Hedging Effect of Low-Quality Capital Assets in Competitive Industries

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Abstract

We highlight the impact of capital quality, i.e., the depreciation rate of capital assets, on firms' investment behavior, endogenous output price dynamics, and industry equilibrium outcomes. To rigorously examine this question, a continuous-time model of dynamic capacity investment under uncertainty is presented where the spot price of a depreciating capital asset is determined in a market equilibrium. The lower-quality (shorter-lived) capital depreciates faster and, thus, requires a higher level of reinvestment. In equilibrium, competitive firms may show a higher willingness to pay for the low-quality capital since depreciation provides an embedded hedge feature for the firm value. In particular, we show that demand elasticity is one of the key determinants of the willingness to pay; ceteris paribus, firms may prefer a high-quality capital in a market with high price elasticity of demand and the low-quality capital in a market with highly inelastic demand. We derive closed-form solutions for the optimal investment policies as well as the steady-state distribution of endogenous output prices and the dynamics of aggregate capital in the economy. We also show that with incremental investment and marked-to-market capital goods prices, the net present value of new investment opportunities is always equal to zero.

1. Introduction

Capital assets are offered in a variety of quality levels. Some capital goods are very long-lasting and can produce for many years. Some other brands or technologies may depreciate much faster and need to be replaced in a few years. In this paper, we are concerned with

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one central question: how does the quality[3] of capital assets affect equilibrium industry outcomes?

To answer this question, we present a continuous-time model of incremental dynamic investment in a depreciating capital asset under uncertainty where the spot price of the asset is determined in a market equilibrium. We analytically derive the optimal aggregate investment threshold of firms facing perfect competition in the output market. Endogenous output-price dynamics, analytic expressions of the long-run steady-state price distributions, and endogenous dynamics of invested capital are also presented.

Absent the reversibility of capacity investments, and when firms operate in an stochastic environment, a competitive industry fails to adjust the aggregate capacity downward when the realizations of random fundamental variables - such as the demand for the output or the cost of inputs - are not favorable. Firms enter a market share game (because they do not internalize the impact of their production decisions on the output price) and produce at full capacity as long as the equilibrium price exceeds marginal production cost. The aggregate industry capacity can, however, decline through a natural capacity *depreciation* channel if the capital quality is less than perfect. Thus, even if the investment is irreversible and competitive firms cannot coordinate to reduce industry output in bad times, the aggregate capacity can gradually decrease due to depreciation.

In a competitive industry, low-quality capital technology – that depreciates quickly – can serve as a natural *hedge* against long-lasting, low output prices.[4] To see this more clearly, note that the impact of industry's capital quality on the value of a competitive firm is non-trivial: higher depreciation is, on the one hand, value-destroying by reducing a firm's own productive asset's life; on the other hand, depreciation also reduces competitors' capacity and helps sustain higher output prices when the demand is sufficiently inelastic (i.e., when the equilibrium price reacts strongly to changes in the quantity supplied to the market.) We refer to the second channel as the hedging value of aggregate depreciation.

In our model, firms purchase their productive assets in a competitive market[5], with a less than perfectly-elastic supply function. The price of capital is not fixed and adjusts as a function of the interaction of firms' willingness to pay per unit and the supply of capital

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[3]Throughout the paper, we use capital quality to refer to the productive life of the capital asset. We assume that other functional aspects of the capital are independent of its productive life.

[4]Firms with market power in monopoly or oligopoly markets, however, do not value the hedging option feature of low-quality capital because they can optimally change the production level.

[5]In this respect we deviate from the existing literature on real options and optimal investment, for seminal papers see Myers (1977) or Tourinho (1979).

goods. In particular, the price of the productive asset increases if a positive shock to the demand for firms' output translates into higher entry rates. The converse is also true when adverse demand shocks drive entry rates down. This is a realistic characteristic of many industries where firms have expertise in servicing their output market but do not have the knowledge to build productive assets by themselves.[6] Within this model specification, we are able to relate optimal entry (investment) decisions not only to demand conditions but also to the characteristics of the supply of productive assets.

Our model considers an incremental type of investment. There are several real-world cases fitting such an incremental model of investment. For example, producing significant quantities of electricity through solar panels requires gradually installing millions of individual units in a large economy such as the US, China, or Germany. In contrast to existing studies on incremental investment (see Pindyck, 1982; Abel, 1983; Abel and Eberly, 1997, for early examples), in our model an endogenously optimal intensity of entry does not require the specification of adjustment costs; instead, price formation in the capital goods market leads to well-defined entry policies.[7]

Within this setup, we analytically determine optimal entry policies and the long-run steady-state distribution of output prices and relate it to capital quality, the supply capacity of productive capital, and demand elasticity. Under some simplifying assumptions on the supply function of the productive capacity, the optimal entry policy is characterized by a critical output-price threshold below which firms do not enter the market. Above this critical level, the intensity of aggregate investment is determined according to the intersection of firms' willingness to pay for productive capital and the marginal cost of supplying capital. We also show that firm valuation under the optimal policy is such that future entry opportunities have a zero net present value.

Among other results, we derive conditions for the existence of a long-run price distribution and its first and second moments. In the limiting case of perfectly elastic supply of productive capital, when the marginal cost of investment is not sensitive to the intensity of investment, a price ceiling for the output price is established.[8] However, once the capacity of the supply

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[6]E.g., a transport company has competence in satisfying their customers demand for transport services but does typically not have the know-how to build trucks. It will simply purchase trucks on a market where it usually has no market power and prices react to the demand for trucks relative to the output capacity of truck producers.

[7]Thus, our model formulation is not subject to the critique stated in Pindyck (1993) that splitting firms in smaller and smaller entities can eliminate adjustment costs.

[8]Our model nests early models of entry (Dixit, 1989; Leahy, 1993; Caballero and Pindyck, 1996) as the limiting case of perfectly elastic supply of productive capital, i.e., infinite capacity of the supply industry.

industry is bounded, the support of the output price is unbounded and episodes of high output prices may occur. We demonstrate that bounded supply capacity leads to early entry since it protects incumbent firms and serves as an entry barrier in future.

According to the above discussed intuition, the competitive firm's entry threshold can increase or decrease with declining capital quality. In particular, we show that, everything else being equal, the entry threshold with low capital quality may indeed be lower (firms enter earlier) than with high-quality capital. In other words, under certain conditions, competitive firms attach great value to the hedging against long-lasting periods of low output prices provided by high industry-wide depreciation rates.

From an asset pricing perspective, the model implies that sectors using high quality capital (i.e., non-depreciating capital) may have a stronger exposure to negative demand shocks. This has major cross-sectional asset pricing implications; it suggests that for industries using high quality of capital the market price of risk associated with firms' equity should on average be higher.

In summary, we contribute to the literature by studying the impact of the capital depreciation rate on industry outcomes. The second contribution is to solve a model of investment dynamics at the industry level, in which the supply of capital is inelastic, and capital prices are endogenous. Finally, we derive the steady-state distribution of prices and solve the model in a rationally consistent fashion. In other words, instead of assuming an exogenously-specified price process, we solve the simultaneous determination of capacity and output prices. The long-run distribution of output prices is derived as a function of the depreciation rate, steepness of the capital goods supply function, i.e., the capacity of the supply industry, and characteristics of output demand. Conditions on the parametrization are stated under which the stationary distribution is non-degenerate (i.e., not converging to zero and not diverging to infinite prices) as well as conditions for the existence of finite first and second moments.

Our model and the numerical exercise provide the following key intuitions: (a) the entry threshold of low quality capital can potentially be lower than the entry threshold of high-quality capital, implying that competitive firms are willing to pay a higher price when the only available capital is a lower-quality one; (b) the volatility of the steady-state prices can have a non-monotone relationship with depreciation; output price volatility can be higher when the depreciation of capital is either very low or very high; c) everything else being

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Then entry takes place at an upper price level and it is impulsive, such that the entry threshold also serves as a price ceiling.

constant, firms may prefer a high-quality capital in a market with low demand elasticity and the low-quality capital in a market with high demand elasticity; d) when firms do not have market power in the capital goods market, firms' optimal entry policy is such that the present value of future installed capital is exactly zero.

The paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 presents the baseline theoretical model as well as model analysis and firm valuation. Section 4 provides results regarding the steady-state distribution of prices. Results from a quantitative exercise are presented in Section 5. Finally, we discuss managerial implications of our model as well as caveats in Section 6.

2. Literature Review

Our paper is related to multiple strands of literature. First, we extended the literature on incremental investment with convex adjustment costs (e.g., Pindyck (1982), Abel (1983), and Abel and Eberly (1994)). Abel and Eberly (1997) offer closed-form solutions for a model which entails both the irreversibility of investment and convex investment costs; they relate the optimal investment to Tobin's Q.

Another closely related area is a strand of papers that considers *endogenous* price dynamics.[9] Several papers (e.g., Dixit and Pindyck (1994), Dangl (1999), Aguerrevere (2003), Roques and Savva (2009), Carlson et al. (2007), and Kogan et al. (2009)) make the output price endogenous by modeling a monopolistic producer facing exogenous shocks to demand function. The modeling framework allows for a feedback effect from the capacity investment of the representative monopolist on the marginal value of the next unit of capacity. Kogan (2001) and Novy-Marx (2007) characterizes the equilibrium investment in a general equilibrium setting – with endogenous prices of capital (capital) and output – Also, Miao (2005) derives the industry equilibrium, output prices, and endogenous capital structure of heterogeneous firms.

Despite its importance, only a few papers explicitly focus on the role of depreciation. Fousekis and Shortle (1995) solve a neo-classic investment problem when the depreciation rate of capital is stochastic. Feichtinger et al. (2001) consider the effect of capital aging on the cost of production and derive the optimal investment problem with multiple vintages of

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[9]Papers that assume an exogenous process for the output price to derive the optimal timing of a lumpy investment with fixed size include McDonald and Siegel (1985), Majd and Pindyck (1987), Pindyck (1988), and Bertola and Caballero (1994). Dangl (1999), Bar-Ilan and Strange (1999), and Hagspiel et al. (2012) extend this literature by solving not only for the optimal stopping time but also determine the optimal intensity of investment.

capital. Evans and Guthrie (2012) build a model with endogenous price and capacity depreciation to study the interaction of scale economy and optimal regulation of firms. Pangburn and Sundaresan (2009) propose a model which accounts for the product life-cycle. In their model, the production capacity does not depreciate but the demand for the output goes through phases of boom and obsolescence. Our paper differs from these papers by highlighting the potential hedging value of depreciating capital.

3. Model

Overview of the Economy Our model follows the standard setup proposed by Leahy (1993) and Baldursson and Karatzas (1996). Time is continuous and the capital stock is infinitely divisible. The market for the output commodity is perfectly competitive. The competitive industry consists of a large number of infinitesimal firms which run the industry's installed capital stock by deciding about the level of production plus a large number of inifinitesimal equal-size firms that decide about entering the market. No firm has any monopsony power in the input market. Demand for the output product is random, and the equilibrium output price is determined through the interaction between the realized demand and the current capital stock of the industry. The capital asset is produced through a convex supply function. This implies that the higher the intensity of aggregate investment is, the higher will be the equilibrium price of the capital good. Output price determines firms' willingness to pay per unit of the productive capital asset. Thus, demand shocks propagate through the production system and translate into price shocks of the capital good. Investment, in turn, determines the future capital stock.

A list of main notations used in the paper is provided in Appendix A.

3.1. Basic Components

Production Technology. We assume firms only need one key production factor, which is capital. Other inputs (e.g., labor) can be flexibly obtained from the spot market and their cost is summarized in a variable cost c per unit of output. Hence, firms' productive capacity is determined by installed capital. We denote aggregate installed capital by K. Capital investment is irreversible; thus, K is a state variable and investment is strictly non-negative. Since investment is irreversible in our model, there is no intentional exit. However, capital depreciation forces a certain percentage of firms to leave the market in each instant.

Industry Investment. We aggregate the behavior of competitive firms regarding their market entry as well as their production decisions into the behavior of one representative firm.

The aggregation of entry decisions of many firms results in the single aggregate investment function, I, for the entire industry.[10]

The representative firm can add productive capacity incrementally by incurring investment costs. In contrast to existing literature (see, e.g., Pindyck, 1982; Abel, 1983; Abel and Eberly, 1997), we analyze the case where the market for productive asset is competitive and no firm has monopsony power over capital goods. I.e., the price of productive capital varies with changing investment intensity of firms. Since the supply curve for productive capital is upward sloping in the investment intensity, lumpy and impulsive investment may only be optimal in the limiting case of infinite supply capacity. Firms have the knowledge to operate the capital good, however, they do not build, install, or adapt it. Since we assume firms receive the capital good ready-to-operate, they do not bear internal adjustment costs. For example, the electricity company pays to receive solar panels in a *turn-key* fashion. The electricity company only needs to operates them. This assumption immediately implies that Pindyck's critique (see Pindyck (1993)) regarding the interaction between firm size and the magnitude of adjustment costs – the incentive to subdivide firms into small entities to avoid convex costs – becomes irrelevant in our case. The convex cost in our model originates from the *aggregate industry investment*, and is independent of the size distribution of firms.

Investment of Competitive Firms. At any point in time, potential entrants observe the current output price and decide on their bid for a marginal unit of the capital asset.[11] Competitive firms do not coordinate their entry. Given the limited supply of the capital asset, the demand of potential entrants will be satisfied only partially.[12] Firms in our model hold rational expectations about aggregate variables: each individual firm takes into account the

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[10]Later, we prove the stability of the derived industry equilibrium. We verify the proposed equilibrium investment / production policy, i.e., we prove that for the aggregate investment policy of the representative firm, the single marginal firm has no incentive to deviate from the implied entry / production policy which aggregates over all firms to the proposed investment / production behavior of the entire industry. Thus, the provided entry policy of competitive firms is a closed-loop Nash equilibrium; see Back and Paulsen (2009) for a critique of existing literature on option exercise games. The current capital stock of the representative firm summarizes the productive capital of all firms which entered the market net of those which exited due to depreciation.

[11]Throughout the paper we adopt the convention that the word *entry* refers to decisions of individual firms; whereas, the word *investment* represents the aggregate behavior of the total industry (through the representative firm). When the discussion is about the optimal behavior of firms, we mainly use "entry" and when the discussion is about aggregate market forces, we use "investment".

[12]We will show later that under equilibrium prices for the capital asset, firms are indifferent between entering the market and staying outside, maybe trying to enter at some later point in time. For this reason, we do not require to specify a selection mechanism deciding which firms are able to enter at a certain point in time.

impact of aggregate investment on the equilibrium price of capital as well as the equilibrium price of the output when making the entry decision. In other words, a single small firm correctly anticipates how the behavior of all other firms in the industry will interact with exogenous demand and supply shocks thereby shaping equilibrium capital and output prices. Thus, the strategy of individual firms is consistent with the aggregate output.[13] Since each firm is very small, there is no preemptive behavior.

Depreciation of Capital. Installed capital, K, gives firms in aggregate a maximum output capacity of K units per time. Capital continuously depreciates at the rate ξ and thus the aggregate *productive* capacity declines over time, unless the representative firm re-invests to offset the depreciation effect. The initial choice of asset quality determines the depreciation rate ξ and is exogenously fixed.[14]

If the representative firm invests an aggregate rate of I dollars per unit of time on capacity, the dynamics of aggregate capacity is given by

$$dK = K\left(\frac{I}{Z(I)K} - \xi\right)dt., \tag{1}$$

with $Z(I)$ the *endogenous* market price of a unit of capital good.[15]

Output Price Dynamics. Firms operate in a stochastic environment, in which the output market is subject to random demand shocks. The demand for the firms' output is iso-elastic such that the price P is a function of aggregate output and the stochastic demand factor,

$$P = \theta^\gamma (qK)^{-\eta}, \quad \gamma > 1, \eta > 0, \tag{2}$$

where $q \in [0,1]$ is the capacity utilization factor and θ is a demand-shift parameter which follows a standard geometric Brownian motion, with the drift parameter α and the volatility parameter σ

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[13]Leahy (1993) shows even if firms are myopic, the aggregate outcome may coincide with the outcome of an industry with perfectly forward-looking rational firms.

[14]In this paper we do not consider mixed capital qualities, i.e., the co-existence of various types of capital in the market.

[15]In what follows we will model supply of productive capital as dependent on the aggregate investment rate, I, as well as on actually installed capital, K. Hence, with an upward sloping supply function, firms will not find lumpy investment optimal; whereas, investment can completely stop if the supply function is bound from below by some minimum price. The case of a perfectly elastic supply and lumpy investment will be discussed as a limiting case.

$$\frac{d\theta}{\theta} = \alpha \, dt + \sigma \, dW. \tag{3}$$

We define a second state variable, $\tilde{P}$, as the capacity-adjusted price. This is the price one would observe at full aggregate capital utilization, $q = 1$,

$$\tilde{P} = Pq^{\eta}, \tag{4}$$

with the dynamics

$$\frac{d\tilde{P}}{\tilde{P}} = [\gamma\alpha + \eta(\xi - \frac{I}{Z(I)K})]dt + \gamma\sigma dW, \tag{5}$$

that is endogenously determined by the optimal aggregate investment rate, I, but is by construction independent of capital utilization. Hence, the dynamics of the observed output price $P = \tilde{P}q^{-\eta}$ is endogenously controlled by the aggregate rate of investment, I, and by the aggregate capital utilization, q.[16]

Demand for Productive Capital. Whenever the price of the capital good, Z, is below the marginal value of capacity, v, all competitive firms seek to enter the market.[17] When v is below Z, none of the firms want to enter. At $v = Z$, firms are indifferent between entering and staying out of the market—reconsidering entry at a later point in time. Thus, aggregate demand for the capital good is perfectly elastic at $Z = Z^* = v$. See Figure 1 for an illustration of the demand for productive capital.

Supply of Capital. Capital is purchased on a competitive market for productive assets. Firms are price takers in this market and do not act strategically. The value function depends on the specific form of the supply function since the supply function determines the entry threshold for new firms as well as the intensity of entry.

In order to allow for a closed-form solution, we assume that the inverse supply function for productive capital has the form

$$Z = G(I, K) = \max\left\{ Z_{\min}, \frac{I}{bK} \right\}, \quad Z_{\min} \geq 0, b > 0. \tag{6}$$

where $Z_{\min}$ constitutes the price floor below which there will be no supply of the capital

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[16]The aggregate investment rate of the industry as well as the price of the capital good are stochastic processes adapted to the filtration generated by random shocks to the demand for firms' output.

[17]Below we show that under some simplifying assumptions on the supply for the capital good, we have $v = v(\tilde{P})$, i.e., the firm value per unit of capacity is a function of the capacity-adjusted price.

asset. The baseline cost floor is determined by physical factors such as materials cost. The unit cost of capital good can not be lower than this floor because for example a solar panel requires a certain amount of rare earth materials.

The total capacity of the supply industry is assumed to be proportional to currently installed capital, K, with the constant of proportionality b. For prices above the cost floor, $Z_{\min}$, the producing sector supplies capital at its maximum capacity which is able to support a growth rate, b, of the base of invested capital, K. The higher the capacity parameter b is, the larger is the *flow* of new capital that can be supported by the supply industry. We assume perfectly inelastic supply of productive capital for prices that exceed some floor $Z_{\min}$.

Whenever firms' willingness to pay for one additional unit of capital exceeds $Z_{\min}$, potential entrants are interested to enter and the supply industry supports aggregate investment of $I = bKZ$.[18] On the other hand, if the valuation of a unit of capital is below $Z_{\min}$, no firm will enter and aggregate investment is zero, $I = 0$. The endogenous demand for capital, characterized by Z^*, will be derived in the following subsection and varies with capacity-adjusted output price $\tilde{P}$.

Figure 1 illustrates the perfectly elastic firm demand for productive capacity together with an upward-sloping supply function. The intersection of supply and demand defines the investment intensity I. From Figure 1, it is immediately clear that under the chosen supply function, firms' entry policy is especially simple, see Section 3.3 for the formal derivation.

3.2. Optimal Capital Utilization

The production technology is flexible and the representative competitive firm can instantaneously choose any aggregate production plan, i.e., capital utilization $q \in [0, 1]$ with aggregate output qK in response to the realization of aggregate demand. In other words, there is no commitment to the utilization of capital. Each competitive firm's marginal unit of capital has negligible influence on the aggregate output and, hence, on the output price. The single firm will simply utilize its marginal unit of capital as long as the gross profit per unit of output, π, is positive, $\pi = P - c > 0$. If the gross profit per unit of output is negative, $\pi = P - c < 0$, the competitive firm will not utilize its capacity. In the special case of $P - c = 0$, the firm is indifferent between producing and not producing one marginal unit of output, since the associated gross profit is zero. Consequently, if $\tilde{P} \geq c$, all firms will produce and we have full aggregate capital utilization, $q^* = 1$, and $P = \tilde{P}$ in this re-

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[18]Remember that bK is the maximum flow (per unit of time) of new productive capital provided by the supply industry.

supply

price Z

demand

v
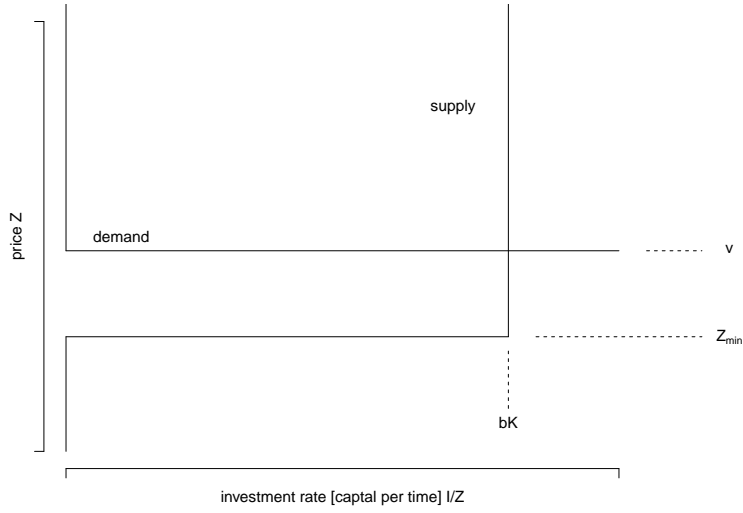
Z_{min}

bK

investment rate [captal per time] I/Z

Figure 1: Demand and Supply of Capital. The equilibrium price [$ per unit of capital] and the equilibrium aggregate investment rate I [$ per unit of time] are determined by the intersection of the perfectly elastic demand of firms and perfectly inelastic supply of productive capital. Please note that v fluctuates with output demand.

gion. If $\tilde{P} < c$, full utilization would lead to negative gross profit for all firms, thus, firms have an incentive to switch off production. However, $q = 0$ is not an equilibrium because given that $P = \tilde{P}q^{-\eta}$ is decreasing in capital utilization, $q \to 0$ will push the output price upwards. When only a fraction $q = (\tilde{P}/c)^{(1/\eta)}$ of firms utilize their capacity, P equals c. At this price, firms are indifferent between producing and not producing, which constitutes an equilibrium.[19]

The optimal capital utilization, q^*, of the representative competitive firm and its effect on the output price, P, can be summarized as follows

$$q^* = \begin{cases} 1, & \text{if } \tilde{P} \geq c \Rightarrow \quad P = \tilde{P} \\ \left(\frac{\tilde{P}}{c}\right)^{1/\eta}, & \text{if } \tilde{P} < c. \Rightarrow \quad P = c \end{cases} \tag{7}$$

In the perfectly competitive market, variable costs, c, serve as a lower bound for the output price, P.

Under optimal capital utilization, the gross profit per unit of capital—for the represen-

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[19]Equilibrium output price, P, will never fall short of marginal costs, c. In such a case, all competitive firms would immediately suspend production, providing an instantaneous positive shock to the output price.

tative firms as well as for each individual marginal firm—is

$$\pi^* = \begin{cases} (\tilde{P} - c) & \text{if} \quad \tilde{P} \geq c, \\ 0 & \text{if} \quad \tilde{P} < c. \end{cases} \tag{8}$$

3.3. Firm Valuation and Optimal Investment

The representative competitive firm is an aggregation of individual marginal firms. When optimizing its investment intensity, I, it does not regard the feedback of investment on the dynamics of the output price process because marginal firms do not take into account the feedback of their entry on the price process. However, aggregate investment is fully determined by the investment policy of the representative firm. Furthermore, the influence of investment intensity on the price of the capital good is also neglected, since marginal firms are price takers in this market. The optimal investment policy, $I(K, \tilde{P})$, is therefore the solution of a fixed-point problem. Given the output-price dynamics and the price of the capital good implied by I, the optimal investment decision must—in each and every state— coincide with I. Finally, we must check that for the given output-price dynamics implied by the investment policy of the representative firm, the single marginal firm's optimal entry decision is such that the aggregate entry results in exactly the investment policy of the representative firm.

The value function of the representative firm can be determined as the discounted expectation of the firm's free cash flow, $K\pi - I$, under the pricing measure. Let $\mathcal{I}(K, \tilde{P})$ denote the conjectured aggregate investment intensity, then the representative firm's optimal investment policy $I(K, \tilde{P})$ must constitute a fixed point of the valuation problem[20]

$$rV(K, \tilde{P}) = \max_I \left\{ \frac{1}{2}\gamma^2 \tilde{P}^2 \sigma^2 \frac{\partial^2 V}{\partial \tilde{P}^2} \right.$$

$$+ \tilde{P} \left[\gamma\alpha + \eta \left(\xi - \frac{\mathcal{I}}{Z(\mathcal{I})K} \right) \right] \frac{\partial V}{\partial \tilde{P}} \qquad \text{(optimization)} \tag{9}$$

$$\left. + K \left(\frac{I}{Z(\mathcal{I})K} - \xi \right) \frac{\partial V}{\partial K} + K\pi - I \right\},$$

$$\mathcal{I}(K, \tilde{P}) = I(K, \tilde{P}), \qquad \text{(verification)} \tag{10}$$

where we substitute the respective π determined in (8) into (9).

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[20]We assume a constant discount rate r and apply the theorem of Feynman-Kac to the stochastic integral of the free cash flow to derive the stated HJB equation which must be satisfied by the firm's value function V.

We observe that the valuation system (9) and (10) allows for a solution which is linear homogeneous in installed capital, K. More formally, $V(K, \tilde{P}) = Kv(\tilde{P})$, where $v(\tilde{P})$ is the value of one unit of installed capacity, i.e., the per unit of capital value of a competitive firm which is already in the market.[21]

Proposition 3.1. *Equilibrium outcome. In equilibrium, firm valuation and optimal entry / investment is the simultaneous solution of the following system of (ordinary differential) equations*

$$V(K, \tilde{P}) = Kv(\tilde{P}) \qquad \text{(linearity)} \qquad (11)$$

$$Z = \max\{Z_{\min}, \frac{\partial V}{\partial K} = v(\tilde{P})\} \qquad \text{(capital price)} \qquad (12)$$

$$\frac{I}{ZK} = \begin{cases} 0, & \text{if} \quad v < Z_{\min}, \\ b, & \text{if} \quad v \geq Z_{\min}, \end{cases} \qquad \text{(opt. investment)} \qquad (13)$$

$$(r + \xi)v = \begin{cases} \dfrac{1}{2}\gamma^2\tilde{P}^2\sigma^2\dfrac{\partial^2 v}{\partial \tilde{P}^2} \\ \quad + \tilde{P}\left[\gamma\alpha + \eta\xi\right]\dfrac{\partial v}{\partial \tilde{P}} + \pi, & \text{if} \quad v < Z_{min} \\[4mm] \dfrac{1}{2}\gamma^2\tilde{P}^2\sigma^2\dfrac{\partial^2 v}{\partial \tilde{P}^2} \\ \quad + \tilde{P}\left[\gamma\alpha + \eta\left(\xi - b\right)\right]\dfrac{\partial v}{\partial \tilde{P}} + \pi. & \text{if} \quad v \geq Z_{min} \end{cases} \qquad \text{(valuation)} \qquad (14)$$

In particular, in the domain of the capacity-adjusted output price, there exists a unique threshold, s, that separates the investment / entry region from the non-investment / non-entry region

$$\begin{aligned} \tilde{P} < s &\Leftrightarrow v(\tilde{P}) < Z_{\min}, & \text{(non-investment / non entry region)} \\ \tilde{P} \geq s &\Leftrightarrow v(\tilde{P}) \geq Z_{\min}, & \text{(investment / entry region)} \end{aligned} \qquad (15)$$

which results in an equilibrium dynamics of the capacity-adjusted output price

$$\frac{d\tilde{P}}{\tilde{P}} = \begin{cases} [\gamma\alpha + \eta\xi]dt + \gamma\sigma dW, & \tilde{P} < s, \\ [\gamma\alpha + \eta(\xi - b)]dt + \gamma\sigma dW, & \tilde{P} \geq s. \end{cases} \qquad \text{(price dynamics)} \qquad (16)$$

Finally, the existence of the value function v requires

$$\gamma\alpha + \eta(\xi - b) < r + \xi. \qquad \text{(constraint on risk-neutral profit growth)} \qquad (17)$$

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[21]Intuitively, we should expect linearity in K since the representative firm is nothing else but the sum of identical individual firms operating a marginal unit of capacity each.

For a proof of Proposition 3.1 and the analytical expression of the value function v see Appendix B.

Proposition 3.1 states that under the proposed optimal entry policy, the value of the representative competitive firm is linear in capital, $V = Kv(\tilde{P})$, i.e., in competitive equilibrium, average and marginal value of capital are identical. Equation (14) is the Hamilton-Jacobi-Bellman equation for the valuation of one unit of installed capital and it explicitly depends only on $\tilde{P}$ (and only indirectly on K via Equation 5). Thus, $\tilde{P}$ contains all relevant information necessary to value productive capital and, consequently, to decide upon entry.[22] Hence, in equilibrium, the dynamics of $\tilde{P}$ determines the evolution of installed capital, K. Capital, K, depends on the historical path of demand shocks and is adapted to information set generated by the process of the capacity-adjusted price, $\tilde{P}$; see below.

Under the equilibrium entry policy, the price of the capital good, Z, is either identical to the marginal value of a unit of capital, $Z = \frac{\partial V}{\partial K} = v$, in which case the capacity of the supply industry supports an aggregate growth rate of installed capital of bK (before depreciation) or equal to $Z_{\min}$. If the marginal value of one unit of capital is below $Z_{\min}$, the price of the capital good is at its minimum $Z = Z_{\min}$, firms will not enter and the aggregate investment is zero. In competitive equilibrium, firms enter the market as long as the marginal value of one unit of capital is larger or equal to the price of capital.

Furthermore, Proposition 3.1 states that there is a uniquely determined price threshold, s, that separates the non-entry region ($\tilde{P} < s$) from the entry region ($\tilde{P} \geq s$). Thus, in equilibrium, the dynamics of the capital adjusted price, $\tilde{P}$, is geometric Brownian with drift rate $\gamma\alpha + \eta\xi$ in the non-entry region and $\gamma\alpha + \eta(\xi - b)$ in the entry region. In the entry region the price has an additional negative drift of $-\eta b$; i.e., through demand elasticity η, capital growth reduces the drift rate of the capital-adjusted output price.

Substituting optimal equilibrium investment (13) into the general valuation equation (9) makes (9) independent of investment and identical to the valuation equation of a firm that operates its existing capital under equilibrium price dynamics and has *no* opportunity to add more capital in the future. Thus, future entry opportunities are zero-NPV as stated in the following proposition.

Proposition 3.2. *Under the optimal entry policy of the competitive firms, the instantaneous rate of aggregate investment is such that the value of the opportunity to enter at some time*

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[22]Obviously, installed capacity influences the output price through the inverse demand function (2). In equilibrium, however, capital, K, contains no information on the price dynamics not already present in the current price $\tilde{P}$.

in the future is zero-NPV.

This proposition needs no separate proof since it can directly be derived from an inspection of the valuation equation (14), as discussed above. It results from the fact that firms purchase capital on a competitive market at an endogenously determined market price. The aggregate investment, I, exactly balances the positive quantity effect of a resulting increase in the capital base with the negative effect of investment on the price dynamics. Whenever an investment strategy implies a positive value of future capital acquisition, firms enter and optimally increase the aggregate investment.

The dynamics of installed capital is derived by substituting the optimal equilibrium investment policy from Proposition 3.1 into the general dynamics of installed capital, (13),

$$dK^* = \begin{cases} -\xi K\, dt & \text{if} \quad \tilde{P} \leq s, \\ -(\xi - b)K\, dt & \text{if} \quad \tilde{P} > s \end{cases} \tag{18}$$

The actual amount of aggregate capital, which is optimally installed is path dependent; the longer historical prices were in the investment region, the more capital is currently installed. If output demand increases sharply after a period of low demand, prices will react sharply since it takes some time until a capital base is built. If the demand recovers sharply after a short period of low demand, output prices will be moderate after the recovery since the existing capital base is still large to serve the demand at moderate prices.

3.4. Special Case: Low Variable Cost

Several industries – such as telecommunication, hotels, and amusement parks – operate with very low (capitalized) variable cost of production, c, compared to minimum initial investment costs, $Z_{\min}$. Another notable example is the solar electricity production, which we use as our base case for the numerical exercise in Section 5. We show in the following subsections that under realistic assumptions on the depreciation of capital and the production capacity of solar panel producers, the stationary output-price distribution will be concentrated around the critical entry price s, which is far above variable cost c. Consequently, the real option to switch off (or reduce) capacity utilization is of negligible value because the industry almost always produces at full capacity (where $q = 1$). The entry threshold can be determined analytically in this case.

Proposition 3.3. *If variable cost c are low such that one can ignore the option to adapt*

capacity utilization, the entry threshold for the competitive firm is

$$s = \frac{\beta_{1,nE} - \beta_{2,E}}{\frac{1-\beta_{2,E}}{r-\gamma\alpha+(1-\eta)\xi+\eta b} - \frac{1-\beta_{1,nE}}{r-\gamma\alpha+(1-\eta)\xi}} \left(Z_{\min} + \frac{c}{r+\xi} \right) \tag{19}$$

Moreover, the competitive entry threshold is bounded by

$$\max\{0, \, r - \gamma\alpha + (1-\eta)\xi\} < \frac{s}{\left(Z_{\min} + \frac{c}{r+\xi} \right)} < r - \gamma\alpha + (1-\eta)\xi + \eta b. \tag{20}$$

Here, $\beta_{1,i}$, $\beta_{2,i}$ are the positive and the negative root of the characteristic polynomial

$$g_{\mathrm{i}}(\beta) = \frac{1}{2}\beta(\beta-1)\gamma^2\sigma^2 + \mu_i\beta - (r+\xi). \tag{21}$$

The subscript $i \in \{E, nE\}$ stands for *Entry* and *No-Entry*, and $\mu_i \in \{\gamma\alpha + \eta(\xi - b), \gamma\alpha + \eta\xi\}$ is a drift term associated with each region.

Proof For a proof of Proposition 3.3 see Appendix C.

The bounds on optimal entry stated in Proposition 3.3 have an intuitive interpretation. First, we reformulate the upper bound to get

$$\frac{s}{r - \gamma\alpha + (1-\eta)\xi + \eta b} - \frac{c}{r+\xi} < Z_{\min} = v(s).$$

The left-hand side is the value of one unit of capital at the threshold $\tilde{P} = s$ under the assumption that new firms enter all the time at the maximum rate supported by the supply industry, independent of the price trajectory. Under this restrictive assumption, the left-hand side of the inequality constitutes the optimal entry threshold.[23] Clearly, the true value of one unit of capital, $v(s)$, exceeds the left-hand side expression, since valuation regards that below s, entry will stop and thus the price drift will be more positive, adding extra value to existing capital. The optimal entry threshold is therefore below the stated threshold.

On the other hand, if $\gamma\alpha + \eta\xi < r + \xi$, the lower bound yields

$$\frac{s}{r - \gamma\alpha + (1-\eta)\xi} - \frac{c}{r+\xi} > Z_{\min} = v(s).$$

Now the left-hand side is the value of one unit of capital at $\tilde{P} = s$ under the assumption that there will be no entry by new firms in the future. Clearly, the true value $v(s)$ is below

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[23]In this section we assume that c is small, hence, the opportunity to adapt output quantity if the output price falls below c has negligible value.

this benchmark value, since for prices exceeding $\tilde{P}$, new firms will enter, reducing the drift rate of the output price and thus dampening capital valuation. From this consideration we can deduce a lower bound on s.

In the alternative case of $\gamma\alpha + \eta\xi > r + \xi$ (when the lower bound is at zero), the drift in the non-entry region (i.e., prices below s) is so large that the present value of one unit of capital under the assumption of no future entry would be infinite. In this case, entry is optimal independent of the output price. Consequently, a price of zero acts as a lower bound to the entry threshold, s, in this case.

We motivate our model with the observation that in a competitive environment, depreciation of productive capital might positively influence market entry. Depreciation negatively influences the effective life of a firm's own invested capital base, which ex-ante reduces the willingness to enter the market. However, firms also anticipate that under episodes of weak demand, other firm's capital will also depreciate. The overall reduction of the aggregate installed capital will in turn mitigate the negative effects on the price. Especially when the output price is highly sensitive to quantities (i.e., when η in (2) is large meaning that the price elasticity is low), the positive effect of depreciation, by hedging firms against extended periods of low output prices, might dominate the negative effect of reducing the effective lifetime of firm's own capital; consequently, lower capital quality might result in earlier entry, i.e., a lower entry threshold s.

This finding is summarized in the following proposition.

Proposition 3.4. *In industries with small operational costs, c, facing a sufficiently inelastic output demand, i.e., sufficiently large η, and when $r - \gamma\alpha > 0$,[24] low capital quality leads to earlier entry, ceteris paribus. More precisely, if*

$$\frac{r - \gamma\alpha}{\beta_{1,nE} - 1} < \gamma^2\sigma^2\sqrt{\left(\frac{1}{2} - \frac{\gamma\alpha}{\gamma^2\sigma^2}\right)^2 + \frac{2r}{\gamma^2\sigma^2}}, \tag{22}$$

then for sufficiently large η

$$\left.\frac{ds}{d\xi}\right|_{\xi=0} < 0.$$

I.e., compared to non-depreciating capital, depreciating capital has a lower entry threshold. Thus, at least in the neighborhood of s, depreciating capital has higher economic value compared to non-depreciating capital.

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[24]This condition means the demand growth is bound from above such that non-depreciating capital has finite value

Proof The proof is in Appendix D.

Proposition 3.4 indeed supports the initial conjecture that shapes the core message of the paper; when the demand is sufficiently inelastic, such that the depreciation of aggregate capacity has a significant impact to keep the price high, the threshold to enter with a low-quality capital is lower, meaning that a competitive industry is willing to pay a higher price for a lower-quality capital, ceteris paribus. In this case, the positive externality of faster depreciation at the industry level dominates the private costs of capacity loss for an individual firm.

If the capacity of the supply industry is large and high investment intensities can be satisfied at a price close to $Z_{\min}$, any positive demand shock that pushes the output price above the entry threshold s will be immediately offset by large investment. So for $b \to \infty$, s serves as a price ceiling and the entry threshold of competitive firms converges to the well known entry trigger first derived in Leahy (1993). I.e., our model nests existing real options impulse-control models.

Lemma 3.5. *If the capacity of the supply industry is large ($b \to \infty$), i.e., arbitrarily high investment rates can be satisfied without any noticeable impact on the price of productive capital, the optimal entry threshold serves as a strict price ceiling. The entry threshold of the competitive firm converges to*

$$s = \frac{\beta_{1,nE}}{\beta_{1,nE} - 1}[r - \gamma\alpha + (1 - \eta)\xi]\left(Z_{\min} + \frac{c}{r + \xi}\right) \tag{23}$$

See Appendix Appendix E for the proof.

4. Stationary Price Distributions

In Proposition 3.1 we present the optimal equilibrium entry strategy which is characterized by an endogenously determined entry threshold, s, such that such that for (capacity-adjusted) output prices $\tilde{P} \leq s$ firms do not enter the market. For $\tilde{P} > s$ firms bid for new capacity; the price of the capital good is determined in a competitive equilibrium, the flow of new capacity is given by the capacity of the supply industry. Thus, the drift rate of the capacity adjusted price $\tilde{P}$ in the non-entry region ($\tilde{P} \leq s$) is driven by demand growth and capital depreciation, in the entry region ($\tilde{P} > s$) it is reduced by the additional inflow of capital which, depending on the price elasticity, creates downward pressure on the price, see (16).

The larger the capacity of the supply industry (the larger b) and the less elastic the output price (the larger η) are, the stronger is the drift-reducing effect of entry. So, intuitively it is

clear that a stationary price distribution exists only if in the non-entry region, $\tilde{P} \leq s$, the drift is sufficiently positive and in the entry region, $\tilde{P} > s$, the drift is sufficiently negative. In summary, the capital must be sufficiently cheap and demand growth and/or depreciation must be sufficiently high in equilibrium to have a price that neither converges to zero nor diverges to infinity, but has a well defined stationary density.

Proposition 4.1. *There exists a non-trivial long-run stationary price distribution if*

$$[\gamma\alpha + \eta\xi] - \frac{1}{2}\gamma^2\sigma^2 \; > \; 0, \tag{24}$$

$$[\gamma\alpha + \eta(\xi - b))] - \frac{1}{2}\gamma^2\sigma^2 \; < \; 0. \tag{25}$$

In this case, the stationary distribution of the capacity adjusted price is

$$f(\tilde{P}) = \begin{cases} A\tilde{P}^{\lambda_1} & if \;\; \tilde{P} \leq s, \\[2mm] B\tilde{P}^{\lambda_2} & if \;\; \tilde{P} > s, \end{cases} \tag{26}$$

with constants A and B determined by the conditions (i) that the integral over the density must equal 1 and (ii) that the density must be continuous at the critical investment boundary, s. The exponents λ_1 and λ_2 are

$$\lambda_1 \;=\; 2\left(\frac{\gamma\alpha + \eta\xi}{\gamma^2\sigma^2} - 1\right) > -1, \tag{27}$$

$$\lambda_2 \;=\; 2\left(\frac{\gamma\alpha + \eta(\xi - b)}{\gamma^2\sigma^2} - 1\right) < -1. \tag{28}$$

Furthermore, the stationary density has finite expectation if

$$\lambda_2 < -2, \tag{29}$$

and finite variance if

$$\lambda_2 < -3. \tag{30}$$

See Appendix F for a derivation of the stationary density from the Kolmogorov forward equation.

According to Proposition 4.1, a stationary price density exists if the drift of log prices, $\log(\tilde{P})$, is positive in the non-entry region ($[\gamma\alpha + \eta\xi] - \frac{1}{2}\gamma^2\sigma^2 > 0$) and negative in the entry region ($[\gamma\alpha + \eta(\xi - b))] - \frac{1}{2}\gamma^2\sigma^2 < 0$). In other words, a non-trivial stationary distribution exists if the mode of the $\tilde{P}$-distribution drifts towards higher prices in the non-entry region and towards lower prices in the entry region. If the drift of the mode has the same sign in

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both regions, the price either converges to zero (when capital is cheap and/or depreciation low), or diverges to infinity (when capital is expensive and/or depreciation high).

The density of the observed price $P = \tilde{P}q^{-\eta} = \max\{c, \tilde{P}\}$ can be directly derived from the dynamics of $\tilde{P}$, considering that optimal capacity utilization prevents P from moving below the lower threshold c (see (7)). In other words, the distribution of P has the entire probability mass corresponding to $\tilde{P} < c$, represented by the grey area, concentrated as an atomic probability mass at c. Figure 2 shows an illustrative example of the shape of the stationary density of $\tilde{P}$ and P. Appendix F derives expressions for the expected price and variance of both P and $\tilde{P}$.
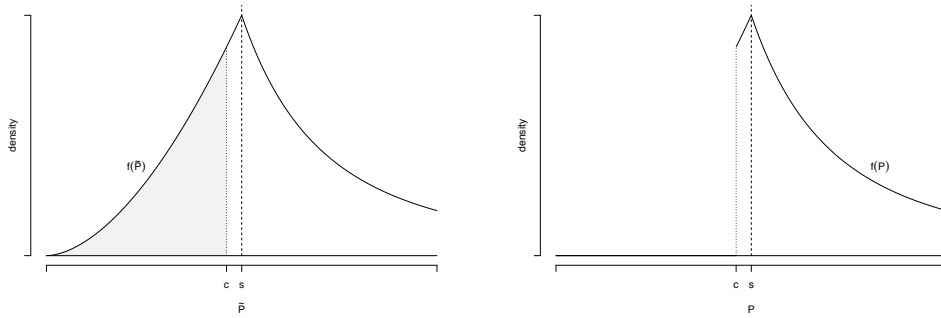

Figure 2: Example of a stationary density of capacity adjusted price $\tilde{P}$ and actual price P

Note that it is not generally true that the border of the entry region, s, lies above the price floor c. If the minimum price of productive capacity, $Z_{\min}$, is low and variable costs, c, are high, market entry starts at a point where capacity is not utilized, $s < c$.

5. Numerical Comparative Statics

In order to provide better insights regarding the fundamental behavior of the model and outcome variables, we present a set of numerical results and comparative statics in this section.

Comparative Statics. We are particularly interested in (i) the optimal entry policy of firms, characterized by the entry threshold s, (ii) the moments of the steady-state price distribution, and (iii) the behavior of the value function. We run comparative statics on key parameters of the model including the depreciation of capital, the elasticity of capital supply, and the elasticity of demand.

Feasible Parameters Range. We only change the parameters in a range such that the conditions expressed in Proposition 3.3, Proposition 3.4, and Proposition 4.1 for the existence of a well-defined value function and price distribution are satisfied. If these conditions are violated, the expected price will either converge to infinity, because the feasible rate of net aggregate investment will always fall behind the demand growth, or collapse to zero, otherwise.

5.1. Baseline Calibration

In order to run the comparative statics in a reasonable range of parameter values, the model is approximately calibrated using available real-world parameters for the US solar panel industry.

Table 1 presents the values of baseline parameters used for numerical analysis. Given the intermittent nature of solar radiation, we assume that a solar panel's average production is 15% of its peak capacity. Thus, we normalize capital and marginal costs by a factor of $\frac{1}{0.15}$ to express parameters in terms of equivalent KW-Year.

Table 1: Base Case Parameters

Variable	Definition	Value	Unit	Source
ξ	depreciation rate of capital	0.05	p.a	Branker et al. (2011)
σ	volatility of demand shift parameter	0.2	p.a	Assumption (Appx GDP Growth Rate Volatility)
α	drift of demand shift parameter	0.022	p.a	US GDP Growth Rate
γ		1	–	–
$-1/\eta$	price elasticity of demand	-0.35	–	Deryugina et al. (2017)
η		2.86		
c	marginal cost of production	260	\$/KWY	Walker (2017)
b	max. rate of capital supply	0.3	–	EIA[25]
$Z_{\min}$	min. price of capital good	13300	\$/KWY	IRENA (2017)
r	discount rate	0.05	p.a.	Common number in the literature

Dynamics of the Industry. We start the numerical investigation by plotting a sample of model-generated equilibrium capacity and price series over time. We simulate a random demand path for 100 periods and then track the dynamics of endogenous capacity and

Capacity over Time

ξ = 0.085
ξ = 0.05
ξ = 0.01
ξ = 0

Capacity (MW)

Period

Price over Time

ξ = 0.085
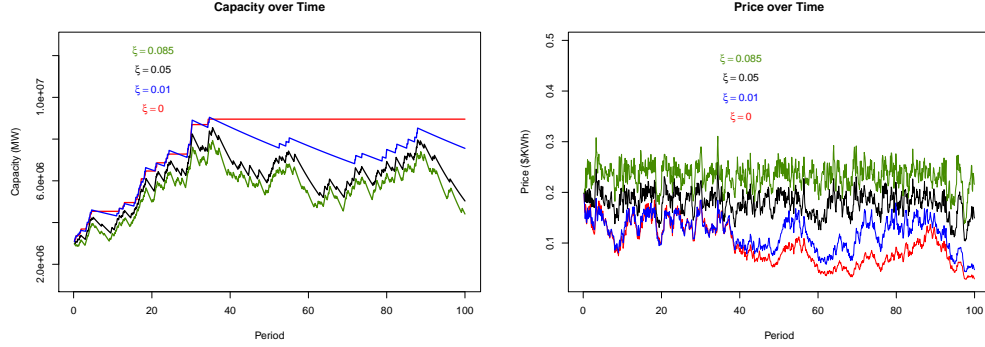ξ = 0.05
ξ = 0.01
ξ = 0

Price ($/KWh)

Period

Figure 3: Samples of Capacity and Price Dynamics Over Time. Plots show the optimal level of capacity and endogenous prices for the same realization of demand shocks.

prices under different parameter values (keeping the realization of demand path the same for all runs). Starting from an initial capital level, the model calculates the market prices of the output and the capital good, and finally the optimal investment strategy of the representative firm. The next period's capital is updated using the optimal investment strategy and the deprecation of capital.

Figure 3 shows the dynamics of endogenous variables associated with different levels of depreciation rate, keeping all other parameters constant. The capacity path of the industry with zero depreciation only changes in the positive direction because the investment is irreversible and there is no depreciation to reduce the capacity. One notes that in several periods (before the period 35), the industry with a 1% depreciation rate (the blue line) builds more capacity than the industry with the zero deprecation (the red line). This is consistent with the key intuition of the paper that a higher depreciation rate may indeed encourage a more aggressive investment because firms using a depreciating capital can feel confident that in times of low demand, the aggregate capacity of the industry gradually fades away and supports the recovery of the output price.

The price recovery effect of a depreciating capital can be observed by looking at the price paths after $t = 40$. The industry faces a low demand after period 40 (which can be inferred from declining capacity of the scenarios with a depreciating capital.) The zero depreciation settings reaches and gets stuck in a high capacity, low price outcome. Since the aggregate capacity does not depreciate, the industry with non-depreciating capital experiences a chronic low price, approximately between periods 40 and 70. Whereas, for the depreciating capital (e.g, the blue line), a price recovery can be noted.

Please note that we assume a fixed supply function, independent of the quality of capital.

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This implies that the effective cost of purchasing a unit of depreciating capital is indeed higher because capital assets with a higher depreciation rate provide a smaller life-time productive capacity. Despite the price bias against the depreciating capital assets, we still observe periods of higher capacity build by the owner of the lower quality capital. The observation of an earlier entry by the lower quality capital, despite a higher effective price, indeed reinforces the key message of the paper.[26]

5.2. Entry Threshold

In this section, we provide comparative statics of the behavior of the entry threshold in response of changes in key parameters such as the demand elasticity, the depreciation rate, and supply elasticity.

Propositions 3.3 and 3.4 provide analytic expressions for the behavior of the entry threshold s when variable costs c are small (which is the case in our setup). The relationship between demand elasticity, depreciation, and the entry threshed is determined through a complex interaction of all parameters. In particular, we have shown that under certain conditions

$$\left.\frac{ds}{d\xi}\right|_{\xi=0} < 0.$$

Depreciation has two opposite effects on the marginal value of capital for a competitive firm: on the one hand, it reduces the firm's own productive capacity over time; on the other hand, it also reduces the aggregate capacity of the industry and helps preserving the equilibrium price.

5.2.1. Demand Elasticity

Price elasticity of demand, $-1/\eta$, plays a crucial role in competitive firms' entry decision and firm valuation. When demand is inelastic, the industry's aggregate depreciation force dominates: favorable price impact of industry depreciation is more important than the lost productive capacity of the firm. When demand is elastic, the impact of firm's own capital depreciation is stronger than the impact on the equilibrium market price because the firm loses more in the lost quantity than its gains from price recovery.

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[26]We can also adjust the supply function such that minimum price becomes a function of the quality of capital. For a more fair comparison, the minimum prices can be set in such a way that, under deterministic demand, the net present values of the productive capacity for all types of capital assets become equal. In that case, lower quality capital will have a smaller minimum price, and one may observe an even more aggressive early entry behavior.

Consistent with the explained intuition, Proposition 3.4 requires the demand to be sufficiently inelastic in order to observe a decreasing entry threshold for lower quality capital. In Figure 4, we observe the theoretically-predicted effect for the demand elasticity: for elastic demand (i.e., when $\eta < 1$), the entry threshold is positively associated with the depreciation rate; the lower quality capital has a higher entry threshold. However, when the demand becomes inelastic (i.e., in the region of $\eta > 1$), the relationship between capital quality and entry threshold becomes non-monotonic.

In particular, we note that when demand is sufficiently inelastic (i.e., when the output price needs to drop a lot in bad times to preserve the equilibrium conditions), the entry threshold is negatively associated with the depreciation rate; higher depreciation may even reduce the entry threshold of new firms. If demand is elastic, higher depreciation always increases the entry threshold.
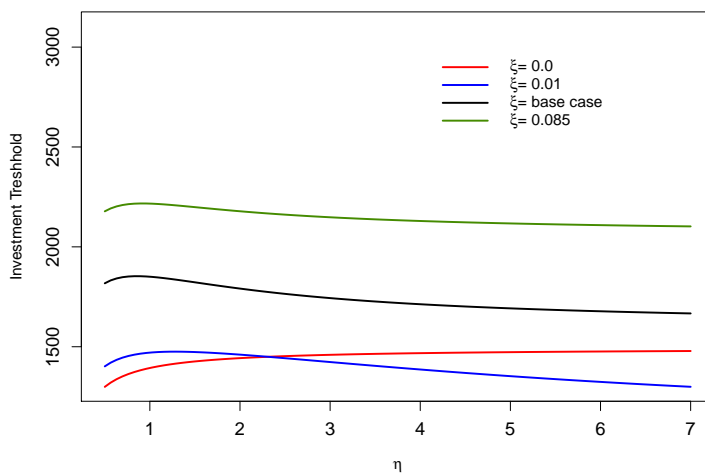

Figure 4: Entry thresholds for different levels of demand elasticity and capital depreciation rates.

A comprehensive illustration is given in Figure 5. We observe that for low values of the current output price, P, the value of the firms operating in less elastic markets is higher; i.e., the there is a negative relationship between the firm value and demand elasticity. On the other hand, for high values of P, above the entry threshold, the relation between firm value and demand elasticity is reversed; firm value is higher when the demand is more elastic.

As we see from Figure 5, inelastic demand serves as an insurance against deteriorating prices, supporting the value of existing capital since depreciation of competitive capital has
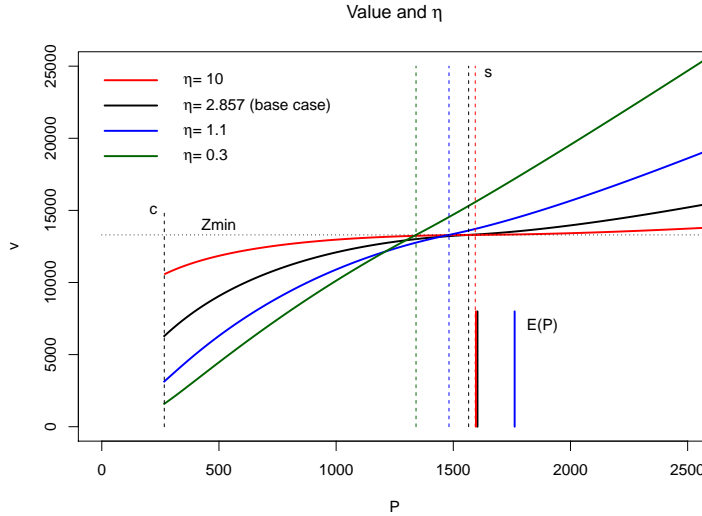
Figure 5: The value function of the competitive firm for different levels of price elasticity of demand, $-1/\eta$. The demand is elastic for $\eta < 1$ and inelastic for $\eta > 1$. Firm value increases in η for low prices and decreases in η for high prices. Entry threshold increases with η, expected price decreases with η.

a strong positive effect. However, inelastic demand makes the market entry of competitors at high prices a larger threat, reducing the value of existing capital at high prices. Overall, more inelastic prices (higher η) leads to later entry but considerably lower expected long-run prices. In particular, the illustrated case with elastic demand (green solid line) has the lowest entry threshold, but the expected long-run price is infinite.

5.2.2. Depreciation Rate and Capital Supply Elasticity

We plot the joint impact of capital depreciation as well as capital supply elasticity on the entry threshold of the competitive industry in Figure 6. Consistent with the predictions of Proposition 3.4, we note that the entry threshold – for the low supply capacity cases – becomes monotonically smaller as the depreciation increases. On the other hand, an inverse-U patter of the entry threshold can be observed for intermediate values of the supply elasticity.

This behavior echos the two opposite effects of depreciation on the entry: on one hand, depreciation increases the user cost of capital and requires a higher level of prices to enter; on the other hand, higher depreciation provides a better hedging for downside risks and encourages an earlier entry. When the supply capacity is low (i.e., the capacity can be built only at slow rates), the implicit hedging value of depreciating capital is pronounced; however, when the elasticity of supply is high, the industry can quickly compensate for

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the depreciating capacity by experiencing a high intensity of entry when the output price exceeds the entry threshold; thus, the hedging effect of depreciation becomes weaker. Since the optimal entry strategy makes future investment opportunity zero-NPV, existing capital does not gain from cheaper investment in the future but only suffer from fiercer competition.
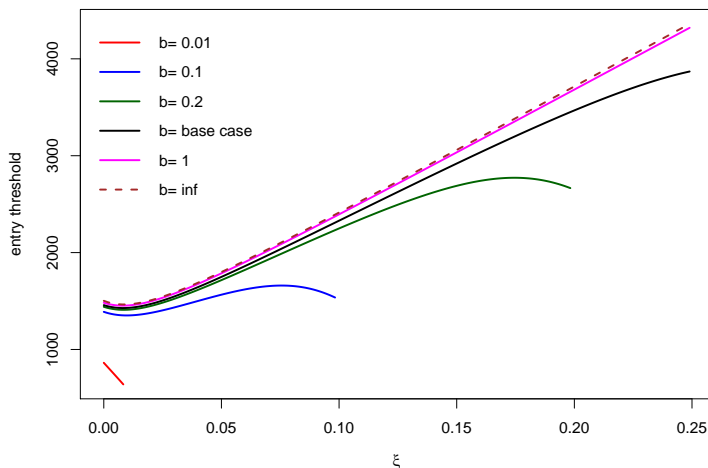

Figure 6: The Joint Impact of Capital Supply Elasticity and Depreciation on the Entry Threshold. A decreasing threshold for low values of the supply elasticity and an inverse U patter of the entry threshold for intermediate values of the supply elasticity can be observed.

If supply capacity is low, existing capital can profit from high prices, since the inflow of new capital is low due to high prices in the supply market. Consequently, the entry threshold increases with supply capacity. With $b \to \infty$, the entry threshold serves as a strict price ceiling, since any positive price innovation will immediately be offset by impulsive aggregate investment.

5.3. Steady-State Price Density

In this section, we further analyze the impact of depreciation rates and demand elasticity on the distribution of steady-state prices.

5.3.1. Distribution of Price

We plot the density of steady-state prices for different levels of capital depreciation in Figure 7. With base-case parameters, the steady-state price density is non-degenerate for $\xi \in [0, 0.0993]$.

Low Depreciation Case. If depreciation is low, the price drift is low even in the non-entry range (prices below entry threshold s). The red line shows the price distribution for non-depreciating capital. The coefficient λ_1 in (26) is -0.9, which implies high probability that the price is at its lower bound c, actually 84.0%. Consequently, the expected stationary price, \$340.1, is only slightly above variable cost c, and the variance of the stationary distribution is low. Slightly higher demand variance ($\sigma > 0.2091$) would make the stationary price distribution degenerate, with 100% probability mass at the lower bound s. Lower demand growth is reducing price drift and, thus, also increases the probability mass at the lower price bound c. With $\alpha < 0.2$ and non-depreciating capital, the stationary price distribution has 100% probability mass at c.

Effect of Higher Depreciation. Increasing capital depreciation increases the overall drift of the price process. As a result, the price density and also the expected price move to higher levels, thereby also increasing the variance of the distribution. The blue line illustrates the price density for $\xi = 1\%$. The long-run probability that prices are at the price floor of c decreases to only 0.073 and expected prices shift upwards so the optimal entry threshold s even decreases and firms enter earlier if depreciation is 0.01 instead of 0.0. With base case depreciation of $\xi = 5\%$ (black line), low prices are pushed towards the entry threshold. The investment rate at high prices is large enough to push prices downwards, such that the variance of the steady-state distribution goes down again.

If depreciation is large compared to the capacity b of capital supply, entry can hardly compensate depreciation and the right tail of the distribution increases considerably, thereby increasing the variance of the steady-state price distribution and pushing expected stationary prices towards infinity. The green line shows the distribution for $\xi = 0.085$, which has an expected value of \$ 2977 despite the fact that the entry threshold is only marginally above the base case entry threshold. Depreciation rates $\xi > 0.0993$ lead to a degenerate price distribution that diverges to infinity.

Our analysis suggests that when the depreciation rate is low, the left tail of the price distribution is large. However, the right tail is small because the effective cost of building capacity is low. When depreciation is high, the price distribution has a tiny left tail and a fat right tail. The model implies that depreciation acts as a hedging mechanism on downside risks; thus, one expects to see a larger concentration of low prices when the depreciation is low. On the other hand, when the depreciation rate is high, the industry will have a limited ability to respond to upside demand shocks; thus, one expects to see a larger concentration of high prices.
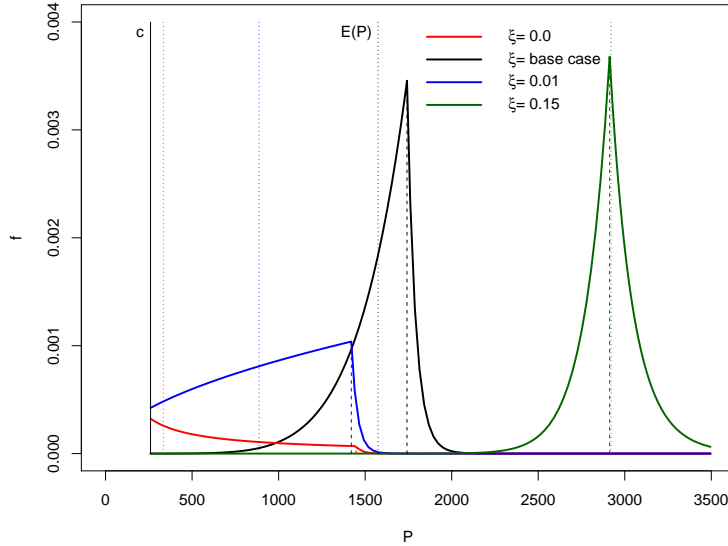
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Figure 7: Distribution of steady-state Prices. Price distribution becomes left or right skewed for very low or high values of depreciation. With $b > \xi$, inelastic demand, i.e., higher levels of η, lead to price density which is more concentrated around the entry threshold

5.3.2. Volatility of Steady-State Prices

Following the argument about right and left tails of the steady-state price distribution, we plot the volatility of steady-state prices versus the depreciation rate in Figure 8. The plots show a non-monotone relationship of steady-state price volatility and capital depreciation for different capacities b of the capital supply industry. Consistent with our theoretical predictions, we observe a U-shaped relationship between the two variables when the supply of capital is not very elastic (see Figure 8). When the supply of capital is sufficiently elastic, a higher depreciation will not increase the upside volatility of prices because the industry is able to quickly respond to positive demand shocks.

As argued above, limited capacity of capital supply leads to a sharp increase of price volatility when ξ is high. Also the expected stationary price increases sharply if ξ becomes large, since the industry has only limited ability to respond to upside demand shocks; thus, one expects to see more frequently large positive shocks. If unlimited entry intensities can be satisfied by the supply industry (at a cost of $Z_{\min}$) the right tail of the price density completely vanishes and there is no increase in price volatility at high depreciation rates.
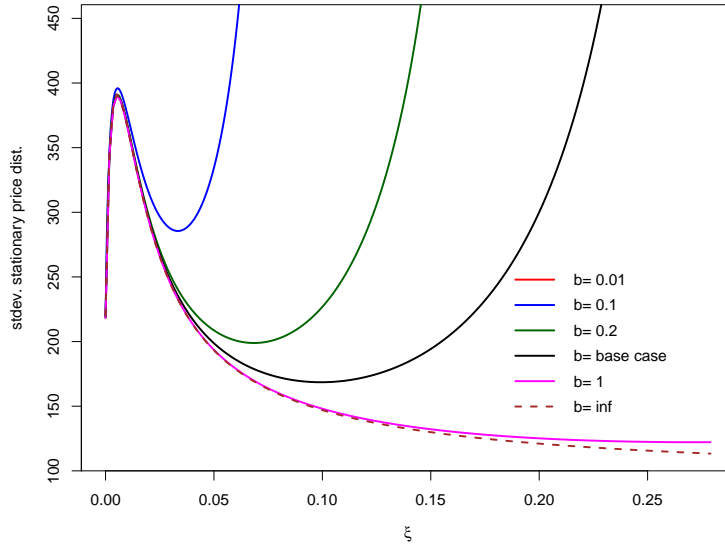
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Figure 8: Impact of Depreciation on the Volatility of Steady-State Price. The price volatility can be U-shaped in depreciation.

6. Implications and Caveats

In its very core, our model enables us to derive the steady-state distribution of endogenous output prices as a function of various features of the industry, including the demand elasticity, the rate of capital depreciation, and the elasticity of capital supply. The distribution of output as well as the behavior of the entry threshold and value function provide several novel managerial and policy implications. We summarize and discuss these implications. Moreover, we provide a short discussion of caveats and limitations of the model.

6.1. Managerial and Policy Implications

Cross-Sectional Asset Pricing. The model implies that sectors using high quality capital (low depreciating capital) have a stronger exposure to low demand shocks. In other words, the low quality capital functions as an implicit hedging mechanism. This result has major cross-sectional asset pricing implications: it suggests that, ceteris paribus, the expected return of the equity of industries using high quality capital should be higher. This effect should be stronger for more competitive industries. These observations provide clear empirical predictions that can be empirically examined by future research.

A lower beta for low quality capital implies that the weighted average cost of capital (WACC) to finance capacity investments will also be lower for firms using lower quality capital. In this paper we assume that WACC is the same for all types of capital. However,

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future research can provide a general equilibrium characterization of the problem in which the cost of capital depends on the quality of capital. A lower WACC for low quality capital will reduce the entry threshold.

Zero-NPV Future Investments. An implication of fierce competition in the output market is that firms always invest up to a point that the marginal benefit of investment is completely offset by the marginal cost of building capacity. Under this assumption, the *expected* NPV of future investment opportunities is always zero because otherwise firms would have explored it. The intuition changes once one of the two key assumptions is relaxed: either there are fixed costs of investment (thus, the investment becomes lumpy) or if the buyer of the capital has some monopsony power in the market for capital goods. In the latter case, the firm will strategically choose the level of capital to influence the input prices, which may violate the zero-NPV condition.

Internal versus External Cost of Capital Adjustment. The aggregate supply of capital good in our model is far from perfectly elastic. Such a supply function implies that the price of capital goods is higher when the aggregate demand for investment is higher. This is a phenomenon which is widely observed in various industries. For example, the price of oil rigs (for drilling new wells) crucially depends on the aggregate rate of investment in the oil industry. The increasing cost of capital in our model is not driven by frictions inside of firms; it is rather a phenomenon related to the market price of the capital goods. To better understand the contrast between internal and external convex adjustment costs one should consider a famous critic of "adjustment cost" literature (e.g., Pindyck (1982), Abel (1983)). The internal adjustment cost typically implies that chopping a firm into smaller units is beneficial because it avoids large adjustments in the capital. However, in our model, the size distribution of firms does not matter as long as the aggregate investment rate remains the same; the industry can not benefit for merging or splitting the firms.

Capital Quality and Entry. The model implies that the riskiness of the industry's profits and the entry threshold crucially depend on capital depreciation as well as the elasticity of capital supply. Thus, a social planner interested in an adoption or capacity building for a new product, may optimally pick a *low quality* capital to encourage an early entry.

Output Price Ceiling. The steady-state distribution results enables us to discuss conditions under which a price ceiling for the output commodity may or may not exist in the market. When $b = \infty$, the industry responds to demand shocks immediately through an impulse

control type behavior, and the capital stock jumps very quickly. The output price is bounded in that limiting case. However, as soon as the supply of capital is far from being fully elastic, i.e., $b < \infty$, the right tail of the stationary price distribution will certainly have unbounded support. However, the lower the sensitivity of capital goods prices to the intensity of investment, i.e., when $b \to \infty$, the lower is the probability that ranges of high prices will be reached. This is a probabilistic counterpart of a price ceiling: for a sufficiently large b, it is very unlikely that prices will exceed the entry threshold dramatically.

Risks for Buyers of Output. The rich set of steady-state price-distribution characteristic (e.g., Figure 7 and 8) enable the consumers of the industry's output (e.g., buyers of electricity) to gain better insights with regard to the degree of riskiness in their input costs. For example, if the long-run distribution of output is tight (the base-case in Figure 7) or when the distribution is mainly left skewed, the downstream industry is less concerned about input price risks.

6.2. Caveats and Limitations

We discuss some of the limitations of our model and propose topics for future research.

Lumpy Investment. We assume there are no fixed costs for investment and firms are allowed to choose any rate of investment (e.g., there is no minimum level of investment). Therefore, it is optimal for firms to follow a continuous investment strategy; in other words, firms will not find it optimal to do lumpy investments. The incremental investment model is very relevant to contexts in which the aggregate capacity of the industry changes via building small units (e.g., installing solar panels, small-scale agriculture processing units). Insights may change if the firm has to make lumpy investment decisions due to factors such as fixed costs or indivisible nature of the project. If for example, the firm is deciding to build a large nuclear power plant, a different model is needed to understand the problem of capital quality choice on which our model is silent. Investment in depreciating assets with fixed costs or limited range of investment size is beyond the scope of the current paper and is proposed as a topic for future research.

Industry with Multiple Capital Types. We allow for only one type of capital in the model. A more sophisticated model allows firms in the industry to choose different types of capital. An interesting question in this case is the aggregate capital composition of the industry. With multiple types of capital, a mixed strategy equilibrium outcome at the industry level may emerge: a fraction of firms may choose low quality capital and the remaining fraction use

high quality capital. If the firm can choose the type of capital to use at each moment (or at the beginning), a new dimension is added to the problem. In the first best (with exogenous capital goods prices), each firm unilaterally prefers a high quality capital for itself and a low quality capital for its competitors and a corner solution may arise in each firms choose high quality capital. However, if the price of capital good is endogenous - as it is in our model -, the relative price of high quality capital may rise. A relatively cheap low-quality capital may result in a mix-type outcome, in which a fraction of firms use high quality and the remaining fraction use low quality capital. Optimizing on the quality of capital and the characterization of industry outcome is an interesting topic which requires a full analysis, beyond the scope of current paper. We propose it for future research.

Strategic Impacts. Our model does not consider strategic aspects of capital quality choice. Firms may use the type of capital as a commitment device to send signals to their competitors. A firm choosing a high quality capital is credibly demonstrating the ability to produce high quantity even in bad times. Future research can consider industrial organization aspects of the problem.

Financial Constraints. We do not model financing aspects of the capital purchase. In reality, firms may have different financing capacities for low and high quality capital, which affects their choice of the optimal capital type (Rampini (2019)). Future research can add aspects such as cost of capital and distress to the current model.

7. Conclusion

In this paper we presented a model of continuous aggregate investment under uncertainty which explicitly accounts for the effect of capital depreciation and convex capital goods supply. We derive closed-form solutions for the entry threshold and firm value of both competitive firms.

We derive the steady-state distribution of output prices and solve the model in a rationally consistent fashion. In other words, instead of assuming an exogenously specified price process, we solve the simultaneous determination of capacity and output prices. We show how different features of the industry (e.g., demand elasticity, the rate of depreciation, and the elasticity of capital supply) influence the distribution of long-run prices. The model implies that the riskiness of the industry's profits and the entry threshold crucially depend on capital depreciation as well as the elasticity of capital supply.

A key intuition of the model is that if the convex adjustment costs are interpreted as the increasing market price of capital (versus internal installation costs), the industry equilibrium will not be affected by the size distribution of firms. This is in contrast with Pyndick's criticism of the adjustment-cost literature in which he suggests that splitting a firm to many smaller firms will cause the adjustment cost effect to disappear.

In summary, our analysis provides the following key insights: a) depreciation functions as a hedging mechanism for the competitive industry; b) the distribution of the steady-state prices depends on a complex interaction between depreciation, the elasticity of capital supply, and demand elasticity; c) the volatility of the steady-state prices can have a non-monotone relationship with depreciation; d) when the price of capital is determined in a competitive market, the value of future investment opportunities is zero;

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Appendix A. Notation

Table A.2 summarizes the key notation used throughout the paper.

Variable	Definition
K	active productive capital
π	gross profit per unit of installed capital
I	aggregate investment of competitive industry
$J = \frac{I}{K}$	investment intensity per unit of existing capital (competitive industry)
ξ	depreciation rate of capital
θ	random demand shift parameter
σ	volatility of demand shift parameter
α	drift of demand shift parameter
γ	sensitivity of price to demand shocks
$-1/\eta$	price elasticity of demand
Z	price of capital good
Z^*	critical willingness to pay for an additional unit of capital
b	max. capital supply
$Z_{\min}$	min. price of capital good
q	Capacity utilization factor
P	Output price
$\tilde{P}$	capacity adjusted output price, price conditional on producing at full capacity
s	Entry threshold of competitive firms
V	value of firm
$v = \frac{V}{K}$	average value of a unit of capital
c	marginal cost of production
r	discount rate
β	Roots of the characteristic polynomial

Appendix B. Proof of Propositions 3.1

The HJB Equation (9) is linear in the instantaneous investment rate I over which the representative firm optimizes. The first-order condition of an internal optimum is $\frac{\partial V}{\partial K} = Z(\mathcal{I})$. In equilibrium and according to our assumptions on the limited capacity of capital supply, see Equation (6), an internal optimum requires

$$\frac{\partial V}{\partial K} = Z(I) = \frac{I}{bK}.$$

The rate of newly installed capacity, $\frac{I}{Z(I)K}$, is in an internal optimum equal to b, the supply capacity. If otherwise the marginal value of a unit of capacity is below the minimum price, $\frac{\partial V}{\partial K} < Z_{\min}$, the derivative of V with respect to I in (6) is negative and the optimal choice is

$I = 0$. Consequently, in this case we have an $\frac{I}{Z_{\min}K} = 0$.

We now substitute the optimal rate of newly installed capacity, $\frac{I}{ZK}$, into (6) and recognize that the resulting partial differential equation allows for a solution which is linear-homogeneous in capital K, i.e., $V(K, \tilde{P}) = Kv(\tilde{P})$. This yields Equation (14), which is a pair of ordinary differential equations in the value per unit of capital, v.

Since π is monotonously increasing in $\tilde{P}$ which follows a diffusion process, $v(\tilde{P})$ is monotonously increasing in $\tilde{P}$. We can rule out a region where v constantly equals $Z_{\min}$, because a constant can not satisfy (14). Thus, we conclude that there is at most one price $\tilde{P}$ at which v equals $Z_{\min}$. If v would be below $Z_{\min}$ for $\tilde{P} \in [0, \infty)$, it would be determined by the upper part of (14) in the whole domain of $\tilde{P}$. This ODE has a standard solution with an asymptotic behavior of v for $\tilde{P} \to \infty$ equal to $\frac{\tilde{P}}{r - \gamma\alpha + (1-\eta)\xi} - \frac{c}{r+\xi}$. I.e., it grows without limit for $\tilde{P} \to \infty$, which is a contradiction to the assumption that v is below $Z_{\min}$ everywhere. If otherwise v would be above $Z_{\min}$ in the whole domain of $\tilde{P}$, only the lower part of (14) would determine the solution. Again, this is a standard ODE and since $\pi = 0$ for $\tilde{P} \to 0$, we have $v \to 0$ for $\tilde{P} \to 0$, which contradicts $v > Z_{\min} \forall \tilde{P}$. Thus, we argue that v crosses the critical level of $Z_{\min}$ at one unique level s of the capacity adjusted price $\tilde{P}$, with $\tilde{P} < s \Leftrightarrow v < Z_{\min}$ and $\tilde{P} \geq s \Leftrightarrow v \geq Z_{\min}$. I.e., the price level $\tilde{P} = s$ separates the non-investment region from the investment region.

Substituting optimal capital investment into the dynamics general dynamics (5) of the capital adjusted price, $\tilde{P}$ yields the equilibrium dynamics of $\tilde{P}$ stated in (16).

Under the equilibrium dynamics of $\tilde{P}$, the value function, v, per unit of capacity can be solved analytically. Since the location of the critical price level s, which determines the investment region, relative to the marginal costs c is not pre-determined, we must distinguish cases with $s \leq c$ and $s > c$.

$$v_c(\tilde{P}) = \begin{cases} A_1 \left(\frac{\tilde{P}}{\min\{c,s\}}\right)^{\beta_{1,\mathrm{nE}}} & \text{if} \quad \tilde{P} \leq \min\{c, s\} \\[2mm] A_2 \left(\frac{\tilde{P}}{s}\right)^{\beta_{1,\mathrm{nE}}} + B_2 \left(\frac{\tilde{P}}{c}\right)^{\beta_{2,\mathrm{nE}}} + \frac{\tilde{P}}{r - \gamma\alpha + (1-\eta)\xi} - \frac{c}{r+\xi} & \text{if} \quad c \leq \tilde{P} < s, \\[2mm] A_2 \left(\frac{\tilde{P}}{c}\right)^{\beta_{1,\mathrm{E}}} + B_2 \left(\frac{\tilde{P}}{s}\right)^{\beta_{2,\mathrm{E}}} & \text{if} \quad s \leq \tilde{P} < c, \\[2mm] B_3 \left(\frac{\tilde{P}}{\max\{c,s\}}\right)^{\beta_{2,\mathrm{E}}} + \frac{\tilde{P}}{r - \gamma\alpha + (1-\eta)\xi + \eta b} - \frac{c}{r+\xi} & \text{if} \quad \tilde{P} > \max\{s, c\} \end{cases} \tag{B.1}$$

where exponents $\beta_{1,\mathrm{nE}}$, $\beta_{2,\mathrm{nE}}$ are the positive and the negative root of the characteristic

polynomial

$$g_{\text{nE}}(\beta) = \frac{1}{2}\beta(\beta - 1)\gamma^2\sigma^2 + \beta(\gamma\alpha + \eta\xi) - (r + \xi). \tag{B.2}$$

Subscript nE stands for "no entry", indicating that the drift rate of price process, $\tilde{P}$, is solely determined by capital depreciation. Exponents $\beta_{1\text{E}}$, $\beta_{2\text{E}}$ are the positive and the negative root of the characteristic polynomial

$$g_{\text{E}}(\beta) = \frac{1}{2}\beta(\beta - 1)\gamma^2\sigma^2 + \beta(\gamma\alpha + \eta(\xi - b)) - (r + \xi). \tag{B.3}$$

Subscript E stands for "entry", where we have $I/(ZK) = b$.

The solution makes economic sense only if the discount rate adjusted for depreciation, $r + \xi$, exceeds the risk-neutral expected growth rate of the firm's gross profit in the entry region, $\gamma\alpha + \eta(\xi - b)$. Otherwise, the firms expected discounted cash flows diverge to infinity. Therefore the constraint on the growth rate stated in (17). Note, in the non-entry region, such a constraint is not binding, since high expected growth will not last but push prices in the entry-region, where entry of new firms dampens profit growth.

The final step of the proof is verifying the equilibrium by proving that under the derived optimal policy, individual competitive firms' entrance decisions lead in aggregate exactly to the joint investment rate which we proposed as the optimum. I.e., the investment policy (13) is a Markovian Nash Equilibrium. Verification is straight forward. If aggregate investment equals the proposed equilibrium investment rate (13), then individual, marginal firms are interested in entering the market only if the value of one unit of capacity, v, exceeds $Z_{\min}$, otherwise they will not enter. Since maximum supply capacity limits the rate of entry to $I/Z = bK$. Hence, aggregation of individual entry decisions yields exactly the proposed equilibrium (13).

Appendix C. Proof of Propostition 3.3

With $c \to 0$, the region $\tilde{P} \leq \min\{c, s\}$ in (B.1) vanishes and second region, $c \leq P \tilde{<} s$, starts at $c = 0$. Since $\beta_{2,\text{nE}} < 0$, v is bounded for $\tilde{P} \to 0$ only if $B_2 = 0$. Furthermore, the third region in (B.1) does not exists for $c = 0$, the value function simplifies to

$$v|_{c=0}(\tilde{P}) = \begin{cases} A_2 \left(\frac{\tilde{P}}{s}\right)^{\beta_{1,\text{nE}}} + \frac{\tilde{P}}{r - \gamma\alpha + (1-\eta)\xi} - \frac{c}{r+\xi} & \text{if } \tilde{P} \leq s, \\ B_3 \left(\frac{\tilde{P}}{s}\right)^{\beta_{2,\text{E}}} + \frac{\tilde{P}}{r - \gamma\alpha + (1-\eta)\xi + \eta b} - \frac{c}{r+\xi} & \text{if } \tilde{P} > s \end{cases} \tag{C.1}$$

We solve for the optimal boundary, s, by imposing value-matching and smooth-pasting conditions at the critical threshold s. Consistency with the optimal entry policy (15) requires $v(s) = Z_{\min}$.

$$
\begin{aligned}
A_2 &= Z_{\min} - \frac{s}{r - \gamma\alpha + (1-\eta)\xi} + \frac{c}{r+\xi} \\
B_3 &= Z_{\min} - \frac{s}{r - \gamma\alpha + (1-\eta)\xi + \eta b} + \frac{c}{r+\xi}
\end{aligned}
\tag{C.2}
$$

Taking the derivatives yields

$$
\frac{\partial v}{\partial \tilde{P}} =
\begin{cases}
\beta_{1,\mathrm{nE}} A_2 \left(\frac{\tilde{P}}{s}\right)^{\beta_{1,\mathrm{nE}}-1} + \dfrac{1}{r - \gamma\alpha + (1-\eta)\xi} & \text{if } \tilde{P} < s, \\[2ex]
\beta_{2,\mathrm{E}} B_3 \left(\frac{\tilde{P}}{s}\right)^{\beta_{2,\mathrm{E}}-1} + \dfrac{1}{r - \gamma\alpha + (1-\eta)\xi + \eta b} & \text{if } \tilde{P} > s
\end{cases}
\tag{C.3}
$$

Plugging in values for A_2 and B_3 from the value-matching conditions

$$
\frac{\partial v}{\partial \tilde{P}} =
\begin{cases}
\beta_{1,\mathrm{nE}}\left[Z_{\min} - \frac{s}{r-\gamma\alpha+(1-\eta)\xi} + \frac{c}{r+\xi}\right]\left(\frac{\tilde{P}}{s}\right)^{\beta_{1,\mathrm{nE}}-1} + \frac{1}{r-\gamma\alpha+(1-\eta)\xi} & \text{if } \tilde{P} \le s, \\[2ex]
\beta_{2,\mathrm{E}}\left[Z_{\min} - \frac{s}{r-\gamma\alpha+(1-\eta)\xi+\eta b} + \frac{c}{r+\xi}\right]\left(\frac{\tilde{P}}{s}\right)^{\beta_{2,\mathrm{E}}-1} + \frac{1}{r-\gamma\alpha+(1-\eta)\xi+\eta b} & \text{if } \tilde{P} > s
\end{cases}
\tag{C.4}
$$

Smooth pasting at s requires the two parts of the derivative to be equal at $\tilde{P} = s$, which results in (19).

Proving the stated bounds of the investment threshold s we start from the observation that s is strictly positive. At s we have $v(s) = Z_{min} > 0$. From (C.1) we see that $v(0) = 0$, monotonously increasing, thus, $v = Z_{\min}$ requires $s > 0$. This lower bound of 0 is binding if $r - \gamma\alpha + (1-\eta)\xi < 0$. Otherwise, we know that $(1 - \beta_{2,\mathrm{E}}) > 0$, $(1 - \beta_{1,\mathrm{nE}}) < 0$. Furthermore,

$$
\frac{1 - \beta_{2,I}}{r - \gamma\alpha + (1-\eta)\xi + \eta b} < \frac{1 - \beta_{2,I}}{r - \gamma\alpha + (1-\eta)\xi},
$$

and consequently

$$
\frac{\beta_{1,nE} - \beta_{2,E}}{\frac{1-\beta_{2,E}}{r-\gamma\alpha+(1-\eta)\xi+\eta b} - \frac{1-\beta_{1,nE}}{r-\gamma\alpha+(1-\eta)\xi}} > \frac{\beta_{1,nE} - \beta_{2,E}}{\frac{1-\beta_{2,E}}{r-\gamma\alpha+(1-\eta)\xi} - \frac{1-\beta_{1,nE}}{r-\gamma\alpha+(1-\eta)\xi}} = r - \gamma\alpha + (1-\eta)\xi.
$$

Independent of the sign of $r - \gamma\alpha + (1-\eta)\xi$ we know that

$$\frac{1 - \beta_{2,E}}{r - \gamma\alpha + (1-\eta)\xi} < \frac{1 - \beta_{2,E}}{r - \gamma\alpha + (1-\eta)\xi + \eta b}.$$

If $r - \gamma\alpha + (1-\eta)\xi > 0$, $(1 - \beta_{2,I}) < 0$, thus, dividing it by a larger positive number increases its value. If otherwise, $r - \gamma\alpha + (1-\eta)\xi < 0$, $(1 - \beta_{2,I}) > 0$. Since $r - \gamma\alpha + (1-\eta)\xi + \eta b > 0$ by constraint (17), the left-hand side of the above inequality is negative and the right-hand side is positive. In both cases the above inequality is satisfied and, consequently,

$$\frac{\beta_{1,nE} - \beta_{2,E}}{\frac{1-\beta_{2,E}}{r-\gamma\alpha+(1-\eta)\xi+\eta b} - \frac{1-\beta_{1,nE}}{r-\gamma\alpha+(1-\eta)\xi}} < \frac{\beta_{1,nE} - \beta_{2,E}}{\frac{1-\beta_{2,E}}{r-\gamma\alpha+(1-\eta)\xi+\eta b} - \frac{1-\beta_{1,nE}}{r-\gamma\alpha+(1-\eta)\xi}} = r - \gamma\alpha + (1-\eta)\xi + \eta b.$$

Appendix D. Proof of Proposition 3.4

To determine the sign of the derivative of s in (19) with respect to ξ, we first determine some derivatives of the characteristic roots $\beta_{1,nE}$ and $\beta_{2,E}$. Depending on whether output price $\tilde{P}$ is located in the non-entry region, nE, or in the entry region, E, the drift rate of the price differs, see (16). We use the following notation

$$
\begin{aligned}
\mu_{\tilde{P},nE} &= \gamma\alpha + \eta\xi, \\
\mu_{\tilde{P},E} &= \gamma\alpha + \eta(\xi - b), \\
\sigma_{\tilde{P}} &= \gamma\sigma, \\
l &= r + \xi.
\end{aligned}
$$

The roots $\beta_{1,nE}$ and $\beta_{2,E}$ can be written as

$$\beta_{1,nE} = \frac{1}{2} - \frac{\mu_{\tilde{P},nE}}{\sigma_{\tilde{P}}^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},nE}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}} \tag{D.1}$$

$$\beta_{2,E} = \frac{1}{2} - \frac{\mu_{\tilde{P},E}}{\sigma_{\tilde{P}}^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},E}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}} \tag{D.2}$$

Next we consider that

$$\frac{d\beta_.}{d\xi} = \frac{\partial\beta_.}{\partial\mu_.}\frac{d\mu_.}{d\xi} + \frac{\partial\beta_.}{\partial l}\frac{dl}{d\xi},$$

which yields

$$\frac{d\beta_{1,\text{nE}}}{d\xi} = \frac{1 - \eta\beta_{1,\text{nE}}}{\sigma_{\tilde{P}}^2 \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},\text{nE}}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}}}, \tag{D.3}$$

$$\frac{d\beta_{2,\text{E}}}{d\xi} = \frac{\eta\beta_{2,\text{E}}}{\sigma_{\tilde{P}}^2 \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},\text{E}}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}}}. \tag{D.4}$$

Using these expressions we calculate in a next step

$$\frac{d}{d\xi}\left[\frac{1 - \beta_{1,\text{nE}}}{l - \mu_{\tilde{P},\text{nE}}}\right] = \frac{\frac{\eta\beta_{1,\text{nE}} - 1}{\sigma_{\tilde{P}}^2 \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},\text{nE}}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}}}(l - \mu_{\tilde{P},\text{nE}}) - (1 - \eta)(1 - \beta_{1,\text{nE}})}{(l - \mu_{\tilde{P},\text{nE}})^2}, \tag{D.5}$$

$$\frac{d}{d\xi}\left[\frac{1 - \beta_{2,\text{E}}}{l - \mu_{\tilde{P},\text{E}}}\right] = \frac{1 - \frac{\eta\beta_{2,\text{E}}}{\sigma_{\tilde{P}}^2 \sqrt{\left(\frac{1}{2} - \frac{\mu_{\tilde{P},\text{E}}}{\sigma_{\tilde{P}}^2}\right)^2 + \frac{2l}{\sigma_{\tilde{P}}^2}}}(l - \mu_{\tilde{P},\text{E}}) - (1 - \eta)(1 - \beta_{2,\text{E}})}{(l - \mu_{\tilde{P},\text{E}})^2}. \tag{D.6}$$

These building blocks are used to determine the derivative of s in (19) with respect to ξ. Some tedious transformations reveal that for $\xi = 0$ and η large,

$$\left.\frac{ds}{d\xi}\right|_{\xi=0,\eta\,\text{large}} \sim a + \eta\left(\underbrace{\frac{\beta}{1-\beta}}_{<0\text{ since }(r-\gamma\alpha>0)}\underbrace{\left[\frac{r - \gamma\alpha}{1-\beta}\frac{1}{A} + 1\right]}_{(*)}\right),$$

with a some constant and

$$A = \gamma^2\sigma^2\sqrt{\left(\frac{1}{2} - \frac{\gamma\alpha}{\gamma^2\sigma^2}\right)^2 + \frac{2r}{\gamma^2\sigma^2}} > 0$$

If $(*) > 0$, sufficiently large η makes $\frac{ds}{d\xi}$ negative. This is the condition stated in Proposition 3.4.

Appendix E. Proof of Lemma 3.5

Take the expression for the entry threshold s from Proposition 3.3 and note that for $b \to \infty$ the coefficient $\beta_{2,\text{E}}, \to 0$. Furthermore $\frac{1 - \beta_{2,E}}{r - \gamma\alpha + (1-\eta)\xi + \eta b} \to 0$. Then (23) immediately follows.

Appendix F. Steady-State Price Density under Perfect Competition

Under perfect competition firms will enter at a rate $I = \frac{Z}{K}$ if the capacity-adjusted price $\tilde{P}$ is above the critical level s at which marginal value v equals the minimum price of a unit of productive capital, $v(s) = Z_{\min}$. Below this threshold s firms will not invest. Thus, the price dynamics on both sides of the threshold s is GBM, but the drift differs, see (16). Let subscripts E and nE indicate the region where firms enter the market ($\tilde{P} > s$) and where they do not enter ($\tilde{P} \leq s$), then the dynamics (16) of $\tilde{P}$ can be written as

$$
\begin{aligned}
\frac{d\tilde{P}}{\tilde{P}} &= \left[\mu_{\tilde{P},\mathrm{nE}} \mathbf{1}_{\{\tilde{P} \leq s\}} + \mu_{\tilde{P},\mathrm{E}} \mathbf{1}_{\{\tilde{P} > s\}} \right] dt + \sigma_{\tilde{P}}\, dW, \\
\mu_{\tilde{P},\mathrm{nE}} &= [\gamma\alpha + \eta\xi], \\
\mu_{\tilde{P},\mathrm{E}} &= [\gamma\alpha + \eta(\xi - b))], \\
\sigma_{\tilde{P}} &= \gamma\sigma.
\end{aligned}
$$

The probability density $f(\tilde{P}, t)$ of this price dynamics evolves according to the Kolmogorov Forward equation (also known as Fokker-Planck equation)

$$
\frac{\partial}{\partial t} f(\tilde{P}, t) = -\mu_{\tilde{P}} \frac{\partial}{\partial \tilde{P}} [\tilde{P} f(\tilde{P}, t)] + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial \tilde{P}^2} [\tilde{P}^2 f(\tilde{P}, t)]
$$

In each of the two ranges, the stationary solution has the form

$$
f(\tilde{P}) = \begin{cases} f_{\mathrm{nE}}(\tilde{P}) & \text{if } \tilde{P} \leq s, \\ f_{\mathrm{E}}(\tilde{P}) & \text{if } \tilde{P} > s, \end{cases} \tag{F.1}
$$

$$
\begin{aligned}
f_{\mathrm{nE}}(\tilde{P}) &= A_{\mathrm{nE}} \tilde{P}^{\lambda_{1,\mathrm{nE}}} + B_{\mathrm{nE}} \tilde{P}^{\lambda_{2,\mathrm{nE}}}, \\
f_{\mathrm{E}}(\tilde{P}) &= A_{\mathrm{E}} \tilde{P}^{\lambda_{1,\mathrm{E}}} + B_{\mathrm{E}} \tilde{P}^{\lambda_{2,\mathrm{E}}}
\end{aligned}
$$

with

$$
\begin{aligned}
\lambda_{1,.} &= \frac{2\mu_{\tilde{P},.} - 3\sigma_{\tilde{P}}^2 + 2\left|\mu_{\tilde{P},.} - \frac{1}{2}\sigma_{\tilde{P}}^2\right|}{2\sigma_{\tilde{P}}^2}, \\
\lambda_{2,.} &= \frac{2\mu_{\tilde{P},.} - 3\sigma_{\tilde{P}}^2 - 2\left|\mu_{\tilde{P},.} - \frac{1}{2}\sigma_{\tilde{P}}^2\right|}{2\sigma_{\tilde{P}}^2}.
\end{aligned}
$$

The boundary conditions to be satisfied by the density function f are

$$f_{\mathrm{nE}}(s) = f_{\mathrm{E}}(s), \tag{F.2}$$

$$\int_0^\infty f(\tilde{P})d\tilde{P} = 1. \tag{F.3}$$

Let us discuss three parameter ranges:

$$\mu_{\tilde{P},.} - \tfrac{1}{2}\sigma_{\tilde{P}}^2 = 0: \quad \begin{aligned} \lambda_{1,.} &= -1, \\ \lambda_{2,.} &= -1. \end{aligned}$$

$$\mu_{\tilde{P},.} - \tfrac{1}{2}\sigma_{\tilde{P}}^2 > 0: \quad \begin{aligned} \lambda_{1,.} &= 2\left(\frac{\mu_{\tilde{P},.}}{\sigma_{\tilde{P}}^2} - 1\right) > -1, \\ \lambda_{2,.} &= -1. \end{aligned}$$

$$\mu_{\tilde{P},.} - \tfrac{1}{2}\sigma_{\tilde{P}}^2 < 0: \quad \begin{aligned} \lambda_{1,.} &= -1, \\ \lambda_{2,.} &= 2\left(\frac{\mu_{\tilde{P},.}}{\sigma_{\tilde{P}}^2} - 1\right) < -1. \end{aligned}$$

Boundary condition (F.3) requires that constants A, B in (F.1) associated with $\lambda = -1$ must be zero. Furthermore, the integral (F.3) converges only if $\lambda_{1,\mathrm{nE}} > -1$ and $\lambda_{2,\mathrm{E}} < -1$, which results in the expression for the price density stated in the proposition.

The density of $P = \tilde{P}q^{-\eta}$ equals the density of $\tilde{P}$ in the region of full capacity utilization, $\tilde{P} > c$. At the lower price bound $P = c$, the distribution of P has an atomic probability mass wich equals the entire probability mass of $\tilde{P} \leq c$.

The expected values of P and $\tilde{P}$ exist, if $\lambda_{2,I} < -2$. Otherwise the integrals $\int_0^\infty Pf(P)dP$ and $\int_0^\infty \tilde{P}f(\tilde{P})d\tilde{P}$ diverge. The expectations are given by

$$E(\tilde{P}) = \frac{A_{1,nE}}{\lambda_{1,nE} + 2}s^{\lambda_{1,nE}+2} - \frac{B_{2,E}}{\lambda_{2,E} + 2}s^{\lambda_{2,E}+2} \tag{F.4}$$

$$E(P) = \begin{cases} \begin{aligned}& \frac{A_{1,nE}c}{\lambda_{1,nE} + 1}c^{\lambda_{1,nE}+1} + \frac{A_{1,nE}}{\lambda_{1,nE} + 2}\left(s^{\lambda_{1,nE}+2} - c^{\lambda_{1,nE}+2}\right) \\ & - \frac{B_{2,E}}{\lambda_{2,E} + 2}s^{\lambda_{2,E}+2}, \end{aligned} & \text{if } c \leq s, \\[4ex] \begin{aligned}& \frac{A_{1,nE}c}{\lambda_{1,nE} + 1}s^{\lambda_{1,nE}+1} + \frac{B_{2,E}c}{\lambda_{2,E} + 1}\left(s^{\lambda_{1,nE}+1} - c^{\lambda_{1,nE}+1}\right) \\ & - \frac{B_{2,E}}{\lambda_{2,E} + 2}c^{\lambda_{1,nE}+2}. \end{aligned} & \text{if } c > s. \end{cases} \tag{F.5}$$

The variance of P and $\tilde{P}$ exist, if $\lambda_{2,E} < -3$. Otherwise the integrals $\int_0^\infty P^2 f(P)dP$ and $\int_0^\infty \tilde{P}^2 f(\tilde{P})d\tilde{P}$ diverge. The variances are given by

$$\text{var}(\tilde{P}) \quad = \quad \frac{A_{1,nE}}{\lambda_{1,nE}+3}s^{\lambda_{1,nE}+3} - \frac{B_{2,E}}{\lambda_{2,E}+3}s^{\lambda_{2,E}+3} - E(\tilde{P})^2 \tag{F.6}$$

$$\text{var}(P) \quad = \quad \begin{cases} \dfrac{A_{1,nE}c^2}{\lambda_{1,nE}+1}c^{\lambda_{1,nE}+1} + \dfrac{A_{1,nE}}{\lambda_{1,nE}+3}\left(s^{\lambda_{1,nE}+3} - c^{\lambda_{1,nE}+3}\right) \\ \qquad -\dfrac{B_{2,E}}{\lambda_{2,E}+3}s^{\lambda_{2,E}+3} - E(P)^2, & \text{if } c \leq s, \\[2em] \dfrac{A_{1,nE}c^2}{\lambda_{1,nE}+1}s^{\lambda_{1,nE}+1} + \dfrac{B_{2,E}c^2}{\lambda_{2,E}+1}\left(s^{\lambda_{1,nE}+1} - c^{\lambda_{1,nE}+1}\right) \\ \qquad -\dfrac{B_{2,E}}{\lambda_{2,E}+3}c^{\lambda_{1,nE}+3} - E(P)^2. & \text{if } c > s. \end{cases} \tag{F.7}$$

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