Two-Sided Markets: The Role of Technological Uncertainty*

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Abstract

This paper examines the effect of technological uncertainty on the optimal pricing and investment decisions in a two-sided market. A platform offers a basic good and a developer offers a complementary good. The performance of the complementary good is stochastic and is endogenously determined by the pricing policy the platform adopts. Heterogeneous consumers join the platform either before uncertainty is resolved or after. In the former case, consumers obtain the basic good and an option to benefit from the complementary good in the future. The platform trades off building an earlier mass of consumer base and extracting profits from late adopters. Consumers are divided into three groups: early adopters, late adopters, and those who never join the platform. A platform's pricing policy depends on the value of the complementary good and the cost of its development. If the cost is small, a price skimming policy is optimal. When the cost is higher, price skimming remains optimal if the value of the complementary good is either small or relatively high. For intermediate values, the platform adopts a price penetration policy. We discuss some examples from the empirical literature in light of the model.

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1 Introduction

The 2020 Porsche Taycan delivers incredible acceleration and aggressive styling. One of the headline features of Porsche's first electric car is its 800-volt charging system. Porsche claims this high-power system allows an 80% charge in just 22.5 minutes, which is much quicker than other current charging systems.[1] But that will not matter if Taycan drivers can't find a compatible charging station. In the United States, Electrify America is working to develop a widespread system of charging stations suitable for Porsche's new electric car. However, some of the charging stations might not receive construction permits from urban areas due to safety, zoning, and environmental regulations. Taycan's success depends on the success of Electrify America, which remains uncertain at this point.

The video game industry faces similar challenges. Console manufacturers' market position and survival depend not only on the console's technological prowess but also on the quality of video games introduced by the game developers. For example, Sega introduced its console Saturn five months before Sony made its entry into the video game market in 1995 with its PlayStation. Saturn immediately faced problems in the competition because Sony was better able to attract third-party game developers.[2] This illustrates that uncertainty surrounding the development of quality games is of utmost importance to console manufacturers.

The smartphone market is yet another one where uncertainty plays a critical role. When Apple launched its iPhone in 2007, it announced that it might provide occasional software updates and upgrades free of charge (see Brochet et al. (2013)). Apple also equipped its phones with only about 500 applications when the App Store was launched, which by 2010 had exploded to 300,000 applications (see Boudreau (2012a)). When purchasing the smartphone back in 2007, early consumers had uncertainty over possible improvements and the availability of some apps in the future.

All the examples above represent a typical two-sided market in which a platform (Porsche/ Sega/Apple) facilitates the interaction between buyers (car users/gamers/smartphone users) and developers (of charging stations/games/apps). A platform offers a *basic* good (electric vehicle/video game console/smartphone) that is durable and can be used for a long time once purchased. Buyers join a platform that provides a product with basic, though not necessarily minimal, functionality. Developers offer a *complementary* good, i.e., features to be added to the basic good whose quality (or even availability) is revealed in the future.

[1]See https://www.digitaltrends.com/cars/2020-porsche-taycan-photos-and-specs/.

[2]Sega is by no means the only console manufacturer to experience such problems. Companies like 3DO and Nintendo have faced problems with high-quality video games. See Cool and Paranikas (2008).

In this paper, we study how uncertainty over successful development of the complementary good affects consumer purchasing decisions and the pricing policy the platform adopts. Our key idea is that, when joining the platform, buyers obtain not only the current basic functionality but also an *option* to benefit from any enhanced functionality if the complementary good is successfully developed in the future. However, the development of the complementary good is both costly and uncertain, and the probability of success depends on the amount of resources devoted to the development process. Therefore, buyers might not necessarily benefit from it if its realization is unsuccessful.

Relative to the nascent literature on two-sided markets, reviewed below, the novelty of our approach is that we allow the probability that the complementary good is successfully developed to be *endogenously* determined. Focusing on a "chicken-and-egg" problem, the literature has vastly ignored the dynamic aspect of the cross-group externalities (see Rysman (2009) for a review). In contrast, we study the impact of technological uncertainty on one side of a market, and how it affects the optimal pricing policy the platform employs. Thus, our analysis highlights the impact of endogenously determined network effect on the equilibrium structure of a two-sided market. We show that the platform's pricing policy is an essential tool for jointly managing consumer expectations and thus their purchasing behavior while, at the same time, incentivizing developers of the complementary good to innovate.

We develop a dynamic model of a two-sided market with three players. The first is a monopolistic platform that offers the basic good. The second is a representative seller (developer) who offers the complementary good and bears the cost of its development. The third is a mass of heterogeneous consumers (buyers) who differ in how much they value the basic good. Development of the complementary good is stochastic, i.e., the seller delivers the complementary good with some probability only. Consumers decide whether to buy the basic good (join the platform) before observing whether the complementary good is successfully developed or after it is. The platform chooses 1) the prices buyers must pay for the basic good before and after uncertainty is realized, and 2) a reward the seller obtains.

As in a typical two-sided market, the interaction between buyers and the seller is characterized by two network effects. First, almost all durable goods exhibit a *direct* network effect in that they draw on common use and repair expertise.[3] For example, gamers' utility from a particular video game console increases as more gamers opt for that console. Second, there is an *indirect* network effect: each group values the participation of the other group.

[3]For example, the so-called "word of mouth" effect: more information is revealed regarding a good and its properties as more consumers buy it.

For example, gamers' utility from a particular console rises as the quality and quantity of available games increase.[4]

Consumers decide whether to join the platform earlier, i.e., before the outcome of the complementary good is realized, or later, when it is known if the complementary good is successfully developed. If consumers join the platform earlier, they can enjoy the basic good longer, and, in addition, they obtain an option to benefit from the complementary good if it is developed successfully. If consumers wait, they can join the platform when the complementary good is developed. The platform implicitly controls the probability of success by choosing the seller's reward. While a higher probability delivers a higher expected surplus to consumers who join the platform earlier, it also changes the optimal timing of joining the platform for some consumers. For a given pricing policy, more consumers are likely to join the platform earlier if the probability goes up. Given the pricing policy announced by the platform, consumers correctly anticipate the probability of success and choose the optimal timing to join the platform that delivers a higher expected utility.

When deciding how many consumers to attract before and after the outcome of the complementary good is realized, the platform is facing the following trade-off. On the one hand, attracting more consumers earlier allows charging buyers who join the platform in the future a higher price due to the direct network effect. On the other hand, to attract a large enough mass of early consumers, the platform must charge a relatively small price earlier. This makes promising the seller a higher reward for success challenging and, as a result, makes it less likely that future buyers can benefit from the indirect effect. Therefore, by fine-tuning the probability of success for the complementary good, the platform trades off building an earlier mass of customer base and extracting profits from early adopters.

We prove that the platform optimally divides all consumers into three groups: 1) low-type who do not join the platform at all, 2) middle-type who wait and join the platform only if the complementary good is successfully developed, and 3) high-type who join the platform before observing the outcome of the complementary good development. These results match phenomena observed in real life. For instance, this is consistent with the market for electric vehicles where, according to the marketing literature, buyers are divided into three groups: environmentally conscious or technology-oriented consumers who would purchase a vehicle even without a sufficiently developed network of charging stations, the

[4] Game developers would also prefer to create games for a console with a larger installed customer base. Strictly speaking, developers are responsible for conceptualizing and designing games while publishers market and distribute games. In practice, developers may choose to market their own games and publishers may employ in-house developers. Therefore, we use publishers and developers interchangeably.

"mass market" represented by consumers who buy a vehicle only if the battery performance is advanced and a high enough number of charging stations is available, and a third group that would rather "walk away" and buy a conventional vehicle rather than an electric one.

We show that the platform employs either a *price penetration* or *price skimming* policy: initially setting a low (high) price and increase (decrease) it after the complementary good is successfully developed in the future. On the one hand, there are two reasons to lower the price of the basic good in the future. First, consumers who join the platform earlier enjoy the basic good over two periods rather than just one. Second, early consumers obtain the basic good and an option to benefit from the complementary good in the future due to the indirect network effect. The platform might extract surplus from them and then use a low price strategy in the future.

On the other hand, there are two reasons to increase the price of the basic good in the future. First, a lower early price increases the number of early consumers, and, as a result, the value of the basic good in the future increases due to the direct network effect. Second, consumers join the platform in the future only if the complementary good is successfully developed. Thus, the future price should be high enough to extract surplus from the combination of both the basic and the complementary good due to the indirect effect.

We prove that if the cost of development of the complementary good is low, the platform employs the price skimming policy. Intuitively, in this case, the complementary good is likely to succeed, and, therefore, consumers are likely to join the platform earlier. The platform is then better off by extracting the surplus from early buyers. This is in agreement with the price dynamics for smartphones and video game console producers consoles.[5]

If the cost of development of the complementary good is relatively high, the optimal pricing policy depends on the value of the complementary good. We show that the platform optimally employs a price skimming (penetration) policy if the value is small (high). Intuitively, for small values of the complementary good, the direct effect dominates, and since early buyers enjoy the basic good longer, the platform lowers the price in the future. We also find another result. When the value of the complementary good becomes sufficiently large, the platform optimally employs the price skimming policy again. Intuitively, if the benefit of the complementary is large, it will be developed almost for sure. Then, consumers who joined the platform earlier are almost certain to benefit from the indirect network effect. The platform is better off by extracting this surplus earlier. This is the case if consumers obtain the basic good mostly to benefit from the complementary one. For example, gamers

[5]See Section 6 for a discussion.

might buy a video game console to play a particular game: Soccer fans may buy PlayStation to mostly play FIFA 20. This price dynamics is consistent with the optimal pricing policy adopted by electric vehicles and media service providers reviewed in Section 6.

We also consider two extensions of the main model. In Section 5.1, we study a version of the main model without commitment. We illustrate that the benefit of commitment for the platform is two-fold. First, commitment allows building a larger early consumer base. Second, commitment to a higher price in the future allows charging a higher earlier price. In Section 5.2, we allow the value of the complementary good to be a function of consumer type. This assumption delivers a novel result: The platform optimally sells the basic good only before the outcome of the complementary good is revealed. Therefore, consumers are divided into two groups only, i.e., high types who join the platform before observing whether the complementary good succeeds, and low types who do not join the platform at all.

The rest of the paper is organized as follows. Section 2 reviews the related literature. The main model and payoffs are introduced in Section 3. Section 4 presents the results of the equilibrium outcomes. Several key extensions are presented in Section 5. Section 6 discusses empirical applications. Section 7 concludes the paper.

2 Related Literature

Our paper contributes to the growing literature on two-sided markets. A central theme of the research in this area has been how the platform pricing policy depends on the competitive settings, magnitudes of network effects and the possibility of multihoming. Armstrong (2006), Rochet and Tirole (2003), and Bolt and Tieman (2008) develop monopoly and imperfect competition models and show that magnitude of indirect network effects, demand elasticity on each side, and the platform's fee structure affect the platform pricing policy.[6]

Most of the papers in the literature that study positive indirect network effects focus on the number of users on each side of the platform: as more users on each side are attracted to the platform, indirect network effects becomes stronger. For instance, Boudreau (2012b) illustrates that the variety of available software grows as the number of software developers increases. However, little is known regarding the factors other than the number of participants (see McIntyre and Srinivasan (2017)).[7] As the video games market demonstrates,

[6]Platforms can charge a combination of registration fees and transaction fees. While registration fees are lumpy, transaction fees depend on the frequency of transactions between the two sides.

[7]Jullien and Pavan (2019) study platform markets in which the information about users' preferences is dispersed.

indirect network externalities arise not simply due to the number of game publishers but also the quality of available games. We contribute to this literature by studying the effect of platform pricing on developers' incentives to innovate.

Our paper also belongs to the stream of the literature that investigates the role of forward-looking consumers and consumer expectations in two-sided markets (see Tan and Zhou (2017), Hagiu and Hałaburda (2014), Hagiu and Spulber (2013), Halaburda and Yehezkel (2013), Zhu and Iansiti (2012)). In these papers, consumers form expectations either over prices or over participation on the other side of the market.[8] We allow for consumer heterogeneity and focus on endogenous uncertainty on the developer side. In our model, consumers form expectations over whether the complementary product of sufficient quality will be available in the future or not.[9]

Finally, our paper relates to the dynamic pricing literature (see Kumar and Sethi (2009), Liu (2010), Nair (2007), Gabszewicz and Garcia (2008)). Several papers study the interaction between dynamic pricing policy and consumers' strategic behavior that arises due to consumers' ability to postpone their purchases in anticipation of future price decreases.[10] A central question in this strand of literature is how to mitigate the negative effect of consumers' strategic behavior on firms' revenues (see Su and Zhang (2008), Aviv and Pazgal (2008), Lai et al. (2010)). In contrast, strategic consumer behavior in our model is a direct result of consumers' anticipation that the complementary products might be available in the future. In turn, we show that the development of the complementary product affects the platform's optimal pricing policy. Therefore, we demonstrate how the platform employs its pricing policy to modify the intertemporal distribution of consumers.

In summary, to the best of our knowledge, ours is the first paper in the literature on two-sided markets that explicitly models investment decisions in a risky environment. Previous literature typically assumes that sellers join a platform either before or simultaneously with the buyers. This simple yet insightful approach, however, might not fully capture and explain the empirical evidence, as buyers in two-sided markets do join a platform expecting a sufficient number of sellers to be available in the future only. We, in contrast, allow buyers

[8]Zhu and Iansiti (2012) develop a model in which consumers take into account the availability of future applications when making their purchase decisions. The consumer anticipation is governed by a discount factor that regulates how much the representative consumer values future complementary products.

[9]Zhou (2017) presents a model in which consumers strategically choose the timing of platform adoption in anticipation of future complementary products likely to be developed by the sellers. Although Zhou (2017) recognizes the option value involved in consumers' timing decision, that paper does not model explicitly the effect of seller side uncertainty on the timing decision.

[10]See, for instance, Dasu and Tong (2010) and Liu and Zhang (2013).

to join the platform having only some expectations that the seller might join the platform in the future. Our paper differs from the existing literature in the following major dimensions. First, we introduce a stochastic development process for the complementary good and a gradual resolution of uncertainty. Second, we characterize the embedded option value for the early buyers of a two-sided market product. Third, we show that the optimal pricing policy critically depends on the additional value brought by the complementary good.

3 Model

Key Players. We consider a model with three players: 1) a monopolistic platform, 2) a mass of heterogeneous consumers, and 3) a representative seller (developer) of the complementary good. The platform offers a *basic* good and the seller offers a *complementary* good, i.e., features to be added on to the basic good. In order to incorporate technological uncertainty into the model, we assume that development of the complementary good is stochastic, i.e., the seller successfully develops the complementary good with some probability only. We assume that all the key parameters of the model are common knowledge for all players at the beginning of the game.

Source of Uncertainty. A key distinguishing feature of our paper is the introduction of endogenous technological uncertainty that is inherent in every innovation process. In most of two-sided markets, development of complementary goods is subject to various technical, legal, financial, organizational, market and even political risks. For example, software developers face various technology risks and often experience failures. Risks might involve economy-wide shocks as well as idiosyncratic ones, i.e., losing to a a rival seller. In the case of new physical products, numerous safety, zoning, and environmental regulations might be a source of uncertainty. For example, charging stations for electric cars might not receive construction permits from urban areas to build a large enough number of stations.

Time-Line. The model is dynamic. The game spans over three time periods $t \in \{0, 1, 2\}$. At $t = 0$, the platform produces the basic good and chooses an optimal pricing policy, i.e., prices p_t that consumers have to pay to join the platform at $t = 1$ and $t = 2$.[11] In addition, the platfrom chooses a reward $b > 0$ the seller receives for the successful development of

[11] In the language of (Rochet and Tirole, 2003), the platform is charging interaction-independent fixed fees.

the complementary good.[12] At $t = 1$, the developer of the complementary good chooses the probability of successful development of the complementary good at $t = 2$, denoted by $0 \leqslant P \leqslant 1$. Increasing the probability of success is costly, and the seller bears the cost. At $t = 2$, the seller either successfully develops the complementary good or fails to do so.

After observing the pricing policy chosen by the platfrom, consumers decide whether to join the platfrom at $t = 1$ or at $t = 2$ after paying price p_1 or p_2, respectively. Crucially, p_1 is the price consumers have to pay *before* observing whether or not the seller is successful in developing the complementary good, whereas p_2 is the price for joining the platform *after* observing success or failure of the seller.[13]

A representative time-line of the game is plotted in Figure 1 below.

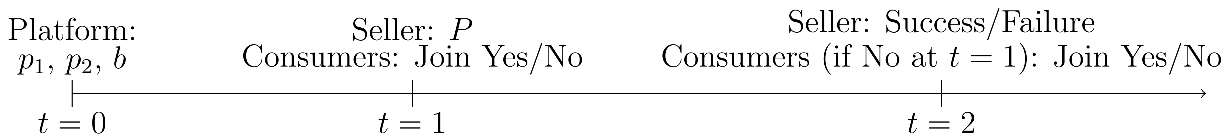

Figure 1: Time-line of the game.

3.1 Payoffs

We now describe the payoff functions for the seller, consumers, and the platform. To simplify the exposition, we assume that all players have a common time discount factor $\delta = 1$.[14]

Seller. The development of the complementary good is a random process with a risky outcome. Therefore, the seller is facing a risky investment problem. With probability P, the seller is successful and is paid b. However, with probability $1 - P$, the complementary good fails and the seller gets nothing. In order to achieve the probability of success P, a certain level of efforts (investment) is required. The cost of these efforts is given by a quadratic function $c(P)$:

$$c(P) = A \times P^2 \tag{1}$$

[12]We normalize the seller's pay-off if the complementary good fails to zero. This is without loss of generality since the seller is risk-neutral.

[13]For example, the price for an iPhone in 2007, when there were only few hundreds applications available in the App Store, was different from the price in 2010, when as many as 300,000 applications became available (Brochet et al., 2013).

[14]None of the results hinge on this assumption.

where $A > 0$ and $c'(P) = 2AP > 0$, $c''(P) = 2A > 0$.[15]

Therefore, given a reward b, the seller chooses the probability of success P to maximize

$$\pi^s = P \times b + (1 - P) \times 0 - c(P) = Pb - c(P).$$

Consumers. We assume that the basic good is durable, i.e., consumers who join the platform at $t = 1$ get some utility form consumption of the basic good in both periods $t = 1$ and $t = 2$.[16] We also assume that the quality of the durable good does not depreciate between the two periods. We introduce two network effects to capture consumer interaction with the platform and the seller of the complementary good.[17]

Direct Network Effect. A consumer gets a higher utility from joining the platform when more consumers are using the platform simultaneously. For example, users of an iPhone enjoy it more if their friends have iPhones too. Consumers are heterogeneous and differ in how much they value the basic good. For example, some consumers might have a higher willingness to pay for an iPhone. A consumer of type θ, $\theta \in [0, 1]$, gets (per period) utility $u(\theta, N)$: $[0, 1] \times [0, 1] \to [0, +\infty)$ from consumption of the basic good, where θ denotes the intrinsic utility form the basic good, and N is the mass of current users of the basic good. In the main part of the paper, we assume that

$$u(\theta, N) = \theta \times N.$$[18]

We normalize the utility of not joining the platform to zero for each consumer type.

Distribution of Types. The distribution of consumer preferences is given by a continuously differentiable distribution function $F(\theta)$ with support $[0, 1]$. A probability distribution $f(\theta) = F'(\theta)$ over the support defines the relative size of customers with valuation $\theta \in [0, 1]$. We assume that the distribution of types is uniform, $f(\theta) = 1$ for $\theta \in [0, 1]$.[19]

Indirect Network Effect. We introduce an *indirect* (positive) network effect between consumers and the seller of the complementary good as follows. Consumer's utility depends on

[15]The cost of investment captures the cost of developing new performance features, adoption by the product, the cost of making that innovation includes marginal cost of development, any marketing costs and/or download purchases (Li et al., 2016).

[16]Most of the network goods are durable. In particular, this is the case for electric vehicles, smartphones, as well as video game consoles.

[17]Almost all durable goods exhibit network effects in that they draw on common use and repair expertise. For example, the so-called "word of mouth" effect: more information is revealed regarding a good and its properties as more consumers buy it. Another example might be the so-called "learning by doing" effect: the seller improves the quality of the product as more consumers submit feedback.

[18]We discuss robustness of our main results to a more general utility function in Section 5.3.

[19]In Section 5.4, we discuss implications of various other distributions.

the bundle, i.e, combination of the basic and complementary goods. In case the complementary good is successfully developed, each type of consumer obtains an additional utility $\alpha(\theta) \geq 0$.[20] For example, the utility that current users derive from their electric cars is likely to increase when more charging stations become available in their area. As another example, the utility that current iPhone users will derive from their phone is likely to increase as new apps are developed and increase the phone's functionality.

3.2 Strategic Choices

Consumer Choice Problem. Consumers are choosing whether to join the platform before observing if the complementary good is successfully developed ($t = 1$), or wait till Success/Failure of the complementary good is revealed ($t = 2$).[21] We denote by N_1 the mass of consumers who purchase the basic good at $t = 1$, by N_2^S the mass of consumers who purchase the basic good at $t = 2$ if the complementary good is successfully developed, and by N_2^F the mass of consumers who purchase the basic good at $t = 2$ even if the complementary good fails to be developed.

When choosing the optimal timing of joining the platform, consumers have to form expectations regarding the probability of success P and the relevant mass of consumers N_1, N_2^S, and N_2^F. Since each consumer is infinitely small, they do not not take into account the effect of their own decision when to join the platform on the equilibrium values of P, N_1, N_2^S, and N_2^F. Following the standard assumption in the network equilibrium models, we assume that each consumer correctly anticipates the the values of P, N_1, N_2^S, and N_2^F in equilibrium.

Optimal Timing to Join the Platform. Consider a type θ consumer who did not join the platform at $t = 1$ and decided to wait till $t = 2$. This consumer's utility at $t = 2$ is[22]

$$\begin{cases} max\{\theta(N_1 + N_2^S) + \alpha(\theta) - p_2, 0\}, & \text{if Complementary good is successfully developed.} \\ max\{\theta(N_1 + N_2^F) - p_2, 0\}, & \text{if Complementary good development fails.} \end{cases}$$

[20]To highlight the effect of technological uncertainty on the optimal pricing policy, we assume the indirect network effect is positive. It is also possible for the indirect network effect to be negative. This direction is left for future research.

[21]Consumers typically make a one-time purchase of a durable good such as a smartphone, an electric vehicle, or a point of sale (POS) terminal that they enjoy over several periods of time.

[22]The function $max\{\}$ reflects that consumers join the platform after the complementary good is successfully developed only if they find optimal to do so, i.e., this decision has to be ex post optimal.

Therefore, at $t = 1$ for a type θ consumer the expected utility of waiting till $t = 2$ is

$$U_2(\theta) \equiv P \times max\{\theta(N_1 + N_2^S) + \alpha(\theta) - p_2, 0\} + \ (1 - P) \times max\{\theta(N_1 + N_2^F) - p_2, 0\} \quad (2)$$

Consider now a consumer of type θ who joins the platform at $t = 1$ after paying price p_1. If the complementary good is successful, the consumer gets $\theta(N_1 + N_2^S) + \alpha(\theta)$ in the future. If the complementary good fails, the consumer gets $\theta(N_1 + N_2^F)$ only. Therefore, the expected utility from joining the platform at $t = 1$ becomes

$$U_1(\theta) \equiv \theta N_1 - p_1 + P \times \big(\theta(N_1 + N_2^S) + \alpha(\theta)\big) + (1 - P) \times \theta(N_1 + N_2^F) \quad (3)$$

The individual rationality constraint implies that a consumer joins the platform at $t = 1$ if

$$U_1(\theta) \geq U_2(\theta),$$

and finds it optimal to wait till $t = 2$ otherwise (if $U_1(\theta) < U_2(\theta)$).[23]

Option Value. The expression for $U_1(\theta)$ also characterizes the option value for consumers who join the platform at $t = 1$. In particular, a consumer who joins the platform at $t = 1$ after paying p_1 obtains an option value equal to

$$P \times \theta(N_2^S - N_2^F) + P \times \alpha.$$

The first term, $P \times \theta(N_2^S - N_2^F)$, represents the direct effect. If the complementary good is successfull, N_2^S consumers will join the platform at $t = 2$ whereas only N_2^F consumers will join the platform at $t = 2$ if the complementary good fails. The second term, $P \times \alpha$, represents the indirect effect, i.e., the additional benefit from the complementary good.

Optimal Pricing. Given the mass of consumers N_1, N_2^S, and N_2^F, the platform collects $p_1 N_1$ from consumers buying the basic good at $t = 1$. In addition, with probability P the complementary good is successful and the platform collects $p_2 N_2^S$ at $t = 2$, whereas the platform collects $p_2 N_2^F$ if the complementary good fails. Without loss of generality, we assume that the cost of producing the basic good is zero. Therefore, the platform chooses prices p_1, p_2, and b to maximize[24]

$$N_1 \times p_1 + \big[P \times N_2^S + (1 - P) \times N_2^F\big] p_2 - P \times b. \quad (4)$$

[23]Without loss of generality, we assume if a consumer is indifferent ($U_1(\theta) = U_2(\theta)$), he joins the platform at $t = 1$.

[24]We assume that all payments are between consumers and the platform owner, i.e, there are no explicit transfers from consumers to the seller. In many markets such as, electric vehicles and video games, it is indeed the case that the seller does not charge consumers directly. For instance, it is typical for the charging

To summarize, the timing of the game is as follows:

- $t = 0$: The platform chooses p_1, p_2, the prices of access to platform in two periods, and b, the reward for a success development of the complementary good. We normalize the marginal cost of producing the platform to zero and assume it does not change over time.

- $t = 1$: Consumers decide whether to join the platform at $t = 1$ or wait till $t = 2$. The seller of the complementary good chooses the probability of successful development of the complementary good at $t = 2$.

- $t = 2$: After observing whether the complementary good was successfully developed or not, consumers (who did not join the platform at $t = 1$) decide to join the platform or never join.

Table 1 below summarizes the key notation used throughout the paper.

Variable	Definition
P	Probability of Success in developing the complementary good
$c(P)$	Seller's cost of efforts to achieve probability of Success P
θ	Consumer type
$F(\theta), f(\theta)$	Distribution and density functions of types, respectively
π^S	Seller's profit
N_1	Mass of consumers buying at $t = 1$
N_2^S	Mass of consumers buying the basic good at $t = 2$ in case of Success
N_2^F	Mass of consumers buying the basic good at $t = 2$ even in the case of Failure
N	Mass of current users of the product (*direct* network effect)
$u(\theta, N)$	Consumer utility
$\alpha(\theta)$	Additional utility from the complementary good (*indirect* network effects)
p_1	Price of the basic good at $t = 1$
p_2	Price of the basic good at $t = 2$
b	Reward (bonus) for a successfull development of the complementary good
$\hat{.}$	The equilibrium value of variables

Table 1: Key Notation.

stations services to be offered without any payment due. Our model is still applicable even when there are consumer-seller transfers if one treats p_t and b as *net* transfers between the platform and consumers and the complementary good seller, respectively.

4 Equilibrium Analysis

We now describe the equilibrium and the trade-offs involved. We present results and the intuition while all the proofs are delegated to the Appendix. In the main part of the paper, we assume the platform *commits* to price p_2 in advance, i.e., the platform announces p_2 at $t = 1$ (before observing outcome of the complementary good development and consumer mass N_1) and cannot change it at $t = 2$.[25] One of many examples of such commitment mechanisms are pre-orders of new products with a fixed price. In Section 5.1, we relax the commitment assumption and allow platform to chose p_2 after observing Success/Failure of the complementary good development and the consumer base N_1. We solve the model using the solution concept of Rational Expectations Equilibrium.[26]

Definition. **Equilibrium** (*with commitment*).
Prices $\hat{p}_1$ and $\hat{p}_2$, a reward $\hat{b}$; a success probability $\hat{P}$; consumer mass $\hat{N}_1$, $\hat{N}_2^S$ and $\hat{N}_2^F$ constitute an equilibrium if

- *Seller.* Given $\hat{b}$, the seller of the complementary good chooses $\hat{P}$ to solve

$$\max_{P\geq 0} \{P \times b + (1-P) \times 0 - c(P)\} \text{ s.t.}$$
$$P \times b + (1-P) \times 0 - c(P) \geq 0.$$

- *Consumers.* Given $\hat{p}_1$, $\hat{p}_2$ and $\hat{P}$,

$$\hat{N}_1 = \int_0^1 \mathbb{1}_{U_1 \geq U_2} dF(\theta),\ \hat{N}_2^S = \int_0^1 \mathbb{1}_{U_1 < U_2} \times \mathbb{1}_{\theta(\hat{N}_1+\hat{N}_2^S)+\alpha(\theta)-p_2\geq 0} dF(\theta),$$
$$\text{and } \hat{N}_2^F = \int_0^1 \mathbb{1}_{U_1<U_2} \times \mathbb{1}_{\theta(\hat{N}_1+\hat{N}_2^F)-p_2\geq 0} dF(\theta).^{27}$$

- *Platform.* Prices $\hat{p}_1$ and $\hat{p}_2$, and a reward $\hat{b}$, are chosen as an optimal solution for

$$\max_{b,p_1,p_2\geq 0} \left\{\hat{N}_1 \times p_1 + \left[\hat{P} \times \hat{N}_2^S + (1-\hat{P}) \times \hat{N}_2^F\right] \times p_2 - \hat{P} \times b\right\}.$$

[25] We assume the platform cannot make the future price, p_2, contingent on success/failure of the complementary good. The platform may not even able to specify in advance what the complementary good is to put it into a contract for the consumers. For example, in the electric vehicles car market where the complementary good is a local charging station, the platform (Tesla) cannot announce pricing policy that is contingent on whether a small local charging station at a particular location is developed or not. A video game platform cannot commit in advance to a future price that is contingent on some video game being developed in the future since the platform might not know what type of game will be eventually developed. Apple had limited abilities to foresee exact types of applications would be available in the future when they released iPhones.

[26] See (Hagiu, 2006) for the details.

[27] A characteristic function $\mathbb{1}_{x\in\mathbb{X}}$ is defined as $\mathbb{1}_{x\in\mathbb{X}} = \begin{cases} 1, & x \in \mathbb{X} \\ 0, & x \notin \mathbb{X} \end{cases}$.

We now characterize the main elements of the equilibrium. In Section 4.1, we study the seller's optimal choice of the success probability for a given reward b. Next, in Section 4.2, we determine the optimal timing of joining the platform for each consumer type for given prices p_1 and p_2 as well as success probability P. Finally, in Section 4.3, we solve for the platform optimal pricing policy and the corresponding mass of consumers who join the platform at different times in equilibrium.

4.1 Seller

Consider the seller's optimization problem for a given reward b. The seller determines the optimal probability of success $\hat{P}$ as a solution to

$$\max_{P\geq 0} \{P \times b - c(P)\} \text{ s.t.}$$
$$P \times b - c(P) \geq 0.$$

The solution has to satisfy the following *F.O.C.*:

$$b = \frac{dc(P)}{dP} = 2AP,$$

which determines the equilibrium probability of success, $\hat{P}(b)$ as a function of the reward b:

$$\hat{P}(b) = \frac{b}{2A}.$$ [28]

Note that because $c(P)$ is convex, $\hat{P}(b)$ is an increasing function, $\frac{d\hat{P}(b)}{db} > 0$. Intuitively, by promising a higher price for the complementary good, the platform makes a successful development more likely.

4.2 Consumers

We now discuss the consumer optimization problem for given p_1, p_2, and P. An arbitrary consumer of type θ, after observing the prices chosen by the platform, decides whether to join at $t = 1$ (before knowing whether the complementary good is successfully developed or not), or wait till $t = 2$ and observe success/failure of the complementary good.

If consumers join the platform at $t = 1$, they obtain both *i*) the basic good that can be used for two periods, and *ii*) an option value of using the complementary good in the future if it is successfully developed. For example, buyers of Porsche's Taycan obtain an electric vehicle as well as an opportunity to benefit from a wide network of charging stations

[28] The First Order Necessary Condition is also sufficient since the objective function is concave.

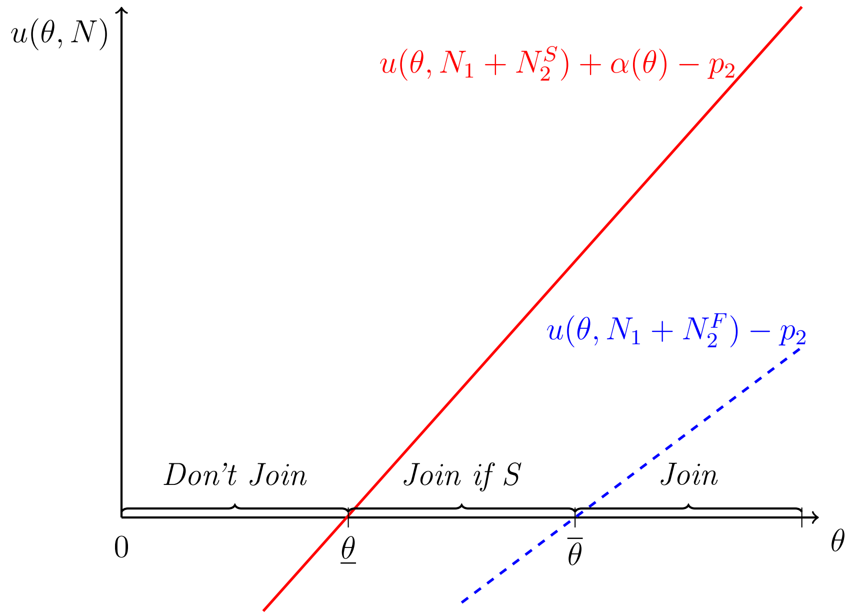

Figure 2: Consumer's decision to join the platform at $t = 2$ for arbitrary N_1, N_2^F and N_2^S. The solid red line represents a consumer utility if the complementary good is successfully developed, and the dashed blue line represent a consumer utility if the complementary good fails. High-type consumers ($\theta \geqslant \overline{\theta}$) join the platform even if the complementary good fails, i.e., as long as $u(\theta, N_1+N_2^F)-p_2 \geqslant 0$. Middle-type consumers ($\underline{\theta} \leqslant \theta < \overline{\theta}$) join the platform only if the complementary good is developed, i.e., only if $u(\theta, N_1 + N_2^S) + \alpha(\theta) - p_2 \geqslant 0$. Low-type consumers ($\theta < \underline{\theta}$) do not join the platform.

that might be developed in the future. Similarly, buyers of a video game console acquire a durable device as well as the opportunity to enjoy games that might be released in the future.[29] If consumers defer their decision till $t = 2$, they might join the platform only if the complementary good is successfully developed.

We perform the analysis backward by studying consumer choice at $t = 2$ first.

For arbitrary $N_1 \geq 0$, $N_2^S \geq 0$ and $N_2^F \geq 0$, consider a consumer of type θ (who did not join the platform at $t = 1$) at period $t = 2$. We denote

$$\underline{\theta} \equiv \tfrac{p_2-\alpha(\theta)}{N_1+N_2^S} \text{ and } \overline{\theta} \equiv \tfrac{p_2}{N_1+N_2^F}.$$

There are three relevant types of consumers (see Figure 2). First, "high" type consumers who join the platform at $t = 2$ even if the complementary good fails to be developed: $\theta \geq \overline{\theta}$. Second, "middle" type consumers who join the platform at $t = 2$ only if the complementary good is successfully developed: $\underline{\theta} \leq \theta < \overline{\theta}$. Third, "low" type consumers who do not join the platform at $t = 2$ even if the complementary good is successfully developed: $\theta < \underline{\theta}$.

[29] As another example, early buyers of iPhone acquired a durable smartphone as well the opportunity to enjoy many features and apps that would be developed later.

Therefore, $U_2(\theta)$ can be rewritten as

$$U_2(\theta) = \begin{cases} 0, & \theta < \underline{\theta} \\ P \times \big(\theta(N_1 + N_2^S) + \alpha(\theta) - p_2\big), & \underline{\theta} \le \theta < \overline{\theta} \\ P \times \big(\theta(N_1 + N_2^S) + \alpha(\theta)\big) + (1-P) \times \theta(N_1 + N_2^F) - p_2, & \theta \ge \overline{\theta} \end{cases}$$

In equilibrium, a consumer of type θ (who correctly anticipates values of N_1, N_2^S, and N_2^F) joins the platform at $t = 1$ if $U_1(\theta) \ge U_2(\theta)$, where

$$U_1(\theta) = \theta N_1 - p_1 + \Big[P \times \big(\theta(N_1 + N_2^S) + \alpha(\theta)\big) + (1-P) \times \theta(N_1 + N_2^F)\Big].$$

By comparing $U_1(\theta)$ and $U_2(\theta)$, a consumer of type θ joins the platform at $t = 1$ if

$$\theta N_1 + P\big(\theta(N_1 + N_2^S) + \alpha(\theta)\big) + (1-P)\theta(N_1 + N_2^F) \ge p_1 \text{ for } \theta < \underline{\theta},$$
$$\theta N_1 + (1-P)\theta(N_1 + N_2^F) \ge p_1 - Pp_2 \text{ for } \underline{\theta} \le \theta < \overline{\theta},$$
$$\theta N_1 \ge p_1 - p_2 \text{ for } \theta \ge \overline{\theta}.$$

In Figure 3 below, we present six possible types of consumer for arbitrary prices p_1 and p_2, success probability P, and consumer mass N_1, N_2^F, and N_2^S.

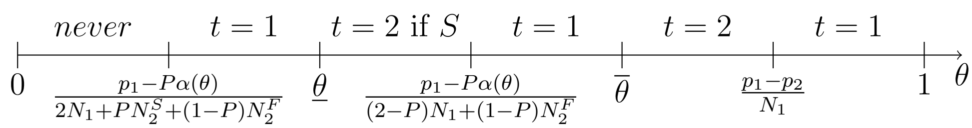

Figure 3: The optimal timing of joining the platform. A consumer might find it optimal to join the platform before observing the development of the complementary good ($t = 1$), wait and join only of the complementary good is developed at ($t = 2$ *if* S), wait and join the platform regardless of the complementary good development ($t = 2$), or not to join the platform (*never*).

In Figure 4, we illustrate the optimal timing of joining the platform for various combinations of prices p_1 and p_2. When both prices are relatively high (the upper-right corner), consumers do not join the platform at all. If p_2 is relatively higher than p_1 (the upper-left corner), consumers join the platform immediately at $t = 1$. Intuitively, for this combination of prices, consumers prefer to join the platform without waiting and observing whether the complementary good is successfully developed or not. Finally, if p_2 is relatively smaller than p_1, consumers wait and joins the platform only after the complementary good is successfully developed at $t = 2$. For these consumers, the option value of postponing the purchase to the subsequent period is high enough.

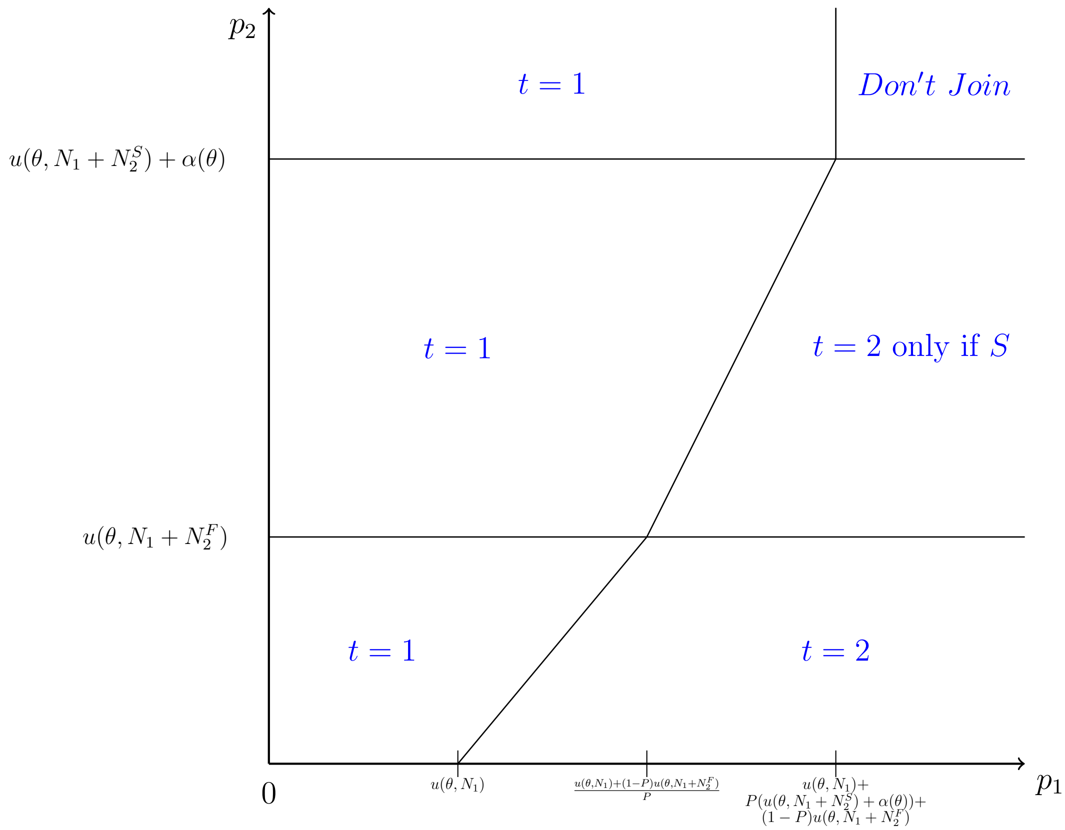

Figure 4: Consumer decision to join the platform for arbitrary prices p_1 and p_2. A consumer might find it optimal to join the platform before observing the development of the complementary good ($t = 1$), wait and join only of the complementary good is developed at ($t = 2$ *only if* S), wait and join the platform regardless of the complementary good development ($t = 2$), or not to join the platform (*never*).

4.3 Platform

We now derive the platform's pricing policy and discuss the trade-offs involved. To highlight the effect of the technological uncertainty on the optimal pricing policy, we consider a case when each type of consumer equally benefits from the complementary good as the main model. In particular, we assume that the indirect network effect is type-independent, i.e.,

$$\alpha(\theta) \equiv \alpha > 0 \text{ for all } \theta \in [0, 1].$$[30]

In Section 5.2, we relax this assumption and allow the additional benefit of the complementary good to be a function of the agent's type.

The Optimal Timing of Joining the Platform. We begin the analysis by characterizing the optimal timing of joining the platform for each consumer type. In particular, we ask 1) whether the platform makes it optimal for some consumers to wait and join the platform only after successful development of the complementary good at $t = 2$, and 2) whether the platform excludes some consumers from the market completely (make it optimal to never buy the basic good).

We prove that in equilibrium all consumers are divided into three adjacent groups according to their type: 1) "high-value" consumers who join the platform at $t = 1$, 2) "middle-value" consumers who wait till $t = 2$ and join the platform only if the complementary good is successfully developed, and 3) "low-value" consumers who never join the platform (see Figure 5).[31]

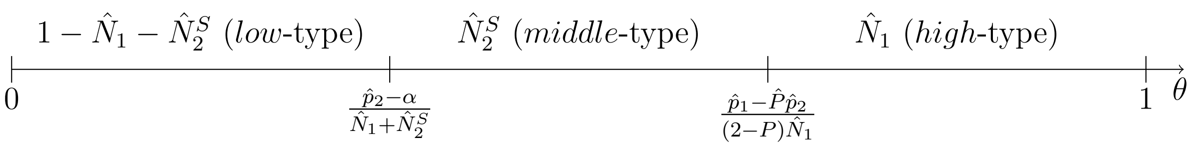

Figure 5: Optimal timing of joining the platform if $\alpha(\theta) = \alpha > 0$.

When deciding how many consumers to attract at $t = 1$, the platform is facing the following trade-off. On the one hand, attracting more consumers at $t = 1$ allows charging future

[30] For example, most of the gamers might benefit from a newly released game regardless of how much they enjoy playing other video games. Another example is a payment card systems, such as MasterCard, that attracts a new merchant (a local cafe) and makes all cardholders (local coffee drinkers) equally better off.

[31] It is straightforward that $\hat{N}_2^F = 0$. Suppose there is an equilibrium with $\hat{N}_2^F > 0$, i.e., where some consumers find it optimal to wait till $t = 2$ and then join the platform even if the complementary good fails. Necessarily, $p_1 > p_2$ in such an equilibrium and at least some of these consumers retain a strictly positive surplus. The platform then can make a higher profit by extracting this surplus by increasing p_2.

consumers who join the platform at $t = 2$ a higher price. This is a consequence of the direct network effect. On the other hand, in order to attract a sufficient mass of early consumers the platform must charge a small enough price at $t = 1$. Therefore, the platform trades off building an earlier mass of consumer base and extracting profits from early adopters. As a result, there is an optimal mass of consumers $\hat{N}_1$ that balances these two forces. In addition, we find that it might be optimal for the platform to exclude some consumers from the market completely. This is the case only if the value of the complementary good is relatively small ($\hat{N}_1 + \hat{N}_2^S < 1$ if $\alpha < 1$). However, if the additional benefit from the complementary good is high ($\alpha \geq 1$), then it is optimal to serve the whole market. We summarize the results in Proposition 1 below.

Proposition 1. The Optimal timing of joining the platform.

$$\hat{N}_1 = \tfrac{2}{3},\ \hat{N}_2^F = 0,\ \hat{N}_2^S = min\left\{\tfrac{\sqrt{1+3\alpha}-1}{3}, \tfrac{1}{3}\right\}.$$

Proof: See Appendix.

Equilibrium Probability of Success. We next discuss the equilibrium probability of success, $\hat{P}$. Since the seller correctly anticipates when each type of consumer joins the platform, the choice of $\hat{P}$ is determined by the equilibrium values of $\hat{N}_1$ and $\hat{N}_2^S$. In Appendix we formally prove that the platform's profit is

$$P\alpha(N_1 + N_2^S) + P(N_1 + N_2^S)(1 - N_1 - N_2^S) + (2 - P)N_1(1 - N_1) - Pc'(P).$$

A higher probability of success for the complementary good changes the optimal timing of joining the platform for some consumer types. For example, consider a type $\tilde{\theta}$ who is just indifferent between joining the platform at $t = 1$ and at $t = 2$. Suppose now the probability increases. Then type $\tilde{\theta}$ consumer strictly prefers joining the platform at $t = 1$ rather than waiting till $t = 2$. The platform then can increase price p_1 accordingly to extract the expected surplus from type $\tilde{\theta}$. Therefore, the platform jointly adjusts prices and the probability to maximize the profit. Intuitively, $\hat{P}$ equalizes the marginal benefit of increasing the probability of success and the marginal cost of efforts. We prove that in equilibrium

$$\hat{P} = \tfrac{\alpha(\hat{N}_1+\hat{N}_2^S)+(\hat{N}_1+\hat{N}_2^S)^2(1-\hat{N}_1-\hat{N}_2^S)-\hat{N}_1(1-\hat{N}_1)}{4A}.$$ [32]

[32] Formally, $\hat{P}$ is determined by

$$\alpha(\hat{N}_1 + \hat{N}_2^S) + (\hat{N}_1 + \hat{N}_2^S) - \hat{N}_1(1 - \hat{N}_1) + (1 - \hat{N}_1 - \hat{N}_2^S) = \hat{N}_1(1 - \hat{N}_1) + c'(P) + Pc''(P),$$

the condition is derived by differentiating the platform's profit given by

We find that $\hat{P}$ is an increasing function of α, and a decreasing function of A. These two results should appear intuitive. The higher the value added by the complementary good (high α), the higher the surplus that can be extracted from the consumers and, therefore, the higher the benefit of success for the seller. When the effort necessary fro achieving success is costlier (high A), a marginal increase in the probability of success becomes less beneficial.

The Optimal Pricing Policy. We now determine prices $\hat{p}_1$ and $\hat{p}_2$ the platform chooses in equilibrium. A critical question for the platform is whether to use a *price penetration* ($\hat{p}_2 > \hat{p}_1$) or a *price skimming* ($\hat{p}_1 > \hat{p}_2$) policy. When choosing whether to charge a smaller price at $t = 2$ than at $t = 1$, the platform is facing the following trade-off.

On the one hand, there are two reasons to lower the price at $t = 2$. First, consumers who join the platform at $t = 1$ enjoy the basic good over two periods rather than just one. Given that early buyers are high-type consumers, the platform may extract surplus from them and then use a low price strategy at $t = 2$. Second, due to the indirect network effect, early consumers at $t = 1$ buy not only the basic good itself, but also an option to benefit from the complementary good if it is successfully developed in the future. The platform extracts this surplus at $t = 1$ by charging a higher price. Then, at $t = 2$, the platform lowers the price and serves the low-type consumers who are willing to join only if the complementary good is successfully developed.

On the other hand, there are as well two reasons to increase the price of the basic good at $t = 2$. First, a lower price at $t = 1$ increases the number of early consumers and, as a result, the value of the basic good in the future increases due to the direct network effect. Second, consumers join the platform at $t = 2$ only if they can benefit from the indirect network effect after the complementary good is successfully developed. Thus, future price $\hat{p}_2$ should be high enough to extract surplus from the combination of both the basic and the complementary goods.

We plot graphically combination of optimal pricing policies for various parameters α (value of the complementary good) and A (cost of development the complementary good) in Figure 6. We can see that the price skimming policy is optimal for relatively small values of A. Intuitively, if the cost of development of the complementary good is sufficiently low then the probability of success is high in equilibrium. Since consumers will benefit from the complementary good almost for sure, they are more likely to join the platform at $t = 1$. The platform is then better off extracting their surplus earlier.

$$\pi^S = P\alpha(N_1 + N_2^S) + P(N_1 + N_2^S)(1 - N_1 - N_2^S) + (2 - P)N_1(1 - N_1) - Pc'(P).$$

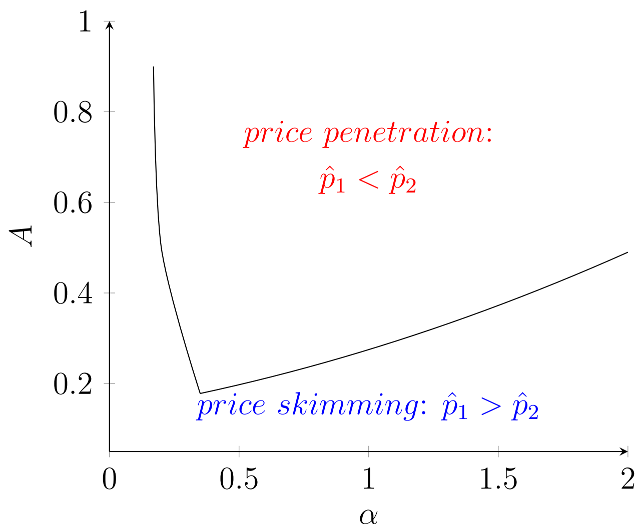

Figure 6: Optimal pricing policy for various combinations of α and A. For all levels of development costs A except sufficiently low, the platform alternates between price skimming and price penetration. For low enough development costs A, price skimming is optimal.

In contrast, if the cost of development of the complementary good is relatively high, the complementary good is less likely to be developed. Then the platform might not charge a high price before uncertainty is realised and, as a result, price penetration policy might become optimal. The magnitude of the indirect effect depends on the equilibrium probability that the complementary good is successfully developed, $\hat{P}$, which, as we proved above, is increasing in α. Therefore, we expect and indeed find that the platform price skimming ($\hat{p}_1 > \hat{p}_2$) policy if the value of the complementary good is relatively small (high), i.e., $\hat{p}_2 < \hat{p}_1$ for small α and $\hat{p}_2 > \hat{p}_1$ as α becomes higher. We illustrate this price dynamics in Figure 7, where $\hat{p}_2 < \hat{p}_1$ for $\alpha < 0.3$ and $\hat{p}_2 > \hat{p}_1$ if $\alpha \geq 0.3$.

Interestingly, we also find another effect. When α becomes sufficiently large ($\alpha \geq 2$ in Figure 7), it is again optimal to use price penetration ($\hat{p}_1 < \hat{p}_2$). Intuitively, if the benefit of the complementary is large, it will be successfully developed with high probability. Since the complementary good is very likely to succeed, consumers who join the platform at $t = 1$ are almost certain to benefit from the indirect network effect. The platform is then better off by extracting this surplus form the option value earlier. Note that this is the case if consumers value the complementary good a lot. For example, gamers may buy a video game console to play a particular game: Soccer fans may buy PlayStation to enjoy FIFA 19.[33]

[33] Another example is a consumer who buys a smartphone to mostly use a particular app (Uber or Lyft).

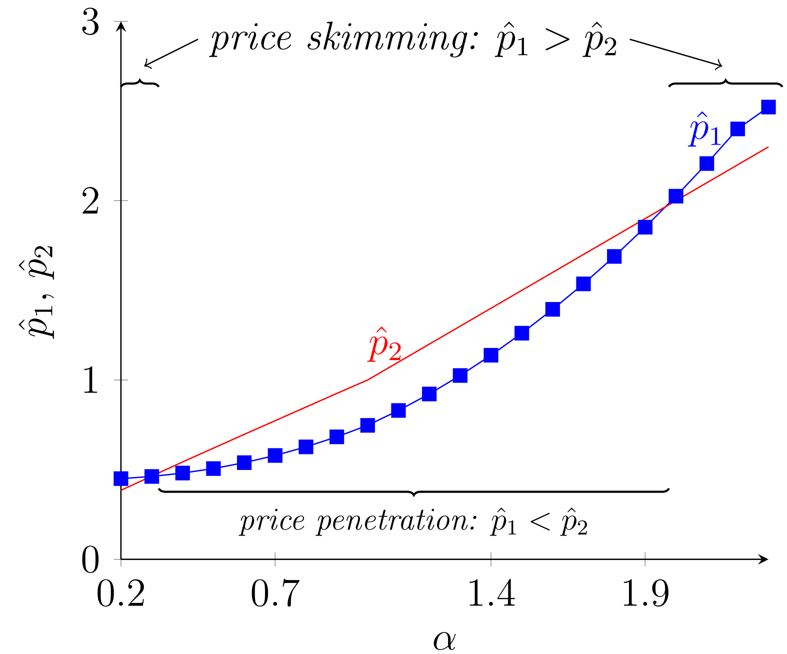

Figure 7: Optimal prices if $A = \frac{1}{2}$.

We summarize all the results in Proposition 2 below.

Proposition 2. The Optimal Pricing Policy.

- $\hat{p}_1 = \hat{P}\alpha + \hat{P}(\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S) + (2 - \hat{P})\hat{N}_1(1 - \hat{N}_1)$,
- $\hat{p}_2 = \alpha + (\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S)$,
- $\hat{b} = \dfrac{\alpha(\hat{N}_1 + \hat{N}_2^S) + (\hat{N}_1 + \hat{N}_2^S)^2(1 - \hat{N}_1 - \hat{N}_2^S) - \hat{N}_1(1 - \hat{N}_1)}{2}$,
- where $\hat{P} = min\left\{\frac{\alpha(\hat{N}_1+\hat{N}_2^S)+(\hat{N}_1+\hat{N}_2^S)^2(1-\hat{N}_1-\hat{N}_2^S)-\hat{N}_1(1-\hat{N}_1)}{4A}, 1\right\}$

Proof: See Appendix.

5 Extensions and Discussion

5.1 Equilibrium without Commitment

In this section, we remove the possibility for the platform to commit to a future price. Therefore, we now require the equilibrium to be subgame perfect.[34] This implies that the

[34]In particular, we are looking for subgame perfect Nash equilibrium. See (Hagiu, 2006) for the details.

platform chooses price p_2 *after* observing a mass of consumers N_1, and this choice must be optimal at that point in time. For consumers, it is also common knowledge at $t = 1$ that the price p_2 is optimally chosen after N_1 has been observed by the platform (given the remaining potential consumer base).

To see what effects an absence of commitment leads to, recall the trade-off the platform is facing when determining the optimal pricing policy.[35] In order to attract enough consumers at $t = 1$, the platform has to "threaten" potential consumers with a higher price in the future.[36] This is exactly what our main model predicts for intermediate values of α (see Figure 6). Commitment to a higher future price p_2 is, therefore, critical for building a necessary mass of consumers at $t = 1$.[37]

We characterize the equilibrium using backward induction. First, given a mass of consumers N_1, we determine the optimal mass of consumers the platform attracts at $t = 2$. Since in equilibrium all consumers will be divided in (at most) three groups, the second period price has to satisfy

$$1 - N_1 - N_2^S = \tfrac{p_2-\alpha}{N_1+N_2^S}.$$[38]

Therefore, at $t = 2$ the platform is choosing $N_2^S(N_1)$ to maximize the second period profit

$$\pi_2 \equiv P \times N_2^S \times p_2 = P \times N_2^S \times \big(\alpha + (N_1 + N_2^S)(1 - N_1 - N_2^S)\big)$$

Consequently, given any N_1, the platform optimally attracts $\hat{N}_2^S(N_1)$ customers at $t = 2$, where

$$\hat{N}_2^S(N_1) = min\left\{\tfrac{1-2N_1+\sqrt{N_1^2-N_1+1+3\alpha}}{3}, 1 - N_1\right\}.$$[39]

[35]In a model with a durable good but without a network effect, a monopolist chooses to lower its price in the second period in order to sell to the residual consumers. This causes the so-called "Coase effect."

[36]Alternatively, a smaller price at $t = 1$, p_1, allows building an early mass of consumers, N_1.

[37]Commitment is critical if $p_2 < p_1$ as well. This is the case when the benefit of the complementary good, α (and, consequently, $\hat{P}$) is sufficiently high. Now the platform commits to make $p_2 < p_1$ in the future, but keep p_2 as close to p_1 as possible.

[38]This follows directly from the fact that, in equilibrium, $\theta < \frac{p_2-\alpha}{N_1+N_2^S}$ never join the platform, $\frac{p_2-\alpha}{N_1+N_2^S} \leq \theta < \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t = 2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t = 1$ (see Figure 5).

[39]The First Order Condition is

$$\alpha + N_1 - N_1^2 - N_1N_2^S + N_2^S - N_1N_2^S - (N_2^S)^2 + N_2^S - 2N_1N_2^S - 2(N_2^S)^2 = 0,$$

which leads to a quadratic equation $3(N_2^S)^2 + (4N_1 - 2)N_2^S + N_1^2 - N_1 + \alpha = 0$.
The second order condition requires $N_1 + N_2^S(N_1) \leqslant 1$ for any $0 \leqslant N_1 \leqslant 1$.

Given the expression for $\hat{N}_2^S(N_1)$, the platform chooses N_1 and P at $t = 1$ (such that $0 \leq \hat{N}_1 + \hat{N}_2^S(\hat{N}_1) \leq 1$) to maximize the total expected profit profit

$$p_1 \times N_1 + P \times \hat{N}^S(N_1) \times p_2 - Pp^S =$$
$$P(N_1 + \hat{N}_2^S(N_1))\big(\alpha + (N_1 + \hat{N}_2^S(N_1))(1 - N_1 - \hat{N}_2^S(N_1))\big) + (2 - P)N_1^2(1 - N_1) - Pc'(P).^{40}$$

While we delegate the formal analysis to the Appendix, we use a numerical example to illustrate the benefit of commitment for the platform. Suppose $\alpha = 1$ and $A = \frac{1}{2}$, then the main model with commitment predicts that $\hat{N}_1 = \frac{2}{3}$, $\hat{N}_2 = \frac{1}{3}$, and $\hat{p}_2 = 1 > \hat{p}_1 = 0.747$ and the total profit is $\pi^S = 0.6276$. In absence of commitment, $\hat{N}_1 = 0.6$, $\hat{N}_2 = 0.4$, and $\hat{p}_2 = 0.79 > \hat{p}_1 = 0.69$ and the total profit is $\pi^S = 0.5935$.

Using the example above, we emphasize that the benefit of commitment is two-fold. First, commitment allows building a larger early consumer base at $t = 1$ (in the example, at $t = 1$ the platform attracts $\frac{2}{3}$ of total consumers with commitment and only 0.6 in absence of commitment).[41] More generally, to see how an absence of commitment alternates the platform's incentives in building a consumer base, recall than with commitment (see Proposition 1) $N_1 = \frac{2}{3}$ and $\hat{N}_2^S = min\{\frac{\sqrt{1+3\alpha}-1}{3}, \frac{1}{3}\}$. Next, for any $\alpha \geqslant 0$, it is the case that

$$\hat{N}_2^S(\tfrac{2}{3}) = min\left\{\frac{1-2\times\frac{2}{3}+\sqrt{(\frac{2}{3})^2-\frac{2}{3}+1+3\alpha}}{3}, 1-\frac{2}{3}\right\} \geqslant min\{\frac{\sqrt{1+3\alpha}-1}{3}, \frac{1}{3}\}.$$

Therefore, with commitment the platform attracts (weakly) fewer consumers at $t = 2$ than in absence of commitment (after attracting $N_1 = \frac{1}{3}$ at $t = 2$).

Second, commitment to a higher price in the future allows charging a higher price at $t = 1$ (in the example, at $t = 1$ the platform charges $\hat{p}_1 = 0.747$ with commitment and only $\hat{p}_1 = 0.69$ in the absence of commitment). Since more consumers buy the basic good at $t = 1$, the platform might charge a higher initial price p_1 due to the direct network effect.

5.2 Type-Dependent Externality, $\alpha(\theta) = \alpha\theta$.

We now consider a modification of the main model and allow the benefit of the complementary good to be a function of consumer type, i.e., $\alpha(\theta) = \alpha\theta$ for each $\theta \in [0, 1]$, where $\alpha > 0$. For example, an additional app released for an iPhone might benefit more those consumers

[40] The expression for p_1 is given by $p_1 = P\alpha + P(N_1 + N_2^S)(1 - N_1 - N_2^S) + (2 - P)N_1(1 - N_1)$, and $b = \frac{dc(P)}{dP}$ is derived from the seller's optimization problem.
[41] Recall that establishing the early consumer base allows the platform to extract a higher surplus in the future due to the direct network effect.

who already use iPhone itself more often.[42] The equilibrium is derived formally in Appendix, and the key results are presented in Proposition 3. We provide the intuition below.

We begin the analysis by describing the equilibrium mass of consumers $\hat{N}_1$, $\hat{N}_2^S$, and $\hat{N}_2^F$. Similarly to the main model (see Proposition 1), it is the case that $\hat{N}_2^F = 0$. However, a novel result of this section is that if the benefit of the complementary good depends on a consumer type, the platform prefers not to sell the basic good at $t = 2$. Therefore, in equilibrium all consumers will be divided into two groups only: 1) "high-type" consumers those who join the platform before observing whether the complementary good succeeds (at $t = 1$), and 2) "low-type" consumers who never join the platform (see Figure 8 below).

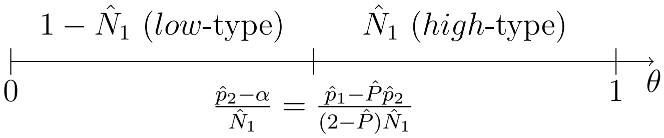

Figure 8: Consumer's decision to join the platform if $\alpha(\theta) = \alpha\theta > 0$.

Intuitively, if the benefit of the complementary good depends on consumer type, the platform cannot charge sufficiently high price at $t = 2$. Recall that consumers who wait and join the platform at $t = 2$ are not the highest types. Therefore, price p_2 must be small enough to attract at least some of them. In addition, $\alpha(\theta) = \alpha\theta$ imposes an upper bound on p_2.[43] As a result, it is not optimal for the platform to sell the basic good at $t = 2$ anymore. We summarize results in Proposition 3 below.

Proposition 3. **Optimal Timing of Joining the Platform if** $\alpha(\theta) = \alpha\theta$.

$$\hat{N}_2^F = \hat{N}_2^S = 0 < \hat{N}_1.$$

Proof: See Appendix.

5.3 General Utility Function

To focus on characterizing the effect of technological uncertainty, we have assumed that the utility function is linear in both consumer type and the current consumer base. We should note that our main results might be extended to a more general utility function that might be relevant in other applications. For example, one may assume that the marginal utility of

[42] Another example might be video game platforms, such as Atari, Nintendo, Sega, Sony Play Station, and Microsoft X-Box. In this case, an additional benefit from a newly released game for a consumer depends on whether she is a frequent player who enjoys the game console a lot or not.

[43] In the main model where $\alpha(\theta) = \alpha$, the platform can charge $p_2 \geqslant \alpha$.

type θ consumer is a non-linear function of his type. Our main results that all consumers are divided into (at most) three groups remain intact with any more general utility function as long the direct externality is non-negative, i.e., $\frac{\partial u(\theta,N)}{\partial N} \geq 0$. However, a more general utility function will not allow us to derive the explicit characterization of the relevant mass of consumers and corresponding price levels.

5.4 Distribution of Consumer Preferences (*Niche* Markets)

We presented the main model assuming the uniform distribution of consumer type. While considering a mass market allows us to derive the explicit solution, our results give some guidance into possible equilibrium in niche markets. We discuss now implications of our model for various distributions of consumer type.

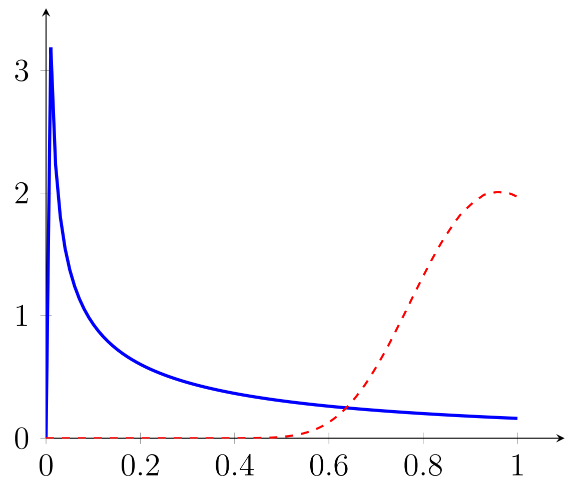

Figure 9: Various distributions of consumer type (niche markets).

First, suppose there are many low-type consumers and higher types are almost uniformly distributed (see the blue solid line in Figure 9). An example might be a low temperature country where only few people find electric cars desirable. This type of distribution makes excluding low-type consumers costlier and, consequently, some of the results from the main model might change. In particular, charging a higher price at $t = 1$ and then lowering it at $t = 2$ may become sub optimal.[44] The reason is that now the platform must make its profit form the low-type consumers. Since charging a higher price earlier can attract only very few high-value consumers, the platform might be better off by lowering the price of the basic good at $t = 1$ in order to establish sufficient consumer base early.

[44]Recall that this was the optimal pricing policy for small and sufficiently high values of α.

Second, suppose consumer type distribution is skewed so that there are many high-type consumers and lower types are almost uniformly distributed (see the red dashed line in Figure 9). An example might be a high temperature country where richer people find electric cars attractive, whereas poorer consumers prefer conventional cars due to cheap gasoline. Then, our intuition and results from the main model still remain intact. In particular, recall that the platform optimally excludes the low value consumers when the value of the complementary good is small (see Proposition 1). This is still optimal since the platform first attracts high-value consumers in order to build an early consumer base. However, when there are many low-type consumers, the platform will find it optimal to charge a relatively smaller price at $t = 2$ to maximize second period profit.

6 Implications and Applications

We now discuss some examples from the empirical literature in light of our model. In particular, we present some evidence that both supports our model and provides an opportunity for explanation. The main testable predictions are generated by Propositions 1 and 2 regarding the optimal timing of joining the platform by consumers and the optimal pricing policy employed by the platform, respectively.

Electric Vehicles. The market for electric vehicles is a case in point. Consider the electric vehicles manufacturers such as Tesla, Toyota, or Renault-Nissan (platform) that must attract consumers who purchase a vehicle with a set of features related both to driving comfort and battery technology, such as, driving range, charging time, and safety. Buyers of electric vehicles express concerns over both battery range and lack of sufficient public charging stations.[45] Given these concerns, we expect that consumers' purchase decisions are not simply a function of the features that an electric vehicle carries at the time of purchase but also any future improvements in the battery technology, availability of charging stations, and even self-driving functionality. The platform must guarantee there is sufficient infrastructure to recharge a vehicle and the battery technology will be developed further. Therefore, companies like Panasonic, LG Chemical, or AESC (seller) must be incentivized to invest in charging stations and battery development. In addition, developers of self-driving functionality (seller) must be incentivized to make significant progress in development in a reasonable time. In this example, the the value of the complementary good (a local charging

[45] See Egbue and Long (2012).

station and a battery) is high relative to the benefit from the basic good (an electric vehicle).

Marketing literature indicates that there is a clear segmentation in the consumer side of the market, and describes the three types of consumers in the electric vehicles market: high-value consumers (e.g., environmentally conscious or technology oriented) who purchase an electric vehicle even before a sufficiently wide network of charging stations is developed, the mass market (consumers who buy an electric vehicle only if the battery performance is sufficiently improved), and a third group who would rather buy a conventional type of car.[46] This is consistent with our model and Proposition 1.

Pricing policies adopted by electric vehicle producers also exhibit features consistent with our model. For example, consider Tesla and its car with full self-driving capabilities. Tesla's cars aren't yet capable of full autonomy, and Elon Musk has said he expects the full set of features to be ready within the next two years. In August 2019, Telsa increased the price of a model with full self-driving capabilities. The company also announced that the price will keep rising as the technology keeps developing further. In April 2019, Alon Musk said the price of full self-driving functionality would "increase substantially over time."[47] This price dynamics is consistent with the optimal pricing policy in Proposition 2 for a high value of the complementary good α and high development costs A. For these parameters, the platform optimally employs price penetration policy and increases the price in the future.

Video Game Consoles. The video game consoles is another example of a two-sided market that exhibits features that can be described with our model. The value of a particular game console (platform) to gamers (consumers) depends on both the number of other gamers who play on the same platform and the number of game titles developed by the third-party developers. Therefore, a video game console is a two-sided market that is characterized by both direct and indirect network externalities.

For a game console producer, it is crucial to build and maintain a gamer base. This is not achieved simply by introducing technologically superior consoles but through a combination of attracting developers and building and managing gamers' expectations about both the number of other gamers and future games that will be offered. The strategy to build and maintain the gamer base is to manage gamers' expectations. When gamers choose a particular console, they not only consider the platform technology, i.e., speed and quality

[46]See McKinsey Global Survey (2017) and references therein.

[47]"As mentioned earlier this year, cost of the Tesla FSD option will increase every few months. Those who buy it earlier will see the benefit," Musk wrote. See https://www.cnbc.com/2019/07/16/tesla-cuts-price-on-model-3-discontinues-versions-of-model-s-model-x.html.

of graphics, but also the plethora of current and future games that will be available on the platform.[48]

Pricing policies adopted by video game console producers are in agreement with predictions of our model. It is typical for video game consoles to become cheaper as more games are developed. Examples are abundant. This was the case for Atari Jaguar released by Atari Corporation in 1993, for Sega Saturn released in 1993, Sony PlayStation introduced in 1995, and many others.[49] This price dynamics is consistent with the optimal pricing policy in Proposition 2 for relatively low development costs A when the platform optimally employs a price skimming policy and lowers the price in the future.

Media Service Providers. Another example we discuss is a media service provider. Consider companies providing streaming services such as HBO, Netflix, or Amazon Prime (platform) that must attract both subscribers (consumers) and developers of visual and audio content (seller). This business model exhibit features that can be described with our model too. There are both direct and indirect positive network effects. Subscribers benefit more from the service when more of their friends and acquaintances watch the same movies and TV series. At the same time, if a platform manages to establish a large subscriber base, visual and audio content producers are more likely to develop the content consistent with the preferences of the platform's clientele.

Our model predicts that the platform's optimal price path can be ascending or descending over time, depending on critical parameters of the model (the value of the complementary good α and development costs A). Netflix pricing policy is an example of an increasing price over time. The platform has steadily increased its standard and premium subscription fees. The standard subscription fee has reached 12.99$/m in 2019, stating from 8.99$/m in 2010, equivalent of a total 44% increase over the past 9 years. The company's biggest price increase happened in Jan 2019, going up from 10.99$/m to 12.99$/m.[50] Consistent with our model, a recent New York Times article mentions that

> *"Spending big on content while keeping prices modest has helped Netflix expand its customer base, about 58 million in the United States and 130 million worldwide."*

[48]Platforms often "vaporware" and spend millions of dollars to build an image of large gamer base and more game titles to appear in the future. For instance, Microsoft allocated a $500 million advertising budget to build an impression of large gamer base (Schilling, 2003).

[49]See Cool and Paranikas (2008) and references therein.

[50]See https://www.usatoday.com/story/money/2019/07/10/has-a-netflix-subscription-finally-gotten-too-expensive/39669723/ and https://www.bbc.com/news/newsbeat-48458598.

While the company was suffering from negative cash-flow in previous years, it did not increase prices much until a large subscribers base was established.[51]

7 Conclusion

This paper examines the effect of technological uncertainty in a two-sided market where the platform offers a basic good and the developer offers a complementary good. The development of the complementary good is stochastic and is successful with some probability only. The early buyers obtain the basic product as well as a real option to benefit from possible enhanced functionality in the future. The platform provides incentives to both consumers and the developer by choosing the optimal dynamic pricing policy.

We show that consumers are divided into at most three groups of early adopters, late adopters, and those who never joiner the platform. We find that the mass of early adopters does not depend on the value of the complementary good, whereas the mass of late adopters increases if the value of the complementary good goes up. When the value of the complementary good is sufficiently large, the platform serves the whole market.

The platform's pricing policy is determined by the value of the complementary good as well as the costs of its development. If the development costs are sufficiently low, then the platform follows a price skimming policy and charges a higher price for those consumers joining the platform earlier. For higher development costs, the platform alternates between price skimming and price penetration. In this case, the seller has less incentives to engage in development of the complementary good and thus the probability of success is also lower. In order to the make development more attractive for the seller, the platform might employ a price penetration strategy and reduces the first period prices.

[51] See https://www.nytimes.com/2019/01/15/business/media/netflix-price-increase.html.

8 Appendix

8.1 The Main Model ($\alpha(\theta) = \alpha > 0$).

Proof of Propositions 1 and 2.

We now characterize the equilibrium if $\alpha(\theta) = \alpha > 0$ for all θ. First, we study the optimal timing of joining the platform for each consumer type for given prices p_1 and p_2, the probability of success P, and N_1, N_2^F, and N_2^S.

we study the sellerâs optimal choice of the success probability for a given reward b. Next, in Section 4.2, Finally, in Section 4.3, we solve for the platformoptimal pricing policy and the corresponding mass of consumers who join the platform atdifferent times in equilibrium.

We first determine the consumer's optimal decision when to join the platform for given N_1, N_2^F, and N_2^S. Then, we solve the platform's optimization problem and determine the equilibrium values of $\hat{N}_1$, $\hat{N}_2^F$, and $\hat{N}_2^S$.

Consumers. Given N_1, N_2^F, and N_2^S, a type θ consumer gets an expected utility from joining the platform at $t = 1$ and waiting till $t = 2$, U_1 and U_2, respectively, that are determined as follows.

$$U_1(\theta) = \theta N_1 - p_1 + \Big[P \times \big(\theta(N_1 + N_2^S) + \alpha\big) + (1 - P) \times \theta(N_1 + N_2^F)\Big], \text{ and}$$

$$U_2(\theta) = P \times max\{\theta(N_1 + N_2^S) + \alpha - p_2, 0\} + (1 - P) \times max\{\theta(N_1 + N_2^F) - p_2, 0\}.$$

Consider first a consumer of type θ at period $t = 2$ who did not join the platform at $t = 1$. We prove next that there are three relevant types of consumers who differ in their optimal decision at $t = 2$. First, a type θ consumer might find it optimal to join the platform even after the complementary good fails. This is the case if their utility is non-negative:

$$\theta(N_1 + N_2^F) - p_2 \geq 0.$$

We denote

$$\overline{\theta} \equiv \tfrac{p_2}{N_1 + N_2^F}.$$

Consequently, if $\theta \geq \overline{\theta}$, then

$$max\{\theta(N_1 + N_2^S) + \alpha - p_2, 0\} = \theta(N_1 + N_2^S) + \alpha - p_2, \text{ and}$$

$$max\{\theta(N_1 + N_2^F) - p_2, 0\} = \theta(N_1 + N_2^F) - p_2.$$

Therefore, for $\theta \geq \overline{\theta}$ types, the expected utility of waiting till $t = 2$ becomes

$$U_2(\theta) = P \times \big(\theta(N_1 + N_2^S) + \alpha - p_2\big) + (1 - P) \times \big(\theta(N_1 + N_2^F) - p_2\big) =$$

$$\theta\big(N_1 + PN_2^S + (1 - P)N_2^F\big) + P\alpha - p_2.$$

Second, a type θ consumer might find it optimal to join the platform only if the complementary good is successfully developed. This is the case if both $i)$ their utility from joining the platform after success is non-negative and $ii)$ their utility from joining the platform after failure is negative:

$$i)\ \theta(N_1 + N_2^S) + \alpha - p_2 \geq 0 \text{ and}$$

$$ii)\ \theta(N_1 + N_2^F) - p_2 < 0.$$

We denote

$$\underline{\theta} \equiv \tfrac{p_2 - \alpha}{N_1 + N_2^S}.$$

Consequently, if $\underline{\theta} \leq \theta < \overline{\theta}$, then

$$max\{\theta(N_1 + N_2^S) + \alpha - p_2, 0\} = \theta(N_1 + N_2^S) + \alpha - p_2, \text{ and}$$

$$max\{\theta(N_1 + N_2^F) - p_2, 0\} = 0.$$

Therefore, for $\underline{\theta} \leq \theta < \overline{\theta}$ types, the expected utility of waiting till $t = 2$ becomes

$$U_2(\theta) = P \times \big(\theta(N_1 + N_2^S) + \alpha - p_2\big) + (1 - P) \times \big(\theta(N_1 + N_2^F) - p_2\big) = P \times \big(\theta(N_1 + N_2^S) + \alpha - p_2\big).$$

Third, a type θ consumer might find it optimal never to join the platform. This is the case if their utility from joining the platform even after success is positive:

$$\theta(N_1 + N_2^S) + \alpha - p_2 < 0.$$

Consequently, if $\theta < \underline{\theta}$, then

$$max\{\theta(N_1 + N_2^S) + \alpha - p_2, 0\} = max\{\theta(N_1 + N_2^F) - p_2, 0\} = 0.$$

Therefore, for $\theta < \underline{\theta}$ types, the expected utility of waiting till $t = 2$ becomes

$$U_2(\theta) = P \times \big(\theta(N_1 + N_2^S) + \alpha - p_2\big) + (1 - P) \times \big(\theta(N_1 + N_2^F) - p_2\big) = 0.$$

We can now rewrite the expected utility of waiting till $t = 2$ as

$$\mathrm{U}_2(\theta)=\begin{cases}0, & \theta<\underline{\theta}\\ P\big(\theta(N_1+N_2^S)+\alpha-p_2\big), & \underline{\theta}\le\theta<\overline{\theta}\\ \theta\big(N_1+PN_2^S+(1-P)N_2^F\big)+P\alpha-p_2, & \theta\ge\overline{\theta}\end{cases}$$

A consumer of type θ joins the platform at $t=1$ if

$$U_1(\theta)\ge U_2(\theta).$$

Given the expression for $U_2(\theta)$ derived above, condition $U_1(\theta)\ge U_2(\theta)$ can be rewritten as

1. $\theta N_1-p_1+P\big(\theta(N_1+N_2^S)+\alpha\big)+(1-P)\theta(N_1+N_2^F)\ge 0$ if $\theta<\underline{\theta}$,
2. $\theta N_1-p_1+P\big(\theta(N_1+N_2^S)+\alpha\big)+(1-P)\theta(N_1+N_2^F)\ge P\big(\theta(N_1+N_2^S)+\alpha-p_2\big)$ if $\underline{\theta}\le\theta<\overline{\theta}$,
3. $\theta N_1-p_1+P\big(\theta(N_1+N_2^S)+\alpha\big)+(1-P)\theta(N_1+N_2^F)\ge\theta\big(N_1+PN_2^S+(1-P)N_2^F\big)+P\alpha-p_2$ if $\theta\ge\overline{\theta}$.

The condition $U_1(\theta)\ge U_2(\theta)$ finally simplifies to:

1. $\theta\ge\frac{p_1-p_2}{N_1}$ if $\theta\ge\overline{\theta}=\frac{p_2}{N_1+N_2^F}$,
2. $\theta\ge\frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ if $\frac{p_2-\alpha}{N_1+N_2^S}=\underline{\theta}\le\theta<\overline{\theta}=\frac{p_2}{N_1+N_2^F}$,
3. $\theta\ge\frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$ if $\theta<\underline{\theta}=\frac{p_2-\alpha}{N_1+N_2^S}$.

We can now describe the optimal timing of joining the platform for a type θ consumer (see Figure 3 in the main part of the paper):

1. never join the platform if $\theta\leqslant\frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$,
2. join the platform at $t=1$ if $\frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}\leqslant\theta\leqslant\underline{\theta}$,
3. join the platform at $t=2$ if the complementary good is successfully developed if $\underline{\theta}\leqslant\theta\leqslant\frac{p_1-P\alpha}{(2-P)N_1+(1-P)N_2^F}$,
4. join the platform at $t=1$ if $\frac{p_1-P\alpha}{(2-P)N_1+(1-P)N_2^F}\leqslant\theta\leqslant\overline{\theta}$,
5. join the platform at $t=2$ (regardless of success/failure) if $\overline{\theta}\leqslant\theta\leqslant\frac{p_1-p_2}{N_1}$,
6. join the platform at $t=1$ if $\frac{p_1-p_2}{N_1}\leqslant\theta\leqslant 1$.

We determine next which of these intervals for θ are relevant in equilibrium. That is, we determine which of the inequalities above are mutually exclusive, i.e., cannot be satisfied simultaneously. In particular, we will prove that in equilibrium type $\theta < \frac{p_2-\alpha}{N_1+N_2^S}$ never join the platform, $\frac{p_2-\alpha}{N_1+N_2^S} \leq \theta < \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=1$.

We introduce the following three conditions $(C1)$, $(C2)$, and $(C3)$:

- $(C1)$: $p_2 < \frac{N_1+N_2^F}{2N_1+N_2^F}p_1 \Leftrightarrow \frac{p_1-p_2}{N_1} > \frac{p_2}{N_1+N_2^F}$,
- $(C2)$: $p_2 < \frac{N_1+N_2^S}{2N_1+PN_2^S+(1-P)N_2^F}p_1 + \alpha\frac{(2-P)N_1+(1-P)N_2^F}{2N_1+PN_2^S+(1-P)N_2^F} \Leftrightarrow \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F} > \frac{p_2-\alpha}{N_1+N_2^S}$,
- $(C3)$: $p_2 > \frac{N_1+N_2^S}{2N_1+PN_2^S+(1-P)N_2^F}p_1 + \alpha\frac{(2-P)N_1+(1-P)N_2^F}{2N_1+PN_2^S+(1-P)N_2^F} \Leftrightarrow \frac{p_2-\alpha}{N_1+N_2^S} > \frac{p_1-I\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$.

Since conditions $(C2)$ and $(C3)$ are mutually exclusive, there are 4 relevant cases.

Case A. $\theta < \frac{p_2-\alpha}{N_1+N_2^S}$ never join the platform, $\frac{p_2-\alpha}{N_1+N_2^S} \leq \theta < \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=2$ only if the complementary good is successfully developed, $\frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F} \leq \theta < \frac{p_2}{N_1+N_2^F}$ and $\theta \geq \frac{p_1-p_2}{N_1}$ join the platform at $t=1$, $\frac{p_2}{N_1+N_2^F} \leq \theta < \frac{p_1-p_2}{N_1}$ join the platform at $t=2$ even if the complementary good fails.

This case cannot be sustained in equilibrium. Since $\frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F} \leq \theta < \frac{p_2}{N_1+N_2^F}$ find it optimal to join the platform at $t=1$, the same must be true for higher types $\frac{p_2}{N_1+N_2^F} \leq \theta < \frac{p_1-p_2}{N_1}$. Therefore, there is a contradiction.

Case B. $\theta < \frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$ never join the platform, $\frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F} \leq \theta < \frac{p_2}{N_1+N_2^F}$ and $\theta \geq \frac{p_1-p_2}{N_1}$ join the platform at $t=1$, and $\frac{p_2}{N_1+N_2^F} \leq \theta < \frac{p_1-p_2}{N_1}$ join the platform at $t=2$ only if the complementary good is successfully developed.

This case cannot be sustained in equilibrium either. The reason is that in this case $N_2^F = N_2^S$ and, therefore, $(C1)$ $p_2 < \frac{N_1+N_2^F}{2N_1+N_2^F}$ contradicts $(C3)$ $p_2 > \frac{N_1+N_2^S}{2N_1+PN_2^S+(1-P)N_2^F}p_1 + \alpha\frac{(2-P)N_1+(1-P)N_2^F}{2N_1+PN_2^S+(1-P)N_2^F}$.

Case C. $\theta < \frac{p_2-\alpha}{N_1+N_2^S}$ never join the platform, $\frac{p_2-\alpha}{N_1+N_2^S} \leq \theta < \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=1$.

Case D. $\theta < \frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$ never join the platform, and $\theta \geq \frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$ join the platform at $t=1$.

Note that case C also includes C *Case D* if $\frac{p_2-\alpha}{N_1+N_2^S} = \frac{p_1-P\alpha}{2N_1+PN_2^S+(1-P)N_2^F}$. Therefore, it is without loss of generality to consider case C only. We prove next that in equilibrium all consumers are divided in at most three groups according to their types.

Proposition 1. $\hat{N}_1 = \frac{2}{3}$, $\hat{N}_2^F = 0$, $\hat{N}_2^S = min\{\frac{\sqrt{1+3\alpha}-1}{3}, \frac{1}{3}\}$.

Proof: If $\theta < \frac{p_2-\alpha}{N_1+N_2^S}$ never join the platform, $\frac{p_2-\alpha}{N_1+N_2^S} \le \theta < \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=2$ only if the complementary good is successfully developed, and $\theta \ge \frac{p_1-Pp_2}{(2-P)N_1+(1-P)N_2^F}$ join the platform at $t=1$, then

- $N_2^F = 0$,
- $1 - N_1 - N_2^S = \frac{p_2-\alpha}{N_1+N_2^S}$,
- $1 - N_1 = \frac{p_1-Pp_2}{(2-P)N_1}$.

We can express prices p_1 and p_2 from the two equations $1 - N_1 - N_2^S = \frac{p_2-\alpha}{N_1+N_2^S}$ and $1 - N_1 = \frac{p_1-Pp_2}{(2-P)N_1}$ as follows

$$p_2(P, N_1, N_2^S) = \alpha + (N_1 + N_2^S)(1 - N_1 - N_2^S), \text{ and}$$
$$p_1(P, N_1, N_2^S) = Pp_2+(2-P)N_1(1-N_1) = P\alpha+P(N_1+N_2^S)(1-N_1-N_2^S)+(2-P)N_1(1-N_1).$$

Therefore, the platform is choosing $0 \le P, N_1, N_2^S \le 1$ such that $N_1+N_2^S \le 1$ to maximize

$$N_1 \times p_1(P, N_1, N_2^S) + PN_2 \times p_2(P, N_1, N_2^S) - P \times b =$$
$$P\alpha(N_1 + N_2^S) + P(N_1 + N_2^S)^2(1 - N_1 - N_2^S) + (2 - P)N_1^2(1 - N_1) - Pc'(P),$$

where $b = \frac{dc(P)}{dP}$ is derived from the seller's optimization problem.

The first order necessary conditions for the optimizations problem are[52]

$[N_1]$: $P\alpha + P\big[2(N_1 + N_2^S)(1 - N_1 - N_2^S) - (N_1 + N_2^S)^2\big] + (2 - P)\big[2N_1(1 - N_1) - N_1^2\big] = 0$;

$[N_2^S]$: $P\alpha + P\big[2(N_1 + N_2^S)(1 - N_1 - N_2^S) - (N_1 + N_2^S)^2\big] = 0$;

$[P]$: $\alpha(N_1 + N_2^S) + (N_1 + N_2^S)^2(1 - N_1 - N_2^S) - N_1(1 - N_1) - c'(P) - Pc''(P) = 0$.

By subtracting $[N_2^S]$ from $[N_1]$, we have

$$2N_1(1 - N_1) - N_1^2 = 0 \implies \hat{N}_1 = \tfrac{2}{3}.$$

Next, $[N_2^S]$ implies

$$\alpha + 2(\tfrac{2}{3} + N_2^S)(1 - \tfrac{2}{3} - N_2^S) - (\tfrac{2}{3} + N_2^S)^2 = 0 \implies N_2^S = \tfrac{\sqrt{1+3\alpha}-1}{3}.$$

Finally, since $\frac{\sqrt{1+3\alpha}-1}{3} > \frac{1}{3}$ if $\alpha > 1$, in equilibrium $\hat{N}_2^S = min\{\frac{1}{3}, \frac{\sqrt{1+3\alpha}-1}{3}\}$. *Q.E.D.*

We next determine the equilibrium prices.

[52]Note that the First Order Conditions are also sufficient since the objective function is concave.

Proposition 2.

$$p_1 = \begin{cases} P\alpha + \frac{2}{9}(2-P), & \alpha \geq 1 \\ P\alpha + P(\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S) + (2-P)\hat{N}_1(1-\hat{N}_1), & \alpha < 1 \end{cases}$$

$$p_2 = \begin{cases} \alpha, & \alpha \geq 1 \\ \alpha + (\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S), & \alpha < 1 \end{cases}$$

Proof: The expressions for the optimal prices directly follow from
$p_2(P, \hat{N}_1, \hat{N}_2^S) = \alpha + (\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S)$, and
$p_1(P, \hat{N}_1, \hat{N}_2^S) = Pp_2 + (2-P)\hat{N}_1(1-\hat{N}_1) = P\alpha + P(\hat{N}_1 + \hat{N}_2^S)(1 - \hat{N}_1 - \hat{N}_2^S) + (2-P)\hat{N}_1(1-\hat{N}_1)$.
Q.E.D.

8.2 The case of no commitment.

If the platform cannot commit to a future price p_2, we must determine a subgame perfect Nash equilibrium.[53] We use backward induction. First, we determine $\hat{N}_2^S(N_1)$, i.e., the optimal consumer mass the platform attracts at $t = 2$ for any given past consumer base N_1. Second, we determine $\hat{N}_1$, i.e., the optimal consumer mass the platform attracts at $t = 1$. Finally, we specify prices the platform chooses in equilibrium.

8.2.1 Optimal choice at t=2.

At $t = 2$ the platform's choice of p_2 must be optimal at that point in time for any given mass of consumers N_1. Following the same steps as in the base model, we establish that in equilibrium all consumers are divided in (at most) three groups. As a result, the second period price has to satisfy

$$1 - N_1 - N_2^S = \frac{p_2 - \alpha}{N_1 + N_2^S}.$$ [54]

Consider the platform choosing optimal price p_2 at $t = 2$. For any given mass of consumers N_1 and past price p_1, the optimal price $\hat{p}_2$ has to maximize the platform's profit at at $t = 2$. The price p_2 can be rewritten from the expression above as $p_2 = (1 - N_1 - N_2^S)(N_1 + N_2^S) + \alpha$.

[53] See (Hagiu, 2006) and references therein for the details.

[54] This follows directly from the fact that, in equilibrium, $\theta < \frac{p_2 - \alpha}{N_1 + N_2^S}$ never join the platform, $\frac{p_2 - \alpha}{N_1 + N_2^S} \leq \theta < \frac{p_1 - Pp_2}{(2-P)N_1 + (1-P)N_2^F}$ join the platform at $t = 2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1 - Pp_2}{(2-P)N_1 + (1-P)N_2^F}$ join the platform at $t = 1$ (see Figure 5).

Therefore, at $t = 2$ the platform is choosing $N_2^S(N_1)$ to maximize the second period expected profit π_2 defined as

$$\pi_2 \equiv P \times N_2^S \times p_2 = P \times N_2^S \times \big(\alpha + (N_1 + N_2^S)(1 - N_1 - N_2^S)\big)$$

The First Order Condition, $\frac{d\pi_2}{dN_2^S} = 0$, can be rewritten as

$$\alpha + N_1 - N_1^2 - N_1 N_2^S + N_2^S - N_1 N_2^S - (N_2^S)^2 + N_2^S - 2N_1 N_2^S - 2(N_2^S)^2 = 0,$$

which leads to a quadratic equation

$$3(N_2^S)^2 + (4N_1 - 2)N_2^S + N_1^2 - N_1 + \alpha = 0,$$

and the relevant root is $\hat{N}_2^S(N_1) = \frac{1-2N_1+\sqrt{N_1^2-N_1+1+3\alpha}}{3}$.[55]

In addition, we have to take into account the second order condition, which requires that for any given N_1, the total mass of consumers who joined the platform by $t = 2$ does not exceed the total feasible consumer base: $N_1 + N_2^S(N_1) \leqslant 1$ for any $0 \leqslant N_1 \leqslant 1$.

Consequently, given any N_1, the platform optimally sells to $\hat{N}_2^S(N_1)$ customers at $t = 2$, where

$$\hat{N}_2^S(N_1) = min\left\{\frac{1-2N_1+\sqrt{N_1^2-N_1+1+3\alpha}}{3}, 1 - N_1\right\}.$$

8.2.2 Optimal choice at t=1.

Given the expression for $\hat{N}_2^S(N_1)$ derived in the previous section, at $t = 1$ the platform is choosing $0 \leq P, N_1 \leq 1$ to maximize the total profit. Since in equilibrium all consumers are divided in (at most) three groups, the first period price has to satisfy

$$1 - N_1 = \frac{p_1 - Pp_2}{(2-P)N_1}.$$ [56]

The price p_1 then can be rewritten from the expression above as

$$p_1 = P\alpha + P(N_1 + N_2^S)(1 - N_1 - N_2^S) + (2 - P)N_1(1 - N_1).$$ [57]

Therefore, at $t = 1$ the platform is choosing $0 \leq P, N_1 \leq 1$ to maximize

$$p_1 \times N_1 + P \times N_2^S \times p_2 - Pb =$$
$$P(N_1 + \hat{N}_2^S(N_1))\big(\alpha + (N_1 + \hat{N}_2^S(N_1))(1 - N_1 - \hat{N}_2^S(N_1))\big) + (2 - P)N_1^2(1 - N_1) - Pc'(P),$$

[55] The second root of the quadratic equation does not belong to the domain, i.e., it violates $0 \leqslant \hat{N}_2^S(N_1) \leqslant 1$ condition.

[56] See Figure 5 in the main section for an illustration.

[57] We also use $p_2 = (1 - N_1 - N_2^S)(N_1 + N_2^S) + \alpha$ from the previous section.

where $b = \frac{dc(P)}{dP}$ is derived from the seller's optimization problem.

The first order necessary conditions are[58]

$[P]$: $\alpha(N_1+\hat{N}_2^S(N_1))+(N_1+\hat{N}_2^S(N_1))^2(1-N_1-\hat{N}_2^S(N_1))-N_1(1-N_1)-c'(P)-Pc''(P)=0$;

$[N_1]$: $P\alpha + P\big[2(N_1+N_2^S)(1-N_1-N_2^S)-(N_1+N_2^S)^2\big]+(2-P)\big[2N_1(1-N_1)-N_1^2\big]=0$.

To see who an absence of alternates the platform's incentives in building a consumer base, recall than with commitment (see Proposition 1) $N_1 = \frac{2}{3}$ and $\hat{N}_2^S = min\{\frac{\sqrt{1+3\alpha}-1}{3}, \frac{1}{3}\}$. Next, for any $\alpha \geqslant 0$, it is the case that

$$\hat{N}_2^S(\tfrac{2}{3}) = min\left\{\frac{1-2\times\frac{2}{3}+\sqrt{(\frac{2}{3})^2-\frac{2}{3}+1+3\alpha}}{3}, 1-\frac{2}{3}\right\} \geqslant min\{\tfrac{\sqrt{1+3\alpha}-1}{3}, \tfrac{1}{3}\}.$$

Therefore, with commitment the platform attracts (weakly) fewer consumers at $t = 2$ than in absence of commitment (after attracting $N_1 = \frac{2}{3}$ at $t = 1$).

8.3 Type-Dependent Externality, $\alpha(\theta) \equiv \alpha\theta > 0$.

Proof of Proposition 3.

Suppose $\alpha(\theta) = \alpha\theta$, where $\alpha > 0$. We first determine the consumer's optimal decision when to join the platform for given N_1, N_2^F, and N_2^S. Then, we solve the platform's optimization problem and determine the equilibrium values of $\hat{N}_1$, $\hat{N}_2^F$, and $\hat{N}_2^S$.

Given N_1, N_2^F, and N_2^S, the type θ consumer gets utility from joining the platform at $t = 1$ and waiting till $t = 2$, U_1 and U_2, respectively, that are determined as follows.

$$U_1(\theta) = \theta N_1 - p_1 + \Big[P \times \big(\theta(N_1+N_2^S)+\alpha\theta\big) + (1-P)\times\theta(N_1+N_2^F)\Big], \text{ and}$$
$$U_2(\theta) = P \times max\{\theta(N_1+N_2^S)+\alpha\theta-p_2, 0\} + (1-P)\times max\{\theta(N_1+N_2^F)-p_2, 0\}.$$

Consider first a consumer of type θ at period $t = 2$ who did not join the platform at $t = 1$. There are three types of customers that differ in their optimal decision. First, the consumer of type θ might find it optimal to join the platform even after the complementary good fails. This is the case if $\theta(N_1+N_2^F) - p_2 \geq 0$. We denote

$$\overline{\theta} \equiv \tfrac{p_2}{N_1+N_2^F}.$$

Consequently, if $\theta \geq \overline{\theta}$, then

$$max\{\theta(N_1+N_2^S)+\alpha-p_2, 0\} = \theta(N_1+N_2^S)+\alpha\theta-p_2, \text{ and}$$
$$max\{\theta(N_1+N_2^F)-p_2, 0\} = \theta(N_1+N_2^F)-p_2.$$

[58]Similarly to the main model, the First Order Conditions are also sufficient since the objective function is concave.

Therefore, if $\theta \geq \overline{\theta}$, then

$$U_2(\theta) = P \times \big(\theta(N_1 + N_2^S) + \alpha\theta - p_2\big) + (1-P) \times \big(\theta(N_1 + N_2^F) - p_2\big) =$$
$$\theta\big(N_1 + PN_2^S + (1-P)N_2^F\big) + P\alpha\theta - p_2.$$

Second, the consumer of type θ might find it optimal to join the platform if and only if the complementary good is successfully developed. This is the case if $\theta(N_1 + N_2^S) + \alpha\theta - p_2 \geq 0$ and $\theta(N_1 + N_2^F) - p_2 < 0$. We denote

$$\underline{\theta} \equiv \tfrac{p_2}{N_1 + N_2^S + \alpha}.$$

Consequently, if $\underline{\theta} \leq \theta < \overline{\theta}$, then

$$max\{\theta(N_1 + N_2^S) + \alpha\theta - p_2, 0\} = \theta(N_1 + N_2^S) + \alpha\theta - p_2, \text{ and } max\{\theta(N_1 + N_2^F) - p_2, 0\} = 0.$$

Therefore, if $\underline{\theta} \leq \theta < \overline{\theta}$, then

$$U_2(\theta) = P\times\big(\theta(N_1+N_2^S)+\alpha\theta-p_2\big)+(1-P)\times\big(\theta(N_1+N_2^F)-p_2\big) = P\times\big(\theta(N_1+N_2^S)+\alpha\theta-p_2\big).$$

Third, the consumer of type θ might find it optimal never to join the platform. This is the case if $\theta(N_1 + N_2^S) + \alpha - p_2 < 0$. Consequently, if $\theta < \underline{\theta}$, then

$$max\{\theta(N_1 + N_2^S) + \alpha\theta - p_2, 0\} = max\{\theta(N_1 + N_2^F) - p_2, 0\} = 0.$$

Therefore, if $\theta < \underline{\theta}$, then

$$U_2(\theta) = P \times \big(\theta(N_1 + N_2^S) + \alpha\theta - p_2\big) + (1-P) \times \big(\theta(N_1 + N_2^F) - p_2\big) = 0.$$

To sum this up, $U_2(\theta)$ can be rewritten as

$$U_2(\theta) = \begin{cases} 0, & \theta < \underline{\theta} \\ P\big(\theta(N_1 + N_2^S + \alpha) - p_2\big), & \underline{\theta} \leq \theta < \overline{\theta} \\ \theta\big(N_1 + PN_2^S + (1-P)N_2^F + P\alpha\big) - p_2, & \theta \geq \overline{\theta} \end{cases}$$

A consumer of type θ joins the platform at $t = 1$ if $U_1(\theta) \geq U_2(\theta)$:
$\theta N_1 - p_1 + P\big(\theta(N_1 + N_2^S) + \alpha\theta\big) + (1-P)\theta(N_1 + N_2^F) \geq 0$ if $\theta < \underline{\theta}$,
$\theta N_1 - p_1 + P\big(\theta(N_1 + N_2^S) + \alpha\theta\big) + (1-P)\theta(N_1 + N_2^F) \geq P\big(\theta(N_1 + N_2^S + \alpha) - p_2\big)$ if $\underline{\theta} \leq \theta < \overline{\theta}$,
$\theta N_1 - p_1 + P\theta(N_1 + N_2^S + \alpha) + (1-P)\theta(N_1 + N_2^F) \geq \theta\big(N_1 + PN_2^S + (1-P)N_2^F + P\alpha\big) - p_2$ if $\theta \geq \overline{\theta}$.

Thus, $U_1(\theta) \geq U_2(\theta)$ is equivalent to:
$\theta \geq \frac{p_1 - p_2}{N_1}$ if $\theta \geq \overline{\theta} = \frac{p_2}{N_1 + N_2^F}$,

$\theta \geq \frac{p_1 - Pp_2}{(2-P)N_1 + (1-P)N_2^F}$ if $\frac{p_2}{N_1 + N_2^S + \alpha} = \underline{\theta} \leq \theta < \overline{\theta} = \frac{p_2}{N_1 + N_2^F}$,
$\theta \geq \frac{p_1}{2N_1 + PN_2^S + (1-P)N_2^F + P\alpha}$ if $\theta < \underline{\theta} = \frac{p_2}{N_1 + N_2^S + \alpha}$.

Repeating the same steps as in case of type-independent externality (see the Proof of Proposition 1), it is straightforward that $N_2^F = 0$ and the only relevant case is $\theta < \frac{p_2}{N_1 + N_2^S + \alpha}$ never join the platform, $\frac{p_2}{N_1 + N_2^S + \alpha} \leq \theta < \frac{p_1 - Pp_2}{(2-P)N_1}$ join the platform at $t = 2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1 - Pp_2}{(2-P)N_1}$ join the platform at $t = 1$. We now determine the equilibrium types of consumers who join the platform at $t = 1$ and at $t = 2$ after the complementary good is successfully developed.

Proposition 3. $\hat{N}_1 = \frac{2 - P\alpha + \sqrt{(2-P\alpha)^2 + 6P\alpha}}{6} > 0$, $\hat{N}_2^F = \hat{N}_2^S = 0$.

Proof: If $\theta < \frac{p_2}{N_1 + N_2^S + \alpha}$ never join the platform, $\frac{p_2}{N_1 + N_2^S + \alpha} \leq \theta < \frac{p_1 - Pp_2}{(2-P)N_1}$ join the platform at $t = 2$ only if the complementary good is successfully developed, and $\theta \geq \frac{p_1 - Pp_2}{(2-P)N_1}$ join the platform at $t = 1$, then

(1) $N_2^F = 0$,
(2) $1 - N_1 - N_2^S = \frac{p_2}{N_1 + N_2^S + \alpha}$,
(3) $1 - N_1 = \frac{p_1 - Pp_2}{(2-P)N_1}$.

We can express p_1 and p_2 from (2) and (3) as follows
$p_2(P, N_1, N_2^S) = (N_1 + N_2^S + \alpha)(1 - N_1 - N_2^S)$, and
$p_1(P, N_1, N_2^S) = Pp_2 + (2-P)N_1(1-N_1) = P(N_1 + N_2^S + \alpha)(1 - N_1 - N_2^S) + (2-P)N_1(1-N_1)$.

Therefore, the platform is choosing $0 \leq P, N_1, N_2^S \leq 1$ such that $N_1 + N_2^S \leq 1$ to maximize

$$N_1 \times p_1(P, N_1, N_2^S) + PN_2 \times p_2(P, N_1, N_2^S) - P \times b =$$
$$P(N_1 + N_2^S)(1 - N_1 - N_2^S)(N_1 + N_2^S + \alpha) + (2-P)N_1^2(1 - N_1) - P\frac{dc(P)}{dP},$$

where $b = \frac{dc(P)}{dP}$ is derived from the seller's optimization problem.

Labeling μ_1 and μ_2 as the Lagrange multipliers of the constraints associated with $(N_1 \geq 0)$ and $(N_2^S \geq 0)$ constraints respectively, the optimization problem has the following Lagrangian:

$$\mathcal{L} = P(N_1 + N_2^S)(1 - N_1 - N_2^S)(N_1 + N_2^S + \alpha) + (2-P)N_1^2(1 - N_1) - P\frac{dc(P)}{dP} + \mu_1 N_1 + \mu_2 N_2^S.$$

The relevant Kuhn-Tucker conditions for the optimization problem with respect to N_1, N_2^S, P are:

$[N_1]$: $(2-P)\left[2N_1(1-N_1) - N_1^2\right] + P\frac{d\left((N_1+N_2^S)(1-N_1-N_2^S)(N_1+N_2^S+\alpha)\right)}{dN_1} + \mu_1 = 0$;
$[N_2^S]$: $P\frac{d\left((N_1+N_2^S)(1-N_1-N_2^S)(N_1+N_2^S+\alpha)\right)}{dN_2^S} + \mu_2 = 0$;
$[P]$: $(N_1 + N_2^S)(1 - N_1 - N_2^S)(N_1 + N_2^S + \alpha) - N_1(1 - N_1) - c'(P) - Pc''(P) = 0$.

Combining $[N_1]$ and $[N_2^S]$ we derive

$$(2-P)\left[2N_1(1-N_1)-N_1^2\right]+\mu_1=\mu_2,$$

which implies $\mu_2>0$ and, consequently, $\hat{N}_2^S=0$. Therefore, the platform is choosing N_1 to maximize $PN_1(1-N_1)(N_1+\alpha)+(2-P)N_1^2(1-N_1)=N_1(1-N_1)(P\alpha+2N_1)$. Consequently, $\hat{N}_1=\frac{2-P\alpha+\sqrt{(2-P\alpha)^2+6P\alpha}}{6}>0$. *Q.E.D.*

Finally, the expressions for the optimal prices directly follow from $p_2(P,\hat{N}_1,\hat{N}_2^S)=(\hat{N}_1+\alpha)(1-\hat{N}_1)$ and $p_1(P,\hat{N}_1,\hat{N}_2^S)=(1-\hat{N}_1)(2\hat{N}_1+P\alpha)$.

Therefore,

$$p_1=(1-\tfrac{2-P\alpha+\sqrt{(2-P\alpha)^2+6P\alpha}}{6})(2\tfrac{2-P\alpha+\sqrt{(2-P\alpha)^2+6P\alpha}}{6}+P\alpha),$$
$$p_2=(\tfrac{2-P\alpha+\sqrt{(2-P\alpha)^2+6P\alpha}}{6}+\alpha)(1-\tfrac{2-P\alpha+\sqrt{(2-P\alpha)^2+6P\alpha}}{6}).$$

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