Using a Certainty Equivalent Equilibrium on a Dual Agent Setting with Multiple Levels of Risk Aversion to Model the Forward Curve of Power in Brazil

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Abstract

The development of a simple and effective mechanism to estimate the value of the forward curve of power could enable market participants to better price hedging position and provide transparency to market outsiders who wish to take a speculative position on the power market. This would, in turn, lead to more liquidity in the market for electricity futures and power derivatives. In this paper we design a model for two market participants, a buyer and a seller of a contract for difference on the future spot price of electricity in southwest Brazil. Those are representatives of all market participants that have need or desire to hedge their future position. We model each participant utility function using a Generalized Extended CVaR Preference and obtain the market equilibrium with the certainty equivalent. The results are compared with prediction of the future spot price of power made by market specialists and found to yield reasonable results when using out of sample data.

Keywords: Forward Curve, Contracts for Difference, CVaR, Certainty Equivalent, Power Market

Introduction

The market for futures of electricity in Brazil is small in face of the dimensions of the country. Being a continental sized nation, the electricity distribution and generation was until very recently state controlled and administered. The liberalization of the power trade started in 1997 with RE-SEB the government's Electric Sector Restructure Project, and the creation of the first wholesale market of electricity (MAE).

Most transactions for power delivery and hedge are made between large market players, who inherited or acquired portions of the national generation infrastructure and large regional distributors. Contracts are signed over-the-counter which results in the price being unavailable to other market participants.

This lack of information creates a difficulty in constructing a forward curve of power prices and the ones commercially available comes from research and review of market participants opinions and analysis. Those products, while most certainly invaluable for market forecast, are made during a span of time and cannot be easily replicated for a specific date or point in time.

The forward curve for other financial and commodity products can be built using an arbitrage free approach: if the expected future price of an asset is higher than the cost of acquiring and storing this asset for future consumption then the agents would start storing to sell later, which would drive the expected future price lower, and vice-versa. In electricity market the cost of storing power for an extended period is prohibitive given the current technology. In some cases, the input can be stored as reservoir water in hydropower plants and oil or natural gas in thermopower plants, but this comes with its own limitations and risks which result in the forward curve of electricity being estimated by market specialists.

This work intends to provide a simple yet effective model to construct the forward curve using data commonly available to all market participants, with clear and transparent process. The model, while not tested enough to warrant commercial usage, could serve as a foundation to the development of a more robust approach that could lead to the design of a forward curve created by market consensus.

The model developed for this paper is an equilibrium of market participants, inspired by the work of Bessembinder and Lemmon (2002). We chose the Generalized Extended CVaR Preference (ECP_G) because it has multiple levels of risk aversion which can behave as a more versatile utility function while maintaining the same basic parameters during the whole period of the forward curve.

As the market for power transactions is expanding so is the need for reliable forecasting and verification tools. The forward curve constructed with this method can be used to set the price for hedge contracts with a common source of knowledge, redistributing the market power between large and small participants and incentivizing the participation. It could also be used as forecast instrument for outside participants that do not possess specific market knowledge but wish to speculate on the power market. Another foreseen usage of a forward curve that can be calculated on demand is the analysis of irregular market behavior or market manipulation.

This paper begins with a brief review of the Brazilian electric sector, the financial instruments used, and the utility function selected. This section is followed by a description of the model designed and its calibration and testing. The results are presented and discussed in the fourth section, which is then followed by the conclusion.

Review of the Brazilian power market

Brazil has a territory of 8.5 million square kilometers and most of its cities and districts are connected to the National Interconnected System (SIN), or national grid. Only 1,7% of the country’s power generation is on isolated systems. Some main powerlines have capacity limits that can cause availability of power to differ between regions, which leads to the formation of four different markets for power, the South, the Southeast, the North and the Northeast, with the Southeast market alone comprising the majority of the national power consumption.

The main source of electric power is hydropower, with 66.6% of all generation, and plants range from large international enterprises such as Itaipu producing up to 14GW, to run of the river plants and to small local hydropower facilities. Thermal power plants constitute 25.2% of the generation, of which 8.6% comes from natural gas plants, 8.5% from biofuels and agricultural sub-products, 3.2% from coal power plants, 2.5% from two nuclear fission plants and 2.4% from heavy oil plants. The remaining 8.2% are generated by solar, wind farms and other alternative sources (EPE, 2019).

Because of the predominance of hydropower, the energy market is closely tied to the rain patterns and the natural affluence of rivers. This led to a centralized planning for water storage and dispatching, which is done by the National System Operator (ONS), based on a model of affluence prediction and minimizing of the marginal cost of operation, the Newave-Decomp software. The outputs of the model in terms of predicted marginal costs of operation are available to market participants and are used are a guideline to calculate the expected future price of power in the spot market. Optimization based models, such as are used in the Newave-Decomp software, do not include risk premium in the calculation of future prices. They might be fit for prediction of future spot price of power but should not be used directly to obtain the future price of power derivatives.

Consumption of power in the Brazilian market was also administered by state agencies until the 1990s, when a process of decentralization took place to compensate for the lack of investment made by the government in the energy sector. In 1998 the restructuring of the sector was concluded with guidelines to reduce vertical integration and increase competition. In the first years of the 2000s a nationwide contingency of power, brought immediate changes to the energy sector, which became law in 2004 and established a new open market for power delivery contract trading.

The power market is divided in two segments: captive consumers, which are protected by the regulated market, with the cost of power mandated by law, and free consumers that can purchase energy with the free-flowing short term power price, the price for difference liquidation (PLD). In the free market large consumers can trade bilateral contracts of power with generators.

Contracts for Difference

Contracts for difference (CfD) are financial instruments or derivatives in the form of a contract between two interested parties, a buyer and a seller, with the objective of settling between the participants the differences of two determined values or prices.

One form of CfD involves setting a specific reference price set for an underlying asset future price. At each time interval the parties will transfer among themselves the difference between the reference price and the observed price. Because the derivative is focused on the differences and there is no deliverable on the end of the contract, no party is obliged to hold the underlying asset. The goal of this operation is to obtain a financial compensation for changes from the price of reference and the spot price, acting as a hedge or leverage instrument.

As a safety measure both parties can set aside a margin account with a monetary amount to warrant the payments for price differences at each period. The amount invested in this account is lower than what would be required if the contract dealt with a position of the asset, since the nominal value of holding the asset is not covered by the contract, only the maximum variation of the price inside the time interval. By using this derivative, the investor can have exposure to the underlying asset by withholding only a fraction of its value, which is a form of leverage.

Since the contract can be signed in directly between the participants the costs of intermediaries are reduced. The risk, on the other hand, is given by each side ability to pay the difference and keep money on the margin account as required. There can be considerable losses even when the price is moving favorably, if the counterpart cannot pay the difference. For this very reason CfD are usually signed by parties with previous relationships and mutual trust, in order to minimize counterpart risk. A CfD market with management of margin deposits and payments can also be created given enough demand.

In the Brazilian power market CfD are common between generators, big consumers, distributors or investors. It can be used as a hedge or to get exposure to future energy prices. Deals can be made without an expiration date and can be interrupted by one of the parties, according to the terms previously set on the contract.

Research in the use of CfD in electric power markets if frequent, specially concerning the Nordic common market. (Kristiansen, 2004; Marckhoff & Wimschulte, 2009; Kozlov 2014)

Generalized Extended CVaR Preference

Utility functions enable the use of analytic tools to situations involving uncertainty. The basis of modern finance theory is the Von Neumann and Morgenstern [Von Neumann, 1944] utility function, that is a direct function of wealth. In the modern portfolio theory, the utility function is quadratic, which provides a risk aversion derived from its convexity. This function provides unexpected results when the wealth level increases and the risk aversion also increases, providing challenges to its interpretation.

Another problem with the utilization of the modern portfolio theory is the use of historical volatility as a risk measure, and not only downside risk [Markovitz, 1959]. The use of semi-variance and measures of downside risk led to the development of the VaR (Jorion, 1996) measure, and following it, the CVaR (Rockafellar and Uryasev, 2016).

A combination of the CVaR with the expected financial outcome provides a utility function with desirable behavior and was named Extended CVaR Preference functional (ECP) by Street (2010). This became the objective function used in the Brazilian centralized hydropower dispatch.

The ECP uses two parameters to express the risk aversion of the individual, the first the weight (λ) and the second the level of CVaR (α). ECP is expressed by the following equation:

$$ECP_{\alpha,\lambda} = (1 - \lambda)\, E[X] + \lambda\, CVaR_{\alpha}(X)\ ;\ \lambda \in [0,1] \qquad (1)$$

The ECP measure combines the simplicity of a linear utility function while at the same time avoiding the Allais paradox because of its non-linearity. Because the utility levels for the individual may vary increasingly with the losses, the ECP functional was generalized by Luz (2016) into having multiple risk aversion coefficients. The ECP_G is described as follows:

$$E[U(X)] = \lambda_0\, E[X] + \sum_{n=1}^{N} \lambda_n\, CVaR_{\alpha n}(X)\ \ ;\ \sum_{n=1}^{N} \lambda_n = 1 \qquad (2)$$

Figure 1 shows the expected behavior of the utility function underlying the ECP_G functional for two levels of risk aversion.

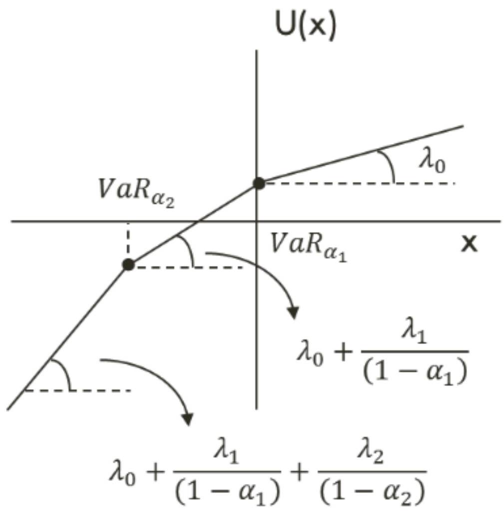

Figure 1 – Utility function underlying the ECP_G functional with N=2. Source: Luz, 2016.

Certainty Equivalent

In an uncertain scenario the risk premium calculated by a utility function can be expressed as a monetary value which would be equivalent to participant as whether to take this value or take the risk. This certainty equivalent can be calculated by applying the invert of the utility function on the expected outcome of the scenario.

The certainty equivalent for the ECP can be obtained using the inverse utility function, which has been proved to exist by Street (2010). For the ECP_G the certainty equivalent is given by Luz (2016) and because of the segmented nature of the underlying utility function of the ECP_G it is expressed by the following system of equations:

$$\begin{cases} Eq = E[X] + \frac{1}{\lambda_0}\sum_{i=1}^{N}\lambda_i\left(CVaR_{\alpha i}(X) - VaR_{\alpha i}(X)\right); \\ \quad \text{for } E[U(X)] \in \left]\lambda_0 VaR_{\alpha 1}(X) + L,\ \lambda_0 VaR_{\alpha 0}(X) + L\right[\\ Eq = \frac{1}{Q}\left(\lambda_0 E[X] + \sum_{i=1}^{N}\lambda_i\left(CVaR_{\alpha i}(X) - VaR_{\alpha i}(X)\right) + \sum_{i=1}^{n}\frac{\lambda_i}{(1-\alpha i)}VaR_{\alpha i}(X)\right) \\ \quad \text{for } E[U(X)] \in \left]\lambda_0 VaR_{\alpha n+1}(X) + L,\ \lambda_0 VaR_{\alpha n}(X) + L\right[\\ L = \sum_{i=1}^{N}\frac{\lambda_i}{(1-\alpha i)}VaR_{\alpha i}(X)\,;\ Q = \frac{1}{\lambda_0 + \sum_{i=1}^{n}\frac{\lambda_i}{(1-\alpha i)}}\,;\ \text{n} \in [1,\ N]\,;\ \sum_{n=1}^{N}\lambda_n = 1 \end{cases} \tag{3}$$

The certainty equivalent can be used as a direct result of the utility function and because it is directly expressed in monetary value can be directly compared with results obtained by different participants with different risk aversion levels or parameters.

Methodology and Model

Data used in this paper consists of 2 sets of 2000 series of estimates of expected future power prices all of which were obtained from Newave-Decomp, the software used by ONS to calculate the marginal cost of power and the future price in the short term market, on the last week of January 2019 and August 2019. In the same dates the forward curve for each concerning year was obtained from DCide.

The model is designed as an equilibrium between market participants that have the intention to hedge against future price variations, and every participant will be part of either the buying or the selling side. The equilibrium will be obtained when the certainty equivalent is equal for both sides. This is similar to the concept drawn by Benth et al. (2008) but using contracts for difference and the ECP_G functional.

The payoff for the CfD uses the following structure:

$$\begin{cases} \pi_S = (p_r - p_s) \times Q \\ \pi_B = (p_s - p_r) \times Q \end{cases} \tag{4}$$

Where π_S is the payoff of the seller, π_B is the payoff of the buyer, p_r the price of reference of the CfD contract, p_s the observed future spot price and Q the quantity of power delivered (which was arbitrated at 100 MWmed).

The ECP_G functional selected has two levels of risk aversion, which allows for significant flexibility while maintaining a straightforward calculation. The equation for the average utility used in this paper is the following:

$$E[U(X)] = \lambda_0 \, E[X] + \lambda_1 \, CVaR_{\alpha 1}(X) + \lambda_2 \, CVaR_{\alpha 2}(X) \tag{5}$$

The equilibrium model was calibrated using the dataset for the year 2020, obtained in January 2019, to arrive at the best fit with the forward curve published in the same date. A discounted cashflow was calculated for each of the 2000 scenarios and based on the NPV of each scenario the significant measures of risk were obtained. The calculation was done in Excel's Solver optimization package Evolutionary algorithm and the target was the minimization of the quadratic difference of the certainty equivalent of the buyer and the seller. The discount rate used in the NPV calculation was 5% yearly.

After the best parameters were selected for λ_1, λ_2, α_1 and α_2 for both buyer and seller, allowing for different levels of risk aversion for each participant, the model was then tested on the August 2019 dataset and compared with the DCide forward curve for the same date.

Results and Discussion

The optimization converged in a best solution for the model, which was the same regardless of the starting values. The results obtained in the calibration step are displayed in tables 1 and 2 below.

Table 1 – Results of the model calibration step. NPV and CEq are expressed in R$ millions

	NPV	**ECP_G**	**Certainty Equivalent**
Buyer	-87	-110	-98
Seller	87	-151	-98

Table 2 – Optimized parameters for the ECP_G model

	λ_0	**λ_1**	**λ_2**	**α_1**	**α_2**
Buyer	0.47	0.21	0.32	0.40	0.99
Seller	0.04	0.39	0.57	0.94	0.99

The model equations using the optimal parameters are as follows:

$$\begin{cases} ECP\_G_B = 0.47 \, E[X] + 0.21 \, CVaR_{40}(X) + 0.32 \, CVaR_{99}(X) \\ ECP\_G_V = 0.04 \, E[X] + 0.39 \, CVaR_{94}(X) + 0.57 \, CVaR_{99}(X) \end{cases} \tag{6}$$

Those results indicate a greater risk aversion on the seller of the CfD contract, which is expected, since the average expected price of power is around 100 R$/MWh and the minimal price is 42.35 R$/MWh, while the maximum price is 513.89 R$/MWh. The distribution of NPV and the associated utility levels for the buyer and seller can be seen in figures 2 and 3 below.

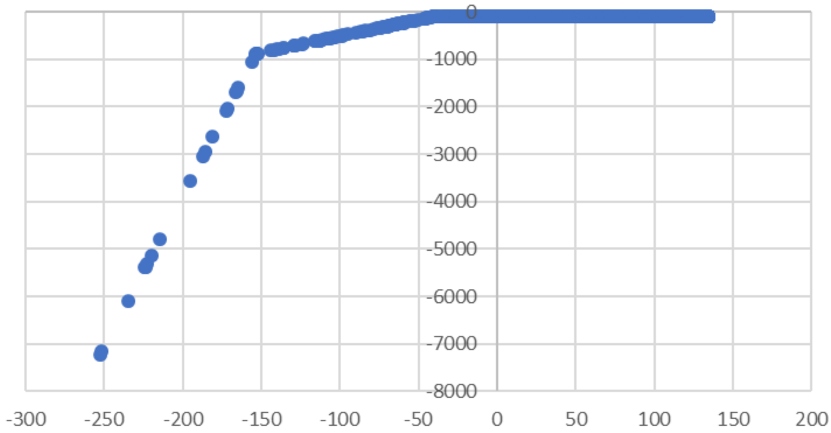

Figure 2 – Utility for the seller (y-axis) and NPV of the CfD (x-axis)

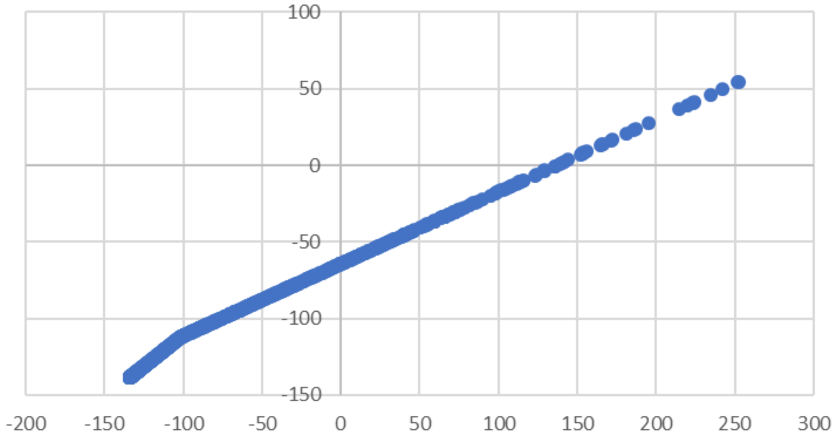

Figure 3 - Utility for the buyer (y-axis) and NPV of the CfD (x-axis)

The relevant data for Figure 2 is the inclination of the utility curve, which is 0.04 in the risk less portion, 57 in the risk most portion and 7 in between and the breakpoints in $VaR_{\alpha1}$ = -41 million Reais and $VaR_{\alpha2}$ = -153 million Reais. For Figure 2 the inclination of the utility curve in the risk less portion is 0.47, 32 in the risk most portion and 0.35 in between and the breakpoints are $VaR_{\alpha1}$ = -102 million Reais and $VaR_{\alpha2}$ = -134 million Reais. The third segment of the curve is not seen in Figure 2 due to an accumulation of the lower price in the same range, and the cut points selected by the model.

It is possible to run the model with several successive reference prices and take note of the NPV and certainty equivalent for each participant at each step. This was done for Figure 4, below.

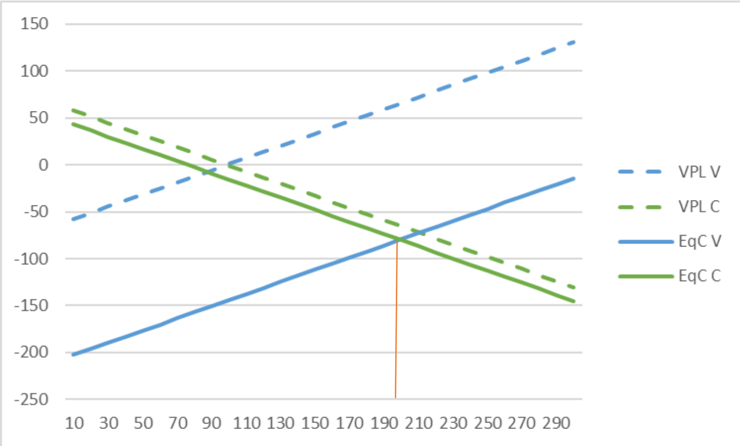

Figure 4 – Step by step analysis of the reference price of the CfD (x-axis) and the impact on both the seller and buyer NPV and certainty equivalent (EqC). The vertical line shows the model equilibrium at 199.32 R$/MWh.

The greater risk aversion of the seller is represented in Figure 4 in the distance between the NPV and the certainty equivalent (blue gap) as opposed to the same distance for the buyer (green gap).

The same model was used in the dataset obtained in August 2019 for the year 2020 and compared with the forecast made by DCide. The model equilibrium was achieved when the CfD price was set in 218.20 R$/MWh, while the forward curve estimation for the same period given by market specialists was 192.75 R$/MWh.

Conclusion

This work intends to use the commonly available market data and translate it to a reasonable forecast of the expected future spot price for the Brazilian power market. This model, if proved to be adherent to reality and behave accordingly to market specialists' estimations, could be used in several applications providing transparency and consistency to electricity futures markets.

This has been an explorational first approach and is limited in the data availability which might have lent nearsightedness to the model and its results out-of-sample. Nevertheless, we believe that the modelling of futures based on available market data could be similar to what analysts are doing with their projections, and if enough data is provided a good model might be obtained.

We will continue to study the model and the findings in this paper and will work on an analytical solution to the equilibrium, which is already on the way. This may be the subject of a future work.

Another important factor to consider is the temporality of market power, described by Benth et al. (2008) which could lead to changes in the risk aversion parameters in time. This was not studied in this paper since allowing for the weights to change with a small amount of data available could lead to overfitting the model.

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