

Petroleum Reserves: Traditional, Probabilistic and with Abandonment Real Options

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Abstract

This paper discusses the estimative of oil reserves volume, by comparing the traditional approach based on the discounted cash flow (DCF), the probabilistic approach, and the real options approach that considers the abandonment option. In all cases, we consider both the residual value and the abandonment cost (environmental recovery costs). The traditional DCF uses *expected* cash flow. The probabilistic approach also uses the DCF rule, but making explicit the probabilistic scenarios and using the DCF rule in each scenario. The real option approach is also probabilistic, but considering the value of waiting before exercising the abandonment option. We use the binomial method considering the oil price uncertainty for both probabilistic and option approaches in order to evaluate both the monetary and the volume values. The option approach gives the higher economic value for the oilfield, followed by the probabilistic approach. I found that, *ex-ante*, the reserve volume is usually (much more) optimistic with traditional DCF than with the options approach. If we consider the options approach the most rigorous, it means that in most cases the reported reserves are optimistic (without considering future new technologies that could extend the oilfield life). However, *ex-post*, the use of real option approach will result in higher recoverable volume (reserves) than the DCF rule because the options approach considers the value of waiting for a better oil price before abandoning, whereas the DCF only consider the expected cash flow. For the volume calculus with abandonment option, the paper introduces the technique of pruned Pascal's triangle to access the probability of continuing to produce in any node from the binomial tree. The conclusion is that oil companies using deterministic DCF are *ex-ante* optimistic regarding oil reserve volume, but *ex-post* abandon too early the oilfields when using the DCF rule.

Keywords: petroleum reserves, real options, abandonment option, petroleum exploration & production, probabilistic reserves, reserves with option to abandon, pruned Pascal's triangle.

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1) Introduction

According to the Society of Petroleum Engineers (SPE, 2007), petroleum resources are the estimated quantities of hydrocarbons naturally occurring on or within the Earth's crust. When these resources are “*anticipated to be commercially recoverable by application of development projects to known accumulations from a given date forward under defined conditions*”, they can be called reserves¹.

In short, petroleum reserve is the economically recoverable volume of hydrocarbons for a given development investment. International efforts to standardize the definitions of petroleum resources and reserves and how they are estimated began in the 1930s. In 1997, in an industry standardization effort, the Society of Petroleum Engineers (SPE) and the World Petroleum Council (WPC) jointly approved the “Petroleum Reserves Definitions” including suggestions for *probabilistic reserves* estimation procedures. In 2007, SPE and WPC, together with the American Association of Petroleum Geologists (AAPG), the Society of Petroleum Evaluation Engineers (SPEE) and Society of Exploration Geophysicists (SEG), released their “Petroleum Resources Management System” (PRMS) document, which consolidates, builds on, and replaces guidance previously published by them (SPE, 2007).

The PRMS prescribes a consistent approach to estimating petroleum quantities, evaluating development projects, and presenting results within a comprehensive classification framework. The definitions and the related classification system provide a measure of comparability between projects, groups of projects, and total company portfolios according to forecast production profiles and recoveries. Therefore, by following PRMS, companies can reduce the subjective nature of resources estimation. However, this is not enough or the best oil companies can do, as will be discussed here.

Companies often estimate reserves using more than one criteria. Stock exchange regulators, like SEC (Security Exchange Commission) in USA, have their own (evolving) rules for oil and gas reporting disclosures. Their goal is to allow the best possible comparison between listed companies, thus their premises are pre-defined and must be followed with little space for particular assumptions. For example, SEC determines the oil price and the discount rate every company should use in economic assessment. Olsen, Lee and Blasingame (2011) talk about *reserves overbooking* and show some improvements made by SEC to avoid this problem.

¹ Discovered and recoverable, but noneconomic petroleum volumes, are named *contingent resources*; whereas undiscovered volumes from exploratory prospects, are named *prospective resources* (SPE, 2007; SPE, 2011).

National regulatory authorities typically require companies to report reserves as well. Although many times the regulation is based in some international guideline, like PRMS, their concern is generally more related to the country petroleum supply. Hence, they usually do not restrict the estimation to contractual limits, as opposed to SEC. In order to increase the reliability about the reported reserves, companies can also hire an independent certification service. A reserves certification is an estimation made by a third-party company, usually a renowned consultancy firm, based on data provided by the hirer. For internal critical analysis of reserves, companies frequently use a different set of assumptions. The estimation of resources involves the interpretation of volumes and values that have an inherent degree of uncertainty. Therefore, a sensibility analysis is often used, and the results are restricted to internal scrutiny in most cases. In order to support corporate planning (e.g., for reserves replacement), companies may use particular premises (like discount rate), scenarios (for example, for oil prices) and more robust methods (like probabilistic approach) to assess the economically recoverable petroleum volume. In this way, some companies use internally and sometimes report more than one reserve number on the same oilfield² (“SPE reserve”, “SEC reserve”, “reserve with corporate premises”, “probabilistic reserve”), reflecting different assumptions or methodologies for assessing the economically recoverable volume. In addition to different geological interpretations, there are market issues with great impact on reserve like the future oil price, the future daily rig rate, and others, whose estimates vary between companies.

Reserves are classified as *proved*, *probable* and *possible*, depending on the degree of geological uncertainty (volume existence and reservoir productivity)³. These uncertainties can be reduced before development with investment in information (more appraisal wells, long-term production test, pilot systems, etc.). However, the market uncertainty remains the same. A “proved” reserve may become non-commercial if the oil price falls; or a contingent resource (noneconomic volume) may become a reserve with improved market condition (e.g., if the oil price rises)⁴. The impact of market conditions on the reserve volume has not been studied with due rigor and depth in the literature that this problem requires. The main goal of this paper is to address this important topic using the modern approach to investment under uncertainty, namely the real options theory, including a comparison with the traditional discounted cash flow (DCF) approach and with the probabilistic DCF approach.

² Example: <http://www.investidorpetrobras.com.br/en/press-releases/proved-reserves-petrobras-2018>

³ See the Petroleum Resources Classification Framework from PRMS (2007, item 1.1).

⁴ E.g., see <http://crudeoilpeak.info/oil-reserves-and-resources-as-function-of-oil-price> (in particular the figures 12-14).

Dermirmen (2007) presents a good overview on the reserve estimation methods used in the industry, including definitions on different kinds of reserves (e.g., proved, probable and possible) and discusses some issues related to a stochastic approach to reserve volume estimation. He highlights the necessity of reliable reserves report by the oil companies: “*Confidence in reserves disclosures became a public issue, and there were calls from investors and lending institutions for more-reliable reserves estimates*”. As in Dermirmen (2007), here “field reserves estimates” are expectation values, not proved or proved + probable volumes. He defends the probabilistic approach “... *generally should be preferred over a deterministic approach*”, but focusing on the geological uncertainties. He didn’t analyze the effect of the oil price uncertainty and the optimal abandonment policy on the reserve volume estimation as done here. In order to take into account the economic/market effect on the reserve volume, instead of explicitly modeling the economic (e.g., oil price) uncertainty, he uses a controversial approach much used in the industry: the truncation of the reserve volume distribution at a minimum economical volume (“economic cutoff”). Of course, the minimum economically recoverable volume depends on the evolving market condition especially the uncertain future oil prices and the specific oilfield operational costs, as considered here. There are some additional problems to work with truncated distributions, e.g., the truncated distribution expected value is higher than the expected value without truncation, which can generate an optimistic information, being necessary to multiply by an economic factor to correct. While Dermirmen (2007) uses the *probabilistic reserves* expression looking only geologic/reservoir uncertainties, here this term is used focusing oil price uncertainty.

Some oil companies report the reserves life indicator named Reserve Life Index (RLI), a reserve-to-production ratio that gives an idea about the years of production with the current developed reserves. RLI indicates how long reserves will last at the current production rate with no additions to reserves. If B is the reserve volume (number of barrels) and Q the current annual production, $RLI = B/Q$, which is typically measured in years. If B and/or Q change with oil price, RLI changes as well. This index can even increase when oil prices fall depending on the operational cost (Opex): if the variable cost is high, but the fixed cost is relatively low as in mature onshore oilfields, can be optimal to stop temporally some wells (instead of permanent abandonment), reducing Q and hence rising RLI. With (permanent) abandonment, may occur the opposite, reducing more B than Q, decreasing the RLI. So, this index is not much appropriated to perform empirical analysis on the oil prices impact on reserve.

Apergis, Ewing & Payne (2016) applied time-series methods to assess the impact of oil prices on the reserve life. With onshore oilfields data from the state of Texas over the years 1977–2013, they found significant evidence of oil price influence on the RLI and “*that the reserve life index and real oil prices are cointegrated, thus exhibiting a long run equilibrium relationship*”. Their results show asymmetric effects, with higher impact from positive oil price shocks. But this specific result may be biased by the onshore data. It would be interesting to do a similar study for offshore oilfields, which have a much higher fixed Opex, so that the definitive abandonment is more frequent than the temporary stopping option when compared with the onshore oilfields used in their study.

After this introduction, the paper is organized as follow. Item 2 performs a review of literature about the project abandonment decisions. Item 3 discusses the DCF and probabilistic approaches using simple examples and anticipating some intriguing results that DCF reports optimistic reserves, but options (and probabilistic) rules ex-post results in higher ultimate recovery volumes. Item 4 details the options approach using binomial method, including the new feature of pruned Pascal triangle to calculate the production probability (hence, the expected reserve) under options rule. Item 5 presents a case study comparing the reserves volume and its economic value for the three decision rules (DCF, probabilistic and options). Item 6 presents the concluding remarks and suggestions for future research.

2) Review of Literature on Abandonment Decisions

This review of the literature on abandonment decisions will also serve to discuss some concepts used in the traditional, probabilistic and options approaches that we are going to analyze in the other items.

Robichek & Van Horne (1967) is a known early research in project abandonment decisions. They used Monte Carlo (MC) simulation to address the uncertain cash flows and for incorporating the effects of abandonment for the investment decision. In the conclusions, they correctly argue “*since having the option to abandon never decreases project value, the typical consequences of ignoring the option would be to underestimate the value of a project*”. Although they used the term “option to abandon”, they did not use the options theory (that only appeared in the next decade) and their approach is more related with the probabilistic method (with MC). Robichek & Van Horne used the following decision rule: in each scenario, the abandonment is optimal if the present value of the

expected future cash flows is lower than the abandonment value (salvage value⁵) of the project. This is, in essence, the DCF approach. However, they did not consider that future abandonment might be more desirable than current abandonment. Dyl & Long (1969) criticized this rule suggesting a rule that consider all possible future abandonment opportunities⁶. This issue will be considered here by working backwards in a dynamic programming fashion. In addition, instead of the expected future cash flows, real options theory considers all future scenarios starting from each scenario. Robichek & Van Horne also realized that the abandonment option reduces the risk: “*much of the downside risk can be eliminated if the Project is abandoned when events turn unfavorable*”. But they didn’t know how this issue changes the discount rate: “*The determination of this effect is extremely complex*”. The option’s discount rate problem was solved later, with the modern option pricing theory. The next important paper did use dynamic programming, but not the fact that the correct option discount rate is not the project’s cost of capital.

Bonini (1977) is other known earlier reference on abandonment decisions under uncertainty in capital projects. Similar to Robichek & Van Horne, Bonini argues “*project would be abandoned in any year if estimated future cash flows (appropriately discounted) did not exceed the current abandonment value*”. In contrast with Robichek & Van Horne, Bonini considered that the future abandonment might be better than current abandonment: his analysis was performed backwards with discrete-time dynamic programming. However, Bonino used the firm’s cost of capital as discount rate. He did not use the real options theory, which had not yet been born at the time (the first real option mathematical model was Tourinho, 1979). The risk-adjusted discount rate for projects with options is different from the case without options: the abandonment option reduces the project risk; it is like a partial insurance. In the section 3 we will also use a discrete-time dynamic programming, but in options style, with risk-penalized probabilities that allows to discount with risk-free interest rate.

The real options literature, more than 20 years after the first textbooks of Dixit & Pindyck (1994) and Trigeorgis (1996), it is now well established in academy and with growing use in firms. Tourinho (1979) was the earliest real option model in petroleum. Dias (2004) gives an overview of petroleum real options models, with focus in exploration and production (E&P). Early real options articles on abandonment decisions include entry-exit models (the firm has the option to reenter after an exit) and models using the analogy with the (financial) put option. The entry-exit real options models include

⁵ The revenue obtained by selling the project assets when abandoning. Here is used the term salvage (or residual) value for all dates, typically decreasing with the time, but Robichek & Van Horne use the term *salvage value* only in the last date to abandon the project. Before, they use the term *abandonment value*.

⁶ Robichek & Van Horne (1969) recognized this problem with their original rule.

Brennan & Schwartz (1985) and Dixit (1989), but they are not well suitable for oilfield abandonment because the option to redevelop a depleted abandoned oilfield is negligible and almost never occurs. Myers & Majd (1990) explores the analogy abandon option with (financial) put option. They used the geometric Brownian motion (GBM), the most used stochastic process in finance, but with negative drift to model the project value V and also analyze (appendix) the case of stochastic salvage value (also a GBM, correlated with V). It is similar to the option to exchange one risk asset (project) by other risk asset (the alternative use of the assets in place, the salvage value). In terms of empirical evidence, Berger, Ofek & Swary (1996) found “*strong support for the predictions of abandonment option theory*” using 1,043 US accounting data from the years 1984-93.

Olsen & Stensland (1988) is closer to the specific petroleum literature on abandonment options. They used two GBM, one for the commodity price $P(t)$ and the other for the production rate $q(t)$. The GBM for the rate $q(t)$ has negative drift, implying that the expected production rate follows an exponential decline. In contrast with this article, they assumed that the option to abandon is perpetual and did not consider abandonment costs or salvage value. They use risk-free discount rate, but without changing the probability measure as here, finding analytical solution for the optimal stopping problem. However, their conclusion is similar to the one of this paper (but in ex-post view): “*Thus, uncertainty will tend to prolong the extraction period compared to the deterministic case.*”

Begg, Bratvold & Campbell (2004) analyze the abandonment option for an oilfield using mean-reversion to model the oil price uncertainty, but their focus is the analysis of different abandonment decision rules as candidates to maximize the oilfield value, not the effect on reserve volume as in this present work. Jafarizadeh & Bratvold (2012) discussed the option to abandon an oilfield, but more as an illustration of two-factor stochastic process for oil prices, combining mean-reversion with GBM.

More recent real options models for oilfield abandonment include Guedes & Santos (2016), Abadie & Chamorro (2017), Borges et al. (2018) and Borges et al. (2019). Guedes & Santos analyze the sequential E&P options: exploration options, appraisal options, investment/scaling options and abandonment options. They found the abandon as the most valuable option, but they consider the abandon in all E&P phases, including failure of exploration/appraisal related with geological uncertainties (existence, quality and volume). Here we are concerned with the abandonment *after* the oilfield development and both the option value (oilfield value considering the option to abandon) and the oilfield volume that results from the optimal management of the abandonment option. Abadie & Chamorro discuss options to delay and to abandon with focus in short-lived oilfields (tight oil

production) using more sophisticated stochastic process for oil price (mean reversion and stochastic volatility). Borges et al. (2018) and Borges et al. (2019) use the fuzzy real options approach to evaluate an oilfield abandonment option, correcting a methodology used in this literature (*fuzzy pay-off method*) with a new approach. However, none of these articles discusses the effect of abandonment option on the reserve volume as here.

3) Discounted Cash Flow and Probabilistic Approaches

The traditional DCF abandonment rule is "*... a project should be abandoned at that point in time when its abandonment value exceeds the net-present value of subsequent expected future cash flows discounted at the cost-of-capital rate*" (Robichek and Van Horne, 1967). In simpler and more precise terms, the correct DCF abandonment rule is to abandon the project on the date that maximizes the NPV (net present value) considering only expected values. The NPV includes all the abandonment costs and salvage values as well as the expected cash flows. Note that the rule is not abandon at the first negative expected cash flow⁷. It may be optimal to produce even with negative cash flow, because postponing the abandonment has the additional benefit of postponing the abandon cost. The benefit of the delayed cost increases with the discount rate and may be more than sufficient to offset the negative cash flow. For example, postponing one year a cost of \$ 100 million, which is applied on the capital market at an interest rate of 5% p.a., means \$ 5 million annual benefit, which offsets a negative cash flow up to - \$ 5 million. The simple example below illustrates the traditional DCF rule and the decision changes if we use different scenarios instead of only expected values.

Consider a mature oilfield that, by the first time (at $t = 0$), is faced with a negative expected cash flow if the production continues from $t = 0$ to $t = 1$. Consider only two possible production periods, so that the abandonment is obligatory at $t = 2$ (end of the concession or end of equipment life). Consider the discount rate $r = 4\%$ p.a., the abandonment cost $A = \$ 100$ million, the corporate tax rate $T_c = 34\%$, and the remaining data in the table below (in million \$). In Table 1, the cash flows, salvage values and abandonment costs are before income tax and in present values at each date t , whereas NPV is showed after the income tax (multiplied by 0.66 to consider the 34% of corporate tax).

⁷ However, the first author experience when working in an oil company showed that some reserve auditors insisted on considering abandonment in the first negative cash flow, whereas others auditors accepted the maximum NPV rule, even producing with negative cash flows. The former case, with suboptimal decision, results in lower reserve reporting, and we name this naïve suboptimal rule as “wrong DCF” to distinguish from the (right) DCF.

Table 1 – Simple DCF Example (in million US\$)

	t = 0	t = 1	t = 2
Expected Net Cash Flow from the Production (CF)	- 1	- 5	-
Salvage Value (SV)	4	2	0
Abandonment Cost (A)	100	100	100
NPV(t = 0) for each date of abandonment	- 63.36	- 62.85	- 64.85

The expected cash flows are net of operational costs, royalties, corporate tax, etc. If abandon at $t = 0$, the negative expected cash flow is avoided, but pays the abandonment cost A and receives the salvage value $SV(t)$. When considering the corporate tax effect, the abandonment payoff at date t is:

$$\text{Abandonment Payoff}(t) = [- A + SV(t)] (1 - T_c) \quad (1)$$

If we produce from $t = 0$ to $t = 1$, we get a negative expected operating cash flow, but we postpone the abandonment cost. If we produce from $t = 0$ to $t = 2$, we get two negative expected cash flows, but we have the benefit from delaying two periods the abandonment cost. What is the abandonment date that maximizes the NPV? It is not the date $t = 0$, even with negative expected cash flow in case of production. The NPV (present value at $t = 0$) for each possible abandonment date τ are:

$$\text{at } \tau = 0 \text{ (immediate): } NPV(\tau = 0) = (- 100 + 4) * (1 - 0.34) = - 63.36;$$

$$\text{at } \tau = 1: NPV(\tau = 1) = \{- 1 + [(- 100 + 2)/(1 + 0.04)]\} * (1 - 0.34) = - 62.85; \text{ and}$$

$$\text{at } \tau = 2: NPV(\tau = 2) = \{- 1 + [- 5/(1 + 0.04)] + [- 100/(1 + 0.04)^2]\} * (1 - 0.34) = - 64.85.$$

So, in this example, the abandonment date that maximizes the NPV is $\tau = 1$. So, it is optimal to produce one year even with negative expected cash flow, because the benefit to postpone the cost of abandon A is higher and compensate this negative operational result. It is not optimal to delay more, because the second year expected cash flow is too negative ($- 5$) compared with the benefit to postpone one year more the abandonment cost ($= r A = 0.04 \times 100 = 4$). However, with a higher discount rate (e.g., 8%), could be even better to produce two years with expected negative cash flows.

Now, consider the same example, but instead of considering only the expected cash flow at $t = 1$, consider that the expected cash flow $E[CF(1)] = - 5$ is due the weighting of two scenarios with 50% chances each, the scenarios $CF(1)^+ = 5$ (oil price rises) and $CF(1)^- = - 15$ (oil price drops). In this case, there are two scenarios for $NPV(\tau = 2)$ that we call $NPV^+(\tau = 2)$ and $NPV^-(\tau = 2)$:

$$NPV^+(\tau = 2) = \{-1 + [5/(1 + 0.04)] + [-100/(1 + 0.04)^2]\} * (1 - 0.34) = -58.51 > NPV(\tau = 1).$$

$$NPV^-(\tau = 2) = \{-1 + [-15/(1 + 0.04)] + [-100/(1 + 0.04)^2]\} * (1 - 0.34) = -71.20.$$

So, instead of producing only from $t = 0$ to $t = 1$ (abandonment at $t = 1$), the optimal rule is: produce the first period; after that, if in $t = 1$ the market improves, produce also in the second period and abandon at $t = 2$, whereas if the market is worse at $t = 1$, stop production and abandon at $t = 1$. With these scenarios, there is 50% chance of producing more than in the case of traditional DCF rule that stops production with 100% chances at $t = 1$. So, in this case the expected reserve volume is higher, when considering scenarios, than with DCF rule (which seems only expected cash flows).

Now, consider the same data, but with a much higher (double) discount rate of $r = 8\%$ per period. For the DCF rule (using expected cash flows), the reader can check that now it is optimal to produce two periods, because the abandonment date that maximizes the NPV is $\tau = 2$: $NPV(\tau = 2) = -60.30$; $NPV(\tau = 1) = -60.55$; and $NPV(\tau = 0) = -63.36$. However, if we expand the expected cash flow into two scenarios, as done above, we get $NPV^+(\tau = 2) = -54.19$ and $NPV^-(\tau = 2) = -66.41$. Now, we get the opposite numerical case: the DCF rule looking only expected values, produce until $t = 2$, whereas with two scenarios at $t = 1$, there are 50% chances to produce only until $t = 1$. So, in this case, the scenarios approach points to lower reserve volume than the DCF approach.

These simple numerical examples show that traditional DCF rule may report higher or lower reserve when compared with the probabilistic approach. The probabilistic approach has some similarities with the options approach (see below the differences) so that the reserve volume may be higher or lower than DCF approach. Questions: overall, is the reserve higher or lower with options? Are the oil companies reporting optimistic, realistic or pessimistic reserves when using DCF? Is there a more important effect on reserve? Yes, as we see below.

In the beginning of the oilfield productive life, the expected revenue is much higher than the Opex because the expected net operational cash flows must be sufficiently positive to pay the investment (if not, the development project has negative NPV and the project is not approved). So, the first effect (DCF indicating higher reserves than probabilistic or option approaches) occurs in the beginning of the oilfield life: DCF indicating production “with certainty”, whereas probabilistic and options indicating some probability of abandonment after some few years. When the oilfield becomes more “geriatric”, with negative cash flows, the second effect occurs: probabilistic and options approach may recommend continuing production whereas DCF may recommend immediate abandonment.

In quantitative terms, due to the production decline of this non-renewable commodity, the production in the first years has a much higher contribution to reserve volume than the last years of production. So, when the firm is reporting for the first time the project reserves, we can say that typically DCF points optimistic reserves when compared with options approach (and probabilistic approach).

Figure 1, from a base case that is detailed in the next item, illustrates this point. Note that from the second year the expected production with options begins to be lower than that of DCF. This is because there is a probability of low oil prices occurring, which results in a positive probability of abandonment. The DCF approach does not “see” these abandonment scenarios. Can a large price change occur in two years? Recall that in June 2014 the average Brent’s oil price was near US\$112/bbl, whereas in June 2016 this average price was around US\$ 50/bbl and even lower some months before: in January 2016, the Brent was around US\$ 32/bbl.

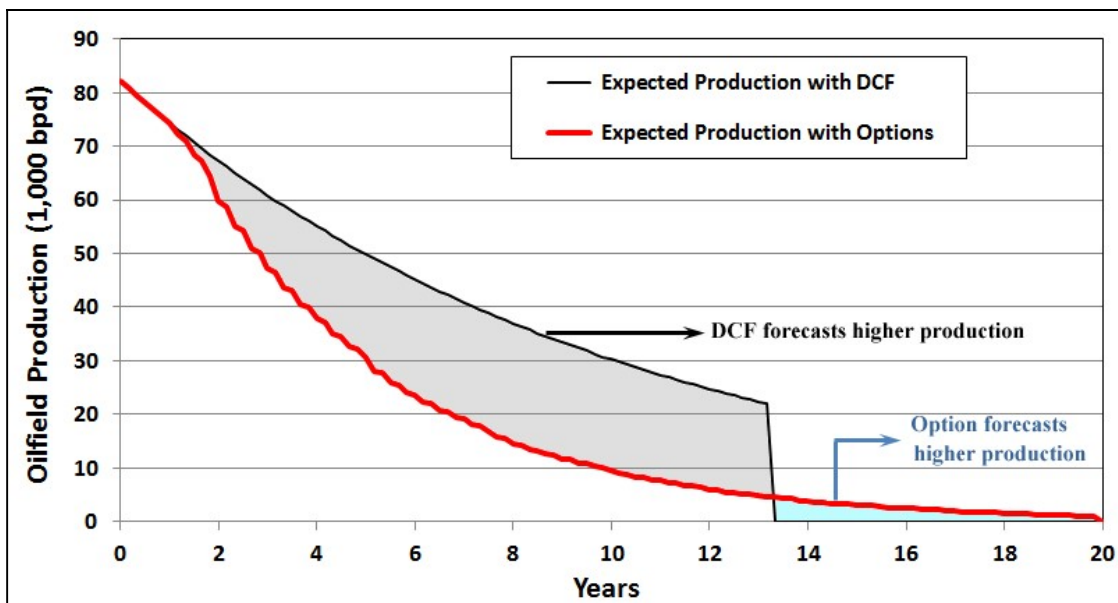


Figure 1 – DCF x Options Production (Reserve) Expectations

The largest shaded area (gray) in Figure 1 shows the DCF additional production forecast in relation to the options’ forecast, whereas the smaller shaded area (blue) shows the opposite case. In this example, the cumulative economical production (reserves) forecast for DCF was 222.77 million bbl, whereas for option was 149.60 million bbl. If the reserves estimate from options perspective is more realistic, the DCF was very optimistic, reporting a reserve number 48.9% higher! Therefore, the difference is very relevant and the reserves reported by oil companies can be too optimistic.

Here we focus on reserve estimate when making (or just after) the development investment, because this is the most important epoch for investors, when large investment are performed and new reserves additions are reported. We call “ex-ante forecast” the reserve report in this period.

Someone could collect empirical data comparing the original reserves reports with ultimate reserves from abandoned oilfields, which could indicate the opposite: the original reserve reports indicated a lower reserve than fulfilled. However, in the vast majority of cases, these oilfields had additional investments (like infill drilling, enhanced oil recovery with fluids injection, etc.) As pointed out in SPE (2007), “*Reserves must further satisfy four criteria: they must be discovered, recoverable, commercial, and remaining (as of the evaluation date) based on the development project(s) applied*” (emphasis added). So, the reserves report does not consider future investment projects.

The Figure 1 shows that if we are reporting reserves between years 12 and 13, the opposite situation occurs. When net operating cash flows approach zero or negative, if the decision maker is using options theory, the oilfield life is extended because options consider scenarios where market conditions improve, the “good news principle” (Dixit & Pindyck, 1994, p.41 n.9). So, while ex-ante DCF reports higher reserves, ex-post the options rule produce more time than the DCF rule.

Hence, in practice, options recommendation results in ultimate higher reserves, even ex-ante reporting lower reserves!

4) The Option to Abandon Approach to Reserve Forecast

In this item, we detail the options approach using the binomial tree method. While for option valuation purposes we must use risk-neutral probabilities, to calculate the expected reserve volume with options we must use real (or true or statistical) probabilities. This is performed below.

We model the uncertainty of oil prices along the time with the most popular stochastic process, the geometric Brownian motion (GBM), used by Black (1976) to model commodities and in pioneering real option models in petroleum extraction (Tourinho, 1979; and Paddock et al, 1988). GBM is much simpler than other models: less parameters to estimate, it is easier to interpret and American options are homogeneous in the underlying asset and in the exercise price⁸. Although other stochastic processes can be used, such us mean-reversion with jumps models, the main conclusions of this paper should not change if other processes are used. As pointed out by Pindyck (1999, section 6),

⁸ So that we can calculate the option F by unity of investment (F/I) or per barrel of reserve (F/B in \$/bbl) and then multiply by the investment I or by the reserve volume B to get the option value. Mean-reversion models do not share this homogeneity (see the textbook of Dias, 2015, chapter 23; and a theorem in Merton, 1973, p.149).

"...generally the size of the error is small, at least relative to errors that would be tolerable in real options applications.", and mention one paper that found errors of 5% when comparing mean-reversion with GBM, "which would be significant for a financial option. In case of a capital investment decision, ... an error of this size is unlikely to be important.". Pindyck (2001, p. 26) reaffirms: "... the GBM assumption is unlikely to lead to large errors in the optimal investment rule.". It is left for a future work to analyze the impact on reserve volume using other stochastic processes.

The risk-neutral GBM equation (see, e.g., McDonald, 2006, eq. 20.26) for the oil price P is:

$$dP^Q = (r - \delta) P dt + \sigma P dz^Q \quad (1)$$

Where r is the risk-free interest rate, δ is the oil price convenience yield⁹, σ is the oil price volatility, and $dz = N(0, 1) \sqrt{dt}$ is the Brownian/Wiener increment. The superscript Q denotes risk-neutral probability measure ("Q-measure") in the random terms dP and dz .

In the binomial method, after one period, the price P can go up to $P^+ = u P$ with risk-neutral probability q or can go down to $P^- = d P$ with probability $1 - q$. The Cox & Ross & Rubinstein (1979) binomial equations for the GBM, but considering the dividends (here the convenience yield), are given by:

$$u = \text{Exp}[\sigma \sqrt{\Delta t}] \quad (2)$$

$$d = 1/u \quad (3)$$

$$q = \frac{e^{(r - \delta) \Delta t} - d}{u - d} \quad (4)$$

The probabilistic and option approaches to estimate the expected volume of reserves must use true (or statistical or real) probabilities, not risk-neutral probabilities used for option pricing. Risk-neutral probability is only a math smart penalization trick to use the risk-free discount rate to calculate the present value of options¹⁰. For the reserves volume expectation is necessary to work with true probabilities. In the binomial, we call the up and down risk-neutral probabilities as "q" and "1 - q". For the true probability we use "p" and "1 - p" for the up and down scenarios, respectively. The true probability of up movement in the binomial is given by:

⁹ Convenience yield is the benefit on holding inventories of a commodity. It is like a non-cash dividend that justifies the investment in storage facilities. It can be measured in futures markets using the relation $F(t) = P \exp(r - \delta) t$, where $F(t)$ is the price of a futures contract to delivery in t years; P is the spot price; r is the risk-free rate. See details in derivatives textbooks such as McDonald (2006), Geman (2005), or Dias (2015).

¹⁰ See any textbook of financial options (e.g., McDonald, 2006) or real options (e.g., Trigeorgis, 1996).

$$p = \frac{e^{(\mu - \delta) \Delta t} - d}{u - d} \quad (5)$$

Where μ is the total rate of return of the oil price (or the risk-adjusted discount rate), which can be estimate using the CAPM. All rates (r , δ and μ) are in continuous time (logarithm returns)¹¹. The difference is only the replacement of the risk-free interest rate r by the risk-adjusted rate μ .

In order to calculate the probability of occurrence of each node of the tree, it is necessary to calculate the number of paths that reach a certain node of the tree. Pascal's triangle¹² is a triangular array of the binomial coefficients very known in mathematics. In the binomial tree context, it is used to calculate the number of paths to reach a specific node in the tree. The triangle is built as follow: start with "1" at first node (here node $x(0, 0)$) and the other nodes follow a rule of adding the two numbers from the previous adjacent nodes to get the node number. Figure 2 below illustrates the Pascal's triangle.

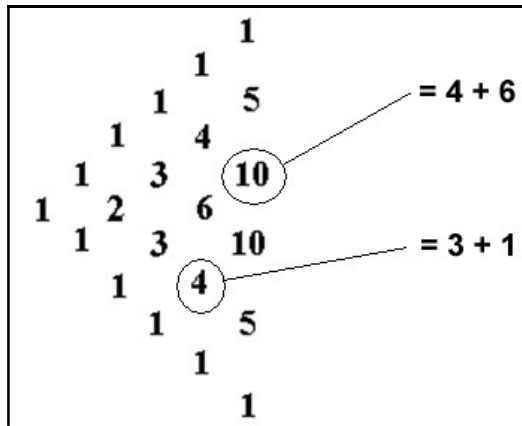


Figure 2 – Pascal's Triangle

Here it will be used to estimate the probability to reach each binomial tree node in order to estimate the expected reserve volume considering only nodes with production. For the case without earlier abandonment, the figure below shows the binomial tree and the Pascal's triangle, which gives the number of paths to reach a node. For example, in $t = 3$ we have four nodes (oil price scenarios), the nodes (from bottom to up) $x(3, 0)$, $x(3, 1)$, $x(3, 2)$ and $x(3, 3)$. The extreme scenarios have only one path to reach each one: (down, down, down) for $x(3, 0)$ and (up, up, up) for $x(3, 3)$. Hence, the probabilities to reach the nodes $x(3, 0)$ and $x(3, 3)$ are p^3 and $(1 - p)^3$, respectively. But the interior nodes we have more than one path to reach. For example, the node $x(3, 2)$ (in $t = 3$, the second node from the top) has three paths: (up, up, down), (up, down, up) and (down, up, up). In all the three

¹¹ If r_d is the discrete-time interest rate and r_c the continuous-time interest rate, we can get r_c using $r_c = \ln(1 + r_d)$.

¹² It is known as the Yanghui triangle in China.

cases, we have two “ups” and one “down”, so the probability for each path is $p^2 (1 - p)$. As we have three paths, the probability to reach the node $x(3, 2)$ is $3 p^2 (1 - p)$, as indicated in the figure below. The Pascal’s triangle gives the number of paths reaching each node (right bottom in the figure).

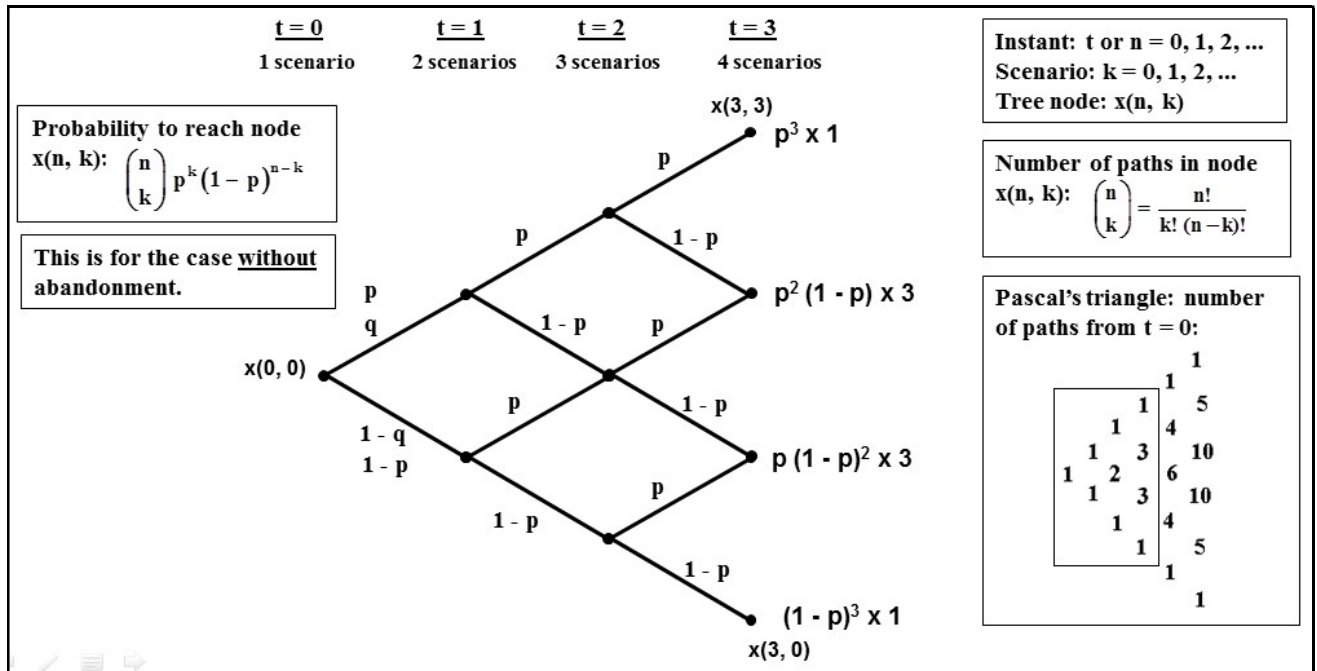


Figure 3 – Binomial Tree and the Node Probabilities without Abandonment Option

So, the number of paths reaching a generic node $x(n, k)$ is given by the binomial coefficient:

$$\text{Number of Paths at } x(n, k) = \binom{n}{k} = \frac{n!}{k! (n - k)!} \quad (6)$$

Because each path at node $x(n, k)$ has probability $p^k (1 - p)^{n - k}$ (k “ups” and $n - k$ “downs”), the probability to reach the generic node $x(n, k)$ is:

$$\text{Probability to reach } x(n, k) = \binom{n}{k} p^k (1 - p)^{n - k} \quad (7)$$

This is known, but it does not apply directly to our case, because we can abandon in one node so that all paths reaching this abandoned node are “killed” and the paths do not continue in terms of oil production. Example: in the Figure 4, if we exercise the option to abandon in the node $x(1, 0)$, all paths from this node will not exist anymore for production purposes (dotted lines in the figure). The nodes $x(2, 0)$ and $x(3, 0)$ will never be reached and the nodes $x(2, 1)$, $x(3, 1)$ and $x(3, 2)$ will be

reached by fewer paths when compared with the case without earlier abandonment. How to calculate the nodes probability in this case?

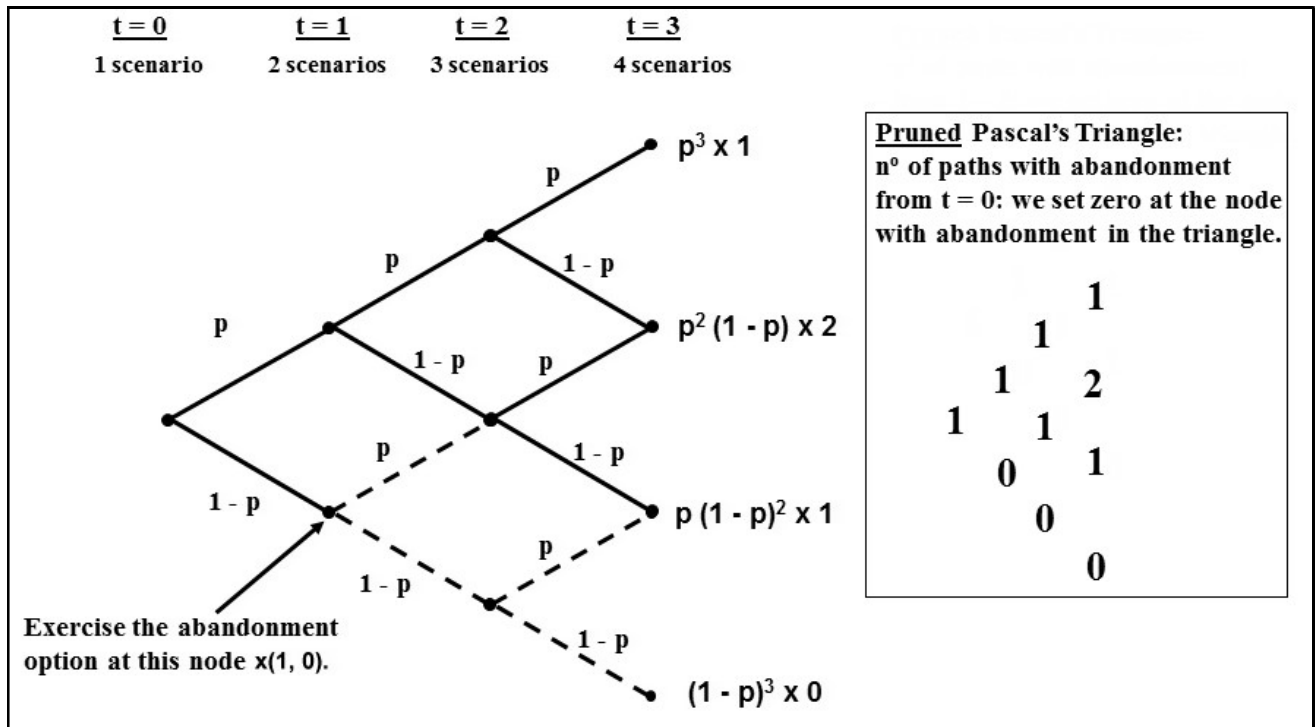


Figure 4 – Binomial Tree with Abandonment Option: Pruned Pascal's Triangle

The probability for one path doesn't change, $p^k (1 - p)^{n - k}$, but the number of nodes change. This paper proposes a very simple recursive approach, the Pruned Pascal's Triangle. In this approach, we first solve the option-pricing problem, so that we know the nodes with abandonment. With this information, we construct the pruned triangle: set zero for each node in which is optimal the abandon. After that, use the Pascal's triangle rule (sum the numbers of the adjacent previous two nodes) as shown in Figure 4. Example, with zero at $x(1, 0)$, the number of paths reaching $x(2, 1)$ is now only one ($1 + 0$ instead of $1 + 1$) and the number of paths reaching $x(3, 2)$ is now only two ($1 + 1$ instead of $1 + 2$). By using this pruned triangle rule, we map all the number of paths in each node of the tree considering the abandonment, denoted by $N(n, k)_{pa}$ and the probability of reach the node $x(n, k)$ with production considering the abandonment option is:

$$\text{Probability to reach } x(n, k) \text{ with option} = N(n, k)_{pa} p^k (1 - p)^{n - k} \quad (8)$$

So, for each instant n we can estimate the probability of producing at this date. Without previous or immediate abandon, the probability of producing is 100%¹³. However, with abandonment, this probability is less than 100% and this issue must be considered in expected reserve volume calculus. Figure 5 illustrates this issue from one numerical case (the same case used in Figure 1).

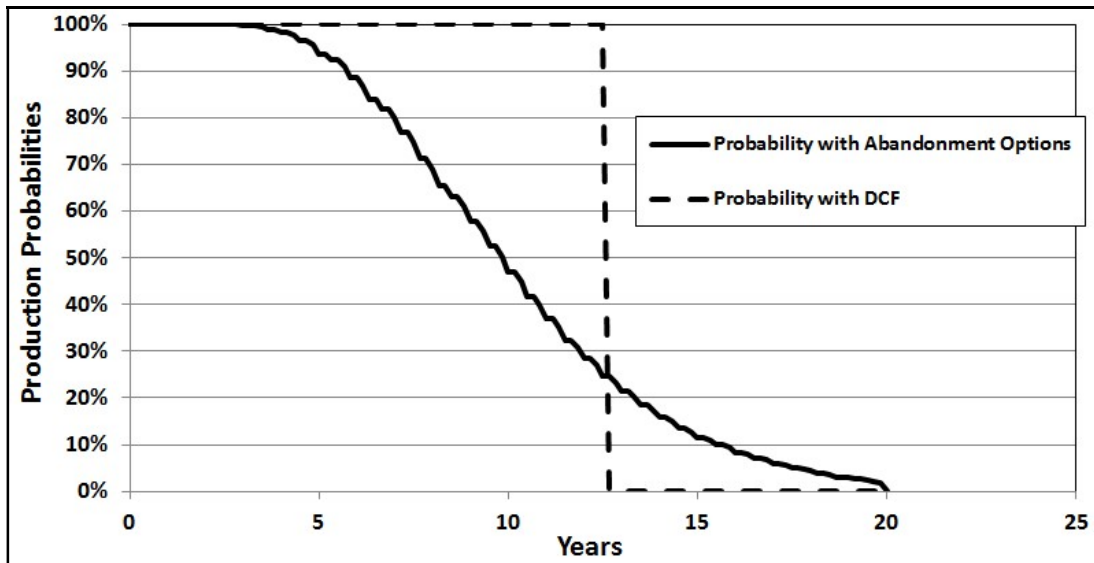


Figure 5 – Oil Production Probabilities: Options x DCF Approaches

5) Case Studies and Numerical Comparison

In this section, we present some numerical simulations in order to compare the three approaches to estimate the reserve volume (traditional DCF, probabilistic, and option) in terms of reserves volumes and the oilfield values. Let us consider a base-case with round numbers. Later, we present a sensitivity analysis for critical parameters like the oil price volatility.

Table 2 presents the economic/financial parameters used in the base-case.

¹³ Without abandonment, the summation in k (for all k in a given date n) for the previous equation is 100%.

Table 2 – Economic/Financial Parameters Used in the Base-Case

Input/Parameter	Value
Oil price at $t = 0$: $P(0)$	US\$ 50/bbl
Oil price volatility: σ	30% p.a.
Risk-free interest rate: r	4% p.a.
Oil convenience yield: δ	6% p.a.
Risk-adjusted discount rate: μ	6% p.a.
Corporate income-tax rate: τ_c	34%
Royalty rate: ROY	10%

The current oil price is based in the price seen in May 2017. The oil price volatility is a rounded value from a 30-years oil price time-series estimative. The risk-free interest rate is based in Treasury bonds from a long-term perspective. The oil price convenience yield is measured in futures markets using WTI contract for delivery in 18 months. The risk-adjusted discount rate considers a risk-premium of 2% p.a. over the risk-free rate. A sensitivity analysis for σ , r , and μ will be presented. The corporate income tax and royalty rates match the Brazilian economy conditions.

Table 3 presents the specific oilfield parameters used in the base-case.

Table 3 – Specific Oilfield Parameters in the Base-Case

Input/Parameter	Value
Initial production: $Q(0)$	30 million bbl/year
Decline rate: λ	10% p.a.
Variable Operational Cost: C	20 US\$/bbl
Fixed Operational Cost: CF	US\$ 200 million/year
Maximum Production Life: T	20 years
Liquidation value at $t = 0$: $K(0)$	US\$ 800 million
Liquidation value at $t = T$: $K(T)$	0
Abandonment Cost: A	US\$ 300 million
Potential Reserves (without abandon): B_p	261,57 million bbl

The initial production corresponds to 82,192 barrels per day. The decline rate is typical of many oilfields. The operational cost inputs are drawn from an offshore oilfield with rounded numbers. The

maximum production life can be either because of the equipment life limit or by legal restriction (in Brazil, the maximum production time of oil concessions is 27 years). The liquidation value at $t = 0$ considers a floating production unit (like a FPSO) that can be used in other oilfield, but with remaining life of 20 years, so that the residual value after 20 years is negligible. The abandonment cost considers the abandon of 12 producing and injection subsea wells. The *potential* reserve is the cumulative production considering the maximum production life (without early abandonment), which is assumed 20 years of production in this numerical example.

We build this binomial model with an Excel spreadsheet, which has several binomial trees: real option trees (with risk-neutral probability), probabilistic DCF tree, and true probability trees for the expected volume calculus. We present the results from the base case in Table 4:

Table 4 – Base Case Results

	Oilfield Value (million \$)	Expected Reserve (million bbl)	Expected Reserve Life (years)
Without Early Abandon	837.81	261.57	20
DCF	1,196.86	222.77	13.33
Probabilistic DCF	1,935.93	112.65	5.93
Options Approach	2,048.53	149.60	8.24

The table above presents the case “without early abandon” for comparison purposes (shows the potential reserve if the oilfield produces all the 20 years). As expected, the options approach has the highest monetary value because the options rule maximizes the oilfield value. The DCF forecast exhibits the highest reserve volume, 48.9 % higher than the one reported by the options approach (but with oilfield value much lower, only 58.4% of the options decision rule case). The probabilistic DCF reports an even lower reserve volume and its rule results in lower oilfield value than the options. However, the probabilistic approach generates a much higher oilfield value than the traditional DCF and a little lower value than the option approach.

We calculate also the optimal oil price to abandon along the oilfield life. This optimal abandonment price is named the option exercise threshold, denoted by $P(t)$. For the base case we generate the following thresholds chart, which shows both the real option and the probabilistic DCF optimal abandonment oil prices.

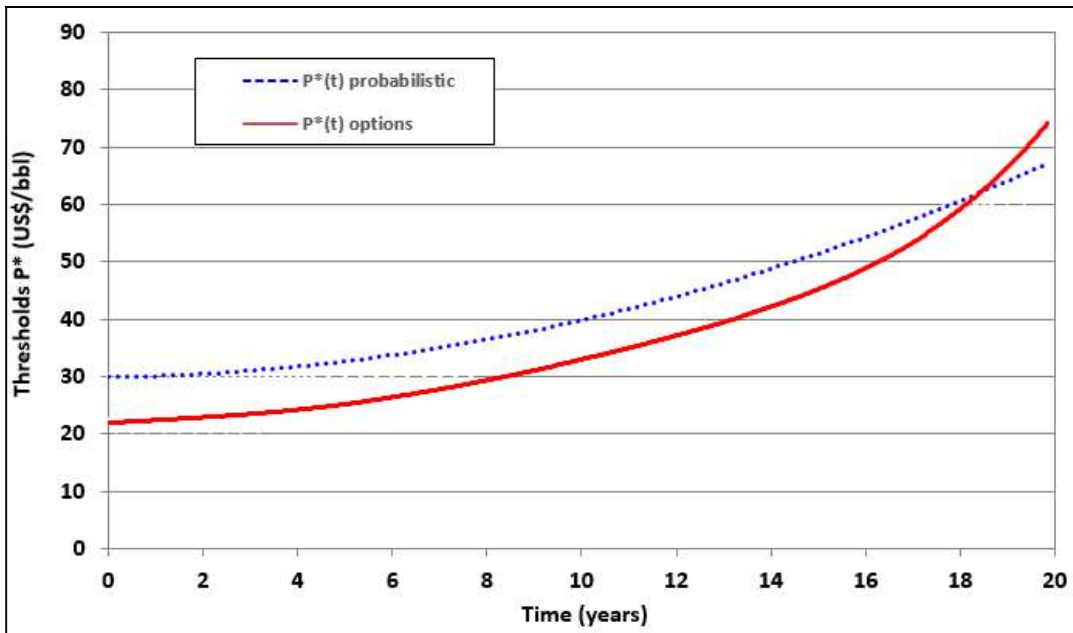


Figure 6 – Oil Price Thresholds for the Base Case

The options threshold curve is below the probabilistic threshold curve for almost all the oilfield life, except near the expiration. But, if we consider a more "pure" case, with no salvage value (only the effect of waiting for a better oil price), the options curve will always be below the probabilistic curve. The Figure 7 shows the base case, except that the salvage value is zero.

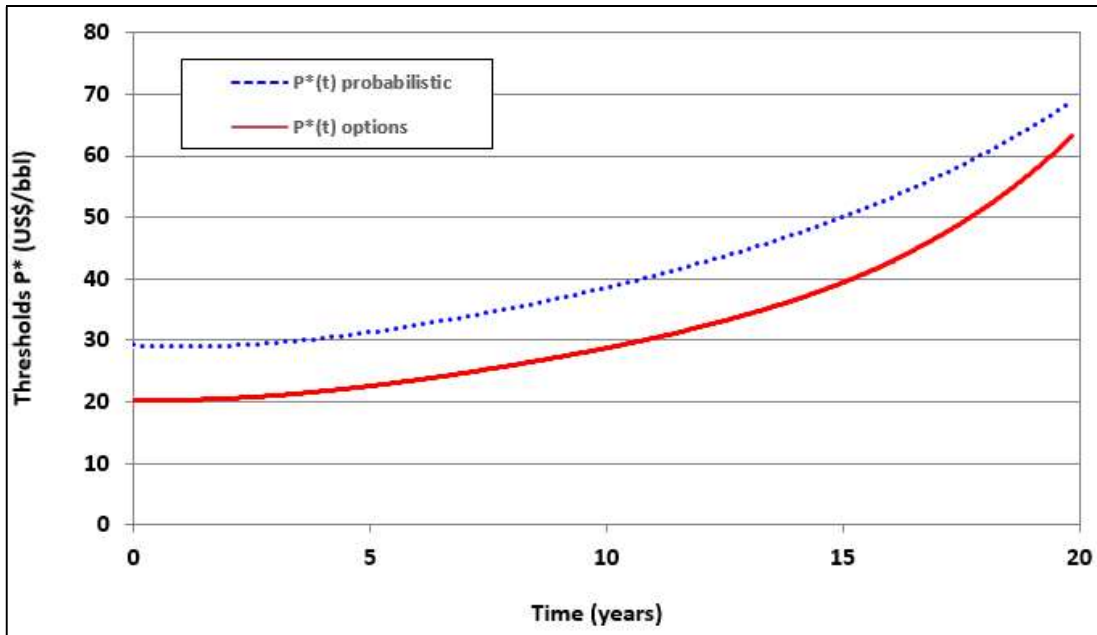


Figure 7 - Oil Price Thresholds for the Base Case without Salvage Value

We perform also a sensitivity analysis for the oil price volatility for several model results (reserve volume, oilfield value and average time to abandon) in the following charts:

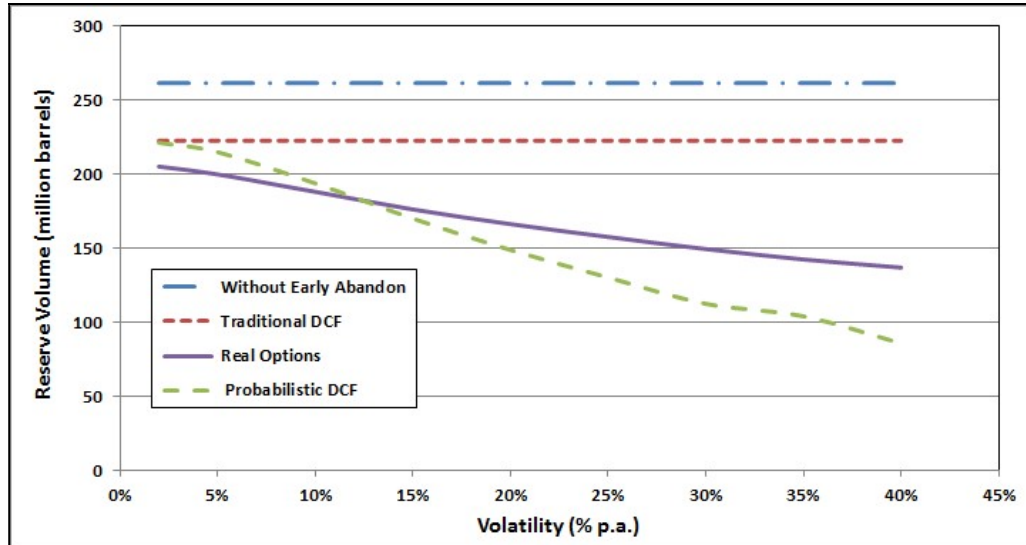


Figure 8 – Effect of Volatility in the Reserve Volume

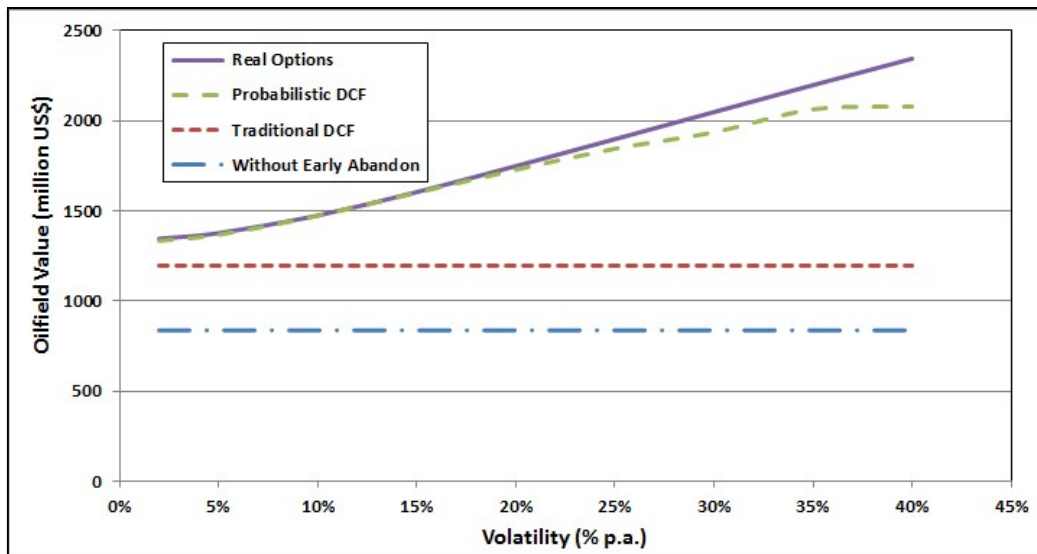


Figure 9 – Effect of Volatility in the Oilfield Value

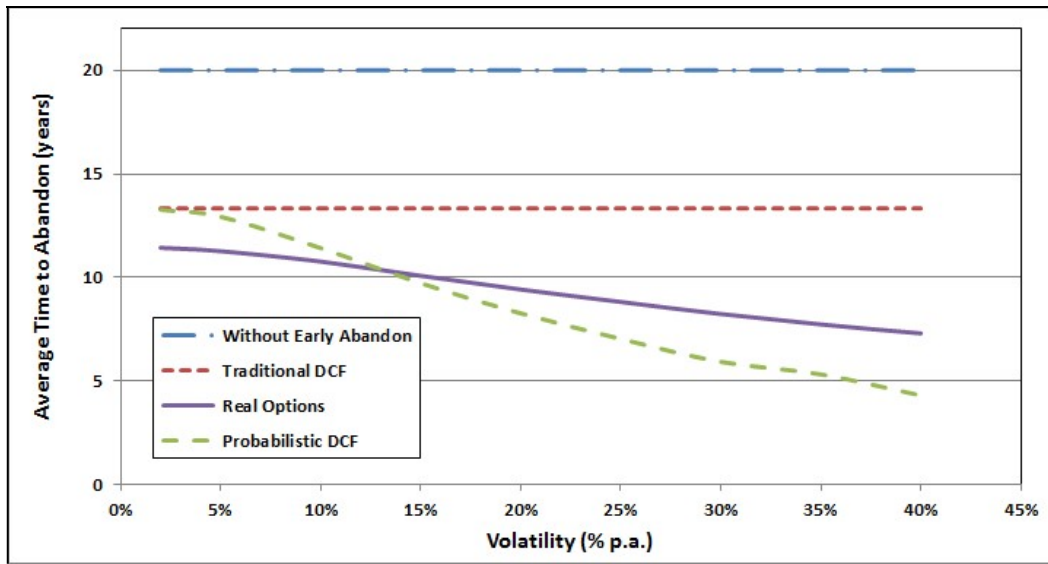


Figure 10 – Effect of Volatility in the Oilfield Expected Life

We can split the expected reserve volume in scenarios. The Figures 11 and 12 show the histograms of oilfield reserve volume for the probabilistic and options cases.

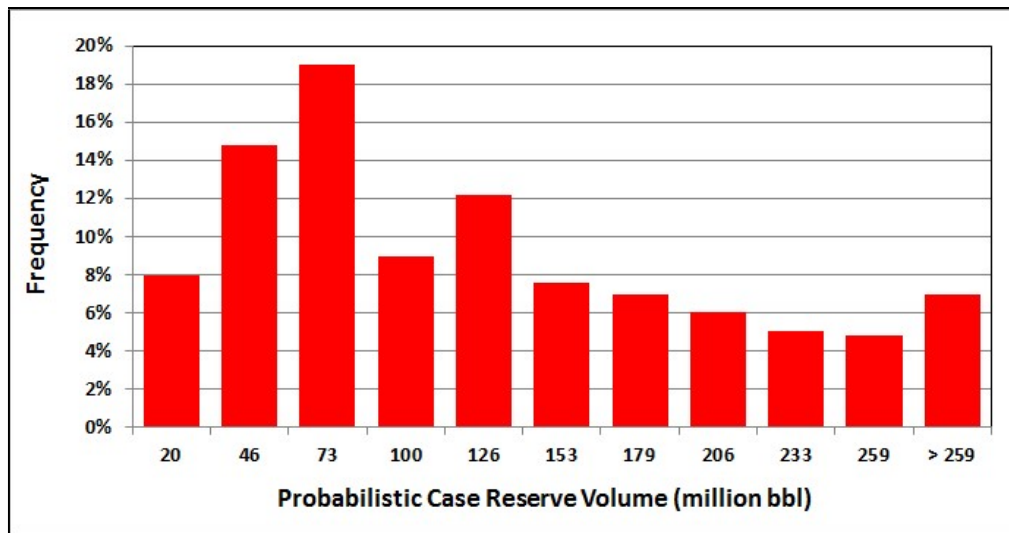


Figure 11 – Histogram of the Reserve Volume for the Probabilistic Case

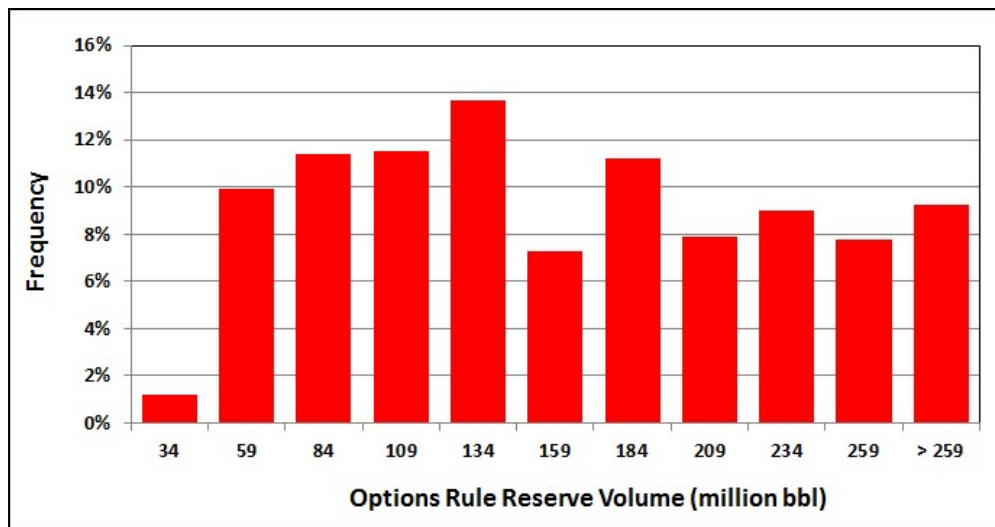


Figure 12 – Histogram of the Reserve Volume for the Options Case

We also use Monte Carlo simulation combined with binomial tree to address the effect of reservoir and geologic uncertainties on reserves, and we found that these technical uncertainties reduce even more the *expected* reserve, although the effect seems *ex-ante* small.

6) Concluding Remarks and Suggestions to Future Research

The paper model, using realistic case studies, shows that oil companies using deterministic DCF are *ex-ante* optimistic regarding oil reserve volume, but *ex-post* abandon too early the oilfields when using the DCF rule. Therefore, it is recommended that oil-sector managers and regulators consider reporting also a more conservative reserve volume using the real options approach in order to provide additional and better information for more sophisticate investors.

We used the GBM for the oil prices because it is the most popular stochastic process and is sufficiently simple for the practitioners to be motivated in the use of option-pricing techniques for the reserves volume calculation. However, it can be easily extended to other stochastic processes like mean-reversion or mean-reversion with jumps (as in Dias & Rocha, 1998). Another extension could be an in depth analyses of the combined effect of oil price and geological uncertainties.

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