frac{\beta_1}{\beta_2} \exp^{(\beta_1 - \beta_2) s_C^*} - \frac{(1 - \alpha)}{\beta_2 (r - \mu)} \quad (57)$$

Equations (12), (16) and (14) becomes respectively:

$$\frac{\exp^{\beta_1 s_F^* + \beta_2 s_S^*} (1 - \alpha) (\beta_1 - \beta_2) + \exp^{\beta_1 s_F^* + \beta_2 s_B^*} \alpha (\beta_1 - \beta_2)}{(\exp^{\beta_1 s_B^* + \beta_2 s_S^*} - \exp^{\beta_1 s_S^* + \beta_2 s_B^*}) \beta_1 \beta_2 (r - \mu)} + \frac{(1 - \alpha) (-1 + (s_F^* + \delta) \beta_2)}{\beta_2 (r - \mu)} \quad (58)$$

$$\frac{\alpha (\beta_1 - \beta_2) \exp^{(\beta_1 + \beta_2) s_B^*} - (1 - \alpha) \beta_2 (1 - \beta_1 s_B^*) \exp^{\beta_1 s_B^* + \beta_2 s_S^*}}{(\exp^{\beta_1 s_S^* + \beta_2 s_B^*} - \exp^{\beta_1 s_B^* + \beta_2 s_S^*}) \beta_1 \beta_2 (r - \mu)} + \frac{(1 - \alpha) \beta_1 (1 - \beta_2 s_B^*) \exp^{\beta_1 s_S^* + \beta_2 s_B^*}}{(\exp^{\beta_1 s_S^* + \beta_2 s_B^*} - \exp^{\beta_1 s_B^* + \beta_2 s_S^*}) \beta_1 \beta_2 (r - \mu)} = 0 \quad (59)$$

$$\frac{(1 - \alpha) (\beta_1 - \beta_2) \exp^{(\beta_1 + \beta_2) s_S^*} - \alpha \beta_2 (1 - \beta_1 s_S^*) \exp^{\beta_1 s_S^* + \beta_2 s_B^*}}{(\exp^{\beta_1 s_S^* + \beta_2 s_B^*} - \exp^{\beta_1 s_B^* + \beta_2 s_S^*}) \beta_1 \beta_2 (r - \mu)} + \frac{\alpha \beta_1 (1 - \beta_2 s_S^*) \exp^{\beta_1 s_B^* + \beta_2 s_S^*}}{(\exp^{\beta_1 s_S^* + \beta_2 s_B^*} - \exp^{\beta_1 s_B^* + \beta_2 s_S^*}) \beta_1 \beta_2 (r - \mu)} \quad (60)$$

For $s_B^* = \frac{1}{\beta_1}$, equation (59) is always negative. It follows that as equation (59) is a monotonic function increasing in s_B^* , $s_B^* > \frac{1}{\beta_1}$.

For $s_S^* = \frac{1}{\beta_2}$, equation (60) is always negative. It follows that as equation (60) is a monotonic function decreasing in s_S^* , $s_S^* < \frac{1}{\beta_2}$.

For $s_C^* = \frac{1}{\beta_2}$ and $\delta = 0$, equation (59) is equal to:

$$\frac{\exp^{\frac{\beta_1}{\beta_2}} \left((1 - \alpha) \exp^{\beta_2 s_S^*} + \alpha \exp^{\beta_2 s_B^*} \right) (\beta_1 - \beta_2)}{\left(\exp^{\beta_1 s_B^* + \beta_2 s_S^*} - \exp^{\beta_1 s_S^* + \beta_2 s_B^*} \right) \beta_1 \beta_2 (r - \mu)} \quad (61)$$

Source	Timeframe	Sample size	Announcement effect	Average Excess Return Subsidiary		
				1 year	2 years	3 years
Anslinger, Carey, Fink, and Gagnon (1997)	1985-1995	119/105/78		23%	14%	10%
Anslinger, Bonini, and Patsalos-Fox (2000)	1988-1996	67			13%	
Michaely and Shaw (1995)	1981-1988	28	1.20%	-13%	-20%	
Powers (2000)	1981-1986	183		6%	-13%	1%
Vijh (1999)	1981-1995	628				8%
Slovin, Sushka, and Ferraro (1995)	1980-1991	32	1.23%			
Schipper and Smith (1986)	1965-1983	76	2%			
Klein, Rosenfeld, and Beranek (1991)	1966-1988	52	2.75%			
Schill and Zhou (2001) [Internet stocks]	1999-2000	Small	25%			
Allen and McConnell (1998)	1978-1993	186	2.12%			

Table 1: Market Performance of Subsidiary

1. Financing	
Enable subsidiary to obtain own financing for anticipated growth	19
Decrease debt of parent	5
Finance capital expenditures	3
Finance growth of parent	5
2. Improve investor understanding of subsidiary	14
3. Restructure asset management	
<i>Change in corporate focus</i>	
Decrease investment in subsidiary's line of business	6
Part of larger restructuring program	5
<i>Re-contract with subsidiary's managers</i>	
Give subsidiary's managers more autonomy	3
Revise incentive contracts of subsidiary's managers	10
4. Other	
Impede a merger in progress	1
Increase flexibility in making acquisitions	6

Source: Schipper and Smith (1986)

Table 2: Stated motives for Equity carve-outs

Second event	Frequency	Number of years between carve-out and second event	
		Average	Range
Re-acquired by parent	22	5.7	2 to 11
Completely divested			
Acquired by another firm	13	6.5	< 1 – 18
Spun-off	4	1.5	1 to 3
Exchange offer or cash sale to Subsidiary	3	9.5	7 to 12
Declared bankruptcy	2	1.7	< 1 – 3
Liquidation	2	3.5	1 to 6
Offer to re-acquire pending	1	2.5	N/A
Offer to divest pending	1	19	N/A
No other restructuring	20		
Total	68		

Source: Schipper and Smith (1986)

Table 3: Subsidiaries whose carve-out was followed by other restructuring

Elapsed time	Number of reacquisition	Number of sell-offs	Total frequency
< 1 year	0	5	5
1-2 years	2	7	9
2-3 years	6	2	8
3-4 years	2	1	3
4-5 years	5	2	7
> 5 years	10	2	12
No second event	0	0	8
Total	25	19	44
Median Elapsed Time	4.5 years	1.33 years	3.17 years

Source: Klein, Rosenfeld, and Beranek (1991)

Table 4: Elapsed time between the carve-out announcement and the subsequent second-event

Type of second event	Frequency	Elapsed time	Number of sell-offs
Spin-off or split-off	38	< 1 year	37
Sell-off	122	1-2 years	23
Re-acquisition	50	2-3 years	19
Bankruptcy, liquidation, or delisting	11	3-4 years	13
None	66	4-5 years	19
No information	5	> 5 years	11

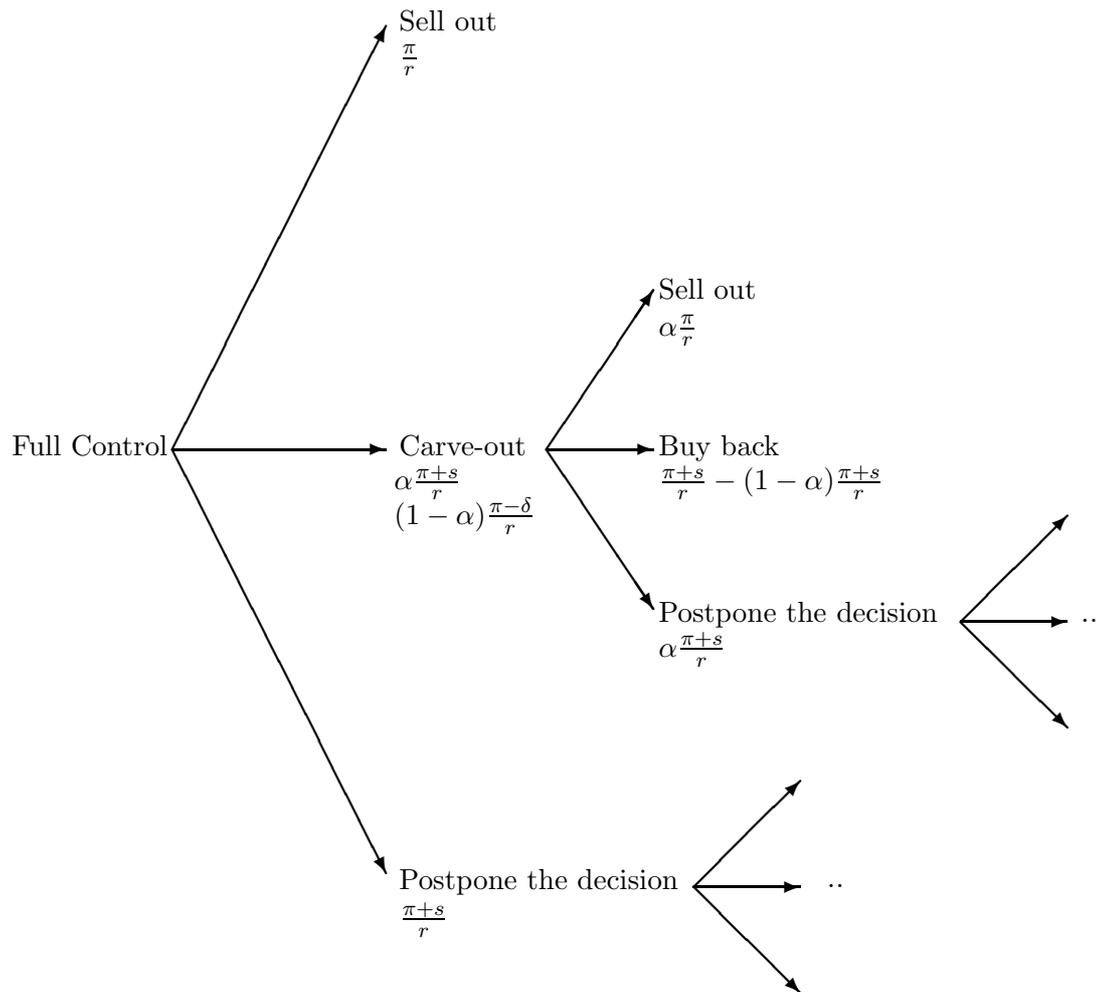
Source: Hand and Skantz (1999a)

Table 5: Frequency of second stage events

Range	Reacquisition	Sell-offs	No secondary event	Total
1%-50%	0	4	4	8
51%-80%	5	7	2	14
> 80%	17	6	2	25
Indetermined	3	2	0	5
Total	25	19	8	52

Source: Klein, Rosenfeld, and Beranek (1991)

Table 6: Percentage of subsidiary shares retained by parent



α : % of retained shares of the subsidiary
 $\frac{\pi}{r}$: subsidiary market value
 $\frac{\delta}{r}$: underpricing
 $\frac{s}{r-\mu}$: synergies

Figure 1: The Decision Tree

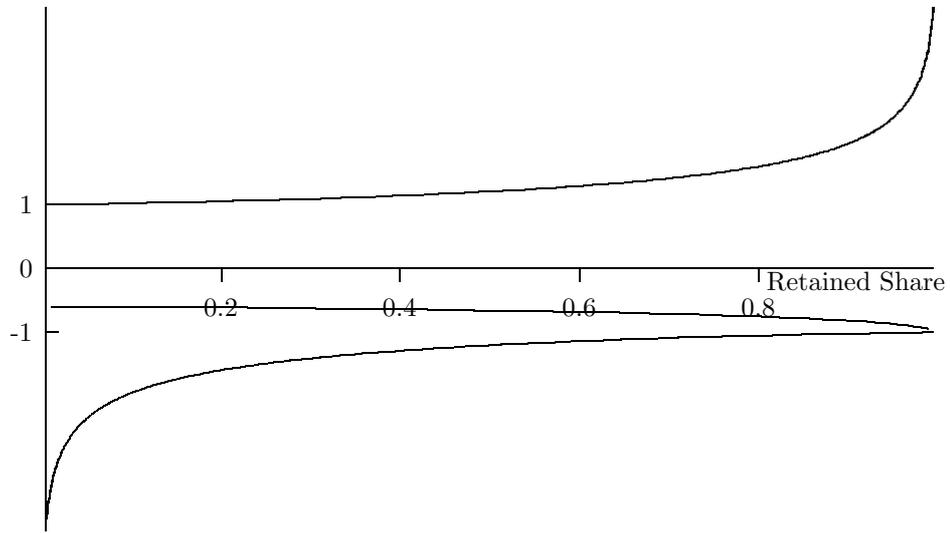


Figure 2: Threshold synergy flow values in case of no underpricing

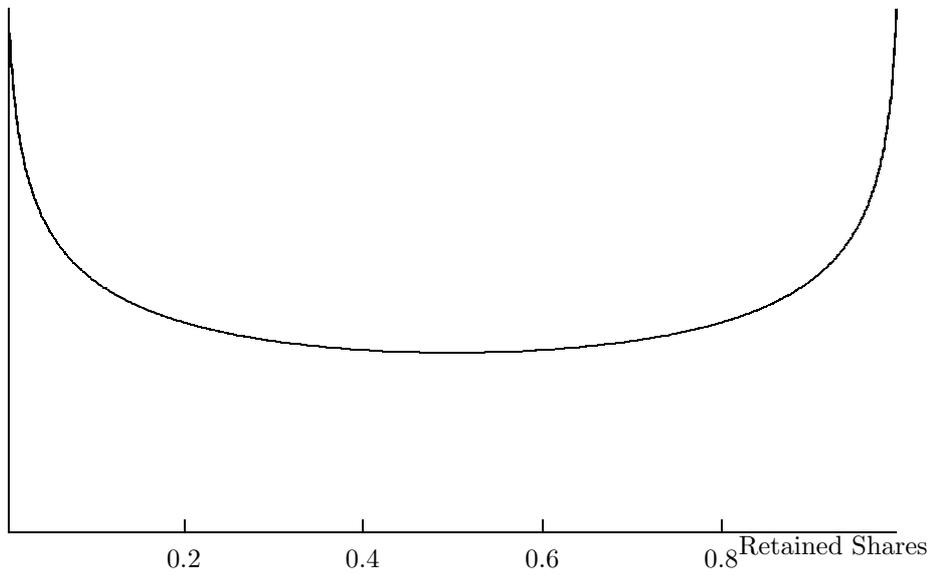


Figure 3: Synergies range for the carve out status for different α

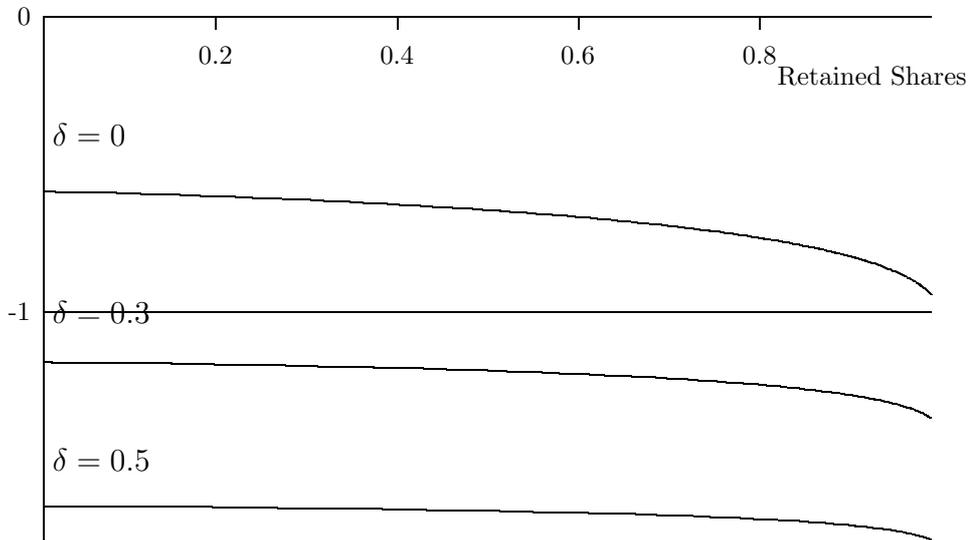


Figure 4: Optimal synergy level to carve out for different underpricing levels
 βs_{SO}^*

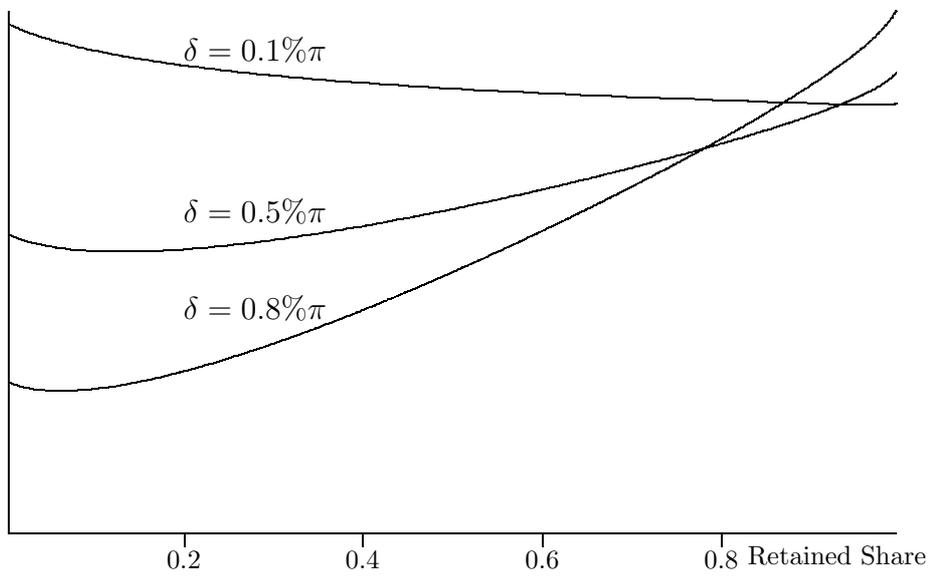


Figure 5: Subsidiary value for the parent for different underpricing values
 $(\sigma = 0.5, r = 0.05, \pi = 1)$

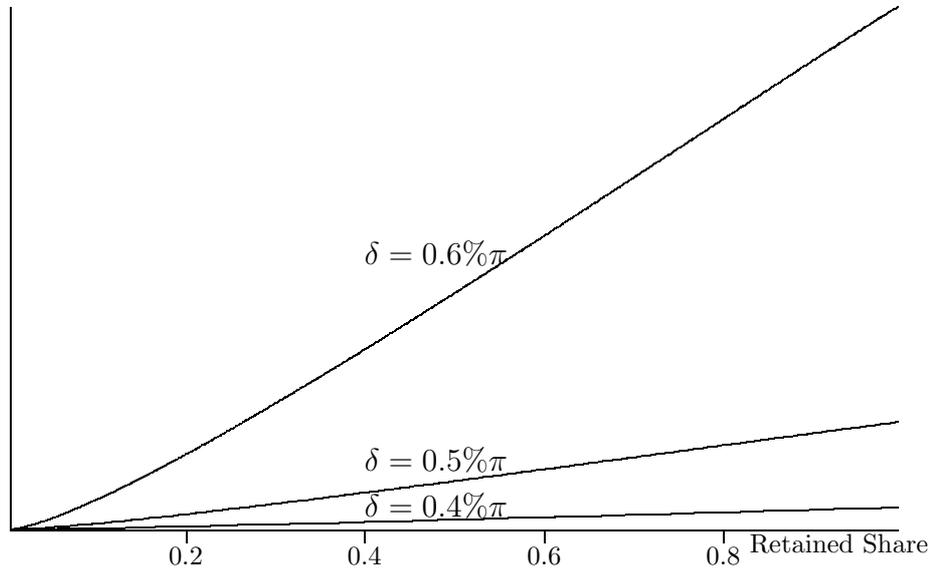


Figure 6: Subsidiary value for the parent for different underpricing values ($\sigma = 0.2, r = 0.05, \pi = 1$)

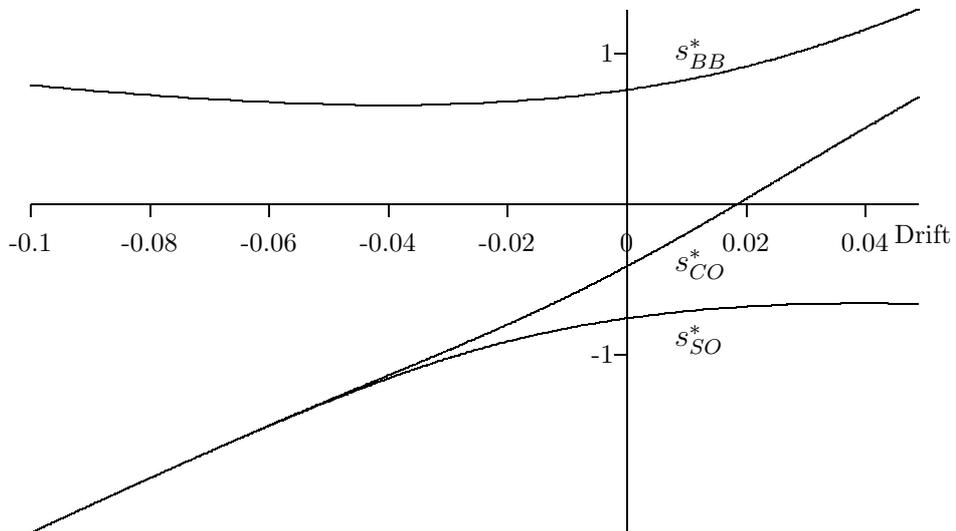


Figure 7: Optimal synergy level to buy back, sell out and carve out as a function of the drift ($\sigma = 0.1, r = 0.1, \delta = 0, \alpha = 0.5$)

