Financially constrained investments: optimal timing and capacity under uncertainty

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Abstract

This paper considers the investment strategy of an entrant that is financially constrained, i.e. it requires to issue debt. We find the optimal investment moment for the firm in an environment with market uncertainty and bankruptcy risk as well as the optimal production capacity. In addition, the optimal debtholder policy is determined. The effect of debt financing on the investment strategy, total welfare, and the debtholder policy is studied.

1 Model

In our baseline model we are studying the situation of a monopolist that has the option to invest and set capacity I in a market where the inverse demand is given by

$$p(t) = X(t)(1 - \eta I).$$

Here, $X(t)$ follows a geometric Brownian motion with trend μ and volatility parameter σ and $\eta > 0$ denotes the price sensitivity parameter. Discounting is done under rate $r > 0$. To finance this project the firm has a sunk cost of δI, for which it needs to acquire debt. Debt holders lend the total sum of δI upon investment to the firm after which the firm will pay the debt holders at rate ρ.

1.1 The Indefinite Project

As a benchmark we will first look at model where there is no bankruptcy risk, i.e. it is assumed that the firm will never default. In a such scenario the firm faces optimization problem, for a given interest rate ρ,

$$\sup_{\tau, I \geq 0} \mathbb{E}_0 \left\{ \int_\tau^\infty e^{-rt} X(t)(1-\eta I) I \mathrm{d}t - \int_\tau^\infty e^{-rt} \rho \delta I \mathrm{d}t \right\},$$

where τ is a first hitting time. Debt holders solve the optimization problem given by

$$\sup_{\rho > 0} \mathbb{E}_0 \left\{ \int_{\tau(\rho)}^\infty e^{-rt} \rho \delta I(\rho) \mathrm{d}t - e^{-r\tau(\rho)} \delta I(\rho) \right\}.$$

Proposition 1 *Define* $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$. *In the scenario without bankruptcy risk we find that*

- *For* $X < \dfrac{\beta_1(\beta_1+1)}{(\beta_1-1)^2}\delta(r-\mu)$ *it holds that investment is delayed. The debt holders set* $\rho^* = \dfrac{r\beta_1}{\beta_1 - 1}$ *and the firm sets* $I^* = \dfrac{1}{\eta(\beta_1+1)}$.
- *For* $X > \dfrac{\beta_1(\beta_1+1)}{(\beta_1-1)^2}\delta(r-\mu)$, *investment is undertaken immediately.*

For the purpose of this extended abstract we refrain from working out the various cases for the last part of Proposition 1. It suffices to mention that the optimal debt holder policy depends on X as does I^* does. The optimal interest rate ρ^* is linear in X.

1.2 Bankruptcy Risk

The firm is paying the debt holders $\rho\delta I$ per time unit. The firm is assumed go bankrupt when the firm is no longer able to pay the debt holders, i.e. when $X(1-\eta I)I < \rho\delta I \Leftrightarrow X < \frac{\rho\delta}{1-\eta I} =: X_B(I)$. Denote by τ_B the first time X drops to X_B. The firm's investment problem becomes

$$\sup_{\tau, I \geq 0} \mathbb{E}_0 \left\{ \int_\tau^{\tau_B} e^{-rt} X(t)(1-\eta I) I \mathrm{d}t - \int_\tau^{\tau_B} e^{-rt} \rho \delta I \mathrm{d}t \right\}.$$

Debt holders solve the optimization problem becomes

$$\sup_{\rho > 0} \mathbb{E}_0 \left\{ \int_{\tau(\rho)}^{\tau_B} e^{-rt} \rho \delta I(\rho) \mathrm{d}t - e^{-r\tau(\rho)} \delta I(\rho) \right\}.$$

Assume $\rho > 0$ is fixed. The optimal investment size I as a function of X follows from

$$\left(\frac{X}{X_B(I)}\right)^{\beta_2} = \frac{r}{\mu}\frac{r-\mu}{\rho\delta}\frac{1-\eta I}{1-(\beta_2+1)\eta I}\left(\frac{X}{r-\mu}(1-2\eta I) - \frac{\rho}{r}\delta\right)$$

The optimal investment trigger X^* is the solution of

$$X = \frac{r-\mu}{rX_B(I(X))}\frac{(\beta_2+1-\beta_1)(1-\eta I(X)) - \eta I\beta_2}{(\beta_2-1)(1-\eta I(X)(\beta_1+1))}.$$