Get Out or Get Down: Competitive Strategies in a Declining Market

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Abstract

We propose solutions for a multi-factor real option duopoly game model which determine the optimal time to divest in an incumbent technology or to switch to a new smaller-scale and lower operating cost technology, with an uncertain output price, and declining output. We use two formulations, one in which the options to divest and switch are treated separately (separate formulation), and another in which these two options are mutually-exclusive (joint formulation). For the technology switch, there is a temporary second-mover revenue market share advantage, whereas for the divestment there is a first-mover salvage value advantage. The joint formulation assumes that the current market revenue is suddenly between the switch and the divest thresholds of the two firms obtained by the separate formulation, as might occur with a pandemic or industry turmoil. Then, we find that the decision thresholds using the joint formulation are quite different from those obtained by the separate formulation: the first-mover divests earlier under the separate formulation, and switches much earlier, and the second-mover also switches earlier. Additionally, the region of inaction (or hysteresis) is greater under the separate formulation. Finally, the sensitivities of the thresholds to changes in the price volatility and in market shares are often of opposite signs for the joint versus separate. Thus, in deciding to get out (divest) or get down (switch to lower operating costs), not only watch the competition but also consider the action options jointly.

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1. Introduction

In markets where output prices are uncertain and output demand is declining, firms often consider simultaneously the option to switch to a new smaller-scale technology, benefitting thereafter from lower operating costs, and the option to divest. In addition, it is assumed that at the moment the firm evaluates the switch/divest problem, the random state variable (e.g., revenue) is above the switch threshold, otherwise it would trigger an immediate switch. However, Décamps et al. (2006) asked a very interesting question: how should firms behave if suddenly, at the beginning of the investment analysis, the state variable is between the divest and switch thresholds? They show that if the switch and the divest thresholds are derived separately, the former threshold is lower than the latter and if (by chance) when the investment analysis is first performed the state variable is in this region, both the option to switch and the option to divest must coexist. Although this scenario is less likely than that in which the state variable is above the switch threshold, it is still possible specially when a pandemic or industry turmoil occurs. In this scenario, if the state variable increases sufficiently, it will trigger the switch, whereas if it decreases enough, it will trigger the divest.

Dias (2004) first raised the mutually exclusive option problem, and provided solutions using finite differences. Décamps et al. (2006) show that in this particular (idle) region, firms hold simultaneously the option to switch and the option to divest. Siddiqui and Fleten (2010) implement the Décamps et al. (2006) model for mutually exclusive projects with an unusual solution. Adkins and Paxson (2019) study this problem, and propose analytical and numerical solutions for a monopoly market. We extend the above literature to a duopoly facing an output declining market. We assume that there is a duopoly market with two active and ex-ante symmetric firms, where there is a first-mover divestment advantage (higher salvage value) and second-mover temporary switch advantage (higher market share), which may involve client inertia.

We note that downscaling during pandemics, change in fashion or technology, or conventional usage patterns, may well inspire first movers to switch technology. But who wants to be first, adapting to temporary client inertia regarding lower-cost operations (online vs on-campus education)? Other contexts are firms, industry or countries facing stagnation or revenue decline, due to natural factors such as in petroleum production, and economic or structural factors, where possible new alternative technologies may validate delaying exit-abandonment, or switching to lower cost production. Due to both pollution concerns and competition from natural gas, coal almost everywhere is being shut down, possibly awaiting cheaper emission control. Book shops and shopping malls in the US (Borders and Barnes & Noble) are being closed, or converted to alternative uses (cafes and reading rooms, rather than book selling). Ceramics and textiles in developed countries faced closure or downsizing. Taxis, accommodation, and universities are experiencing competition from mobile-digital technologies.

Décamps et al. (2006) study irreversible investments in alternative projects and show that when firms hold the option to switch from a smaller scale to a larger scale project, a hysteresis region between the investment region can persist even if the uncertainty of the output price increases. Bobtcheff and Villeneuve (2010) examine investments in two mutually exclusive projects with two sources of uncertainty, and conclude that when these uncertainties hold simultaneously, the project payoffs are not sufficient criteria for deciding on the investment timing. Kwon (2010) looks at a declining profit stream following an arithmetic Brownian motion process, so the exit threshold decreases as volatility increases. Adkins and Paxson (2011) investigate optimal capital replacement and abandonment decisions considering that both revenues and costs are uncertain and their value declines over time. Chronopoulos and Siddiqui (2015) study the timing of the replacement of an incumbent technology, assuming that there is technological uncertainty, and the ex-post revenues which the adoption of the new technology generate are uncertain. This investment analysis is examined under three different strategies, compulsive, laggard, and leapfrog. Their results reveal that, under the compulsive strategy, technological uncertainty has a non-monotonic impact on the optimal investment decision.

Hagspiel et al. (2016) look at investment decisions in a new technology under uncertainty in profit declining markets, where firms hold the option to invest in a new technology with which they produce a new product, holding the option to exit the market and considering that the firms also decide on the capacity size. Among other findings, they show that a higher potential profitability of the new product market accelerates the investment timing, but the capacity choice can alter this result, reversing the above intuitive result, if the choice of the investment capacity is smaller.

Støre et al. (2018) study an irreversible switch from oil to gas production, with both oil and gas production declining over time. They provide analytical solutions for the switching threshold and the real option value of the switching opportunity. Huberts et al. (2019) show that entry may be deterred, possibly in a war of attrition or pre-emption, following interesting strategies. Adkins and Paxson (2019) study the appropriate rescaling for a monopoly from an incumbent large-scale technology assuming that market revenue is declining. They also consider the case of abandonment and treat the two investments both separately and jointly, showing different implications for government policies.

Several authors focus on the uncertainty of new technologies, which should provide interesting extension of our current approach. Farzin et al. (1998) assume both the speed of arrival and degree of improvement of future technologies are uncertain. Doraszelski (2004) allows for future technologies with improvements. Hagspiel et al. (2015) also consider changing arrival rates for new technologies.

We have set up a context where the first-mover advantage is small, dependent on only obtaining full salvage value, so some of the option values and thresholds are very sensitive to small changes in the ex-post “market share”. These market sharing assumptions constitute quasi-pre-emptive games, where the second-mover is not immediately motivated to adopt the cost reduction technology in the second stage (or perhaps not motivated because of the alternative temporary larger market share, maybe a management delusion). Eventually, the second-mover is allowed to adopt the new technology (but with an equal market share). Lieberman and Montgomery (1988) focus on technological leadership (which we adopt), pre-emption of scarce assets, and customer switching costs. Joaquin and Butler (2000) consider the first mover advantage of lower operating costs. Tsekrekos (2003) suggests both temporary and pre-emptive permanent market share advantages for the leader in a sequential investment pattern. Paxson and Pinto (2003) model a leader with an initial market share advantage, which then evolves as new customers arrive (birth) and existing customers depart (death). Paxson and Melmane (2009) provide a two-factor model where the leader starts with a larger market share, applied to show that (by foresight) Google was likely to be undervalued compared to Yahoo at the Google IPO. Bobtcheff and Mariotti (2010) consider a pre-emptive game of two innovative competitors, whose existence may be revealed only by first mover investment.

The rest of the paper is organized as follows. Section 2 presents both the divestment and the switching models for a separate formulation and their respective sensitivity analyses. Section 3 derives the divestment and the switching models for a joint formulation, and presents their respective sensitivity analyses. Section 4 concludes the work and provides some suggestions for further research.

2. Base Model

Let us consider a duopoly market with two active and ex-ante symmetric firms operating with an incumbent high-cost technology (policy *X*), where there is output price ($P(t)$) uncertainty and the output quantity ($q(t)$) is declining over time. Each firm holds the option of either continuing operations with policy X or abandoning production and receiving a salvage value. A parameter $\lambda \epsilon (0,1)$ ensures that there is a first-mover divestment advantage, where Z is the divestment value of the first-mover and λZ the divestment value of the second-mover. Moreover, there is an alternative lower-cost technology (policy *Y*) available, so the two firms can also consider switching from policy *X* to policy *Y*. The operating costs associated with policies *X* and *Y* are given by f_k, with $k \epsilon \{x, y\}$. f_x and f_y are the operating costs for policies *X* and *Y*, respectively.

A motivation for exiting the market first (apart from the higher divestment value) is because there is output price uncertainty and the output is declining over time, so the high-cost business can become unprofitable soon. Furthermore, the two firms are assumed to be ex-ante symmetric, so they face the same periodic operating costs, which vary with the size of their market share.

This section studies the optimal time to divest in policy *X*, or to switch to policy *Y*, relying on two distinct model formulations. One formulation, named "separate" in which the option to divest in policy *X* and the option to switch to policy *Y* are treated independently and, another, named "joint" in which these two options co-exist. The joint formulation follows that of Décamps et al. (2006) and Adkins and Paxson (2019) derived for a monopoly market. We show the differences between the duopoly investment thresholds for these two model formulations.

The output price follows a geometric Brownian motion (GBM) process given by:

$$dP = \alpha_P P dt + \sigma_P P dW \tag{1}$$

where, α_P is the instantaneous conditional expected percentage changes in P per unit of time, σ_p is the instantaneous conditional standard deviation of P per unit of time, and dz is the increment of a standard Wiener process. For convergence of the solution $r - \alpha_P > 0$, where r is the riskless interest rate. For simplicity of notation, we assume that the asset or convenience yield is given by $\delta_p = r - \alpha_P$. The output quantity flow q declines over time according to:

$$dq = -\theta q dt \tag{2}$$

where $\theta > 0$ denotes a known constant depletion rate.

Therefore, firm *i*'s revenue flow, if operating with policy *k*, is given by:

$$P(t)q(t).D_{k_i k_j} \tag{3}$$

where $D_{k_i k_j}$ is a deterministic competition factor that represents the percentage of the firm *i*'s market revenue share for a given scenario, with $i, j = \{L, F\}$, where L means "first-mover" and F "second-mover" and $k = \{0, X, Y\}$, where "0" indicates inactive, "*X*" indicates policy *X*, and "*Y*" indicates policy *Y*.[2]

For the first-mover and the second-mover, inequalities (4a) and (4b) hold for the divestment, whereas inequalities (4c) and (4d) hold for the switching:

$$D_{X_L X_F} > D_{0_L X_F} \tag{4a}$$

$$D_{X_F 0_L} > D_{X_F X_L} \tag{4b}$$

$$D_{X_L X_F} = D_{Y_L Y} > D_{Y_L X_F} \tag{4c}$$

$$D_{X_F Y_L} > D_{X_F X_L} = D_{Y_F Y_L} \tag{4d}$$

Specifically, for the divestment, if $\mathrm{D}_{\mathrm{X_F 0_L}} = 1.0$ and $\mathrm{D}_{\mathrm{X_F X_L}} = \mathrm{D}_{\mathrm{Y_F Y_L}} = 0.5$, it means that the second-mover gets 100 percent of the market share when alone in the market with policy *X*, and each firm gets 50% of the market share when they are active either with policy *X* or with policy *Y*, respectively. For the switching, in our base case scenario, we assume that $\mathrm{D}_{\mathrm{Y_L X_F}} = 0.4$ and $D_{\mathrm{X_L X_F}} = \mathrm{D}_{\mathrm{Y_L Y_F}} = 0.5$, which means that, when the first-mover switches to policy *Y* (with the second-mover still operating

[2] As an illustration about how these competition factors work: for the divest scenario, $D_{X_F 0_L}$ represents the second-mover's market share when it operates with policy *X* alone (after the first-mover has left the market) and $D_{X_F X_L}$ represents the second-mover's market share for when both firms operate with policy *X*. A similar rationale applies to the competition factors of the switch scenario. For instance, $D_{Y_L Y_F}$ represents the first-mover's market share when both firms operate with policy *Y*, whereas $D_{Y_L X_F}$ represents the first-mover's market share when the first-mover is active with policy *Y* and the second-mover is active with policy *X*. Notice that $D_{k_F k_L} + D_{k_L k_F} = 1$, the full market. Also, we can express the first-mover's competition factors as a function of the second-mover's (and vice-versa), that is: $D_{k_L k_F} = 1 - D_{k_F k_L}$.

with policy *X*), its market share drops (from 50% to 40%), although it will get back to 50% again as soon as the second-mover also switches to policy *Y*.[3]

Using a risk-neutral framework and Ito's Lemma, we find that the value of an active second-mover with the option to divest and operating costs f_k satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_k^{i,j}}{\partial p^2} + (r-\delta)p\frac{\partial F_k^{i,j}}{\partial p} - \theta q \frac{\partial F_k^{i,j}}{\partial q} + pqD_{X_F X_L} - f_k - rF_k^{i,j} = 0 \tag{5}$$

where $F_k^{i,j}$ denotes the option value of firms *i* and *j* when they are active with policy *k*.

Based on the American perpetuity solution, the value function $f_k^{i,j}(p,q)$ satisfying the differential equation (5) takes the form:[4]

$$f_k^{i,j}(p,q) = A_1^{i,j} p^{\beta_1} q^{\gamma_1} + A_2^{i,j} p^{\beta_2} q^{\gamma_2} + \frac{pqD_{1_F 1_L}}{\delta+\theta_k} - \frac{f_k}{r} \tag{6}$$

where A_1 and A_2 are two non-negative coefficients to be determined, with β_1 and β_2, and γ_1 and γ_2 related through the following characteristic equation:

$$Q(\beta_k, \gamma_k) = \frac{1}{2}\sigma^2 \beta_k(\beta_k - 1) + (r-\delta)\beta_k - \theta\gamma_k - r = 0 \tag{7}$$

By examining the respective smooth-pasting conditions, we conclude that the principle of similarity can be applied, which implies that $\gamma_{1k} = \beta_{1k}$ and $\gamma_{2k} = \beta_{2k}$ (see, Paxson and Pinto, 2005). Therefore,

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[3] Competition is embedded in our model through two factors. One is the firms' market share ($D_{k_i k_j}$) that is governed by the inequalities 4a, 4b, 4c, and 4d, and another is the second-mover's (first-mover's) divestment disadvantage (advantage), since the divestment value of the second-mover is only a percentage λ of that of the first-mover.

[4] See Adkins and Paxson (2011) and Adkins and Paxson (2017).

$$\beta_{1(2)k} = \left(\frac{1}{2} - \frac{r-\delta-\theta_k}{\sigma^2}\right) + (-)\sqrt{\left(\frac{1}{2} - \frac{r-\delta-\theta_k}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \quad (8)$$

where $\beta_{1k} > 1$ and $\beta_{2k} < 0$.[5]

We note that β_{1k} and β_{2k} vary with the depletion rates θ_k, which we assume are the same for the two policies, so $\theta_k = \theta$. Additionally, because the similarity principle holds, the analysis can be framed in terms of a single variable, using the following variable change: $\nu = p.q$. Therefore, Equation (6) becomes:

$$f_k^{i,j}(\nu) = A_1^{i,j}\nu^{\beta_{1k}} + A_2^{i,j}\nu^{\beta_{2k}} + \frac{\nu D_{k_F k_L}}{\delta+\theta} - \frac{f_k}{r} \quad (9)$$

2. Separate Formulation

A key assumption underlying the separate formulation is that the options to switch and divest are exclusive options, that is, if the firm exercises one of these options it will not exercise the other. In addition, for simplicity, we assume that, if firms switch from policy *X* to policy *Y*, they will operate with policy *Y* forever, and that the investment game ends after the firms has divested. For a duopoly game, there are two additional implicit assumptions: i) the first-mover switches and divests before the second-mover, and ii) at the beginning of the game, the state variable ν is above $\hat{\nu}_{SS}^L$ for the switching and above $\hat{\nu}_{SD}^L$ for the divesting. Finally, the second-mover gets 100% of the market revenue after the first-mover has divested but (we assume) only a percentage λ of the first-mover's divestment value (Z).

2.1 Divesting

2.1.1 Second-mover

[5] Henceforth, when no confusion is possible, we drop the subscript "k" from the solution of the quadratic characteristic equation (7): $\beta_{1(2)k}$.

The value function is given by:

$$f_X^F(\nu)\big|\nu(0) > \hat{\nu}_{SD}^L \begin{cases} \frac{\nu D_{X_F X_L}}{\delta+\theta} - \frac{D_{X_F X_L} f_X}{r} + A_{2D}^F\, \nu^{\beta_2} + \frac{\hat{\nu}_{SD}^L \left(D_{X_F 0_L} - D_{X_F X_L}\right)}{\delta+\theta}\left(\frac{\nu}{\hat{\nu}_{SD}^L}\right)^{\beta_2} & if\ \nu > \hat{\nu}_{SD}^L \\ \frac{\nu D_{X_F 0_L}}{\delta+\theta} - \frac{D_{X_F 0_L} f_X}{r} + A_{2D}^F\, \nu^{\beta_2} & if\ \ \nu \in [\hat{\nu}_{SD}^L, \hat{\nu}_{SD}^F) \\ \lambda Z & if\ \ \nu \leq \hat{\nu}_{SD}^F \end{cases} \tag{10}$$

The economic interpretation for (10) is the following: in the first row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with policy *X,* the third term represents the option value to divest, and the fourth term is the second-mover's expected gain due to the fact that the first-mover divests first and, when it divests (ν reaches $\hat{\nu}_{SD}^L$), the market share of the second-mover increases from $D_{X_F X_L}$ to $D_{X_F 0_L}$; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the second-mover is alone in the original market, and the third term is the option value to divest; the term in the third row is the second-mover's divestment value which is a proportion (λ) of the divestment value of the first-mover.

The option coefficient A_{2D}^F and divestment threshold $\hat{\nu}_{SD}^F$ are obtained from the value-matching and smooth-pasting conditions, evaluated at $\hat{\nu}_{SD}^F$, given by Equations (11) and (12), respectively:

$$\frac{\hat{\nu}_{SD}^F D_{X_F 0_L}}{\delta+\theta} - \frac{D_{X_F 0_L} f_X}{r} + A_{2D}^F\, {\hat{\nu}_{SD}^F}^{\ \beta_2} = \lambda Z \tag{11}$$

$$\frac{D_{X_F 0_L}}{\delta+\theta} + \beta_2 A_{2D}^F\, {\hat{\nu}_D^F}^{(\beta_2-1)} = 0 \tag{12}$$

Using Equations (11) and (12), we determine the constant A_{2D}^F and the divestment threshold ν_{SD}^F:

$$A_{2D}^F = \frac{-D_{X_F 0_L}}{(\delta+\theta)\beta_2 {\hat{\nu}_{DX}^F}^{(\beta_2-1)}} \tag{13}$$

$$\hat{\nu}_{SD}^F = \frac{\left(D_{X_F 0_L} f_X + r\lambda Z\right)\beta_2(\delta+\theta)}{D_{X_F 0_L} r(\beta_2-1)} \tag{14}$$

2.1.2 First-mover

The value function is given by:

$$f_X^L(\nu)\big|\nu(0) > \hat{\nu}_{SD}^L = \begin{cases} \frac{\nu D_{X_L X_F}}{\delta+\theta} - \frac{D_{X_L X_F} f_X}{r} + A_{2D}^L\, \nu^{\beta_2} & if\ \nu > \hat{\nu}_{SD}^L \\ Z & if\ \nu \in [\hat{\nu}_{SD}^L, \hat{\nu}_{SD}^F) \\ Z & if\ \nu \le \hat{\nu}_{SD}^F \end{cases} \quad (15)$$

The economic interpretation of (15) is the following: the first two terms in the first row represent the revenue stream less the operating costs for the region where the two firms operate with policy *X*, and the third term is the option value to divest; the term in the last two rows represents the divestment value.

The option coefficient A_{2D}^L and divestment threshold $\hat{\nu}_{SD}^L$ are obtained from the value-matching and smooth-pasting conditions, evaluated at $\hat{\nu}_{SD}^L$, given by Equations (16) and (17), respectively:

$$\frac{\hat{\nu}_D^L D_{X_L X_F}}{\delta+\theta} - \frac{D_{X_L X_F} f_X}{r} + A_{2D}^L\, \hat{\nu}_{SD}^{L\ \beta_2} = Z \quad (16)$$

$$\frac{D_{X_L X_F}}{\delta+\theta} + \beta_2 A_{2D}^L \hat{\nu}_{SD}^{L\ (\beta_2-1)} = 0 \quad (17)$$

Using Equations (16) and (17), we determine the constant A_{2D}^L and the divestment threshold $\hat{\nu}_{SD}^L$ that are given by:

$$A_{2D}^L = \frac{-D_{X_L X_F}}{(\delta+\theta)\beta_2 \hat{\nu}_D^{L\ (\beta_2-1)}} \quad (18)$$

$$\hat{\nu}_{SD}^L = \frac{(D_{X_L X_F} f_X + rZ)\beta_2(\delta+\theta)}{r D_{X_L X_F}(\beta_2-1)} \quad (19)$$

2.2 Switching

This section evaluates a policy switching problem where the two firms have the option to switch from policy X to policy Y for which they have to invest k. Policy X is based on a large-scale technology whereas policy Y is based on a small-scale technology. Therefore, the switching to policy Y enables firms to reduce operating cost from f_X to f_Y . The operating costs are a function of the firm's market share, so these are multiplied by $D_{k_i k_j}$. For the sake of simplicity, the option to divest in policy Y is neglected. Notice that because the incumbent technology is large-scale whereas the new technology is small-scale, so there is a temporary second-mover's market share advantage during the period when the first-mover operates with policy Y and the second-mover operates with policy X. This market share advantage disappears when the second-mover also switches to policy Y, after which each firm gets 50 percent of the market.[6] As for the above section, the first-mover benefits from a higher salvage value.

The value functions for the two firms are based on these supposed advantages/disadvantages, the economics of the incumbent versus downsized state, and the explicit options held by, and serendipity options granted by, the first and second movers. The option values are based on the value-matching and smooth-pasting conditions at the thresholds which justify first and second mover actions (permanent single transitions from one state to another), which result in analytical solutions.[7]

2.2.1 Second-Mover

The value function is given by:

[6] Although we neglect ex-post first-mover's advantages, they often exist in sequential investment games, when there is a learning advantage to be the first to adopt a new technology-related policy.
[7] Multiple switching among several states is covered in Paxson (2005), with simultaneous solution of several value matching and smooth pasting equations.

$$f_Y^F(\nu)\big|\nu(0) > \hat{\nu}_{SS}^L = \begin{cases} \frac{\nu D_{X_F X_L}}{\delta+\theta} - \frac{D_{X_F X_L} f_X}{r} + A_{2S}^F \nu^{\beta_2} + \frac{\hat{\nu}_{SS}^L (D_{X_F Y_L} - D_{X_F X_L})}{\delta+\theta} \left(\frac{\nu}{\hat{\nu}_{SS}^L}\right)^{\beta_2} & if\ \nu > \hat{\nu}_{SS}^L \\ \frac{\nu D_{X_F Y_L}}{\delta+\theta} - \frac{D_{X_F Y_L} f_X}{r} + A_{2S}^F \nu^{\beta_2} & if\ \nu \in [\hat{\nu}_{SS}^L, \hat{\nu}_{SS}^F) \\ \frac{\nu D_{Y_F Y_L}}{\delta+\theta} - \frac{D_{Y_F Y_L} f_Y}{r} - (k - \lambda Z) & if\ \nu \leq \hat{\nu}_{SS}^F \end{cases} \quad (20)$$

The economic interpretation for (20) is the following: in the first row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with policy *X,* the third term represents the option value to switch, and the fourth term is the second-mover's expected gain due to the fact that the first-mover switches first and, when it switches (ν reaches $\hat{\nu}_{SS}^L$), the market share of the second-mover increases from $D_{X_F X_L}$ to $D_{X_F Y_L}$; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the second-mover operates with policy *X* and the first-mover operates with policy *Y*, and the third term is the option value to switch; in the third row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with policy *Y*, and the third term is the switching investment cost less the salvage value related to policy *X* which is a proportion (λ) of that of the first-mover.

The option coefficients A_{2S}^F and the switching threshold $\hat{\nu}_{SS}^F$ are obtained through the boundary conditions: the value-matching condition which is obtained by equalizing the second and the third rows of (20), evaluated at $\nu = \hat{\nu}_{SS}^F$, and the smooth-pasting condition which is the first derivative of the value matching condition. These equations are given by:

$$\frac{\hat{\nu}_{SS}^F D_{X_F Y_L}}{\delta+\theta} - \frac{D_{X_F Y_L} f_X}{r} + A_{2S}^F {\hat{\nu}_{SS}^F}^{\beta_2} - \frac{\hat{\nu}_{SS}^F D_{Y_F Y_L}}{\delta+\theta} + \frac{D_{Y_F Y_L} f_Y}{r} + (k - \lambda Z) = 0 \quad (21)$$

$$\frac{D_{X_F Y_L}}{\delta+\theta} + \beta_2 A_{2S}^F {\hat{\nu}_{SS}^F}^{(\beta_2 - 1)} - \frac{D_{Y_F Y_L}}{\delta+\theta} = 0 \quad (22)$$

From (21) and (22), we obtain:

$$A_{2S}^{F} = \frac{\left(D_{Y_F Y_L} - D_{X_F Y_L}\right) \hat{v}_{SS}^{F\ (1-\beta_2)}}{\beta_2(\delta+\theta)} \tag{23}$$

$$\hat{v}_{SS}^{F} = \frac{\beta_2(\delta+\theta)\left(r(k-\lambda Z) + D_{Y_F Y_L} f_Y - D_{X_F Y_L} f_X\right)}{r(\beta_2 - 1)\left(D_{Y_F Y_L} - D_{X_F Y_L}\right)} \tag{24}$$

2.2.2 First-Mover

The value function is given by:

$$f_Y^L(\nu)\big|\nu(0) > \hat{v}_{SS}^L = \begin{cases} \frac{\nu D_{X_L X_F}}{\delta+\theta} - \frac{D_{X_L X_F} f_X}{r} + A_{2S}^L\, \nu^{\beta_2} + \frac{\hat{v}_{SS}^F\left(D_{Y_L Y_F} - D_{Y_L X_F}\right)}{\delta+\theta}\left(\frac{\nu}{\hat{v}_{SS}^F}\right)^{\beta_2} & if\ \ \nu > \hat{v}_{SS}^L \\ \frac{\nu D_{Y_L X_F}}{\delta+\theta} - \frac{D_{Y_L X_F} f_Y}{r} - (k - Z) & if\ \ \nu \in [\hat{v}_{SS}^L, \hat{v}_{SS}^F) \\ \frac{\nu D_{Y_L Y_F}}{\delta+\theta} - \frac{D_{Y_L Y_F} f_Y}{r} & if\ \nu \le \hat{v}_{SS}^F \end{cases} \tag{25}$$

The economic interpretation for (25) is the following: in the first row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with policy *X,* the third term represents the option value to switch, and the fourth term is the first-mover's expected gain due to the fact that the second-mover will eventually switch after the first-mover, and when it switches (ν reaches $\hat{v}_{SS}^F$) the market share of the first-mover increases from $D_{Y_L X_F}$ to $D_{Y_L Y_F}$; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the first-mover operates with policy *Y* and the second-mover operates with policy *X*, the third term is the switching cost less the salvage value related to policy *X*; the terms in the third row represent the present value of the revenue stream less the operating costs for the region where both firms are operating with policy *Y*.

The option coefficient A_{2S}^L and the switching threshold $\hat{v}_{SS}^L$ are determined from the value-matching condition obtained by equalizing the first and the second rows of (25) and the associated smooth-pasting condition, both evaluated at $\nu = \hat{v}_{SS}^L$:

$$\frac{\hat{v}_{SS}^{L} D_{X_L X_F}}{\delta+\theta} - \frac{D_{X_L X_F} f_X}{r} + A_{2S}^{L}\,\hat{v}_{SS}^{L}{}^{\beta_2} + \frac{\hat{v}_{SS}^{F}\left(D_{Y_L Y_F} - D_{Y_L X_L}\right)}{\delta+\theta}\left(\frac{\hat{v}_{SS}^{L}}{\hat{v}_{SS}^{F}}\right)^{\beta_2} - \frac{\hat{v}_{SS}^{L} D_{Y_L X_F}}{\delta+\theta} + \frac{D_{Y_L X_F} f_Y}{r} + (k - Z) = 0 \tag{26}$$

$$\frac{D_{X_L X_F}}{\delta+\theta} + \beta_2 A_{2S}^{L}\,\hat{v}_{SS}^{L}{}^{\beta_2 - 1} + \frac{\beta_2\left(D_{Y_L Y_F} - D_{Y_L X_L}\right)\hat{v}_{SS}^{F}{}^{(1-\beta_2)}\hat{v}_{SS}^{L}{}^{(\beta_2-1)}}{(\delta+\theta)} - \frac{D_{Y_L X_F}}{\delta+\theta} = 0 \tag{27}$$

From Equation (27), we obtain A_{2S}^{L}:

$$A_{2S}^{L} = \frac{\left(D_{Y_L X_F} - D_{X_L X_F}\right) - \beta_2\left(D_{Y_L Y_F} - D_{Y_L X_L}\right)\hat{v}_{SS}^{F}{}^{(1-\beta_2)}\hat{v}_{SS}^{L}{}^{(\beta_2-1)}}{\beta_2(\delta+\theta)\hat{v}_{SS}^{L}{}^{(\beta_2-1)}} \tag{28}$$

There is no closed-form solution for $\hat{v}_{SS}^{L}$, but it can be obtained numerically using Equation (26).

The absolute advantage of being the leader, given the parameter values, is the present value of the leader at each stage plus the option to switch at the first stage (both firms with policy *X*) and the value enhancement that results from an increase of its market share at the moment the second-mover also switches.

2.3 Results

This section shows a sensitivity analysis for the effect of changes in the (divest and switch) model parameters on the investment thresholds of the first-mover and the second-mover. We show a sensitivity analysis of the effect of the first-mover's market share when it operates with policy *Y* and the second-mover operates with policy *X* on the option coefficients of both firms, starting with the base case model parameter values provided in Table 1.

Table 1: Base case model parameter values

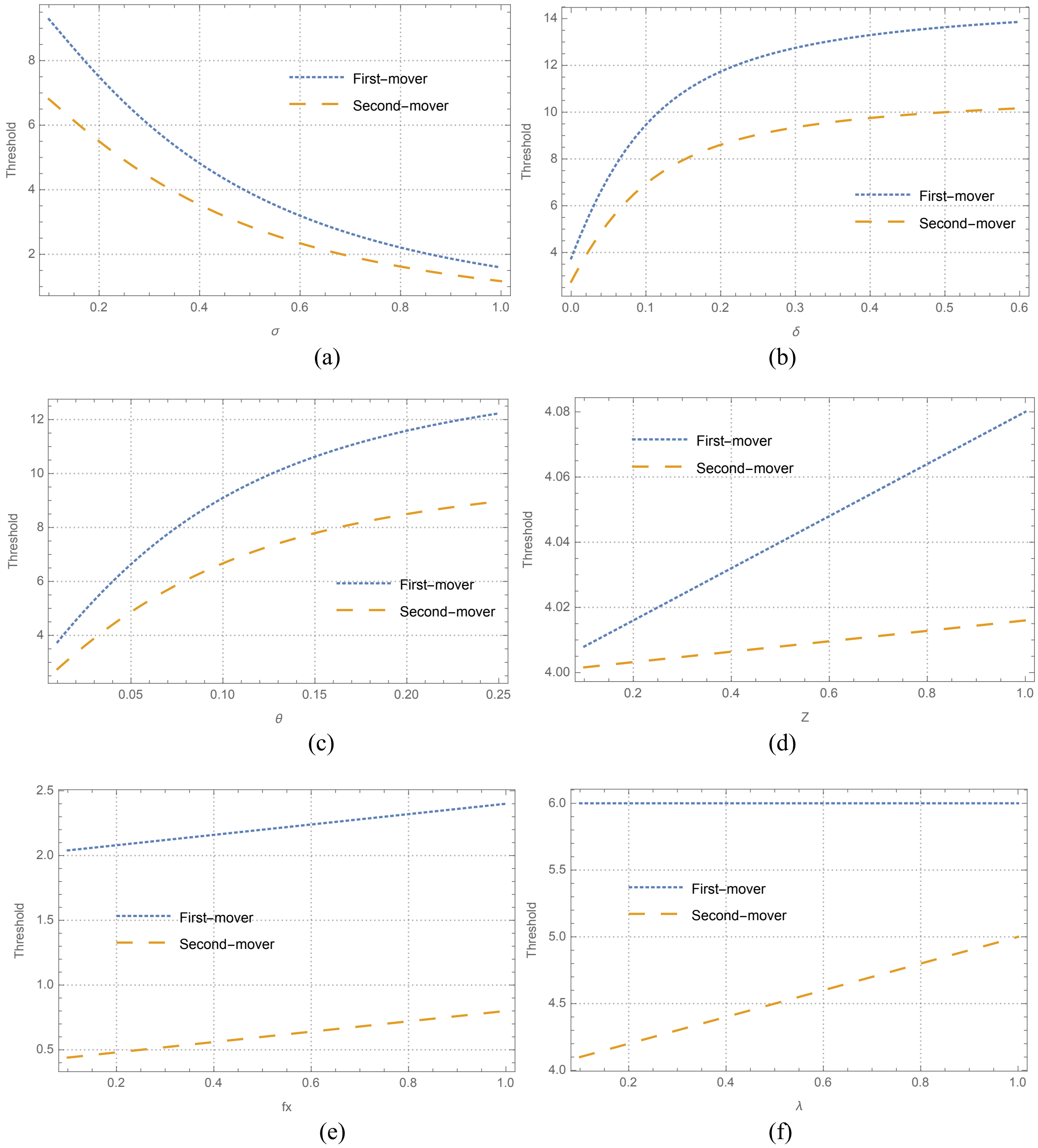
Notation	*Definition*	*Value*
p	Output price	1.00
q	Output quantity	10.00
ν	Revenue value	10.00
r	Risk-free rate	0.10
δ	Convenience yield	0.03
θ	Depletion rate for both policy *X* and policy *Y*	0.04
σ	Output price volatility	0.30
λ	% drop in divestment value for the second-mover	0.40
f_X	Periodic operating costs for policy *X*	10.00
f_Y	Periodic operating costs for policy *Y*	1.00
Z	First-mover's divestment value	25.00
k	Switching investment cost to policy Y	32.00
	Competition Factors	
	DIVEST	
$D_{X_F X_L}$	SM's market share if active with the FM, both firms with policy *X*	0.50
$D_{X_F 0_L}$	SM's market share if active alone with policy *X*, after the leader divests	1.00
	SWITCH	
$D_{X_L X_F}$	FM's market share if active with the SM, both firms with policy *X* (FIRST-STAGE)	0.50
$D_{X_F Y_L}$	SM's market share if active with policy *X* and FM is active with policy *Y* (SECOND-STAGE)	0.60
$D_{Y_L X_F}$	FM's market share if active with policy *Y* and SM is active with policy *X* (SECOND-STAGE)	0.40
$D_{Y_F Y_L}$	SM's market share if active with the FM, both firms with policy *Y* (THIRD-STAGE)	0.50
$D_{Y_L Y_F}$	FM's market share if active with the SM, both firms under policy *Y* (THIRD-STAGE)	0.50

Note: FM stands for "first-mover" and SM stands for "second-mover". In the sensitivity analysis, for the output price volatility (σ_P) and the output price drift (α_P), we drop the subscript "P". In the second-stage, there is a technology-symmetry between the FM and the SM and we assume that SM's market share increases from 0.50 to 0.60 because policy *Y* is based on a small-scale technology.

2.3.1 Sensitivity analysis

In Figure 1, we provide a sensitivity analysis which shows the effect of changing in some of model key parameter values on the first-mover and second-mover divestment thresholds.

Figure 1: Sensitivity of the divestment thresholds to changes in the model parameters.



The above findings are in line with what we would expect. Specifically, a higher output price uncertainty delays the divestment for both firms (figure 1a), a higher convenience yield, output declining rate, divestment value, or fixed operating costs, accelerate the divestment for both firms (figures 1b to 1e), whereas a lower second-mover's salvage value disadvantage (as λ increases) accelerates the divestment of the second-mover and has no effect on the divestment threshold of the first-mover (figure 1f).

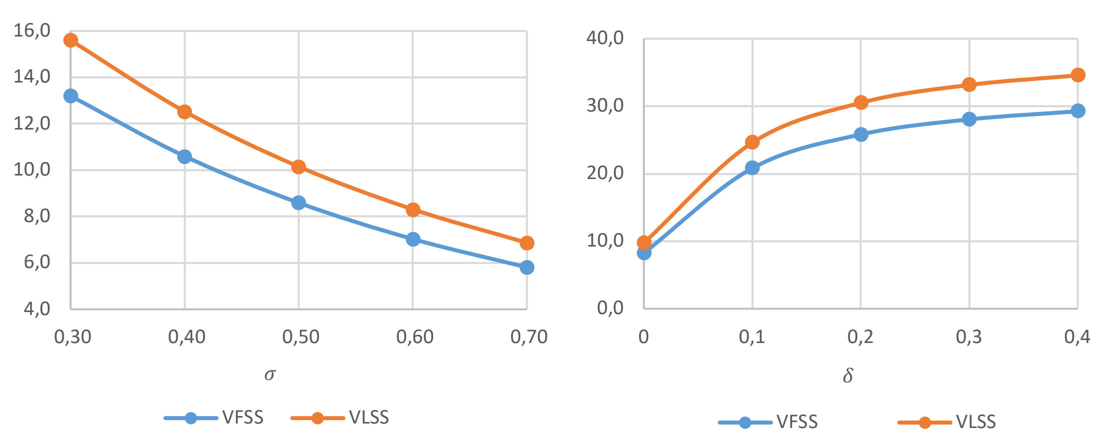
In Table 2, we show the option coefficients and divestment thresholds for the first and the second movers. Notice that the state variable approaches the thresholds from above, so the first-mover divest before the second-mover as it is expected. The lag time between the divesting of the two firms decrease however with the output uncertainty and second-mover's salvage value disadvantage, and increase with the convenience yield, output declining rate, divestment value, or fixed operating costs.

Table 2: Base case scenario - option coefficients and thresholds for the Separate-Divest Formulation

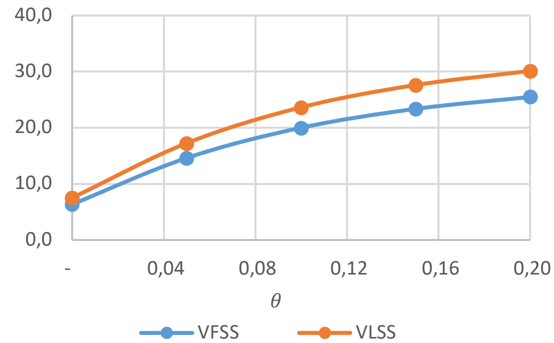
Coefficient and Divestment Thresholds	Value
A_{2D}^{F}	339.90
A_{2D}^{L}	350.45
$\hat{v}_{SD}^{L}$	6.00
$\hat{v}_{SD}^{F}$	4.40

In Figure 2, we provide a sensitivity analysis which shows the effect of changing some parameter values on the switching thresholds of the first-mover and the second-mover.

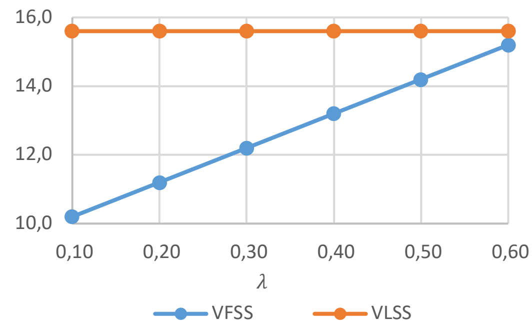
Figure 2 - this figure shows the sensitivity of the switching thresholds of the two firms to changes in some of the model parameter values. VFSS and VLSS are the switching thresholds of the second-mover and the first-mover, respectively, for the SEPARATE formulation.



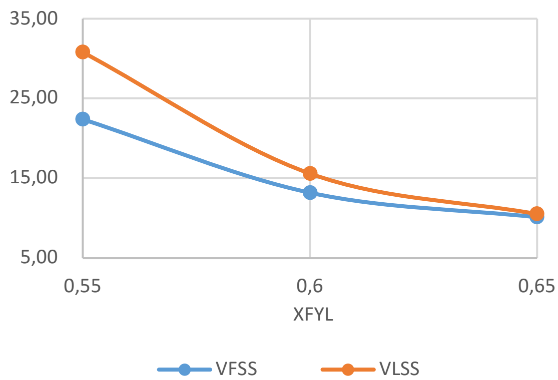
(a) (b)



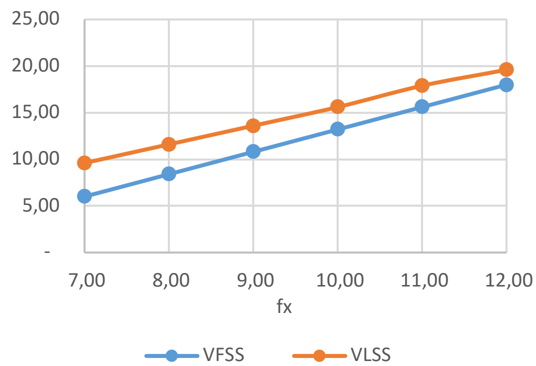
(c)



(d)



(e)



(f)

Our results above show that a higher output price uncertainty or second-mover advantage delays the switching for both firms (figures 2a and 2e, respectively), whereas a higher convenience yield, output declining rate, or operating costs associated with policy X accelerates the switching (figures 2b, 2c, and 2f, respectively). A lower second-mover's salvage value disadvantage (as λ increases) accelerates the switching of the second-mover and has no effect on the switching threshold of the first-mover (figure 2f).

In Table 3, we show the option coefficients and switching thresholds for the first and the second movers. Notice that the state variable approaches the thresholds from above, so the first-mover switches before the second-mover as it is expected. The lag time between the switching of the two firms decrease however with the output uncertainty, the second-mover's salvage value, the second-mover's market share advantage, and the operating costs associated with the policy X (figures 2a, 2d, 2e, and 2f) and increase with the convenience yield and output declining rate.

Table 3: Base case - thresholds and option coefficients for Separate-Switch Formulation

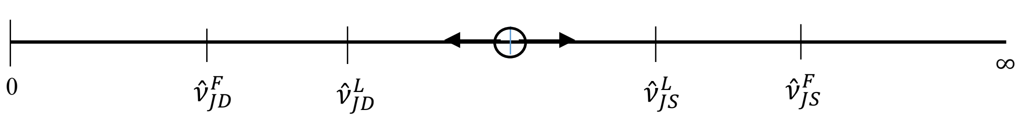
Coefficient and Switching Thresholds	Value
A_{2S}^{F}	441,29
A_{2S}^{L}	63,20
$\hat{v}_{SS}^{L}$	15,60
$\hat{v}_{SS}^{F}$	13,20

3. Joint Formulation

A key assumption underlying the duopoly joint formulation (à la Décamps et al., 2006) is that, at the beginning of the investment game, the state variable ν (revenue) is between the threshold to divest and switch ($\hat{v}_{JD}^{L}$ and ($\hat{v}_{JS}^{L}$) that are determined independently using a separate formulation. Hence, the two firms have two simultaneous options: the option to divest and the option to switch. We assume that, after the divestment from policy *X* the game is over (i.e., firms are not allowed to re-enter the market after the exit) and after the firms have switched from policy *X* to policy *Y* they will operate with policy *Y* forever.

Figure 3 illustrates the threshold sequence of the decision game - notice that at the beginning of the game the state variable $\nu(0)\ \epsilon\ (\hat{v}_{JS}^{F}, \hat{v}_{JD}^{F})$.

Figure 3: ν is randomly declining: Décamps et al. (2006), at the beginning of the decision game, $\nu(0)\ \epsilon\ (\hat{v}_{JS}^{L}, \hat{v}_{JD}^{L})$ for the first-mover and $\nu(0)\ \epsilon\ (\hat{v}_{JS}^{F}, \hat{v}_{JD}^{F})$ for the second-mover



Our derivation formulation is grounded on the continuity of the firm's value function, over the domain of revenue (ν) values, and its differentiability at the decision thresholds.

3.1 Second-Mover

The value function is given by (ν is randomly declining):

$$f_{X,Y}^{F}(\nu)\big|\nu(0)\,\epsilon\,(\hat{\nu}_{JD}^{F},\hat{\nu}_{JS}^{F})\begin{cases}\frac{\nu D_{Y_F Y_L}}{\delta+\theta}-\frac{D_{Y_F Y_L} f_Y}{r}-(k-\lambda Z) & if\ \nu\geq\hat{\nu}_{JS}^{F}\\ \frac{\nu D_{X_F Y_L}}{\delta+\theta}-\frac{D_{X_F Y_L} f_X}{r}+A_S^F\nu^{\beta_1}+A_D^F\nu^{\beta_2} & if\ \ \nu\,\epsilon(\hat{\nu}_{JS}^{F},\hat{\nu}_{JS}^{L}]\\ \frac{\nu D_{X_F X_L}}{\delta+\theta}-\frac{D_{X_F X_L} f_X}{r}+a_{211}\nu^{\beta_1}+a_{222}\nu^{\beta_2} & if\ \ \nu\,\epsilon(\hat{\nu}_{JS}^{L},\hat{\nu}_{JD}^{L})\\ \frac{\nu D_{X_F 0_L}}{\delta+\theta}-\frac{D_{X_F 0_L} f_X}{r}+A_S^F\nu^{\beta_1}+A_D^F\nu^{\beta_2} & if\ \ \nu\,\epsilon\left[\hat{\nu}_{JD}^{L},\hat{\nu}_{JD}^{F}\right)\\ \lambda Z & if\ \ \nu\leq\hat{\nu}_{JD}^{F}\end{cases}\tag{29}$$

The economic interpretation for (29) is the following: in the first row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with policy *Y*, the third term is the switching cost less the salvage value related to policy *X*, which for the second-mover is a percentage (λ) of the salvage value of the first-mover; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the first-mover operates with policy *Y* and the second-mover operates with policy *X*, the third term represents the value of the option to switch to policy *Y*, and the fourth term represent the value of the option to divest; in the third row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms operate with policy *X*, the third term comprises the value of the option to switch to policy *Y* plus the second-mover's expected gain because the first-mover will switch first and, when it switches (ν reaches $\hat{\nu}_{JS}^{L}$), the market share of the second-mover increases from $D_{X_F X_L}$ to $D_{X_F Y_L}$, the fourth term comprises the value of the option to divest plus the second-mover's expected gain because the first-mover will divest first and, when it divest (ν reaches $\hat{\nu}_{JD}^{L}$), the market share of the second-mover increases from $D_{X_F X_L}$ to $D_{X_F 0_L}$; in the fourth row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the second-mover is alone in the original market, and the third and the fourth terms are the options to switch and divest, respectively (the former option is exercised if ν increases reaching $\hat{\nu}_{JS}^{F}$ and, the latter, is exercised if ν decreases reaching $\hat{\nu}_{JD}^{F}$); the term in the fifth row is the divestment value that is a percentage (λ) of the divestment value of the first-mover.

From expression (29), we can obtain four value-matching conditions that ensure the continuity of the second-mover's value function over the domain of revenue (ν) values and two smooth-pasting conditions that ensure the differentiability of the value function at the decision thresholds ($\hat{v}^F_{JD}$ and $\hat{v}^F_{JS}$). Therefore, we obtain an equation system with six equations and eight unknown variables (A^F_S , A^F_D, $a_{211}, a_{222}, \hat{v}^F_{JS}, \hat{v}^F_{JD}, \hat{v}^L_{JS}, \hat{v}^L_{JD}$).

The value-matching conditions are given by:

$$\frac{\hat{v}^F_{JS} D_{Y_F Y_L}}{\delta+\theta} - \frac{D_{Y_F Y_L} f_Y}{r} - (k - \lambda Z) - \frac{\hat{v}^F_{JS} D_{X_F Y_L}}{\delta+\theta} + \frac{D_{X_F Y_L} f_X}{r} - A^F_S \hat{v}^F_{JS}{}^{\beta_1} - A^F_D \hat{v}^F_{JS}{}^{\beta_2} = 0 \tag{30}$$

$$\frac{\hat{v}^L_{JS} D_{X_F Y_L}}{\delta+\theta} - \frac{D_{X_F Y_L} f_X}{r} + A^F_S \hat{v}^L_{JS}{}^{\beta_1} + A^F_D \hat{v}^L_{JS}{}^{\beta_2} - \frac{\hat{v}^L_{JS} D_{X_F X_L}}{\delta+\theta} + \frac{D_{X_F X_L} f_X}{r} - a_{211} \hat{v}^L_{JS}{}^{\beta_1} - a_{222} \hat{v}^L_{JS}{}^{\beta_2} = 0 \tag{31}$$

$$\frac{\hat{v}^L_{JD} D_{X_F X_L}}{\delta+\theta} - \frac{D_{X_F X_L} f_X}{r} + a_{211} \hat{v}^L_{JD}{}^{\beta_1} + a_{222} \hat{v}^L_{JD}{}^{\beta_2} - \frac{\hat{v}^L_{JD} D_{X_F 0_L}}{\delta+\theta} + \frac{D_{X_F 0_L} f_X}{r} - A^F_S \hat{v}^L_{JD}{}^{\beta_1} - A^F_D \hat{v}^L_{JD}{}^{\beta_2} = 0 \tag{32}$$

$$\frac{\hat{v}^F_{JD} D_{X_F 0_L}}{\delta+\theta} - \frac{D_{X_F 0_L} f_X}{r} + A^F_S \hat{v}^F_{JD}{}^{\beta_1} + A^F_D \hat{v}^F_{JD}{}^{\beta_2} - \lambda Z = 0 \tag{33}$$

The smooth-pasting conditions are the first derivatives of the value-matching conditions that include the second-mover's thresholds $\hat{v}^F_{JS}$ and $\hat{v}^F_{JD}$ – Equations (30) and (33).

$$\frac{D_{Y_F Y_L}}{\delta+\theta} - \frac{D_{X_F Y_L}}{\delta+\theta} - \beta_1 A^F_S \hat{v}^F_{JS}{}^{\beta_1 - 1} - \beta_2 A^F_D \hat{v}^F_{JS}{}^{\beta_2 - 1} = 0 \tag{34}$$

$$\frac{D_{X_F 0_L}}{\delta+\theta} + \beta_1 A^F_S \hat{v}^F_{JD}{}^{\beta_1 - 1} + \beta_2 A^F_D \hat{v}^F_{JD}{}^{\beta_2 - 1} = 0 \tag{35}$$

3.2 First-Mover

The value function is given by (ν is declining):

$$f_{X,Y}^{L}(\nu)\big|\nu(0)\,\epsilon\,(\hat{\nu}_{JD}^{L},\hat{\nu}_{JS}^{L})=\begin{cases}\frac{\nu D_{Y_LY_F}}{\delta+\theta}-\frac{D_{Y_LY_F}f_Y}{r}-(K-Z) & if\ \nu\geq\hat{\nu}_{JS}^{F}\\ \frac{\nu D_{Y_LX_F}}{\delta+\theta}-\frac{D_{Y_LX_F}f_y}{r}-(K-Z)+a_{112}\nu^{\beta_1} & if\ \nu\,\epsilon(\hat{\nu}_{JS}^{F},\hat{\nu}_{JS}^{L}]\\ \frac{\nu D_{X_LX_F}}{\delta+\theta}-\frac{D_{X_LX_F}f_X}{r}+A_S^L\nu^{\beta_1}+A_D^L\nu^{\beta_2} & if\ \nu\,\epsilon\left(\hat{\nu}_{JS}^{L},\hat{\nu}_{JD}^{L}\right)\\ Z & if\ \nu\leq\hat{\nu}_{JD}^{L}\end{cases} \tag{36}$$

The economic interpretation for (36) is the following: in the first row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms operate with policy *Y*; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the first-mover operates with policy *Y* and the second-mover operates with policy *X*, the third term is the switching cost less the salvage value related to the divestment in policy *X*, the fourth term represents the first-mover's expected value due to the fact that the second-mover switches after the first-mover and, when it switches (if ν reaches $\hat{\nu}_{JS}^{F}$), the market share of the first-mover increases from $D_{Y_LX_F}$ to $D_{Y_LY_F}$; in the third row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms operate with policy *X*, the third term is the option value to switch to policy *Y*, and the fourth term is the option value to divest; the term in the fourth row is the salvage value.

From expression (36), we can obtain three value-matching conditions that ensure the continuity of the first-mover's value function over the domain of revenue (ν) values and two smooth-pasting conditions that ensure the differentiability of the value function at the first-mover's thresholds ($\hat{\nu}_{JD}^{L}$ and $\hat{\nu}_{JS}^{L}$). Therefore, we obtain an equation system with 5 equations and 8 unknown variables ($a_{112}, A_S^L, A_D^L, \hat{\nu}_{JD}^{F}, \hat{\nu}_{JS}^{F}, \hat{\nu}_{JD}^{L}, \hat{\nu}_{JS}^{L}$).

The value-matching conditions are given by:

$$\frac{\hat{\nu}_{JS}^{F}D_{Y_LY_F}}{\delta+\theta}-\frac{D_{Y_LY_F}f_Y}{r}-\frac{\hat{\nu}_{JS}^{F}D_{Y_LX_F}}{\delta+\theta}+\frac{D_{Y_LX_F}f_y}{r}-a_{112}\hat{\nu}_{JS}^{F\ \beta_1}=0 \tag{37}$$

$$\frac{\hat{v}_{JS}^{L} D_{Y_L X_F}}{\delta+\theta} - \frac{D_{Y_L X_F} f_y}{r} - (K - Z) + a_{112} \hat{v}_{JS}^{L\ \beta_1} - \frac{\hat{v}_{JS}^{L} D_{X_L X_F}}{\delta+\theta} + \frac{D_{X_L X_F} f_X}{r} - A_S^L \hat{v}_{JS}^{L\ \beta_1} - A_D^L \hat{v}_{JS}^{L\ \beta_2} = 0 \quad (38)$$

$$\frac{\hat{v}_{JD}^{L} D_{X_L X_F}}{\delta+\theta} - \frac{D_{X_L X_F} f_X}{r} + A_S^L \hat{v}_{JD}^{L\ \beta_1} + A_D^L \hat{v}_{JD}^{L\ \beta_2} - Z = 0 \quad (39)$$

The smooth-pasting conditions are:

$$\frac{D_{Y_L X_F}}{\delta+\theta} + \beta_1 a_{112} \hat{v}_{JS}^{L\ \beta_1 - 1} - \frac{D_{X_L X_F}}{\delta+\theta} - \beta_1 A_S^L \hat{v}_{JS}^{L\ \beta_1 - 1} - \beta_2 A_D^L \hat{v}_{JS}^{L\ \beta_2 - 1} = 0 \quad (40)$$

$$\frac{D_{X_L X_F}}{\delta+\theta} + \beta_1 A_S^L \hat{v}_{JD}^{L\ \beta_1 - 1} + \beta_2 A_D^L \hat{v}_{JD}^{L\ \beta_2 - 1} = 0 \quad (41)$$

Combining Equations (30) to (35) and (37) to (41), we obtain an equation system with 11 equations and 11 unknown variables, from which we can obtain numerically the solutions for the unknown variables.

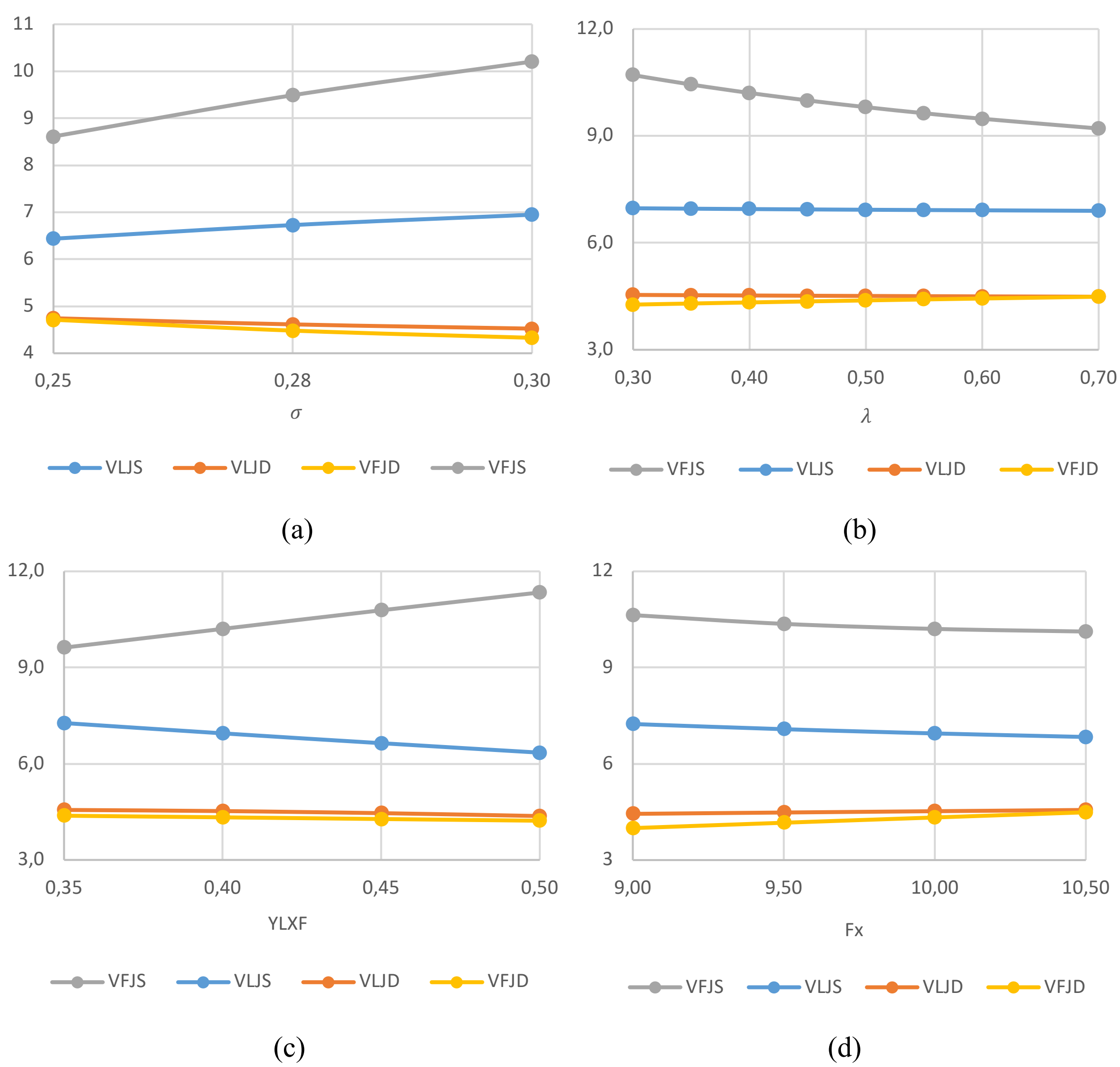
3.3 Results

This section reports our main findings and a sensitivity analysis on the effect of changes in the key model parameters on the decision thresholds and the option coefficients. Our results show that the thresholds of the two firms for both switch and divest obey the sequence time condition described in Figure 3. The base case model inputs are those provided in Table 1.

3.4 Sensitivity Analysis

Figure 4a shows that the "idle" region (the region where the state variable is between the switch and the divest thresholds) increase with the uncertainty for both firms - that is, the switching thresholds of the two firms increase whereas the divesting thresholds decrease. Figure 4b shows that the idle region decreases as λ increases. Figure 4c shows the effect of the second-mover's market share advantage on the decision thresholds. Notice that, YLXF=0.35 means that when the first-mover switches to policy *Y* the second-mover will get a temporarily market share advantage (the first-mover gets 35% of the market whereas the second-mover gets 65%). Therefore, we conclude that, an increase in YLXF delays significantly the switching of the second-mover and accelerates slightly the switching of the first mover. It also delays very slightly the divestment of both firms. Figure 4d shows that the idle region decreases for both firms as the operating costs associated with the incumbent policy increase, and that both firms divest or switch earlier as if the operating costs increase.

Figure 4: this figure shows the sensitivity of the decision thresholds (switch and divest) of the first and the second movers to changes in some of the key variables of the Joint Formulation. VLJS and VFJS are the switching thresholds of the first-mover and the second-mover for the JOINT formulation, whereas VLJD and VFJD are the divesting thresholds of the first-mover and the second-mover for the JOINT formulation.



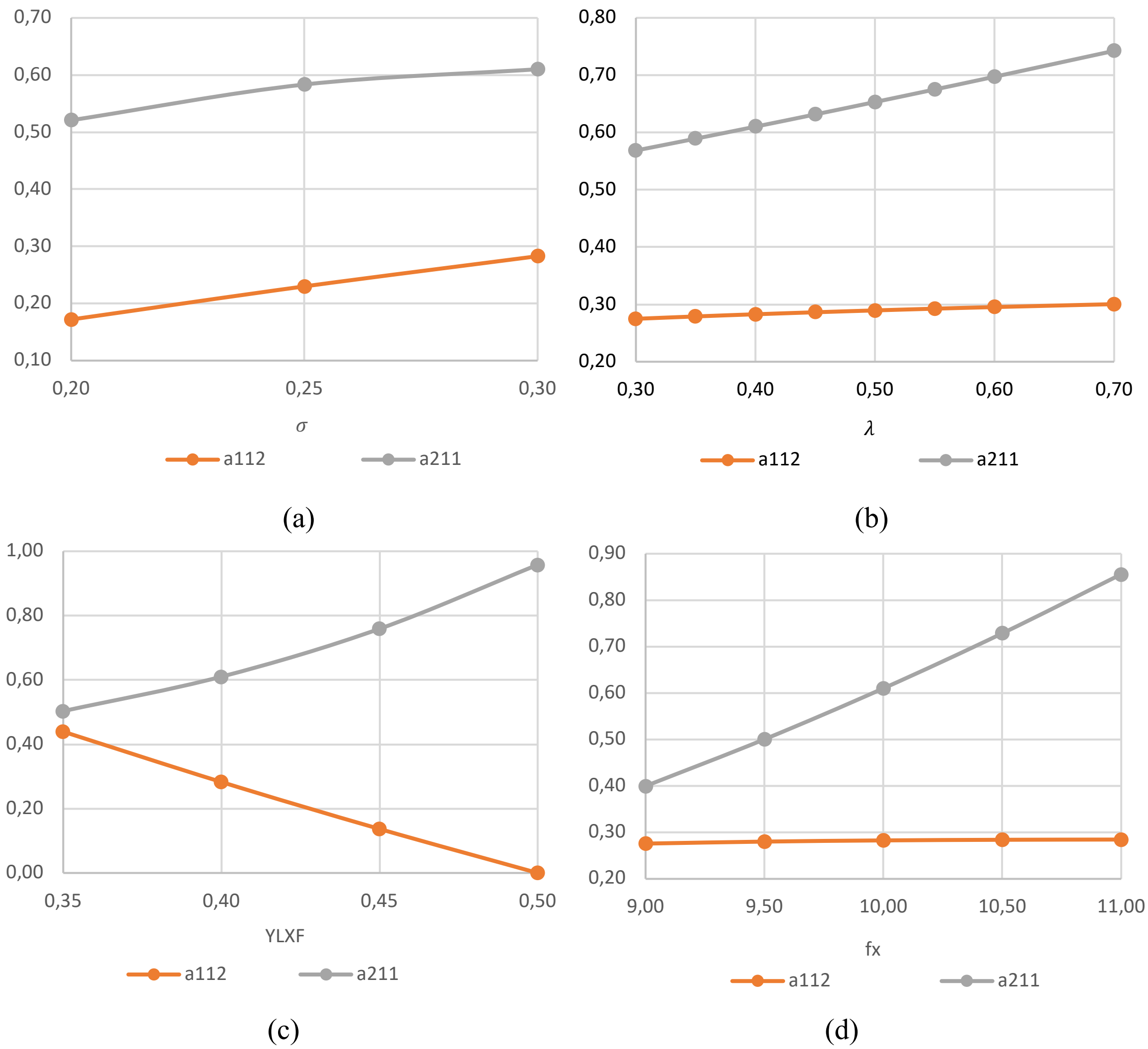
For the joint formulation, there are three important option coefficients which are new to the literature and, therefore, deserve our careful attention (a_{211}, a_{222}, and a_{112}).

Figure 5 shows the effect of changes in our model parameters on each of these option coefficients. Since the size of a_{112} and a_{211} is significantly lower than that of a_{222}, we show the sensitivity of the former two option coefficient in figures 5a to 5d and sensitivity of the latter option coefficient in figures 5e to 5h. Notice that, the coefficient a_{211} comprises both the value of option to switch of the second-mover plus the second-mover's expected gain due to the fact that the first-mover switches first to a small-scale policy and, when it switches (that is, ν reaches $\hat{\nu}_{JS}^{L}$), the market share of the second-mover increases from $D_{X_F X_L}$=0.5 to $D_{X_F Y_L} = 0.6$.

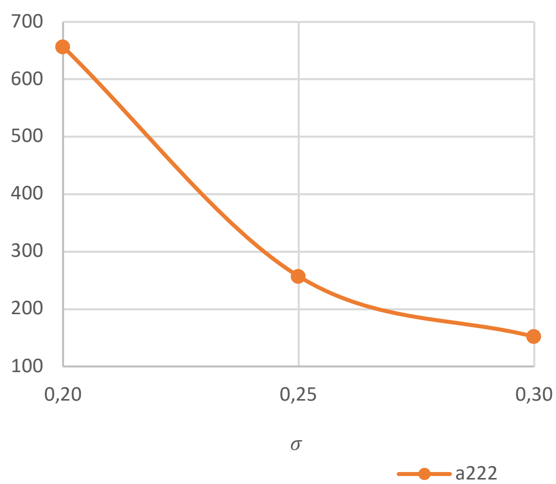
The coefficient a_{112} comprises the value of the coefficient of the option to switch plus the first-mover's expected gain due to the fact that the second-mover will also eventually switch (after the first-mover) and, when it switches (if ν reaches $\hat{\nu}_{JS}^{F}$), the market share of the first-mover increases from $D_{Y_L X_F}$ (40 percent) to $D_{Y_L Y_F}$ (50 percent).

Figure 5a shows that both of the above option coefficients increase with uncertainty, which is in line with our expectations, and similar results are shown in figure 5b for the association of each of these coefficients with the second-mover's salvage value disadvantage (λ) and with the operating costs of the incumbent policy *X*, although the sensitivity of a_{211} is significantly higher than that of a_{112}. The effects of the first-mover's market share when it operates with policy *Y* and the second-mover operates with policy *X* ($D_{Y_L X_F}$) on the a_{112} and a_{211} are, however, quite distinct. While the former coefficient decreases significantly with $D_{Y_L X_F}$, the latter increases significantly. Finally, figures 5e and 5f, and 5g and 5h, respectively, show that a_{222} decreases significantly with the uncertainty and the first-mover's market share ($D_{Y_L X_F}$), and increase in an almost linear way with both a decrease in the second-mover's salvage value disadvantage (notice that $\lambda = 1$ means that the salvage value related to the divestment in policy X is the same for both firms) and an increase in the operating costs associated with policy X.

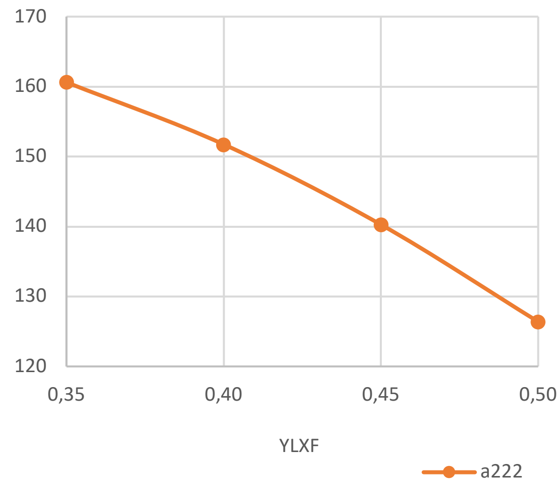
Figure 5: this figure shows the sensitivity of the options coefficients (a_{211}, a_{222}, and a_{112}) of the first and the second movers to changes in some of the key variables of the Joint Formulation. Because the size of a_{112} and a_{211} is significantly lower than the size of a_{222}, we show the sensitivity of the former two option coefficient in figures (a) to (d), and the sensitivity of the latter in figures (e) to (h).



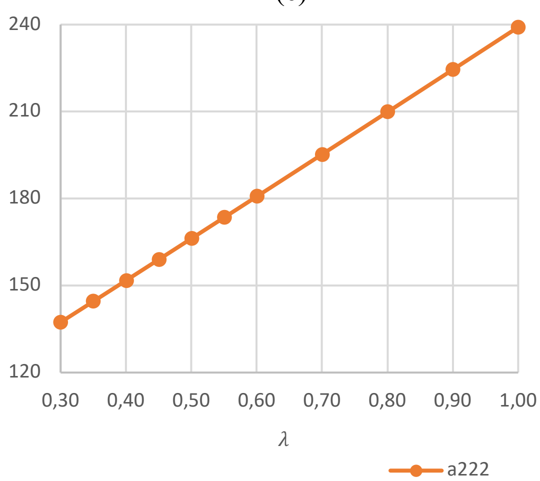
(a) (b) (c) (d)



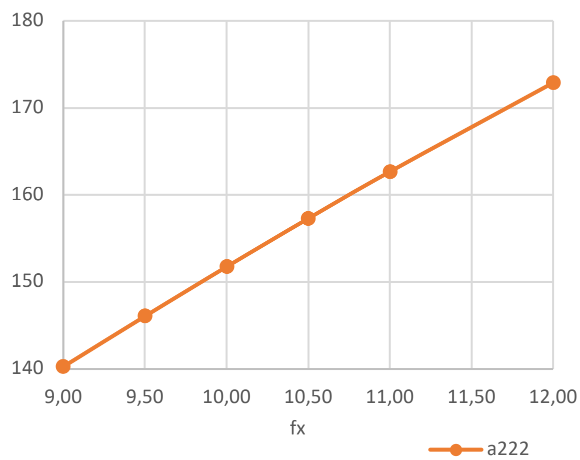
(e)



(f)



(g)



(h)

Figure A1 in the Appendix relies on the base case parameter values and compares the decision thresholds of the separate and the joint formulations and these thresholds with those we obtain from a separate and a joint formation when applied to a monopoly market. In Tables A1, Panels A, B, and C, we show a comparative sensitivity analysis between the two formulations. Panel C also shows the base case option coefficients and thresholds using the joint formulation.

4. Conclusion

Is a joint formation model feasible, with a solution using joint option coefficients, which should be consider in mutually-exclusive option contexts? Are the results using a joint formulation model significant in capital budgeting, that is are the thresholds justifying immediate action (divestment, or switching in our case) different from using the separate formulation? Can the joint formulation be extended to a duopoly with first mover advantages? What is the role of the serendipity options used herein? What are the implied partial derivatives of thresholds with respect to market shares at each stage?

Relying on the above described theoretical findings, we conclude that: i) the joint formulation with option coefficients and thresholds is feasible and perhaps should be considered in many other contexts; ii) the action thresholds for capital budgeting for mutually-exclusive opportunities are 1/2 or 3/4 of those determined using conventional real option theory; iii) extending the joint formulation to a duopoly is feasible and interesting, and introduces market share deltas for both option coefficients and thresholds; iv) the serendipity options are a novel concept with interesting implications; and v) market share derivatives are a new consideration, offering new perspectives on capital budgeting and management.

A key contribution of this paper is the relevance and specification of the joint model, with proposed implementation, adding new coefficients in the value functions of the two firms (a_{112} and a_{211} and a_{222}) and to the appropriate value matching conditions. This was not shown in Décamps et al (2006), or other applications of the joint model such as Bobtcheff and Villeneuve (2010). Moreover, the thresholds are significantly different for the separate and joint formulation, as shown in our sensitivity analysis. Suppose capital budgeting using net present value (NPV) is 100 percent wrong as noted by Pindyck (1993). Using real options jointly the thresholds are 1/2 for switching of the separate Dixit (1993), and 3/4 for divestment for these parameter values. Therefore, the joint formulation makes a difference! The serendipity option values, and the other conventional option values, for both the separate and joint versions, are innovations.

The limitations of the model are that the joint formulation is hard to solve using some numerical techniques. The sensitivities to changes in a full range of parameter values has not yet been calculated for the joint formulation. There are extensions for the partial derivatives of all of the separate formulation thresholds and option coefficients, with plausible illustrations (and indicated hedging opportunities). Can this joint formulation approach be used for many other mutually-exclusive option contexts? The treatment of the serendipity options seems to us as a fertile area for future research.

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Appendix

Table A1: this table shows a sensitivity analysis on the effect of changes in our model parameters on the leader's and the follower's option coefficients and switching and divesting thresholds for the Separate formulation (Panel A) and the Joint formulation (Panel B). In the third row of Panel A, second row of Panel B, and third row of Panel C are our results for the base case, whereas in the rows after these rows are our results for the cases where ewe increase the model base case parameters values by 10%.

SEPARATE FORMULATION (Panel A)		***First-Mover***		***Second-Mover***	
	Divest	A_{2D}^{L}	$\hat{v}_{SD}^{L}$	A_{2D}^{F}	$\hat{v}_{SD}^{F}$
	Base case	*350.45*	*6.00*	*339.90*	*4.40*
Notation	Increase by 10%				
r	0.110	412.22	5.93	356.74	4.25
δ	0.033	351.30	6.20	343.83	4.54
θ	0.044	351.34	6.26	344.90	4.59
σ	0.333	253.43	4.12	260.25	5.61
λ	0.440	350.44	6.00	347.16	4.44
f_X	11.000	407.40	6.40	416.41	4.80
Z	27.500	378.31	6.20	347.16	4.44
Panel B	***Switch***	A_{2S}^{L}	$\hat{v}_{SS}^{L}$	A_{2S}^{F}	$\hat{v}_{SS}^{F}$
	Base case	*63.20*	*15.60*	*441.20*	*13.20*
Notation	Increase by 10%				
r	0.110	101.01	14.66	461.70	11.79
δ	0.033	71.49	16.11	432.18	13.63
θ	0.044	81.28	16.78	418.90	14.20
σ	0.330	82.00	15.08	273.21	12.76
λ	0.440	31.35	16.11	462.96	14.05
f_X	11.000	10.37	16.11	635.08	18.18
f_y	1.100	76.06	15.95	417.25	13.43
Z	27.500	129.10	17.15	462.96	14.05
K	35.200	75.83	14.79	341.67	12.31
D_{XFYL}	0.660	-92.30	10.23	344.07	10.07

JOINT FORMULATION (Panel C)		First-Mover					Second-Mover					
	Divest	A_D^L	A_S^L	a_{112}	$\hat{v}_{JD}^L$	$\hat{v}_{JS}^L$	A_D^F	A_S^F	a_{211}	a_{222}	$\hat{v}_{JD}^F$	$\hat{v}_{JS}^F$
	Base case	*258.0164*	*0.6628*	*0.2828*	*4.5238*	*6.9480*	*334.1445*	*0.0693*	*0.6101*	*151.7397*	*4.3283*	*10.2062*
Notation	Increase by 10%											
r	0.110	*289.0554*	*0.7378*	0.3310	4.4734	*6.7766*	*350.3832*	0.0704	0.5539	168.4225	4.1857	9.4503
δ	0.033	264.1679	0.5517	0.2392	4.7194	7.3061	339.0275	0.0517	0.5133	156.1793	4.4814	10.7963
θ	0.044	266.0167	0.5178	0.2260	4.7847	7.4270	340.3979	0.0467	0.4842	157.5551	4.5317	10.9984
σ	0.333	196.8939	0.6456	0.3133	4.3696	7.3652	253.7841	0.0129	0.6121	120.5774	4.0799	11.6929
λ	0.440	257.6464	0.6667	0.2856	4.5182	6.9401	339.9555	0.0848	0.6272	157.5162	4.3515	10.0366
f_X	11.000	284.1741	0.7858	0.2847	4.6003	6.7439	402.7385	0.1298	0.8558	162.6596	4.6488	10.0938
f_y	1.100	260.6983	0.6342	0.2776	4.5656	7.0737	335.0458	0.0582	0.5727	154.3828	4.3395	10.4033
Z	27.500	270.8408	0.7266	0.2856	4.5600	6.8335	339.9555	0.0848	0.6586	153.7530	4.3515	10.0366
K	35.200	276.1909	0.4835	0.2613	4.8080	7.9661	339.4706	0.0051	0.3829	170.4168	4.3946	11.6472
D_{XFYL}	0.666	256.3181	0.6813	0.22962	4.4973	6.9105	339.5045	0.0047	0.5481	157.3687	4.3950	9.4356

Figure A1: this figure shows our model divest and switch thresholds of the first-mover and second-mover for the separate and joint formulations (the two figures at the right-hand side), and the divesting and the switching thresholds for a monopoly, relying on the Dixit (1993) separate formulation and the Décamp et. al. (2006) joint formulation (the two figures at the left-hand side). For the separate formulation, $\hat{v}_{SD}^{L}$ and $\hat{v}_{SD}^{F}$ are, respectively, the first-mover and the second-mover divesting thresholds, and $\hat{v}_{SS}^{L}$ and $\hat{v}_{SS}^{F}$ are, respectively, the first-mover and the second-mover switching thresholds. $\hat{v}_{SX}$ and $\hat{v}_{SD}$, and $\hat{v}_{SSX}$ and $\hat{v}_{SSX}$ are the decision thresholds for the divest and switch of the Separate formulation of Dixit (1993) and the Joint formulation of Décamps et al. (2006).