# Strategic Relationships between Buyers and Sellers under Uncertainty

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* We wish to thank Rajesh Chakrabarti, Cheol Eun, Jayant Kale, David Nachman, Mike Rebello, Husayn Shahrur, and Anand Venkateswaran for valuable comments and suggestions.

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## Abstract

We analyze strategic relationships between buyers and sellers in markets with relationship-specific costs and exogenous dynamic uncertainty by investigating the scenario wherein a representative buyer trades with two sellers in a different country. We show that under exchange rate uncertainty, relationship-specific costs may either *raise* or *lower* competition and the level of prices in long-term contracts between buyers and sellers. Low levels of exchange rate uncertainty facilitate competition by allowing the sellers to co-exist. However, if the level of uncertainty is beyond a threshold, the only viable equilibria are those where one of the sellers captures the market.



**Key words**: Strategic Relationships between Buyers and Sellers, Dynamic Uncertainty, Relationship Specific Costs, International Trade

**JEL Classification Codes:** L11, D43, F10, C72

**Strategic Relationships between Buyers and Sellers under Uncertainty**

## 1. Introduction

One of the important themes in the industrial organization literature is the understanding of how differences in the relative bargaining power of firms arise and affect their strategic relationships (Tirole 1988). In particular, there is a well-developed body of literature that examines the role of *relationship-specific costs* in influencing the relative bargaining power of buyers and sellers and the resulting effect on their strategic relationships[1]. However, the extant literature has not fully explored the impact of relationship-specific costs on buyer-seller strategic relationships in the presence of some form of *exogenous dynamic uncertainty* (for example, exchange rate uncertainty when buyers and sellers trade across different countries). Our paper analyzes the impact of such uncertainty on the strategic relationships between buyers and sellers.

There are several economic scenarios where the interplay between both relationship-specific costs and exogenous dynamic uncertainty significantly affects strategic relationships between buyers and sellers.[2] However, we focus on the scenario wherein buyers in one country trade with sellers in a foreign country. Hence, both buyers and sellers are exposed to exchange rate uncertainty. An investigation of this scenario is particularly relevant due to the dramatic acceleration in *globalization* that has led to increasing trade between buyers and sellers in different countries.[3] This scenario also highlights the central economic issues we wish to address, that is, the impact of both relationship-specific costs and exogenous dynamic uncertainty on the strategic relationships between buyers and sellers.

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[1] See Klemperer (1995) for a survey of this literature.

[2] Firms with multiple potential suppliers, banks that screen or monitor customers, consumers in search of the best bargain on a product, have to incur relationship-specific, sunk/set-up/search costs that are offset by lower variable costs. The prices quoted by the suppliers, loan payments made by bank customers, and prices of competing products are all affected by the presence of some form of exogenous, dynamic uncertainty.

[3] According to the Worldwatch Institute (2002), world exports have increased 17-fold from 1950 to 1998 (from $311 billion to $5.4 trillion); volume of FDI has increased 15-fold since 1970 to $644 billion in 1998; and the number of transnational corporations has increased from 7000 in 1970 to 60,000 today. Total FDI flows of developed countries increased from $72 billion in 1981 to $369 billion in 1990 (Froot 1993).

We analyze this problem within a continuous time equilibrium framework that leads to predictions about strategic relationships between buyers and sellers that are significantly different from those in the extant literature. We show that, depending on the exchange rate volatility, the presence of relationship-specific costs may either increase or decrease competition among sellers and either increase or decrease the level of prices in a mature market.[4] When exchange rate volatility levels are low, multiple sellers may co-exist in the market, but the number of co-existing sellers in equilibrium decreases with exchange rate volatility. This implies that *seller concentration* in the foreign market increases with exchange rate volatility. We also demonstrate that if the exchange rate volatility is above a critical level, the possibility of unrestrained *tacit collusion* among the sellers may necessitate the imposition of price ceilings for trade to occur. Our results therefore show that the interplay between relationship-specific costs and exchange rate uncertainty has a significant impact on strategic relationships between buyers and sellers across different countries.

We obtain these results within a parsimonious framework that considers a single, representative, risk-neutral buyer with two potential, risk-neutral sellers in a foreign country. Specifically, we analyze equilibria of the *three-player game* between the buyer and the sellers where the buyer incurs different relationship-specific *sunk* costs vis-à-vis these non-identical sellers that are observable to both sellers. The buyer has a constant, inelastic demand for the product at any instant of time and derives constant utility from each unit of the product. Either seller can completely fulfill the buyer's demand so that the buyer will not be in relationships with both sellers simultaneously. The sellers' strategies are to quote constant prices *in their currency* for each unit of a product (for example, the sellers could be American firms selling to firms in another country with long-term dollar-denominated contracts).[5] The buyer responds to the sellers' quoted prices by choosing to be

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[4] We refer to a mature market as one where relationship-specific costs are already present (Klemperer 1995).

[5] Joskow (1987) finds that in markets with significant relationship-specific costs, buyers and sellers both prefer to enter into long-term contracts ex ante and rely less on repeated negotiations over time and Carlton (1986) observes long-term rigidity in the prices of intermediate goods.

either out of the foreign market or in a relationship with one of the sellers whenever it is in the foreign market at any instant of time.[6]

As a benchmark, we first consider the situation where there is only one seller in the foreign market. We show that a *trading equilibrium* (where a relationship is established between the buyer and the seller) exists only if the exchange rate volatility is below a threshold. If the volatility is above this threshold, no trading equilibrium exists. The intuition for this result is that the seller is faced with a tradeoff *between* quoting a lower price in its currency, thereby hastening the buyer's entry into a relationship with it, but obtaining lower profits in its currency when in business *and* quoting a higher price, delaying the buyer's entry, but obtaining higher profits. When the exchange rate volatility is below a threshold, these effects balance each other at a finite price. However, when the volatility is beyond this threshold, the second effect in the seller's tradeoff predominates at any finite price so that it quotes unbounded prices thereby precluding the buyer's entry. This outcome can be averted if a price ceiling is imposed on the seller. Our results are in sharp contrast with the classical analysis of entry and exit under uncertainty (Dixit 1989). If the price quoted by the foreign seller is exogenously fixed (in the seller's currency) so that the seller does not behave strategically, it can be shown that as exchange rate volatility increases, the domestic buyer behaving strategically delays entry into the foreign market, but *always* enters it with positive probability (Dixit and Pindyck 1994).[7]

Next, we consider the scenario where there are two sellers in the foreign market. *Given seller prices*, we show that the buyer's *switching option* to switch between the sellers over time has positive value if and only if the ratio of seller prices lies in a non-empty bounded interval that depends on the relationship-specific costs and the exchange rate process. The existence of this non-

[6] For example, the *buyer* could be a firm that resells the product at a constant price per unit in its domestic market and faces a constant demand per unit time for the product or it could be a consumer with a constant inelastic demand and constant utility for the product.
[7] We assume here, of course, that the relationship-specific costs are lower than the maximum possible expected payoffs to the buyer from entering the foreign market so that entry into the foreign market is feasible for the buyer.

empty interval is therefore a *necessary* condition for *switching equilibria* where the sellers *co-exist*, that is, each has a nonzero probability of being in business with the buyer over time. If the interval is non-empty and the sellers have constant variable costs of production in their currency, we show that a *sufficient* condition for the sellers to co-exist in any equilibrium with the buyer is that the ratio of their variable costs lies within this interval. The intuition for this result is the following. Since each seller quotes a constant price (in the sellers' currency) to the buyer, neither can quote a price below its variable cost. If the ratio of the sellers' variable costs lies in the interval, then each seller can prevent the other from capturing the market by quoting a price such that the buyer always obtains positive value from switching between them.

One of our main analytical results is a demonstration of the existence of a critical exchange rate volatility level above which the buyer's switching option has *zero value* for all possible prices quoted by the sellers. Therefore, above this level, the only viable equilibria are *no-switching* equilibria where one of the sellers *captures* the market, that is, obtains all possible business with the buyer. The level of prices when the seller with the relationship-specific cost *advantage (disadvantage)* captures the market is typically *higher (lower)* than the level of prices if there were no relationship-specific costs. Therefore, in the presence of exchange rate uncertainty, relationship-specific costs may either raise or lower the level of prices. In the absence of uncertainty, one of the two non-identical sellers, in general, captures the market. The presence of uncertainty makes co-existence feasible, but this feasibility disappears when the volatility is too high. The intuition for this result is as follows. With increasing exchange rate volatility, the buyer delays its entry into the foreign market. Therefore, it spends too little time in the market where it makes profits with either seller to exploit the potential tradeoff between the lower relationship-specific costs vis-à-vis one seller and the lower variable costs vis-à-vis the other. Hence, the buyer will establish relationships with only a single seller depending on the relationship-specific cost/variable cost tradeoff and the exchange rate volatility. Extending our framework to the scenario with more than two sellers leads to the conclusion that seller *concentration* in the foreign market increases with exchange rate volatility.

Von Weizsacker (1984) considers a framework with long-term constant price contracts between buyers and sellers in the same country and concludes that relationship-specific costs *increase* competition and *lower* the level of prices. On the other hand, Klemperer (1987a), Beggs and Klemperer (1992), and Padilla (1995) use multi-period deterministic frameworks, where sellers compete in each period, to conclude that, in general, relationship-specific costs *raise* the level of prices in a mature market and *relax* competition. Our results that, in the presence of exchange rate uncertainty, relationship-specific costs may either raise or lower competition and either raise or lower the level of long-term prices in a mature market contrast with the above results. The fact that the exchange rate volatility plays a crucial role in determining the actual equilibrium outcome (that is, co-existence or market capture) and the corresponding level of prices is a new insight offered by the explicit incorporation of dynamic uncertainty within our framework.[8]

We demonstrate the viability of equilibria with both sellers co-existing or either seller capturing the market even when the exchange rate volatility is so high that the buyer would not enter the foreign market had it negotiated with only one seller (not subject to price ceilings). However, if the sellers' variable costs are "close" to each other, we show that the possibility of unrestrained tacit collusion between the sellers may prevent the buyer's entry into the foreign market unless there is an exogenously imposed price ceiling. The intuition for this result is that when the sellers' variable costs are "close" to each other, one of them can potentially capture the market *only* by quoting a

[8] Froot and Klemperer (1989) examine the effects of exchange rates on buyers and sellers in different countries by considering a two-period deterministic framework where a *foreign* and a *domestic* seller compete in each period. They show that foreign firms may either raise or lower their *dollar* export prices when the dollar appreciates temporarily. However, we differ from them not only in our framework, but also in the economic issues we address. They perform comparative static analyses of the expectations of changes in exchange rates on *short-term prices* in the *buyer's currency*, but they do not consider *dynamic* exchange rate uncertainty. One of our goals is to examine how exchange rate *volatility* affects competition between foreign sellers and the *long-term prices* they quote in equilibrium. On the other hand, they are concerned with how the level of the exchange rate (rather than the level of its volatility) affects import prices. Moreover, they do not explicitly derive optimal pricing strategies of the sellers and are therefore not directly concerned with the investigation of actual equilibrium outcomes under different conditions.

price “close” to its variable cost. Therefore, both can potentially increase their expected profits by accommodating each other in equilibrium. However, under increasing uncertainty, both prefer to quote increasingly higher prices thereby delaying the buyer’s entry, but obtaining higher profits when in business. This leads to tacit collusion. When the exchange rate volatility is beyond a threshold, neither seller is satiated at a finite price. This leads to unrestrained tacit collusion and no trading equilibrium exists unless price ceilings are imposed. Therefore, even with significant competition in the foreign market, the intervention of a regulator may be required to ensure that trade occurs[9].

Klemperer (1987b) and Farrell and Shapiro (1988) also note the possibility of tacit collusion whereas Padilla (1995) concludes that tacit collusion may be hard to sustain in equilibrium. These papers consider deterministic frameworks where contracts between buyers and sellers are *short-term*. Von Weizsacker (1984), whose framework considers long-term constant price contracts between buyers and sellers, does not note the possibility of tacit collusion. We show that, under exchange rate uncertainty, tacit collusion may occur with long-term contracts between buyers and sellers. Moreover, we obtain the additional insight that exchange rate volatility beyond a critical threshold may lead to unrestrained tacit collusion requiring the imposition of price ceilings for trade to occur.

Methodologically, our paper is related to the emerging literature that investigates strategic irreversible investment under uncertainty in a multi-period or continuous time framework. This literature was pioneered by Dixit (1989, 1991) and Dixit and Pindyck (1994). Trigeorgis (1996), Grenadier (2000), and Huisman (2001) provide comprehensive surveys of this literature. The papers in this stream of the literature have largely focused on the strategic behavior of sellers where the demand is exogenously specified and the sellers are Cournot competitors. Moreover, they have primarily investigated *duopolistic timing games* where two sellers strategically enter the market

[9] Dixit (1991) analyzes the effects of price ceilings on irreversible investment in a framework where identical firms are in Cournot competition and the demand for their product is exogenously specified.

sequentially at possibly random times. A distinguishing feature of our framework is that *both* buyers and sellers behave strategically. Moreover, in our model, the sellers are Bertrand competitors. This entails the analysis of a three-player game between non-identical players that is very different from the games considered so far in this stream of the literature.

The rest of the paper is organized as follows. **Section 2** outlines the model used in our analysis. In **Section 3**, we derive the optimal policies of the buyer for given seller prices. In **Section 4**, we analyze the equilibrium problem between the buyer and the sellers. **Section 5** concludes and indicates directions for future research. All detailed proofs appear in the **Appendix.**

## 2. The Model

We consider independent and identical buyers in a single country with two potential sellers, seller 1 and seller 2, in a foreign country. The sellers' production technologies have constant returns to scale so that we may, without loss of generality, focus on a single, representative buyer. The risk-neutral buyer has a constant, inelastic demand of 1 unit of the product per unit time and derives a constant utility of 1 from each unit of the product. The sellers quote constant prices $Q_1, Q_2$ in their own currency to the buyer.[10] Therefore, the buyer and sellers are exposed to the uncertainty in the foreign exchange rate $q(.)$ that is assumed to evolve as follows[11]:

(2.1) $$dq(t) = q(t)[m dt + s dB(t)].$$

We assume that all agents are risk-neutral with uniform beliefs about the process $q(.)$, and are discounted expected utility maximizers in their respective currencies. The price (per unit) of the

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[10] Von Weizsacker (1984) assumes constant price contracts and MacLeod and Malcomson (1993) show the efficiency of fixed-price contracts with exogenous switching costs. Both consider buyers and sellers in the same country. Farrell and Shapiro (1989) show that when there are relationship-specific setup costs and unobservable switching costs and sellers choose the price and quality of the product offered, *long-term* contracts may outperform *short-term* contracts. In our scenario, the presence of exchange rate uncertainty and the fact that buyers and sellers maximize expected profits in *different* currencies makes the analysis of the allocative efficiency of equilibria, especially the comparison between the efficiency of long-term and short-term contracts a nontrivial issue (see, for example, Adler and Dumas 1983).

[11] $B(.)$ is a Brownian motion defined on a filtered probability space $(\Omega, F, F_t, P)$.

product $p(.)$ demanded by seller 2 in the *buyer's currency* is given by $p(.) = Q_2 q(.)$ and also evolves as in (2.1) with drift $m$ and volatility $s$. Therefore, the price per unit of the product demanded by seller 1 is proportional to the price demanded by seller 2 and is given by $lp$ where $l = Q_1 / Q_2$. The buyer incurs different relationship specific *sunk costs* $k_1, k_2$ vis-à-vis sellers 1 and 2 respectively *each time* it enters into relationships with the sellers with

(2.2) $$0 \le k_1 < k_2,$$

and these costs are common knowledge.[12] Given this difference in relationship specific sunk costs, seller 2 can compete with seller 1 only by quoting a lower price. Therefore, it suffices to consider situations where the ratio of the seller prices $l$ satisfies

(2.3) $$l > 1.$$

**Remark 1:** At this stage, we assume that the prices $Q_1, Q_2$ quoted by the sellers are exogenously specified. In **Section 4,** these prices will be endogenously determined in equilibrium.

For simplicity of exposition, we assume throughout this paper that the sellers do not obtain any portion of the relationship-specific costs $k_1, k_2$ incurred by the buyer. We also assume that the buyer does not bear an exit cost or penalty for exiting a relationship with a seller[13]. At any time $t$, the buyer may either be idle[14] (denoted by $0$), in a relationship with seller 1 (denoted by 1) or in a

[12] The differences in relationship-specific costs could arise, for example, due to seller attempts to differentiate themselves from each other, one of the sellers being more "developed" than the other, one of the sellers being the incumbent with whom the buyer has already established a relationship and the other being a potential entrant. Although we focus on identical buyers in this paper, it is easy to extend our framework to consider *non-identical buyers*, that is, different buyers have different pairs of relationship-specific costs vis-à-vis the sellers (Farrell and Shapiro 1989). If these costs are *observable* to the sellers, then they can quote different prices to different buyers so that our analysis of a representative buyer is without loss of generality. If the costs are *unobservable* (Farrell and Shapiro 1989), each seller quotes a price rationally responding to the *distribution* of buyer relationship-specific costs.

[13] These can be easily incorporated within our framework without qualitatively affecting our results. We also assume that each seller has operations that are independent of its business with the buyer and that these continue regardless of whether it is in business with the buyer.

[14] The idle state may also represent the scenario where the buyer has a domestic seller who charges a constant price in the buyer's currency and with whom the buyer has no relationship-specific costs.

relationship with seller 2 (denoted by 2). We use the variable $s$ to denote these three possibilities so that $s$ takes on values in the set $\{0,1,2\}$. The feasible policies of the buyer are given by

(2.4) $$\Gamma \equiv \{t_1, t_2, \ldots,\}$$

where $\{t_n\}$ is an increasing sequence of $F_t$ – stopping times representing the instants at which the buyer switches between the various states. The discounted expected utility of the buyer from following policy $\Gamma$ is given by (where $1_A$ is the indicator of the set $A$)

(2.5) $$U_\Gamma(p, s_0) = E\sum_{i=0}^{\infty}\Big\{ 1_{s=1}[-\exp(-b\,t_i)k_1 + \int_{t_i}^{t_{i+1}} \exp(-bs)(1-l\,p(s))ds] + 1_{s=2}[-\exp(-b\,t_i)k_2 + \int_{t_i}^{t_{i+1}} \exp(-bs)(1-p(s))ds]\Big\}.$$

In the above, $b > m$ is the discount rate of the buyer, $p$ is the initial price offered by seller 2, and $s_0$ is the initial state of the buyer. Each term in the summation above represents the total discounted cash flows of the buyer from using either of the sellers over the time interval $(t_i, t_{i+1})$. If it decides to use either seller, it pays a relationship-specific sunk cost (either $k_1$ or $k_2$) and variable costs (given by $p(.)$ or $l\,p(.)$ ). The goal of the buyer is to choose its switching policy $\Gamma$ so as to maximize its discounted expected utility $U_\Gamma$.[15]

**Remark 2:** Since the variable costs incurred by the buyer with the two sellers are proportional to each other, the "state of the buyer" is described by the price $p$ demanded by seller 2 and the value

[15] In this paper, we consider the scenario wherein the buyer may re-enter the foreign market after exiting it. We can easily modify our framework to analyze the situation where the buyer, after exiting the foreign market from a relationship with either seller, does not re-enter it. Our main results and economic insights are not qualitatively affected. We can also consider the repeated game wherein the buyer, after exiting the foreign market from a relationship with either seller, must renegotiate with both sellers before re-entering the market. The sellers' prices are constant (in their currency) *between* successive renegotiations, but may change after a renegotiation. Under altemative simplifying assumptions, we can show that our main results are again not qualitatively affected. We discuss this in more detail in Section 5.

of the variable $s$. For subsequent expositional convenience we shall refer to the buyer being in "state 0, state 1 or state 2" as the buyer being idle, with seller 1 or with seller 2 respectively.

From (2.5), it is clear that at any time $t$, the optimal decision of the buyer does not depend on time, but only on the current value of the variable $s$ of the buyer and the price $p$ demanded by seller 2. Therefore, it suffices to consider policies of the buyer that are described as follows:

(2.6) $$\Lambda \equiv \{p_{01}, p_{10}, p_{12}, p_{21}, p_{02}, p_{20}\}$$

where $p_{ij}$ is the *switching point* for switching from state $i$ to state $j$, i.e. $p_{ij}$ is the price of seller 2 at which the buyer will switch from state $i$ to state $j$. We now observe that it is never optimal for the buyer to switch from state 2 to state 1. Intuitively, when the buyer is in state 2, it has already incurred a sunk cost $k_2$ so it would be sub-optimal for the buyer to switch to state 1 paying an additional sunk cost of $k_1$ and obtaining a higher variable cost in return. It therefore suffices to consider policies of the buyer that are described as follows:

(2.7) $$\begin{array}{l} \{p_{01}, p_{10}\} \text{, i.e. the buyer only uses seller 1 (Case 1)} \\ \{p_{02}, p_{20}\}\text{, i.e. the buyer only uses seller 2 (Case 2)} \\ \{p_{01}, p_{10}, p_{12}, p_{20}\}\text{, i.e. the buyer may use both sellers over time (Case 3).} \end{array}$$

Since we have assumed that the buyer is initially in the idle state and the initial price $p > 1$, it follows that in the Case 3 above, it suffices to consider policies where

(2.8) $$p_{12} \le p_{01} \le 1$$

i.e. the switching point from state 1 to state 2 is below the switching point from state 0 to state 1. If it is optimal for the buyer to use both sellers over time, then our argument preceding (2.7) implies that the buyer will only enter state 2 via state 1. Moreover, since it is clearly never optimal for the buyer to switch into state 0 from state 1 or state 2 when its variable cost is favorable, it suffices to consider policies where

(2.9) $$p_{20} \ge 1, l p_{10} \ge 1 .$$

We denote the optimal *value functions* of the buyer, (i.e. the buyer's optimal expected utilities when it uses policies described by (2.7)) by $v_1, v_2, v_{12}$ respectively. The optimal value function $v$ of the buyer over all feasible policies is therefore given by

(2.10) $$v = \max(v_1, v_2, v_{12}) .$$

For the sellers to co-exist, the buyer's value function $v$ must be equal to $v_{12}$ and be *strictly greater* than $\max(v_1, v_2)$ so that its corresponding optimal policy must involve switching between both sellers over time as described by Case 3 in (2.7). One of the goals of our paper is the elucidation and characterization of the situations where the buyer will optimally switch between both sellers over time and the corresponding implications for equilibria between the buyer and the sellers.

***Functional Forms for the Value Functions***

If $u$ is the value function of a policy (not necessarily optimal) of the buyer, then it is well known that $u$ satisfies the following system of ordinary differential equations:

(2.11) $$\begin{aligned} & -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} = 0 \text{ in state } 0 \\ & -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} + 1 - lp = 0 \text{ in state } 1 \\ & -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} + 1 - p = 0 \text{ in state } 2 \end{aligned}$$

with appropriate boundary conditions for the transitions between different states. Any solution to the system of equations above is of the form;

(2.12) $$\begin{aligned} & u(p) = Ap^{h_1^+} + Bp^{h_1^-} \text{ in state } 0 \\ & u(p) = Cp^{h_1^+} + Dp^{h_1^-} + \frac{1}{b} - \frac{lp}{b - m} \text{ in state } 1 \\ & u(p) = Ep^{h_1^+} + Fp^{h_1^-} + \frac{1}{b} - \frac{p}{b - m} \text{ in state } 2 \end{aligned}$$

where $A, B, C, D, E, F$ are constants determined by the boundary conditions and $h_1^+, h_1^-$ are the positive and negative root respectively of the quadratic equation :

(2.13) $$\frac{1}{2}s^{2}x^{2}+(m-\frac{1}{2}s^{2})x-b=0.$$

We can now write down the functional forms for the value functions corresponding to the various types of policies the buyer may choose. For the sake of brevity, we only illustrate the case where the buyer uses policies where it switches between both sellers over time, i.e. Case 3 in (2.7). The other situations follow as special cases. Using (2.12) we see that the value function of a policy $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ is given by

(2.14)
$$\begin{aligned} u_{12}(p) &= A_{12}p^{h_1^-};\ p > p_{01} \text{ and the buyer is in state 0} \\ &= B_{12}p^{h_1^+} + C_{12}p^{h_1^-} + \frac{1}{b} - \frac{lp}{b-m};\ p_{12} < p < p_{10} \text{ and the buyer is in state 1} \\ &= D_{12}p^{h_1^+} + \frac{1}{b} - \frac{p}{b-m};\ p < p_{20} \text{ and the buyer is in state 2} \end{aligned}$$

with the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$ determined by *value matching* (continuity) conditions at the switching points. If $v_{12}(p_0)$ is the *optimal value function* of the buyer over the class of policies where it may switch between both sellers over time, we have

(2.15) $$v_{12}(p_0) = \sup_{(p_{01}, p_{12}, p_{10}, p_{20})} u_{12}(p_0).$$

If the policy defined by $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ is optimal within the class of policies where both sellers are used, then $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ are determined by additional *smooth pasting* or differentiability conditions at the switching points.

***The Value of the Switching Option***

As stated earlier, the buyer holds the option of switching between the two sellers over time. It is therefore interesting to determine the *value* of this switching option. We can use the notation introduced above to define this value as follows:

(2.16) $$\text{Value of Switching Option} = (v(p_0) - \max(v_1(p_0), v_2(p_0)),$$

where $p_0 > 1$ is the initial price demanded by seller 2. In the above equation, $v(p_0) = \max(v_1(p_0), v_2(p_0), v_{12}(p_0))$ is the maximum value to the buyer from using both sellers and $\max(v_1(p_0), v_2(p_0))$ is the maximum value from using only one of the two sellers. The switching option of the buyer has strictly positive value if and only if there exists a solution $(p_{01}, p_{12}, p_{10}, p_{20})$ of the value matching and smooth pasting conditions. The value of the switching option of the buyer need not always be positive. We provide necessary and sufficient conditions on the sunk and variable costs for the switching option value to be positive. The analysis of these conditions and an examination of their relationship with viable equilibria between the buyer and the sellers is one of the goals of the paper. This completes the formulation of the model.

## 3. Optimal Policies of the Buyer

In this section, we present our primary analytical results characterizing the optimal switching policies of the buyer *given* its relationship-specific costs vis-à-vis the two sellers and the prices they quote (in the sellers' currency). This analysis is crucial to the consideration of *equilibria* of the game between the buyer and the sellers where the sellers respond competitively by quoting prices to the buyer. We derive explicit conditions on the relationship-specific costs and the sellers' prices for the buyer's switching option to have strictly positive value. If the buyer's switching option does not have strictly positive value, it is optimal for the buyer to use only one of the two sellers whenever it is in the foreign market.

Our primary interest is in the scenario where the relationship-specific cost incurred in using seller 2 is larger than the relationship-specific cost incurred in using seller 1. For analytical convenience, we assume that the sunk cost of using seller 1 is zero, i.e. $k_1 = 0$ in the notation of the previous section. We relax this assumption in our numerical simulations that illustrate the generality of our analytical results. The optimal policies of the buyer are completely characterized in the following theorem.

**Theorem 3.1**

*a) For each* $k_2 > 0$*, there exists an interval of seller price ratios* $(l_{min}, l_{max})$ *such that the buyer's optimal policies have the following form: If* $l \le l_{min}$*, the buyer will use seller 1 alone; if* $l_{min} < l < l_{max}$*, the buyer will switch between both sellers over time; if* $l \ge l_{max}$*, the buyer will use seller 2 alone. We may have* $l_{min} = l_{max}$ *in which case the buyer's switching option has zero value for all* $l$ *.*

*b) For each* $l > 1$*, there exists an interval of relationship-specific costs* $(k_{min}, k_{max})$ *vis-à-vis seller 2 such that the buyer's optimal policies have the following form: if* $k_2 \le k_{min}$*, the buyer will use seller 2 alone; if* $k_{min} < k_2 < k_{max}$*, the buyer will switch between both sellers over time; if* $k_2 \ge k_{max}$*, the buyer will use seller 1 alone. We may have* $k_{min} = k_{max}$ *in which case the buyer's switching option has zero value for all* $k_2$*.*

*Proof.* The proof follows from the result of Proposition 3.1 later in the section.

The intuition for Theorem 3.1 is that for given relationship specific costs, if the seller price ratio $l$ is very high, seller 1's price is much higher than that of seller 2 so that the buyer is willing to pay the higher relationship specific costs and use seller 2 alone. On the other hand, for $l = 1$, i.e. equal variable costs, the buyer will only use seller 1 due to the lower relationship specific costs. Therefore, we would intuitively expect the existence of thresholds $l_{min}$ and $l_{max}$ such that for $l < l_{min}$, the buyer will only use seller 1, and for $l > l_{max}$, the buyer will only use seller 2. In the intermediate region, i.e. if $l_{min} < l < l_{max}$, the buyer will enter the market with seller 1 and switch to seller 2 if the price becomes more favorable so that both sellers will be used over time. The intuition for part b) of the theorem is analogous.

Although the existence of the thresholds $l_{\min}, l_{\max}$ can be understood intuitively, the conditions under which the interval $(l_{\min}, l_{\max})$ is non-empty are not obvious. The non-emptiness of the interval guarantees the existence of seller prices for which the optimal policies of the buyer involve switching between the sellers over time. When the prices quoted by the sellers are endogenously determined in equilibrium between the buyer and the sellers, the non-emptiness of the interval is therefore, a *necessary* (but not sufficient) condition for the sellers to co-exist in equilibrium with the buyer.

***Analytical Characterization of Switching Interval***

For each $l$, let $z_1^l(p_0)$ denote the optimal value the buyer obtains from the class of policies where it always enters the market with seller 1, but may optimally switch to seller 2 if the exchange rate becomes more favorable. Since $k_1 = 0$, it is easy to show that the optimal policy within this class must involve the buyer entering and exiting the market from a relationship with seller 1 when $p(.) = 1/l$. However, the buyer may optimally switch to seller 2 if the process $p(.)$ falls to a level $p_{12}$ lower than $1/l$ and exit the market from a relationship with seller 2 when $p(.)$ rises to some level $p_{20} > 1$. The value function $z_1^l(p_0)$ has the functional form given by (2.14) with $p_{01} = p_{10} = 1/l$, and the entry and exit triggers $p_{12}, p_{20}$ may be determined using value matching and smooth pasting conditions. We denote the optimal value functions of the buyer from policies where it *only* uses either seller 1 or seller 2 by $v_1^l(p_0), v_2(p_0)$ respectively where the superscript in $v_1^l(p_0)$ indicates the explicit dependence of the value function on the seller price ratio $l$. The following proposition characterizes the switching interval thresholds $l_{\min}, l_{\max}$ and thereby proves Theorem 3.1.

**Proposition 3.1**

*a)* $l_{\max} = \sup( l : z_1^l(p_0) > v_2(p_0))$

*b)* $l_{\min} = \min( l_0, l_{\max})$ *where* $l_0 = \inf( l : z_1^l(p_0) > v_1^l(p_0))$

*Proof.* In the Appendix.

We can use analogous arguments to characterize the interval $(k_{\min}, k_{\max})$ analytically and prove part b) of Theorem 3.1. We shall omit these for the sake of brevity.

We have solved the buyer's optimal switching problem numerically to obtain the optimal value functions of the buyer. To illustrate the generality of our conclusions, we have considered scenarios where both the relationship specific costs $k_1, k_2$ are non-zero. **Figures 1a** and **1b** graphically illustrate the result of part a) of Theorem 3.1**.** From the figures, we see that $l_{\min}$ is the point at which the buyer's optimal value function from using seller 1 alone $v_1$ equals the buyer's optimal value function over all feasible policies $v$ while $l_{\max}$ is the point at which the buyer's value function from using seller 2 alone $v_2$ equals $v$. For $l \le l_{\min}$ and $l \ge l_{\max}$, $v = v_1$ and $v = v_2$ respectively and for $l_{\min} < l < l_{\max}$, $v > \max(v_1, v_2)$, so that the switching option of the buyer given by (2.16) has strictly positive value. **Figure 1b** illustrates a scenario where the interval $(l_{\min}, l_{\max})$ is empty so that, depending on the value of $l$, one of the two sellers always captures the market. In this case, the value of the buyer's switching option is zero for all values of $l$.

**Figures 2a** and **2b** graphically illustrate the intuition underlying part b) of Theorem 3.1. In this case, for a fixed seller price ratio $l$, the switching option of the buyer has strictly positive value if and only if the relationship specific costs due to seller 2 lie in the interval $(k_{\min}, k_{\max})$ that may be empty. In **Figures 3a** and **3b**, we study the variation of the price triggers that define the stationary optimal policies of the buyer. As described in the previous section, four price triggers, i.e. the price where the buyer enters the market with seller 1, $p_{01}$, switches to seller 2, $p_{12}$, exits from seller 1, $p_{10}$, and exits from seller 2, $p_{20}$, come into play in the regions where the buyer's switching option has strictly positive value. In the regions where either seller 1 or seller 2 captures the market, only the corresponding entry and exit price triggers $p_{01}, p_{10}, p_{02}, p_{20}$ appear.

***Dependence of Buyer's Switching Option on Exchange Rate Volatility***

As discussed above, the buyer's switching option having nonzero value for some values of the sellers' price ratio $l$ is a necessary condition for equilibria where both sellers co-exist. This occurs if and only if the switching interval $(l_{\min}, l_{\max})$ is nonempty. We now determine economic conditions under which the buyer's switching option has zero value by studying the variation of the switching option value with exchange rate volatility. The following result shows that if the exchange rate volatility is beyond a critical threshold, the buyer's switching option has zero value for all values of the sellers' price ratio $l$, that is, for all possible seller prices.

**Proposition 3.2**

*There exists an exchange rate volatility level* $s_T$ *such that, for all values of the sellers' price ratio* $l$*, the buyer's switching option has zero value if* $s > s_T$*. Hence, for* $s > s_T$*, the only viable equilibria are those where one of the sellers captures the market.*

*Proof*. In the Appendix.

The intuition for this result is the following. The range of values of the exchange rate for which it is profitable for the buyer to be in business with either seller is bounded. As the exchange rate volatility increases, the buyer delays entry into the foreign market. Therefore, it spends less time in the market where it makes profits with either seller to exploit the potential tradeoff between the lower relationship-specific costs vis-à-vis seller 1 and the lower variable costs vis-à-vis seller 2. Above a critical volatility level, the buyer spends too little time in the foreign market to justify switching so that the tradeoff has zero value to the buyer. Therefore, the buyer will only establish relationships with one seller over time. The seller who captures the market may be either one of the two sellers. In the absence of uncertainty, one of the two sellers captures the market in general. The presence of uncertainty makes their co-existence feasible, but this feasibility disappears if the level of uncertainty is "too high". Our framework may be extended to the scenario where the foreign market is an oligopoly with multiple sellers. In this case, the result predicts that *seller concentration* increases with exchange rate volatility, or that fewer sellers will co-exist in equilibrium.

**Figures 4a-4c** graphically illustrate these results (where $k_1$ is chosen to be nonzero to illustrate their generality). They show the variation of $l_{\min}$ and $l_{\max}$ with the exchange rate volatility $s$ for different values of $k_2$. The results clearly show that as the volatility $s$ is increased ceteris paribus, there exists a threshold value $s^*$ below which the switching interval $(l_{\min}, l_{\max})$ is non-empty and above which it becomes empty. [16] Therefore, the buyer's switching option has positive value for $s < s^*$ and zero value for $s > s^*$.[17]

## 4. Equilibria between the Buyer and Sellers

In this section, we explicitly investigate equilibria of the game between the buyer and its sellers incorporating the variable costs of the sellers. We assume that the relationship specific costs $k_1, k_2$ the buyer incurs with the sellers are exogenously specified. However, the prices quoted by the

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[16] It is interesting to compare these results with those of Farrell and Shapiro (1989). The presence of exchange rate uncertainty in our framework erodes the "lock-in" effect on buyers due to the incurrence of relationship-specific costs in developing sellers so that a buyer may always exit a relationship with a particular seller. However, we may extend their notion of "lock in" to refer to the situation where the buyer is always in business with a particular seller *if it is in the foreign market*. For low exchange rate volatilities, the non-emptiness of the buyer's switching interval implies the viability of switching equilibria where the buyer is never locked in to a particular seller. However, if the volatility is above a critical threshold, the only viable equilibria are those where the buyer is locked in to one of the two sellers, that is, if the buyer is ever in the foreign market, it will only be with one of the sellers.

[17] We may obtain analogous results for the switching regions $(k_{\min}, k_{\max})$ that we do not present for the sake of brevity.

sellers (i.e. the buyer's variable costs with either seller) are determined competitively. In the game between the buyer and its sellers, the sellers' strategies are to quote constant prices per unit of the product in the sellers' currency and the buyer's response is to choose its optimal switching policy. Each seller has constant variable costs of production (in the sellers' currency) and therefore adopts a "markup pricing" policy by quoting a price at a premium to its cost. We denote the variable costs of the sellers by

(4.1) $$C_1, C_2 > 0,$$

where seller 2's costs may well exceed seller 1's costs. The prices set by the two sellers are given by $Q_1, Q_2$ with $Q_1 > C_1, Q_2 > C_2$. The sellers are risk-neutral and both have the same opportunity cost of capital or discount rate $b'$.

***Equilibria between the Buyer and a Single Seller***

As a benchmark, we first consider the case where there is a single foreign seller. This allows us to evaluate the benefit derived by the buyer from negotiating with multiple sellers in the foreign market. Moreover, this directly generalizes the classical analysis of the entry and exit decision of the buyer (see, for example, Dixit 1989) to the case where the seller responds strategically to the buyer. Moreover, in some situations, the equilibrium outcome of the two-seller game may reduce to that of the one-seller game. We can state the following result that provides explicit conditions for the existence of equilibrium between the buyer and the seller.

**Proposition 4.1**

*a) If the buyer has only one potential seller in the foreign market, equilibrium exists where the buyer and seller may establish a relationship if*

(4.2) $$b' + m > s^2$$

*The buyer and seller will not establish a relationship, that is, the buyer will not enter the market, if*

(4.3) $\beta' + \mu < \sigma^2$.

*b) If (4.3) holds and there is an exogenous price ceiling* $Q_{ceil}$ *on the prices the seller may quote, then a trading equilibrium where the buyer and seller may establish a relationship exists where, if* $Q_{ceil}$ *is sufficiently high, the seller quotes* $Q_{ceil}$.

*Proof.* In the Appendix.

The conditions (4.2) and (4.3) *are independent* of the relationship-specific costs of the buyer. The intuition for this result is that the seller is faced with the tradeoff of either quoting a lower price in its currency, thereby hastening the buyer's entry into a relationship with it, but obtaining lower profits in its currency *or* quoting a higher price, delaying the buyer's entry, but obtaining higher profits. When the exchange rate volatility is below a threshold, these effects balance each other at a finite price. However, when the volatility is beyond this threshold, the second effect in the seller's tradeoff predominates at any finite price so that it quotes unbounded prices thereby precluding the buyer's entry. This outcome can be averted if a regulator imposes a price ceiling on the seller. In the absence of strategic behavior by the seller, it is well known that the buyer delays its entry into a relationship with the seller with increasing uncertainty but there is always a nonzero probability that a relationship will be established (Dixit 1989). Incorporating strategic behavior by the seller alters this result significantly.

**The Structure of the Game between the Buyer and the Sellers**

We begin by defining the structure of the game between the buyer and the sellers. The costs of both sellers are common knowledge between all the players. The negotiating process begins at time 0. *At its discretion*, the buyer first elicits a price quote from either one of the two sellers, and then obtains a price quote from the other seller after revealing the first quote. Thus, we clearly have a leader-follower game structure where one of the sellers is chosen as the leader and the other the follower *at the behest* of the buyer. The sellers rationally anticipate the buyer's optimal policies (as determined in previous sections) in response to their quoted prices.

As we have seen in the previous sections, given prices $Q_1, Q_2$ quoted by the sellers, the buyer's optimal policy is either to use only one of the two sellers or to switch between the sellers over time. Therefore, the equilibrium outcome of the game is capture of the market by either seller or the co-existence of both sellers in the market. Alternatively, there may exist no equilibrium at all, i.e. the buyer and the sellers may never reach an agreement in which case market failure occurs. We shall now introduce some analytics essential to a detailed analysis of the game described above. The sellers' and buyer's value functions given the seller prices and the initial value of the exchange rate are denoted by $V_1(Q_1, Q_2, q(0)), V_2(Q_1, Q_2, q(0)), V(Q_1, Q_2, q(0))$ respectively. The buyer's value function $V$ has been derived in earlier sections. We now derive the sellers' value functions that depend on cash flows in the sellers' currency.

***The Value Functions of the Sellers***

Given exogenous relationship-specific costs $k_1, k_2$ and prices $Q_1, Q_2$ quoted by the sellers, we have seen that both sellers co-exist in the buyer's market if and only if $l_{\min} < Q_1 / Q_2 < l_{\max}$ where $(l_{\min}, l_{\max})$ is the interval of seller price ratios where the buyer's switching option has strictly positive value. The optimal policies of the buyer are described by the entry and exit points (expressed in terms of seller 2's quoted price $Q_2 q(.)$ )

$$\{p_{01}(Q_1 / Q_2), p_{12}(Q_1 / Q_2), p_{10}(Q_1 / Q_2), p_{20}(Q_1 / Q_2)\},$$

Thus, the entry, exit and switching points are functions of the ratio of seller prices $Q_1 / Q_2$. When the buyer uses only one of the two sellers, only the corresponding entry and exit price triggers appear.

We can now use standard arguments to show that the value functions $V_1(p), V_2(p)$ of the sellers as a function of the price $p$ quoted by seller 2 *in the buyer's currency* must satisfy the following system of differential equations:

$$-b^{'} V_i + mp\frac{dV_i}{dp} + \frac{1}{2}s^{2}p^{2}\frac{d^2V_i}{dp^2} = 0; \text{ when the buyer is } \textit{not in} \text{ state } i \in \{1,2\}$$

$$-b^{'} V_i + mp\frac{dV_i}{dp} + \frac{1}{2}s^{2}p^{2}\frac{d^2V_i}{dp^2} + Q_i - C_i = 0; \text{ when the buyer is } \textit{in} \text{ state } i \in \{1,2\}.$$

The first equation arises from the fact that the seller $i$ obtains no cash flows when it is not in business with the buyer and the second arises from the fact that the seller obtains cash flows at the rate $(Q_i - C_i)$ when it is in business with the buyer. For the sake of brevity, we indicate the functional forms of the value functions only for the scenario where the buyer's policy involves switching between both sellers, i.e. Case 3 in (2.7). Given the optimal policies of the buyer, the sellers' value functions are therefore given by

(4.4)

$$\begin{aligned} V_1(p) &= A_1 p^{r_1^-};\ p \geq p_{01}(l) \text{ and the buyer is in state 0} \\ &= B_1 p^{r_1^+} + C_1 p^{r_1^-} + \frac{Q_1 - C_1}{b^{'}};\ p_{10}(l) \geq p \geq p_{12}(l) \text{ and the buyer is in state 1} \\ &= D_1 p^{r_1^+};\ p \leq p_{20}(l) \text{ and the buyer is in state 2} \end{aligned}$$

(4.5)

$$\begin{aligned} V_2(p) &= A_2 p^{r_1^-};\ p \geq p_{12}(l) \text{ and the buyer is in state 0 or state 1} \\ &= D_2 p^{r_1^+} + \frac{Q_2 - C_2}{b^{'}};\ p \leq p_{20}(l) \text{ and the buyer is in state 2} \end{aligned}$$

with $r_1^+, r_1^-$ being the positive and negative roots of (2.13) with $b$ replaced by $b^{'}$ and $l = Q_1 / Q_2$.

The value functions of the sellers $V_1(Q_1, Q_2, q(0)),\ V_2(Q_1, Q_2, q(0))$ may be *discontinuous* functions of the arguments $Q_1, Q_2$. The only possible discontinuities of the value functions are at the *indifference* points where $Q_1/Q_2 = l_{\min}$ or $Q_1/Q_2 = l_{\max}$. Due to the nature of the optimal policies of the buyer, if the sellers' value functions are discontinuous at $l_{\min}$ and $l_{\max}$, then seller 1's value function *falls* at these points and seller 2's value function *rises.*

In order to ensure that the value functions of the sellers are well defined we introduce an additional rule of the game. At $l = l_{\min}$ and $l = l_{\max}$, the buyer always chooses the policy that maximizes the expected profits of the follower. Therefore, if seller 1 is chosen as the leader and seller 2 is chosen as the follower, then at $l = l_{\min}$, the buyer would choose the policy that uses both sellers over time. At $l = l_{\max}$, the buyer would choose the policy that uses seller 2 alone. Correspondingly, if seller 2 is chosen as the leader and seller 1 is the follower, then at $l = l_{\min}$, the buyer would choose the policy that uses seller 1 alone. At $l = l_{\max}$, the buyer would choose the policy that uses both sellers over time. It is not difficult to check that the rule stated above ensures that the value functions $V_1(q(0),.,Q_2)$ and $V_2(q(0),Q_1,.)$ are *left continuous*[18]. The additional rule of the game ensures the existence of equilibria between the buyer and both sellers under broader conditions. This benefits the buyer since it is able to use its bargaining power with both sellers effectively.

If seller 1 is chosen as the leader and seller 2 the follower, then, for each price quote $Q_1$ of seller 1, let $y_2(Q_1)$ be the best response of seller 2, i.e. $y_2(Q_1) = \arg\max_{Q_2} V_2(q(0),Q_1,Q_2)$. The left continuity of $V_2(q(0),Q_1,.)$ ensures that $y_2(Q_1)$ always exists. Since seller 1 is the leader, it will quote a price $Q_1^*$ satisfying

(4.6) $$Q_1^* = \arg\max_{Q_1} V_1(q(0),Q_1,y_2(Q_1)).$$

If (4.6) has no solution, i.e. $Q_1^*$ does not exist, then seller 1 is unable to quote a price and is therefore not in the market. Hence, the problem reduces to the case where the buyer negotiates with

[18] The notation $V_1(q(0),.,Q_2)$ means that the second argument is varied while the first and third arguments of the function are kept fixed.

only a *single seller*, i.e. seller 2 in the foreign market. This provides additional motivation for our detailed consideration of the one-seller scenario earlier in the section. Similarly, if seller 2 is chosen as the leader and seller 1 the follower, then for each price quote $Q_2$ of seller 2, let $y_1(Q_2)$ be the best response of seller 1, i.e. $y_1(Q_2) = \arg\max_{Q_1} V_1(q(0), Q_1, Q_2)$. The left continuity of $V_1(q(0),., Q_2)$ ensures that $y_1(Q_2)$ exists. Since seller 2 is the leader, it will quote a price $Q_2^*$ satisfying

(4.7) $$Q_2^* = \arg\max_{Q_2} V_2(q(0), y_1(Q_2), Q_2).$$

If $Q_2^*$ does not exist, seller 2 is unable to quote a price so that seller 1 is the only seller in the market and we are again in the single seller scenario discussed earlier. The buyer will choose seller 1 (seller 2) as the leader and seller 2 (seller 1) as the follower in equilibrium if and only if

$$V(q(0), Q_1^*, y_2(Q_1^*)) > (<) V(q(0), y_1(Q_2^*), Q_2^*).$$

***Equilibria between the Buyer and Sellers***

We can now state the following result that provides *sufficient* conditions for both sellers to co-exist in any possible equilibrium with the buyer or for either seller to capture the market.

**Proposition 4.2**

*Suppose a solution to either (4.6) or (4.7) exists.*

*a) If $(l_{\min}, l_{\max})$ is non-empty and $l_{\min} < C_1/C_2 < l_{\max}$, then the capture of the market by either seller cannot be an equilibrium outcome, that is, the sellers must co-exist in any possible equilibrium.*

*b) Suppose $(l_{\min}, l_{\max})$ is empty so that $l_{\min} = l_{\max} = l^*$. If $C_1/C_2 > l^*$, seller 2 captures the market in any equilibrium. If $C_1/C_2 < l^*$, seller 1 captures the market in any equilibrium. If $C_1/C_2 = l^*$, the sellers quote prices equal to their marginal costs in the unique equilibrium, and the buyer is indifferent between them.*

*Proof.* In the Appendix.

The above proposition provides a precise connection between the analysis of the optimal policies of the buyer presented in the previous sections and the equilibrium analysis of the present section. We argued previously that the non-emptiness of the switching region $(l_{\min}, l_{\max})$ is a *necessary* but not *sufficient* condition for both sellers to co-exist in any possible equilibrium with the buyer. For the sellers to co-exist in equilibrium with the buyer, the ratio of the prices they quote must lie in the interval $(l_{\min}, l_{\max})$. The result of Proposition 4.2 a) says that if the costs of the sellers are "aligned" so that their ratio lies within $(l_{\min}, l_{\max})$, then the ratio of the prices they quote in equilibrium must also lie within $(l_{\min}, l_{\max})$. On the other hand, if $(l_{\min}, l_{\max})$ is empty, then the seller with the higher bargaining power (depending on whether $C_1 / C_2 > l^*$ or $C_1 / C_2 < l^*$ ) captures the market. However, as the following elementary result (whose proof we omit) shows, the level of prices when seller 1 captures the market may be very different from the level when seller 2 captures it.

**Proposition 4.3**

*Suppose a solution to either (4.6) or (4.7) exists. If seller 1 captures the market in equilibrium, then the equilibrium price* $Q_1^*$ *must satisfy* $C_1 < Q_1^* \leq l_{\min} C_2$. *If seller 2 captures the market, then the equilibrium price* $Q_2^*$ *must satisfy* $C_2 < Q_2^* \leq \dfrac{C_1}{l_{\max}} < C_1$.

From the above proposition, we see that if seller 2 captures the market so that $C_2 < C_1$, it does so by quoting a price *lower* than seller 1's variable cost $C_1$. In the absence of relationship-specific costs, the usual result of Bertrand competition would lead to seller 2 capturing the market by quoting a price $C_1$. Therefore, the presence of relationship-specific costs leads to *lower* prices. On the other hand, if $C_1 \leq C_2$ and seller 1 captures the market, it does so by quoting a price that is, typically *higher* than seller 2's variable cost $C_2$, the price it would quote to capture the market in the absence of relationship-specific costs. Therefore, relationship-specific costs may either raise or lower the level of prices depending on which seller captures the market.

It is interesting to combine the results of the previous two propositions and the result that the buyer's switching option has zero value for all possible seller prices when the exchange rate volatility is above a critical level. In this scenario, seller 1 may capture the market if it has higher bargaining power (that is, $C_1 / C_2 < l^*$) by "exploiting" the buyer, that is, by quoting prices *higher* than it would quote in the absence of relationship-specific costs. On the other hand, if seller 2 has higher bargaining power (that is, $C_1 / C_2 > l^*$), it captures the market by "undercutting" its rival, that is, by quoting prices *lower* than it would quote in the absence of relationship-specific costs[19]. Therefore, the exchange rate volatility plays an important role in determining whether relationship-specific costs raise or lower competition among the sellers and whether the overall level of prices is higher or lower[20].

***Numerical Derivation of Equilibria***

We have implemented a numerical procedure to derive equilibria between the buyer and the sellers. Our numerical algorithm assumes that the prices quoted by the sellers can only be multiples of a fixed currency unit. With this additional assumption, since prices take values in a discrete set, it is not difficult to see that the only scenarios where equilibrium *may* fail to exist are those where the maxima in *both* (4.6) and (4.7) are attained at $\infty$. We have computed equilibria for various combinations of underlying parameter values and for different costs of the sellers.

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[19] It is interesting to interpret our results when our framework is applied to the scenario where seller 1 is the incumbent and seller 2 is the potential market entrant (Farrell and Shapiro 1988). We provide explicit conditions for when the incumbent is able to *deter* the potential entrant, the incumbent *accommodates* the entrant, or the entrant *captures* the market from the incumbent. If the uncertainty is beyond a threshold, either the incumbent succeeds in deterring the entrant or the entrant captures the market.

[20] These results may be compared with those of Klemperer (1995) who finds that switching costs hamper competition and raise the level of prices in a mature market and Von Weizsacker (1984) who concludes that switching costs increase competition and lower the level of prices.

*Equilibria with Cost Differentials*

**Figure 5** shows the variation of the equilibrium prices quoted by the sellers with the ratio of the costs of the two sellers. The figure depicts all the three types of equilibria between the buyer and the sellers: the region where seller 1 captures the market, an intermediate region where the sellers co-exist when the costs are comparable and a third region where seller 2 captures the market. Consistent with the result of Proposition 4.2**,** we note that both sellers co-exist when $l_{\min} = 1.04 < C_1 / C_2 < 1.09 = l_{\max}$. However, as the figure indicates, the sellers may co-exist in equilibrium even when $C_1 / C_2$ does not lie in $(l_{\min}, l_{\max})$ since the condition of Proposition 4.2 is a *sufficient* but not *necessary* condition for co-existence. We notice that the level of prices quoted by seller 1 when it captures the market is higher than the level of prices quoted by seller 2 when it captures the market. This is consistent with the result of Proposition 4.3**.**

**Figure 6** illustrates a scenario where the parameters chosen satisfy the condition (4.3). Therefore, by the result of Proposition 4.1, in the monopolistic situation with a single seller, the buyer would not enter the foreign market. When there are two potential sellers, the figure indicates that there may be four possible outcomes: either seller capturing the market, both sellers co-existing and, most interestingly, an intermediate region (where the costs of the two sellers are "close" to each other) where the sellers quote infinite prices so that the buyer will not enter the foreign market. The existence of equilibria with both sellers clearly shows that the buyer may enter the foreign market when it negotiates with both sellers, that it would not otherwise have entered if it had negotiated with only one seller. Interestingly, the equilibrium outcome may well be the capture of the market by either seller. However, if the sellers' costs are "close" to each other, the buyer enters the foreign market only if there is an exogenously imposed ceiling on seller prices due to the possibility of *unrestrained tacit collusion* between the sellers where the sellers prefer to accommodate each other in equilibrium by quoting increasingly higher prices[21]. Thus, the intervention of a regulator may be required to ensure that the buyer enters the foreign market.

**Strategic Relationships with Relationship-Specific Costs and Exchange Rate Uncertainty**

We will now combine all the results we have obtained to describe the effects of the interplay between exchange rate uncertainty and differing relationship-specific costs on the strategic relationships between the respective players.

First consider the monopolistic situation with a single seller. In the absence of exchange rate uncertainty, we may see that if a trading equilibrium exists (conditional on the seller's cost), the seller quotes the monopoly price at which the buyer's profits are zero. The seller, however, obtains positive profits in general. In the presence of exchange rate uncertainty, the buyer and the seller both possess "values of waiting" to enter into business. If the uncertainty is "low" (that is, condition (4.2) of Proposition 4.1 holds), then equilibrium exists where, in general, both the buyer and the seller obtain positive expected profits. However, if the uncertainty is "high" (that is, condition (4.3) of Proposition 4.1 holds), the seller is never satiated at a finite price and the buyer will not enter the foreign market *unless* there is an exogenously imposed ceiling on the prices the seller may quote.

Now consider the scenario where the buyer negotiates with both sellers. In the absence of exchange rate uncertainty, we may see that the seller with the more favorable combination of its variable costs and buyer relationship-specific costs, captures the market. In the presence of "low" uncertainty (that is, (4.2) holds), we may have equilibria where either seller captures the market or both sellers co-exist (as Figure 5 indicates) depending on the relative magnitudes of their respective costs and the relationship-specific costs of the buyer. However, if the costs of the sellers are "aligned" as in the hypothesis of Proposition 4.2, the sellers *must* co-exist in equilibrium with the

[21] Therefore, one of the equilibrium outcomes may be *tacit* collusion. Farrell and Shapiro (1988) and Klemperer (1987a) both report the possibility of tacit collusion in markets with switching costs in frameworks where sellers compete in each period. We show that tacit collusion may occur with long-term contracts between buyers and sellers.

buyer. Therefore, the presence of relationship-specific costs and exogenous uncertainty may allow both sellers to co-exist in equilibrium.

Finally, consider the situation where the exchange rate uncertainty is "high" (that is, (4.3) holds). When the relationship-specific costs of the buyer vis-à-vis the sellers are different, the switching option of the buyer may have positive value provided the exchange rate volatility is below the critical threshold (Proposition 3.2) above which the buyer's switching option has zero value. In general, either seller may be able to quote a price that captures the market or both may choose to quote prices in equilibrium that allow their co-existence. If the sellers' costs are identical or very "close" to each other, the *market capture* price for either seller, if one exists, would be very close to its cost so that the seller would obtain low expected profits from capturing the market. The presence of high uncertainty may therefore induce both sellers to accommodate each other wherein they quote prices that allow their co-existence. However, very high uncertainty levels may induce both sellers to quote increasingly higher prices thereby delaying the buyer's entry into the market, but obtaining higher profits when in business and this may lead to unrestrained tacit collusion by the sellers. Hence, the buyer may not enter the foreign market when the sellers' costs are "close" to each other with differing relationship-specific costs as seen in Figure 6. The entry of the buyer is possible only if a regulator imposes a price ceiling on the sellers.

If the sellers' costs are significantly different from each other and the buyer relationship-specific costs are different, we may have equilibria where either seller captures the market or both sellers co-exist even with high exchange rate volatilities as illustrated in Figure 6. However, as Proposition 3.2 and Figures 4a, 4b, 4c illustrate, above the critical volatility level where the buyer's switching option has zero value, both sellers *cannot* co-exist in equilibrium regardless of their costs. By the results of part b) of Proposition 4.2 and Proposition 4.3, seller 1 captures the market if it has higher bargaining power by quoting a price that is typically higher than it would quote without relationship-specific costs. On the other hand, if seller 2 has higher bargaining power, it captures the market by quoting a price that is lower than it would quote without relationship-specific costs.

Therefore, the presence of relationship-specific costs and exogenous dynamic uncertainty may either increase or decrease competition among the sellers and either increase or decrease the level of prices. The complexity of the equilibrium dynamics between the buyer and the sellers described above indicates that the presence of both differing relationship-specific costs and exogenous uncertainty, causes very significant changes in the strategic relationships between the respective players.

## 5. Summary and Conclusions

In this paper, we have proposed and investigated an equilibrium framework to analyze strategic relationships between buyers and sellers in different countries exposed to exchange rate uncertainty. An investigation of this scenario is particularly relevant due to the dramatic acceleration in globalization over the past decade. Our objective was to analyze the impact of differences in the relationship-specific costs of the buyers and exchange rate uncertainty on the relative bargaining power of the respective players and the resulting effect on the nature of their strategic relationships.

We showed that the presence of relationship-specific costs may lead to various possible equilibrium outcomes: switching equilibria where the sellers co-exist, no-switching equilibria where either seller may capture the market, or tacit collusion between the sellers that may necessitate the imposition of price ceilings to induce the entry of the buyer. The level of exchange rate uncertainty plays a crucial role in determining the actual equilibrium outcome. Contrary to the extant literature, we find that, in the presence of exchange rate uncertainty, relationship-specific costs may either raise or lower competition and either raise or lower the level of prices in a mature market.

As mentioned in Section 2 (footnote 15), we may modify our framework to consider the situation where the buyer, after exiting the foreign market from a relationship with either seller, must renegotiate with both sellers before re-entering the market. The prices the sellers quote are constant (in their currency) between successive renegotiations. For tractability, we may make alternative simplifying assumptions.

One alternative is to assume that the buyer is *myopic*, that is, after each renegotiation, the buyer responds to the sellers' quoted prices by adopting a policy that maximizes its expected profits till it exits the market *without* considering the profits it may derive as a result of future re-entries into the market. Hence, each "round" of the renegotiation game is independent of other rounds. This may be a reasonable description of the behavior of a buyer contemplating trade with foreign sellers where the buyer's relevant decision horizon ends at the time it exits the foreign market. The renegotiations may occur at different values of the exchange rate that may be exogenously specified or endogenously determined. After each renegotiation, the buyer adopts an entry, exit, and switching policy assuming that it will not re-enter the foreign market once it has exited it. The sellers quote prices rationally anticipating the buyer's myopic policy. Each round of this renegotiation game is now a straightforward modification of our existing framework to the scenario where the buyer does not re-enter the foreign market after exiting it (see footnote 16). We can then show that the results of this paper qualitatively apply to each round of this renegotiation game where the notions of market capture and co-existence pertain to each round. The sellers' quoted prices may now differ across rounds of the game depending on the exchange rate levels at which renegotiation occurs.

Another alternative is to replace the assumption that the buyer is myopic with the assumption that renegotiations may occur only when the exchange rate has a specific value. The value of the exchange rate at which renegotiations may occur can be exogenous or can be chosen by the buyer. This may be a reasonable description of the behavior of a buyer that contemplates entering a foreign market and negotiates with foreign sellers only when the exchange rate is at a favorable level. For example, once a buyer has exited a foreign market because the exchange rate is unfavorable, it is reasonable to suppose that the buyer will contemplate re-entry and renegotiate with foreign sellers only when the exchange rate returns to a favorable level. It may easily be seen, given the process (2.1) for the exchange rate, that equilibria of this repeated game are renegotiation-proof, that is, sellers quote constant prices in their currency that are never renegotiated. Our main results are not qualitatively affected by this modification of our framework although the equilibrium prices the sellers quote in this game are, in general, different from the equilibrium prices we obtain within the framework assumed in this paper.

As discussed in the introduction (footnote 2), our model is applicable to other economic scenarios. In this paper, we have considered the scenario where the buyer's profits are influenced by the stochastic variation of its costs due to the sellers. It is easy to modify our model to investigate economic scenarios where the costs are constant while the payoffs are stochastic. Such a model may be appropriate in the investigation of the strategic behavior of a buyer that is investing in new technology, incurring expenditure on research and development, training human capital, developing natural resources, etc. In each case, the buyer may choose to dynamically incur different levels of sunk costs (analogous to the "relationship specific" costs in our model) to obtain proportionally higher random payoffs.

Several important issues can be considered in future research. We may easily extend our framework to consider *non-identical buyers,* that is, different buyers may have different pairs of relationship-specific costs vis-à-vis the sellers that may be unobservable to the sellers (see footnote 12). We can also include the possibility of new buyers entering the market periodically with the sellers quoting different prices to new buyers (see, for example, Klemperer 1995, Farrell and Shapiro 1988). It would be interesting to consider the general dynamic game between the buyer and the sellers where each seller may periodically change the price it quotes in response to actions by the buyer and by the other seller. It would also be important to examine the influence of information asymmetry amongst the players on the existence and nature of the equilibria between them.

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**APPENDIX**

**Proof of Proposition 3.1**

a) We begin by noting that $z_1^l(p_0)$ is a monotonically decreasing continuous function of $l$. For $l=1$, i.e. both sellers quote equal prices, it is clearly optimal for the buyer to use seller 1 alone. Therefore, $z_1^1(p_0)=v_1^1(p_0)>v_2(p_0)$. Moreover, $\lim_{l\to\infty} z_1^l(p_0)=0<v_2(p_0)$. Hence, $l_{\max}$ exists and $l_{\max}>1$. Since, $z_1^l(p_0)=v_1^l(p_0)$ for $l\le 1$, we must have $l_0>1$. Therefore, $l_{\min}\ge 1$[22]. Since $v_2(p_0)$ does not depend on $l$ it follows by the definition of $l_{\max}$ that it is always optimal for the buyer to enter the market with seller 1 for $l<l_{\max}$ and use seller 2 alone for $l>l_{\max}$ with $l_{\max}$ being the point of *indifference* between these two policies.

b) Since, by definition, $l_{\min}\le l_{\max}$, it follows from the proof of part a) that for $l<l_{\min}$, it is optimal for the buyer to enter the market with seller 1. Moreover, by the definition of $l_{\min}$ in the statement of the proposition, it is optimal for the buyer to use seller 1 alone for $l<l_{\min}$.

It remains to prove that if $l_{\min}=l_0<l_{\max}$ and $l_{\min}<l<l_{\max}$, it is never optimal for the buyer to use seller 1 alone, i.e. the buyer will always enter the market with seller 1 and optimally switch to seller 2 when the price falls further. It is important to emphasize here that this is *not* obvious at the outset. Even though the buyer is indifferent between the policy of using seller 1 alone and the policy of using both sellers at $l=l_0$, there may exist some $l>l_0$ for which it is optimal to use seller 1 alone. We can prove the result by the following arguments.

By the definition of $l_0$ and the fact that $z_1^l(p_0), v_1^l(p_0)$ are continuous functions of $l$, $z_1^{l_0}(p_0)=v_1^{l_0}(p_0)$, that is, for $l=l_0$, the buyer is *indifferent* between the policy of using seller 1

[22] However, we may have $l_0=\infty$ so that $l_{\min}=l_{\max}$, if $z_1^l(p_0)=v_1^l(p_0)$ for all $l$, that is, in the situation where it is never optimal for the buyer to switch to seller 2 if it enters the market with seller 1.

alone and the policy of entering the market with seller 1 and optimally switching to seller 2 if the price falls further. Let the optimal entry and exit points for state 2 when $l = l_0$ be given by $p_{12}, p_{20}$ respectively. Then $l_0$ is the indifference point if and only if

$$E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)(1-p(s))ds - E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)[1_{p(s)\leq\frac{1}{l_0}}(1-l_0 p(s))]ds = k_2$$

In the above, the first term on the left is the expected value (conditional on the current price being $p_{12}$) of switching to seller 2 and continuing with seller 2 until the exit trigger $p_{20}$ is reached and the second term on the left hand side is the corresponding expected value if the buyer were to instead follow the policy of entering and exiting with seller 1 at the threshold $1/l_0$ without ever switching to seller 2. The relation above basically states that the buyer is able to exactly recover its relationship-specific cost of switching to seller 2 through its resultant variable cost savings. These arguments would repeat for each subsequent re-entry of the buyer into the foreign market after exiting it from a relationship with seller 2.

Suppose it were optimal to use seller 1 alone for some $l > l_0$. Since $l > l_0, 1/l < 1/l_0$, we see that

$$E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)1_{p(s)\leq\frac{1}{l_0}}(1-l_0 p(s))ds > E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)1_{p(s)\leq\frac{1}{l}}(1-l\ p(s))ds.$$

Therefore, we see that

$$E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)(1-p(s))ds - E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)1_{p(s)\leq\frac{1}{l}}(1-l\ p(s))ds > k_2,$$

Therefore, the policy of switching to seller 2 at $p_{12}$ and exiting at $p_{20}$ has strictly greater value than the policy of using seller 1 alone. Therefore, the policy of using seller 1 alone cannot be optimal. This completes the proof of the proposition. ♦

**Proof of Proposition 3.2**

For analytical simplicity, recall that we consider the case where $k_1 = 0, k_2 > 0$. We begin by first explaining the intuition of the proof. Our goal is to show the existence of a volatility level $s_T$ such that the buyer's switching option has zero value for $s > s_T$ and *all* values of $l$. We have previously argued that if it is optimal for the buyer to enter the foreign market with seller 2, it will never switch to seller 1 so that its switching option trivially has zero value. Therefore, it suffices for us to show the existence of $s_T$ such that for $s > s_T$, if the buyer enters the market with seller 1, it will never subsequently switch to seller 2 for any value of $l$.

Hence, we consider the scenario where the buyer enters the market with seller 1 when the price process $p(.) = \frac{1}{l}$. We then evaluate the *additional value* the buyer obtains from switching to seller 2 when $p(.) = p_e \leq \frac{1}{l}$ and continues with seller 2 until it exits the market at some $p_q > 1$. The additional value is the difference between the value of switching to seller 2 at $p_e$ and continuing with seller 2 until $p(.) = p_q$ AND the value of continuing with the policy of entering and exiting the market with seller 1 whenever $p(.) = \frac{1}{l}$ until $p(.) = p_q$. The additional value is therefore the difference between the variable cost savings of switching to seller 2 and the relationship-specific cost $k_2$ incurred, over a *single* entry-exit cycle with seller 2. We show the existence of an exchange rate volatility level $s_T$ beyond which the additional value the buyer obtains from switching is strictly negative for all values of $l$ and all possible switching points $p_e \leq 1/l$. Therefore, the buyer is unable to recover its relationship-specific costs of switching to seller 2 over each entry-exit cycle with seller 2. These arguments may be repeated each time the buyer re-enters the foreign market after exiting it from a relationship with seller 2. It follows that it is never optimal for the buyer to switch to seller 2 from seller 1 for $s > s_T$. Intuitively, the reason why this happens is that as the exchange rate volatility increases, the higher risk causes the threshold price at which the buyer gains nonzero value from switching to seller 2 to decrease. But this decreases the time the buyer is able to spend with seller 2 and reduces the potential variable cost savings from switching to seller 2. Above a critical volatility level, the buyer is unable to recover the relationship-specific costs it incurs in switching to seller 2. We now proceed with the actual proof.

<u>Step 1</u> *Characterization of Buyer's Additional Value from Switching*

The buyer enters the market with seller 1 when the price process $p(.) = 1/l$. Without loss of generality, we may assume that $l > 1$ since it is optimal for the buyer to use seller 1 alone if $l \leq 1$. We characterize the maximum possible additional value $w(p_e, l, s)$ the buyer obtains from switching to seller 2 at some $p_e \leq 1/l$ and continuing with seller 2 until it exits the market at some $p_q(l,s) > 1$. The additional value from switching and the optimal exit point both depend on $l$ and $s$. The function $w(p_e, l, s)$ can be expressed as follows:

(A1)
$$w(p_e, l, s) = E_{p_e}\left\{ \int_0^{t_{p_q(l,s)}} e^{-bs}[(1-p(s)) - 1_{p(s)\leq 1/l}(1-lp(s))]ds \right\} - k_2$$
$$= x(p_e, l, s) - k_2$$

The first term in the integrand above is the discounted profit from switching to seller 2 and the second term is the discounted profit from continuing with the policy of entering and exiting the market with seller 1 at $1/l$, until the exit point $p_q(l,s)$ is reached. We can use Ito's lemma to evaluate the expectation $x(p_e, l, s)$ by determining the *function* $x(p, l, s)$ where the argument $p$ varies. In particular, we may see that $x(p, l, s)$ satisfies the following differential equation for

*each* $l, s$ : $\frac{1}{2}s^2 p^2 \frac{\partial^2 x}{\partial p^2} + mp\frac{\partial x}{\partial p} - bx + (1-p) - 1_{p\leq 1/l}(1-lp) = 0$

with the boundary conditions $x(p_q(l,s),l,s)=0, \frac{\partial x(p,l,s)}{\partial p}|_{p=p_q(l,s)}=0$. These arise from the fact that $x(p,l,s)$ attains its maximum possible value at $p=p_q(l,s)$. From the above, we see that $x(p,l,s)$ has the following functional form:

$$
\begin{aligned}
x(p,l,s) &= A(l,s)p^{h_1^+(s)} + \frac{(l-1)p}{b-m}; \text{ for } p<1/l \\
&= B(l,s)p^{h_1^+(s)} + C(l,s)p^{h_1^-(s)} + \frac{1}{b} - \frac{p}{b-m}; \text{ for } 1/l \le p < p_q(l,s) \qquad \text{(A2)} \\
&= 0 \qquad ; \text{ for } p \ge p_q(l,s)
\end{aligned}
$$

In the above, $h_1^+(s), h_1^-(s)$ are the positive and negative roots of the quadratic equation (2.13) where we have indicated their explicit dependence on the exchange rate volatility $s$ .

<u>Step 2</u> *Coefficients and Exit Point* $p_q(l,s)$

The coefficients above and the exit point $p_q(l,s)$ are obtained from value matching (continuity) and smooth pasting (differentiability) conditions at the boundaries of the different regions. We omit writing down these equations for the sake of notational brevity. We can solve these equations after some tedious algebra to obtain the following equation for $p_q(l,s)$:

$$
\text{(A3)} \quad (mh_1^+(s)-b)(lp_q(l,s))^{h_1^-(s)} + bp_q(l,s)(1-h_1^+(s)) = (m-b)h_1^+(s)
$$

It is not difficult to check that there is exactly one root of the above equation that is *greater than 1* and this is the required exit point.

<u>Step 3</u> *Properties of roots* $h_1^+(s), h_1^-(s)$ *and coefficients* $A(l,s), B(l,s)$

The coefficients $B(l,s), A(l,s)$ in (A2) are given by

$$
\begin{aligned}
B(l,s) &= \frac{h_1^-(s)}{b(h_1^+(s)-h_1^-(s))}(p_q(l,s))^{-h_1^+(s)} + \frac{1-h_1^-(s)}{(b-m)(h_1^+(s)-h_1^-(s))}(p_q(l,s))^{1-h_1^+(s)} \qquad \text{(A4)} \\
A(l,s) &= B(l,s) + \frac{l^{h_1^+(s)}}{(h_1^+(s)-h_1^-(s))}\left(\frac{mh_1^-(s)-b}{b(b-m)}\right)
\end{aligned}
$$

We may easily check that the roots $h_1^+(s), h_1^-(s)$ of equation (2.13) have the following properties:

(A5) $$h_1^+(s) > 1, h_1^-(s) < 0, \ \lim_{s\to\infty} h_1^+(s) = 1, \lim_{s\to\infty} h_1^-(s) = 0$$

From (A4) we see that

(A6) $$|B(l,s)| \le \left|\frac{h_1^-(s)}{b(h_1^+(s)-h_1^-(s))}\right| |(p_q(l,s))^{-h_1^+(s)}| + \left|\frac{1-h_1^-(s)}{(b-m)(h_1^+(s)-h_1^-(s))}\right| |(p_q(l,s))^{1-h_1^+(s)}|$$

Since $p_q(l,s) > 1$ and $l > 1$ and $h_1^+(s) > 1$, we see that $(p_q(l,s))^{-h_1^+(s)} < 1$ and $(p_q(l,s))^{1-h_1^+(s)} < 1$.

Hence, from (A5) and (A6), we see that

(A7) $$\lim_{s\to\infty} |B(l,s)| \le \frac{1}{b-m}$$

where the inequality above holds *uniformly* in $l$. We now see from (A1), (A2), (A4), (A5), (A7),

$$\lim_{s\to\infty} w(p_e, l, s) = \lim_{s\to\infty} x(p_e, l, s) - k_2 = \lim_{s\to\infty} A(l,s)(p_e)^{h_1^+(s)} + \frac{(l-1)p_e}{b-m} - k_2$$

$$= \lim_{s\to\infty} B(l,s)(p_e)^{h_1^+(s)} + \frac{(lp_e)^{h_1^+(s)}}{(h_1^+(s)-h_1^-(s))}\left(\frac{mh_1^-(s)-b}{b(b-m)}\right) + \frac{(l-1)p_e}{b-m} - k_2$$

$$= \frac{(1-l)p_e}{(b-m)} + \frac{(l-1)p_e}{b-m} - k_2 < 0$$

Since $p_e \le \frac{1}{l}$ so that $lp_e \le 1$, we see that the inequality above holds *uniformly* in $p_e \in [0,1]$ and $l \in [1, \frac{1}{p_e}]$. Since $w(p_e, l, s)$ is a continuous function of its arguments, this clearly implies the existence of an exchange rate volatility level $s_T$ beyond which the additional value of switching to seller 2 from seller 1 is non-positive for all $l$ and all possible switching points $p_e \le 1/l$. Since, as we have noted earlier, it is sub-optimal for the buyer to switch to seller 1 after entering the market with seller 2, it follows that the buyer's switching option has zero value for $s > s_T$. Therefore, for $s > s_T$, the buyer either only uses seller 1 or only seller 2 whenever it is in the foreign market. This completes the proof. ♦

**Proof of Proposition 4.1**

a) Without loss of generality, let us assume that the seller in the foreign market is seller 2. Given a price quote $Q_2 \geq C_2$ of seller 2, the buyer's long term stationary switching policies are to enter a relationship with seller 2 when the price process $Q_2 q(.)$ in the buyer's currency hits a level $p_e^2$ and to exit a relationship with seller 2 when the price process hits a level $p_q^2$ where $p_e^2$ and $p_q^2$ depend only on the sunk cost of using seller 2, i.e. $k_2$, the drift and volatility of the exchange rate process and the buyer's discount parameter $b$. Using standard dynamic programming arguments (see the derivation of the value functions of the sellers in **Section 4,** especially equations (4.4) and (4.5)), we may check that seller 2's initial value function $V_2(p)$ as a function of the initial price $p$ *in the buyer's currency* is given by

(A8)
$$\begin{aligned} V_2(Q_2 q(0)) &= A_{Q_2}(Q_2 q(0))^{r_1^-} \text{ if } Q_2 q(0) \geq p_e^2 \text{ and the buyer is idle} \\ &= D_{Q_2}(Q_2 q(0))^{r_1^+} + \frac{Q_2 - C_2}{b'} \text{ if } Q_2 q(0) < p_q^2 \text{ and the seller sells to the buyer} \end{aligned}$$

where $r_1^+, r_1^-$ are the positive and negative root of (2.13) with $b$ replaced by $b'$ and the coefficients above are determined by the following value matching conditions:

(A9)
$$A_{Q_2}(p_e^2)^{r_1^-} = D_{Q_2}(p_e^2)^{r_1^+} + \frac{Q_2 - C_2}{b'}; \; A_{Q_2}(p_q^2)^{r_1^-} = D_{Q_2}(p_q^2)^{r_1^+} + \frac{Q_2 - C_2}{b'}$$

From (A9), we easily see that

(A10)
$$A_{Q_2} = \frac{Q_2 - C_2}{b'}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right] > 0 \text{ since } p_q^2 > p_e^2$$

The price quote $Q_2^*$ is an equilibrium price if and only if seller 2's value function is maximized at $Q_2^*$, i.e. $Q_2^* = \arg\max_{Q_2} V_2(Q_2 q(0))$. From (A8), (A10), we see that

$$\lim_{Q_2\to\infty} V_2(Q_2 q(0)) = \lim_{Q_2\to\infty} A_{Q_2}(Q_2 q(0))^{r_1^-}$$

(A11)
$$= \lim_{Q_2\to\infty} \left(\frac{Q_2 - C_2}{b'}\right)(Q_2)^{r_1^-}(q(0))^{r_1^-}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right]$$

$$= \lim_{Q_2\to\infty} \left(\frac{(Q_2)^{1+r_1^-} - C_2(Q_2)^{r_1^-}}{b'}\right)(q(0))^{r_1^-}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right]$$

We now recall that $r_1^-$ is the negative root of the equation $-b'x + (m - \frac{1}{2}s^2)x + \frac{1}{2}s^2x^2 = 0$.

We see that $r_1^- > -1$, i.e. –1 is less than both the roots of the above equation, if and only if the left hand side above is positive when evaluated at $x = -1$ and this is exactly condition (4.3) of the proposition. We therefore see that if (4.3) holds, $r_1^- > -1$ and we easily see from (A11) that

(A12) $$\lim_{Q_2\to\infty} V_2(Q_2 q(0)) = \infty$$

(A12) clearly implies that the seller is never satiated at a finite price so that the buyer will not enter the foreign market. On the other hand, condition (4.2) is equivalent to $r_1^- < -1$. In this case, it is easy to see that equations (A8), (A9), (A11) imply that

$$\lim_{Q_2\to\infty} V_2(Q_2 q(0)) = \lim_{Q_2\to C_2} V_2(Q_2 q(0)) = 0$$

Since $V_2(.)$ is a continuous function, we see that its maximum exists and is attained at some $Q_2^*$ which is the required equilibrium price. Moreover, we can analytically determine the equilibrium price $Q_2^*$ from the explicit analytical expressions for $V_2(.)$. Therefore, an equilibrium between the buyer and the seller exists. This completes the proof of part a).

b) If there is an exogenously imposed ceiling $Q_{ceil}$ on seller prices, then the fact that $V_2(Q_{ceil}q(0))$ is finite clearly implies that the seller quotes a finite price in equilibrium that may be $Q_{ceil}$ if it is sufficiently high. ♦

**Proof of  Proposition 4.2**

For the sake of concreteness, we will assume that (4.6) has a solution but (4.7) may or may not have a solution. The arguments are very similar for the other case.

a) The proof proceeds by contradiction. Suppose seller 2 captures the market in some equilibrium and let the equilibrium price quoted by seller 2 be $Q_2 > C_2$. Seller 2 may capture the market if it were chosen as the leader or the follower by the buyer. Suppose first that it were chosen as the follower. Since $l_{\min} < \frac{C_1}{C_2} < l_{\max}$, seller 1 can always guarantee itself strictly positive expected profits by quoting a price in the interval $(C_1, l_{\max}C_2)$. Since a solution to (4.6) exists by hypothesis, capture of the market by seller 2 cannot be an equilibrium outcome when seller 1 is chosen as the leader and seller 2 the follower.

Suppose seller 2 captures the market being chosen as the leader. This can only occur if a solution to (4.7) exists. In this case, the previous argument applies to show that seller 1 may always obtain strictly positive expected profits by quoting a price in the interval $(C_1, l_{\max}C_2)$. Hence, conditions (4.6) ensure that seller 1 has an optimal response $y_1(Q_2)$ at which it obtains strictly positive expected profits. Hence, the capture of the market by seller 2 cannot be an equilibrium outcome.

On the other hand, suppose seller 1 captures the market in some equilibrium and let the equilibrium price quoted by seller 1 be $Q_1 > C_1$. Suppose first that seller 1 was chosen as the leader and seller 2 the follower. Since $C_1 > l_{\min}C_2$ by hypothesis, in response to the price $Q_1$ quoted by seller 1, seller 2 may always guarantee itself strictly positive expected profits by quoting a price in the interval $(C_2, C_1 / l_{\min})$. Hence, conditions (4.7) ensure that seller 2 has an optimal response $y_2(Q_1)$ at which it obtains strictly positive expected profits. Since a solution to (4.6) exists by hypothesis, capture of the market by seller 1 cannot be an equilibrium outcome if it were chosen as the leader and seller 2 the follower.

On the other hand, suppose seller 2 were chosen as the leader and seller 1 the follower. If (4.7) has a solution, then the argument above applies to show that seller 1 cannot capture the market in equilibrium. If (4.7) does not have a solution, then seller 2 is unable to quote a price so that seller 1 captures the market. The equilibrium price $Q_1$ quoted by seller 1 in this case is the price derived in the proof of **Proposition 4.1**. However, the buyer's value function in this case is clearly lower than its value function from choosing seller 1 as the leader and seller 2 as the follower (for which an equilibrium exists by hypothesis) since the buyer benefits from competition[23]. Since we have already shown that both sellers must co-exist in such an equilibrium outcome, we have shown that the capture of the market by seller 1 cannot be an equilibrium outcome. Therefore, both sellers must co-exist, i.e. both sellers must obtain strictly positive expected profits, in any possible equilibrium of the game between the buyer and the sellers. This completes the proof.

b) The proof basically follows from the fact that both sellers must follow markup pricing policies and the buyer's optimal policy involves the use of either seller 1 over time or seller 2 over time depending on the ratio of the prices they quote. We omit the details here for the sake of brevity.

[23] Note that the buyer always has the option of negotiating with only one seller. Therefore, its value from negotiating with both sellers is always at least as great as its value from negotiating with either seller alone.

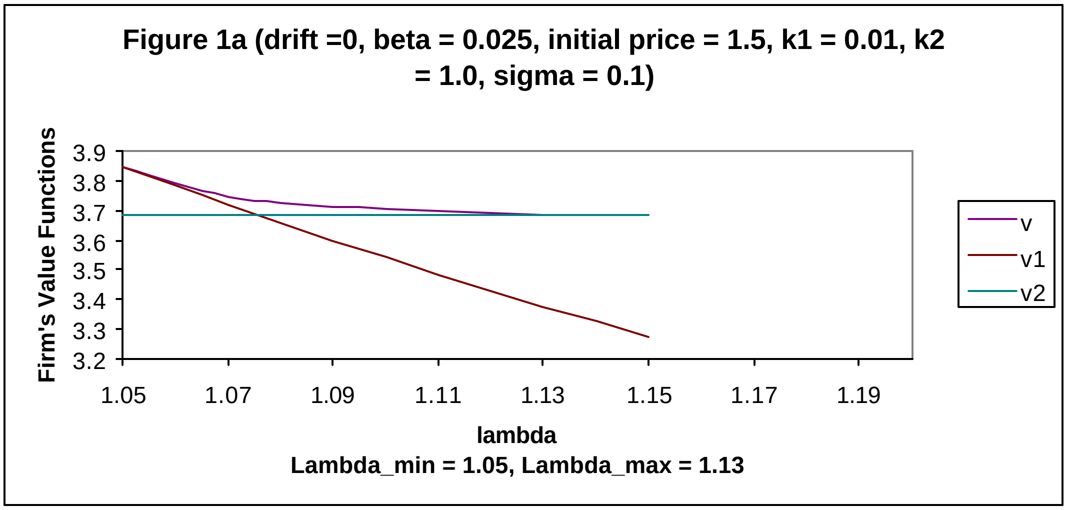


Figure 1a (drift =0, beta = 0.025, initial price = 1.5, k1 = 0.01, k2 = 1.0, sigma = 0.1)
Firm's Value Functions
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
1.05
1.07
1.09
1.11
1.13
1.15
1.17
1.19
lambda
Lambda_min = 1.05, Lambda_max = 1.13
v
v1
v2

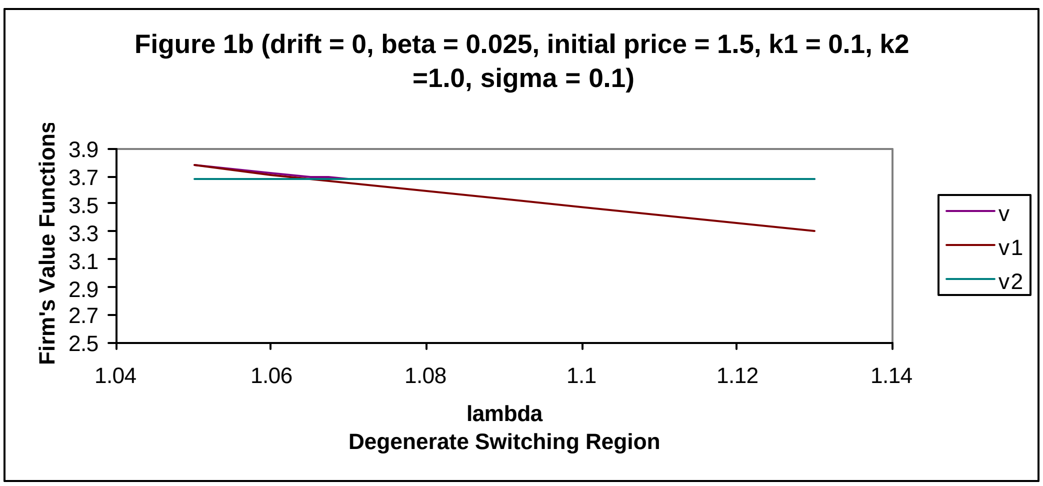


Figure 1b (drift = 0, beta = 0.025, initial price = 1.5, k1 = 0.1, k2 =1.0, sigma = 0.1)
Firm's Value Functions
3.9
3.7
3.5
3.3
3.1
2.9
2.7
2.5
1.04
1.06
1.08
1.1
1.12
1.14
lambda
Degenerate Switching Region
v
v1
v2

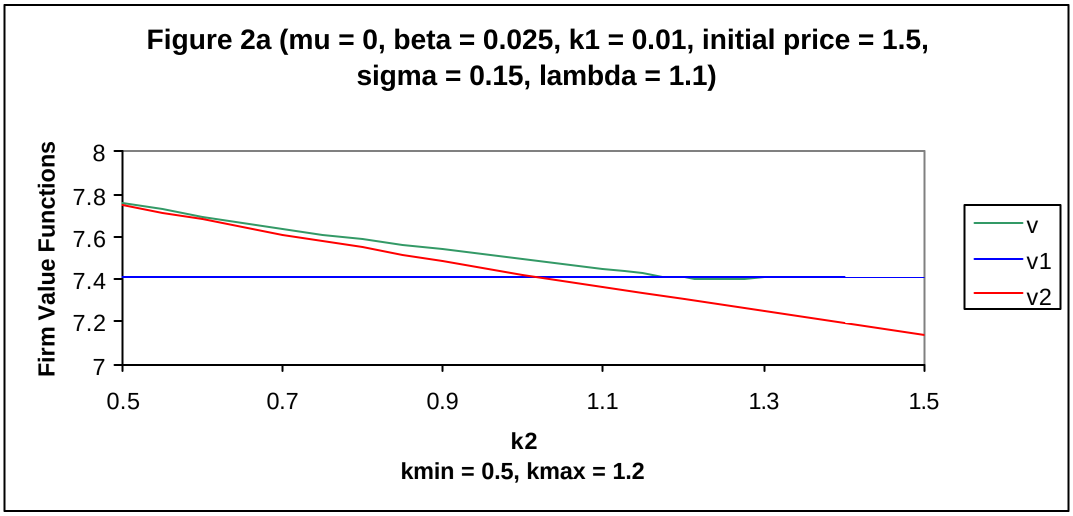


Figure 2a (mu = 0, beta = 0.025, k1 = 0.01, initial price = 1.5, sigma = 0.15, lambda = 1.1)
Firm Value Functions
8
7.8
7.6
7.4
7.2
7
0.5
0.7
0.9
1.1
1.3
1.5
k2
kmin = 0.5, kmax = 1.2
v
v1
v2

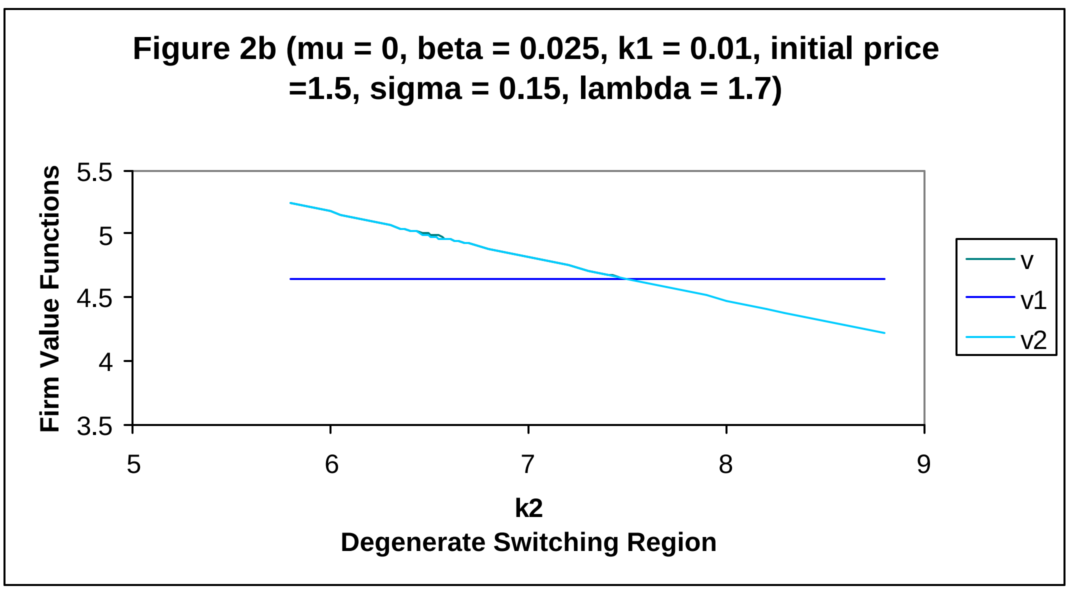
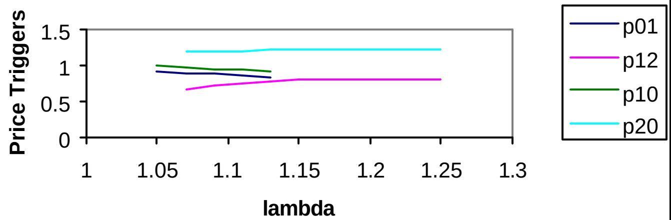


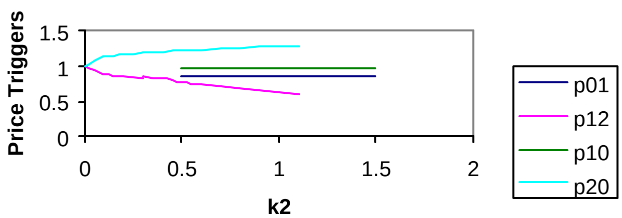
Figure 2b (mu = 0, beta = 0.025, k1 = 0.01, initial price =1.5, sigma = 0.15, lambda = 1.7)
Firm Value Functions
5.5
5
4.5
4
3.5
5
6
7
8
9
k2
Degenerate Switching Region
v
v1
v2

**Figure 3a (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01, k2 = 1.0, sigma = 0.1)**




**Lambda_min = 1.07, lambda_max = 1.13**

**Figure 3b (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.15, lambda = 1.1)**




**kmin = 0.5, kmax = 1.1**

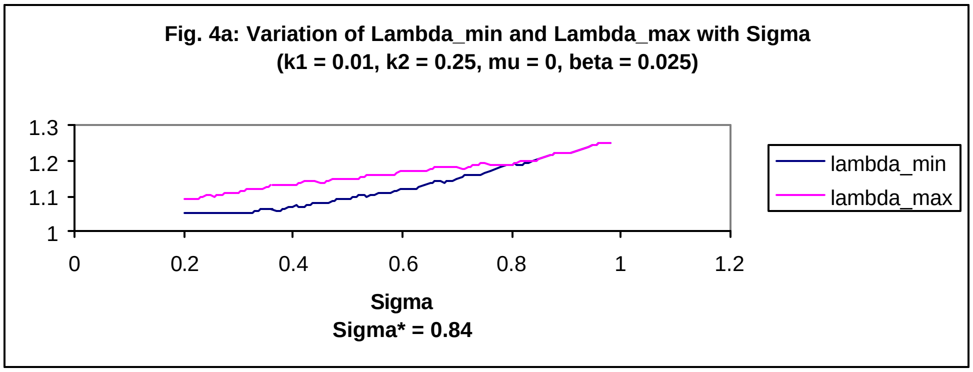


Fig. 4a: Variation of Lambda_min and Lambda_max with Sigma
(k1 = 0.01, k2 = 0.25, mu = 0, beta = 0.025)
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1
1.2
Sigma
Sigma* = 0.84
lambda_min
lambda_max

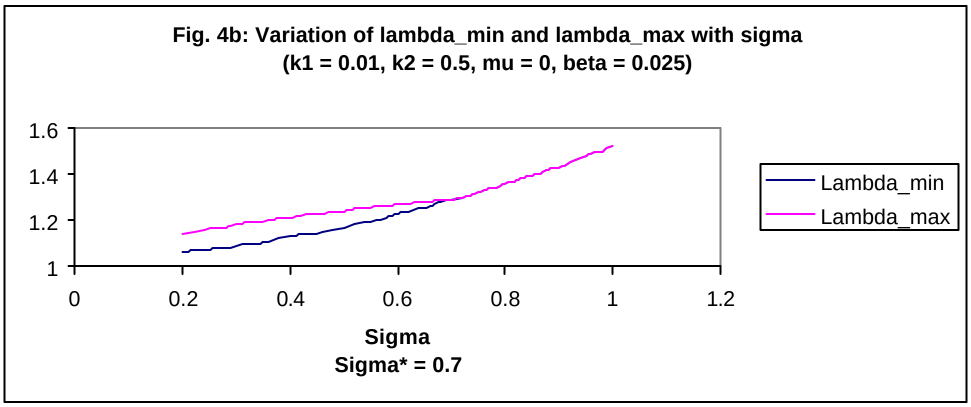


Fig. 4b: Variation of lambda_min and lambda_max with sigma
(k1 = 0.01, k2 = 0.5, mu = 0, beta = 0.025)
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
1.2
Sigma
Sigma* = 0.7
Lambda_min
Lambda_max

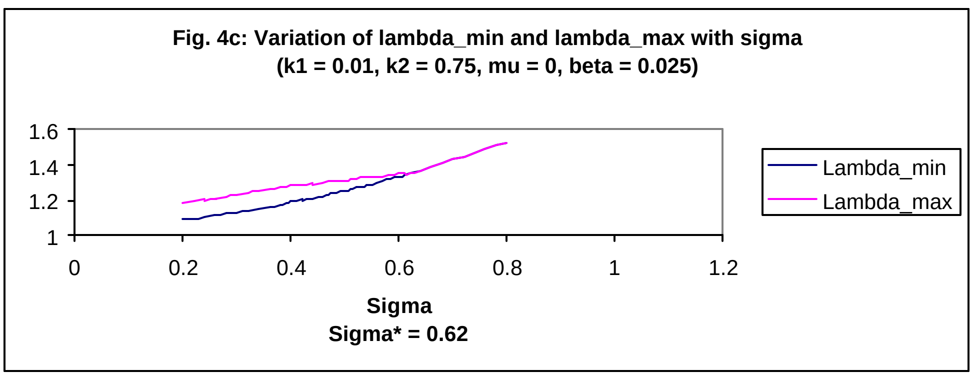


Fig. 4c: Variation of lambda_min and lambda_max with sigma
(k1 = 0.01, k2 = 0.75, mu = 0, beta = 0.025)
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
1.2
Sigma
Sigma* = 0.62
Lambda_min
Lambda_max

**Figure 5: Variation of Equilibrium Prices with C1/C2 (mu = 0, beta = beta' = 0.025, sigma = 0.1, k1 = 0.01, k2 = 0.5, C1 = 0.8)**
**lambda_min = 1.04, lambda_max = 1.09**

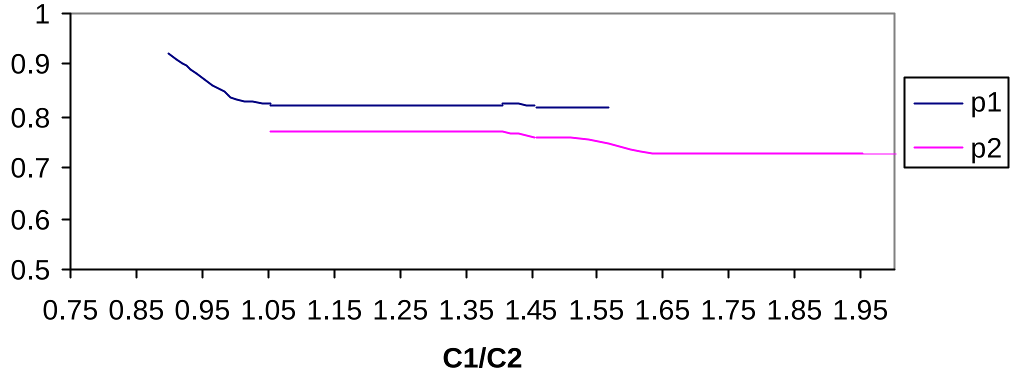
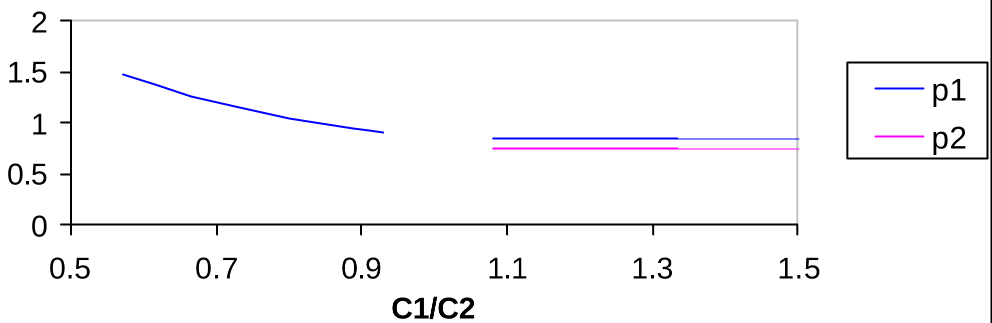



**Figure 6: Variation of Equilibrium Prices with C1/C2 (mu = 0, beta = beta' = 0.025, sigma = 0.4, k1 = 0.01, k2 = 0.5, C1 = 0.8)**
**lambda_min = 1.13, lambda_max = 1.2**




**Unrestrained Tacit Collusion occurs when c1/c2 is in the interval (0.95, 1.05)**