Analyzing a Real Option on a Petroleum Property

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July 1992, Revised 1997, 1999

Data Needed for a Real Options Analysis

There are two important driving variables in a real options analysis: **volatility** (total risk of spot price) and **convenience yield** (expected growth rate of the spot price relative to the cost of capital).

A real option that has not been exercised gives its owner upside potential, while mitigating downside risk. By separating the upside risk from the downside risk, the real option turns risk into something that creates value. As soon as the project is adopted, the owner becomes locked into downside risk. Thus, the desire to avoid downside risk motivates the owner of the option to delay development. The separation of upside risk from downside risk also creates economic value for the owner of the option, over and above the value measured by traditional NPV calculations. Thus the volatility (standard deviation of rates of change) of spot commodity prices is important. The presence of high volatility tends to make delayed development of the project optimal.

On the other hand, early development of the project is optimal to the extent that a dollar today is more valuable than a dollar tomorrow. Let us temporarily assume that there is no risk. Let NPV_t denote the future payoff in year t if the project is developed at time t. The value today (time 0) of the policy of developing at time t is $\frac{\mathrm{NPV}_t}{(1+k)^t}$, where k is the cost of capital. Clearly, if the NPV is growing at a rate faster than the cost of capital, it is optimal to delay the project indefinitely. Generally, the NPV grows at a rate less than the cost of capital, so early development of the project is generally optimal. The strength of this tendency to develop early depends on the relationship between the cost of capital and the forecast of growth in economic value of the benefits from delayed development. This difference is called the convenience yield. In the commodity business, it is a measure of the value of having a spot commodity available today as opposed to a commodity for delivery in the future. Thus, we can measure convenience yield by comparing futures and spot prices for a commodity.

Now we can see that there is a tradeoff between volatility and convenience yield. Volatility pushes for delayed development and convenience yield pushes for early development.

In summary, the useful market data are: current futures and futures options prices for petroleum commodities (especially crude and natural gas) plus gold futures (to infer the riskless rate of interest for commodity traders). A limitation of these data are that they reflect short term horizons, rather than long-term futures. These can be extended to longer terms by using:

1. Long term bond yields and forward interest rates implied by them.

2. The corporate cost of capital.

3. The corporate forecast of the nominal escalation rate of petroleum spot prices.

Calculate Implied Forward Interest Rates from Eurodollar Futures

Eurodollar futures are quoted as the difference between 100 and the annual yield of a 90-day LIBOR deposit. Thus, on August 27, 1997, the September 1997 Eurodollar future is quoted at 94.26. At delivery, it would generate a 90 deposit for the months of October, November and December that pays interest at the rate 100 – 94.26 = 5.74%. Annualized, this yield is

$$(1 + (100 - 94.26)/400)^{(365/90)} - 1 = 5.95\%$$

or

$$\ln(1.0595) = 5.78\%.$$

In this way, we can calculate the the continuously compounded forward interest rates r_t from Eurodollar futures, which are quoted out to about t = 10 years.

The Convenience Yield for Petroleum Contracts

Many commodities, including petroleum, have a *convenience value*, which is sometimes measured as a convenience yield, as discussed above. That is, the spot contract is more valuable than the present value of the futures contract. This is because there is value associated with the convenience of being able to deliver the spot commodity on short notice for emergencies, changing weather, etc. The convenience value is offset by the cost of storage (interest carrying cost, physical storage costs, insurance costs, etc.). Many financial economists treat the convenience value as a yield, or value per unit of commodity value, per unit of time. Thus, the convenience yield is ananalogous to the dividend yield on a stock. Call options should not be exercised early unless the stock pays dividends—the option is exercised to "capture" the dividend. Thus, determining the convenience yield is critical to the exercise or project adoption decision.

One problem here is that convenience yields have a term structure—they vary over term to maturity of the contract. They also seem to vary over time (seasonality) and with spot price. Futures markets will only give short-term convenience yields, but it is long-term convenience yields that are critical to real options. Thus, we have to do the best with the market data we have, while recognizing their limitations.

The formula for convenience yields is a modification of the formula for forward interest rates. The convenience yield merely offsets the interest rate in the futures contract. Thus, for petroleum futures, we have:

$$F_t = e^{\delta_t - r_t} F_{t+1}.$$

where

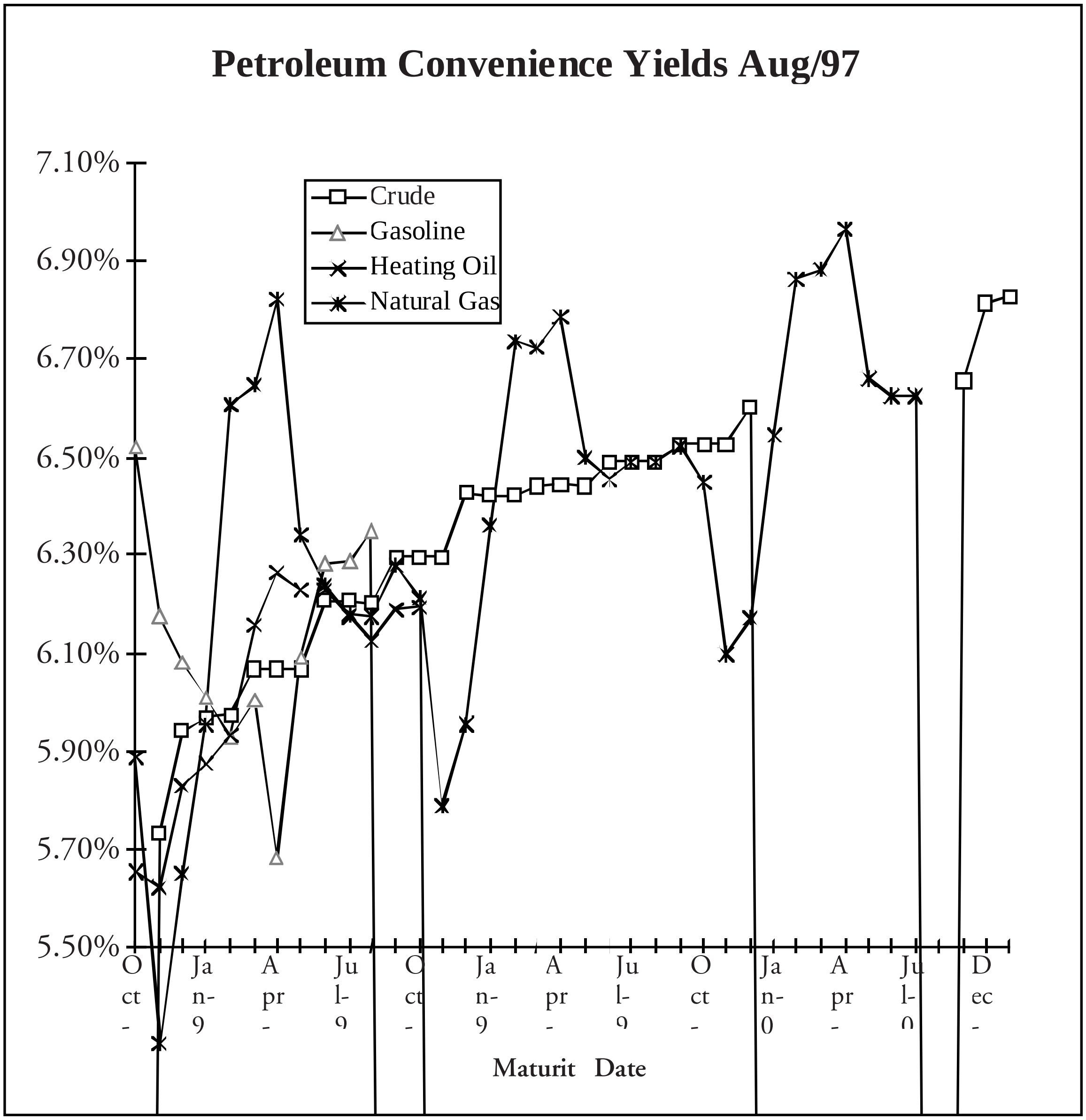
F_t = futures price for delivery at time t.

δ_t = convenience yield (dividend rate) for period t.

Thus, $$\delta_t = r_t - \ln (F_{t+1}/F_t).$$

Quote Date27 Aug 1997

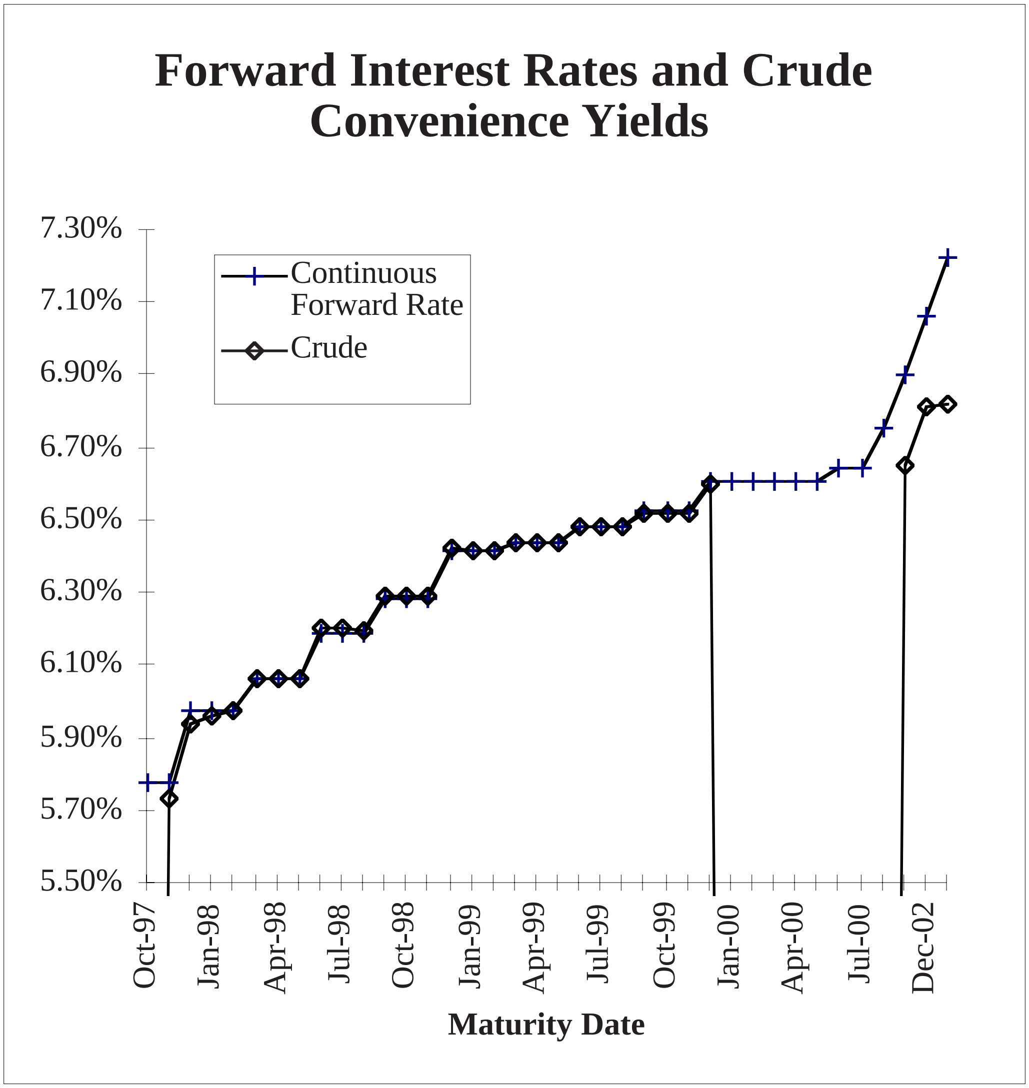
Maturity	Euro-dollar	Maturity	Crude Oil	Gasoline	Heating Oil	Natural Gas
Sep-97	94.26	Sep-97		66.40	0.5326	2.515
Dec-97	94.06	Oct-97	19.73	60.66	0.5408	2.482
Mar-98	93.97	Nov-97	19.83	57.90	0.5508	2.625
Jun-98	93.85	Dec-97	19.92	57.20	0.5608	2.732
Sep-98	93.75	Jan-98	19.95	57.00	0.5678	2.740
Dec-98	93.62	Feb-98	19.96	57.35	0.5708	2.545
Mar-99	93.60	Mar-98	19.96	57.85	0.5643	2.360
Jun-99	93.55	Apr-98	19.96	60.55	0.5513	2.160
Sep-99	93.51	May-98	19.96	60.40	0.5408	2.090
Dec-99	93.43	Jun-98	19.92	59.75	0.5383	2.078
Mar-00	93.43	Jul-98	19.88	59.05	0.5393	2.080
Jun-00	93.39	Aug-98	19.85	57.95	0.5433	2.083
Sep-00	93.35	Sep-98	19.83		0.5498	2.085
Dec-00	93.28	Oct-98	19.81		0.5563	2.105
Mar-01	93.28	Nov-98	19.79			2.233
Jun-01	93.24	Dec-98	19.77			2.363
Sep-01	93.20	Jan-99	19.76			2.380
Dec-01	93.13	Feb-99	19.75			2.293
Mar-02	93.13	Mar-99	19.74			2.210
Jun-02	93.09	Apr-99	19.73			2.121
Sep-02	93.05	May-99	19.73			2.106
Dec-02	92.97	Jun-99	19.73			2.115
Mar-03	92.97	Jul-99	19.73			2.115
Jun-03	92.93	Aug-99	19.73			2.115
Sep-03	92.89	Sep-99	19.74			2.117
Dec-03	92.81	Oct-99	19.75			2.137
Mar-04	92.81	Nov-99	19.76			2.248
Jun-04	92.77	Dec-99	19.77			2.371
Sep-04	92.73	Jan-00				2.389
Dec-04	92.66	Feb-00				2.319
Mar-05	92.66	Mar-00				2.241
Jun-05	92.62	Apr-00				2.148
Sep-05	92.58	May-00				2.134
Dec-05	92.50	Jun-00	19.79			2.140
Mar-06	92.50	Jul-00				2.146
Jun-06	92.46	Dec-00	19.81			
Sep-06	92.42	Dec-01	19.86			
Dec-06	92.35	Dec-02	19.91			
Mar-07	92.35	Dec-03	19.99			
Jun-07	92.32					



Note that natural gas has a large and cyclical convenience yield because of storage limitations. Heating oil and unleaded gasoline are countercyclical to each other, leaving a largely flat convenience yield for upstream crude oil.

Convenience Yields on Crude Oil in Closer Detail

Below, we drop the cyclical natural gas, gasoline and heating oil contracts and focus on crude oil. In this closer detail, there is a secular trend in the crude oil convenience yield. The futures term structure was essentially flat on this date, so the convenience yield equals the forward interest rate and the term structure of interest rates trended upward.



Inferring Convenience Yields from Commodity Price Forecasts

We have suggested the use of futures or forward prices F_t to infer the commodity convenience yields δ_t with the formula:

$$F_t = e^{\delta_t - r_t} F_{t+1}.$$

The analyst may desire to compute the convenience yields implied by her time-0 forecasts $E_0[\tilde{S}_T]$ of time-T future spot prices $\tilde{S}_T$. (The tilde denotes risk.) These forecasts are typically drawn up to facilitate NPV calculations for traditional capital budgeting techniques. (The analyst may want to follow this procedure either because long-term forward prices are not available, or because she wants to achieve consistency with her NPV calculations used to calculate payoffs of exercising the option in the overall option analysis.) However, some care must be taken to adjust for the fact that forward prices and expected future spot prices are not the same thing. A common research agenda for various commodity markets is to test the "unbiased expectations hypothesis" of equality between F_t and $E_0[\tilde{S}_T]$. (Empirically, the *ex ante* expected spot prices are usually proxied by the *ex post* average spot prices.) Often a bias is found between these two items.

The theoretical relationship between forward (or futures) prices F_T and expected future spot prices $E_0[\tilde{S}_T]$, is best understood by noting that these two items are discounted at different rates. Futures and forward prices are certainty equivalents, and are discounted at the riskless discount rates (r_t), while the expected spot price is discounted at the risk-adjusted discount rates or costs of capital (k_t), as in a standard NPV analysis.

Thus, if we wanted to determine the value today of a financial claim to $\tilde{S}_T$ in the future we could discount the expected value at the risk-adjusted discount rates (k_t), just as we would in a traditional NPV calculation. This gives a value today of

$$e^{-k_0}\, e^{-k_1}\, e^{-k_2} \cdots e^{-k_{T-1}} E_0[\tilde{S}_T] = \exp\left(-\Sigma_{t=0}^{T-1}\ k_t\right) E_0[\tilde{S}_T].$$

On the other hand, consider a forward contract. It is a contract to pay a certain amount F_T at time T in the future in exchange for a risky amount $\tilde{S}_T$, that would also be paid at time T in the future. This is the definition of a certainty equivalent. Thus, the value today of the risky payoff $\tilde{S}_T$ can also be determined by discounting its certainty equivalent F_T at the discount rates we use for riskless cash flows, (r_t). This gives a value today of

$$e^{-r_1}\, e^{-r_2}\, e^{-r_3} \cdots e^{-r_{T-1}}\, F_T = \exp\left(-\Sigma_{t=0}^{T-1}\ r_t\right) F_T.$$

Equating these two formulas for the value of a claim to $\tilde{S}_T$ at time T gives:

$$\exp\left(-\Sigma_{t=0}^{T-1}\ k_t\right) E_0[\tilde{S}_T] = \exp\left(-\Sigma_{t=0}^{T-1}\ r_t\right) F_T$$

or the futures-spot relationship:

$$F_T = \exp\left(-\Sigma_{t=0}^{T-1}\ (k_t - r_t)\right) E_0[\tilde{S}_T].$$

The difference between the risk-adjusted discount rate and the riskless rate ($k_t - r_t$) is the risk premium required for bearing the risk of the commodity price. It could be calculated with the capital asset pricing model, for example. (Or one could take k_t as the cost of capital as used in a traditional NPV analysis.) Thus, the forward or futures price is the expected spot price discounted for the risk premiums ($k_t - r_t$), but not for the time value of money (r_t).

With this relationship between futures and expected spot prices, we can use our earlier formula for the convenience yield:

$$\delta_t = r_t - \ln\,(F_{t+1}/F_t)$$

and substitute our the futures/spot relationship to get the convenience yield as:

$$\delta_t = k_t - \ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right).$$

Note that $\ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right)$ is the growth rate in the expected spot price. If the growth rate is expressed in nominal terms, the cost of capital should also be nominal. The same convenience yield comes from using a real cost of capital and a real expected growth rate in spot prices.

Thus, we can calculate the convenience yields from the forecasts of future spot prices for various future points in time with the same formula that we use to calculate the convenience yields from the forward prices, with one additional modification: the risk-adjusted discount rate is used instead of the riskless rate of return. This accounts for the fact that the futures prices are certainty equivalents and have been adjusted for risk.

Examples of Different Convenience Yields

It is useful to consider the implications of this last formula on two traditional NPV models. First, suppose the expected commodity price is constant (in real terms). Then, $E_0[\tilde{S}_t] = E_0[\tilde{S}_{t-1}] = \cdots = E_0[\tilde{S}_0]$. This implies that the convenience yield equals the risk-adjusted discount rate: $\delta_t = k_t$ for t. Certainly this would generate a large enough convenience yield to induce early exercise of many real options. If costs of developing the property are also constant in real terms, then the NPV of developing the property is constant (viewed from the starting date of development). But, since a dollar today is worth more than a dollar later, a positive-NPV resource property should be developed immediately in this situation. The NPV analysis agrees with the options analysis.

On the other hand, it is interesting to see what is implied by a zero convenience yield. Setting $\delta_t = 0$ and rearranging yields $k_t = \ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right)$. That is, a zero convenience yield is equivalent to assuming that spot prices will grow at an average rate that equals the cost of capital, k_t. In option-pricing theory, a call option on an asset that has a zero convenience yield should never be exercised, because a higher and higher discounted value can be obtained by following a policy of later and later exercise. In NPV terms, this means that the PV (as of time 0) of the revenue stream (per unit of production) $e^{-kt} E_0[\tilde{S}_t]$ is constant for all t. That is, delaying exercise does not reduce the value today of the future revenue, because revenue growth matches the discounting factor. On the other hand, the present value of the exercise price or development cost $e^{-kt} K$ declines as the project is delayed. Hence infinite delay is optimal. With a little thought, it is clear that this same result will arise if we simply use traditional NPV software to compare later and later development scenarios.

Of course, if the expected growth rate in the commodity price exceeds the discount rate, the convenience yield is negative, and development clearly should be delayed from an options perspective, as well as an NPV perspective, in order to increase value. Nevertheless, in the late 1970s and early 1980s, many bullish forecasts of oil price increases exceeding the discount rate were accompanied by heavy investment in petroleum projects. This is at variance with optimal behaviour, from both an options perspective and an NPV perspective.

Calculating Implied Variance Rates of Oil Prices

Option value comes from risk or volatility because options separate upside potential from downside risk. There are two ways of estimating the risk of the price of a barrel of oil, or volume of gas. One way is to estimate the standard deviation of commodity prices over time. Another way is to look at short-term traded options to infer the standard deviation from Black-Scholes European option prices. The first method will miss changes in risk because it looks backwards rather than forward. The second method measures short-term risk when the relevant risk for a real option is long-term risk.

Traded Crude Oil Call Option Prices (Oct/92 maturity)

Strike Price	19	20	21	22	23	24	25	26
Call Price	2.96	2.06	1.26	0.66	0.28	0.12	0.06	0.04

Black-Scholes Call Option Prices

This Table assumes no early exercise of short-term options. Interest rates and convenience (dividend yields inferred from futures prices)

	Strike Price							
Sigma	19	20	21	22	23	24	25	26
10%	2.89	1.92	1.05	0.43	0.13	0.03	0.00	0.00
11%	2.89	1.93	1.08	0.48	0.16	0.04	0.01	0.00
12%	2.89	1.95	1.12	0.53	0.20	0.06	0.01	0.00
13%	2.90	1.96	1.16	0.58	0.23	0.08	0.02	0.00
14%	2.90	1.99	1.20	0.62	**0.27**	0.10	0.03	0.01
15%	2.91	2.01	**1.24**	**0.67**	0.31	**0.13**	0.04	0.01
16%	2.92	2.03	**1.28**	0.72	0.36	0.15	**0.06**	0.02
17%	2.94	**2.06**	1.32	0.77	0.40	0.18	0.08	0.03
18%	**2.95**	2.09	1.37	0.81	0.44	0.22	0.10	**0.04**
19%	2.97	2.12	1.41	0.86	0.48	0.25	0.12	0.05

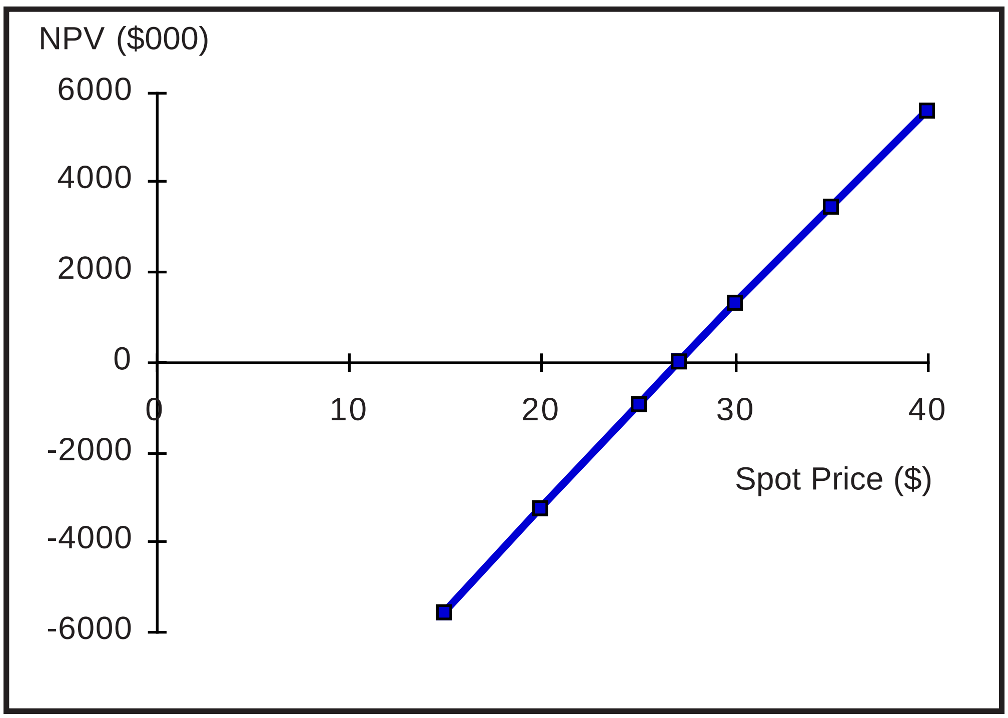
The bold-face options Black-Scholes options prices are those that are the closest fit to the traded option prices. Thus, the evidence implies a short-term standard deviation of price of 14% to 18% on crude oil, each year.

Determining the Payoff to Exercising the Real Petroleum Call Option

The real call option payoff is calculated by running an oil and gas economic evaluation program for an oil reservoir with various spot prices. The reservoir has a 3000 MSTB of oil and produces at 1000 BOPD per day, declining at 20% per annum. Operating costs are initially $5/Bbl. Trucking costs are $2/Bbl and other costs are $500/month. Land costs are $3 million and development costs are $8 million. Federal tax is at 28% and provincial tax is at 10%.

The nominal future spot oil prices and operating costs are expected to rise at the inflation rate of 4% per year. These programs use a risk-adjusted discount rate, so a nominal cost of capital of 10% was used. This is consistent with a convenience yield of 10% – 4% = 6%.

The riskless real rate of interest is taken to be 3%.



The NPV plotted against the spot oil price resembles a call option payoff with an exercise price *K* of approximately $27.35 per barrel. The graph is almost a straight line, with data as shown in the "payoff" section of the spreadsheet below. The slope of the line is approximately 440,000, so that the project is effectively a call option on 440,000 barrels of oil at an exercise price of $27.35/bbl. At a spot price of $26.51, the project has an NPV of approximately –$441,000. To some, this might suggest that the property is worthless. However, it has value as a real option, as seen below.

Using a Binomial Spreadsheet to Analyze the Real Option

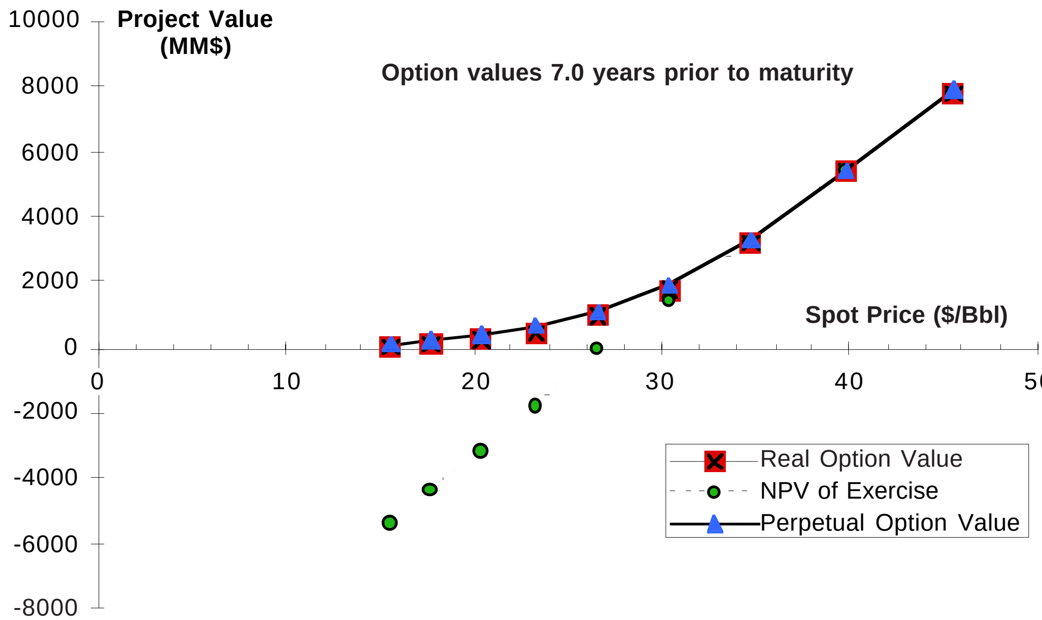
A spreadsheet for a 40-period binomial option lattice model has been developed by the author (realopt7.xls). Analyzing the real option to acquire *one* barrel of oil over a 7-year lease and over a perpetual (freehold) period yields the following results from the spreadsheet:

Input		**Payoff Structure: Net Present Value**		
		Spot Price (Real$/Unit)	**Payoff (MM Real$)**	**Slope**
Underlying asset price, pNought =	***$26.51***			
		$0.00	-$12,474	460.20
Continuously comp. real interest, i =	***3.00%***	***$15.00***	***-$5,571***	460.20
Years to expiry of option, T =	***7***	***$20.00***	***-$3,270***	460.20
Annual volatility s =	***16.00%***	***$25.00***	***-$969***	450.60
Continuously comp. div. yield, delta =	***6.00%***	***$27.15***	***$0***	450.60
Intermediate		***$30.00***	***$1,284***	429.00
This is a real development option.		***$35.00***	***$3,429***	428.40
Break even (exercise price, K) =	**$27.35**	***$40.00***	***$5,571***	408.30
Effective number of units of commodity =	**439.87**	***$45.00***	***$7,613***	408.30
("Reserves" = slope of NPV versus underlying spot price.)		$1,672.06	$671,942	
Output (Project values in MM Real$/Bbl)		***Input variables in Bold Italic***		
American binomial option value =	**$976.91**	**Output in Bold**		
Perpetual American value =	**$1,142.76**			
Black-Scholes European option value =	**$640.77**	Revised by Gordon Sick May 28/99		
Perpetual American hurdle, PStar =	**$36.65**	Printed May 28/99 6:56 PM		

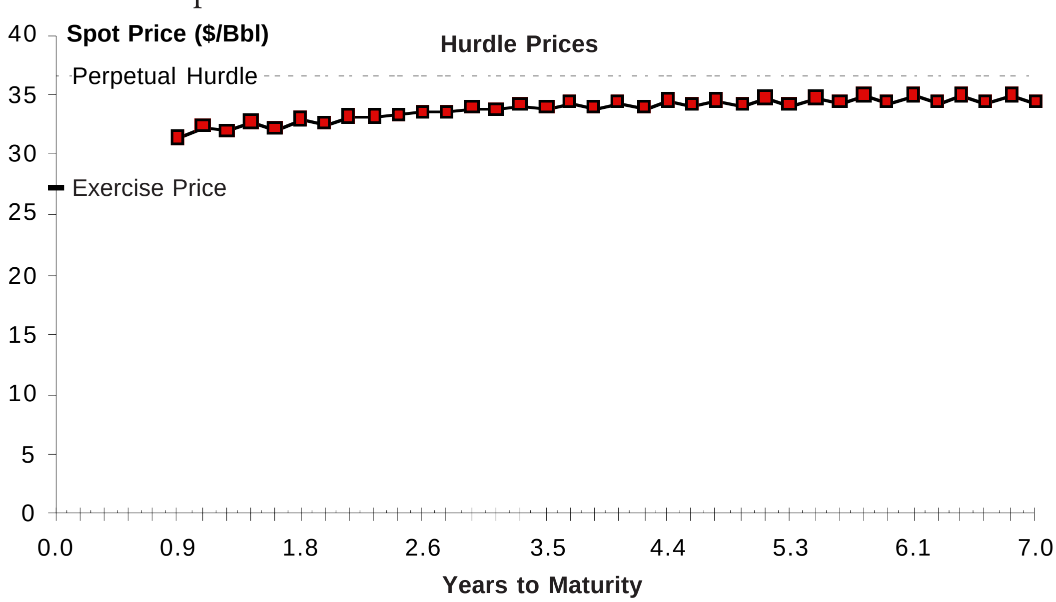
Thus, the undeveloped property, when developed as a real option over a 7-year lease, has a value of $976,910. If the land were owned outright as freehold land, the value of the real option is $1,142,760. In this case, we can see that the amount of value lost because the option is offered as a lease rather than freehold property is almost $200,000. This economic loss is borne by the owner of the lease (residents of a province) and must be weighed against any perceived benefits resulting from the early (but sub-optimal) exercise decisions induced by the lease.

There is a significant probability that prices will rise high enough before the 7-year lease expires that early exercise becomes optimal. The European option value shows the option value when early exercise is not allowed, and it is only $640,770. Thus, we can see that there are substantial probabilities that the property should optimally be developed prior to the maturity of the 7-year lease, as well as after the maturity. Of course, if the constraints on development are in the other direction, and the lease has 0 years to maturity, the value of the real option is the NPV of the develop-or-walkaway decision. Since the NPV at a spot price of $26.51 is almost nil, this value is zero. Thus, recognition of the real option value has turned an otherwise worthless property into one with a value of $976,910.

Values of the option for other spot prices are shown below:



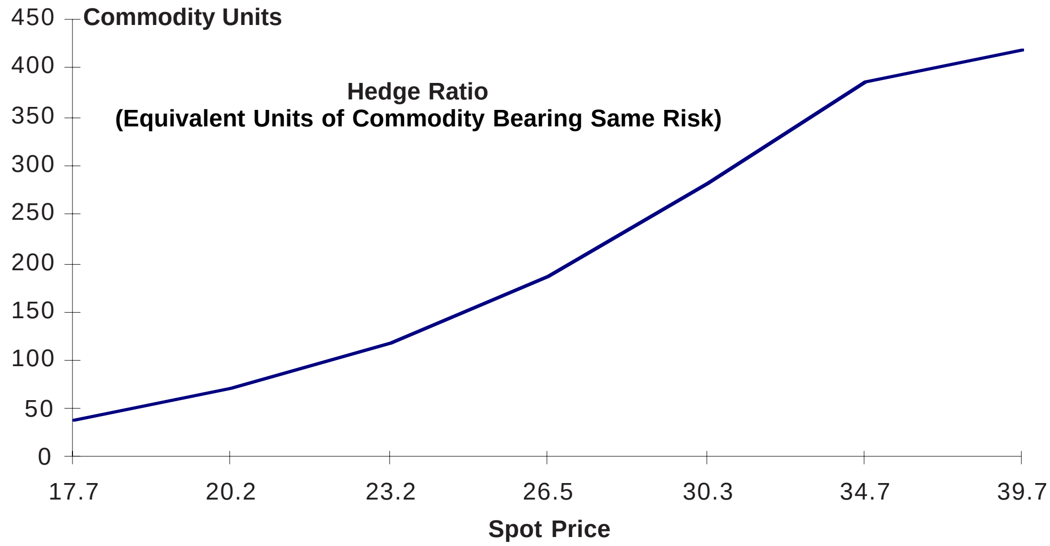
The critical decision variable is the spot price of oil, which must be compared with the hurdle price to make the exercise decision. The perpetual option should be exercised the first time the spot price of oil rises above $36.65. The oil-price hurdles for various intermediate dates in the 7-year lease can be plotted from the spreadsheet as below:



Note how the hurdle price rises as the life of the lease increases. This is how the decision-maker takes advantage of the greater flexibility afforded by the longer-lived option.

The granularity of the hurdle price graph comes from the granularity of the binomial decision tree in the spreadsheet. This has no effect on the accuracy of the computation of option values. The true exercise price is a smooth curve starting at the exercise price and increasing to an asymptote at the perpetual hurdle. The curve lies inside the range of the zig-zag plot of exercise prices shown above. The graph doesn't show the exercise hurdle for short-lived options because the tree is not wide enough to bracket the potential hurdle prices. Adjusting the initial spot price up or down in the Input section will often allow a more complete graph.

It is also useful to see how the real option absorbs risk by lowering the firm's exposure to the price of oil, or its oil-price hedge ratio. The following graph shows how the hedge ratio changes with the underlying price of oil:



From the spreadsheet, adopting the project is equivalent to acquiring a sensitivity to the price changes of 439.87 barrels of oil (this precise number varied from 408 to 460 because the NPV is not linear in oil price.) However, the real option is not as sensitive to oil price changes. The hedge ratio varies from a low of under 50 barrels of oil when the oil price is $17.70 (and the option is deep out of the money) to a high of 450 barrels when the oil price is $39.70 (and the option is in the money).