Investments in Thermopower Generation: A Real Options Approach for the New Brazilian Electrical Power Regulation*

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ABSTRACT

One of the main questions in electricity market deregulation is the aptitude of private capital for investments in power generation. This is especially important in Brazil, whose load has a strong growth trend (≈5% per year). Thermopower is an attractive alternative for expanding generation, as it is complementary in many aspects to hydropower, which supplies most Brazil's power at a very low price most of the time, but makes the system vulnerable to seasonal water variations. This paper studies the competitiveness of thermopower generation in Brazil under current regulations and assesses under the real options theory approach the conditions for investments in thermopower generation.

Keywords: Stochastic Dynamic Programming; Real Options Theory; Investments in Power Generation.

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1. Introduction

The Brazilian generation dispatch is based in a minimum-cost scheme, using a stochastic dynamic programming model (COD: centralized optimal dispatch) (Pereira et al. 1984, 1985 and 1998).

The COD model used in actual generation dispatch considers only the uncertainty of water affluence, disregarding the randomness of future power demand and thermopower operation (fuel) cost. That model takes the expectation of the minimum operating cost in a range of scenarios, in contrast to the minimization of expected operation cost given the dispatch decision made today, which corresponds to the "stochastic solution."

At present (2002), most (95%) of Brazil's electrical power is supplied by hydroelectric stations, exploiting a large, complex and highly integrated hydrological and power systems that provide power at low price for long time periods, but are very vulnerable to water affluence uncertainty.

Thermopower generation is complementary to hydropower in many aspects, as shown in Table 1, which compares medium-size (200 MW) power plants.

Table 1 - Hydropower x Thermopower Complementarities

Characteristic	Hydro	Thermo
Investment (US$ / kW)	~ 1500 … 2500	~ 650
Operation Cost (US$ / MWh)	2(1)	~ 22.3(2)
Machinery Lifetime (years)	~ 40	~ 20
Installation time (years)	~ 8	~ 2
Production Regimen	Uncertain	Controlled
Operation Flexibility	High	Low

(1) The hydropower direct cost corresponds to operation and maintenance

(2) Natural gas turbines using combined-cycle (gas and steam) efficiency = 7000 MMBTU/MWh; natural gas price = US$2.9/MMBTU

The electric power regulatory agency, ANEEL[3] sets some values that have a decisive influence on the attractiveness of investments in power generation:

- The "energy deficit cost" raises the operation cost in "dry" scenarios and also when the system capacity is becoming too tight, signaling the need for system expansion.
- The "normative value", which is the maximum price of forward supply contracts that can be charged to consumers and therefore acts as a cap for those contracts, which provides the largest share of the utilities' revenues.

Besides the spot price volatility, the thermopower generators are also exposed to fuel cost volatility, both due to normal fluctuations in the prices of carbon fuels and also exchange rate volatility between the dollar and Brazil's currency, the *real.* This makes an investment decision even more difficult.

This paper presents a COD model for a simplified system. This model considers the load and fuel price and other economic uncertainties. The results of this model are used to: 1) calculate the investment attractiveness for power generators using real options theory; and 2) assess the regulatory effect on the investment attractiveness.

This paper is organized as follows: Section 2 presents the extended COD model; Section 3 presents the real options approach (Dixit and Pindyck 1994; Trigeorgis 1996) to appraise the attractiveness of investment in power generation, showing the connection with COD model results; Sections 4 and 5 discusses regulation policy, analyzing how the energy deficit cost and normative value affect investment attractiveness.

[3] Agência Nacional de Energia Elétrica - ANEEL: the National Electric Power Agency

2. The Centralized Operation Dispatch Model

To ease the computation and also to focus on the generation dispatch problem, we have adopted a simplified model of the power system where all power generation (hydro and thermo) and the load are concentrated at one point. This model ignores the transmission constraints and losses. Therefore, the generation scheduling aims only to minimize the expected long-term system operation cost for all possible natural states "$z_t = \{v_t, a_t, c_t, d_t\}$" (combinations of reservoir level, water affluence, natural gas price and electricity demand):

$$W_t(z_t) = \mathrm{Min}_{u_t} \; E\left[w_N(z_N).e^{-rN} + \sum_{i=t}^{N-1} w_i(z_i, u_i(z_i)).e^{-ri} \right] \tag{1}$$

Where "$w_i(z_i,u_i(z_i))$" is the system operation cost; "$u_t(z_t) = \{u_0(z_t), u_1(z_t), \ldots, u_{N-1}(z_t)\}$" for each t = 0,1,…, N-1, denotes the water depletion for hydropower generation in a given time period "t" and state of nature "z_t", and "ρ" is the social discount rate.

At each time period "t" and state of the nature "z_t", the system operation cost "$w_t(z_t,u_t(z_t))$" is approximately equal to the thermopower generation cost, since the direct cost of hydropower generation is negligible (≈ US$2/MWh).

Thermopower generation complements hydropower generation to meet current demand. If the electricity demand exceeds the available generation, the excess demand is supplied by a fictitious power supply, to balance the generation–demand equation. The marginal cost of this fictitious supply is very high, corresponding to the energy deficit cost.

Therefore, the system operation cost "$w_t(z_t,u_t(z_t))$" in each time period "t" and state of the nature "z_t" can be expressed as:

$$w_t(z_t,u_t(z_t)) = c_t\, g_t(z_t) + \beta.f_t(z_t) \tag{2}$$

Where "c_t" is marginal cost of the thermopower generation; "$g_t(z_t)$" is the thermopower generation, "$f_t(z_t)$" is the energy deficit and "β" is the energy deficit penalty.

The required generation is set by the generation–load balance constraint:

$$d_t = \eta\, u_t(z_t) + g_t(z_t) + f_t(z_t) \qquad (3)$$

Where "d_t" is the electricity demand in each time period, "$u_t(z_t)$" is the hydropower generation (stored water usage), "η" is the hydroelectric generation efficiency, "$g_t(z_t)$" is the thermopower generation and "$f_t(z_t)$" is the energy (generation) deficit.

The marginal cost of this generation-demand balance constraint is the electricity price "ω_d" on the spot market[4]:

$$\omega_d(t, z_t) \equiv \partial W_t(z_t) / \partial d_t \mid z, t \qquad (4)$$

The stored water availability for hydropower generation is set by the water storage balance constraint, i.e. inflow + previous storage = outflow (hydropower generation + spillage) + remaining storage:

$$\Delta v_t(z_t) = (a_t - u_t(z_t) - s_t(z_t)) \qquad (5)$$

Where "$\Delta v_t(z_t)$" is the water storage increment in each time period, "a_t" is the affluence, and "$s_t(z_t)$" is the reservoir spillage.

The hydroelectric generation conversion is a function of the reservoir level (in turn a function of the stored water quantity), which is highly non-linear. To simplify the calculations, the affluence "a_t", output "$u_t(z_t)$" and spilled "$s_t(z_t)$" water volume were previously converted into

[4] The final price ($/MWh) on the spot market includes some other components to pay for ancillary services and taxes.

their energy equivalents, supposing that the reservoir is currently at mid-storage level (approx. 65% of its maximum height).

The water value "ω_a" is defined as the marginal cost of this water storage balance constraint, i.e., the decrement on system operation cost due to an increase in the affluence:

$$\omega_a(t, z_t) \equiv \partial W_t(z_t) / \partial a_t \mid z, t \tag{6}$$

Hydropower and thermopower generation are subject to the respective capacities (U and G), and the water storage is subject to the reservoir capacity (V):

$$0 \leq u_t(z_t) \leq U \qquad 0 \leq g_t(z_t) \leq G \qquad 0 \leq v_t(z_t) \leq V \tag{7}$$

Although the direct cost of hydropower generation is negligible, the finite availability of stored water imposes an opportunity cost on water usage (depletion). As our stochastic model is Markovian, we can apply the Bellman recursive method (stochastic dynamic programming - SDP) to solve it. The problem of minimizing the long-term system operation cost can be decomposed into a series of short-term (single period) problems subjected to the same constraints:

$$W_t(z_t) = \mathrm{Min}_{u_t(z_t)} \{ w_t(z_t,u_t(z_t)) + E\,[W_{t+1}(z_{t+1})\, e^{-\rho\,\Delta t}] \} \quad t = N\text{-}1,....0 \tag{8a}$$

$$W_N(z_N) = w_N(z_N) \tag{8b}$$

Where "$E[W_{t+1}(z_{t+1})]$" is the expected system operation cost expressed as a function of the current-period hydropower generation ("hydropower generation opportunity cost"). The expectation is taken over the transition conditional probabilities from each possible current state to the next period state.

The hypothesis of Markovian dynamics of the system states is true only if the system (generation) configuration expansion is predefined. This premise means that the investors do not react to the price evolution over time and the system is expanded according to a predefined plan, whatever the energy price is.

The SDP solution also requires the system cost definition at the final time period "T" (planning horizon, set to 10 years). This value is set to zero and the planning horizon is extended longer to lessen the effect of this boundary condition in the solution.

To have a more realistic model for long-term expansion planning, we have considered the uncertainties in the power demand forecast and in natural gas price, besides the uncertainty in water affluence, as the load and gas price are correlated to the macro-economical variables gross national product growth and world economic growth. The stochastic process models for water affluence "a_t", demand "d_t" and thermopower generation operative cost "c_t" are described in the Appendix.

The combination of these three uncertainties would lead to a huge number of cases. To curb the computation complexity, we have grouped them into the following models:

1. Standard Model (Std): considers only the water affluence uncertainty
2. Load Model (Load): considers the demand (load) uncertainty and the water affluence uncertainty
3. Thermopower Cost Model (Fuel): considers the thermopower generation cost (natural gas price) uncertainty and the water affluence uncertainty

3. Investment Attractiveness

The power utility's revenue is partly earned on long-term forward contracts, which set a price "P" and an energy amount "q^c". The difference between the promised and the measured energy "g^d" (the thermopower generation "$g_t(z_t)$"), is settled on the "spot" market, at its current price "S" ("spot market adjustment"), where S is the spot price "ω_d" given in equation 4.

The thermopower utility's net revenue "R_g" is the balance of this income minus the operating costs, which is the product of production "g^d" and operating cost "c":

$$R_g = P.q^c + (g^d - q^c).S - g^d.c \quad (9)$$

Thermopower plants driven by turbines have a limited operational range, typically from 70 to 100% of rated capacity. To allow for this limitation, the thermopower utility may declare a fraction $(1 - q^f)$ of its capacity as "inflexible", which is always dispatched as if it had no cost. Therefore, the thermopower plant total generation "g^d" is equal to the dispatch of the inflexible and flexible portions:

$$g^d = (1 - q^f)\ Th + q^f.g/G \quad (10)$$

where "Th" is the rated capacity of the thermopower plant, "G" is the flexible fraction of the thermopower plant capacity and "g" is the dispatched portion of "G".

Thus the thermopower plant's net revenue can be expressed as:

$$R_g = P.q^c + ((1 - q^f) + q^f.g/G - q^c).S - g^d.c \quad (11)$$

For a hydropower plant, the net revenue "R_u" is equal to its income, since there are no operating (fuel) costs:

$$R_u = P.q^c + (u^d - q^c).S \quad (12)$$

where u^d is the dispatched hydropower.

Table 2 shows the power utility's net revenue as a function of its contracted capacity "q^c" and its flexibility "q^f".

Table 2 - Utility's Net Revenue (Limit Cases)

q^c	q^f	Thermopower	Hydropower
1	0	P – c	-
1	1	P – S + (S - c).g/G	P – S.(1-u^d)
0	0	S – c	-
0	1	(S - c).g/G	S.u^d
g^d, u^{d} (1)	1	(P - c).g/G	P.u^d

(1) This is a theoretical case where the generators are exposed only to dispatch uncertainty.

The basic condition to commit an investment is that its net present value NPV, defined as $V_t(z_t)$, i.e., its expected discounted cash flow, be positive:

$$V_t(z_t) = \{\Sigma_t\ E[R_t(z_t)]\ /(1 + r)^t\} – I_t \geq 0 \tag{13}$$

where "$E\{R_t(z_t)\}$" is the expected value of future net revenue "R" (t periods ahead), "r" is the risk adjusted interest rate, "z_t" is defined in the sections above, i.e, all possible states of nature "$z_t = \{v_t, a_t, c_t, d_t\}$" (combinations of reservoir level, water affluence, natural gas price and electricity demand), and "I_t" is the investment sunk cost. It should be emphasized that the net revenue is calculated from the COD model results {c, S, g, u} as shown in Table 2.

However, a positive NPV does not guarantee the immediate exercise of the investment option, as in most cases the investment may be postponed if the future scenarios are too uncertain and investors can wait for a better opportunity to commit to the investment. This is where Real Option valuation takes place.

Upon the final period of the planning horizon "T" the investment option is exercised i.e., the investment is committed, if its NPV is positive; otherwise it is not. Therefore at the end of the planning horizon, the investment option value for each scenario "$O_T(z_T)$" is:

$$O_T(z_T)= \max \{ V_T(z_T), 0\} \tag{14}$$

where $V_T(z_T)$ is the expected net present value of the investment if committed at the end of the planning horizon (investment opportunity time window).

In the previous period (T-1), the investor would make the investment if its $V_{T-1}(z_{T-1})$ is greater than the expected discounted value of the investment option at the last period "$E\{O_T(z_T)\}$":

$$O_{T-1}(z_{T-1}) = \max \{V_{T-1}(z_{T-1}) , E\{O_T(z_T)\}/(1 + r)\} \quad (15)$$

This reasoning is applied recursively going backwards to evaluate the investment option at each time period:

$$O_t(z_t) = \max \{V_t(z_t) , E\{O_{t+1}(z_{t+1})\}/(1 + r)\} \quad (16)$$

The investment attractiveness is assessed by the proportion "$p_t(z_t)$" of scenarios in which the investment would be committed, i.e., the investment option would be exercised:

$$p_t(z_t) = \Sigma_z I_t(z_t) / Z \quad (17)$$

where "$I_t(z_t)$" is the indication of the investment option exercise ($I_t(z_t) = 1$ if the investment is committed in scenario "z" at time "t"; $I_t(z_t) = 0$ otherwise) and "Z" is the number of scenarios considered at time "t".

If $p_t(z_t) = 1$, the investment would be committed for a given time "t" in all states of nature z considered[5]. If $p_0(z_0) = 1$, then the investment would be committed immediately in all considered states of nature "z" at the start of the planning period (t = 0), i.e., the investment would not be delayed nor postponed.

4. Attractiveness of Investments and the Energy Deficit Cost

The energy deficit cost "β" (Section 2) is a critical parameter in the centralized operation dispatch model, as it defines the risk aversion of the system operator to a future energy deficit. As the probability of a deficit increases due to low affluence forecasts (dry season), especially

[5] All scenarios occurred in the last 40 years for the state variables.

if the reservoirs levels are already low, the deficit cost raises the future system cost, making thermopower generation a more economical alternative.

As mentioned before (Section 1), the energy deficit cost is set by the regulatory agency. Its current[6] value is equivalent[7] to US$342/MWh.

Since the chance of a future deficit depends also on future generation capacity, to assess the response of attractiveness of investment in thermopower generation to the energy deficit cost, we have to model expansion of the system. This is done by adjusting a factor "λ" that multiplies the required system expansion to vary the future system supply balance:

$$H^* = H_0 + \lambda (H - H_0) \tag{18a}$$

$$G^* = G_0 + \lambda (G - G_0) \tag{18b}$$

where "H_0" and "G_0" are the current hydro and thermopower supply capacities; "H" and "G" are the planned hydro and thermopower supply capacities (balanced system) and "H*" and "G*" are the alternative future hydro and thermopower supply capacities.

When $\lambda = 1$, the system is balanced, i.e., generation is equal to expected demand, and the energy deficit probability is $\leq 5\%$. If $\lambda < 1.0$ the system capacity is under the required level and contrarily if $\lambda > 1$.

We assessed the attractiveness of investment in hydro and thermopower generation for three alternative expansion factors (λ = 0.9, 1.0 and 1.1) and for three values of energy deficit cost (US$342/MWh, US$684/MWh and US$1368/MWh).

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[6] As of February 2002

[7] Considering the current exchange rate = R$ 2 /US$ 1

Table 3 – Proportion (%) of Immediate Investment

	Thermopower			Hydropower		
Deficit Cost (US$/MWh)	λ = 0.9	λ = 1.0	λ = 1.1	λ = 0.9	λ = 1.0	λ = 1.1
342	23	20	13	15	10	10
684	25	21	18	24	21	18
1368	27	22	19	26	22	19

From the results shown in Table 3 we can draw the following conclusions:

1. The energy deficit cost, as considered in the current COD model, is not effective to attract investments in generation, either thermopower or hydropower.

2. The system expansion factor, which is a function of the investment level, has a strong influence on investment decision – a 10% variation from equilibrium is equivalent to a 100% variation in energy deficit cost.

5. Attractiveness of Investments and the Normative Value

The forward contracts price "P" is limited by the normative value "NV", set by the regulatory agency (ANEEL) as the maximum energy rate that can be charged to consumers. So "NV" has a major influence on the investment decision: a low value would depress the generator's revenue, discouraging investments, while a high value would allow the exercise of market power by the dealers, overburdening the final consumers.

To protect consumers while not impairing investments, the "NV" must be set as the minimum value for which the required investments would be committed in due time for all expected scenarios:

$$NV = \min P_c \text{ such that } p_0(z_0|P^*) = 1 \qquad (19)$$

where "P*" is the forward contract price and "$p_0(z_0|P^*)$" is the proportion of scenarios where the given "P*" would guarantee the required immediate investment.

Therefore, the effect of the normative value "NV" on investments can be observed by inspecting the critical (minimum) forward contract price "P" required for the exercise of the investment option and comparing this to its current value to check how effective it is to attract new investments. The simulations were done setting the energy deficit cost an its value as of January 2001(≈US$342/MWh).

Table 4 shows the critical (minimum) forward contract price "P*" required for immediate investments in thermopower plants for the three types of model uncertainty. The results are shown for a balanced system (λ = 1).

Table 4 - Critical Price "P*" for a Thermopower Plant - (US$/MWh)

q^c	q^f	Revenue Equation	Std	Fuel	Load
1.0	0	P – c	38	53	38
1.0	1	P - S + (S - c).g/G	39	54	46
0.5	1	0.5.(P - S) + (S - c).g/G	54	69	57
g^d	1	(P - c).g/G	66	81	> 100

From them we can see that:

1. The lowest critical price (US$38/MWh), which occurs in the case with fully inflexible operation and fully contracted revenue and taking the affluence as the only economical uncertainty, is roughly equal to the current normative value (US$38.69/MWh). Hence, the current policy does not prompt thermopower producers to deal in the spot market and have flexible operation. These consequences depress the energy market and system efficiency.

2. The highest critical prices occur when the contracted energy is equal to the dispatched generation, showing the revenue uncertainty burden for utility investors.

3. The second lowest critical price (US$39/MWh) is almost equal to the lowest one, and occurs for a fully flexible, fully contracted revenue. The only difference between this case and the one with the lowest critical price is the flexibility level, showing that the utility's

operational flexibility does not affect the critical price, and that long-term contracts effectively hedge the utility's exposure to the spot price.

4. The critical price that is required to exercise the investment option increases with the utility's exposure to the spot market.

5. The uncertainty in natural gas price ("Fuel" model) increases the critical prices. The first line of Table 4 shows an increase of 40% compared to the lowest critical price (US$38/MWh).

6. The uncertainty of demand ("Load" model) increases the critical price for the flexible operation mode. The second line of Table 4 shows an increase of 18% compared to the lowest critical price (US$38/MWh).

Table 5 shows the critical forward energy price "P*" required for an immediate investment in hydropower plants, considering water affluence as the only source of uncertainty and a balanced system (λ=1). The results are presented for several investment cost levels (α), compared to a thermopower critical value P*(Thermo).

Table 5 - Critical Price "P∗" for a Hydropower Plant - (US$/MWH)

q^c	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	P*(Thermo)
1.0	117	141	166	39
0.5	85	133	182	54
u^d	63	94	126	66

These results show that:

1. The hydropower / thermopower investment cost ratio (α) increases with the uncertainty of the revenue, i.e., it increases when the contract level (q^c) decreases and also when other economic uncertainties are considered, as for the "Fuel" model.

2. Considering fully contracted supply ($q^c = 1$), hydropower is not competitive to thermopower, since it is very unrealistic that the investment to build a hydropower station would be smaller than that to build a thermopower plant. For that case, Table 5 shows that a critical price P* for hydropower is more than three times the value required for a thermopower plant with the same capacity.

3. The spot market adjustment can be viewed as a hedge mechanism that benefits thermopower but not hydropower.

6. Conclusions

Bearing in mind the limitations of our simplified COD model, we can draw some interesting conclusions from this study:

1. A regulation policy based only on an increase in the energy deficit cost is not effective to attract the required investments in power generation;

2. A utility totally exposed to the spot market, i.e., without long-term forward contracts, is vulnerable to the system shortage degree, which is sensitive to other investors' actions.

3. The flexible operation of thermopower plants does not increase the critical price, and optimizes the system operation. The revenue uncertainty can be hedged by long-term forward contracts.

4. The normative value must match the critical price required for immediate investment.

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Appendix

The water affluence, natural gas price and electricity demand are stochastic processes, whose parameters can be estimated from historical data. Those processes are presumed uncorrelated and the random error is generated by zero-mean Normal distributions.

1. Water Affluence

Based on long-term historical data series (some collected since 1930), the water affluence series for the main Brazilian hydrological basins is suitably modeled by a stationary periodic autoregressive process PAR (1).

Removing the periodic (seasonal) component, the remaining autoregressive process AR (1) can be also modeled as a mean reverting process:

$$\Delta a_t = \varphi\,(\bar{a}_t - a_{t-1}) + \sigma_a\,\varepsilon_a \qquad \varepsilon_a \sim N(0,1) \tag{A1}$$

where "Δa_t" is the incremental water affluence between consecutive periods, "φ" is the autoregressive correlation coefficient, "$\bar{a}_t$" is the average water affluence; a_{t-1} is water affluence in the previous period, "σ_a" is the standard deviation of the water affluence stochastic process distribution and "ε_a" is a standard normal random variable.

The water affluence was also converted into its energetic equivalent, supposing that the reservoirs were at half filled.

To represent system expansion, the mean affluence level "$\bar{a}_t$" was incremented accordingly, with all other parameters of the affluence stochastic process held constant.

Using the historical series for the main Brazilian basins, we have estimated $\varphi = 0.11$ and $\sigma_a = 87$.

2. **Natural Gas Price**

Most new high output, high performance thermoelectric plants are driven by gas turbines, using natural gas.

Due to a lack of historical data on natural gas in the Brazilian market, we have used historical crude oil prices as a proxy. The following mean reverting process can approximate this commodity price series:

$$\Delta c_t = \eta\ (\bar{c} - c_t) + \sigma_c\, \varepsilon_c \qquad \varepsilon_c \sim N(0,1) \tag{A2}$$

Where "Δc_t" is the increment in natural gas price, η is the long term mean reverting coefficient, "$\bar{c}$" is the long-term mean price; "c_t" is natural gas price in the current period, "σ_c" is the standard deviation of the natural gas price stochastic process distribution and "ε_c" is a standard normal random variable. Using the historical series for crude oil price, we have estimated $\eta = 7.5$ and $\sigma_c = 0.86$.

3. **Electricity Demand**

Electricity demand is related to the economic activity level and to electricity price. However, in Brazil the electricity price elasticity is very low and in most cases can be ignored. So, in Brazil the electricity demand growth is closely related to economic growth, with electricity demand often being used as a proxy for GDP.

Electricity demand has a strong periodic component. Removing that seasonal modulation, the remaining process can be modeled as a random walk with a positive drift.

$$\Delta d_t\ /\ d_t = \mu + \sigma_d\, \varepsilon_d \qquad \varepsilon_d \sim N(0,1) \tag{A3}$$

where “$\Delta d_t/d_t$“ is the electricity demand marginal increment, “μ“ is the demand growth (drift), “σ_d“ is the standard deviation of the electricity demand stochastic process distribution and “ε_d“ is a standard normal random variable.

Using the historical series for electricity demand, we have estimated μ = 6% and σ_d = 3.5%.

The initial value (as of 2000) of the yearly power demand is 330 TWh.