"PETROLEUM CONCESSIONS WITH EXTENDIBLE OPTIONS USING MEAN REVERSION WITH JUMPS TO MODEL OIL PRICES"

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ABSTRACT

The holder of a petroleum exploration concession has an investment option until the expiration date fixed by the governmental agency, in some countries these rights can be extended by additional cost. The value of these rights and the optimal investment timing are calculated by solving a stochastic optimal control problem of an American call option with extendible maturities. The uncertainty of the oil prices is modeled as a mix jump-diffusion process. Normal information generates continuous mean-reverting process for oil prices, whereas random abnormal information generates discrete jumps of random size. Comparisons are performed with the popular geometric Brownian process and also the quantification and analysis of alternative timing policies for the petroleum sector.

Suggested JEL Classification: G31, G12 and Q39

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1) Introduction

Petroleum firms acting in exploration and production (E&P), routinely need to evaluate concessions and to decide the investment timing for its project portfolio. In some countries, the exploration concession has features such as the possibility of *extension of the exploratory period* if the holder of the concession spend some fee (extra-tax) to governmental agency and/or additional exploratory or appraisal investment. This *extendible option* feature for petroleum E&P concessions is the case of Brazil[1], but also exists in Europe (see Kemna, 1993). The adequate concession timing policy in the exploratory phase is one polemic point in the industry debate, which this paper intends to contribute with some quantification of alternative timing policies under conditions of market uncertainty.

This paper is related to the real options theory (or theory of irreversible investment under uncertainty). As new contribution, we use the framework of options with *extendible maturities*, known before only for financial options[2], not for real assets. In addition, we use a mix stochastic process (mean-reversion with jumps) to model the petroleum prices which, despite of its economic logic, has not been used before in petroleum economic literature.

The model is useful for both, firms facing investment decisions, and government evaluating sectoral policy (mainly timing policy) considering that firms have rational expectations and will act optimally.

2) The Concession and the Stochastic Model for Petroleum Prices

Consider an oilfield, discovered in the concession area, which can be totally or partially delineated. The model presented here identifies the optimal investment rule and the oilfield value.

In the exploratory phase, it is important to consider technical uncertainty (existence, size and quality of petroleum reserves). The integration of this development decision model (market uncertainty oriented) with exploratory factors (technical uncertainty) can be performed easily with simple models[3], but also with more complex models[4]. Development decision models are useful for a good exploratory valuation model: the evaluation of the development project is a necessary step to evaluate the exploratory prospect itself. In the spirit of *dynamic programming*, calculations are performed *backwards*, and for this purpose we need to know the terminal value (option to develop a delineated oilfield) in order to estimate the initial value (exploratory prospect with some success probability).

Oil price, the main source of market uncertainty, is modeled with a special stochastic process: a combined *jump-diffusion* process. This model follows Merton's (1976) concept on asset prices oscillations. The arrival of *normal information* over an infinitesimal time interval generates only marginal adjustment of the prices, which is modeled by a continuous diffusion process, whereas the arrival of *abnormal information* (very important news) generates a discrete stochastic shock (jump), which is modeled as a Poisson process. The model that combines both, the jump-diffusion one is also known as *Poisson-Gaussian* model.

The adopted diffusion process for petroleum prices is the *mean-reversion process* because it is considered the natural choice for commodities[5]. Normal information means smoothly or marginal interaction between production versus demand (inventory levels is an indicator) and depletion versus new reserves discoveries (ratio reserves/production as indicator). Basic microeconomics theory tells that, in the long run, the price of a commodity ought to be tied to its long-run marginal production cost or, "*in case of a cartelized commodity like oil, the long-run profit-maximizing price sought by cartel managers*" (Laughton & Jacoby, 1995, p.188). In other words, although the oil prices have sensible short-term oscillations, it tends to revert back to a "normal" *long-term equilibrium* level. Production cost varies largely across the countries, mainly due to the geologic features, and most of the lower cost countries belong (or are influenced) by the OPEC cartel. Hence, even with a growing non-OPEC production, the OPEC role remains very important in the production game of the petroleum industry.

Other important mean-reverting evidence comes from *futures market*, as pointed out by Baker et al (1998, pp.124-127). First, the *term structure* of futures prices are decreasing (toward the "normal" long-run level, in *backwardation*) if the spot prices are "high", and are increasing (in *contango*) if prices are "low". Second, if the prices are random walk, the *volatility* in the futures prices should equal the volatility of the spot price, but the data show that spot prices are much more volatile than futures price. In both cases, the mean-reverting model is much more consistent with the futures prices data than random walk model. In addition, the econometric tests from futures term structure performed by Bessembinder et al (1995, p.373-374) also reveals strong mean-reversion for oil prices and agricultural commodities (but weak reversion for precious metals and financial assets).

The Poisson-jump[6] can be either positive or negative for petroleum prices, depending of the kind of economic/politic abnormal news. In petroleum history there were abnormal news causing large jumps in petroleum prices, along few weeks. For example: jumps in 1973/1974 (Iom Kipur war and Arabian oil embargo), in 1979/1980 (Iran revolution and Iran-Iraq war), in 1986 (Saudi Arabia price war), in 1990 (Kuwait invasion by Iraq) and in 1991 (the Iraq defeat). At least three large jump-ups and two jump-downs for oil prices can be identified in these events. This feature is incorporated into the model, which allows either direction for the jumps and stochastic size for the jump. We follow Merton (1976), except that he used log-normal distribution for the jumps size instead of the two truncated-normal distribution that we assumed, and he used geometric Brownian (because he models financial assets, not commodities) instead of mean reversion for the continuous process. The mean-reverting+jump model was used before to model interest rate process (see Das, 1998, p.4), but despite its economic logic, was not used before for oil prices.

Let P be the spot price of one oil barrel. Most of times the prices change continuously as a mean-reverting process and sometimes change discretely by jumps. The oil prices follow the equation:

$$\frac{dP}{P} = [\eta\,(\bar{P} - P) - \lambda\, k]\, dt + \sigma\, dz + dq$$

$$dq = \begin{cases} 0, \text{ with probability } 1 - \lambda dt \\ \phi - 1, \text{ with probability } \lambda dt \end{cases}$$

$$k = E(\phi - 1) \qquad \textbf{Eq.(1)}$$

Eq.(1) for the rate of variation of oil prices (dP/P) has three terms in the right side. The first term is the mean-reverting drift: the petroleum price has a tendency to go back to the long-run equilibrium mean $\bar{P}$ with a reversion speed η. The second one presents the continuous time uncertainty represented by volatility σ, where dz is the Wiener increment. The last one is the jump term, with the Poisson arrival parameter λ (there is a probability λ.dt to occur a discrete jump). The jump has random size: ϕ has a special probability distribution with mean k+1, represented by two truncated-normal distributions, one normal distribution for the jump-up and the other one for the jump-down, (see Fig.1 below). In case of jump, this abnormal movement has the same chances to be up or down.

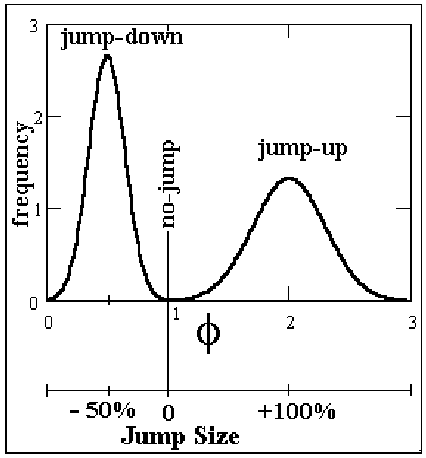


Figure 1 - Random Jumps Distribution

The figure shows that the exact size of each jump is uncertain. The same figure points that, in case of jump-up, the price is expected to double, whereas in case of jump-down the price is expected to drop to the half. We assume large expected jumps but with low frequency, of 0.15 per annum.

By taking expectations in Eq.(1), is easy to see that $\mathbf{E(dP/P) = \eta\ (\bar{P} - P)\ dt}$, that is, the process is expected to revert, and this tendency is higher as far the current price is from the long run mean. This is like an elastic/spring force. In this process is useful the concept of *half-life* of the oil price process, which is a more practical measure for reversion speed (it points the *slowness to revert*). Half-life (H) is the time that is expected the oil price to reach the intermediate value between the current price and the long-run average price, and is given by the following equation (proof: see Appendix A).

$$\mathbf{H = \frac{\ln(2)}{\eta\,\bar{P}}}$$ **Eq.(2)**

The jump-diffusion model has economic logic appeal and is a good mapping for the probability distribution along the time for the oil prices. The model presents complex empirical problems due to the additional parameters estimation, when comparing with a more popular (and simpler) model, the geometric Brownian motion (GBM). However, GBM models are less rigorous than the jump-diffusion stochastic process presented above, and this disadvantage can be important to model long-maturities options like undeveloped reserves. Other important models for oil prices are the two and three factors models, and models with stochastic long run price, that we discuss briefly.

The two-factor model (Gibson & Schwartz, 1990) generally uses GBM for spot oil prices and a stochastic mean-reverting convenience yield δ. This additional factor corrects the main bias from the one-factor GBM model, becoming more consistent with the market data from the futures term structure. The three-factor model is presented in a recent article of Schwartz (1997a, pp.929-931), allowing the interest rate to be the third stochastic factor, also modeled as mean-reverting. The three stochastic processes are correlated, was performed a complex empirical job (using Kalman filter) to estimate several parameters of these processes. He compares it with the one and two-factor models.

Another important class of models allows the equilibrium long-term price level to be stochastic, presented in Baker et al (1998, pp.134-135) and in Schwartz & Smith (1997). Both papers argue that this model is equivalent to the two-factor stochastic convenience yield model. The model has economic logic because it is likely that the equilibrium level changes with the evolution of variables like the marginal cost for price takers producers, the correlation of forces between OPEC and non-OPEC, new environmental regulations, new technologies, etc. The equilibrium price is likely to be positively correlated with spot prices in that model. In our model this equilibrium price is assumed constant, which is reasonable in the context because our stochastic process describes the oil prices only until the exercise of the option, assuming a market value of reserves after the exercise. For models that describe also the cash flow *after* the option exercise, this improvement could be more important.

3) The Timing of Investment and the Optimization Problem

Let the instant $t = T_1$ be the primary (or the first) expiration of the concession option. At this time the owner has three alternatives: to develop the field immediately, to pay a fee (and/or additional exploratory investment) to extend the maturity of the option (looking for better conditions to invest), or to give up the concession, returning the tract to the agency. So the firm has, in addition to the classic option model with the decision for the maximum between NPV and zero, the decision to buy another option paying the fee. Let the instant $t = T_2$ be the second and definitive expiration of the concession option. At this time the firm will choose the maximum between NPV and zero. This second expiration is like the classic option case. We consider that the operating project value W(P), that is, the project value *after* the investment, can be conveniently given by the following equation:

$$\mathbf{W(P) = B \, . \, V(P) = B. \, q \, P} \qquad \textbf{Eq.(3)}$$

Where B is the quantity of barrels of reserves in the ground (the reserve volume) and V is the market value of one barrel of reserve. We assume that this value is proportional to oil prices, which has been used as assumption in real options models[7]. Consequently, V follows the same stochastic process of P. The proportion factor q is, in average, 33% of oil price ("one-third" rule of the thumb), but can be a different proportion for different cases of reserves. This proportion is named *economic quality of a developed reserve*[8], because the higher q implies the higher operational profit from this underlying asset.

The value of q is assumed constant and independent of the price, which could be viewed as one critical assumption for "pure reversion thinking". But we consider with the observed high positive correlation between V and P[9] and the value of q itself can be estimated using the expected oil prices trend from a mean-reverting model or using information from futures market (decreasing the bias) as in Schwartz (1997a, eq.18 or eq.30). In addition, due the effect of depletion and discounting, the operating cash flows from the first 5 years has higher importance in the reserve value than distant cash flows.

Schwartz (1997a, p.971) results give us another important argument, using very different stochastic models that driven heavily on futures markets insights (the two and three-factor models mentioned before). These models imply an underlying *project value that is linear with the spot price* (Schwartz, 1997a, Fig.13), the inclination of his two or three factor NPV is exactly our economic quality q, and hence can be reproduced with our Eq.3. This contrasts with the predictability of the "pure reversion model" that undervalues the project in high spot prices scenario and overvalues the project in the low price case. In practice, the simplification of q constant corrects some bias from "pure reversion"[10].

If we consider the extension of our stochastic process for the time horizon of the operating cash flows (that is, after the exercise of the development option), would be necessary to consider additional features. For a more complete model could be important allow for the operational options (expansion or speed up, temporary stopping, abandon) and, perhaps more relevant, improving our stochastic process by allowing the long run equilibrium price to be stochastic instead constant. These upgrade features are left to a further work, but based in the above reasons, we think that our error is not much important to justify going deeper by now. For example, in the high price case by taking account the option to speed up production (with additional wells or early production systems)[11], we get some offsetting effect over the expected reduction in V due to the expected price reversion. In addition, managers periodically can revise the value of q to be used in Eq.(3).

We shall currently use values *per-barrel* (of course is also possible to work in total values), then afterwards we will use NPV to express net present value per barrel, so:

$$\mathbf{NPV = V(P) - D = q\,P - D} \qquad \textbf{Eq.(4)}$$

Where D is the development investment *per barrel* of reserve.

Even being non-stochastic in our model, the investment value D in the first period (0-T_1) can be different from the second period (T_1-T_2) if we consider some benefit derived from that extension cost. For example, suppose that part of the extension fee (K) is an additional exploratory well. If this well could be used as a development well (as producer or as water injector), the extension investment can be reduced by a certain quantity due to this well use[12]. So we use D_1 for the investment (per barrel) until the first expiration and D_2 for the investment in the extension period ($D_1 \geq D_2$). If the additional exploratory well is a good investment independently of the extension benefit, is possible to consider the traditional option model (instead of extendible options) with a single maturity at T_2 (because the additional exploratory cost will be done anyway).

It is necessary to derive both the value of the concession (the value of the option to invest) F(P, t), and the optimal decision rule thresholds. The decisions are to develop, or to wait, or to extend the option, or even to give up. The solution procedure can be view as a maximization problem under uncertainty. We use the Bellman-dynamic programming framework (see Dixit & Pindyck, 1994, chapter 4) to solve the stochastic optimal control problem. We want to maximize the value of the concession option F(P, t) seeking the instant when the price reach a level P* (the threshold) in which is optimal one type of action (investment or pay to extension). The Bellman equations are:

$$F_1(P,t) \equiv \max_{P_1^*(t)} \begin{Bmatrix} [V(P) - D_1\ , E[F_1(P+dP, t+dt)\, e^{-\rho dt}\ ],\ \text{for all}\ \ t < T_1 \\ [V(P) - D_1, E[F_2(P+dP, t+dt)\, e^{-\rho dt} - K, 0],\ \text{for}\ \ t = T_1 \end{Bmatrix} \qquad \textbf{Eq.(5)}$$

$$F_2(P,t) \equiv \max_{P_2^*(t)} \begin{Bmatrix} [V(P) - D_2\ , E[F_2(P+dP, t+dt)\, e^{-\rho dt}\ ],\ \text{for all}\ \ T_1 < t < T_2 \\ [V(P) - D_2, 0],\ \text{for}\ \ \ t = T_2 \end{Bmatrix} \qquad \textbf{Eq.(6)}$$

where ρ is an exogenous discount rate, that can be a CAPM[13] like risk-adjusted discount rate for the underlying asset if the market is sufficiently complete, or an arbitrary exogenous discount rate in case of incomplete markets. In the first case is also possible to use "risk-neutral" valuation, by using a risk-free interest rate instead ρ, but is necessary to change the drift of the stochastic process[14]. The risk-neutral approach relies in the absence of arbitrage opportunities or dynamically complete markets[15].

Let us consider a more general assumption in the model: the jump-risk is *systematic* (correlated with the market portfolio) so it is not possible to build a riskless portfolio[16], the market is not complete for this model with non-diversified jump risk. Instead using an exogenous discount rate, the alternative for incomplete markets models is a more restrictive assumption, using single-agent optimality framework and/or detailed equilibrium description, as performed in Naik & Lee (1990) for jumps in the market portfolio itself. Bates (1991) uses a risk-neutral approach for jump-diffusion with systematic risk, but via restrictions on preferences. These more complex approaches need to specify the investor utility. In petroleum corporations there are hundreds of thousands of stockholders, with different levels of wealth and so with different utilities. So, a more complex approach trying to specify utility has practical disadvantage, without to be much more rigorous than the adopted dynamic programming framework as Dixit & Pindyck (1994), using an exogenous (e.g. corporate discount rate, capital cost) or a "market-estimated/proxy" discount rate ρ.

We are interested in find out the optimal path $P_1^*(t \leq T_1)$, $P^E(T_1)$ and $P_2^*(T_1 < t \leq T_2)$, as well as the value of concession $F(P,t)$ in each of these periods. Using the Bellman equation and the Itô's Lemma, is possible to build the following partial differential-difference equation (PDE)[17]:

$$\tfrac{1}{2}\,\sigma^2 P^2 F_{PP} + \{\eta(\bar{P} - P) - \lambda\, E[\phi - 1]\}\, P\, F_P + F_t + \lambda\, E[F(P\,\phi, t) - F(P, t)] = \rho\, F \qquad \textbf{Eq.(7)}$$

With the following boundary conditions:

$$F(0, t) = 0 \qquad \textbf{Eq.(8)}$$

$$F_1(P, T_1) = \max\,[V(P) - D_1,\ F_2(P, t) - K,\ 0] \qquad \textbf{Eq.(9)}$$

$$F_i\,(P^*, t) = V(P^*) - D_i\,, \qquad i = 1, 2 \qquad \textbf{Eq.(10)}$$

$$F_2\,(P, T_2) = \max\,[V(P) - D_2,\ 0] \qquad \textbf{Eq.(11)}$$

$$F_P\,(P^*, t) = V_P(P^*) = q \qquad \textbf{Eq.(12)}$$

The Eq.(7) is a PDE of parabolic type and is solved using the numerical method of finite differences in the explicit form (see Appendix B). A C++ program with a graphical interface (see Appendix D) was developed to solve this model and to perform the comparative statics analysis.

The boundary conditions (Eqs. 8 to 12) are typical for an American call option with extendible maturities. Eqs. 8 and 11 are standard for call options, whereas Eqs.10 and 12 addresses the early exercise feature of American options. The second condition (Eq.9) for the first expiration T_1 is the extendible feature condition, and means to choose between the alternatives: to develop, to extend and to give up (respectively in the max. parenthesis). The lowest price at T_1 that we choose to extend the option paying K is the *extension threshold* P^E. The last equation (12), known as "smooth pasting condition" (or "high-contact"), is equivalent to the optimum exercise condition, so alternatively can be performed the earlier exercise test (the maximum between the lived option and the payoff V – D).

Figure 2 shows the extendible option at the first expiration moment ($t = T_1$) identifying the three possible range of petroleum prices associated to different decisions (give up, extend or develop now) at the first expiration. The threshold values are also displayed in the chart. This graph is close to traditional option payoff chart, except for the region between 11.9 and 19.7 US$/bbl, where the optimal action is to extend the option (see the curve with option shape for the interval where is optimal to extend the option). This graph is typical for the geometric Brownian motion (GBM), and the shape is similar to the presented in the mentioned paper of Longstaff (1990, Fig.1, p.939). For the jump-diffusion case we will get a similar chart, with some difference in the shape of the option curve.

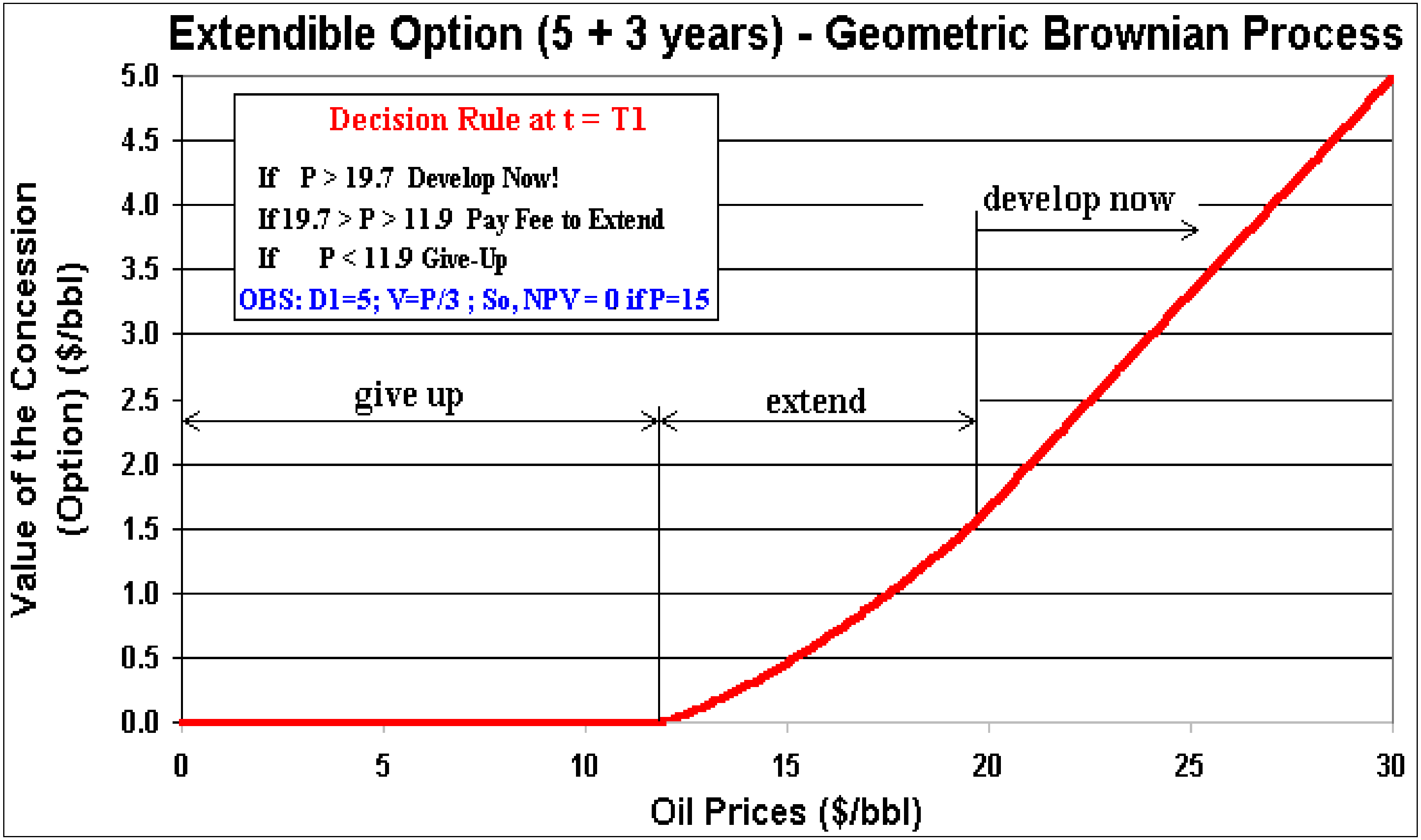


Figure 2 – Extendible Option at the First Expiration (for Brownian Motion Base Case)

4) Comparative Statics

4.1) Base Case: The Parameters

The Table 1 (see below) shows the parameters value used for the base case. Some values were estimated using available data about oil prices and/or using available related literature, such as the volatility, the long-run average oil price, the reversion speed, the jump size and jump frequency. Others were assumed as representative values for Brazilian offshore oilfields, such as the investment at both expirations, the cost to extend the option and the economic quality of the reserves.

The assumed times to expiration consider an average international practice[18]. The (per-barrel) investment cost, the current spot oil prices and the economic quality of the reserves are set so that in the base case the NPV of the project is zero. The extension cost, of US$ 0.3/bbl means US$ 30 million for a 100 million barrels of reserve, which is approximately the cost of two deepwater exploratory wells. A preliminary and simplified empirical job to estimate the parameters for the jump-reversion stochastic process using oil prices time series (mainly for the volatility) is shown in the Appendix C, that used market data from the Brent oil, the main oil reference in Europe. Comparison of this jump-reversion process base case with the popular geometric Brownian motion is presented in section 4.3.

Table 1 – Parameters from the Base Case for Jump+Mean-Reverting Model

Parameter	Notation	Base Case Value
Volatility of the Diffusion Process (% p.a.)	σ	22
Exogenous Discount Rate (% p.a.)	ρ	10
Reversion Speed ; [Half-Life (years)]	η ; [H]	0.03 ; [1.16]
Annual Frequency of Jumps (per annum)	λ	0.15
Economic Quality of Developed Reserve	q	0.333
Long-Run Average Oil Price (US$/bbl)	$\overline{P}$	20
Average Jump-Up (%)	μ_u	100
Standard Deviation of the Jump-Up (%)	s_u	30
Average Jump-Down (%)	μ_d	– 50
Standard Deviation of the Jump-Down (%)	s_d	15
First Expiration (years)	T_1	5
Second Expiration (years)	T_2	8
Investment up to T1 (US$/bbl)	D_1	5
Investment after T1 until T2 (US$/bbl)	D_2	4.85
Cost to Extend the Option (US$/bbl)	K	0.3

For the *half-life* value, although some values from literature are higher (2+ years) than ours, the values that Bessembinder et al (1995, pp.373-374) found in futures market (data from March 1983 to December 1991) are very close to ours. Extrapolating the values from their Table IV, we find an implicit half-life of 1.1 years, practically the same of our higher estimate (1.16 years, corresponding to η = 0.03). Bradley (1998, p.59) also finds a half-life close to our base case (of 1.39 years). Anyway, several sensibility analyses for the parameters were performed, including the reversion speed case, and can be supplied by the authors to interested people upon request.

The long run equilibrium price is hard to obtain. One reference is a long run OPEC price goal of about US$ 21/bbl, but the long run marginal cost from non-OPEC countries, under US$ 19/bbl, could be used as lower bound. The increasing non-OPEC production has been offsetting by the rising costs experimented by oil companies going to deepwater and ultra-deepwater to find new reserves. Perhaps the best value is in between the OPEC and the non-OPEC marginal cost[19].

Baker et al (1998, p.129) estimate of the long run oil price was $18.86/bbl (in 1995 dollars) and used (pp.138-140) $20/bbl as initial long run level in their model[20]. In the same article, one graph (p.127) of term structure of futures prices suggests a long-run price between 18-21 $/bbl. We adopt $20/bbl (in 1998 dollars) for the Brent crude. This value is also adopted in Bradley (1998, pp.59-61) and shown in Cortazar & Schwartz (1996, Figure 4). Our $\overline{\mathbf{P}}$ value is constant along the option term.

We assume an exogenous discount rate ρ of 10% p.a. (which is also the official discount rate to report the present value of proven reserves to stock market investors) in the base case. In reality, with our general assumption of systematic jump risk, is not possible to use the non-arbitrage way to build a riskless portfolio because market information is not sufficient to spawn *all* the risk. In this case there is no theory for setting the "correct" discount rate (CAPM doesn't hold), unless we make restrictive assumptions about investors' utility functions (without guarantee of more reliable results).

One practical "market-way" to estimate ρ is taking the net convenience yield (δ) time series (calculated by using futures market data from longest maturity contract with liquidity)[21], together with spot prices series, estimating ρ by using the equation: $\rho = \delta + \eta(\overline{\mathbf{P}} - \mathbf{P})$. Here δ in general is just the difference between the discount rate (total required return) and the expected capital gain E(dP/P), like a dividend. The parameter δ is endogenous in our model and, from a market point of view, is used in the sense of Schwartz (1997b, p.2) description: "*In practice, the convenience yield is the adjustment needed in the drift of the spot price process to properly price existing futures prices*". High oil prices P in general means high convenience yield δ (positive correlation)[22], and for very low P the net convenience yield can be even negative. There is an offsetting effect in the equation (even being not perfect), so we claim as reasonable the approximation of ρ constant. In compensation, we don't need to assume constant interest rate (because it doesn't appear in our model) or constant convenience yield (here implicitly changes with P)[23]. The series (P, δ) permits to estimate an *average* "market" ρ (from the ρ time series that we will get with this approach) or by looking the intercept from the simple regression P x δ. In this way, the value of ρ depends heavily of the assumed values for η and $\overline{\mathrm{P}}$. This is only a bound for ρ in the general model.

The alternative, using the same market data, is to estimate the return ρ on this commodity by running a cointegrating regression of the temporal series (P, δ) or by estimating the risk premium running a simple regression of futures and spot prices, see Pindyck (1993, pp.514-517)[24].

Figure 3 shows the option value for the base case at the current data (t = 0, upper/thinner line) and the payoff line (bottom line) at the first expiration (T_1). The option curve shape is different of the Brownian motion case (Fig.2), here the option graph exhibits a typical shape for mean-reverting process. See the option curve smooth pasting on the payoff line: the tangency point is the threshold for immediate investment. The main thresholds of the base case are showed in the chart.

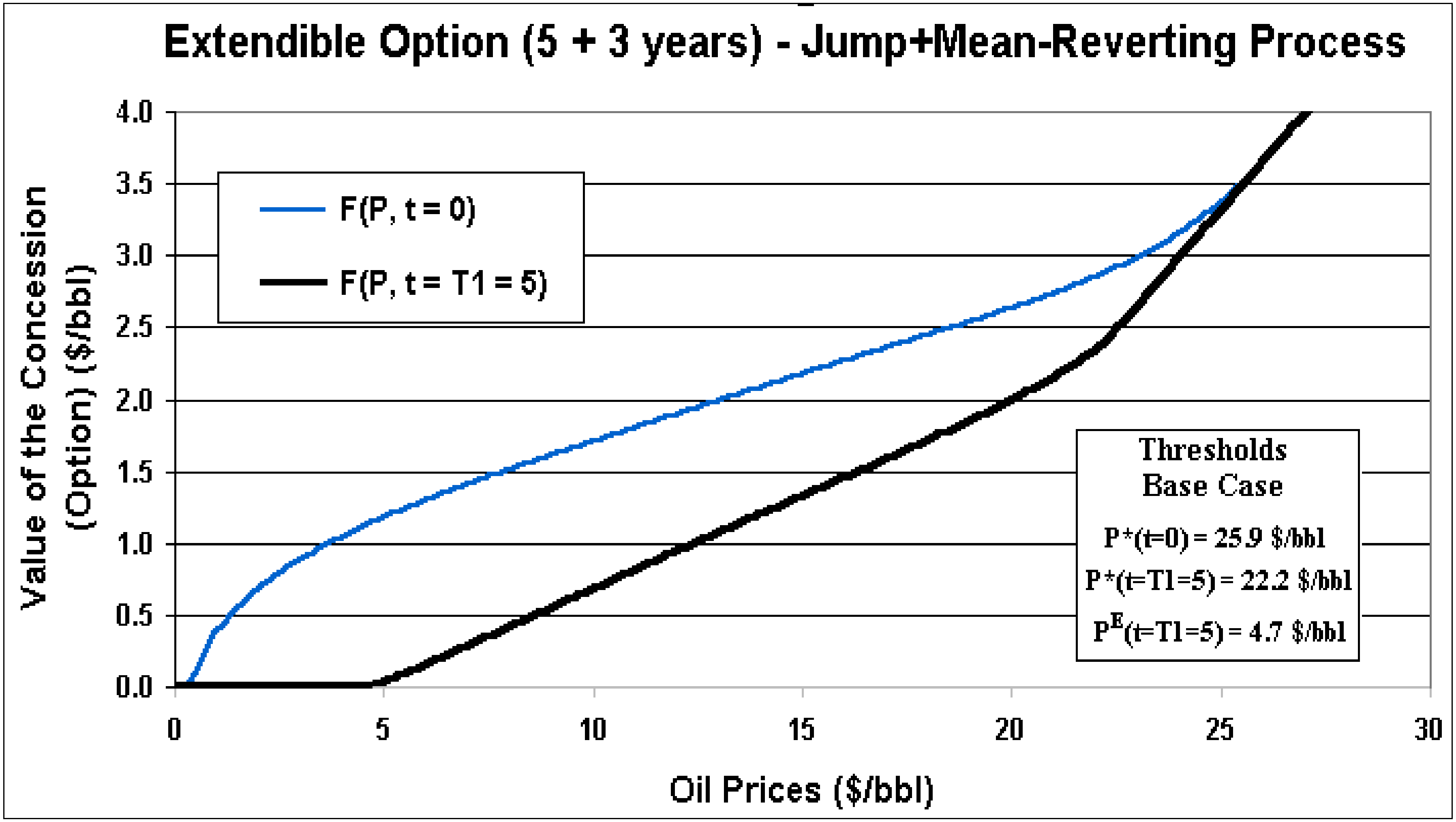


Figure 3 - Extendible Option at t = 0 and at the First Expiration (Mean-Reverting + Jump)

Figure 4 shows the options value at the first expiration (payoff, bottom line) and the option curve (upper line) just after the first expiration, for a case slight different of the base-case (with higher fee to extension, K= 0.5 $/bbl, in order to highlight the effect). Note that the payoff and the option curve are parallel in the interval that is optimal to extend the option and also that the distance between the parallel lines is K, the fee to be paid in order to extend the option.

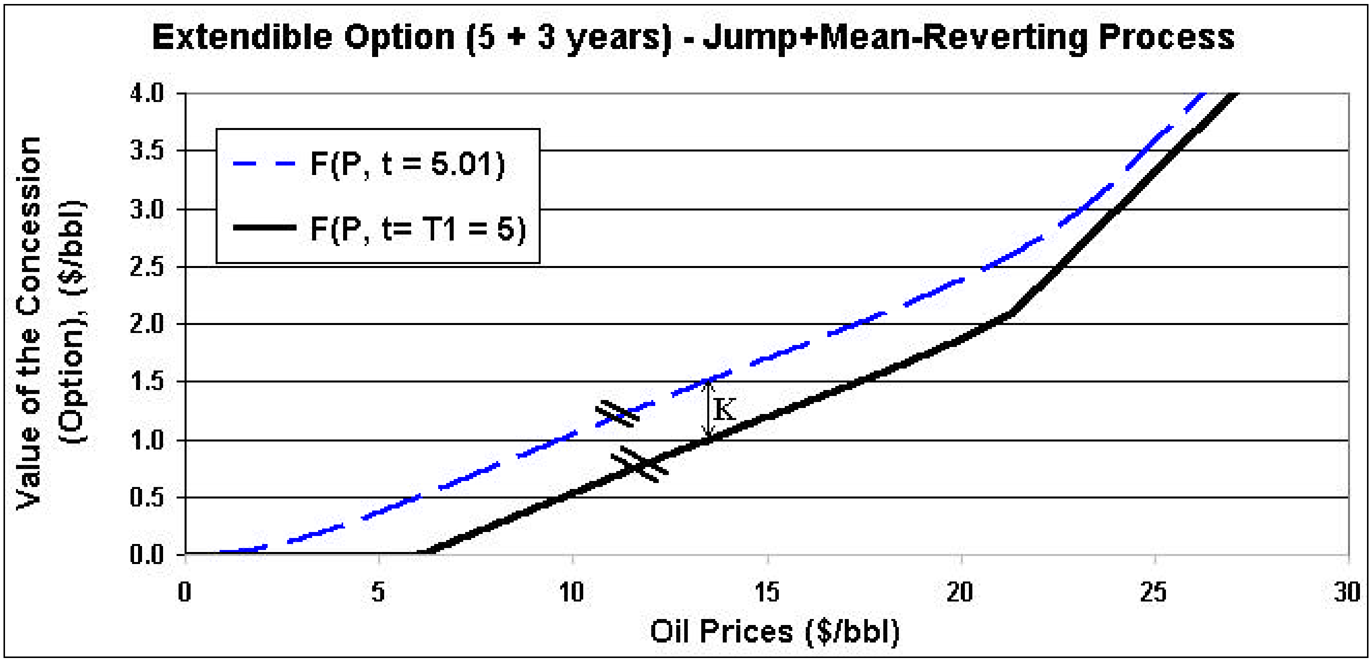


Figure 4 – The Options Value at the First and Just after the First Expirations (for K = 0.5)

Figure 5 displays the threshold lines for both terms of the option. On or above the threshold lines, is optimal the immediate investment.

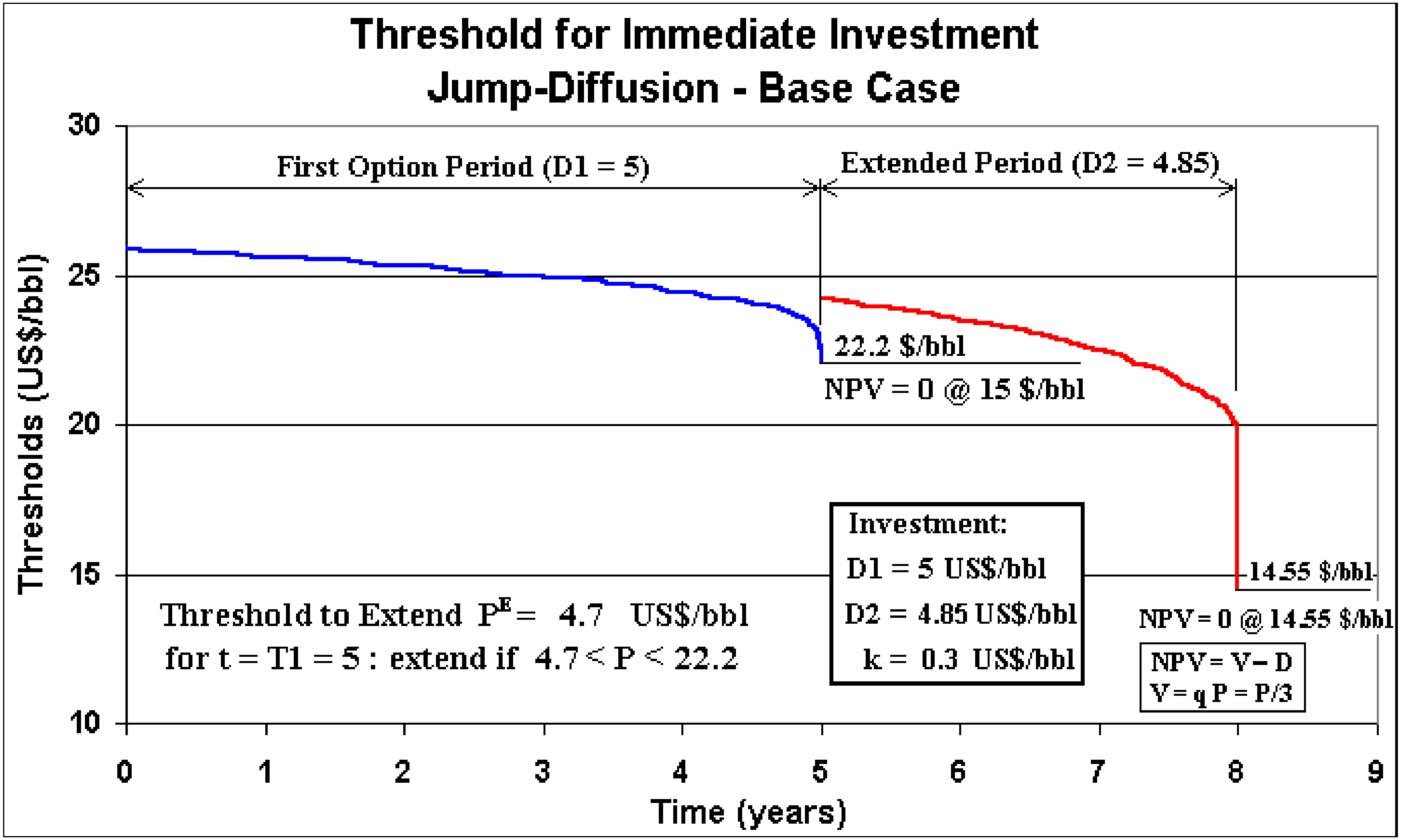


Figure 5 – The Threshold Lines for Jump-Diffusion Process (Base Case)

4.2) Sensibility Analysis

Several sensibility analyses were performed for each parameter of this process. Some parameters show higher impact on both option value and thresholds than other. For example, the economic quality of the developed reserves (q) has large impact, mainly in the option value: from q = 0.2 to 0.45, the option F(P=15) rises from 0.46 to 4.00 $/bbl and the threshold P*(t=0) drops from 30.9 to 24.4 $/bbl. However, for the standard deviations of the jump size (both up and down) simulations have shown a minor impact on the results.

One interesting analysis in this transition phase of Brazilian petroleum sector is about time to expiration policy. The table below shows that an increase in the time to expiration has major impact over the option value than over the threshold. The table points that rising the total expiration (T_1+T_2) from 5 years to 8 years, the option value increases near 20%, whereas the threshold value increases less than 4%. So, 8 years instead 5 years attract higher bid bonus (~ proportional to option value) without delaying (looking the thresholds) too much good investment projects.

In the base case, the option value reach US$ 2.178/bbl, which is significantly higher than NPV value (NPV is zero for P = 15 $/bbl). For a 100 million barrels oilfield, means an option value of US$ 217.8 million.

Table 2 – Sensibility of the Time to Expiration Value for Option and Threshold

T_1 (years)	T_1 + T_2 (years)	F(P=15) ($/bbl)	% in F	P*(0) ($/bbl)	% in P*
2	3	1.440	-	24.1	-
3	5	1.828	26.9	25.1	4.1
5	8 (base case)	2.178	19.2	25.9	3.2
6	10	2.314	6.2	26.2	1.2
8	12	2.417	4.5	26.4	0.8

Moreover, higher time to expiration presents other benefits (so higher bonus-bid) that were not considered in this paper. For example: (a) “Bayesian” gain of sequential exploratory investment (rather parallel) using information gathered for correlated prospects; (b) low attractiveness for one firm to bid several tracts, if the time is too small to perform optimal sequential investment (according the “auction theory”, less bidders per tract means lower expected bonus value); (c) an economic/optimal planning of resources (e.g. deepwater rigs) allocation is impaired if the timing is too short, losing business opportunities that are available on specific timing like seasonal rates of special service ships, etc.; and (d) *revelation* of exploratory work in the basin (see Dias, 1997, p.143) that reduces technical uncertainty and points out new geologic plays currently not considered (which leverage the tract value and so the winner bid, if there is time to wait and to use this information).

Figure 6 shows the thresholds sensibility with the Poisson arrival factor λ. For higher jump frequency the threshold level for the immediate investment is higher, which has economic logic because the investor is less willing to invest due to the risk of jump-down. However the threshold for the extension decreases, because jump-up increase the possibility of a not good project to transform into a good one. Hence, in most cases, firms should pay a small cost to extend the option rights.

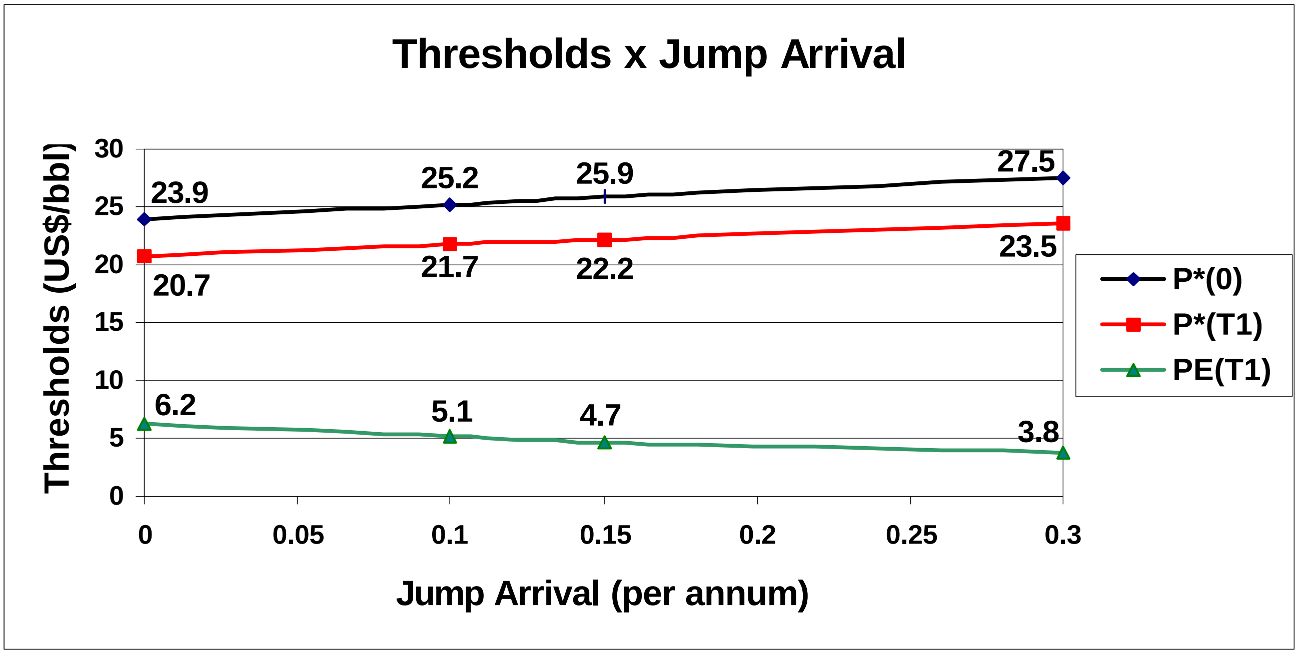


Figure 6 – Thresholds and the Arrival Jump Frequency λ

The comparative statics results in general were: option values increased for higher reversion speed, lower discount rate[25], higher volatility, higher jump arrival, higher jump-up mean, lower extension cost, higher long-run mean, higher economic quality of the reserve, and higher expiration time.

4.3) Comparing Geometric Brownian with Jump+Mean-Reversion

Geometric Brownian Motion (GBM) also known as drifted random walk model, is the most popular stochastic process and is generally a very good stochastic process in financial economics, although far from perfect, mainly for commodities. The GBM model for the oil prices is shown below.

$$dP = \alpha\, P\, dt + \sigma\, P\, dz \qquad \text{Eq.(13)}$$

where $\alpha = \rho - \delta$ and dz is the Wiener increment.

The parameters for GBM base case are $r = \delta = 5\%$, $\sigma = 23\%$. The comparison of the GBM with our more rigorous jump-diffusion model (Eq.1) for oil prices is summarized in the table below.

Table 3 – Option Values at P = US$ 18.3/bbl

Jump + Mean Reversion Process: F(P = 18.3 \$/bbl, t = 0)					
Base	No-Jump ($\lambda = 0$)	No volatility $\sigma = 0\%$	$\sigma = 5\%$	No reversion $\eta = 0$	$\sigma = 23\%$, $\lambda = 0$ and $\eta = 0$
2.4768	1.8979	2.0225	2.2592	1.8237	1.4162

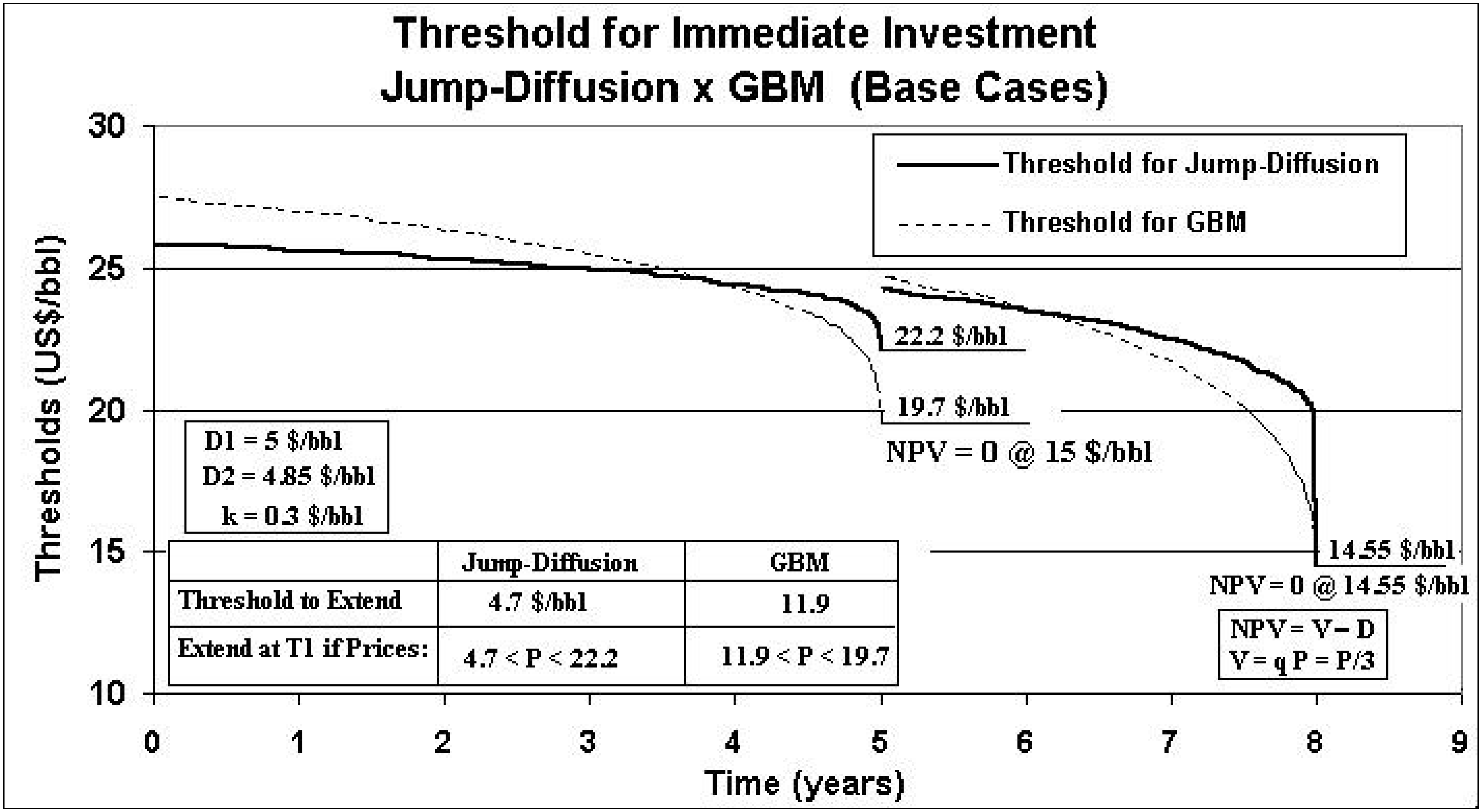
Geometric Brownian Motion: F(P = 18.3 \$/bbl, t = 0)		
Base ($r = \delta = 5\%$)	$r = 10\%$ and $\delta = 5\%$	$r = 10\%$ and $\delta = 10\%$
1.5739	2.0831	1.4162

The table was built with a convenience yield δ of 5% for both processes. In the case of GBM, δ is a parameter input of the model, and is constant. In the case of Jump+Mean-Reversion, δ is not constant, is not direct parameter input (it is implicit, endogenous of the model) and depends of the price level: $\delta(P)$. In order to compare in the same basis, let us *choose a petroleum price* to compare options *so that dividend yield is 5%*. This price is 18.3 US$/bbl because it implies a convenience yield of 5% for our jump-mean-reverting process, as shown using the equation for δ:

$$\delta = \rho - \eta(\bar{P} - P) \Rightarrow 0.05 = 0.1 - 0.03(20 - P) \Rightarrow P = 18.3 \text{ \$/bbl}$$

Comparing jump+mean-reversion and GBM (Table 3), jump+mean-reversion in general presents higher option values. The GBM has higher option value only for higher interest rate case (r = 10%, the same value of ρ in jump-diffusion setting) *and* when comparing with no jumps (λ = 0), no volatility (σ = 0) or no reversion (η = 0) cases. The option value is closer of GBM in case low uncertainty (5%) for the reversion process. However, the rôle of interest rate r in the GBM and the ρ in the jump-diffusion are very different. The option value increases with r in the GBM and decreases with ρ in the jump-diffusion (see last endnote). In the GBM r is independent of δ, so the only effect is to increase the waiting benefit, but in the jump-diffusion model ρ is not independent of δ. In other words, for the same drift $\eta(\bar{P} - P)$ a change in the value of ρ, implicitly means change in the value of δ. For this reason, if we use a lower value for ρ (e.g. ρ = 5% = r), we get a higher option value (2.5814, not shown in the table), and the option values from jump-diffusion process become still higher than GBM. For η = 0, implying $\rho = \delta$, we can compare the case of GBM with r = δ and jump-diffusion for no reversion, no jump and with the same volatility (η = 0, λ = 0, σ = 23%). In this case, as expected, the values are the same, equal to 1.4162 (see Table 3). For a jump-diffusion with ρ = 5%, and also with σ = 23%, η = 0, λ = 0 (not shown in table), the option is again the same of the GBM base case (which has r = δ = 5%), that is, 1.5739.

Figure 7 shows both thresholds, for the jump-diffusion process and for the GBM. The threshold curve is smoother for GBM than for jump-diffusion process near expirations. The reason is the *effect of the convenience-yield* δ. In case of GBM, δ is constant and positive, whereas for jump-diffusion process, δ is not constant (depends of oil prices P). In jump-diffusion process δ is positive for higher oil prices and *negative* for lower prices. A well known property from American options is that earlier exercise only can be optimal if $\delta > 0$. So, earlier exercise is possible only if P is higher than 16.7 $/bbl (in the base-case) and this explain the discontinuity of the threshold curve at the expiration[26].



	Jump-Diffusion	GBM
Threshold to Extend	4.7 $/bbl	11.9
Extend at T1 if Prices:	4.7 < P < 22.2	11.9 < P < 19.7

Figure 7 – Thresholds Comparison: Jump-Diffusion x Geometric Brownian Motion

The other important observation is that the threshold to undertake the project in the beginning of the term is higher for the GBM (although in general the option values are lower). This is coherent with the results of Schwartz (1997a, p.972)[27] when comparing GBM with their two and three-factor models even being very different from our jump-diffusion model, but that systematically produce results qualitatively close of our model when he compares these models with the GBM.

Figure 8 presents the thresholds curves for the first period (at t = 0 and t = T_1) in function of the volatility. Higher volatility favors waiting and the extension (higher spread between P* and P^E).

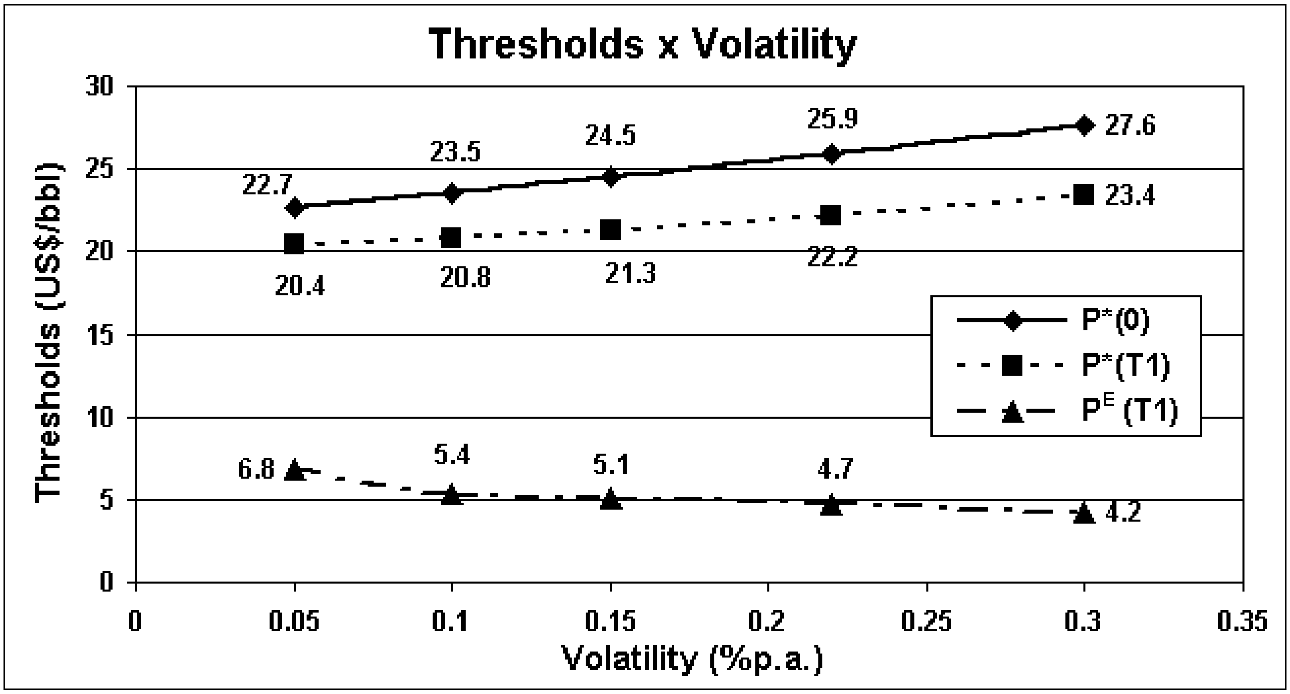


Figure 8 –Effect of the Volatility on the Jump-Reversion Thresholds

5) Conclusions

This paper develops a model for extendible options embedded in the petroleum offshore E&P contracts that happen in some countries. The model incorporate the possibility of the extension cost (e.g. exploratory wells) to be used partially as a benefit (to reduce the development cost).

Sensibility analysis of the parameters, suggest a higher option value (and so expected higher bonus bid) to a higher time to expiration without a significant additional delay of investment in good projects. Moreover, there are other benefits (so even higher bonus-bid) from higher time to expiration that was not quantified in this paper, such as *revelation* and portfolio optimization.

The stochastic model of jump+mean-reversion for the oil prices has more economic logic than previous models used in real options literature, considering that normal news causes continuous small mean-reverting adjustment in oil prices, whereas abnormal news causes abnormal movements in these prices (jumps). A future improvement is to allow for stochastic long run equilibrium price (mainly for longer terms), calculating the initial equilibrium price (of the industry players) by the game theory.

The comparison of this more rigorous model with the more popular Geometric Brownian Motion pointed a higher option value for the jump-diffusion case. Hence, a higher expected bid in the lease-sale process is a consequence of using this more rigorous model. Other good models from literature like

Gibson & Schwartz (1990) and Schwartz (1997a) two and three-factor, that rely more heavily on the futures markets, despite being very different, present results qualitatively very similar with ours.

Several extensions are possible for our main model. For example: (a) allowing the equilibrium price level to be stochastic; (b) using a correlated stochastic process for the operational cost, instead the adopted linear function V(P); (c) incorporating the technical uncertainty and exploratory revelation; (d) considering other options like sequential development (extendible call on a call) and/or abandon (extendible call on a put); and (e) portfolio planning, quantifying the expected first hitting time for a project that currently is optimal to wait, in order to estimate when the investment is expected to start.

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APPENDIXES

APPENDIX A – MEAN REVERSION AND HALF-LIFE

The model of oil prices reversion known as Geometric Ornstein-Uhlenbeck is used in Dixit & Pindyck (1994) and in Metcalf & Hasset (1995):

$$\frac{dP}{P} = \eta(\bar{P} - P)\,dt + \sigma\,dz \qquad \textbf{(A1)}$$

This model (A1) has the same forecasting expected value (A3) of our actual jump-diffusion model (A2):

$$\frac{\mathbf{dP}}{\mathbf{P}} = [\boldsymbol{\eta}(\bar{\mathbf{P}} - \mathbf{P}) - \boldsymbol{\lambda}\mathbf{k}]\,\mathbf{dt} + \boldsymbol{\sigma}\,\mathbf{dz} + \mathbf{dq} \qquad \textbf{(A2)}$$

$$\mathbf{E}(\frac{\mathbf{dP}}{\mathbf{P}}) = \boldsymbol{\eta}(\bar{\mathbf{P}} - \mathbf{P})\ \mathbf{dt} \qquad \textbf{(A3)}$$

We define ***half-life*** (H) of the petroleum prices[28] as the time for the expected oil prices to reach the middle price between the current price and the long run mean. This oil price half-life is deducted below:

From eq.A3: $dP/[P(\bar{\mathbf{P}} - P)] = \eta\, dt$

Integrating from P_0 (t_0) to $P_1(t_1)$, and letting $\Delta t = t_1 - t_0$, we get:

$$\ln\left(\frac{P_1 - \bar{P}}{P_0 - \bar{P}}\right) = -\,\eta\,\bar{\mathbf{P}}\,\Delta t \qquad \text{(A4)}$$

For Δt = half-life H, by definition we have that $(P_1 - \bar{\mathbf{P}}) = 0.5\,(P_0 - \bar{\mathbf{P}})$, hence:

$$\boxed{\mathbf{H} = \frac{\mathbf{ln(2)}}{\eta\,\bar{\mathbf{P}}}} \qquad \text{(A5)}$$

From A4 we can get the expected oil price at the generic instant t_1:

$$\mathbf{E(P_1)} = \bar{\mathbf{P}} + (\mathbf{P_0} - \bar{\mathbf{P}})\ \mathbf{e}^{-\bar{\mathbf{P}}\,\eta\,\Delta\mathbf{t}} \qquad \text{(A6)}$$

In some papers appeared a slightly different half-life equation: $H' = \ln(2)/\kappa$, where κ is a reversion speed. This equation comes from models like in Smith & McCardle (1997). They model the *logarithm* of the oil prices (instead the prices itself) as mean-reverting, $\pi = \ln(P)$, with the Ornstein-Uhlenbeck process:

$$d\pi = \kappa\,(\bar{\pi} - \pi)\,dt + \sigma\,dz$$

Following the same procedure above, is easy to show that the half-life of this process is $H' = \ln(2)/\kappa$. This logarithm model has some advantages (for example the long-run mean doesn't appear in the half-life equation), whereas our model has the practical advantage of the half-life interpretation because we use oil prices half-life instead half-life of ln(P): as reversion parameters the user (manager) enter the long-run average and the number of years which is expected the oil price to reach the half distance towards the long run mean.

APPENDIX B – EXPLICIT FINITE DIFFERENCE NUMERICAL SOLUTION

To solve the partial differential equation (PDE) of parabolic type we use the finite difference method (FDM) in the explicit form. It consists of transforming the continuous domain of P and t state variables by a network or mesh of discrete points. The PDE is converted into a set of finite difference equations, which can be solved iteratively using the appropriated boundary conditions ($t = T_1$ and $t = T_2$). The solution is reached by proceeding backwards through small intervals ΔPs until we find the optimal path P*(t) to every t. The use of finite difference method for jump-diffusion processes appeared before at least in Bates (1991).

Suppose the following discretization for two variables:

$$F(P,t) \equiv F(\,i\Delta P,\ j\Delta t\,) \equiv F_{i,j} \qquad \text{, where } 0 \le i \le m \ \text{ e } 0 \le j \le n_1 \text{ or } n_2 \text{ with } n_{1,2} = T_{1,2} / \Delta t.$$

The choice of the discrete steps must be done in a way that all the coefficients of the finite difference equation be always positive to any value inside the grid to ensure the convergence of explicit FDM. So the convergence of the FDM settles the choice of ΔP and Δt. The partial derivatives are approximated by the differences:

$F_{PP} \approx [\, F_{i+1,j} - 2F_{i,j} + F_{i-1,j} \,] / (\Delta P)^2$; $F_P \approx [\, F_{i+1,j} - F_{i-1,j} \,] / 2\Delta P$; $F_t \approx [\, F_{i,j+1} - F_{i,j} \,] / \Delta t$

We use the "central-difference" approximation for the P variable and the "forward-difference" for the t variable. Applying these approximations to the PDE and its respective boundary conditions we have the following difference equation:

$$F_{i,j} = p^{+}F_{i+1,j-1} + p^{0}F_{i,j-1} + p^{-}F_{i-1,j-1} + p_{jump}E\left[F_{[i.\tilde{\phi},\, j-1]}\right]$$

$$p^{+} = \frac{\Delta t}{\Delta t.\rho + 1}\left[\frac{\sigma^2 i^2}{2} + \frac{i.(\eta.\overline{P})}{2} - \frac{i^2.\eta.\Delta P}{2} - \frac{i.\lambda.k}{2}\right] \quad ; \quad p^{0} = \frac{\Delta t}{\Delta t.\rho + 1}\left[\frac{1}{\Delta t} - \sigma^2 i^2 - \lambda\right]$$

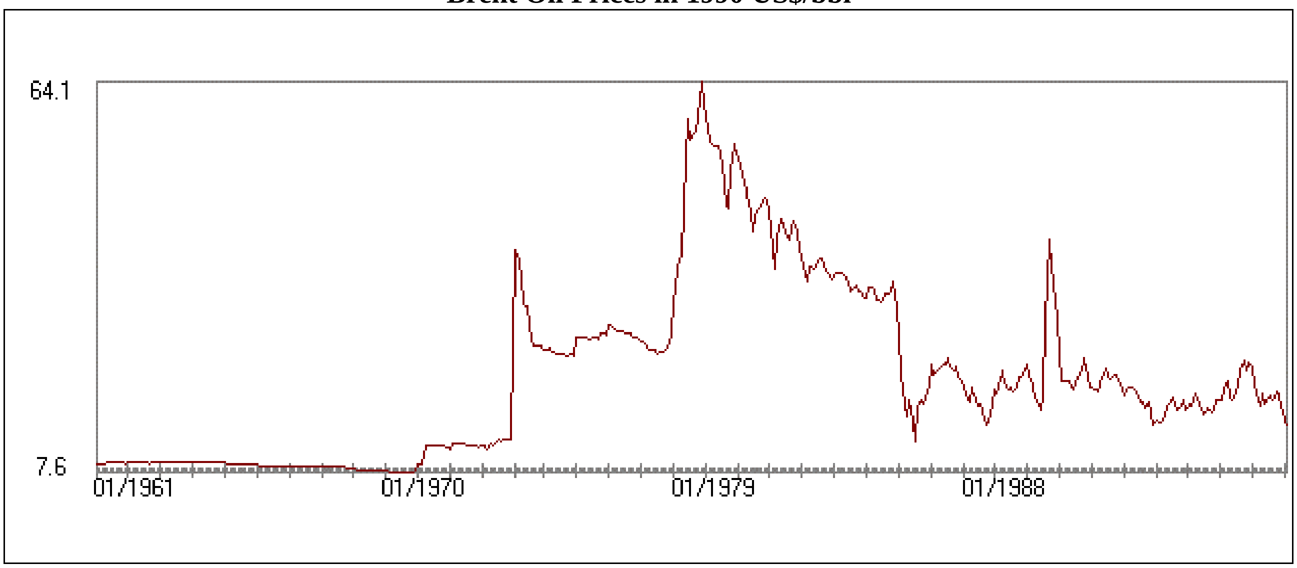
$$p^{-} = \frac{\Delta t}{\Delta t.\rho + 1}\left[\frac{\sigma^2 i^2}{2} - \frac{i.(\eta.\overline{P})}{2} + \frac{i^2.\eta.\Delta P}{2} + \frac{i.\lambda.k}{2}\right] \quad ; \quad p_{jump} = \frac{\Delta t}{\Delta t.\rho + 1}\lambda \quad ; \quad k = E[\tilde{\phi} - 1]$$

More about the FDM can be found in Brennan & Schwartz (1978) or Smith (1971).

APPENDIX C – VOLATILITY AND OTHER PARAMETERS ESTIMATIVE

The oil price[29] time series (see graph below for real prices at 1990 US$) suggests that the price of oil is subject to permanent and transitory instabilities

Brent Oil Prices in 1990 US$/bbl



The model instability can be considered on 2 ways. One, simpler, admits that there was a structural change from a certain moment, say the middle of the 70s, which would recommend discarding the older periods. This view is particularly arbitrary due to the fact that it depends on the cut point and the assessment that only a model change occurred. Alternatively the assessment of a structural change can be incorporated into the model explaining the hypothesis that the parameters follow a random walk.

Even though this last approach is more elegant, it implies in the incorporation of non-linear elements to the model, and in the use of more complex estimation models. Two models will be considered for $(dP/P)_t = p_t$. Model 1 refers to the geometric Brownian motion and Model 2 to the mean reversion process. In both models the parameters (δ, ψ) control the adaptability degree of estimations of the pair (a, b), introduce non-linearity and do not allow the finding of analytical forms for its estimates, requiring the use of numerical methods for it.

Model 1: Geometric Brownian Motion

$p_t = a_t + e_t \qquad e_t \sim N(0, s_t) \qquad$ (C1)

$a_t = a_{t-1} + e_{1t} \qquad e_{1t} \sim N(0, \delta s_t)$

Model 2 admits that the change in oil price in relation to a local average (A) generates tensions in the market that pressure the price toward the average. Suppose the next equation C2 is a simplified discrete time representation of the continuous mean-reverting process. Since price level A is unknown, this representation implies in the product between the speed reversion estimator parameter (b_t) and the medium price (A_t). The model (C2) can be parameterized in the form (C3), where the medium price A = a/b.

Model 2 : Mean Reversion Process:

$p_t = b_t (A_{t-1} - P_{t-1}) + e_t \qquad e_t \sim N(0,s_t) \qquad$ (C2)

$p_t = a_t - b_t P_{t-1} + e_t \qquad e_t \sim N(0,s_t) \qquad$ (C3)

$a_t = a_{t-1} + e_{1t} \qquad e_{1t} \sim N(0, \delta s_t) \qquad ; \qquad b_t = b_{t-1} + e_{2t} \qquad e_{2t} \sim N(0, \psi s_t)$

The change in the instability standard of the prices along the sample will be considered using an adaptive model for estimation of volatility(ies). In this specification, the choice of the parameter (θ), which controls the adaptability degree of volatility, is arbitrary. The variance equation s_t follows (C4), with (C5) as solution.

$s_t = \theta s_{t-1} + (1-\theta) e^2_{t-1} \qquad$ (C4)

$s_t = \sum_i \theta^i \, e^2_{t-i} \; / \; (1-\theta) \qquad$ (C5)

The difference equation (C4) can be solved in the form (C5) that shows the effect of errors of the last (i) periods in the estimation of the (t) period volatility. The value of (i) for (θ^i)=0.5 is denominated half-life of *information* (different of oil prices half-life presented before) and can be used to suggest the relevant values of (θ). The table below shows the estimated values for the volatility of the model (2) – which shows results similar to the ones from model (1) – for many values of (θ), together with their corresponding half-life.

Half-Life (years)	1	2	5	10	∞
θ	0.90	0.95	0.98	0.99	1
Volatility (monthly)	7.11	6.68	6.17	5.75	4.39

The characteristics of prices suggest that the volatility has the same instability behavior since 1979, and because of that we consider relevant the values with half-life smaller than 10 years. For these values the volatility is in the interval [6.17, 7.11], therefore we chose (θ = 0.95) to calculate the estimates.

In this calculation we considered as belonging to the sample the numbers (dP/P) that were in the interval [–0.15, 0.15]. All the others were considered a consequence of the jump, which were removed of the sample. The models (1) and (2) were estimated using the method MCMC (Markov Chain Monte Carlo) – see West and Harrison (1997) – and obtained the after the mode and the interval of maximum density *a posteriori* (IMDP) for 65% level.

	Model 1		Model 2	
	Mode	IMDP(65%)	Mode	IMDP(65%)
δ	.067	[.022, .136]	.031	[0, .047]
ψ	-	-	.003	[.001, .006]
volatility	6.684	[5.742, 6.72]	6.286	[4.85, 6.62]

The relevant values for the degree of adaptability of the model are in table above. Each of these corresponds to a possible description for the medium price and the effect of the deviation with respect to the medium price. Using the most likely value of both models we obtain the results below.

	Model 1		Model 2	
	Average	Average/S. Deviation	Average	Average/S. Deviation
a	-.63	.52	7.08	1.94
b	-	-	.487	2.56
Medium Price	-	-	14.5	-
Volatility	6.684	-	6.286	-

The table below presents a summary of the obtained results from the Model (2) with different combinations of medium prices and the deviation effect obtained (reversion) for data from 1979 to 1998.

Factor*	δ , ψ	Volatility (monthly)	b_t	a/b
1	0.04, 0.004	6.50	.72	16
8	0.005, 0.0005	6.73	.13	17.8
40	0.001, 0.0001	6.77	.043	19.7

* Factor 1 indicates the most adaptive model, which gives more weight to the recent observations, and Factor 40 represents the less adaptive one with almost the same weight to all observations.

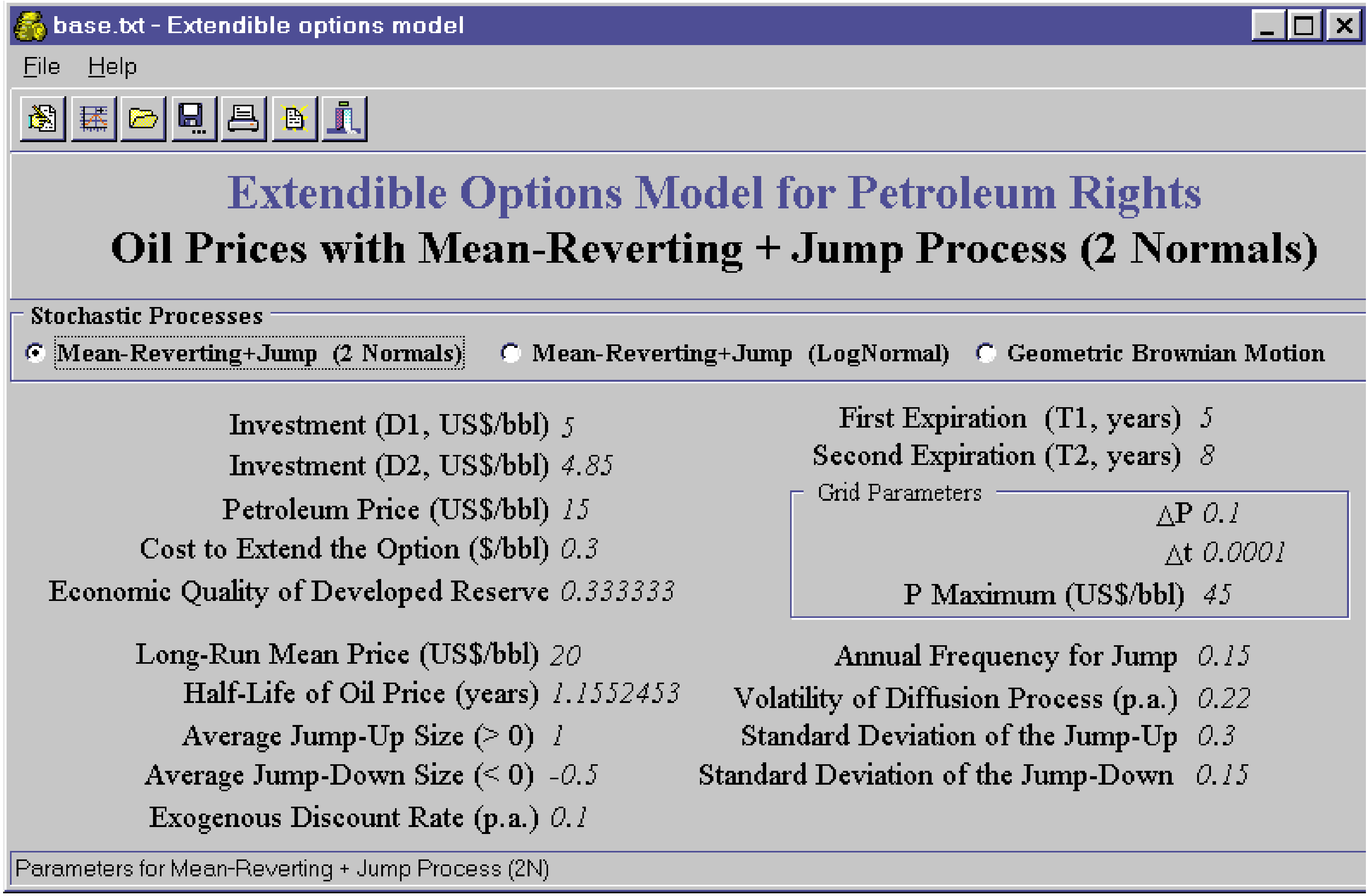
Every alternative belongs to the interval of maximum density a posteriori (IMDP) to the level of 65%, therefore your choice can be realized using non-statistical approaches.

APPENDIX D - THE SOFTWARE INTERFACE

The software interface was built using Borland C++ Builder. The main screen is shown in the figure below.

The interface have three stochastic processes available to choose to perform the calculus for the extendible option problem: (a) mean-reversion+jump, with the random jump using two truncated-normal distribution (2 Normals); (b) mean-reversion+jump, with the random jump using log-normal distribution (LogNormal); (c) geometric Brownian motion. The software has two alternatives for the reversion-jump process because our first version of the model, presented in Stavanger (May 1998), used the log-normal distribution for jumps (like Merton, 1976) instead the two-truncated normal distribution, so it remained in the software.

The parameters from the base case for the first stochastic process are shown in the figure (including the grid density parameters for the finite difference method: ΔP, Δt and P maximum). There are others interactive windows in the software, integrating the user-friend interface.


base.txt - Extendible options model
File
Help
Extendible Options Model for Petroleum Rights
Oil Prices with Mean-Reverting + Jump Process (2 Normals)
Stochastic Processes
Mean-Reverting+Jump (2 Normals)
Mean-Reverting+Jump (LogNormal)
Geometric Brownian Motion
Investment (D1, US$/bbl) 5
Investment (D2, US$/bbl) 4.85
Petroleum Price (US$/bbl) 15
Cost to Extend the Option ($/bbl) 0.3
Economic Quality of Developed Reserve 0.333333
First Expiration (T1, years) 5
Second Expiration (T2, years) 8
Grid Parameters
ΔP 0.1
Δt 0.0001
P Maximum (US$/bbl) 45
Long-Run Mean Price (US$/bbl) 20
Half-Life of Oil Price (years) 1.1552453
Average Jump-Up Size (> 0) 1
Average Jump-Down Size (< 0) -0.5
Exogenous Discount Rate (p.a.) 0.1
Annual Frequency for Jump 0.15
Volatility of Diffusion Process (p.a.) 0.22
Standard Deviation of the Jump-Up 0.3
Standard Deviation of the Jump-Down 0.15
Parameters for Mean-Reverting + Jump Process (2N)

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NOTES

[1] The fiscal regime of Brazilian E&P sector is the concession contract (lease), which firms offer bonus to National Petroleum Agency (ANP) in first-price sealed bidding process. The first version of the concession contract was published in Feb/98, some new guidelines in Jan/99 by ANP, and still can evolve until the bid. The model presented here is not Brazilian specific, but the underlying motivation is the Brazilian case.

[2] For a discussion on extendible maturities options, see Longstaff (1990) and Briys et al (1998, chapter 16). The payoff of an extendible option is the maximum of two risky payoffs: the payoff from a standard call option and a compound option (call on a call) less the cost to get it. So, extendible options are more general than compound options.

[3] See Dias (1997) for a simple model integrating three kind of uncertainties (technical, economic and strategic) using a decision-tree plus a game-tree for the exploratory phase, and a continuous-time model for the development phase.

[4] See Dixit & Pindyck book (1994, section 4, chapter 10) for a continuous time model combining both technical and market uncertainty. Although the model is drawn to nuclear industry, it can be adapted to the petroleum one.

[5] See for example the econometric tests of D. Pilipovic (1998, table 4-9, p.78, WTI petroleum). Pindyck & Rubinfeld (1991, chapter 15) using a Dickey-Fuller unit root test, rejected the random walk hypothesis for a very long time series (more than 100 years). But they point that the oil price reversion to a long-run equilibrium level is likely to be slow.

[6] In real options literature, most jump processes has been used to model random competitors arrival (see Trigeorgis, 1996, pp.273, 284-288, 328-329), and in outcomes from R&D projects (see Pennings & Lint, 1997).

[7] See Paddock, J.L. & D. R. Siegel & J. L. Smith (1988) and Dixit & Pindyck (1994, chapter 12, section 1).

[8] Dias presented this concept in the "Workshop on Real Options", May 1998, Stavanger, Norway.

[9] The main reference for the market value of reserve, published by the traditional John S. Herold since 1946 (see data and discussion in Adelman et al., 1989, mainly Table 2), shows a large positive correlation between P and V, including jumps. Examples: between 1981-85, V was in the range of 8-10 \$/bbl, whereas in 1986 dropped to 5.88 \$/bbl; for the 70’s oil prices shocks, jumps in V were still more pronounced. The volatility of V has been slight lower than P.

[10] The Schwartz’s models assume that the operational costs (OC) are deterministic and independent of the commodity price P. However, for oilfields, the correlation between OC and P has been very high as shown the data from Adelman et al. (1989, Table 2). By the other side, our model simplifies assuming perfect correlation. The truth is in between. A more realistic but more complex model should allow for stochastic costs with a positive correlation with P process.

[11] Early Production Systems were exactly what happened to Brazil in the high prices times from early 80’s.

[12] In Brazil frequently an exploratory well is used in the development project. Even if the well is not the better location for the project, the investment reduction due to the already drilled well can be a good compensation. Kemna (1993, based in her consulting for Shell) presented a model for extendible options, but not allow for any benefit derived from the fee/additional exploratory extendible cost. She developed a more simplified model, using European style option.

[13] Capital Asset Pricing Model, a mean-variance equilibrium model, is used to set discount rates for assets and projects.

[14] The equivalent alternative (largely used in derivatives pricing) is a probability transformation, using an artificial probability (or *martingale* measure) instead of the real probability process. *For the mean-reversion process case*, see the *risk-neutral probabilities* in Sick (1995, pp.676-677), and the drift changing in Dixit & Pindyck (1994, p.162). In both cases the main difference (when comparing with GBM) is that the convenience yield δ is a function of P instead constant.

So, risk-neutral probabilities and risk-neutral drift are function of P, and appear a risk adjusted discount rate. For mean-reversion both the drift and the risk adjusted discount rate ρ are specified, so is not possible by-pass the estimation of ρ even discounting with the risk-free discount rate (risk-neutral world). This rate ρ appears just due to δ. For the GBM all that matters is the difference $\delta = \rho - \alpha$, and with δ known and constant, is not necessary to estimate ρ.

[15] For a discussion of dynamic programming versus risk neutral/*contingent claims* approach, see Dixit & Pindyck (1994, chapter 4). For an extension of risk neutral valuation to nontraded assets, see Trigeorgis (1996, pgs.101-103).

[16] For oil prices, is hard to say if jumps are or not really systematic, but theoretically a jump in the oil demand by the market (mainly in crashes/recessions case) could cause a price jump. Nietert (1997, p.1-4) distinguishes three types of jumps for stocks: firm-specific, industry-specific (with systematic component) and market jumps.

[17] For the contingent claims framework, the Eq.(7) is similar: the right term ρF is replaced with rF, and the second term $\{\eta\ (\bar{\mathbf{P}} - P) - \lambda\ E[\phi - 1]\}P\ F_P$ is replaced with $\{r - \rho + \eta\ (\bar{\mathbf{P}} - P) - \lambda\ E[\phi - 1]\}P\ F_P$. See also the endnotes 14, 15.

[18] The preliminary version of the concession contract in Brazilian pointed out 3 years plus 2 years of extension, but the new guidelines in January/99 points that the total time can reach 9 years.

[19] A suggestion for further research is to set the long-run equilibrium price modeling with the game theory, seeking a Nash-Cournot equilibrium or even modeling a Stackelberg leader-follower duopoly: OPEC and eventual allied as follower maximizing the profit (choosing the production level) given the production of the price takers producers (leader). Pure statistical approach could be noisy, misleading the evolving forces correlation between the players.

[20] That paper uses an uncertain long run equilibrium price modeled with geometric Brownian model, with this equilibrium price growing exponentially.

[21] The known formula for a commodity futures prices is $F(t) = e^{(r - \delta)\ t}\ P$. This equation is deducted by arbitrage and assumes that δ is deterministic, so it looks contradictory with our assumption of systematic jump and with our model that implies that δ is as uncertain as P. But we want an implicit value for δ and so for ρ, to get a market reference (a bound) to set ρ. It is only a practical "market evaluation" for the discount rate that is assumed constant in our model.

[22] Schwartz (1997a, p.943, Table IX) finds strong correlation between the spot price and the convenience yield (+0.915 for 259 samples and + 0.809 for other 163 samples). The correlation between spot price and interest rate (r) were slight negative (–0.0293 and –0.0057), whereas between δ and r seem to be independent (-0.0039 and +0.0399).

[23] Much less realistic is the standard GBM assumption of δ constant. Even the superior two-factor model of Gibson & Schwartz (1990) assumes that both the interest rate and the market price of convenience yield risk (p.967) are constant.

[24] The Pindyck (1993) model is based in the "fundamentals" (present value of δ stream), so is also not strictly coherent with our model with systematic jumps, but again his suggested market way to estimate ρ could be a good reference.

[25] Increasing the discount rate ρ, decrease both the option and the threshold at t=0 because, given a fixed drift, the convenience (dividend) yield δ has to adjust to the chances in ρ due to the relation $\rho = \eta\ (\bar{\mathbf{P}} - P) + \delta$. Increasing the convenience yield, the waiting value decreases and so the threshold and the option value. See Dixit & Pindyck (1994, chapter 5) for further explanation of mean-reversion process and sensibility analysis for the discount rate.

[26] At the expiration ("now-or-never") the option is the maximum between NPV and zero. NPV is zero for P = 14.55 \$/bbl (= threshold at expiration T2). In the threshold curve there is a gap because a minute before the expiration, a *necessary* condition to exist an optimal exercise is $P > 16.7$ (in order to get $\delta > 0$) and δ is *sufficiently* positive to optimal earlier exercise only at around the level of \$20/bbl.

[27] Schwartz (1997a, eq.52 and footnote 35) compares thresholds using perpetual options for the GBM and 10 years maturity for the 2 and 3 factors models. However for the volatility used, the threshold for perpetual and 10 years maturity are very close (threshold asymptotic property for long term), permitting the comparison.

[28] The original concept comes from the physics: measuring the rate of decay of a particular substance, half-life is the time taken by a given amount of the substance to decay to half its mass.

[29] Light Brent Blend oil (before 1984 were used other similar quality oil from North African, Libya and Qatar). Oil series source: IMF, International Financial Statistics.