Analysis of Investment Opportunities for Network Expansion Projects: A Real Options Approach*

Work in Progress

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May, 2001

Abstract

This paper provides a conceptual framework to analyze investment decisions for network expansion problems. Specifically, we develop an algorithm to find the optimal time to open a new segment in an already existing network. Segment demand is uncertain and capital investments are high; hence, the investment decision is non-trivial.

Due to the network environment, the decision to open a new segment cannot be analyzed as an independent one: the network externalities that arise both in the price and cost functions influence the project value and the optimal investment policies. Furthermore, the inclusion of a new segment also increases the value of the network as a whole, which augments the benefits of expansion. Finally, future growth should also be taken into account in the expansion model; that is, the option to open further segments branching out from the current one not only adds strategic value to the segment itself, but will in general also generate new network effects.

The model starts by quantifying the effects that network structures have on segment economics, including both price and cost. For a given network, a real options approach is then taken to derive expressions for the optimal time to add a segment, the option value of the expansion opportunity, and the sensitivity of these results to the key parameters of the analysis. We show that, for positive network externalities, an increase in the network size both raises the option value and lowers the demand level at which it is optimal to add the segment. Future growth options are then incorporated by expanding the analysis to a sequential capacity expansion framework with different underlying stochastic processes for each new segment.

*I would like to thank Blake Johnson for his useful comments and guidance through the writing of this paper.

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1 Introduction

One of the most strategic decisions in network industries is that of determining the value and optimal timing of expanding their existing network through a new segment. Industry examples of these decision processes include airline routing expansions to new markets, investments in utility distribution infrastructure, and expansion of data networks. While the nature of these industries might seem quite different from each other, the basic elements in the investment decision are common to all: Customer demand to fulfill capacity is uncertain throughout time, and the capital investment required is high; hence, the decision to invest is non-trivial. Throughout this article we present a conceptual framework to analyze this kind of investment decision within network environments; as such, we focus on the investment aspect of the problem and leave aside the network flow optimization issues, for which efficient algorithms have already been developed. While most of the analysis is framed at a general conceptual level, we use airline routing examples to clarify some concepts and help the reader gain more insight to the problem.

Specifically, we assume the network owner has already identified a certain segment as a potentially good investment opportunity, and that the opportunity is proprietary. The capital expenditures required are the infrastructure costs needed to open a new segment for some fixed capacity. Under uncertain demand levels, the question is then whether the expansion project is a good investment, and if so, what is the optimal time to undertake it.

While the scenario described so far is common to many investment decision processes, the presence of a network environment plays a critical role: In such an environment, the decision of opening a segment cannot be analyzed as an individual one, since networks exhibit consumption and production externalities. Economides (1996) defines a positive consumption externality as an effect that causes the value of a unit of a certain good to increase with the expected number of units to be sold. For our case, we can restate it as an effect that causes the customer to be willing to pay a higher price to use a segment in a larger network than for the same segment in a smaller one. Also, due to the presence of economies of scale, networks might exhibit positive production externalities: an effect that causes the network owner to pay a lower cost to operate a segment in a larger network than for the same segment in a smaller one. In the following section we explore and describe in more detail the sources of these externalities.

We start building the investment decision model by understanding the effects that network structures have on segment economics. In Section 2, we define profit function formulas to include general pricing and cost functions, as well as the already mentioned network effects. In Section 3, the decision to open a new segment is modeled using a stochastic dynamic programming approach, following Dixit and Pindyck (1994); we develop expressions for the project value for a certain network size, and the optimal investment policies for the segment. Section 4 explores the changes in these policies that result from a network size increase, and provides analytic results for the increase of the network value as a result of expansion. Finally, Section 5 is devoted to the analysis of future expansions; that is, once the new segment is opened, the firm inherits the option to open further segments branching out from the current one — expansion that can be thought of as compound options embedded in the first one.

2 Investment Elements

In this section we lay out the basic profit elements that drive the investment value function. We examine the form of the demand function, the profit elements, and investment costs that affect the financial decision to open a new segment. These functions are then modified to include the network effects mentioned previously.

2.1 Demand Process

The segment demand, as mentioned earlier, is the main source of uncertainty. It is assumed that it follows a stochastic process whose mean grows linearly in time. As such, we model it as a Geometric Brownian Motion:

$$dx_t = \alpha x_t dx_t + \sigma x_t dz, \tag{1}$$

where x_t represents the customer demand for the segment at time t; α and σ are the corresponding mean and standard deviation for the stochastic process.

2.2 Profit Function

To get the desired profit expression, we start by defining the *basic* unit price function $\dot{p}(x_t)$. From a basic microeconomic perspective, we model price as a function of demand, and for generality, we assume that it can take any polynomial form on x_t. This gives us the flexibility to include increasing, constant, or decreasing returns to scale, depending on the specific nature of the business. That is,

$$\dot{p}(x_t) = \sum_{k=0}^{m} \dot{p}_k x^k, \tag{2}$$

where m represents the order of the polynomial, and $\dot{p}_k$ $k = 0, 1, \ldots, m$ are deterministic constants.

This *basic* price function then has to be modified to include network externalities. As mentioned in the introduction, the presence of a network environment affects the pricing of each of the segments in the system. The pricing of a segment within a larger network can be different to that in a smaller one due to one or more of the following factors:

1. **Connectivity:** In a larger network, customers (or the network traffic owner) will have access to more segments. In addition, they can use more than one routing path to reach their destiny in the network; this might have advantages in scheduling, timing and user-preference. For airline routing, it can be interpreted as the set of different flight combinations and schedules that a passenger can take to get to a certain destination.

2. **Brand Recognition:** A larger network will generate greater awareness and brand recognition, which is viewed as added value from the customer's perspective. Also, a network with strong presence in a region or network node could generate economies of scale that allow the network owner to provide better customer service or technical support compared to that of a smaller network. We can say then that a larger network will *attract* more customers than a smaller network would.

3. **Switching Costs:** A larger network makes it more likely that a customer can use the same network in the future, since more segments are available. Customers can then use the same network for different segment needs at different times. By doing this, they benefit from the switching costs or technology incompatibilities that they would have incurred otherwise when switching to a different network. These costs might include the learning process of the network technology or operation, or even the acquisition of new equipment to use the network. Also, because of the same network availability, the reward that customers get from customer loyalty programs - such as frequent flyer programs in airlines - might be larger in a larger network than in a smaller one. We can say then that a larger network will *retain* more customers than a smaller network would.

These effects are captured in a single network effects function $pe(N)$, which is a function of the network size N. Following Economides and Himmelberg (1995), the network effects function is increasing for a small network of size N if either one of the two conditions hold: (i) the network effect of a network of zero size is zero, or (ii) there are immediate and large profit benefits to network expansions for small networks. We cannot expect however to have an increasing function as $N \to \infty$. While the network effects might be positive as the network grows in size, it must be that after a certain large size N, the marginal effects of an additional segment are less significant. To represent this behavior, we model $pe(N)$ as some type of convex function; then:

$$\frac{\partial pe(N)}{\partial N} > 0 \quad \text{and} \quad \frac{\partial^2 pe(N)}{\partial N^2} < 0.$$

The *modified* price function then, is a combination of the *basic* price and the network effect, and as such, is a function of both demand and network size:

$$p(x_t, N) = \sum_{k=0}^{m} \dot{p}_k x^k + pe(N). \tag{3}$$

Note that the two functions that comprise the price expression are separable, which is common for "indirect" networks. Indirect networks are those in which the number of customers in the network cannot be directly identified with the network size [Economides (1996)], and hence one variable cannot be expressed in terms of the other; as a counter-example, telephony networks are of the direct type, since as the number of customers grows, the links between them grow at an exponential rate. As we will see, the separability of functions will actually be an advantage through the mathematical development of this paper.

Following the development of the *basic* price function, the total variable costs of operating the segment are represented by $c(x_t)$, again, a polynomial expression dependent on demand:

$$c(x_t) = \sum_{k=0}^{l} c_k x^k, \tag{4}$$

where l represents the order of the polynomial, and c_k $k = 0, 1, \ldots, l$ are deterministic constants. For modeling purposes, the fixed cost component will be included later.

Combining the price and cost functions, the profit function is

$$\pi_t(x_t, N) = x_t p(x_t, N) - c(x_t). \tag{5}$$

2.3 Investment Costs

Investment cost $I(N)$ is a function that represents the initial outlay necessary to open the segment; it has three main components: The first one is the capital costs of opening the segment, including both the cost of the segment link, and the node to which it connects. In the case of the airline industry, this is both the aircraft allocation expense, and the investment at the airports the segment connects.

The second element in the investment function is the discounted value of the fixed costs associated with the segment throughout its service time. For most networks, the fixed costs present economies of scale — that is, every time a new segment is launched, the marginal cost of doing so is lower; this can be viewed as network effects on the cost side. For increasing returns to scale, $\frac{\partial I(N)}{\partial N} < 0$.

The third element of investment cost is the lost profit from demand already flowing through the network that will be re-directed through the new segment. The purpose of this is twofold: from a model perspective, it avoids double counting; and from a strategy perspective, it places a penalty to avoid cannibalization within segments; although we realize this is a simplistic approach to network flows, it has proven to be a good heuristic method, and is widely used in airline expansion analyses.

A final point on network externalities: Depending on the nature of the network, it might be that actually these externalities appear locally rather than globally in the network; that is, it might be that network effects are a result of a subset of N, rather than N as a whole; as we will see, however, this has little influence in the model as long as the profit elements are correctly described.

3 Investment Policies

Once the investment elements have been described and quantified, it is possible to derive expressions for the segment value and its optimal investment policy. By investment policy we refer to the optimal time to realize the network expansion, at which the project value is maximized. The problem can be framed with a real options approach, in which the network owner has the right, but not the obligation to open the prospective segment at each period of time, by paying the associated investment costs. Hence, the owner can decide whether to invest at each point in time or wait for the next period. Since demand is uncertain, the goal is to find the timing to invest that maximizes the value of the expansion opportunity. For a good literature review on project valuation using real options, refer to McDonald and Siegel(1989), Dixit and Pindyck (1994) and Trigeorgis (1998).

3.1 Value function

The value of the segment is a function of demand. Since the latter is characterized by a stochastic process, the expression for the project's value is given by

$$V(x_t, N) = \pi_t(x_t, N)dt + (1 + \rho dt)^{-1}\mathcal{E}[V_t(x_{t+dt}, N|x_t, N)], \tag{6}$$

where the first term represents the instantaneous cashflows at time t, and the second, the project appreciation in the next interval dt. Here, ρ is the discount rate, to be chosen exogenously. Note also that it is assumed the segment is operated in the network for an infinite period of time.

Using Ito's Lemma, and taking expectations, we find

$$\rho V(x_t, N)dt = \pi(x_t, N) + \frac{\partial V}{\partial x_t}\alpha x_t dt + \frac{1}{2}\frac{\partial^2 V}{\partial x_t^2}\sigma^2 x_t^2 dt. \tag{7}$$

Now, assume we have limited production capacity Q; this equation is split into three cases: One for the case where $x_t = 0$ (trivial case), one for partially met demand —$0 < x_t \leq Q$—, and one for the case where demand is above capacity $x_t > Q$ (where the option is already deep in the money). Equation (7) becomes three differential equations governing the project value $V(x_t, N)$:

$$\frac{1}{2}\sigma^2 x_t^2 \frac{\partial^2 V}{\partial x_t^2} + \alpha x_t \frac{\partial V}{\partial x_t} - \rho V = 0 \qquad x_t = 0 \tag{8}$$

$$\frac{1}{2}\sigma^2 x_t^2 \frac{\partial^2 V}{\partial x_t^2} + \alpha x_t \frac{\partial V}{\partial x_t} - \rho V + [x_t p(x_t, N) - c(x_t)] = 0 \qquad 0 < x_t \leq Q \tag{9}$$

$$\frac{1}{2}\sigma^2 x_t^2 \frac{\partial^2 V}{\partial x_t^2} + \alpha x_t \frac{\partial V}{\partial x_t} - \rho V + [Qp(x_t, N) - c(Q)] = 0 \qquad x_t > Q. \tag{10}$$

The general solution for (8) is given by

$$V(x_t, N) = A_1 x^{\beta_1} + A_2 x^{\beta_2}, \tag{11}$$

where β_1 and β_2 are the roots to the fundamental equation

$$Q \equiv \frac{1}{2}\sigma^2 \beta_i(\beta_i - 1) + \alpha\beta_i - \rho = 0. \tag{12}$$

Since we would expect α to be smaller than ρ,

$$\beta_1 = \frac{1}{\sigma^2}\left[-\left(\alpha - \frac{\sigma^2}{2}\right) + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2\rho}\right] \qquad > 1$$

$$\beta_2 = \frac{1}{\sigma^2}\left[-\left(\alpha - \frac{\sigma^2}{2}\right) - \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2\rho}\right] \qquad < 0$$

Equations (9) and (10) are non homogeneous versions of (8). The general solution of these are

$$V(x_t, N) = \begin{cases} B_1 x^{\beta_1} + B_2 x^{\beta_2} + B(x_t, N) & \text{if} \quad 0 < x_t \leq Q \\ , C_1 x^{\beta_1} + C_2 x^{\beta_2} + C(x_t, N) & \text{if} \quad x_t > Q. \end{cases} \tag{13}$$

$B(x_t, N)$ and $C(x_t, N)$ are particular solutions to the differential equations, and are given by

$$B(x_t, N) = \sum_{k=0}^{m} \frac{p_k(N) x^{k+1}}{\delta(k+1)} - \sum_{k=0}^{l} \frac{c_k x^k}{\delta(k)}, \tag{14}$$

$$C(x_t, N) = \sum_{k=0}^{m} \frac{p_k(N) Q x^k}{\delta(k)} - \sum_{k=0}^{l} \frac{c_k Q^k}{\delta(0)}. \tag{15}$$

where $\delta(k) = 0.5\sigma^2 k(1-k) - \alpha k + \rho$. The term $\delta(k)$ represents the risk-adjusted discounting rate for the different terms in the profit function; this rate depends on the stochastic process parameters and on the polynomial order of each term with respect to demand x. Then, the deterministic terms — those which implicitly contain x^0— will be discounted simply by ρ; the first-order terms will be discounted by the risk-adjusted rate $\rho - \alpha$, the second-order terms will be discounted by $\sigma^2 - 2\alpha + \rho$, and so forth. Therefore, we can interpret the last term in (13) as the actual perpetuity of the project cashflows, and hence, it is the fundamental component of the value of the project. The first two other terms, on the other hand, represent the value of holding the investment option.

3.2 Boundary Conditions

The value of the constants for the solutions to the differential equations above can be found by using the right boundary conditions. We know that the value of the project should be zero for a demand of zero. Given this it must be that $A_1 = A_2 = B_2 = C_1 = 0$.

We are left only with the task of determining B_1 and C_2. We consider the boundary at which $x_t = Q$. From Dixit (1993), the solution $V(x_t, N)$ must be continuously differentiable across Q. Equating the values and derivatives of the two expressions at $x_t = Q$, we find a system of linear equations. By solving these, we find that the values for the constants B_1 and C_2 are a function of the capacity Q, the network size N, and the stochastic demand process parameters:

$$\begin{aligned} B_1(N) = \ & \tfrac{Q^{1-\beta_1}}{\beta_1-\beta_2} * \left[\sum_{k=0}^{m} p_k(N) Q^k \left(\tfrac{\beta_2-(k+1)}{\delta(k+1)} - \tfrac{\beta_2-k}{\delta(k)}\right)\right] \\ & -\tfrac{Q^{1-\beta_1}}{\beta_1-\beta_2} * \left[\sum_{k=0}^{l} c_k Q^{k-1} \left(\tfrac{(\beta_2-k)}{\delta(k)} - \tfrac{\beta_2}{\delta(0)}\right)\right]. \end{aligned}$$

Similarly,

$$\begin{aligned} C_2(N) = \ & \tfrac{Q^{1-\beta_2}}{\beta_1-\beta_2} * \left[\sum_{k=0}^{m} p_k(N) Q^k \left(\tfrac{\beta_1-(k+1)}{\delta(k+1)} - \tfrac{\beta_1-k}{\delta(k)}\right)\right] \\ & -\tfrac{Q^{1-\beta_2}}{\beta_1-\beta_2} * \left[\sum_{k=0}^{l} c_k Q^{k-1} \left(\tfrac{(\beta_1-k)}{\delta(k)} - \tfrac{\beta_1}{\delta(0)}\right)\right]. \end{aligned}$$

With these constants, the expression for the project value is complete.

To provide insight into the expressions derived, we present the expression for the project value assuming the simplest form the profit function can take; that of constant cost and unit prices. Then,

$$\begin{aligned} p(x_t, N) &= p_0(N) \\ c(x_t) &= c_0 \\ \pi_t(x_t, N) &= [p_0(N)]\, x_t - c_0. \end{aligned}$$

The project value equation for the region where $0 < x_t \le Q$ becomes

$$\begin{aligned} V(x_t, N) =& B_1 x^{\beta_1} + B(x_t, N) \\ =& \frac{Q^{1-\beta_1}}{\beta_1 - \beta_2} \left(\frac{\beta_2 - 1}{\rho - \alpha} - \frac{\beta_2}{\rho} \right) p_0(N) x^{\beta_1} + \left[\frac{p_0(N) x_t}{\rho - \alpha} - \frac{c_0}{\rho} \right]. \end{aligned}$$

3.3 Option Value and Optimal Investment Timing

With the expression for the segment value, we can now calculate the value of the option to invest. At each period of time, the option holder can decide whether to exercise the option or wait until the next period. In a dynamic programming equation, this relationship can be expressed as follows:

$$F(x_t, N) = \max \left\{ V(x_t, N) - I(N), (1 + \rho dt) \, E \left\{ F(x_{t+dt}, N | x_t, N) \right\} \right\}. \tag{16}$$

The first term of the equation represents the value of exercising the option — that is, the segment value minus the investment cost, while the second expression refers to the continuation region. Notice that for the latter, we only have the appreciation of the option value and not any cashflows, since the option does not yield any cashflows until exercised.

In the continuation region

$$F(x_t, N)(1 + \rho dt) = \mathcal{E}[dF] + F(x_t, N). \tag{17}$$

Using Ito's Lemma, we find

$$\frac{1}{2} \frac{\partial^2 F}{\partial x_t^2} \sigma^2 x_t^2 + \frac{\partial F}{\partial x_t} \alpha x_t - \rho F(x_t, N) = 0. \tag{18}$$

The differential equation above has the same form as the one governing the value of the project, and hence, the form of the solution is the same. Namely,

$$F(x_t, N) = D_1 x^{\beta_1} + D_2 x^{\beta_2}, \tag{19}$$

where again β_1 and β_2 are the roots to the fundamental equation in (12).

When x_t is very small, the possibility of growing to the threshold level x^* is very remote, and hence the option should be almost worthless. To ensure $F(0, N) = 0$ as $x_t \to 0$, it must be that $D_2 = 0$.

At the threshold demand level x^*, the stopping and continuation functions should meet, since the investor is indifferent between waiting and investing at that time; to ensure continuity and differentiability:

$$F(x^*, N) = D_1 x^{*\beta_1} = V(x^*, N) - I(N), \quad (20)$$

$$\beta_1 D_1 x^{*(\beta_1 - 1)} = \frac{\partial V(x^*, N)}{\partial x}. \quad (21)$$

We know that the optimal demand threshold that will maximize the investment should be in the range $0 < x_t \leq Q$. By substituting the value function in the equations above and solving for x^*, we can find the optimal timing to expand the network.

For the general profit function, we are left with a polynomial function that can be easily solved for x^* once such functions have been selected:

$$\sum_{k=0}^{m} \frac{(\beta_1 - (k+1))p_k(N)x^{*k+1}}{\delta(k+1)} - \sum_{k=0}^{l} \frac{(\beta_1 - k)c_k x^{*k}}{\delta(k)} - I(N)\beta_1 = 0. \quad (22)$$

As an example, for the case of constant price and cost functions, the polynomial is a simple first order one:

$$\frac{(\beta_1 - 1)p_0(N)x^*}{\rho - \alpha} - \frac{(\beta_1)c_0}{\rho} - I(N)\beta_1 = 0$$

Once the optimal demand trigger is computed, it is straightforward to find the constant D_1. From (20),

$$D_1 = B_1 + (x^*)^{-\beta_1} \left(B(x_t, N) - I(N) \right). \quad (23)$$

Finally, it is just a matter of substituting D_1 in the option expression to find the value of holding the option to expand the network into that specific segment. For the constant price and cost functions, the expression for the option value is given by

$$\begin{aligned} F(x_t, N) &= D_1 x^{\beta_1} \\ &= \left[\frac{Q^{1-\beta_1}}{\beta_1 - \beta_2} \left(\frac{\beta_2 - 1}{\rho - \alpha} - \frac{\beta_2}{\rho} \right) p_0(N) \right] x^{\beta_1} \\ &\quad + \left[x \beta_1^{-1} (\beta_1 - 1)^{1 - \frac{1}{\beta_1}} \left(\frac{p_0(N)}{\rho - \alpha} \right) \left(\frac{c_0}{\rho} + I(N) \right)^{\frac{1}{\beta_1} - 1} \right]^{\beta_1}. \end{aligned}$$

4 Network Value

4.1 Sensitivity to Network Size

In this section, we develop closed form solutions for the changes in the optimal investment policies and project value as the network size increases. Intuition suggests that, for strictly positive network externalities, the change in the project value will be positive as the network size increases. With a more valuable project, and keeping the rest of the parameters constant, this represents a better market opportunity, and hence, the network owner would expand earlier — that is, the demand trigger to enter the market diminishes. At the same time, the option value to expand should increase as well.

In the following, we summarize the comparative statics for the segment value and investment policies with respect to a change in network size. As the reader can verify, the separability of the profit function greatly simplifies the calculations.

- Comparative statics for the segment value:

$$\begin{aligned}\frac{\partial V}{\partial N} &= \frac{\partial B_1}{\partial N} x^{\beta_1} + \frac{\partial pe}{\partial N} \frac{x}{\delta(1)} \\ &= \left[\frac{Q^{1-\beta_1}}{\beta_1 - \beta_2} \frac{\beta_2 \alpha - \rho}{\rho(\rho - \alpha)} x^{\beta_1} + \frac{x}{\rho - \alpha} \right] \frac{\partial pe}{\partial N} > 0. \end{aligned} \tag{24}$$

- Comparative statics for the demand trigger:

$$\begin{aligned}\frac{dx^*}{dN} &= -\left[x \frac{\beta_1 - 1}{d(1)} \frac{\partial p}{\partial N} - \beta_1 \frac{\partial I}{\partial N} \right] \cdot \left[\sum_{k=0}^{m} \frac{(k+1)(\beta_1 - (k+1)) p_k(N) x^{*k}}{\delta(k+1)} \right. \\ &\quad \left. - \left[x \frac{\beta_1 - 1}{d(1)} \frac{\partial p}{\partial N} - \beta_1 \frac{\partial I}{\partial N} \right] \cdot \left[- \sum_{k=0}^{l} \frac{k(\beta_1 - k) c_k {x^*}^{k-1}}{\delta(k)} \right] \right]^{-1} < 0.\end{aligned}$$

- Comparative statics for the expansion option value:

$$\begin{aligned}\frac{\partial F}{\partial N} &= \frac{\partial B_1}{\partial N} + (x^*)^{-\beta_1} \left(\frac{\partial B}{\partial N} - \frac{\partial I}{\partial N} \right) + \frac{d}{dN} \left((x^*)^{-\beta_1} \right) (B(x_t, N) - I(N)) \\ &> 0.\end{aligned}$$

As shown above, the magnitude of the project and option value, and the optimal deployment trigger x^* are a function of the network size, of the parameters that describe the underlying stochastic process, and of the profit function. As stated at the beginning of this section, positive network externalities increase the value of the new segment and of the network itself. Higher value translates into an earlier expansion — compared to that if such effects were not in place.

So far, we have talked only about positive network externalities. As networks increase in size, the network effects might cease or actually become negative; this is the case of congested networks that suffer an efficiency decline with network growth. While it is difficult to draw general conclusions for this general case, the expressions for the segment value and the investment policies, as well as those for the comparative statics still apply, as long as the network effect functions are correctly described.

4.2 System network effects

It is as important as to explore the effects that the network has on a potential segment, as to account for the effects that the new segment has on the existing network. Every time a new segment is added into the network, the network size increases, and therefore, the value of each component of the network increases correspondingly. The value of each will grow according to a formula of the same form of the value V of a segment. Notice however that each segment will have its own stochastic demand process x_t.

Hence, we can view the total value of a segment expansion as the combination of the value of the segment itself, as analyzed above, plus the impact that the addition of this segment will have in the rest of the network. Accordingly, to model the expansion decision correctly, we must also determine the impact of the addition of the segment on the value of the existing network. In the case where the stochastic processes governing the demands for the different network segments are independent, the expression for the exercise region of (16) would read

$$F(x_t, N) = V(x_t, N) - I(N) + \sum_{i=1}^{N-1} [\Delta V_i(x_i, N-1, \cdot) - \Delta I_i(N-1)] \tag{25}$$

In this case, $\Delta V_i(x_i, N-1, \cdot)$ represents the change in value of segment i due to a unit change in the network size, given by (24), with appropriate substitution of each segment's capacity and demand process parameters. $\Delta I_i(N-1)$ represents the change in the investment costs for segment i due to a unit change in the network size; for positive network effects, this change will be negative. Therefore, the first two terms represent the value of the segment to be opened, while the summation term encompasses the change in the value of the system due to the addition of the new segment.

In reality however, we would expect segment demands to be correlated, and in many cases, this correlation could be very strong. The economic growth in a certain region, for example, would affect such segment demands. Given the mathematical complexity of dealing with a potentially large number of correlated stochastic processes, a dynamic programming simulation approach is recommended to find the network expansion policies and values.

5 Future Network Growth

The next element to add into the model is future network growth. That is, by opening the new segment, the firm naturally acquires the option, without obligation, to open further segments branching out from the new segment. In this sense, we can think about future expansions as compound options. Hence, when computing the optimal policies for the segment value, we have to include the option value of future segments. The modeling for this is very similar to earlier work by Benavides and Johnson (1998) for sequential capacity expansions. Potential extensions of this work include determining the optimal sequence of segment expansions from a set of potential segment additions.

6 Conclusions

This article developed an analytical framework to understand and quantify the value that investing in a new network segment would bring, given uncertain demand and the presence of a network effects both in price and cost. Through a real options approach, we found expressions for the optimal timing of such investment, and the sensitivity of such policies to network size. Given positive network externalities, the value of a network expansion increases with the network size, and earlier expansion is optimal relative to the same expansion without network effects. The general and analytic nature of the results make them relevant to a range of network industries, such as the airline, electricity distribution, and data network industries with very little modification.

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