Ownership Structure and Optimal Field Development

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ABSTRACT

The paper presents a model for the valuation of marginal offshore oil fields that are geographically close to an existing major oil production installation. The correct development strategy for these satellite oil fields is given by optimally exercising American real options to develop the fields. The most cost efficient way of developing the fields is to connect the marginal fields, e.g. via a sub-sea development, to the production unit of the main field. A problem arises by the fact that the ownership structures of the satellites are not identical for all fields. Hence, the order and timing of the development of the fields are a question of negotiations between the licensees. A non-efficient outcome of the negotiations reduces the total value of the oil fields.

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Introduction

In some oil production areas all major oil fields are already developed. However, new technology has made it economically viable to develop small satellite fields if these fields exploit free capacity in existing infrastructure. In offshore oil production this has become a very successful way of boosting production and increasing the profitability and life of very expensive existing infrastructure. Problems have arisen from the fact that different licensees own the fields that are geographically closely connected. Often each marginal field often has a number of owners.

The optimal field development is most easily obtained, from an organisation point of view, in the case that one owner controls all the fields in a region. This paper presents a model that, by using real option theory, can help us to get a better understanding of the efficiency loss from different ownership structures. This problem is closely related to a major political question in the Norwegian oil industry at the moment. Norwegian offshore oil fields typically have a number of licensees. One of the major licensees is normally the operator of the field. The development of the field is often carried out by the operator or by one of the other licensees. In most cases the Norwegian State is the major licensee, either via the 100% state owned oil company Statoil, via the 50% owned Norsk Hydro or mainly via the state's direct investment in oil fields. Statoil will shortly be partly privatised. In connection to this it is argued that the Norwegian state should sell off parts of its direct investments. The main argument for this is to rearrange the inefficient ownership structure of the oil field licenses. The model offers a guideline to the potential gains from this potential reorganisation.

Overview of the paper

Section one introduces the model. A fixed number of minor oil fields or satellites are geographically closely situated to an existing infrastructure. The optimal strategy and the value of the options to develop these satellites, by connecting them to the existing infrastructure are derived. The main structure of the model is as follows: the licensees have an American option to invest in a development of each of the minor satellite oil fields. The most efficient way to develop the fields is to connect them to the existing structure of an adjacent main oil field. The satellites cannot be developed at the same time because the installation on the major oil field has a limited production capacity.

That is, as the production level of the main field and already connected and developed satellites is being reduced the likelihood of investment in another marginal field increases. Uncertainty is modelled by a stochastic oil price. The size of reservoirs, the quality of the oil etc is, for simplicity, assumed to be known. The efficiency loss from the game of deciding the order and timing of the development of the satellites is studied.

In section two the problem of a mixed ownership structure is introduced. A stylised, but close to reality, case is presented where an inefficient order of development is chosen. The cost of this chose is calculated. In section three some final comments are made.

Section 1: A model of options to develop oil field satellites

A geographical restricted offshore oil field region consists of one major oil fields and a number, *n*, of minor satellite oil fields. The main oil field has already been developed and is producing oil. The satellite oil fields are not yet developed.

Let the remaining oil reserves of the oil fields at time *t* be given by y_{it} $, i = 1,...n$, where field *1* is the main oil field which already is in production, and field *2* to *n* are satellite fields. The fields are given a number according to the order the fields are developed. The satellite oil fields can technically be developed individually, but it is assumed that the cost of individual development is prohibitively high for a realistic development of the oil price and technical progress. Hence, the oil fields will be developed by connecting the reservoirs to the existing infrastructure of the main oil field. To develop a new field, therefore, means to increase the oil output of the installation of field 1. Let the total oil output from field *1* be given by $y_t = \kappa \sum_{i=1}^{k} y_i$, were *k* is the number of developed oilfields and κ is the rate of production. That is, extraction of oil is assumed to be given by a fixed rate, κ, of the total developed oil reserve y_t.

The oil price is assumed to be stochastic and with incremental changes given by a geometrical Brownian motion

$$\frac{dp_t}{p_t} = \tilde{\mu}dt + \sigma dz_t \tag{1}$$

where $\tilde{\mu}$ and σ are the drift and the standard deviation of the relative change of the oil price p_t [1].

The revenue, at time *t*, from the oil exported from the main installation is partly derived from the extraction of oil from the main oil field and parts are from oil field 2 to field *k*. Hence, the dynamics of the developed oil reserve, between development of new oil field satellites, are given by

$$\frac{dy_t}{y_t} = \kappa dt \tag{2}$$

For simplicity assume that there is no production cost related to the oil infrastructure of the main field. However, there are costs related to developing the satellite oil fields and connecting them to the main oil field. It is reasonable to assume that these costs are high if the already existing oil production is high and low if there is lot of available capacity at the main field. If the already existing production is very high, adding another satellite field will imply an extension of the production and transport unit or marginal costs will increase dramatically as capacity is exploited close to the limit. At very high levels of production the cost of adding another field to the main field is almost prohibitively high. Let the cost of developing and connecting a satellite oil field, *i*, to the major unit, at time *t*, be dependent of the present level of production

$$I_t^i = f_t^i(y_t) \tag{4}$$

where $f_t^i(y_t)$ is an increasing function of y_i.

The value of the oil field region is dependent on the oil price development and an optimal strategy for developing the satellite oil fields regarding timing and order.

Assume that the optimal order is given by $o \in (1,...,j,k,...,n))$. The value of the oil field region, under the certainty equivalent probability measure *Q*, is then in the case that the oil fields are developed in the most efficient order and timing given by the value function $\Phi(x_t)$, where $x_t = p_t \kappa y_t$.

$$\Phi(x_t) = \sup_{\tau} E_t^Q \left[\int_t^{\infty} e^{-r(s-t)} x_s ds - \sum_{i=2}^{n} I^i (y_{\tau_t}) \right] \tag{5}$$

where *r* is the risk free rate of return and the vector $\tau \in (\tau_1, \tau_2, ..., \tau_j, \tau_k, ..., \tau_N)$ is the point of time of development of the satellite oil field in the case that the order of the development of the fields are optimal. That is, τ^* is the optimal time for exercising the options to develop each of the satellite oil fields.

For energy markets futures market often exists and can be used, via no-arbitrage arguments, to establish the market value of risk. However, in the oil market the futures contracts normally are restricted to settlement up to about two years. For applications of this model the perspective is in most cases longer. An equilibrium approach to the pricing of risk is, therefore, applied. In accordance with this approach let λ be the unit price of risk of the stochastic process p_t. By Ito's lemma it follows that

$$\begin{aligned} dx_t &= (\tilde{\mu} - \kappa + \tfrac{1}{2}\sigma^2) x_t dt + \sigma x_t dZ_t = (\tilde{\mu} - \lambda - a + \tfrac{1}{2}\sigma^2) x_t dt + \sigma x_t d\tilde{Z}_t \\ &= \mu x_t dt + \sigma x_t d\tilde{Z}_t \end{aligned} \tag{6}$$

where $\mu = (\tilde{\mu} - \lambda - \kappa + \tfrac{1}{2}\sigma^2)$ is the trend of the process x_t under the certainty equivalent probability measure Q. The state of the system (5), i.e. the vector of time and x_t, is evidently a Markov process. Between each point of time a new field is developed the value function must satisfy the following stochastic differential equation

[1] Discussion of the proper stochastic oil price and the implications of using a geometric process

$$\frac{\partial \Phi_t}{\partial t} + \mu \frac{\partial \Phi_t}{\partial x} + \tfrac{1}{2}\sigma^2 \frac{\partial^2 \Phi_t}{\partial x^2} - e^{-rt} x_t = 0 \tag{7}$$

That is, at any time the change in the value function must be equal to the change in the instantaneous revenue from the oil production. The solution to the differential equation is given by

$$A x^{\beta_1} + B x^{\beta_2} + \frac{x}{r - \mu} \tag{8}$$

where the two roots are given by

$$\beta_k = \frac{-(\tilde{\mu} - \lambda - \kappa) \pm \sqrt{(\tilde{\mu} - \lambda - \kappa) - 2r\sigma^2}}{\sigma^2} \quad \text{for } k = 1, 2 \tag{9}$$

and $\beta_1 = \beta > 1$, $\beta_2 < 0$ and *A* and *B* are constants. The value of the oil fields at time *t*, which is assumed to be after the development of field *j* and before the development of field k, i.e. $\tau_j \le t \le \tau_k$, is given by the risk adjusted present value of expected future cash flow if no further fields are developed plus the value of the option to develop new fields. The value of the option to develop new fields is positive and becomes close to zero in the case that the oil price becomes very low. From this it follows that B must be equal to zero and the value of the option to develop new fields in the future at time *t* is given by $A_k x^{\beta}$, where satellite *k* is the next field development in line, given an optimal order of the developments. The following value matching and condition must hold if development of field *k* is optimal

$$A_k p^{\beta} y_k^{\beta} + \frac{\kappa p y_j}{r - \mu} = A_{k+1} p^{\beta} y_k + \frac{\kappa p y_k}{r - \mu} - f_k(y_j) \tag{10}$$

The first part of the equation is the value of the option to develop satellite *k*. The second part is the value of existing fields if no further investments are made. The first part to the right of the equation is the value of the option to develop field *k + 1*. The second part is the value of the fields if investing in field *k* is the final development. The last term is the development cost of field *k*.

To develop field *k* implies that the oil companies reduce the option values from $A_k p^\beta y_k^\beta$ to $A_{k+1} p^\beta y_k$ but increases the expected net present value of future cash flow from $\frac{\kappa p y_j}{r-\mu}$ to $\frac{\kappa p y_k}{r-\mu}$ if no further investments are made. This change comes at a cost of $f_k(y_j)$, which depends on the present oil production level.

For the strategy to be optimal the high contact condition must hold[2]

$$\beta A_k p^{\beta-1} y_k^\beta + \frac{\kappa y_j}{r-\mu} = \beta A_{k+1} p^{\beta-1} {y_k}^\beta + \frac{\kappa y_k}{r-\mu} \tag{11}$$

Equation (10) and (11) can be used to solve for p^* and A_k given the level of production y_j and y_k, before and after the development of field *k*, and A_{k+1}.

The value of A_{k+1} is dependent on the optimal strategy for developing field *k + 1*. That is, the value can be derived from similar optimality conditions as (10) and (11). To actually solve for A_i, $i = 2,..., j, k,...n$ we start by solving for the least attractive field among the *n* satellites. The value matching condition for field *n* is

$$A_n p^\beta y_n^\beta + \frac{\kappa p y_{n-1}}{r-\mu} = \frac{\kappa p y_n}{r-\mu} - f_n(y_{n-1}) \tag{12}$$

In this equation the second term to the right is missing because after field n is developed no options for further investments remains. The high contact condition is

$$\beta A_n p^{\beta-1} y_n^\beta + \frac{\kappa y_{n-1}}{r-\mu} = \frac{\kappa y_n}{r-\mu} \tag{13}$$

[2] See e.g. Dixit and Pindyck for a discussion of this condition

Equation (12) and (13) give the optimal trigger price of development p_n* and the value of A_n.

$$P_n* = \frac{\beta}{\beta-1} f_n(y_{n-1}) \frac{r-\mu}{\kappa(y_n - y_{n-1})} \tag{14}$$

$$A_n = (\beta-1)^{\beta-1}\beta^{-\beta}\kappa^{\beta}(r-\mu)^{-\beta} f_n(y_{n-1})^{1-\beta}\left(\frac{y_n - y_{n-1}}{y_n}\right)^{\beta} \tag{15}$$

The trigger price, p_n*, changes deterministically given a cut in the cost of development from a reduction in the developed reservoir size y_{n-1}. The size of the new development, $y_n - y_{n-1}$, is assumed fixed. From the last term of (15) it seems to be a more complicated relation between A_n and y_{n-1}, than between p_n* and y_{n-1}. However, by solving for A_n from (15) in the first part of relation (12) it follows that the option value of developing field *n* is given by

$$V_n = A_n p^{\beta} y_{n-1}^{\beta} = (\beta-1)^{\beta-1}\beta^{-\beta}\kappa^{\beta}(r-\mu)^{-\beta} f_n(y_{n-1})^{1-\beta}(y_n - y_{n-1})^{\beta} p^{\beta} \tag{16}$$

From (16) it follows that the option value only depends on the oil price, p_t, for a given level of remaining oil reserves y_{n-1}.

Using equation (10), (11) and (15) we can solve for the optimal trigger oil price for the development of field *n – 1*, $p_{n-1}*$, given the remaining oil reserves before the investment, y_{n-2}. The result, A_{n-1}, of this calculation can then be used to calculate $p_{n-2}*$ and A_{n-2}. By solving the system of equations recursively we get generally

$$P_k* = \frac{\beta}{\beta-1} f_k(y_{k-1}) \frac{r-\mu}{\kappa(y_k - y_{k-1})} \tag{17}$$

$$A_k = (\beta - 1)^{\beta - 1} \beta^{-\beta} \kappa^{\beta} (r - \mu)^{-\beta} y_k^{-\beta} \sum_{i=k}^{n} \left(f_i (y_{i-1})^{1-\beta} (y_i - y_{i-1})^{\beta} \right) \tag{18}$$

From (14) it follows that the optimal trigger oil price $p_k *$ is independent of future investment choices. It only depends on the change in the investment cost of the project *k*. According to equation (15) the value of the option depends on the cost structures of all the remaining developments. The value decreases if the cost is high, i.e. high $f_i(y_{i-1})$ for all $j \geq k$. The value of y_{i-1} at time *t* is given by

$$y_{i-1} = \sum_{l=k}^{j-1} (y_l - y_{l-1}) + y_t \tag{19}$$

Hence, the value of A_n is given by

$$A_n = (\beta - 1)^{\beta - 1} \beta^{-\beta} \kappa^{\beta} (r - \mu)^{-\beta} y_k^{-\beta} \sum_{i=k}^{n} \left(f_i \left(\sum_{l=k}^{j-1} (y_l - y_{l-1}) + y_t \right)^{1-\beta} (y_i - y_{i-1})^{\beta} \right) \tag{20}$$

The option value of developing oil field number *k* is then given by

$$V_{k,t} = (\beta - 1)^{\beta - 1} \kappa^{\beta} \beta^{-\beta} (r - \mu)^{-\beta} \sum_{i=k}^{n} \left(f_i \left(\sum_{l=k}^{j-1} (y_l - y_{l-1}) + y_t \right)^{1-\beta} (y_i - y_{i-1})^{\beta} \right) p_t^{\beta} \tag{21}$$

The value of the option (21) is the sum of the options to develop every satellite. Strictly speaking, to exercise on option does not exclude any other options. The only effect is that the cost of development for the other options, represented by $f_i(y_{i-1})$, is increased.

Section two; The effect of an inefficient exercise order

The order of the development that is given above, $o \in (1, ..., j, k, ..., n))$, is assumed to maximise the option value in (21). That is, the most cost efficient field is developed first. If the size of the reservoirs are identical then $f_k(y_t) \geq f_j(y_t)$ for all $k \geq j$. If the less cost efficient field *k* is developed before j, then the value of the options are reduced. This follows from (21) since $\sum_{l=k}^{j-1}(y_l - y_{l-1}) > \sum_{l=k}^{k-1}(y_l - y_{l-1})$ if $j \geq k$ and $\beta > 1$.

A main reason why the satellites may not be developed in the most efficient way is that different oil companies own the fields or that more than one oil company owns a fields but the ownership structure are not identical for all of the fields. With a mixed ownership structure let the option value of oil company α be given by $V_{k,t}^{\alpha}$

$$V_{k,t} = (\beta - 1)^{\beta-1}\beta^{-\beta}\kappa^{\beta}(r-\mu)^{-\beta}\sum_{i=k}^{n}\left(\lambda_j^{\alpha} f_i\left(\sum_{l=k}^{j-1}(y_l - y_{l-1}) + y_t\right)^{1-\beta}(y_i - y_{i-1})^{\beta} p_t^{\beta}\right) \quad (22)$$

where λ_j^{α} is the owner share of oil company α in oil field *j*. From (22) it follows that oil company α may prefer an early development of field *k* before field *j*, even if fields *k* is less cost efficient, if oil company α has a larger owner share in *k* than in *j*, i.e. if $\lambda_k^{\alpha} > \lambda_j^{\alpha}$.

To exemplify this assume three oil companies, A, B, and C, one main oil filed, and three satellite oil fields, 1, 2, and 3. Every oil company owns a share of each satellite, but the shares are not identical. Table 1 gives the owner shares.

Table 1: Oil companies and owner shares, λ_j^{α}

	Oil company A	*Oil Company B*	*Oil Company C*
Satellite 1	0.6	0.3	0.1
Satellite 2	0.4	0.3	0.3
Satellite 3	0.4	0.2	0.4

Let the investment cost function be given by $f_j(y) = y_t^{\eta_j}$ for *j = 1, 2, 3*. Satellite *1* is assumed to be the most cost efficient development with an investment cost given by

$\eta_1 = 1.5$. Let the cost functions of satellite 2 and 3 be given by $\eta_1 = 1.6$ and $\eta_1 = 1.7$, respectively. Hence, satellite *2* is more cost efficient to develop than satellite *3*. Let the size of each field be identical and equal to 10. Other parameter values are listed in table 2.

Table 2, parameters

Variable	*Value*
p_t	20
y_t	10
$\tilde{\mu}$	0.15
σ	0.3
λ	0.05
κ	0.05
r	0.15
μ	0.005

Given these parameter values the optimal trigger price for development of field 1 is 23.6, the trigger price for field 2 is 29.7 and for field 3 it is 37.4. Table 3 gives the total value of the option of developing the oil field and the value of the options from the point of view of each of the oil companies given the order of development.

Table 3, Value of option to develop the satellites, given order of development

Order	*1, 2, 3*	*2, 1, 3*	*2, 3, 1*	*3, 1, 2*	*3, 2, 1*	*1, 3, 2*
Totalt	**9.37**	9.11	8.88	8.76	8.63	9.27
Oil company A	**5.45**	4.52	4.15	4.38	4.05	5.41
Oil Company B	2.12	**2.31**	2.27	1.91	1.97	2.01
Oil Company C	1.81	2.28	2.47	2.46	**2.62**	1.85

In total, the most preferable order is naturally to start by developing field 1, then field 2 if oil prices rises or the remaining reservoir drops significantly, and finally develop fields 3. The aggregated option value for this strategy is 9.37. Oil company A is evidently in favour of this strategy. A's share of the option value is 5.44. However, B prefers to wait and develop fields 2 before field 1 and then eventually field 3. This alternative, 2, 1, 3, gives B an option value of 2.31, which is 9% higher than in the 1, 2, 3 case. Oil company C prefers to develop field 3 before field 2 and then eventually field 1. C’s share of the option value in the 3, 2, 1 case is 2,62, which is 44% higher than the 1, 2, 3 case.

Which order of development that is chosen depends on the way negotiations are carried out. Side-payments may help securing an efficient development of the fields. However, in many cases the process of reaching an agreement is regulated by law or by contracts. A process that often has been used historically is a majority vote, but if one of the field owner has more than 50% of the owner shares, then at least one of the minority owners has to agree with the major owner. The table below ranks the different orders according to the preferences of the oil companies as given by table 3.

Table 4, Ranking of the development order according to preferences

Preference	*Oil company A*	*Oil Company B*	*Oil Company C*
1	1,2,3	**2,1,3**	3,2,1
2	1,3,2	2,3,1	2,3,1
3	**2,1,3**	1,2,3	3,1,2
4	2,3,1	1,3,2	**2,1,3**
5	3,1,2	3,2,1	1,3,2
6	3,2,1	3,1,2	1,2,3

Oil company A has a 70% owner share in oil field 1. However, oil company B and C will not vote for a development of this oil field. Oil company B prefers field 2 and oil company C prefers field 3. Oil company A is not able to get support for his second alternative either, which is 1, 3, 2. However, oil company B and C have a common second alternative, which is 2, 3, 1. Oil company A will oppose this alternative as it is ranked as low as place four from A’s perspective. B and C controls together a 60% of the owner shares in field 2 and 3. The value of the option in the 2, 3, 1 case is 8.88 compared to 9.37 if the optimal 1, 2, 3 alternative is chosen.

To avoid the 2, 3, 1 alternative oil company A should support company B’s first chose of 2, 1, 3. This gives an option value of 4.52 to A compared to 4.15 in the 2, 3, 1 case. Given the parameter values above, the value of the main oil field without the options to connect the satellites is 68.97. This gives a total value of 78.34 if the optimal development order is chosen. The alternative preferred by the majority of the companies, 2, 1, 3, reduces the value of the oil field cluster to 78.08, which is 0.3% lower. The total option value is reduced by 2.8%.

Section three; Final comment

The paper offers a model that illustrates a major problem related to the ownership structure of many oil fields. Historically, the government granted shares in each oil field to a number of owners. The ownership structure has often ended up very different from oil field to oilfield. With technological development, there are substantial cost savings potential from connecting marginal oil fields, which economically cannot be developed individually, to the existing infrastructure of major oil field. The existing oil infrastructure and new technology create options to develop marginal fields that may be very valuable. However, the complicated ownership structure may imply that options to invest are not efficiently exercised. This is illustrated by this paper. The loss from an inefficient ownership structure may be substantial. This paper, therefore, is an argument for promoting a uniform ownership structure in each oil field region. This process seems to be going on in many areas at the moment, especially in mature oil regions, e.g. the North Sea. Oil companies are swapping ownership shares to focus on restricted regions. Governments are also taking part in this process in order to maximise the social benefit and taxation potential from oil production.

References

Brekke, K. A. and B. Øksendal, 1994, Optimal switching in an economic activity under uncertainty, SIAM Journal of Control and Optimization, 7, 1021-1036

Brennan, M. J & E. S. Schwartz (1985), "Evaluating Natural Resource Investments", *Journal of Business*, 58, 135 – 157

Brennan, M. J. & L. Trigeorgis (2000), “Project Flexibility, Agency, and Competition – New Developments in the Theory and Application of Real Options” *Oxford University Press*

Dixit, A. and R. S. Pindyck, 1994, Investment under Uncertainty, Princeton University Press

Dixit, A., 1989, Entry and Exit Decisions under Uncertainty, Journal of Political Economy, 97, 3, 620-638

Harrison, J. M., T. M. Sellke and A. J. Taylor, 1983, Impulse control of Brownian motion, Mathematics of Operations Research, 8, 454-466

Leslie, K. and M. Michaels (1997), "The real power of real options", *The McKinsey Quarterly*, 3, 5 - 22

Lund, M. (1999), "Real Options in Offshore Oil Field Development Projects", *Statoil, article presented at the International Conference on Real Options, Leiden*

Paddock, J, D. Siegel and J. Smith, 1988, Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases, Quarterly Journal of Economics, 103, 479-508

Schwartz, E., 1997, The Stochastic Behaviour of Commodity Prices: Implications for Valuation and Hedging, Journal of Finance, LII, no.3, 923-973

Trigeorgis, L. (1996), *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, Cambridge, MA: The MIT Press