Do not let policy termination harm growth or welfare: Incremental investment under subsidy termination risk

Roel L. G. Nagy[a], Stein-Erik Fleten[a], and Lars H. Sendstad[a]

[a]*Department of Industrial Economics and Technology Management, NTNU, 7491 Trondheim*

June 24, 2021

Abstract

Investments are frequently subsidized, but once a subsidy is close to reaching its goal or putting a strain on fiscal budgets, a subsidy may be terminated. An important question for policy makers is how to minimize the negative impact of an industry's risk of subsidy termination on the policy maker's goals of attracting investment or maximizing welfare. We study a competitive industry in which firms eligible for a subsidy are faced with an option to invest. The subsidy can be withdrawn due to unexpected events, such as depletion of public budgets or a change in political support. We assume the social planner sets a subsidy, and the firms decide on when to invest. We contribute to the literature by analyzing how the subsidy retraction risk impacts the firms' optimal investment timing, an industry's total investment and total surplus. We find that firms faced with a lump-sum subsidy subject to withdrawal risk expand sooner and, as a result, install a larger capacity than firms without a subsidy. Furthermore, a subsidy can increase total surplus if set correctly. The social planner's optimal subsidy size increases with the industry's capacity, and decreases with the subsidy withdrawal risk.

Keywords— investments, capacity expansion, policy uncertainty, real options

1 Introduction

Subsidies are commonly used to mitigate market imperfections and increase welfare as a result. Alternatively, subsidies can be used to encourage investment in a technology that provide a social good, but the technology is not economically viable yet. As subsidies are used as a tool to solve market imperfections or help new technologies become viable, they are terminated at some point in the future. The profitability of investors' projects depend largely on the subsidy's lifetime, thus it is important the investor accounts for the risks related to termination of a subsidy. Furthermore, it is also important for a policy maker to account for an industry's response to the risk of subsidy termination as the industry's investment decisions are key to reach the policy maker's targets. Examples of such transitions in which subsidy and subsidy termination play a role are renewable energy[1] and agriculture[2].

We consider an industry consisting of many risk-neutral, profit-maximizing firms, each holding an option to invest. The investment is eligible for a subsidy, and we study the effect of the risk of subsidy termination on the firms' investment decision. The cost of investment is dependent on the availability of a subsidy. We consider a lump-sum investment subsidy, which represents a general class of investment subsidies including investment tax credits and capital subsidies. After investing, a firm has a fixed-size project of which the production is immediately sold (i.e. no stock can be created). The future revenue stream of the project is uncertain. Each firm has to determine when it when to invest, resulting in an incremental investment problem from the

[1]The annual installed wind capacity in the US in the period 1997 – 2005 strongly depended on the production tax credit. Investment increased in the years before the tax credit expired, and has been low in the following years (The Economist, 2013). Stokes (2015) and Stokes and Warshaw (2017) point out the important role of the public opinion on renewable energy policy in the US. Stokes and Warshaw (2017) address the withdrawal risk caused by public opinion: "Since 2011 several [US] states have weakened their renewable energy policies. Public opinion will probably be crucial for determining whether states expand or contract their renewable energy policies in the future".

[2]The amount of subsidies in agriculture has gone down compared from the late 90s to the period 2009-11 in Europe (The Economist, 2012), and the European Union has recently debated to limit spending on agriculture (The Economist, 2019). In the UK, farmers are concerned with the consequences of missing out on the £3bn of annual subsidy under the European Union's common agricultural policy after the UK has left the EU (The Economist, 2020). The uncertainty regarding the farmers' income is affecting their investment decisions.

point of view of the entire industry. This interpretation is applicable to, for example, a country's renewable energy capacity, in which many projects initiated by different investors increase the country's or industry's total capacity. With this paper, we seek to answer the following open research questions: (i) how does the prospect of policy termination affect investment behavior under subsidy and after subsidy termination, (ii) how does the prospect of policy termination affect total surplus, and (iii) how should a social planner set its subsidy size optimally to maximize total surplus?

In answering the first question, we find that a competitive industry faced with a lump-sum subsidy subject to withdrawal risk expands sooner while the subsidy is available and, as a result, installs a larger capacity during the subsidy lifetime than an industry with firms without subsidy. Once the subsidy is withdrawn, an industry stops with investment until the prices have grown sufficiently to attract investment without subsidy. While the subsidy is in effect, it can influence the industry's investment decisions such that it can increase total surplus.

An optimally set subsidy has a positive effect on welfare. The subsidy incentivizes the firms to invest earlier and more during the subsidy lifetime. However, if the social planner implements a too large subsidy, the total surplus can be lower compared to the scenario without subsidy. The optimal subsidy size increases with an industry's capacity, and decreases with the subsidy withdrawal risk.

As support schemes are considered crucial to induce investments, it is also important from the policy maker's point of view to implement appropriate policies. Both theoretical (Ritzenhofen and Spinler, 2016) and empirical (García-Álvarez et al., 2018) research attempt to determine the optimal subsidy design. The literature has studied the effect of subsidies on investments - with a strong focus on investments in renewable energy - during a subsidy lifetime, see, for example, Boomsma et al. (2012), Boomsma and Linnerud (2015), Chronopoulos et al. (2016), Dixit and Pindyck (1994, Chapter 9), Hassett and Metcalf (1993) and Nagy et al. (2021). Chronopoulos et al. (2016) study the effect of subsidy withdrawal risk of a price premium on the investment timing and size of a monopolist. When investment is lumpy, the effect of subsidy withdrawal risk on investment is ambiguous, and depends on whether the price premium is expected to be there for a long time after investment or not. They also compare a lumpy investment strategy with a sequential strategy, and find the firm invests in a larger capacity under the sequential strategy as the firm as more flexibility to adjust its capacity over time. However, if the firm does not have the flexibility over time, the firm's investment can be studied as a lumpy investment problem. Dixit and Pindyck (1994, Chapter 9), Hassett and Metcalf (1993) and Nagy et al. (2021) study a monopolist facing a one-time investment decision under a lump-sum subsidy subject to withdrawal risk. They all conclude the firm invests sooner under subsidy if the risk of subsidy withdrawal is larger. Nagy et al. (2021) conclude that this earlier investment goes at the cost of a lower investment size, as the firm invests at a lower output price. One question that has received little attention in the literature so far is the question what the consequences are on the level of investment in an industry after a subsidy has been terminated.

Another branch of literature studies the effect of support schemes on welfare, e.g. Kydland and Prescott (1977) and Willems and Zwart (2018), and their effect on a policy maker's targets, e.g. Nordhaus (2007) and Stern (2018). Kydland and Prescott (1977) conclude that a policy maker's flexibility to adjust its policy over time reduces welfare as forward-looking rational firms anticipate these decisions today. Willems and Zwart (2018) study the regulation of irreversible capacity expansion and focus on the role of private information of a firm on capacity costs. The social optimal policy depends on whether the private information has a small or a large support. However, Willems and Zwart (2018) do not study the effect of the risk of termination of a subsidy on investment behavior. The work by Nordhaus (2007), Stern (2018) and Keen (2020) is motivated by climate change and studies the effect of policy on specific climate targets.

This paper contributes to the literature by showing that a subsidy can increase total welfare in a dynamic oligopoly. We show that the policy maker can use the subsidy to achieve the so-called first-best solution when firms account for the risk of a subsidy being unavailable in the future. In other words, the subsidy works as tool to align the industry's investment with the social optimal investment. Current literature discusses the challenge of obtaining the first-best solution via the use of a policy (see, e.g., Dobbs (2004), Willems and Zwart (2018)), and often focuses on obtaining the second-best solution after concluding the first-best solution is unattainable for the policy maker. Furthermore, we provide new insights by studying the long-term effects of a subsidy, also studying what happens after subsidy withdrawal. A lump-sum subsidy is an effective tool of speeding up investment early, but this effect tapers off over time.

The remainder of this paper is structured as follows. The model is presented in Section 2. In Section 3, we solve the model with and without subsidy withdrawal risk. We study the optimal decision of an industry of profit-maximizing firms and provide comparative statics. Section 4 studies the optimal investment from a social planner perspective, as well as deriving the optimal subsidy. Section 5 provides a numerical case study. Section 6 concludes.

2 Model

We propose a theoretical framework that studies an industry's optimal investment decision under uncertain subsidy support. We consider an industry that currently produces K units. There are many risk-neutral, profit-maximizing firms that can invest in small, fixed-size project with a future revenue stream that is uncertain. The industry's capacity increases gradually and incrementally as firms install their projects.

The output price is denoted by $P(X, K)$. We assume the output price is given by:

$$P(X, K) = X(1 - \eta K), \tag{1}$$

where η is a positive constant[3]. The output price depends on both K, the industry's total production capacity, and $X(t)$, which represents exogenous shocks. The exogenous shocks are assumed to follow a geometric Brownian motion process given by:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t),\ X(0) = x, \tag{2}$$

where μ is the drift rate, σ the uncertainty parameter and $dW(t)$ the increment of a Wiener process.

The cost of one unit of investment is set equal to κ. Hence, increasing the production capacity from K to $K + dK$ yields an investment cost of $\kappa \cdot dK$ when no subsidy is in effect. A subsidy provides a discount at rate θ on the investment cost, so that the investment costs are then equal to $(1 - \theta)\kappa \cdot dK$.

Initially, the subsidy is assumed to be available, but can be withdrawn due to a random event such as the depletion of the public budget or a change in government. We assume that the industry's perceived risk of subsidy retraction due to randomness is an exponential jump process with parameter λ. This implies that the industry's perceived probability that the subsidy will be retracted in the next time interval dt is equal to λdt.

Next, we derive the objective of an industry, which is to maximize its total profit. Without loss of generality, we assume a current production capacity of $K(0) = k$. Each firm chooses when to install its project, i.e. the times τ_i, $i \in \mathbb{N}$, where $\tau_i \leq \tau_j$ for all $i \leq j$. We denote the capacity after the i-th increment by K_i:

$$K_i = K_{i-1} + dK = k + i \cdot dK. \tag{3}$$

We assume that when the firm installs its project, the industry's total capacity increases by fixed amount dK. An industry maximizes the producer surplus (PS) and its objective is given by:

$$\begin{aligned} F = \sup_{\tau_1, \tau_2, \ldots} PS(X, K) = \sup_{\tau_1, \tau_2, \ldots} \Bigg\{ \sum_{i=1}^{\infty} \mathbb{E}\Bigg[\int_{\tau_{i-1}}^{\tau_i} & P(X(t), K_i) \cdot K_i \cdot \exp(-rt)dt \\ & - (1 - \theta \cdot \mathbb{1}_{\xi(\tau_i)})\kappa \cdot dK \cdot \exp(-r\tau_i) \,\Bigg|\, X(\tau_{i-1}), \xi(\tau_{i-1}) \Bigg] \Bigg\}, \end{aligned} \tag{4}$$

where $\tau_0 = 0$ indicating the start of the planning horizon. Furthermore, $P(.)$ is some demand function and $\mathbb{1}_{\xi(t)}$ is an indicator function which takes value one if the subsidy is still available at time t and zero if not. As the subsidy is available at the start of the planning horizon, we have $\xi(0) = 1$.

In Corollary 1, we show that the problem in which the industry is maximizing its PS as defined in equation (4) is equivalent to the problem in which the industry maximizes the added value of each extra unit of capacity. Therefore, we solve the industry's problem to maximize its total profit by solving multiple, independent optimization problems of maximizing the added value of each capacity increment. This approach is preferred as it avoids dealing with dependencies between different capacity increments and, hence, is easier than directly solving (4). We provide the solution to this problem in the next section.

Corollary 1 *The industry's objective in equation* (4) *can be rewritten to:*

$$\begin{aligned} F = {} & \mathbb{E}\Bigg[\int_0^{\infty} P(X(t), k) \cdot k \cdot \exp(-rt)dt \,\Bigg|\, X(0) = x, \xi(0) = 1 \Bigg] \\ & + \sum_{i=1}^{\infty} \sup_{\tau_i} \Bigg\{ \mathbb{E}\Bigg[\int_{\tau_i}^{\infty} \Big(\Delta P_i(X(t)) \cdot K_{i-1} + P(X(t), K_i) \cdot dK \Big) \cdot \exp(-rt)dt \\ & - (1 - \theta \cdot \mathbb{1}_{\xi(\tau_i)})\kappa \cdot dK \cdot \exp(-r\tau_i) \,\Bigg|\, X(\tau_{i-1}), \xi(\tau_{i-1}) \Bigg] \Bigg\}, \end{aligned} \tag{5}$$

where $\Delta P_i(X(t))$ is the price change from increasing the capacity for the i-th time, i.e. $\Delta P_i(X(t)) = P(X(t), K_i) - P(X(t), K_{i-1})$.

[3] The inverse demand function (1) is a special case of the one used by Dixit and Pindyck (1994, Chapter 11), which assumes $P = XD(K)$ with an unspecified demand function $D(K)$, and is frequently used in the literature (see, e.g., Pindyck (1988), He and Pindyck (1992), and Huisman and Kort (2015)).

The proofs of all corollaries and propositions can be found in Appendix A.

Corollary 1 implies that maximizing the total project value is equivalent to maximizing the marginal revenue of each capacity increment. The intuition behind this is that when capacity is increased, there are only three relevant factors:

(i) the price change decreasing the marginal revenue of every unit of the current capacity, captured by the term $\Delta P_i(X(t)) \cdot K_{i-1}$;

(ii) the additional revenue from the capacity increment, captured by the term $P(X(t), K_i) \cdot dK$; and,

(iii) the investment cost of expanding the capacity, dependent on the availability of the subsidy and captured by the term $(1 - \theta \cdot \mathbb{1}_{\xi(\tau_i)})\kappa \cdot dK$.

As long as factors (i) and (iii) together outweigh (ii), it is optimal for an industry to wait with increasing its capacity.

Note that we did not use the fact that the demand function is assumed to be given by equation (1) in Corollary 4 nor in the corresponding proof. Therefore, the equivalence of the objective functions in equations (5) and (4) holds for any demand function.

In the remainder of this section, we derive the expression for the objective of the social planner, who maximizes the total surplus (TS). The total surplus consists of the sum of the producer surplus and the consumer surplus (CS):

$$TS = PS + CS. \tag{6}$$

Corollary 2 *The consumer surplus is given by:*

$$\begin{aligned} CS(X,K) &= \mathbb{E}\Big[\int_0^\infty \frac{1}{2}\eta X(t) \cdot k^2 \cdot \exp(-rt)dt \,\Big|\, X(0) = x\Big] \\ &\quad + \sum_{i=1}^{\infty} \mathbb{E}\Big[\int_{\tau_{i-1}}^\infty \frac{1}{2}\eta X(t)\big(2k + (2i-1)dK\big)dK \cdot \exp(-rt)dt \,\Big|\, X(\tau_{i-1})\Big]. \end{aligned} \tag{7}$$

The producer surplus under any demand function is derived in Corollary 1. The producer surplus under the demand function given by (1) is given by:

$$\begin{aligned} PS(X,K) &= \mathbb{E}\Big[\int_0^\infty X(t) \cdot (1 - \eta k) \cdot k \cdot \exp(-rt)dt \,\Big|\, X(0) = x\Big] \\ &\quad + \sum_{i=1}^{\infty} \Big\{ \mathbb{E}\Big[\int_{\tau_i}^\infty \Big(-\eta X(t) \cdot dK \cdot K_{i-1} + X(t) \cdot (1 - \eta K_i) \cdot dK \Big) \cdot \exp(-rt)dt \\ &\quad - \kappa \cdot dK \cdot \exp(-r\tau_i) \,\Big|\, X(\tau_{i-1})\Big]\Big\}. \end{aligned} \tag{8}$$

The social planner's objective is given by

$$\begin{aligned} TS(X,K) &= \mathbb{E}\Big[\int_0^\infty X(t) \cdot \Big(1 - \frac{1}{2}\eta k\Big) \cdot k \cdot \exp(-rt)dt \,\Big|\, X(0) = x\Big] \\ &\quad + \sum_{i=1}^{\infty} \Big\{ \mathbb{E}\Big[\int_{\tau_i}^\infty \Big(\eta X(t) \cdot dK^2 + X(t) \cdot (1 - \eta K_i) \cdot dK \Big) \cdot \exp(-rt)dt \\ &\quad - \kappa \cdot dK \cdot \exp(-r\tau_i) \,\Big|\, X(\tau_{i-1})\Big]\Big\}. \end{aligned} \tag{9}$$

From equation (9), note that the problem of the maximization of the total surplus can be rewritten as the sum of the maximization of added value of each independent extra extra unit of capacity. Therefore, the maximization of the total surplus can be rewritten into multiple independent maximization problems problems, which are easier to solve.

Furthermore, we find from equation (9) that the problem of maximization of the total surplus can be rewritten into maximizing the added surplus value of each independent capacity increment. Breaking down the expression for the total surplus, there is the total surplus if capacity remains at capacity $K = k$ forever. Note that the second term of the right-hand side of expression (9) consists of two parts:

(i) the consumer surplus increases as supply increases, while the producer's marginal revenue for its current production decreases as supply increases. The increase in consumer surplus dominates this negative effect on the producer surplus, and both effects are captured in the term $\eta X(t) \cdot dK^2$;

(ii) the producer obtains an additional profit from the additional unit of capacity dK that is sold against the price $X(t) \cdot (1 - \eta K_i)$.

A social planner maximizing total surplus will increase its capacity when the effects in (i) and (ii) jointly outweigh the investment cost of increasing capacity decreases the total surplus (third term of the right-hand side of (9)). Compared to the considerations of the profit-maximizing industry outlined in the discussion of Corollary 1, effects (ii) and (iii) are the same. The social planner and industry have different optimal decisions due to a difference in the effect of the increase of supply discussed in (i). For an industry, increasing the supply has a negative effect on the value of the current production (i.e. production at the level before the capacity increase) as it decreases the output price. For the social planner, this negative effect is offset by an increase in consumer surplus.

3 Investment and subsidy from an industry's point of view

3.1 The industry's investment without subsidy

We maximize the marginal revenue of the incremental investment, which also maximizes the value of the entire project, as shown in Corollary 1. Let V_1 (V_0) denote the value of the option to expand capacity with(out) the subsidy. The value of the industry's investment without subsidy under the demand function (1) is given by:

$$V_0(X_0, K) = \frac{x(1-\eta k)k}{r-\mu} + \sum_{i=1}^{\infty} \left(\frac{x}{X_0^i}\right)^{\beta_{01}} \cdot \left(\frac{X_0^i\big(1-\eta(2K_i - dK)\big)}{r-\mu} - \kappa\right) dK, \tag{10}$$

where X_0^i denotes the industry's optimal timing threshold without subsidy to do the i-th capacity increment. For convenience, we denote X_0 as the vector containing all X_0^i. Moreover, β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. β_{01} can be interpreted as a measure of the wedge between the industry's optimal threshold and the investment costs. Therefore, $\beta_{01} > 1$ holds and the value of β_{01} depends on the market uncertainty σ, market growth rate μ and the discount rate r.

We derive the option value for the gradual capacity expansion without subsidy using the same approach as (Dixit and Pindyck, 1994, Chapter 11). Using Itô calculus and the Bellman equation, it follows that

$$\frac{1}{2}\sigma^2 X^2 \cdot \frac{d^2 V_0(X,K)}{dX^2} + \mu X \cdot \frac{dV_0(X,K)}{dX} - rV_0(X,K) = 0 \tag{11}$$

should hold for the value of the option to expand capacity without the subsidy for the current value X of the GBM and K for the capacity. In this ODE, r is the discount rate. The solution to this ODE is given by:

$$V_0(X,K) = A_{01}(K) \cdot X^{\beta_{01}} + \frac{X(1-\eta K)K}{r-\mu}, \tag{12}$$

where $A_{01}(K)$ is a positive expression to be determined. The marginal revenue of the option with respect to capacity is given by:

$$\frac{dV_0(X,K)}{dK} = \frac{dA_{01}(K)}{dK} \cdot X^{\beta_{01}} + \frac{X(1-2\eta K)}{r-\mu}. \tag{13}$$

We follow the approach by Dixit and Pindyck (1994) and apply the value matching and smooth pasting condition to the objective (13) to derive the optimal investment threshold. The value matching condition tells us that when an industry decides to expand, its marginal revenue equals marginal costs. The smooth pasting guarantees the expression we value match is smooth with respect to the timing threshold X. The value matching and smooth pasting condition for the expansion threshold without subsidy are given by:

$$\frac{dA_{01}(K)}{dK} \cdot \left(X_0^i\right)^{\beta_{01}} + \frac{X_0^i(1-2\eta K_i)}{r-\mu} = \kappa, \tag{14}$$

$$\beta_{01} \cdot \frac{dA_{01}(K)}{dK} \cdot \left(X_0^i\right)^{\beta_{01}-1} + \frac{1-2\eta K_i}{r-\mu} = 0. \tag{15}$$

Dixit and Pindyck (1994) solve this system of equations and conclude that the optimal investment threshold without subsidy is given by:

$$X_0^i(K_i) = \frac{\beta_{01}}{\beta_{01}-1} \cdot \frac{(r-\mu)\kappa}{1-2\eta K_i}. \tag{16}$$

The expression $A_{01}(K)$ has to satisfy:

$$\frac{dA_{01}(K_i)}{dK} = -\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1} \cdot \left(\frac{1-2\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}}. \tag{17}$$

By integration, we get

$$A_{01}(K_i) = \left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1} \cdot \frac{1-2\eta K_i}{2\eta(\beta_{01}+1)} \cdot \left(\frac{1-2\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}} \tag{18}$$

$$= \frac{\kappa(1-2\eta K_i)}{2\eta(\beta_{01}-1)(\beta_{01}+1)} \cdot \left(\frac{(\beta_{01}-1)(1-2\eta K_i)}{\beta_{01}\kappa(r-\mu)}\right)^{\beta_{01}}. \tag{19}$$

3.2 The industry's investment under a subsidy subject to subsidy withdrawal risk

Similarly as in Section 3.1, the value of the industry's investment with subsidy is given by:

$$V_1(X_1, K) = \frac{x(1-\eta k)k}{r-\mu} + \sum_{i=1}^{\infty} \left(\frac{x}{X_1^i}\right)^{\beta_{11}} \cdot \left(\frac{X_1^i\big(1-\eta(2K_i - dK)\big)}{r-\mu} - (1-\theta)\kappa\right) dK, \tag{20}$$

where X_1^i denotes the industry's optimal timing threshold under subsidy to do the i-th capacity increment. For convenience, we denote X_1 as the vector containing all X_1^i. Furthermore, β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0$. β_{11} can be interpreted as a measure of the wedge between the industry's optimal investment threshold and the investment costs when the subsidy is available. The equation is similar to the expression for β_{01}, with the only difference being that β_{11} depends on λ. We have that $\beta_{11} > \beta_{01}$ as the risk of subsidy withdrawal decreases the wedge as there is a risk that investment costs significantly increase.

Next, we applying Ito's lemma to the value of the option to expand capacity with the subsidy and using the Bellman equation. The equation is given by:

$$\begin{aligned}&\frac{1}{2}\sigma^2 X^2 \cdot \frac{d^2V_1(X,K)}{dX^2} + \mu X \cdot \frac{dV_1(X,K)}{dX} - rV_1(X,K) + \\ &\qquad \lim_{dt\to 0} \mathbb{P}[\text{Subsidy withdrawal occurs in time interval } dt] \cdot \frac{1}{dt} \cdot (V_0(X,K) - V_1(X,K)) = 0.\end{aligned} \tag{21}$$

In this case, we also have to account for the risk of random policy withdrawal. We get:

$$\frac{1}{2}\sigma^2 X^2 \cdot \frac{d^2V_1(X,K)}{dX^2} + \mu X \cdot \frac{dV_1(X,K)}{dX} - rV_1(X,K) + \lambda(V_0(X,K) - V_1(X,K)) = 0. \tag{22}$$

The solution to (22) is given by:

$$V_1(X,K) = A_{11}(K) \cdot X^{\beta_{11}} + A_{01}(K) \cdot X^{\beta_{01}} + \frac{X(1-\eta K)K}{r-\mu}, \tag{23}$$

where $A_{11}(K)$ is a positive expression to be determined.

Similar to the case without subsidy, the optimal investment threshold follows from solving the system consisting of the value matching and smooth pasting condition. This gives us the following two equations:

$$\frac{dA_{11}(K)}{dK} \cdot X_1^{\beta_{11}} + \frac{dA_{01}(K)}{dK} \cdot X_1^{\beta_{01}} + \frac{X_1(1-2\eta K)}{r-\mu} = (1-\theta)\kappa, \tag{24}$$

$$\beta_{11} \cdot \frac{dA_{11}(K)}{dK} \cdot X_1^{\beta_{11}-1} + \beta_{01} \cdot \frac{dA_{01}(K)}{dK} \cdot X_1^{\beta_{01}-1} + \frac{1-2\eta K}{r-\mu} = 0. \tag{25}$$

Combining these two equations results in an implicit equation for our investment threshold X_1:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot \frac{dA_{01}(K)}{dK} \cdot X_1^{\beta_{01}} + \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1(1-2\eta K)}{r-\mu} - (1-\theta)\kappa = 0. \tag{26}$$

To our knowledge, implicit equation (26) does not have an analytical solution. In Section 5, we will solve this expression numerically.

We can derive how the optimal investment threshold changes with respect to the policy parameters. The following proposition states how the optimal investment decision is affected by subsidy retraction risk.

Proposition 1 *The optimal investment timing X_1 is negatively affected by the subsidy retraction risk λ.*

From Proposition 1, one may be tempted to conclude that a social planner interested in maximizing investment during the subsidy lifetime should maximize the subsidy withdrawal risk. However, increasing the withdrawal risk does not only lower the firms' investment threshold under subsidy, but it also shortens the expected lifespan of the subsidy. The shorter the lifespan of the subsidy, the less time there is for capacity to grow. Therefore, the actual impact of the increase subsidy withdrawal risk on an industry's capacity is ambiguous. The impact of increasing the subsidy withdrawal risk on an industry's capacity is studied in detail in Section 5.

Proposition 2 *The optimal investment timing X_1 is negatively affected by the subsidy size θ.*

An important advice for a social planner interested in maximizing investment during the subsidy lifetime follows from Proposition 2. This social planner should set the subsidy as large as possible, as this incentivizes the firms to invest at a lower threshold. Even though this maximizes the investment during the subsidy lifetime, it is also important for a policy maker to know what the impact of its policy is after withdrawal. The impact of a subsidy on investment after the subsidy lifetime is studied in detail in Section 5.

4 Investment and subsidy from the social planner's point of view

4.1 The social planner's investment

This subsection solves the investment problem from the perspective of the social planner, i.e. investment is made such that it maximizes total surplus. As noted at the end of Section 2, the maximization of the total surplus can be rewritten into smaller optimization problems in which the added value of each capacity increment is maximized.

Let V_S denote the value of the option to expand capacity for the social planner. The total surplus of the investment under the demand function (1) is given by:

$$V_S(X_S, K) = \frac{x(2-\eta k)k}{2(r-\mu)} + \sum_{i=1}^{\infty} \left(\frac{x}{X_S^i}\right)^{\beta_{01}} \cdot \left(\frac{X_S^i\big(2-\eta(2K_i - dK)\big)}{2(r-\mu)} - \kappa\right) dK, \tag{27}$$

where X_S^i denotes the social planner's optimal timing threshold to do the i-th capacity increment.

Repeating the steps in Section 3.1, it follows that the value of the social planner's option satisfies the following ODE:

$$\frac{1}{2}\sigma^2 X^2 \cdot \frac{d^2 V_S(X,K)}{dX^2} + \mu X \cdot \frac{dV_S(X,K)}{dX} - rV_S(X,K) = 0 \tag{28}$$

The marginal added surplus of the option with respect to capacity is given by:

$$\frac{dV_S(X,K)}{dK} = \frac{dA_S(K)}{dK} \cdot X^{\beta_{01}} + \frac{X(1-\eta K)}{r-\mu}, \tag{29}$$

in which V_S is the value of the social planner's option, $A_S(K)$ is some positive function.

We apply the value matching and smooth pasting condition to the objective (29) to derive the social optimal investment threshold. The value matching and smooth pasting condition for the social optimal expansion threshold (denoted by X_S) are given by:

$$\frac{dA_S(K)}{dK} \cdot X_S^{\beta_{01}} + \frac{X_S(1-\eta K)}{r-\mu} = \kappa, \tag{30}$$

$$\beta_{01} \cdot \frac{dA_S(K)}{dK} \cdot X_S^{\beta_{01}-1} + \frac{1-\eta K}{r-\mu} = 0. \tag{31}$$

We find that the optimal investment threshold without subsidy is given by:

$$X_S(K) = \frac{\beta_{01}}{\beta_{01}-1} \cdot \frac{(r-\mu)\kappa}{1-\eta K}. \tag{32}$$

The expression $A_S(K)$ has to satisfy:

$$\frac{dA_S(K)}{dK} = -\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1} \cdot \left(\frac{1-\eta K}{\beta_{01}(r-\mu)}\right)^{\beta_{01}}. \tag{33}$$

As before, we integrate to get

$$A_S(K) = \left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1} \cdot \frac{1-\eta K}{\eta(\beta_{01}+1)} \cdot \left(\frac{1-\eta K}{\beta_{01}(r-\mu)}\right)^{\beta_{01}} \tag{34}$$

$$= \frac{\kappa(1-\eta K)}{\eta(\beta_{01}-1)(\beta_{01}+1)} \cdot \left(\frac{(\beta_{01}-1)(1-\eta K)}{\beta_{01}\kappa(r-\mu)}\right)^{\beta_{01}}. \tag{35}$$

4.2 The social optimal policy

In practice, the social planner is not the decision maker regarding the investment, but it can only influence the firm making the investment decisions via setting a subsidy. This subsection studies the question how the subsidy should be set in order to maximize the total surplus under the industry's investment decisions.

If the social planner has the flexibility to change the subsidy size over time, the optimal subsidy size θ for a given capacity level can be derived. The following proposition states how the social planner maximizing total surplus should set its subsidy size.

Proposition 3 *The social planner maximizing surplus sets its subsidy size equal to:*

$$\theta^*(K) = 1 - \frac{1}{\beta_{11}(\beta_{01}-1)} \cdot \left[\beta_{01}(\beta_{11}-1) \cdot \frac{1-2\eta K}{1-\eta K} - (\beta_{11}-\beta_{01}) \cdot \left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}}\right], \tag{36}$$

where $K < \frac{1}{2\eta}$ is the industry's current capacity.

If there is no subsidy withdrawal risk (i.e. the subsidy is never withdrawn), the expression for the optimal subsidy size simplifies. The result is shown in Corollary 3.

Corollary 3 *Assuming no subsidy withdrawal risk (i.e. $\lambda = 0$), the social planner maximizing surplus sets its subsidy size equal to:*

$$\theta^*(K) = \frac{\eta K}{1-\eta K}, \tag{37}$$

where $K < \frac{1}{2\eta}$ is the industry's current capacity.

Corollary 3 implies that the social planner maximizing total surplus should increase its subsidy as the capacity grows. An industry has little incentive to increase its capacity when it is high as the output price is then low. From the social planner's perspective, the consumer surplus does still increase with capacity, so the social planner has an incentive to increase capacity. The subsidy then works as a tool to decrease investment costs that much such that the firm still has an incentive to increase capacity when the output prices are low.

Furthermore, Corollary 3 shows that the optimal subsidy rate is increasing in the market power parameter η. A firm with a lot of market power invests only very little to keep prices high. The social planner wants to see more capacity as it would increase consumer surplus, thus a significant subsidy is used to incentivize the firm to invest more.

Proposition 4 shows the effect of the industry's capacity on the socially optimal subsidy size for any level of subsidy retraction risk.

Proposition 4 *The socially optimal subsidy size $\theta^*(K)$ is positively affected by the industry's capacity K.*

From Proposition 4, it follows that a social planner optimally should install a larger subsidy when an industry's capacity is larger. The social planner maximizes the sum of consumer and producer surplus, while an industry deciding on investment only maximizes the latter. The marginal revenue for an industry from one additional unit of capacity is lower when the capacity is larger, as then the output price decreases with capacity. However, an additional unit of capacity does increase consumer surplus, and thus, the larger the capacity, the more the industry's optimal investment threshold exceeds the social planner's optimal investment threshold. A larger subsidy is necessary to bridge this gap.

Proposition 5 discusses the effect of the subsidy retraction risk on the socially optimal subsidy size for any level of industry's capacity.

Proposition 5 *The socially optimal subsidy size $\theta^*(K)$ is negatively affected by the subsidy retraction risk λ.*

It follows from Proposition 5 that a social planner optimally should install a smaller subsidy when the withdrawal risk is larger. When the withdrawal risk is larger, an industry installs an additional unit of capacity sooner, as it is afraid to lose the subsidy if it waits longer. This means that the optimal decision by an industry and the social planner are better aligned, as the social planner prefers early investment as it accounts for consumer surplus and producer surplus. As a result, the optimal subsidy size is set lower to completely align an industry's and the social planner's decision.

5 Numerical study

In this section, we discuss the effect of subsidy and the likelihood of subsidy withdrawal on an industry's investment decision. The parameter values are displayed in Table 1.

Notation	Parameter	Value
r	Risk-free interest rate	3%
μ	Price trend	1%
σ	Price volatility	5%
x	Demand intercept at $t = 0$	10
η	Demand parameter	0.005
κ	Variable investment cost	500
dK	Capacity increment	1

Table 1: Parameter values used in the iso-elastic demand scenario

5.1 Investment by an industry and capacity growth

In Figure 1(a), we plot the optimal investment threshold X_1 as a function of the current production capacity K for different levels of the subsidy termination intensity λ. For comparison, we also plot X_0, which is the optimal investment threshold without a subsidy and without subsidy termination risk, as well as X_S, the social optimal investment threshold. Apart from the investment threshold for a given level op capacity, this figure also has a second interpretation. When prices develop over time, the supremum of the demand intercept X gives us an industry's current investment capacity. Note that an industry's capacity is capped at $\frac{1}{\eta}$, as the price is negative for a capacity exceeding that level. With the parameter values in Table 1, the maximum capacity equals 100.

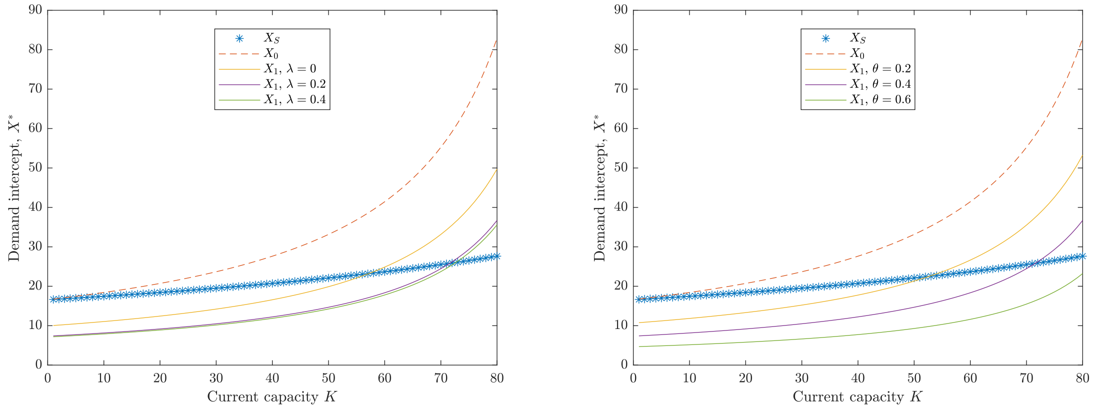
Our results indicate that an industry installs a larger capacity for a given path of the process X when the subsidy withdrawal risk is larger. As investment capacity is cheaper as long as the subsidy is in effect, an industry commits to a larger capacity sooner if it is threatened the subsidy will disappear soon. Therefore, a policy maker that aims to increase an industry's capacity can increase an industry's incentive to invest by threatening to withdraw the subsidy soon. However, the policy maker should keep in mind that the subsidy needs to be financed, hence it increases the policy maker's payouts via subsidy increase. Furthermore, if the policy maker threatens to take away the subsidy soon, but keeps the subsidy alive much longer than planned, an industry may change its behavior in the future. A future threat of withdrawing subsidy becomes less effective, as an industry learns from experience that the subsidy will be available longer than is announced.

we observe that an industry invests sooner with subsidy than without subsidy for a given capacity. A subsidy lowers the investment costs for an industry, hence lowering the threshold to investment. We also conclude that an industry's optimal capacity for a given demand intercept level is higher under subsidy than without. An industry has an incentive to already increase capacity early to guarantee the capacity increment is subsidized. This is in stark contrast with Nagy et al. (2021), who study a single firm having the option to do a lump-sum investment subject to an investment cost subsidy. The firm eligible for a subsidy optimally invests in a *smaller* capacity than a firm without subsidy as the firm can only invest once. In case of an incremental investment decision, the firm still has the flexibility to extend capacity after investment, while a firm facing one-time investment decision has not, which leads to a difference in results.

Considering the social optimal investment threshold X_S, we observe that it is less sensitive to the current capacity compared to the industry's threshold. The difference in objectives between an industry and the social planner is that the latter also includes the consumer surplus (CS). The CS increases with capacity as long as the GBM X has a positive value. Therefore, the social planner has an incentive to install a larger capacity already at lower output prices compared to an industry.

The effect of different subsidy sizes is studied in Figure 1(b). Firms that are eligible for a subsidy have a lower investment threshold than firms without subsidy. Firms with subsidy invest sooner as a unit of capacity is cheaper under subsidy. Furthermore, the larger the subsidy, the lower a firm's investment threshold.

A social planner interested in aligning an industry's investment threshold with the social optimal threshold can use a subsidy to do so. However, when the capacity is low, an industry's investment threshold is equal to the social optimal investment threshold. When the current capacity increases, the investment threshold of an industry increases, but the social planner's threshold remains approximately equal. The industry needs a larger output price to incentive investment as the marginal revenue of a unit increase of capacity decreases when the current capacity is larger. The social planner also accounts for CS, hence, the aforementioned effect for an industry is less dominant in the social planner's decision. The social planner can use a subsidy to align an industry's decision with the social optimal decision when the current capacity is larger. The subsidy incentivizes the industry to increase capacity at a lower value of the GBM X, while without subsidy an industry would wait longer than the social planner wants. The larger the current capacity K, the larger the subsidy required to align the industry's and social planner's threshold.



((a)) Subsidy size $\theta = 0.4$. ((b)) Subsidy termination risk $\lambda = 0.2$.

Figure 1: Investment timing as a function of the current production capacity K for different levels of subsidy termination risk λ (left) and for different subsidy size θ (right); both figures including social optimal decision X_S. [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 500$, $dK = 1$.]

Next, we run 10,000 simulations[4] to see how an industry invests over time, and how this depends on subsidy withdrawal risk, subsidy size and the time of subsidy withdrawal. An example of two simulation runs, labelled A and B respectively, of the demand intercept X and the corresponding industry's capacity over time are shown in Figure 2.

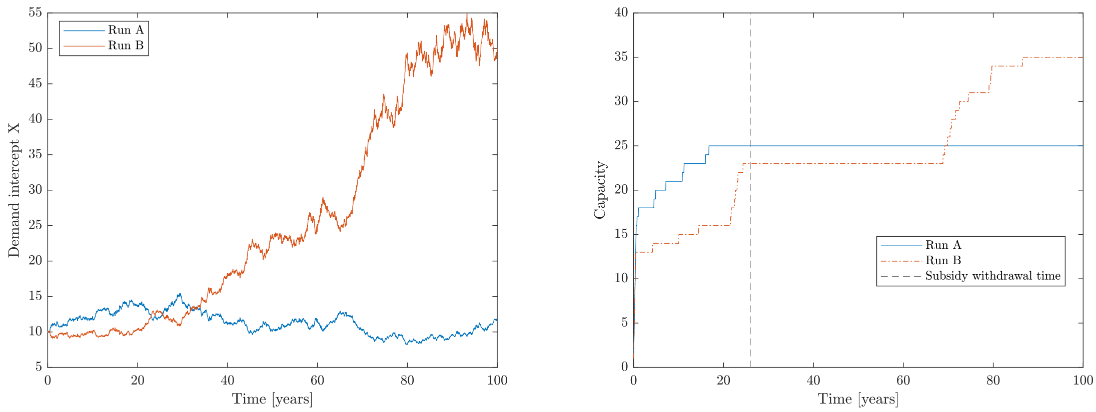


Figure 2: An example of simulated GBM X (left) and the industry's capacity (right). [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 500$, $dK = 1$, $x = 10$, $\lambda = 0.2$, $\theta = 0.4$.]

Most simulation runs can be broken down in three parts: (1) an industry's total capacity grows while the subsidy is available, (2) the capacity stagnates after subsidy withdrawal, and (3) once the output prices

[4] The simulation of the geometric Brownian motion in (2) is done using $X(t_i) = X(t_{i-1}) \cdot \exp(\mu - \frac{\sigma^2}{2}) \cdot dt + \sigma\sqrt{dt} \cdot W_i$ for all moments in time t_i. W_i a draw from the standard normal distribution, and t_i, t_{i-1} are two consecutive moments in time with step size dt. We use antithetic variables for the simulation of the geometric Brownian motion, thus for a simulation with draws W_1, W_2, ..., a run with $-W_1, -W_2, \ldots$ is performed. The time of subsidy withdrawal, τ_S, is randomly regenerated via the inverse cdf of a Poisson jump: $\tau_S = -\frac{\log(1-Z)}{\lambda}$, where Z is a draw from the standard normal distribution. We drew 5,000 simulations of the subsidy withdrawal and used the same withdrawal time for the two runs that are linked via the usage of antithetic variables.

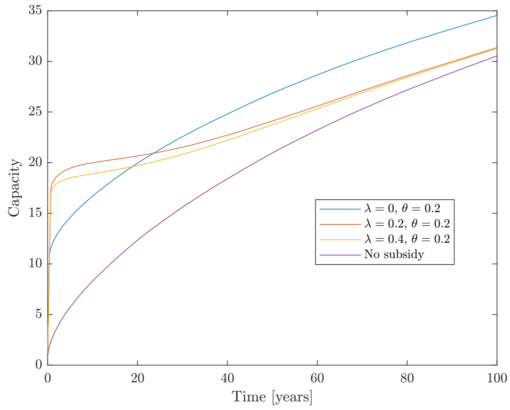
reach a sufficiently high level, the capacity grows while the subsidy is unavailable. In Figure 2, Simulation run A does not reach sufficiently high output prices to attract investment during the time after subsidy withdrawal. These three parts result from an industry increasing the capacity sooner under a subsidy than without. An industry's investment threshold increases at the time the subsidy is withdrawn, as its marginal cost of investment increases. As a result, it does not invest directly after subsidy withdrawal and waits with investment until the output prices are significantly larger.

In Figure 3, we show the average capacity over time of 10,000 simulations for different subsidy sizes and withdrawal risk. On the left (right), we show the expected capacity of the simulations for different subsidy withdrawal risk λ for a subsidy size of $\theta = 0.2$ ($\theta = 0.4$).

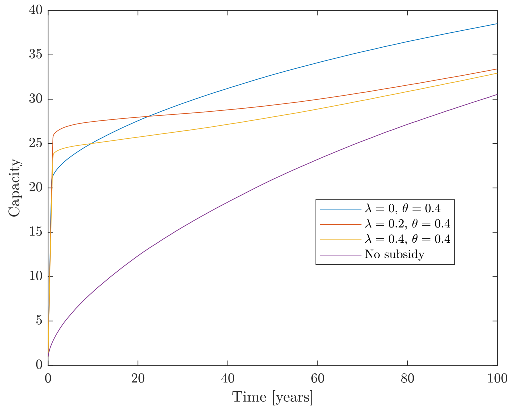
Comparing the scenarios with and without subsidy first, we observe that the industry's capacity under subsidy is larger than without subsidy. This effect is most prominent if the subsidy is provided forever, i.e. there is no withdrawal risk. In case of a subsidy subject to withdrawal risk, the positive effect on the capacity also remains for some time after subsidy withdrawal, but fades after some time.

Secondly, looking at the role of subsidy size, an industry increases its capacity more during the subsidy lifetime when the subsidy is larger in size. However, an industry's total capacity grows at a slightly lower rate once the subsidy is withdrawn. This is the result from the fact that an industry invests more when building capacity is cheaper; it will invest less at the time when the subsidy has been withdrawn and investment costs have increased as a result.

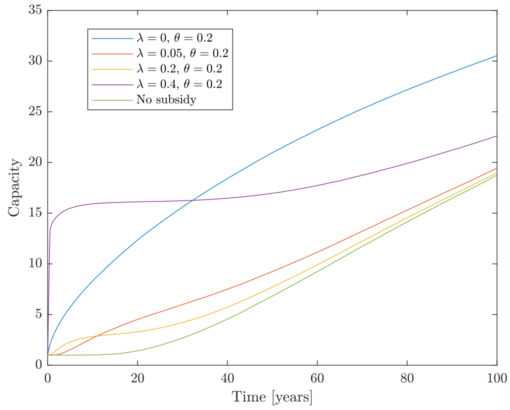
Finally, we discuss the role of subsidy withdrawal risk on an industry's total capacity over time. An industry increases its capacity more during the subsidy lifetime when the subsidy withdrawal risk is higher. As an industry anticipates the future withdrawal of the subsidy and the resulting increase of the investment cost, an industry moves investment it would usually do at the mid-term (10 – 20 years) to the short term (less than 10 years). As a result, the capacity on the short term under subsidy is higher when the expected life span of the subsidy is shorter. However, the threat of the subsidy being unavailable at the mid-term results in little to no investment in expectation on the midterm. Hence, the capacity in the mid-term under a subsidy with a longer life span is higher than under a subsidy with a shorter life span. This effect remains for the long-term as well, until the time at which the effect of the subsidy completely has faded after approximately 40 to 50 years.



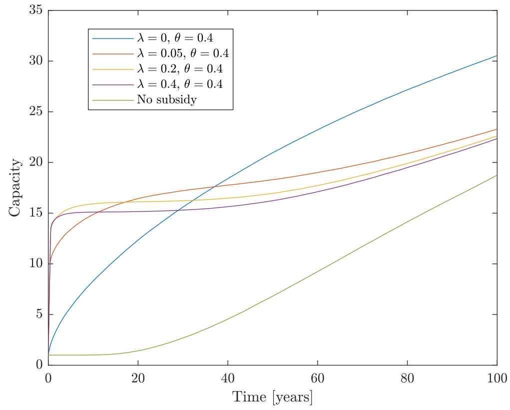
((a)) $\kappa = 300$.



((b)) $\kappa = 300$.



((c)) $\kappa = 500$.

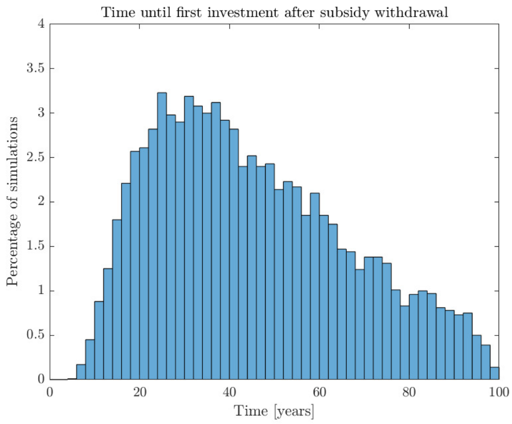


((d)) $\kappa = 500$.

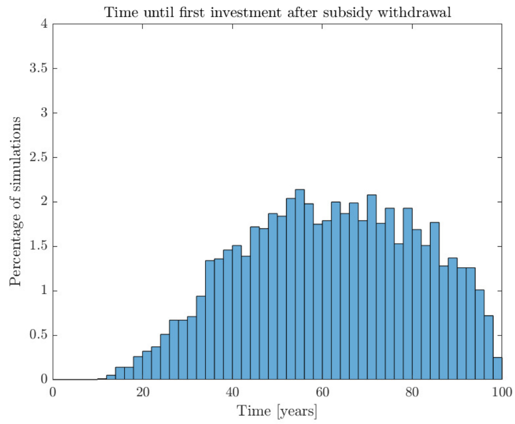
Figure 3: Expected industry's total capacity over time for different subsidy characteristics. [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $dK = 1$, $x = 10$.]

In Figure 4, we show the histograms of the amount of years it takes an industry to increase its capacity for the first time after the subsidy has been withdrawn. As shown in Figure 2, an industry does not increase its capacity for quite some time right after subsidy withdrawal, as the investment costs for the firms has increased, while the output prices do not jump upwards at that time. This happens especially especially when the subsidy size is large. When $\theta = 0.2$, only 1706 (1625) simulations result in no investment when $\lambda = 0.2$ ($\lambda = 0.4$). The effect is more dominant when $\theta = 0.4$: 4232 (3980) simulations result in no investment when $\lambda = 0.2$ ($\lambda = 0.4$).

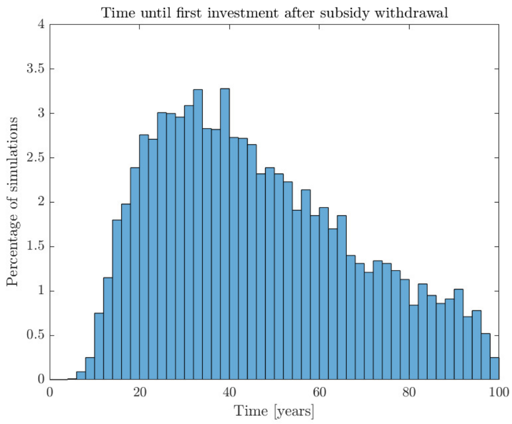
The larger the subsidy size or the larger the likelihood of subsidy retraction, the longer the period without investment after subsidy withdrawal. Both a larger subsidy size and a larger subsidy withdrawal risk increase an industry's investment during the subsidy lifetime. As a result, an industry's capacity is at a higher level at the time of subsidy withdrawal. Once the subsidy is withdrawn, the investment costs for an industry jumps upwards, while the output prices remain at approximately the same level as at the end of the subsidy lifetime. The higher an industry's capacity at the time of subsidy withdrawal, the longer it takes the output prices to grow to level to attract investment without subsidy.



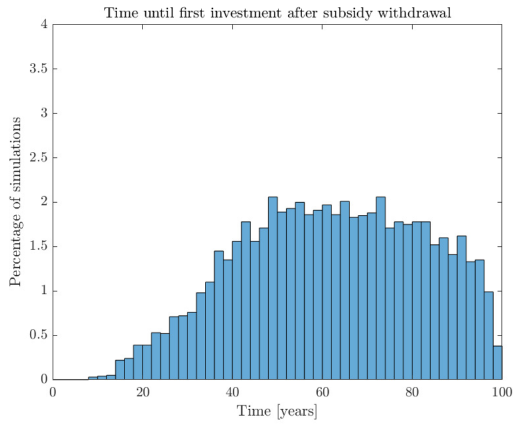
((a)) $\lambda = 0.2$, $\theta = 0.2$.



((b)) $\lambda = 0.2$, $\theta = 0.4$.



((c)) $\lambda = 0.4$, $\theta = 0.2$.



((d)) $\lambda = 0.4$, $\theta = 0.4$.

Figure 4: Histograms of the time until the first investment after subsidy withdrawal for different levels of subsidy termination risk λ and subsidy size θ. [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 500$, $dK = 1$, $x = 10$.]

5.2 Total surplus

Next, we look at the total surplus (TS) and the optimal subsidy size. The optimal subsidy size as a function of the industry's capacity K for different levels of subsidy withdrawal risk λ are plotted in Figure 5. Note that the optimal subsidy size increases in the industry's capacity and decreases with the subsidy withdrawal risk, hence, the results are consistent with Propositions 4 and 5.

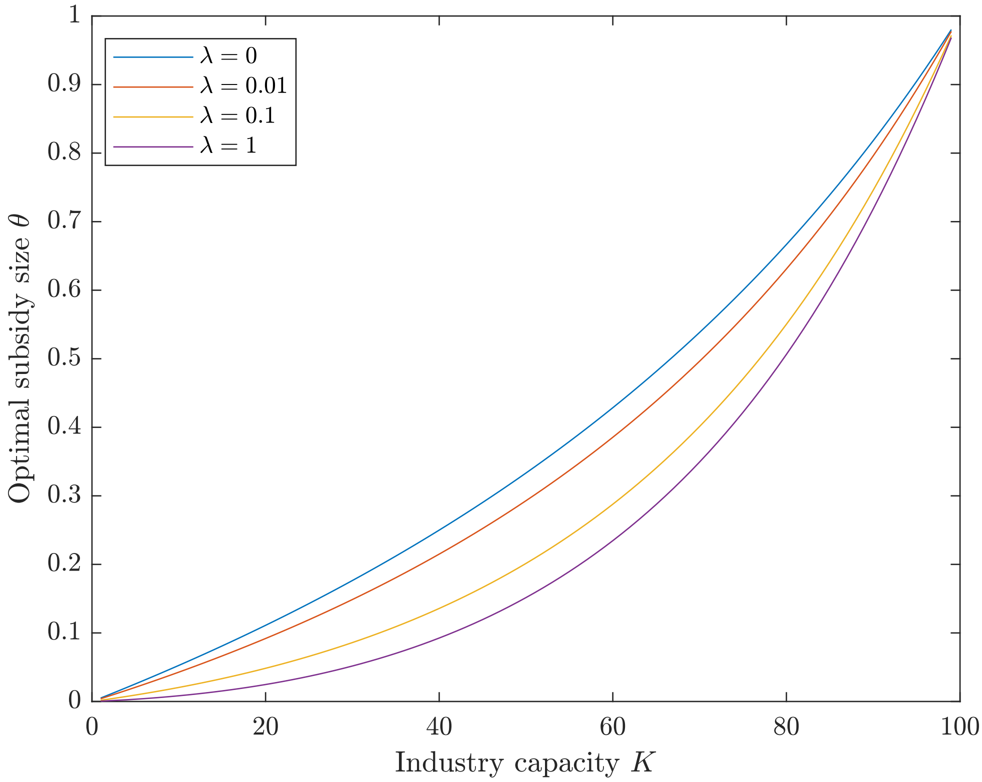


Figure 5: Optimal subsidy size θ as a function of the industry's total capacity for different subsidy retraction risk λ. [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 350$, $dK = 1$, $x = 10$.]

In Figure 6, the TS under the decisions by competitive firms without a subsidy is shown on the left, and the TS under the decisions by a social planner is shown on the right. The TS is broken down into producer surplus (PS) and consumer surplus (CS) in both figures. Under the social planner's decisions, the CS is larger than under the firm's decisions and the PS is around zero. When the firms in the competitive industry make the decisions, the maximize PS and the CS is much smaller than the PS; the firms increase capacity at a lower rate than the social planner to keep output prices higher than desirable from a social optimal point of view.

In Figure 7 shows the TS under the decisions by competitive firms for a given subsidy on the left (subsidy size $\theta = 0.2$, subsidy withdrawal risk $\lambda = 0.2$), and the gain in TS compared to the no subsidy case on the right. In 68.93% of the simulations, the TS increases due to a subsidy. The firm invests sooner under a subsidy, hence the CS increases compared to no subsidy. The subsidy does also increase the PS, but the increase in PS are mainly financed from the subsidy, hence the subsidy payouts of the social planner increase at approximately the same rate. As the firm invests sooner under a subsidy, the CS increases compared to no subsidy.

However, also note that the subsidy is not successful at increasing the TS in all simulations. In 30.55% of the simulations the TS decreases due to the subsidy and in 0.52% no changes to the TS occur. As the subsidy causes the firm to increase capacity sooner, there are cases in which the prices go down quickly after the firm has increased capacity. This may lead to significant losses to the firms that have invested, leading to a negative PS. It also keeps out new investors, causing an industry's capacity to be low, hence the CS is low as well.

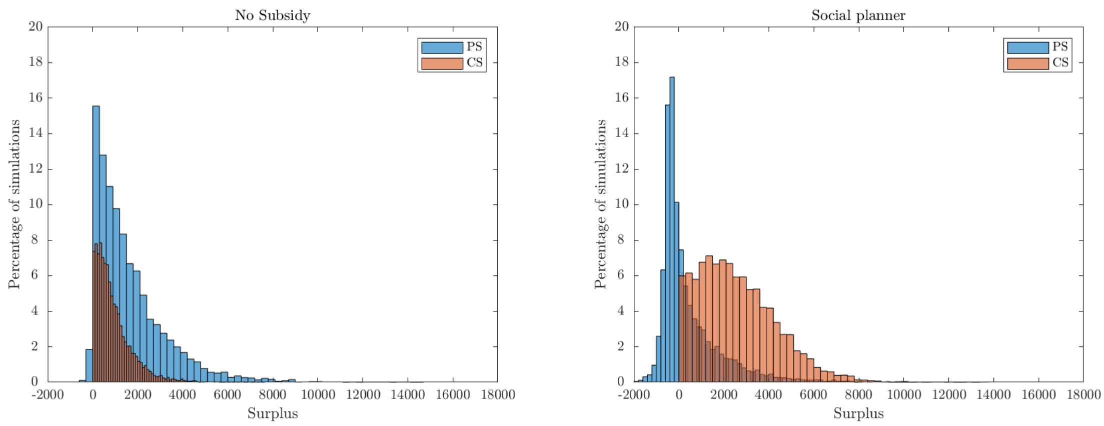
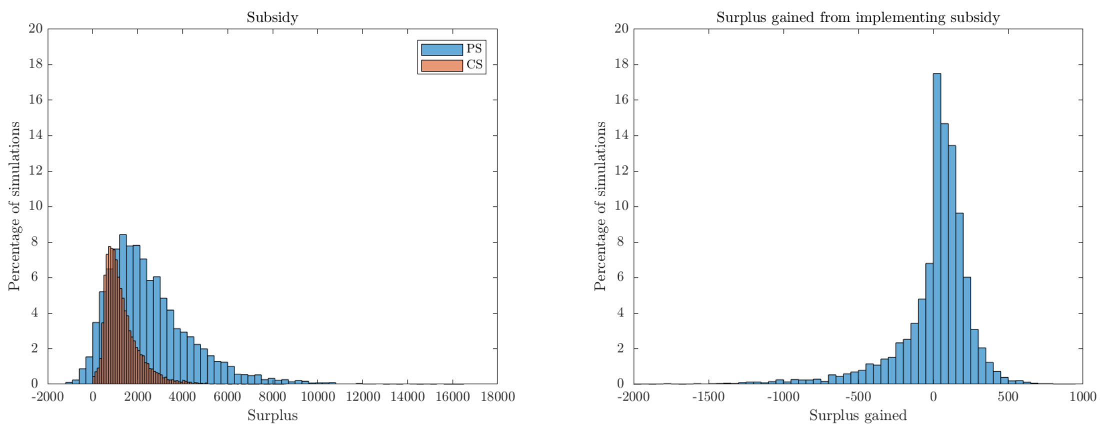


Figure 6: Distribution of PS and CS in the simulations when investment decisions are made by the industry without subsidy (left) and by the social social planner (right). [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$.]



((a)) CS and PS under $\lambda = 0.2$ and $\theta = 0.2$.

((b)) TS gained from subsidy.

Figure 7: Distribution of PS and CS in the simulations when investment decisions are made by the industry under subsidy (left) and the gain in TS compared to a not subsidized industry (right). [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$.]

6 Conclusion

This paper studies the effect of a lump-sum subsidy subject to withdrawal risk on an industry's investment. The social planner determines the size of the subsidy, but not the timing of its termination. The firms in the industry have to determine when to invest. The investment is irreversible and incremental. We study both the problem of the profit-maximizing firms, as well as the social planner that aims to maximize welfare.

Studying the firms' problem, we find that a firm invests sooner when the likelihood of subsidy withdrawal or the subsidy size is larger. Compared to a scenario in which no subsidy is implemented, an industry invests more during the lifetime of the subsidy, but investment slows down after the subsidy has been withdrawn. Once the subsidy is withdrawn, an industry stops with investment until the prices have grown sufficiently to attract investment without subsidy. This means that a policy maker can use subsidy to attract investment in the short-term, but this effect of the subsidy tapers off over time.

Looking at the social planner's problem of welfare maximization, we conclude that a subsidy increases expected total welfare if set optimally. When firms account for the risk of a subsidy being withdrawn in the future, the policy maker can use the subsidy as tool to align the industry's investment with the social optimal investment. However, if the social planner implements a too large subsidy, the total surplus can be lower compared to the scenario without subsidy. The optimal subsidy size increases with an industry's capacity.

Total welfare increases with capacity, but the firm's marginal revenue decreases with capacity, and the subsidy compensates the firm for this gap in revenue. Furthermore, the optimal subsidy size decreases with the subsidy withdrawal risk. The subsidy is a tool that aligns the decisions of the firm with those of the social planner. The discrepancy between the firm and the social planner decreases when the subsidy withdrawal risk increases.

A Proofs of theorems and propositions

A.1 Proof of Corollary 1

Let $\Delta P_i(X)$ be the price change from increasing the capacity for the i-th time, i.e. $\Delta P_i(X) = P(X\cdot, K_i) - P(X\cdot, K_{i-1})$. The objective can be rewritten as follows:

$$\begin{aligned} F &= \sup_{\tau_1,\tau_2,\dots} \mathbb{E}\Bigg[\sum_{i=0}^{\infty}\int_{\tau_i}^{\tau_{i+1}} P(X,K_i)\cdot K_i \cdot \exp(-rt)dt - \sum_{i=1}^{\infty}(1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot \exp(-r\tau_i)\,\Bigg|\, X(\tau_i),\xi(\tau_i)\Bigg] \\ &= \sup_{\tau_1,\tau_2,\dots} \mathbb{E}\Bigg[\int_0^{\infty} P(X,k)\cdot k\cdot \exp(-rt)dt + \sum_{i=1}^{\infty}\int_{\tau_i}^{\infty}\Big(\Delta P_i(X)\cdot K_{i-1} + P(X,K_i)\cdot dK\Big)\cdot \exp(-rt)dt \\ &\qquad - \sum_{i=1}^{\infty}(1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot \exp(-r\tau_i)\,\Bigg|\, X(0)=x,\xi(0)=1\Bigg] \end{aligned} \tag{38}$$

$$\begin{aligned} &= \mathbb{E}\Bigg[\int_0^{\infty} P(X,k)\cdot k\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x,\xi(0)=1\Bigg] \\ &\quad + \sum_{i=1}^{\infty}\sup_{\tau_i}\Bigg\{\mathbb{E}\Bigg[\int_{\tau_i}^{\infty}\Big(\Delta P_i(X)\cdot K_{i-1} + P(X,K_i)\cdot dK\Big)\cdot \exp(-rt)dt \\ &\quad - (1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot \exp(-r\tau_i)\,\Bigg|\, X(0)=x,\xi(0)=1\Bigg]\Bigg\}. \end{aligned} \tag{39}$$

A.2 Proof of Corollary 2

The consumer surplus is calculated by taking the expectation and the integral over the instantaneous consumer surplus (ICS) (see, e.g. Huisman and Kort (2015)). The instantaneous consumer surplus is given by:

$$ICS(X,K) = \int_{P(X,K)}^{X} D(P)dP \tag{40}$$

$$= \frac{1}{2}\eta X K^2. \tag{41}$$

where $D(P)$ is the demand function, i.e. the inverse of (1). The consumer surplus can be derived as follows:

$$CS(X,K) = \mathbb{E}\Bigg[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot K_{i-1}^2\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg] \tag{42}$$

$$= \mathbb{E}\Bigg[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot \big(k^2 + 2k(i-1)dK + (i-1)^2 dK^2\big)\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg] \tag{43}$$

$$\begin{aligned} &= \mathbb{E}\Bigg[\int_0^{\infty}\frac{1}{2}\eta X(t)\cdot k^2\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg] \\ &\quad + \mathbb{E}\Bigg[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot \big(2k(i-1) + (i-1)^2 dK\big)dK\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg] \end{aligned} \tag{44}$$

$$\begin{aligned} &= \mathbb{E}\Bigg[\int_0^{\infty}\frac{1}{2}\eta X(t)\cdot k^2\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg] \\ &\quad + \sum_{i=1}^{\infty}\mathbb{E}\Bigg[\int_{\tau_{i-1}}^{\infty}\frac{1}{2}\eta X(t)\cdot \big(2k + (2i-1)dK\big)dK\cdot \exp(-rt)dt\,\Bigg|\, X(0)=x\Bigg]. \end{aligned} \tag{45}$$

The producer surplus under any demand function is derived in Corollary 1. The producer surplus under the demand function given by (1) is given by:

$$\begin{aligned}\sup_{\tau_1,\tau_2,\ldots} PS(X,K) &= \mathbb{E}\bigg[\int_0^\infty X(t)\cdot(1-\eta k)\cdot k\cdot \exp(-rt)dt \,\bigg|\, X(0)=x\bigg]\\ &+\sum_{i=1}^{\infty}\sup_{\tau_i}\bigg\{\mathbb{E}\bigg[\int_{\tau_i}^\infty\Big(-\eta X(t)\cdot dK\cdot K_{i-1}+X(t)\cdot(1-\eta K_i)\cdot dK\Big)\cdot\exp(-rt)dt \\ &-\kappa\cdot dK\cdot\exp(-r\tau_i)\,\bigg|\,X(0)=x\bigg]\bigg\}.\end{aligned}\tag{46}$$

A.3 Proof of Proposition 1

We refer to the implicit equation (26) as $f(X_1)$:

$$f(X_1)\equiv\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K)}{dK}\cdot X_1^{\beta_{01}}+\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-2\eta K)}{r-\mu}-(1-\theta)\kappa=0.\tag{47}$$

By total differentiation, we derive:

$$0=\frac{df}{d\lambda}=\frac{\partial f}{\partial\lambda}+\frac{\partial f}{\partial X}\cdot\frac{\partial X}{\partial\lambda}\iff\frac{\partial X}{\partial\lambda}=-\frac{(\frac{\partial f}{\partial\lambda})}{(\frac{\partial f}{\partial X})}.\tag{48}$$

We are going to show that $\frac{df}{d\lambda}<0$ by showing that both $\frac{\partial f}{\partial\lambda}>0$ and $\frac{\partial f}{\partial X}>0$. First, we prove $\frac{\partial f}{\partial\lambda}>0$. By directly differentiating (47) with respect to λ, we derive:

$$\begin{aligned}\frac{\partial f}{\partial\lambda}&=\frac{1}{\beta_{11}^2}\cdot\frac{d\beta_{11}}{d\lambda}\cdot\bigg(\beta_{01}\cdot\frac{dA_{01}(K)}{dK}\cdot X_1^{\beta_{01}}+\frac{X_1(1-2\eta K)}{r-\mu}\bigg)&(49)\\ &=-\frac{1}{\beta_{11}}\cdot\frac{d\beta_{11}}{d\lambda}\cdot\frac{dA_{11}(K)}{dK}\cdot X_1^{\beta_{11}},&(50)\end{aligned}$$

where $\frac{d\beta_{11}}{d\lambda}=\frac{1}{\sigma(\beta_{11}-1)+\mu}>0$. $\frac{\partial f}{\partial\lambda}>0$ follows from $\frac{dA_{11}(K)}{dK}<0$.

Secondly, it remains to be proven that $\frac{\partial f}{\partial X}>0$ for any λ. The expression for $\frac{\partial f}{\partial X}$ is given by:

$$\frac{\partial f}{\partial X}=\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K)}{dK}\cdot\beta_{01}\cdot X_1^{\beta_{01}-1}+\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{1-2\eta K}{r-\mu},\tag{51}$$

where $\frac{dA_{01}(K)}{dK}=-\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\left(\frac{1-2\eta K}{\beta_{01}(r-\mu)}\right)^{\beta_{01}}<0$. We rewrite the condition $\frac{\partial f}{\partial X}>0$ using the expressions for $\frac{\partial f}{\partial X}$ and $\frac{dA_{01}(K)}{dK}$ to:

$$(\beta_{11}-\beta_{01})\cdot\left(\frac{\beta_{01}-1}{\beta_{01}}\cdot\frac{1-2\eta K}{\kappa(r-\mu)}\cdot X_1\right)^{\beta_{01}-1}<\beta_{11}-1.\tag{52}$$

By recognizing the expression for the investment threshold without subsidy X_0 on the left hand side, we can rewrite this as:

$$(\beta_{11}-\beta_{01})\cdot\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1}<\beta_{11}-1.\tag{53}$$

Note that this expression holds for $\lambda=0$, as then $\beta_{11}=\beta_{01}>1$ and $X_1=(1-\theta)X_0<X_0$. Therefore, $\frac{\partial f}{\partial\lambda}<0$ at $\lambda=0$. For $\lambda>0$ (hence $\beta_{11}>\beta_{01}$), we can rewrite the condition to:

$$\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1}<\frac{\beta_{11}-1}{\beta_{11}-\beta_{01}}.\tag{54}$$

This condition always holds for positive λ as then both $\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1}<1$, while $\frac{\beta_{11}-1}{\beta_{11}-\beta_{01}}>1$. To see $\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1}<1$, note that at $\lambda=0$, we have $X_1<X_0$ and $\frac{df}{d\lambda}<0$. Therefore, at some small positive λ, we see that X_1 is lower, hence $X_1<X_0$ still holds and condition (54) holds, leading to $\frac{df}{d\lambda}<0$ at that positive value of λ.

A.4 Proof of Proposition 2

Similar to the proof of Proposition 1, the derivative of the optimal investment threshold with respect to subsidy size θ can be written as:

$$\frac{\partial X}{\partial \theta} = -\frac{(\frac{\partial f}{\partial \theta})}{(\frac{\partial f}{\partial X})}. \tag{55}$$

We directly derive $\frac{\partial f}{\partial \theta}$ by differentiation of the implicit equation (47):

$$\frac{\partial f}{\partial \theta} = \kappa. \tag{56}$$

Therefore,

$$\frac{\partial X}{\partial \theta} = -\frac{\kappa}{(\frac{\partial f}{\partial X})}, \tag{57}$$

and

$$\frac{\partial X}{\partial \theta} < 0 \Longleftrightarrow \frac{\partial f}{\partial X} > 0. \tag{58}$$

Proving $\frac{\partial f}{\partial X} > 0$ for any θ can be done in the same way as proving $\frac{\partial f}{\partial X} > 0$ for any λ, see the second half of the proof of Proposition 1.

A.5 Proof of Proposition 3

Solving the industry's optimal investment threshold for a subsidy of size θ for any level of withdrawal risk follows from the implicit equation (26). Substituting the social planner's optimal investment threshold for maximizing total surplus, defined in (32), into equation (26) and solving for θ gives:

$$\theta^*(K) = 1 - \frac{1}{\kappa} \cdot \left(\frac{\beta_{11} - \beta_{01}}{\beta_{11}} \cdot \frac{dA_{01}(K)}{dK} \cdot X_S^{\beta_{01}} + \frac{\beta_{11} - 1}{\beta_{11}} \cdot \frac{X_S(1 - 2\eta K)}{r - \mu} \right). \tag{59}$$

Plugging in the social optimal investment threshold X_S yields:

$$\theta^*(K) = 1 - \frac{1}{\beta_{11}(\beta_{01} - 1)} \cdot \left[\beta_{01}(\beta_{11} - 1) \cdot \frac{1 - 2\eta K}{1 - \eta K} - (\beta_{11} - \beta_{01}) \cdot \left(\frac{1 - 2\eta K}{1 - \eta K} \right)^{\beta_{01}} \right]. \tag{60}$$

A.6 Proof of Corollary 3

Solving the industry's optimal investment threshold for a subsidy of size θ without any withdrawal risk yields:

$$X_1(K) = \frac{\beta_{01}}{\beta_{01} - 1} \cdot \frac{(r - \mu)(1 - \theta)\kappa}{1 - 2\eta K}. \tag{61}$$

As shown in Section 4.1, the social planner's optimal investment threshold for maximizing total surplus is given by:

$$X_S(K) = \frac{\beta_{01}}{\beta_{01} - 1} \cdot \frac{(r - \mu)\kappa}{1 - \eta K}. \tag{62}$$

Solving $X_0(K) = X_S(K)$ for θ gives the stated expression.

Alternatively, one can derive the stated expression by substituting $\lambda = 0$ into (36), using that $\beta_{11} = \beta_{01}$ when $\lambda = 0$.

A.7 Proof of Proposition 4

Taking the derivative with respect to K of the optimal subsidy size $\theta^*(K)$ defined in (36) gives:

$$\frac{d\theta^*}{dK} = \frac{\beta_{01}}{\beta_{11}} \cdot \frac{\beta_{11} - 1}{\beta_{01} - 1} \cdot \frac{\eta}{(1 - \eta K)^2} \cdot \left[1 - \frac{\beta_{11} - \beta_{01}}{\beta_{11} - 1} \cdot \left(\frac{1 - 2\eta K}{1 - \eta K} \right)^{\beta_{01} - 1} \right]. \tag{63}$$

As $\beta_{11} > \beta_{01} > 1$ and $\left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1} \in (0, 1]$, we have that $\frac{d\theta^*}{dK} > 0$.

A.8 Proof of Proposition 5

Taking the derivative with respect to λ of the optimal subsidy size $\theta^*(K)$ defined in (36) gives:

$$\frac{d\theta^*}{d\lambda} = \frac{d\beta_{11}}{d\lambda} \cdot \frac{\beta_{01}}{\beta_{01}-1} \cdot \frac{1}{\beta_{11}^2} \cdot \frac{1-2\eta K}{1-\eta K} \cdot \left[\left(\frac{1-2\eta K}{1-\eta K} \right)^{\beta_{01}-1} - 1 \right]. \tag{64}$$

As $\frac{d\beta_{11}}{d\lambda} = \frac{1}{\sigma(\beta_{11}-1)+\mu} > 0$ and $\left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1} \in (0,1]$, we have that $\frac{d\theta^*}{d\lambda} \leq 0$.

References

Boomsma, T. K. and Linnerud, K. (2015). Market and policy risk under different renewable electricity support schemes. European Journal of Operations Research, 89:435–448.

Boomsma, T. K., Meade, N., and Fleten, S.-E. (2012). Renewable energy investments under different support schemes: A real options approach. European Journal of Operations Research, 220(1):225–237.

Chronopoulos, M., Hagspiel, V., and Fleten, S.-E. (2016). Stepwise green investment under policy uncertainty. International Association for Energy Economics, 37(4):87–108.

Dixit, A. K. and Pindyck, R. S. (1994). Investment under uncertainty. Princeton university press.

Dobbs, I. M. (2004). Intertemporal price cap regulation under uncertainty. The Economic Journal, 114(495):421–440.

García-Álvarez, M. T., Cabeza-García, L., and Soares, I. (2018). Assessment of energy policies to promote photovoltaic generation in the European Union. Energy, 151:864–874.

Hassett, K. A. and Metcalf, G. E. (1993). Energy conservation investment: Do consumers discount the future correctly? Energy Policy, 21(6):710–716.

He, H. and Pindyck, R. S. (1992). Investments in flexible production capacity. Journal of Economic Dynamics and Control, 16:575–599.

Huisman, K. J. and Kort, P. M. (2015). Strategic capacity investment under uncertainty. RAND Journal of Economics, 46(2):376–408.

Keen, S. (2020). The appallingly bad neoclassical economics of climate change. Globalizations, pages 1–29.

Kydland, F. E. and Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. Journal of Political Economy, 85(3):473 – 492.

Nagy, R. L. G., Hagspiel, V., and Kort, P. M. (2021). Green capacity investment under subsidy withdrawal risk. Energy Economics, 98.

Nordhaus, W. D. (2007). A review of the stern review on the economics of climate change. Journal of Economic Literature, 45(3):686–702.

Pindyck, R. S. (1988). Irreversible investment, capacity choice, and the value of the firm. American Economic Review, 78:969–985.

Ritzenhofen, I. and Spinler, S. (2016). Optimal design of feed-in-tariffs to stimulate renewable energy investments under regulatory uncertainty - a real options analysis. Energy Economics, 53:76 – 89.

Stern, N. (2018). Public economics as if time matters: Climate change and the dynamics of policy. Journal of Public Economics, 162:4–17.

Stokes, L. C. (2015). Power Politics: Renewable Energy Policy Change in US States. PhD thesis, Massachusetts Institute of Technology.

Stokes, L. C. and Warshaw, C. (2017). Renewable energy policy design and framing influence public support in the united states. Nature Energy, 2.

The Economist (2012). Agricultural subsidies.

The Economist (2013). Blown away.

The Economist (2019). Some farmers are especially good at milking European taxpayers.

The Economist (2020). British farmers fret about losing their protection and their subsidies.

Willems, B. and Zwart, G. (2018). Optimal regulation of network expansion. The RAND Journal of Economics, 49(1):23–42.