Preemptive Patenting under Uncertainty and Asymmetric Information

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Abstract

This paper examines the investment behaviour of an incumbent and a potential entrant that are competing for a patent with a stochastic payoff. We incorporate asymmetric information into the model by assuming that the challenger has complete information about the incumbent whereas the latter does not know the precise value of its opponent's investment cost. We find that even a small probability of being preempted gives the informationally-disadvantaged firm an incentive to invest at the breakeven point where it is indifferent between investing and being preempted. By investing inefficiently early to protect its market share, the incumbent gives up not only its option to delay the investment, but also reduces the value of the firm by an amount that increases with the investment cost incurred and the potential loss of market share. The investment behaviour of the challenger is the same as under complete information, namely the challenger 'epsilon preempts' the incumbent, if optimal to do so.

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1 Introduction

Seminal papers by Brennan and Schwartz (1985), McDonald and Siegel (1986) and Dixit (1989) stress that investing under uncertainty and irreversibility involves sacrificing the option to invest in the future. The optimal point to invest should balance the profits foregone by delaying the investment against the option value relinquished when the investment is made.[1] This leads to a decision rule under which an investment is made when its net present value (NPV) is strictly positive. The execution of the investment is delayed beyond the traditional neoclassical Marshallian breakeven threshold. Managers that are guided by the traditional breakeven rule have therefore at times been criticised of acting in a myopic 'now or never' fashion whereby they destroy the company's flexibility and option value of waiting.

A number of recent papers have, however, argued that the potential for firms to delay investments may be limited if they are operating in an environment where first mover's advantages, preemption and competition are important (e.g., Smets (1991), Grenadier (1996), Kulatilaka and Perotti (1998), Mason and Weeds (2000) and Weeds (2000), among others). Consequently, the threat of preemption may reduce the firms' flexibility to delay and the option value of waiting. Smets (1991) and Grenadier (1996), for example, show that, when two firms contemplate to enter in a market, the more efficient of the two will preempt the weaker firm by entering an instant before the breakeven threshold of the less efficient firm.[2] This means that if both firms are identical all option value of waiting to invest will be destroyed and entry happens at the competitive breakeven threshold. If one firm is more efficient than the other, then the more efficient firm will, by 'epsilon preempting', still preserve some option value of waiting.[3] In a fairly competitive environment, it is to be expected that the threat of preemption will lead to inefficiently early investment and destroy most option value of waiting to invest. Preemptive considerations therefore seem to bring the investment rule under uncertainty and irreversibility more or less back in

[1] The optimal timing of an investment is therefore similar to the optimal exercise of an American option on a dividend paying stock.

[2] They also show that in some cases simultaneous entry is possible (even when both firms are not identical), but this equilibrium is less relevant for our paper.

[3] The concept of epsilon preemption is the one proposed, for example, in Fudenberg and Tirole (1985): if a firm can invest at time t and thus put its rival at a disadvantage, then the latter has an incentive to preempt the former by investing just prior to time t.

line with the neoclassical Marshallian investment rule, and results are in the spirit of some influential papers on technological innovation. For example, Dasgupta and Stiglitz (1980), after examining firms engaged in games with complete information, reach the conclusion that competition may even result in *excessive* speed in R&D. In essence, the incentive to preempt rivals and thus deter their entries can drive a firm to invest earlier in order to gain the first-mover advantage. Reinganum (1983) notes that 'actual and potential new entrants play a crucial role in stimulating technical progress, both as direct sources of innovation and as spurs to existing industry members.' Gilbert and Harris (1984) show that preemptive equilibria are characterised by *zero* profit on every new investment. That is, in the absence of a binding contract between the agents, competition can completely dissipate the economic rent.

The above papers share, however, the common feature that firms are assumed to be completely informed about each other. As this assumption may often be violated in reality, some recent papers have started analyzing the exercise of real options under alternative information structures. The model closest to ours is by Lambrecht and Perraudin (2003).[4] They introduce incomplete information and preemption into an equilibrium model of firms that have the opportunity to enter into a new market. Firms know their own cost of entry but only know the distribution of their competitors' investment costs. They show that the optimal investment trigger may lie anywhere between the zero-NPV trigger (the so-called Marshallian trigger) and the firm's optimal monopolistic, non-competitive trigger (referred to as the non-strategic option trigger), depending on the fear of preemption implied by the distribution of the competitors' costs. Furthermore, higher product market uncertainty leads to more delay, conform to what is predicted by the real options paradigm. The main implication therefore is that incomplete information preserves some option value of waiting

[4]Grenadier (1999) describes a model in which firms learn about the investment payoff from the actions of other agents, i.e., each firm has a private signal about the payoff of the investment that is revealed when it acts. Information revelation allows firms that have not yet acted to update their information about the value of the underlying investment. This paper is, however, not directly relevant for our discussion as the firms' investment payoff is not affected by the order in which firms exercise. Moreover, his model is more relevant to describe second mover's advantages. A standard application of his model is the exploration of oil when two or more firms have adjacent tracts of land that may contain an oil deposit. Firms then face a trade-off between the benefits of drilling and potentially obtaining oil earlier and the benefits of waiting for other firms to drill first and reveal information about the size of the oil deposit.

and therefore tends to restore, at least to some extent, the real options paradigm for capital budgeting in the presence of preemption and competition.

This paper aims to contribute further to the debate on the validity of real options theory in the presence of preemption by considering the case of asymmetric information. In particular, we consider the situation where one firm has complete information about its rival, whereas the latter has incomplete information about its opponent's investment cost. As such the asymmetric information case is an intermediate case between complete and incomplete information, and the interesting question is whether investment behaviour under uncertainty and competition in such an environment brings us closer to the real options paradigm, or closer to the traditional zero-NPV rule. From the above discussion, one would expect to obtain the former (latter) outcome if the incomplete (complete) information component dominates.

A second differentiating factor from many previous real options papers is that we consider an asymmetrical race.[5] Similar to Gilbert and Newbery (1982) we consider a patent race between an incumbent and a potential entrant. Entry into the monopolised market can be gained only by patenting a substitute for the incumbent's present product.[6] The incumbent can therefore prevent entry by preemptively patenting the substitute. The model allows us to examine how informational asymmetries affect the persistence of a monopoly and therefore contributes to previous work on monopoly persistence by Newbery (1978), Gilbert and Newbery (1982), Reinganum (1983), Harris and Vickers (1985), and Leininger (1991), among others.

In this paper we assume that the potential entrant knows everything about the incumbent, whereas the incumbent does not know the entrant's exact investment cost but only the distribution it is drawn from. We could, of course, reformulate the model to allow the entrant to be the informationally disadvantaged firm but chose not to do so for the following reasons.[7] First, the incumbent company may be a larger, possibly listed company. Information about the incumbent may therefore be more publicly available than for the entrant. Second, the incumbent may be facing a whole pool

[5]Note that by choosing the model parameters appropriately, we can obviously obtain the symmetrical race as a special case of our model.

[6]The reader should bear in mind that the market cannot be really monopolised as the incumbent faces a potential entrant.

[7]In fact, solving the case where the entrant is informationally disadvantaged is slightly easier, as will become clear.

of potential challengers, all with varying costs of entry. However, only one of these challengers may effectively seek entry. The incumbent may not be able to identify this challenger and may therefore have to make a conjecture about the distribution of the investment cost of the challenger that will effectively seek entry. Finally, note that most of the existing literature on oligopolistic entry deterrence has typically assumed asymmetric information with respect to the market or the production process (e.g., the level of demand or cost of production) rather than the characteristics of the market players.[8] Those papers have therefore typically assumed the incumbent to be better informed.

Our results are as follows. We find that the slightest possibility of preemption will make the informationally disadvantaged firm act at the point where it is indifferent between investing and being preempted (referred to as the strategic Marshallian trigger). The incumbent's investment trigger is therefore not influenced at all by economic uncertainty, which lends support to the neoclassical Marshallian theory. If the incumbent's second patent is not put to commercial use then its investment cost is a pure entry deterrence cost, and it may weaken the incumbent if it has to deter entry on a regular basis. Therefore, the informational disadvantage makes it more difficult for the incumbent to protect its monopoly position in the long run. We further show that in some cases the incumbent may be able to learn about its opponent's entry cost if the latter does not act when market conditions become increasingly more favorable. However, when challengers are reasonably efficient, the value of learning will typically be small or non-existent. A comparative analysis shows that the informationally disadvantaged firm is worse off under asymmetric information than under complete or incomplete information. For the completely informed entrant, the outcome is the same as under complete information since it will try to epsilon preempt the incumbent. Some of the entrant's option value of waiting may be preserved if the entrant is sufficiently efficient compared to the incumbent.

The paper proceeds as follows. In section 2.1 we first specify the normal form of the game by deriving the payoff to the incumbent and the challenger in case either of them wins the patent race. Section 2.2 derives each firm's value and investment threshold when it faces no competition. Section 2.3 formalises the issues of asymmetric information, learning and competition, and solves for each firm's strategic

[8] See Chatterjee and Sugita (1990), Harrington (1987), and Mailath (1991).

investment threshold and its corresponding firm value in equilibrium. Section 3 analyses the main results. Section 4 compares the results under asymmetric information with the ones that would be obtained under complete and incomplete information. The last section concludes.

2 A Strategic Investment Model

Consider a patent race game played by an incumbent and a potential entrant. Specifically, the incumbent is serving a monopolised market under the threat of a challenger who wants to enter the market.[9] Assume that before entry occurs, the incumbent produces only one product that has a patent of infinite duration.[10] Entry into the monopolised market can be gained only by patenting a substitute for the incumbent's present product. The costs of acquiring this new patent are K_i for the incumbent and K_e for the entrant.[11] As soon as the patent is acquired, the second product will be launched without any further cost. Depending on whether and by whom the second patent has been acquired, the market structure will be (1) a monopoly with only one product, (2) a monopoly with two products, or (3) a duopoly with two products. In order to focus purely on the role of the patent in the persistence of the monopoly, we assume that there are no other barriers to entry. In particular, there are no capacity constraints, no production costs, and hence no returns to scale. Furthermore, since there are no production costs, no firm will want to exit the market once it has started producing the first or second product.[12]

Assume that before the second product is launched the incumbent makes a profit

[9]This line of research is also studied by, for example, Gilbert and Newbery (1982) and Harris and Vickers (1985).

[10]In the US, patents are granted for a term of 17 years (14 years for design patents). Since the model developed in this paper uses option pricing techniques to value patents, patents can, from a practical viewpoint, safely be regarded as infinite maturity (perpetual) options. Moreover, the additional complexity required of finite maturity models is not warranted by the small gain in accuracy.

[11]We denote parameters, variables or probability distributions related to the incumbent and the potential entrant by the subscripts i and e respectively.

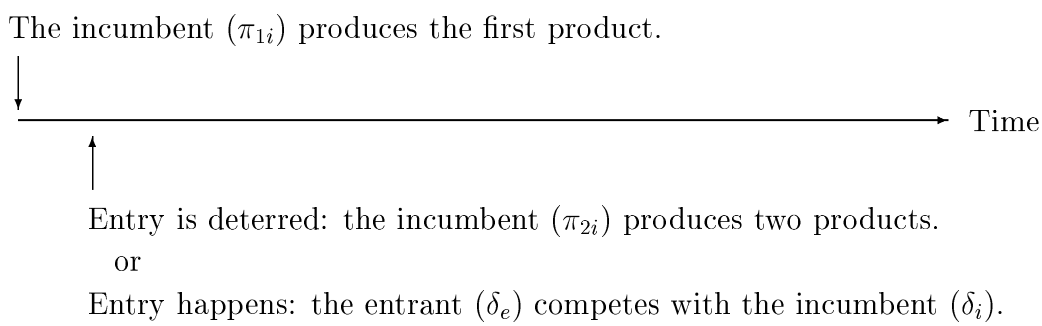
[12]For a model of market exit in a monopolistic or duopolistic environment, we refer respectively to the papers by Dixit (1989) and Lambrecht (2001).

of $\pi_{1i}x_t$ per period of time, where x_t is a stochastic variable representing demand shocks and π_{1i} is a strictly positive constant. Suppose x_t follows a geometric Brownian motion:

$$dx_t = x_t \mu dt + x_t \sigma dW_t, \tag{1}$$

where μ is the growth rate parameter, σ is the proportional variance parameter, and dW_t is the increment of the standard Wiener process. If the incumbent succeeds in patenting the substitute, its profit flow becomes $\pi_{2i}x_t$. On the other hand, if an entrant acquires the new patent, the incumbent's profit flow becomes $\delta_i x_t$ and the entrant will accumulate profits at a rate $\delta_e x_t$. We focus on the case where the incumbent (i) becomes worse off if entry happens and (ii) makes more profit by producing two products than only one. In other words, $\pi_{2i} > \pi_{1i} > \delta_i$. This is the economically more interesting and relevant case.

The above patent race game can be summarised as follows (with the profit flows of the agents shown in parentheses).



2.1 Derivation of the Payoff

In this subsection we derive the payoff matrix of the patent game. To begin, we calculate the incumbent's and entrant's values of the patents on products 1 and 2, both of which are assumed to have infinite duration.

The Value of the Patent on Product 1

The patent on the first product is already owned by the incumbent and therefore relevant to the incumbent only. Assume that the value of the incumbent producing product 1 is comprised completely of the patent. This value is denoted by N_{1t}. Under the assumption of risk neutrality, investors are willing to invest in the incumbent's firm only when its total return (i.e., the profit flow plus the anticipated capital gain) equals the return they can get from investing the same amount of money in a riskless bond which yields a constant interest rate r. Therefore,

$$rN_{1t} \;=\; \pi_{1i}x_t \;+\; \lim_{dt\to 0}\frac{\mathrm{E}_t[dN_{1t}]}{dt}, \tag{2}$$

where $\pi_{1i}x_t$ denotes the profit flow of the incumbent as defined above. Suppose that N_{1t} is a twice continuously differentiable function of the state variable x_t. We can then use Ito's lemma to obtain the differential equation

$$rN_1(x_t) \;=\; \pi_{1i}x_t \;+\; \mu x_t\frac{\partial N_1(x_t)}{\partial x_t} + \frac{\sigma^2}{2}x_t^2\frac{\partial^2 N_1(x_t)}{\partial x_t^2}. \tag{3}$$

Assuming that $\mu < r$, the general solution of this second order non-homogeneous differential equation is given by:

$$N_1(x_t) = A_1 x_t^{\zeta} \;+ A_2 x_t^{\beta} + \frac{\pi_{1i}x_t}{r-\mu}, \tag{4}$$

where ζ and β are, respectively, the negative and positive roots of the characteristic equation $y(y-1)\sigma^2/2 + y\mu \;=\; r$. Applying the no-bubble conditions as $x_t \to 0$[13] and $x \to \infty$,[14] one obtains the solution

$$N_1(x_t) \;=\; \frac{\pi_{1i}x_t}{r-\mu}. \tag{5}$$

Note that the solution of N_{1t} may be obtained alternatively in a probabilistic way by directly working out the expectation $\mathrm{E}_t\,[\,\int_t^{\infty}\,\pi_{1i}x_s\exp(-r(s-t))ds\,]$.

[13] If x_t ever touched 0, the state variable would remain at that level forever (i.e., 0 is an absorbing barrier). Correspondingly, the value of the patent on product 1 would also be 0. Since $\zeta < 0$, x_t^{ζ} goes to infinity as x_t tends to zero. To avoid the value on the right-hand side of equation (4) diverging, we must therefore set $A_1 = 0$. Note that if we introduced a fixed production cost then the term A_1x^{ζ} would represent the value of future suspension or exit options. Since there is no such cost in our model, it is never optimal for firms to leave the market. That is, the option value of exit is 0.

[14] The component $A_2x_t^{\beta}$ represents the possibility of speculative bubbles as $x_t \to \infty$. People might price an asset more than its fundamental value if they expect to be able to resell it later and realise a sufficiently large capital gain. We rule out this possibility of speculative bubbles, so $N_1(x_t)$ consists only of the fundamental value $\frac{\pi_{1i}x_t}{r-\mu}$.

The Value of the Patent on Product 2

Both the incumbent and the potential entrant want to acquire the patent on the second product when the state variable is high enough. We now calculate the value of this patent to each agent.

Following the argument above, it is easy to show that in a duopoly the value of the second patent to the entrant is

$$\frac{\delta_e x_t}{r-\mu}. \tag{6}$$

The incumbent's incremental profit flow from the new patent is $\pi_{2i}x_t - \pi_{1i}x_t = (\pi_{2i} - \pi_{1i})x_t$. Therefore the incremental value of the second patent to the incumbent is

$$\frac{(\pi_{2i} - \pi_{1i})x_t}{r-\mu}. \tag{7}$$

The incumbent's loss due to new entry is $\delta_i x_t - \pi_{1i}x_t = (\delta_i - \pi_{1i})x_t$ per period. This corresponds to a negative present value of

$$\frac{(\delta_i - \pi_{1i})x_t}{r-\mu}. \tag{8}$$

Taking into account the cost of acquiring the patent, we can summarise in the following matrix the payoffs to the incumbent and the challenger, depending on who acquires the new patent:

	Incumbent receives	Challenger receives
Incumbent acts	$\frac{(\pi_{2i}-\pi_{1i})x_t}{r-\mu} - K_i$	0
Challenger acts	$\frac{(\delta_i-\pi_{1i})x_t}{r-\mu}$	$\frac{\delta_e x_t}{r-\mu} - K_e$

where $\pi_{2i} \geq \pi_{1i} \geq \delta_i > 0$, $\delta_e > 0$ and $K_i, K_e > 0$. Recall that K_i and K_e are the costs of the incumbent and the entrant to acquire the new patent, respectively.

2.2 Value of the Non-strategic Claim to Innovate

In his pioneering article, Arrow (1962) asks: "What is the gain from innovation to a firm that is the only one to undertake R&D, given that its innovation is protected by a patent of unlimited duration?" Here we will attempt to isolate the 'pure' incentive to innovate, i.e., that which is independent of any strategic considerations of preemptive innovation. In the next subsection we discuss the case where the firm is competing with a rival.

Proposition 1 *The non-strategic 'pure' value from innovation to the incumbent and the entrant is given by:*

$$V_j(x_t|\overline{x}_{jn}) \ = \ [\Theta_j \overline{x}_{jn} - K_j] \left(\frac{x_t}{\overline{x}_{jn}}\right)^{\beta} \quad (j \ = \ i, e) \tag{9}$$

where $\Theta_i \equiv \frac{\pi_{2i} - \pi_{1i}}{r-\mu}$ *and* $\Theta_e \equiv \frac{\delta_e}{r-\mu}$*. The optimal non-strategic exercise trigger is given by:*

$$\overline{x}_{jn} \ = \ \frac{\beta K_j}{(\beta - 1)\Theta_j} \quad (j \ = \ i, e) \tag{10}$$

In particular, the respective non-strategic triggers for the incumbent and entrant are:

$$\overline{x}_{in} \ = \ \frac{\beta(r-\mu)K_i}{(\beta-1)(\pi_{2i} - \pi_{1i})} \qquad \overline{x}_{en} \ = \ \frac{\beta(r-\mu)K_e}{(\beta-1)\delta_e} \tag{11}$$

Proof: See Appendix.

Proposition 1 illustrates how the option valuation approach leads to an investment trigger which is drastically different from the neoclassical Marshallian trigger, which is defined as:

Definition 1 *The Marshallian trigger,* $\overline{x}_{jm}$*, is defined as the trigger at which the investor breaks even, ignoring strategic behaviour, i.e. the trigger for which* $\Theta_j \overline{x}_{jm} - K_j = 0$*, or equivalently* $\overline{x}_{jm} = \frac{K_j}{\Theta_j}$*. Such triggers for the incumbent and the challenger respectively are given by:*

$$\overline{x}_{im} \ = \ \frac{K_i}{\frac{(\pi_{2i} - \pi_{1i})}{r-\mu}} = \frac{(r-\mu)K_i}{\pi_{2i} - \pi_{1i}} \qquad \overline{x}_{em} \ = \ \frac{K_e}{\frac{\delta_e}{r-\mu}} = \frac{(r-\mu)K_e}{\delta_e} \tag{12}$$

Indeed, since $\beta > 1$, uncertainty makes the investor delay beyond the Marshallian breakeven trigger, i.e. $\overline{x}_{jm} < \overline{x}_{jn}$. It is easy to show that as uncertainty disappears (i.e. $\sigma \to 0$), the option trigger converges towards $\frac{K_j}{\Theta_j}$ if $\mu \leq 0$ and towards $\frac{rK_j}{(r-\mu)\Theta_j}$ if $\mu > 0$.

The triggers illustrate an important difference between the incumbent's and the entrant's incentives to undertake R&D. One can easily verify that $\frac{\partial \overline{x}_{in}}{\partial \pi_{1i}} > 0$. Hence, increasing the incumbent's pre-innovation profit reduces its incentive to undertake R&D. This is the familiar Arrow (1962) *'replacement effect'*. It tends to lower the incumbent's R&D expenditures compared to the entrant's, who has no pre-innovation profits. Finally, note that if firms were to act in the above non-strategic way, then the monopoly would persist if $\overline{x}_{in} < \overline{x}_{en} \Leftrightarrow \frac{\pi_{2i}-\pi_{1i}}{K_i} > \frac{\delta_e}{K_e}$. The determining factor for winning the innovation 'race' is therefore the ratio of the coefficient of incremental profit flow to the cost of patenting.

2.3 Value of the Strategic Claim to Innovate

In this subsection we incorporate fear of preemption into the firms' investment decisions. The patent game considered in this paper has the following extensive form: two players are involved (the incumbent and the challenger), and the player that acts first acquires the patent (note that action is triggered at a sufficiently high level of x_t). The game can therefore be in two possible states: either nobody has acted yet, or one firm has acted (i.e. patented the innovation) and the game ends. The normal (or strategic) form of the game has been given in the payoff matrix of Section 2.1, which summarises the payoffs for both agents according to their actions.

Analogous to the previous subsection, we now define for comparative purposes a 'strategic' or 'competitive' Marshallian trigger which takes into account the cost of preemption.

Definition 2 *Firm j's ($j = i, e$) strategic Marshallian trigger, $\overline{x}_{jc}$, is defined as the state at which the investor is indifferent between investing or being preempted, i.e. the trigger for which:*
$\Theta_j \, \overline{x}_{jc} - K_j = \theta_j \, \overline{x}_{jc}$ *or equivalently* $\overline{x}_{jc} = \frac{K_j}{\Theta_j - \theta_j}$, *where* $\theta_i = \frac{\delta_i - \pi_{1i}}{r-\mu}$ *and* $\theta_e = 0$. *Such triggers for the incumbent and the challenger are respectively given by:*

$$\overline{x}_{ic} = \frac{K_i}{\frac{(\pi_{2i}-\delta_i)}{r-\mu}} \qquad \overline{x}_{ec} = \frac{K_e}{\frac{\delta_e}{r-\mu}} = \overline{x}_{em} \tag{13}$$

Definition 2 illustrates how the strategic Marshallian triggers depend critically on the parameters π_{2i}, δ_i and δ_e. It is reasonable to assume that a monopolist does not make less profits than two non-colluding duopolists, i.e. $\pi_{2i} \geq \delta_i + \delta_e$.[15] This property is called the *'efficiency effect'* and is discussed by Gilbert and Newbery (1982), for instance. It states that the 'preemptive' payoff to innovation, that is the difference between winning the patent and letting the rival win it, is bigger for the incumbent $(\pi_{2i} - \delta_i)$ than for the entrant $(\delta_e - 0)$. Therefore, the efficiency effect gives the incumbent a bigger incentive to innovate because a monopoly is more efficient at making profits.

The impact of the efficiency effect can be illustrated by comparing both agents' strategic Marshallian triggers. The monopoly will persist if $\overline{x}_{ic} < \overline{x}_{em}$, or equivalently if $\frac{\pi_{2i}-\delta_i}{K_i} > \frac{\delta_e}{K_e}$. Hence, the determining factor for winning the innovation race is now the ratio of the coefficient of the preemptive profit flow (and not the incremental profit flow as in the non-strategic case) to the cost of patenting. The efficiency effect implies that $\pi_{2i} - \delta_i \geq \delta_e$ and therefore gives a comparative advantage to the incumbent, which contributes to the persistence of the monopoly.

We now consider the effect of asymmetric information on a duopolistic firm's investment behaviour. The information is asymmetric in the sense that the incumbent's investment cost is known by the challenger but the incumbent only knows the probability distribution of the challenger's cost. This is formalised in Assumption 1:

Assumption 1 *The incumbent's investment cost, K_i, is public knowledge and known to the potential challenger. However, the challenger's cost, K_e, is known only to itself. The incumbent knows that this cost, K_e, is drawn from a probability distribution $G(K_e)$ that has a continuous probability density function (pdf) $G'(K_e)$ and a positive support $[K_L, K_U]$.*

The following assumption merely defines the outcome of the race for the case where both firms want to invest at the same time and there is a tie.

Assumption 2 *If both firms act simultaneously, each firm acquires the patent with probability 1/2.*

[15] Indeed, if the monopolist wishes it can always duplicate the actions of non-colluding duopolists.

As is usual in games with asymmetric information, we first analyse the optimal investment rule for the informed agent (the challenger), and subsequently derived the optimal strategy of the incumbent.

2.3.1 Investment Rule for the Entrant

The investment rule for the entrant is very simple: the entrant will always act before the incumbent and patent the second product if it is profitable to do so. The condition ensuring that the entrant makes a profitable investment is that it must not act at a state lower than its Marshallian trigger, $\overline{x}_{em}$. If the incumbent's strategic trigger, $\overline{x}_{is}$, is higher than the entrant's non-strategic trigger, $\overline{x}_{en}$, then the entrant will act at the non-strategic trigger, $\overline{x}_{en}$, and fully exploit all option value of waiting. If, however, the incumbent's strategic trigger, $\overline{x}_{is}$, is lower than the entrant's non-strategic trigger but higher than the entrant's Marshallian trigger, then the entrant will ϵ-preempt the incumbent by acting at $\overline{x}_{is}^{-} = \overline{x}_{is} - \epsilon$ (where ϵ is an infinitesimally small number).

Figure 1 illustrates the entrant's investment rule. We plot the entrant's non-strategic, strategic and Marshallian triggers as a function of its entry cost. For the moment we assume that the incumbent's strategic trigger, $\overline{x}_{is}$, is exogenously given. Both the non-strategic and the Marshallian triggers of the entrant are, as usual, strictly increasing linear functions of the entry cost, i.e., $\overline{x}_{en} = \frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_e$ and $\overline{x}_{em} = \frac{(r-\mu)}{\delta_e}K_e$. The thick solid kinked line represents the entrant's strategic trigger, $\overline{x}_{es}$.

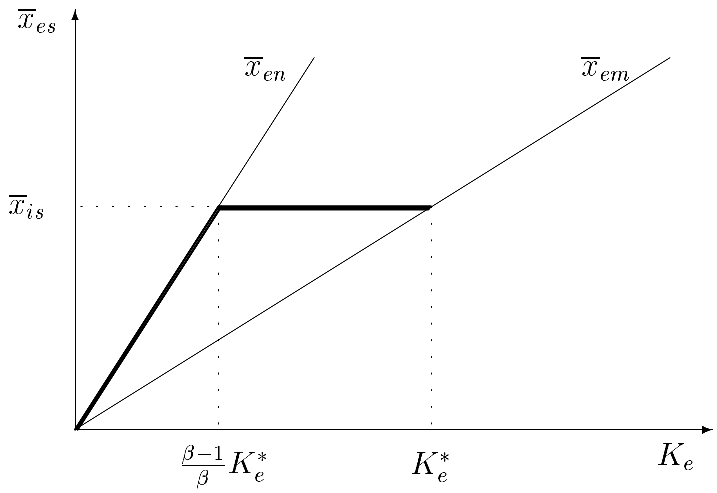


Figure 1: Entrant's trigger as a function of its cost, K_e.

The thick kinked line represents the entrant's strategic investment trigger, $\overline{x}_{es}$, as a function of its own investment cost, K_e, when the incumbent's investment trigger, $\overline{x}_{is}$, is exogenously given. The trigger, $\overline{x}_{es}$, falls within the cone formed by the entrant's Marshallian breakeven threshold, $\overline{x}_{em}$, and its non-strategic option threshold, $\overline{x}_{en}$. Given the incumbent's strategic investment trigger, $\overline{x}_{is}$, the entrant will act at the non-strategic trigger, $\overline{x}_{en}$, if $K_e < \frac{\beta-1}{\beta}K_e^*$. If $\frac{\beta-1}{\beta}K_e^* \leq K_e < K_e^*$, the entrant will act preemptively at $\overline{x}_{is}^-$. If $K_e = K_e^*$, both the entrant and the incumbent act at $\overline{x}_{is}$. No entry will occur if $K_e^* < K_e$. Note that $\overline{x}_{en} = \frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_e$, $\overline{x}_{em} = \frac{(r-\mu)}{\delta_e}K_e$ and $K_e^* = \frac{\delta_e \overline{x}_{is}}{r-\mu}$.

Figure 1 also illustrates the relationship between the strategic triggers of the incumbent and the entrant. By choosing an investment threshold, $\overline{x}_{is}$, the incumbent effectively sets a cut-off level of entry cost, K_e^*, above which entry cannot happen, i.e. $K_e^* \equiv \frac{\delta_e \overline{x}_{is}}{r-\mu}$. An entrant with cost lying between $\frac{\beta-1}{\beta}K_e^*$ and K_e^* will act at $\overline{x}_{is}^-$.[16] A more cost-efficient entrant whose cost is smaller than $\frac{\beta-1}{\beta}K_e^*$ will act at its non-strategic trigger, $\overline{x}_{en}$.

Before the state variable, x_t, reaches the incumbent's strategic trigger, the incumbent might be able to improve its conjecture about the challenger's cost distribution by monitoring the maximum level of the stochastic process, $\hat{x}_t$ (i.e. $\hat{x}_t \equiv max\{x_\tau | 0 \leq \tau \leq t\}$).

Consider the case where $K_L < \frac{\beta-1}{\beta}K_e^*$. As the state variable increases and explores new territory, $\hat{x}_t$ moves up as well. If $\hat{x}_t$ becomes so high that $\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)} > K_L$ and

[16]For the rest of this paper, we will denote $\overline{x}_{is} - \epsilon$ by $\overline{x}_{is}^-$ for simplicity.

no challenger has acted (at its non-strategic trigger, $\overline{x}_{en}$), then the incumbent learns that the challenger's cost cannot be situated in the interval $\left[K_L, \frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}\right]$, otherwise entry would have happened. With this learning, the incumbent updates its conjecture about the probability distribution of the rival's cost by Bayes' rule:

$$G'(K_e|\hat{K}) = \frac{G'(K_e)}{1 - G(\hat{K})} \tag{14}$$

or equivalently,

$$G(K_e|\hat{K}) = \frac{G(K_e) - G(\hat{K})}{1 - G(\hat{K})} \tag{15}$$

where $\hat{K} \equiv \min\left(\max\left(K_L, \frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}\right), \frac{\beta-1}{\beta}K_e^*\right)$ is the updated lower boundary of the support of the challenger's cost distribution.[17] In essence, equation (14) says that the updated pdf of the challenger's cost is the original pdf scaled up by the factor $1-G(\hat{K})$, so that the updated pdf integrates to unity. Note that $1-G(\hat{K})$ corresponds to the sum of the probability masses in the interval $[\hat{K}, \infty)$.

The entrant's optimal investment rule is now summarised as follows:

Proposition 2 *Given that the incumbent's investment trigger is $\overline{x}_{is}$, $K_e^* \equiv \frac{\delta_e \overline{x}_{is}}{r-\mu}$ and $\Theta_e \equiv \frac{\delta_e}{r-\mu}$, the potential entrant's strategic claim value is:*

$$V_{es}(x_t|\overline{x}_{es}) = p\left[\Theta_e \overline{x}_{es} - K_e\right]\left(\frac{x_t}{\overline{x}_{es}}\right)^{\beta}, \tag{16}$$

where the entrant's strategic investment trigger, $\overline{x}_{es}$, and the probability of the entrant winning the patent game, p, are given by:

if $K_e < \frac{(\beta-1)K_e{}^*}{\beta}$	:	*entry happens at* $\overline{x}_{es} = \overline{x}_{en}$ *(i.e.,* $p=1$*)*
if $\frac{(\beta-1)K_e{}^*}{\beta} \leq K_e < K_e{}^*$	:	*entry happens at* $\overline{x}_{es} = \overline{x}_{is}^{-} = \frac{K_e{}^*(r-\mu)}{\delta_e} - \epsilon$ *(i.e.,* $p=1$*)*
if $K_e = K_e{}^*$	:	*entry happens at* $\overline{x}_{es} = \overline{x}_{is}$ *or the monopoly persists, each with probability* $1/2$ *(i.e.,* $p=1/2$*)*
if $K_e{}^* < K_e$	:	*entry is not possible (i.e.,* $p=0$*)*

Proof: See Appendix.

[17] Alternatively, we will use $G(K_e|\hat{x}_t)$ to represent $G(K_e|\hat{K})$ when emphasising that the incumbent's conjecture is improved by observing $\hat{x}_t$.

2.3.2 Investment Rule for the Incumbent

We now discuss how to find the investment trigger and claim value of the incumbent. In our model the incumbent's strategic trigger, $\overline{x}_{is}$, is always known by the rival, and the incumbent knows this fact. If the incumbent chooses a higher trigger (but still lower than its non-strategic trigger, $\overline{x}_{in}$,) it can enjoy more of the option value of waiting. However, with a higher trigger the incumbent also suffers a higher probability of being preempted by the entrant. Therefore one might expect that, as suggested by the existing literature, the incumbent would strike a balance between the benefit of delaying the investment and the cost of being preempted.

However, we find that fear of ϵ-preemption usually spurs the incumbent into acting sooner until its limit – the breakeven trigger, $\overline{x}_{ic}$ – is reached. That is, a positive probability of being preempted, no matter how small that probability is, will always make the incumbent act earlier. Only when the incumbent is sure of getting the new market will it act at a trigger higher than $\overline{x}_{ic}$. This finding is formalised in Proposition 3 below and derived in the proof.

So far we have implicitly assumed that the incumbent's investment has not been triggered since the beginning of the patent game. To complete our discussion, in Proposition 3 we also consider the possibility that the game starts from a state higher than the incumbent's strategic investment trigger. If there is a non-zero probability of being preempted and the initial state, x_0, is higher than $\overline{x}_{ic}$, then the incumbent should act immediately at the beginning of the game. In contrast, if the patent game starts from a state lower than $\overline{x}_{ic}$, then the incumbent's optimal strategy is to wait for its investment to be triggered, even at the risk of being preempted by the entrant. It is under this scenario that the incumbent may be able to learn about the challenger's cost parameter.

Proposition 3 *Let x_0 be the state variable at the start of the patent game considered in this paper, and $\overline{x}_{ic}$ denotes the incumbent's strategic Marshallian trigger as previously defined.*

The incumbent's strategic investment trigger is given by[18]

$$\overline{x}_{is} = \min\left(\overline{x}_{in}, \max\left(\overline{x}_{ic}, \frac{(r-\mu)K_L}{\delta_e}\right)\right). \tag{17}$$

except for Case B-2 (see below) where $\overline{x}_{is}$ equals x_0.

The incumbent's firm value is given as follows:

***Case A**: suppose $x_0 < \overline{x}_{ic}$.*

***Case A-1**: If $\frac{r-\mu}{\delta_e}K_L \leq \overline{x}_{ic} \leq \frac{r-\mu}{\delta_e}K_U$ then the incumbent faces a non-zero probability of being preempted. The strategic claim value of the incumbent is*

$$\begin{aligned} V_{is}(x_t, \hat{x}_t|\overline{x}_{is}) \;=\; & \left[\int_{K_L}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i - \pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e)\right] I_{\overline{x}_{ic} \leq \frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L} + \\ & \left[\int_{\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}}^{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}} \frac{(\delta_i - \pi_{1i})\overline{x}_{en}(K_e)}{r-\mu}\left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta} dG(K_e|\hat{x}_t) + \right. \\ & \left. \int_{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i - \pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t)\right] I_{\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L < \overline{x}_{ic}} + \\ & \int_{\frac{\delta_e \overline{x}_{is}}{r-\mu}}^{K_U} \left[\frac{(\pi_{2i} - \pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right]\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t) + \frac{\pi_{1i}x_t}{r-\mu}, \end{aligned} \tag{18}$$

where $\overline{x}_{en}(K_e) \equiv \frac{\beta(r-\mu)K_e}{(\beta-1)\delta_e}$ and I denotes the indicator function, i.e. $I = 1$ if the condition stated in the subscript holds, and $I = 0$ otherwise.

***Case A-2**: If $\overline{x}_{ic} < \frac{r-\mu}{\delta_e}K_L$ then the incumbent's cost is so low that entry will be deterred completely. The strategic claim value of the incumbent is*

$$V_{is}(x_t|\overline{x}_{is}) \;=\; \left(\frac{(\pi_{2i} - \pi_{1i})\,\overline{x}_{is}}{r-\mu} - K_i\right)\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} + \frac{\pi_{1i}x_t}{r-\mu}. \tag{19}$$

***Case A-3**: If $\frac{r-\mu}{\delta_e}K_U < \overline{x}_{ic}$ then the challenger's cost is so low that the incumbent can never win the new patent. The incumbent's strategic claim value is*

$$V_{is}(x_t, \hat{x}_t|\overline{x}_{is}) \;=\; \left[\frac{(\delta_i - \pi_{1i})\,\overline{x}_{en}(K_e)}{r-\mu}\left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta}\right] I_{\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_U \leq \overline{x}_{ic}} +$$

[18] Note that $\overline{x}_{in}$ is always greater than $\overline{x}_{ic}$.

$$\left[\int_{\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}}^{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}} \frac{(\delta_i - \pi_{1i})\overline{x}_{en}(K_e)}{r-\mu}\left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta} dG(K_e|\hat{x}_t) + \int_{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i - \pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t)\right] I_{\overline{x}_{ic} < \frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_U} + \frac{\pi_{1i}x_t}{r-\mu}. \tag{20}$$

Case B: *suppose* $\overline{x}_{ic} \le x_0 < \overline{x}_{in}$.

Case B-1: *If* $x_0 < \frac{r-\mu}{\delta_e}K_L$ *then (as in Case A-2) there will be no entry. Again the incumbent's strategic claim value is given by equation (19).*

Case B-2: *If* $\frac{r-\mu}{\delta_e}K_L \le x_0$ *then the incumbent will act as soon as the game starts. Its strategic claim value is*

$$\begin{aligned} V_{is}(x_0|\overline{x}_{is}) &= \frac{\pi_{1i}x_0}{r-\mu} + \frac{1}{2}\frac{(\delta_i - \pi_{1i})x_0}{r-\mu}G\left(\frac{\delta_e x_0}{r-\mu}\right) + \\ &\quad \left(\frac{(\pi_{2i}-\pi_{1i})x_0}{r-\mu} - K_i\right)\left(1 - \frac{1}{2}G\left(\frac{\delta_e x_0}{r-\mu}\right)\right). \end{aligned} \tag{21}$$

Proof: See Appendix.

The terms on the right-hand side of equation (18) are explained as follows: (i) As shown in the Appendix, $\overline{x}_{is}$ should be equal to $\overline{x}_{ic}$ for Case A-1. Therefore, the condition $\overline{x}_{ic} \le \frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L$ is equivalent to $\frac{\beta-1}{\beta}K_e^* \le K_L$. If this condition is satisfied then there will be no learning for the incumbent. The integral $\int_{K_L}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e)$ represents the expected present value of the loss incurred by the incumbent if $K_e < K_e^*$ and the entrant preempts at $\overline{x}_{is}^-$. (ii) When the condition $\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L < \overline{x}_{ic}$ holds, the incumbent is able to learn by observing $\hat{x}_t$. The integral $\int_{\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}}^{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}} \frac{(\delta_i-\pi_{1i})\overline{x}_{en}(K_e)}{r-\mu}\left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta} dG(K_e|\hat{x}_t)$ is the expected loss to the incumbent if $K_e < \frac{\beta-1}{\beta}K_e^*$ and the entrant invests at $\overline{x}_{en}$, while the integral $\int_{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t)$ represents the expected loss to the incumbent if $\frac{\beta-1}{\beta}K_e^* \le K_e < K_e^*$ and the entrant preempts at $\overline{x}_{is}^-$. (iii) The integral $\int_{\frac{\delta_e \overline{x}_{is}}{r-\mu}}^{K_U} \left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right]\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t)$ is the expected net present value of the new patent to the incumbent if $K_e^* < K_e$ and the incumbent acts at $\overline{x}_{is}$. (iv) The last term $\frac{\pi_{1i}x_t}{r-\mu}$ is the value of the incumbent's existing patent on product 1.

We now discuss the broad intuition of Proposition 3. First there is the distinction between whether (case A) or not (case B) there is learning. Since competitive fear makes the incumbent act at its strategic Marshallian trigger (unless the challenger is dominated), learning can only happen if the initial level of the state variable, x_0, is below the incumbent's strategic breakeven point, $\overline{x}_{ic}$. In those cases the challenger will enter at a level below $\overline{x}_{ic}$ only if its non-strategic entry threshold is below $\overline{x}_{ic}$ (otherwise the entrant would prefer to wait till $\overline{x}_{ic}^{-}$ to exploit its option value of waiting to invest). Consequently, if the challenger does not act, then the incumbent can infer that the challenger must be less efficient than previously thought, and consequently the lower bound of the entrant's cost parameter distribution gets updated to $\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}$.

Once the state variable equals or exceeds the incumbent's strategic Marshallian trigger, $\overline{x}_{ic}$, then the incumbent will act unless it can be sure that there is *no* chance of being preempted.[19] We show in the proof that a trigger above $\overline{x}_{ic}$ cannot be credible, as the incumbent can always do better by slightly acting earlier in order to surprise and preempt the entrant. As a consequence, any equilibrium trigger above the incumbent's strategic Marshallian trigger unravels. This is quite a strong result as it means that under asymmetric information the less informed incumbent sacrifices all its option value of waiting because of the fear of being preempted.[20] This is the case even if *ex post* it appears that the challenger is rather inefficient and the incumbent could have delayed investing well beyond its strategic Marshallian trigger without being preempted. The exception to the rule is the rather extreme case where the incumbent dominates the entrant (i.e., where the breakeven threshold of the most efficient challenger still exceeds the incumbent's strategic Marshallian trigger, i.e., when $\overline{x}_{ic} < \frac{(r-\mu)K_L}{\delta_e}$). In those cases the incumbent can afford to wait beyond its breakeven threshold up to the point $\frac{(r-\mu)K_L}{\delta_e}$ where the most efficient firm could enter. In an extreme case the challenger can be so inefficient that the incumbent can act at its non-strategic option trigger, $\overline{x}_{in}$, just as if there is no competitor. This happens if $\overline{x}_{in} < \frac{(r-\mu)K_L}{\delta_e}$.

For completeness' sake we conclude by briefly discussing each of the individual cases of Proposition 3:[21]

[19]The probability of the incumbent being preempted is zero if the breakeven threshold of the most efficient entrant exceeds $\overline{x}_{ic}$, or equivalently if $\overline{x}_{ic} < \frac{(r-\mu)K_L}{\delta_e}$.

[20]This result is in sharp contrast with the complete and incomplete information cases where the stronger firm preserves some option value of waiting. We will return to this important point later.

[21]Further details and proofs can be found in the Appendix.

Case A-1: neither the incumbent nor the challenger dominates, i.e. $\frac{(r-\mu)K_L}{\delta_e}<\overline{x}_{ic}<\frac{(r-\mu)K_U}{\delta_e}$

In this case, the incumbent will let a challenger whose Marshallian trigger is lower than $\overline{x}_{ic}$ act at $min\{\overline{x}_{en}, \overline{x}_{ic}^{-}\}$, since it does not pay for the incumbent to act at a state lower than $\overline{x}_{ic}$. However, if the state variable reaches $\overline{x}_{ic}$ and no entry has occurred, the incumbent will act immediately at $\overline{x}_{ic}$.

Case A-2: the incumbent dominates, i.e. $\overline{x}_{ic} < \frac{r-\mu}{\delta_e} K_L$

Case A-2-a: the incumbent weakly dominates, i.e. $\overline{x}_{ic} < \frac{(r-\mu)K_L}{\delta_e} \leq \overline{x}_{in}$

In this case, the incumbent will act at $\frac{(r-\mu)K_L}{\delta_e}$ to prevent any challenger from entering the market. The claim values of the incumbent and the entrant are, respectively,

$$V_{is}(x_t|\overline{x}_{is}) = \left[\frac{(\pi_{2i} - \pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right]\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} + \frac{\pi_{1i} x_t}{r-\mu}, \tag{22}$$

$$V_{es}(x_t) = 0, \tag{23}$$

where $\overline{x}_{is} = \frac{(r-\mu)K_L}{\delta_e}$.

Case A-2-b: the incumbent strictly dominates, i.e. $\overline{x}_{in} < \frac{(r-\mu)K_L}{\delta_e}$

In this case, even the most cost-efficient challenger has a Marshallian trigger higher than the incumbent's non-strategic trigger. That is, the challenger does not pose any threat to the incumbent, so the latter will act at the non-strategic trigger, $\overline{x}_{in}$.

Case A-3: the challenger dominates, i.e. $\frac{(r-\mu)K_U}{\delta_e} < \overline{x}_{ic}$

When the Marshallian trigger of the most cost-inefficient challenger is lower than the incumbent's strategic Marshallian trigger, it will always be the challenger who patents the second product. However, entry cannot occur at a state higher than $\overline{x}_{ic}$, otherwise the incumbent would have a chance to act at $\overline{x}_{ic}$. Therefore, the entrant's investment trigger is $min\{\overline{x}_{en}, \overline{x}_{ic}^{-}\}$.

Case B-1: the incumbent dominates, i.e. $\overline{x}_{ic} \leq x_0 < \frac{r-\mu}{\delta_e} K_L$

Although $\overline{x}_{ic} \leq x_0$, the incumbent will not act immediately when the game is started. Instead, the incumbent should wait until a non-zero probability of being preempted comes into existence or $\overline{x}_{in}$ is reached, so that the benefit of waiting can be exploited as much as possible.

Case B-2: the incumbent acts at the beginning of the game, i.e. $\frac{(r-\mu)K_L}{\delta_e} \leq x_0$

In this case, the patent game starts from a state higher than $\overline{x}_{ic}$ and the incumbent is uncertain whether the challenger's Marshallian trigger is lower than x_0. Driven by the fear of preemption, the incumbent will thus act at $t = 0$. Consequently, if the challenger's cost is so low that $\overline{x}_{em} \leq x_0$, the challenger should also act at $t = 0$ (and each player will win the new patent with probability 1/2); otherwise the challenger should give in and let the incumbent win the new patent.

3 Analysis and Discussion of the Results

In this section we analyse in some further detail the results from Proposition 3. Equation (17) shows that the incumbent's strategic trigger is independent of the probability distribution of the challenger's cost. The probability distribution, $G(K_e)$, is needed only when we are calculating (i) the probability of being preempted by the entrant (if $\overline{x}_{ic}$ is higher than $\frac{r-\mu}{\delta_e}K_L$) or (ii) the incumbent's claim value. When $G(K_e)$ is required, we use the Pareto distribution:[22]

$$G(K_e) = \frac{K_e^{-\alpha} - K_L^{-\alpha}}{K_U^{-\alpha} - K_L^{-\alpha}} \quad \text{for} \quad K_e \in [K_L, K_U] \tag{24}$$

where $0 < K_L < K_U < \infty$, and $\alpha \neq 0$. If $\alpha = -1$, $G(K_e)$ reduces to the uniform distribution over $[K_L, K_U]$. When α is positive, the probability distribution $G(K_e)$ is skewed towards K_L. In contrast, the more negative α is, the more the probability distribution is skewed towards K_U.

In Figure 2 we plot the incumbent's strategic trigger as a function of its own cost, together with the corresponding probability of being preempted by the entrant.

[22]Note that a common phenomenon known as the 80/20 rule (i.e., 80% of the probability is concentrated on 20% of the support) can be represented by the Pareto distribution. However, as shown by Equation (17), the particular shape of the cost distribution $G(K_e)$ does not enter into the incumbent's investment decision.

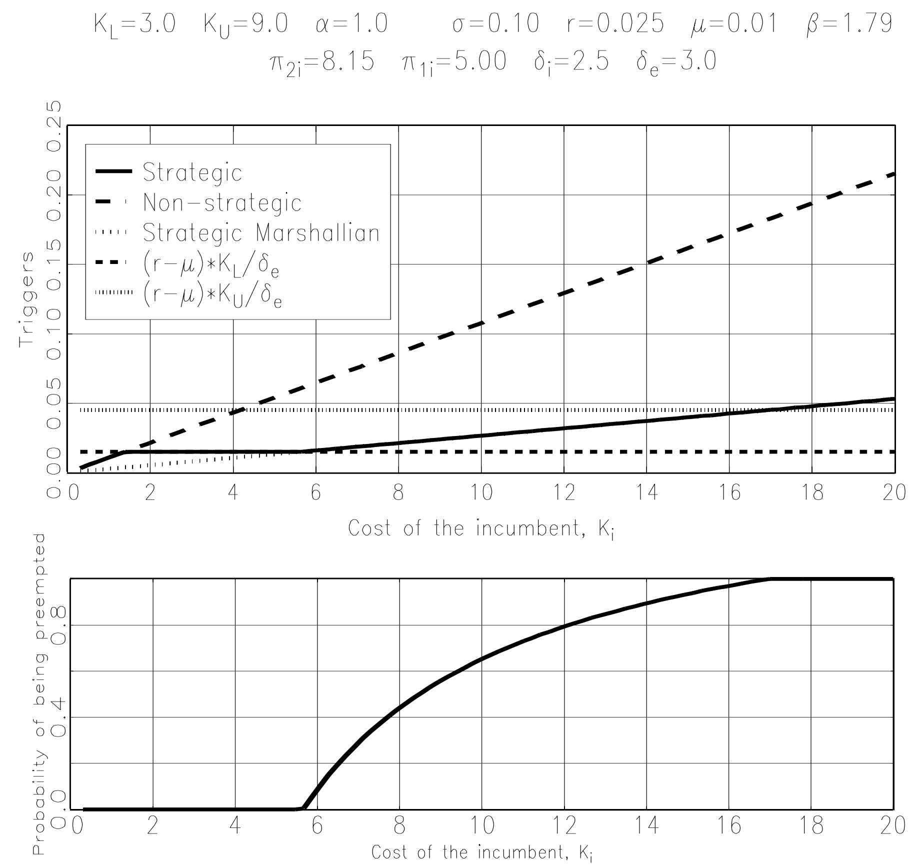


Figure 2: The incumbent's triggers as a function of K_i and the corresponding probability of being preempted.

The curves in the upper panel are discussed in descending order of the legend. The solid line denotes the incumbent's strategic entry trigger, $\overline{x}_{is}$. The (single) dashed line plots the incumbent's non-strategic option entry trigger, $\overline{x}_{in}$. The (single) dotted line shows the incumbent's strategic Marshallian trigger, $\overline{x}_{ic}$. The double dashed (dotted) line represents the breakeven threshold of the most (least) efficient entrant.

The lower panel plots the incumbent's probability of being preempted as a function of its investment cost, K_i. If $\overline{x}_{in} < \frac{r-\mu}{\delta_e}K_L$, the incumbent can fully enjoy the option value of waiting by acting at $\overline{x}_{in}$. If $\overline{x}_{ic} \leq \frac{r-\mu}{\delta_e}K_L \leq \overline{x}_{in}$, the incumbent will act at $\frac{r-\mu}{\delta_e}K_L$ to prevent any entry. If $\frac{r-\mu}{\delta_e}K_L < \overline{x}_{ic} < \frac{r-\mu}{\delta_e}K_U$, the incumbent faces a positive probability of being preempted. If $\frac{r-\mu}{\delta_e}K_U \leq \overline{x}_{ic}$, entry will certainly occur.

As shown in Figure 2, the probability of being preempted is 0 when the incumbent dominates, in which case it will act at either the non-strategic trigger or the lowest possible entry trigger ($\frac{(r-\mu)K_L}{\delta_e}$), whichever is lower. On the other hand if the challenger dominates, the probability of being preempted equals 1. This happens when the incumbent's strategic Marshallian trigger exceeds the breakeven threshold of the most inefficient entrant (i.e. $\frac{(r-\mu)K_U}{\delta_e} < \overline{x}_{ic}$). In the intermediate case where the probability of being preempted is strictly between 0 and 1, the incumbent will act as soon as its strategic Marshallian trigger is hit, provided that, of course, the new market still exists.

It is also clear from equation (17) that the incumbent's strategic investment trigger will usually be independent of product market uncertainty (σ). This result is in line with the Marshallian investment rule, but diametrically opposed to the real options paradigm. From Proposition 2 it follows that, if entry occurs, the entrant is able to exploit some option value of waiting. However, since the entrant's trigger is usually determined by the incumbent's strategic Marshallian trigger, it follows that the entrant's trigger is not influenced by product market volatility.

Figure 3 shows a typical case in which the incumbent's firm value is seriously reduced due to the threat of entry.

This is illustrated by the fact that the firm's value (given by the 'Strategic claim' curve) is only a little higher than the value it would have after entry (i.e., the 'Post-entry value' curve) but substantially below the incumbent's value when there is only one product (given by the 'no new product curve', $\frac{\pi_{1i}x_t}{r-\mu}$). Hence, even if the incumbent acquires the second product, it will be worse off compared to the case where no second product exists. The reason is that the competitive threat forces the incumbent to adopt the second product inefficiently early in order to protect its market. In particular, if the incumbent's investment threshold is given by its strategic Marshallian breakeven trigger then upon adopting the second product, the incumbent's firm value drops by an amount:

$$\frac{(\pi_{2i} - \pi_{1i})\,\overline{x}_{ic}}{r-\mu} - K_i \tag{25}$$

Substituting for $\overline{x}_{ic}$ the loss is given by:

$$-K_i\left(\frac{\pi_{1i} - \delta_i}{(\pi_{2i} - \pi_{1i}) + (\pi_{1i} - \delta_i)}\right) \tag{26}$$

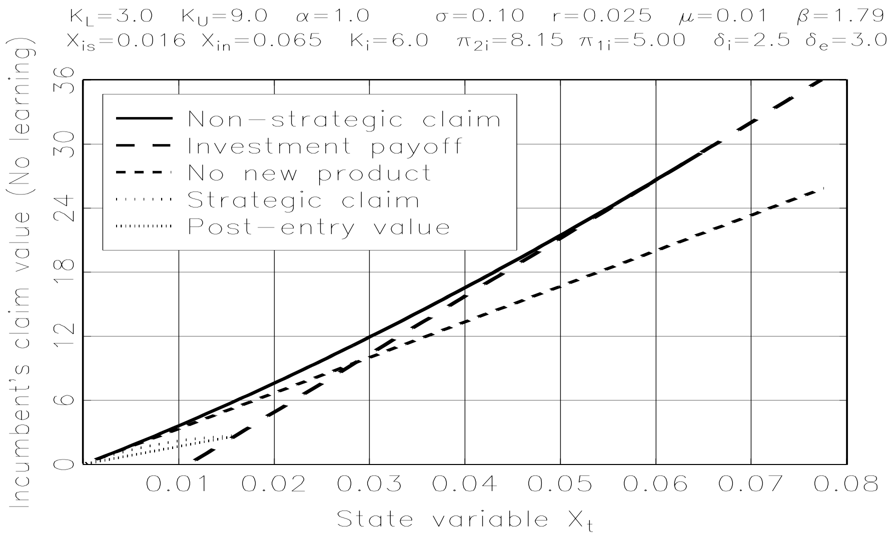


Figure 3: The incumbent's claim values under different conditions are plotted against the state variable for the no learning case.

The incumbent has the highest claim value if it can invest at the non-strategic trigger, the second highest if there is no new patent at all, and the lowest if entry has occurred. Even a slight possibility of entry can reduce the incumbent's claim value to almost the post-entry value.

Consequently, the incumbent's value reduction increases with the investment cost (K_i) and the profit fall that would be caused by entry ($\pi_{1i} - \delta_i$). The value reduction becomes, however, less severe as the profit contribution from the second product ($\pi_{2i} - \pi_{1i}$) rises. If the incumbent has no market share to lose (i.e. $\pi_i = \delta_i$) then the value reduction is zero. The maximal loss in value is the investment cost, K_i. This scenario happens when the second product does not generate any new profits to the incumbent (i.e. $\pi_{2i} = \pi_{1i}$). It also follows from the definition of the strategic Marshallian trigger that upon acquiring the second product at $\overline{x}_{ic}$ it will be the case that:

$$\frac{(\pi_{2i} - \pi_{1i})\,\overline{x}_{ic}}{r-\mu} - K_i = \frac{(\delta_i - \pi_{1i})\,\overline{x}_{ic}}{r-\mu} \tag{27}$$

or equivalently, upon acquiring the second patent at $\overline{x}_{ic}$ the incumbent's firm value is given by:

$$\frac{\pi_{2i}\overline{x}_{ic}}{r-\mu} - K_i = \frac{\delta_i\overline{x}_{ic}}{r-\mu} \tag{28}$$

Hence, at the strategic Marshallian trigger, $\overline{x}_{ic}$, the incumbent is indifferent between acquiring the second product, or becoming a duopolist. The proof of Proposition 3 shows that for all levels above $\overline{x}_{ic}$ the incumbent strictly prefers to invest rather than being preempted. This implies that the option value created by delaying the investment is always outweighed by the cost of being preempted.

In Figure 3, the incumbent's 'Strategic claim' value is obtained using Proposition 3. The 'Non-strategic claim' curve represents the incumbent's total claim value $\frac{\pi_{1i}x_t}{r-\mu} + \left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{in}}{r-\mu} - K_i\right]\left(\frac{x_t}{\overline{x}_{in}}\right)^{\beta}$, which consists of both the existing investment on the first product and the new investment opportunity, supposing that there was no potential entrant. The 'Investment payoff' curve represents the incumbent's total claim value, $\frac{\pi_{2i}x_t}{r-\mu} - K_i$, after it has patented the second product. The value of the incumbent's existing investment, $\frac{\pi_{1i}x_t}{r-\mu}$, is represented by the 'No new product' curve. Finally, if the entrant has patented the second product, the incumbent's claim value is $\frac{\delta_i x_t}{r-\mu}$, which corresponds to the 'Post-entry value' curve. The incumbent's non-strategic option trigger, $\overline{x}_{in}$, is the point where the 'non-strategic claim' curve (solid line) smooth-pastes to the 'investment payoff' curve (single dashed line). The value for this trigger, $\overline{x}_{in}$, is shown to be around 0.06. The incumbent's non-strategic Marshallian trigger, $\overline{x}_{im}$, is given by the intersection of the two dashed lines, i.e. the 'investment payoff curve' and the 'no new product' curve. Its value is slightly below 0.03. Finally, the strategic option trigger, $\overline{x}_{is}$, corresponds to the strategic Marshallian trigger, $\overline{x}_{ic}$, and is given by the point where the 'investment payoff curve' value matches (intersects with) the 'post-entry value' curve. The value for $\overline{x}_{ic}$ is about 0.015. The figure illustrates that the threat of entry and market share loss makes the incumbent invest so inefficiently early that it substantially reduces its firm value.

Figure 4 illustrates the impact of learning on the incumbent's strategic claim value. Whenever the state variable, x_t, rises to a new high, $\hat{x}_t$, and the challenger does not act, the incumbent learns that the challenger has a higher cost distribution than previously conjectured. This is good news for the incumbent as the probability of being preempted diminishes. Consequently, each time the state variable reaches a new high, the incumbent's firm value moves along the envelope onto a higher value function.[23] Lambrecht and Perraudin (2003) also find a similar learning effect on the firm's claim value, but in their model both firms gain from learning as both players are incompletely informed about the opponent. In contrast, in our model learning is relevant to the incumbent only.[24]

4 Symmetric, Asymmetric and Incomplete Information: A Comparative Analysis

In this last section we compare our results with those in the existing literature on real options and preemption. Existing models have focussed on preemptive entry under complete (e.g. Smets (1991) and Grenadier (1996)) and incomplete (Lambrecht and Perraudin (2003)) information. The existing papers typically specialise the problem to symmetrical races where two potential entrants contemplate to move into a new market. In order to make our asymmetrical race between an incumbent and an entrant comparable with previous models, we assume in what follows that $\pi_{1i} = \delta_i$. In that case each firm has a zero payoff if it is preempted, and a non-negative payoff if it wins

[23]The envelope is obtained by setting x_t equal to $\hat{x}_t$ in the strategic claim value and by varying $\hat{x}_t$, i.e. the envelope is given by $V_{is}(\hat{x}_t, \hat{x}_t|\overline{x}_{is})$.

[24]Note that in Lambrecht and Perraudin (2003), the 'envelope' curve smooth-pastes to the 'Investment payoff' curve at the optimal investment threshold, while in our model these two curves do not meet tangentially. This difference stems from their model assumption that the agents have symmetric information (therefore the players follow the same optimising rule given by the smooth-pasting condition). Also note that under incomplete information the optimal investment trigger strikes a balance between the cost of being preempted and the benefit of waiting to invest. The solution is an interior optimum in that the threshold is situated in between the Marshallian breakeven threshold, and the non-strategic option trigger, i.e. some but not all option value of waiting is destroyed. In our paper, however, we obtain a corner solution (the strategic Marshallian trigger) since the cost of being preempted always outweighs the benefit of waiting. This corner solution does not satisfy the traditional high contact (smooth-pasting) condition.

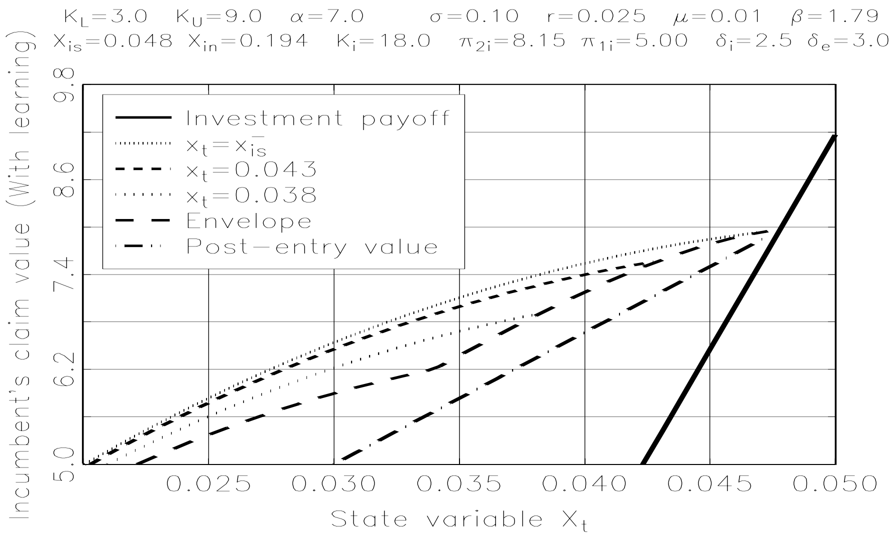


Figure 4: Impact of learning on the incumbent's strategic claim value.

This figure illustrates that learning can enhance the incumbent's strategic claim value. Whenever x_t hits a new high (i.e. a larger $\hat{x}_t$) and the potential entrant does not act, the incumbent learns that the challenger has a higher cost distribution. This raises the incumbent's probability of winning the patent race and consequently the incumbent's strategic claim value shifts along the 'envelope' to a higher level. The 'Envelope' curve (given by $V_{is}(x_t = \hat{x}_t, \hat{x}_t|\overline{x}_{is})$) represents the trace of this learning process. The incumbent's claim value is evaluated at three different values for $\hat{x}_t$ namely: 0.038, 0.043 and $\overline{x}_{is}$.

the race (consequently the non-strategic and strategic Marshallian trigger coincide, i.e. $\overline{x}_{jc} = \overline{x}_{jm}$ $(j = i, e)$). To make the analysis non-trivial we assume that neither firm strictly dominates (i.e. neither firm can act at its non-strategic option trigger without being preempted).

We know that under complete information the more efficient firm typically enters just (epsilon) before its opponent's Marshallian breakeven trigger. Ignoring the epsilon component this means that firm i and e's triggers under complete information are respectively given by:[25]

$$\overline{x}_{iscomp} \;=\; \max[\overline{x}_{im}\,,\,\overline{x}_{em}] \qquad and \qquad \overline{x}_{escomp} \;=\; \max[\overline{x}_{im}\,,\,\overline{x}_{em}] \tag{29}$$

Firm i (e) wins the race if $\overline{x}_{im} \;<\; (>)\, \overline{x}_{em}$. It follows that the weaker firm chooses the Marshallian breakeven trigger and sacrifices all option value of waiting, whereas the more efficient firm retains some option value of waiting depending on the distance between both firms' Marshallian triggers ($|\overline{x}_{im} - \overline{x}_{em}|$).

With incomplete information both firms have to decide on their entry threshold knowing that their opponent's entry cost is drawn from a distribution, $G(K)$. Lambrecht and Perraudin (2003) show that each firm's entry threshold is situated between its Marshallian threshold and its non-strategic option threshold, i.e.:

$$\overline{x}_{im} \leq \overline{x}_{isincomp} \leq \overline{x}_{in} \qquad and \qquad \overline{x}_{em} \leq \overline{x}_{esincomp} \leq \overline{x}_{en} \tag{30}$$

Firm i (e) wins the race if $\overline{x}_{isincomp} \;<\; (>)\, \overline{x}_{esincomp}$. The optimal investment threshold strikes a balance between the cost of being preempted and the option value of delaying to invest. This means that under incomplete information *both* firms preserve some option value of delay.[26]

Finally, we found that under asymmetric information the uninformed firm, i, acts at its Marshallian trigger, whereas the informed firm, e, epsilon preempts its competitor, if profitable to do so. Hence,

$$\overline{x}_{isasym} = \overline{x}_{im} \qquad and \qquad \overline{x}_{esasym} \;=\; \max[\overline{x}_{im}, \overline{x}_{em}] \tag{31}$$

Firm i (e) wins the race if $\overline{x}_{im} \;<\; (>)\, \overline{x}_{em}$. It follows that the uninformed firm, i, always sacrifices its option value of waiting, even if it is more efficient and ex-post ends up being the winner. The cost of the informational disadvantage therefore corresponds to the option value of waiting to invest.[27]

[25]In what follows, the subscripts *comp*, *incomp* and *asym* respectively stand for complete, incomplete and asymmetric information.

[26]The bounds $\overline{x}_{jn}$ or $\overline{x}_{jm}$ ($j = i, e$) are only reached for the extreme cases where respectively the hazard of being preempted is zero or infinitely high.

[27]We showed in previous section that if the uninformed firm loses some of its existing profits when being preempted (i.e., $\pi_{1i} > \delta_i$), then not only the option value of waiting to invest is destroyed but also part of the uninformed firm's existing standalone value. The additional loss can amount up to the full investment cost, K_i.

The triggers under complete, incomplete and asymmetric information (assuming neither firm strictly dominates) are summarised in the following table:

	Firm i	Firm e
Complete information	$\overline{x}_{iscomp} = \max[\overline{x}_{im}, \overline{x}_{em}]$	$\overline{x}_{escomp} = \max[\overline{x}_{im}, \overline{x}_{em}]$
Incomplete information	$\overline{x}_{im} \leq \overline{x}_{isincomp} \leq \overline{x}_{in}$	$\overline{x}_{em} \leq \overline{x}_{esincomp} \leq \overline{x}_{en}$
Asymmetric information	$\overline{x}_{isasym} = \overline{x}_{im}$	$\overline{x}_{esasym} = \max[\overline{x}_{im}, \overline{x}_{em}]$

We conclude from the above table that competition under asymmetric information leads to a more inefficient outcome for the uninformed firm (i) than competition under complete or incomplete information. For the informed firm (e), complete and asymmetric information lead to the same outcome in that the informed firm invests just before the uninformed firm's Marshallian breakeven trigger, provided that this is profitable to do so.

5 Conclusion

This paper models a patent race between an incumbent and a potential entrant. Entry into the monopolised market happens by patenting a substitute for the incumbent's present product. The payoff of the substitute is stochastic. Asymmetric information is incorporated by assuming that the challenger has complete information about the incumbent, whereas the latter does not know the precise value of the challenger's investment cost but only the distribution it is drawn from. The model allows us to examine the impact of asymmetric information on the incumbent's and challenger's investment decision and on the persistence of the monopoly.

Even though, at first sight, the investment model with asymmetric information seems to be an intermediate case between complete and incomplete information, we find that the resulting investment behaviour is more in line with the one that would prevail in a world of complete information.

We show that a small probability of being preempted gives the incumbent a sufficient incentive to invest at the breakeven point where it is indifferent between investing and being preempted. The informationally disadvantaged incumbent therefore sacrifices all option value of waiting. Furthermore, if entry leads to some dissipation of the incumbent's profits then the incumbent will be prepared to destroy some of its existing monopoly value in order to protect its market. We show that this value reduction increases with the incumbent's investment cost and the potential loss of monopoly profits. If the second product does not generate any new profits when adopted by the incumbent then the reduction in the incumbent's value can equal the full investment cost. If, however, entry does not cause any profit dissipation then preemptive patenting by the incumbent will not destroy any of its existing value (apart of the option value of waiting to invest).

We show that the entrant will try to 'epsilon preempt' the incumbent, if optimal to do so. It follows that the entrant's and the incumbent's trigger are independent of product market uncertainty (unless one firm strictly dominates the other) and that that the corresponding investment rules are closer to the neoclassical Marshallian breakeven rules than to the real options investment rule. One would therefore expect that the qualitative nature of previous results from the industrial organisation literature on the timing of technological innovation is likely to remain unaltered by uncertainty and asymmetric information.

Next, we show that if the challenger is relatively inefficient then the incumbent may be able to learn about the rival's investment cost: whenever the profit state variable hits a new high and the challenger does not act, then the incumbent may be able to infer that the challenger's investment cost is higher than previously assumed.

Finally, we show that if the incumbent has an informational disadvantage compared to its challenger then deterring entry will become much costlier than under complete information, as the slightest threat of preemption will lead the incumbent preemptively to invest and to incur entry deterrence costs. On the other hand, if the incumbent were to have an informational advantage (compared to its challengers) then this would put the incumbent in a very powerful position as it would epsilon preempt its rival (if optimal to do so), whereas any rival would end up having to invest at the breakeven threshold, and therefore dissipate all rents.

Appendix

Proof of Proposition 1

See Dixit and Pindyck (1994, Chapter 5) for a standard derivation of this proposition. Note that, as shown in Dixit and Pindyck (1994, pp. 315-316), $\left(\frac{x_t}{\overline{x}_{jn}}\right)^\beta$ represents the discount factor $E_t[e^{-rT}]$, where T is the first time the stochastic process reaches a fixed level $\overline{x}_{jn}$ starting from the initial position x_t.

Proof of Proposition 2

If the entrant can win the patent with certainty (i.e. $p = 1$), then its strategic claim value is simply the investment payoff, $\Theta_e \overline{x}_{es} - K_e$ multiplied by the discount factor, $\left(\frac{x_t}{\overline{x}_{es}}\right)^\beta$. However, to reflect the possibility that the entrant may not win the game if its cost is too high, the factor p must be included in equation (16). The value of p follows directly from the derivation of Figure 1.1 and Assumption 2.

Proof of Proposition 3

Case A: We now consider the situation where $x_0 < \overline{x}_{ic}$.

Case A-1: $\frac{r-\mu}{\delta_e} K_L \leq \overline{x}_{ic} \leq \frac{r-\mu}{\delta_e} K_U$. In this case, the incumbent can not be certain whether its strategic Marshallian trigger is higher or lower than the Marshallian trigger of a potential entrant.

We will show later that the optimal $\overline{x}_{is}$ should be $\overline{x}_{ic}$ in this case. Therefore, $K_L \leq \frac{\delta_e \overline{x}_{is}}{r-\mu} \equiv K_e{}^* \leq K_U$ and only the challenger whose cost is lower than $\frac{\delta_e \overline{x}_{is}}{r-\mu}$ can preempt the incumbent by acting at $\overline{x}_{is}^-$. Under the circumstances, we consider the following two possibilities:

Case A-1-a: $\overline{x}_{ic} \leq \frac{\beta(r-\mu)}{(\beta-1)\delta_e} K_L$, i.e., even the most cost-efficient challenger does not have a non-strategic trigger lower than $\overline{x}_{ic}$. Therefore, the incumbent cannot improve its conjecture about the probability distribution of the potential entrant's cost by observing $\hat{x}_t$.

By using the payoff matrix in Section 2.1, the value of the incumbent is found to be

$$V_{is}(x_t|\overline{x}_{is}) = \frac{\pi_{1i} x_t}{r-\mu} + E_t\left[\frac{(\delta_i - \pi_{1i})\,\overline{x}_T}{r-\mu} e^{-r(T-t)} G(\frac{\delta_e \overline{x}_T}{r-\mu})\right] +$$

$$E_t\left[\left(\frac{(\pi_{2i}-\pi_{1i})\,\overline{x}_T}{r-\mu}\;-\;K_i\right)e^{-r(T-t)}\left(1\;-\;G(\frac{\delta_e\overline{x}_T}{r-\mu})\right)\right], \quad (32)$$

where $T \equiv \inf\{\tau | x_\tau \geq \overline{x}_{is}\}$ is the first time that x_t hits $\overline{x}_{is}$. The second and third terms on the right-hand side of equation (32) are the incremental expected present value to the incumbent when the entrant and the incumbent, respectively, preempt their rival. Calculating these expectations gives:

$$\begin{aligned} V_{is}(x_t|\overline{x}_{is}) &= \int_{K_L}^{\frac{\delta_e\overline{x}_{is}}{r-\mu}} \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}\left(\frac{x_t}{\overline{x}_{is}}\right)^\beta dG(K_e) + \\ & \int_{\frac{\delta_e\overline{x}_{is}}{r-\mu}}^{K_U}\left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu}-K_i\right]\left(\frac{x_t}{\overline{x}_{is}}\right)^\beta dG(K_e)+\frac{\pi_{1i}x_t}{r-\mu} \\ &= \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}(\frac{x_t}{\overline{x}_{is}})^\beta G(\frac{\delta_e\overline{x}_{is}}{r-\mu}) + \\ & \left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu}-K_i\right](\frac{x_t}{\overline{x}_{is}})^\beta\left[1-G(\frac{\delta_e\overline{x}_{is}}{r-\mu})\right]+\frac{\pi_{1i}x_t}{r-\mu} \end{aligned} \quad (33)$$

We now prove that the equilibrium strategy of the incumbent is to act at $\overline{x}_{ic}$. That is, the strategic trigger, $\overline{x}_{is}$, that optimises V_{is} should be $\overline{x}_{ic}$.[28]

Firstly, we show that the equilibrium strategic trigger can not be higher than $\overline{x}_{ic}$. Note that as long as entry has not occurred, the incumbent has the option of patenting the second product. Upon exercising this option, the incumbent's firm value, $V_{iex}(x_t)$, is $\frac{\pi_{2i}x_t}{r-\mu} - K_i$. Therefore,

$$\begin{aligned} V_{is}(x_t|\overline{x}_{is})\;-\;V_{iex}(x_t) &= \left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu}-K_i\right](\frac{x_t}{\overline{x}_{is}})^\beta-\left[\frac{(\pi_{2i}-\pi_{1i})x_t}{r-\mu}-K_i\right] \\ & -\left[\frac{(\pi_{2i}-\delta_i)\overline{x}_{is}}{r-\mu}-K_i\right](\frac{x_t}{\overline{x}_{is}})^\beta G(\frac{\delta_e\overline{x}_{is}}{r-\mu}). \end{aligned} \quad (34)$$

From equation (34), one can easily verify that (i) for any $\overline{x}_{is}$, $V_{is}(0|\overline{x}_{is}) > V_{iex}(0)$, and (ii) $\overline{x}_{is} > \overline{x}_{ic}$ if and only if $V_{is}(\overline{x}_{is}|\overline{x}_{is}) < V_{iex}(\overline{x}_{is})$. In other words, if the incumbent chooses a strategic trigger higher than $\overline{x}_{ic}$, then the value of the function V_{is} will be higher (lower) than that of the function V_{iex} when the state variable is 0

[28]Note that we restrict our discussion to pure strategies in this paper. A pure strategy for a player is a complete plan of action – it describes how the player will act in every contingency in which the player has to move. A mixed strategy for a player is a probability distribution over some or all of his pure strategies. Therefore, a player's pure strategies are simply the limiting cases of his mixed strategies (see, for example, Eichberger (1993, Section 1.2)).

($\overline{x}_{is}$.) This means that the function curve of $V_{is}(x_t|\overline{x}_{is})$ crosses that of $V_{iex}(x_t)$ from above at some state between 0 and $\overline{x}_{is}$, because V_{is} and V_{iex} are continuous functions of the state variable x_t.

Therefore, if $\overline{x}_{is} > \overline{x}_{ic}$, the incumbent's claim value without patenting the second product, $V_{is}(x_t|\overline{x}_{is})$, will be smaller than that with patenting, $V_{iex}(x_t)$, for some state lower than $\overline{x}_{is}$. This implies that a trigger higher than $\overline{x}_{ic}$ can not be credible, because the incumbent has an incentive not to keep to that trigger. Indeed, a trigger at $\overline{x}_{is}$ means there is a probability of $G(\frac{\delta_e \overline{x}_{is}}{r-\mu})$ that the entrant ϵ-preempts the incumbent at $\overline{x}_{is} - \epsilon$. Faced with this risk, the incumbent is better off acting at $\overline{x}_{is} - 2\epsilon$, causing the entrant to act even earlier at $\overline{x}_{is} - 3\epsilon$, and so on.

Secondly, it is clear that a trigger lower than $\overline{x}_{ic}$ can not be optimal, since $\overline{x}_{ic}$ is by definition the state at which the incumbent is indifferent between being preempted and acting. In other words, in a state lower than $\overline{x}_{ic}$ the incumbent would rather be preempted than patent the second product.

The only remaining candidate for the equilibrium trigger is $\overline{x}_{is} = \overline{x}_{ic}$. For such a trigger the value-matching condition is satisfied because

$$V_{is}(\overline{x}_{ic}|\overline{x}_{ic}) = \frac{\pi_{2i}\overline{x}_{ic}}{r-\mu} - K_i = V_{iex}(\overline{x}_{ic}).$$

Furthermore, $V_{is}(x_t|\overline{x}_{ic}) > V_{iex}(x_t)$ for any state x_t lower than $\overline{x}_{ic}$, so that the incumbent does not have an incentive to act before the trigger $\overline{x}_{ic}$ is reached. We have thus proved that $\overline{x}_{ic}$ is the equilibrium strategic trigger of the incumbent.

Case A-1-b: $\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L < \overline{x}_{ic}$. In this case, the incumbent's learning happens whenever $\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_L < \hat{x}_t < \overline{x}_{ic}$ and a higher $\hat{x}_t$ is observed.

Again, using the payoff matrix in Section 2.1, the value of the incumbent is

$$\begin{aligned} V_{is}(x_t, \hat{x}_t|\overline{x}_{is}) = & \int_{\frac{(\beta-1)\delta_e \hat{x}_t}{\beta(r-\mu)}}^{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}} \frac{(\delta_i - \pi_{1i})\overline{x}_{en}(K_e)}{r-\mu} \left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta} dG(K_e|\hat{x}_t) + \\ & \int_{\frac{(\beta-1)\delta_e \overline{x}_{is}}{\beta(r-\mu)}}^{\frac{\delta_e \overline{x}_{is}}{r-\mu}} \frac{(\delta_i - \pi_{1i})\overline{x}_{is}}{r-\mu} \left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t) + \\ & \int_{\frac{\delta_e \overline{x}_{is}}{r-\mu}}^{K_U} \left[\frac{(\pi_{2i} - \pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right] \left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} dG(K_e|\hat{x}_t) + \frac{\pi_{1i}x_t}{r-\mu}. \end{aligned} \tag{35}$$

The first term on the right-hand side of equation (35) is the expected present value of the incumbent's loss due to the entrant's entry, which happens at $\overline{x}_{en}$ if $K_e < \frac{\beta-1}{\beta}K_e^*$. Similarly, the second term is the incumbent's loss due to being preempted at $\overline{x}_{is}^-$ if $\frac{\beta-1}{\beta}K_e^* \leq K_e < K_e^*$. The third term is the incremental value of the incumbent patenting the second product at $\overline{x}_{is}$, which happens if $\overline{x}_{is} < \overline{x}_{em}$.

We now prove that the equilibrium strategy of the incumbent is to act at $\overline{x}_{ic}$. Suppose $\overline{x}_{is} > \overline{x}_{ic} \equiv \frac{K_i(r-\mu)}{\pi_{2i}-\delta_i}$. Let x_t and $\hat{x}_t$ be infinitesimally close to $\overline{x}_{is}^-$ (so $\hat{K}$ tends to $\frac{\beta-1}{\beta}K_e^*$.) Then

$$\begin{aligned}
&\lim_{x_t=\hat{x}_t\to\overline{x}_{is}} [V_{iex}(x_t) - V_{is}(x_t,\hat{x}_t|\overline{x}_{is})] \\
= &\lim_{x_t=\hat{x}_t\to\overline{x}_{is}} \left[(\frac{\pi_{2i}x_t}{r-\mu} - K_i) - V_{is}(x_t,\hat{x}_t|\overline{x}_{is})\right] \\
= &\left[\frac{\pi_{2i}\overline{x}_{is}}{r-\mu} - K_i\right] - \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}G(K_e|\hat{K})\Big|_{\frac{\beta-1}{\beta}K_e^*}^{K_e^*} - \\
&\left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right]G(K_e|\hat{K})\Big|_{K_e^*}^{K_U} - \frac{\pi_{1i}\overline{x}_{is}}{r-\mu} \\
= &\left[\frac{\pi_{2i}\overline{x}_{is}}{r-\mu} - K_i\right] - \frac{(\delta_i-\pi_{1i})\overline{x}_{is}}{r-\mu}G(K_e^*|\hat{K}) - \\
&\left[\frac{(\pi_{2i}-\pi_{1i})\overline{x}_{is}}{r-\mu} - K_i\right]\left[1 - G(K_e^*|\hat{K})\right] - \frac{\pi_{1i}\overline{x}_{is}}{r-\mu} \\
= &\left[\frac{(\pi_{2i}-\delta_i)\overline{x}_{is}}{r-\mu} - K_i\right]G(K_e^*|\hat{K}) > 0.
\end{aligned}$$

Therefore as the state variable, x_t, approaches $\overline{x}_{is}$, the incumbent is better off if it acts earlier instead of sticking with the trigger $\overline{x}_{is}$. That is, any $\overline{x}_{is}$ that is higher than $\overline{x}_{ic}$ is not a credible trigger. On the other hand, we have shown in Case A-1-a that a trigger lower than $\overline{x}_{ic}$ is not optimal, so the only possible equilibrium trigger is $\overline{x}_{ic}$. Indeed, the value-matching condition is satisfied if $\overline{x}_{is} = \overline{x}_{ic}$, because

$$\lim_{x_t=\hat{x}_t\to\overline{x}_{ic}} [V_{iex}(x_t) - V_{is}(x_t,\hat{x}_t|\overline{x}_{ic})] = \left[\frac{(\pi_{2i}-\delta_i)\overline{x}_{ic}}{r-\mu} - K_i\right]G(K_e^*|\hat{K}) = 0.$$

Case A-2: $\overline{x}_{ic} < \frac{r-\mu}{\delta_e}K_L$, i.e., the incumbent's strategic Marshallian trigger is lower than the Marshallian trigger of the most cost-efficient challenger.

Case A-2-a: $\frac{r-\mu}{\delta_e}K_L \leq \overline{x}_{in}$

Following a similar argument to that in Case A-1-a above, one can show that, for any $\overline{x}_{is}$ such that $\overline{x}_{is} > \frac{r-\mu}{\delta_e}K_L$, the claim value $V_{is}(x_t|\overline{x}_{is})$ becomes smaller than the exercise value, $V_{iex}(x_t)$, as x_t approaches $\overline{x}_{is}$. Again the reason is because at $\overline{x}_{is}^-$ the incumbent's claim value is a probability-weighted average of the exercise value $\frac{\pi_{2i}x_t}{r-\mu} - K_i$ (in case the incumbent patents the second product) and the duopoly value $\frac{\delta_i x_t}{r-\mu}$ (in case entry occurs).[29] At $\overline{x}_{is}$, the incumbent's firm value will be either $\frac{\pi_{2i}\overline{x}_{is}}{r-\mu} - K_i$ or $\frac{\delta_i \overline{x}_{is}}{r-\mu}$, depending on who patents the second product.

Consequently, the incumbent has an incentive to act earlier instead of keeping to the trigger $\overline{x}_{is}$, so any trigger higher than $\frac{r-\mu}{\delta_e}K_L$ can not be credible. In equilibrium, the incumbent will set its strategic trigger $\overline{x}_{is}$ equal to $\frac{r-\mu}{\delta_e}K_L$. (Note that in this case $\overline{x}_{ic} < \overline{x}_{is} < \overline{x}_{in}$.) Therefore, no entry will occur and the incumbent's claim value is

$$V_{is}(x_t|\overline{x}_{is}) \; = \; \left(\frac{(\pi_{2i} - \pi_{1i})\,\overline{x}_{is}}{r-\mu} - K_i\right)\left(\frac{x_t}{\overline{x}_{is}}\right)^{\beta} \; + \; \frac{\pi_{1i}x_t}{r-\mu}. \tag{36}$$

Case A-2-b: $\overline{x}_{in} < \frac{r-\mu}{\delta_e}K_L$

In this case, even the most cost-efficient challenger does not pose a threat to the incumbent, so the latter can act at the non-strategic trigger $\overline{x}_{in}$. That is, $\overline{x}_{is} = \overline{x}_{in}$ and the incumbent's firm value is

$$V_{is}(x_t|\overline{x}_{in}) \; = \; \left(\frac{(\pi_{2i} - \pi_{1i})\,\overline{x}_{in}}{r-\mu} - K_i\right)\left(\frac{x_t}{\overline{x}_{in}}\right)^{\beta} \; + \; \frac{\pi_{1i}x_t}{r-\mu}. \tag{37}$$

Case A-3: $\frac{r-\mu}{\delta_e}K_U \; < \; \overline{x}_{ic}$, i.e., even the most cost-inefficient challenger can preempt the incumbent. In a non-cooperative patent game, entry will occur at $\min\left(\overline{x}_{ic}^-, \overline{x}_{en}\right)$ because the incumbent cannot credibly commit himself to acting at a trigger higher than $\overline{x}_{ic}$. Although in this case the incumbent cannot patent the second product, for expositional simplicity we will abuse the notation slightly by stating that $\overline{x}_{is} = \overline{x}_{ic}$.

Case A-3-a: $\frac{\beta(r-\mu)}{(\beta-1)\delta_e}K_U \leq \overline{x}_{ic}$

In this case, even the most cost-inefficient challenger can act at its non-strategic trigger $\overline{x}_{en}$. Therefore, the incumbent's claim value is

$$V_{is}(x_t|\overline{x}_{is}) \; = \; \frac{(\delta_i - \pi_{1i})\,\overline{x}_{en}(K_e)}{r-\mu}\left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^{\beta} \; + \; \frac{\pi_{1i}x_t}{r-\mu}. \tag{38}$$

[29] See the right-hand side of equation (33).

Case A-3-b: $\overline{x}_{ic} < \frac{\beta(r-\mu)}{(\beta-1)\delta_e} K_U$

In this case, the challenger will act at its non-strategic trigger $\overline{x}_{en}(K_e)$ if its cost is so low that $\overline{x}_{en}(K_e) < \overline{x}_{ic}$; otherwise it will preempt the incumbent by acting at $\overline{x}_{ic}^-$. Therefore, the incumbent's claim value is

$$\begin{aligned} V_{is}(x_t, \hat{x}_t|\overline{x}_{is}) &= \int_{\frac{(\beta-1)\delta_e\hat{x}_t}{\beta(r-\mu)}}^{\frac{(\beta-1)\delta_e\overline{x}_{is}}{\beta(r-\mu)}} \frac{(\delta_i - \pi_{1i})\overline{x}_{en}(K_e)}{r-\mu} \left(\frac{x_t}{\overline{x}_{en}(K_e)}\right)^\beta dG(K_e|\hat{x}_t) + \\ & \int_{\frac{(\beta-1)\delta_e\overline{x}_{is}}{\beta(r-\mu)}}^{\frac{\delta_e\overline{x}_{is}}{r-\mu}} \frac{(\delta_i - \pi_{1i})\overline{x}_{is}}{r-\mu} \left(\frac{x_t}{\overline{x}_{is}}\right)^\beta dG(K_e|\hat{x}_t) \; + \; \frac{\pi_{1i}x_t}{r-\mu}. \end{aligned}$$

Case B: We now consider the situation where $\overline{x}_{ic} \leq x_0 < \overline{x}_{in}$.

Case B-1: $x_0 < \frac{r-\mu}{\delta_e} K_L$, i.e., even the most cost-efficient challenger cannot act at the start of the patent game.

In this case, since $\overline{x}_{ic} \leq x_0 < \frac{r-\mu}{\delta_e} K_L$, we can make use of the results obtained in Case A-2. Therefore, the incumbent will act at $\overline{x}_{is} = \min\left(\frac{(r-\mu)K_L}{\delta_e}, \overline{x}_{in}\right)$ and its claim value is given by equation (36).

Case B-2: $\frac{r-\mu}{\delta_e} K_L \leq x_0$

In this case, any $\overline{x}_{is}$ higher than x_0 is not credible. Again, this is because if no entry has occurred at $\overline{x}_{is}^-$, the incumbent should act immediately to avoid the possibility of being preempted at $\overline{x}_{is}$. Therefore $\overline{x}_{is} = x_0$.

If the challenger's cost is such that $\overline{x}_{em} \equiv \frac{(r-\mu)K_e}{\delta_e} \leq x_0$, then it will want to act at x_0 as well. Given Assumption 2, the incumbent and the challenger will each win the game with probability 1/2. On the other hand, if $\frac{(r-\mu)K_e}{\delta_e} > x_0$ then the incumbent will certainly succeed in patenting the second product. So the incumbent's claim value is

$$\begin{aligned} V_{is}(x_0|\overline{x}_{is}) &= \left[\frac{1}{2}\frac{(\delta_i - \pi_{1i})x_0}{r-\mu} + \frac{1}{2}\left(\frac{(\pi_{2i} - \pi_{1i})x_0}{r-\mu} - K_i\right)\right] G(\frac{\delta_e x_0}{r-\mu}) + \\ & \left(\frac{(\pi_{2i} - \pi_{1i})x_0}{r-\mu} - K_i\right)\left(1 - G(\frac{\delta_e x_0}{r-\mu})\right) + \frac{\pi_{1i}x_0}{r-\mu} \qquad (39) \\ &= \frac{\pi_{1i}x_0}{r-\mu} + \frac{1}{2}\frac{(\delta_i - \pi_{1i})x_0}{r-\mu} G(\frac{\delta_e x_0}{r-\mu}) + \qquad (40) \\ & \left(\frac{(\pi_{2i} - \pi_{1i})x_0}{r-\mu} - K_i\right)\left(1 - \frac{1}{2}G(\frac{\delta_e x_0}{r-\mu})\right). \qquad (41) \end{aligned}$$

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