Oversupply of Offshore Drilling Rigs and Optimal Strategy:

Invest, Operate, Lay-Up and Scrap

Nuran Cihangir Martin
Pontifícia Universidade Católica - PUC Rio de Janeiro
nurancihangir@gmail.com
+55 21 99623-0893
Rua Marquês de São Vicente, 225- Gávea - Rio de Janeiro – Brazil

Luiz Eduardo Teixeira Brandão
Pontifícia Universidade Católica - PUC Rio de Janeiro
brandao@iag.puc-rio.br
+55 21 2138-9304
Rua Marquês de São Vicente, 225- Gávea - Rio de Janeiro – Brazil

Abstract

Offshore drilling rigs are leased to oil and gas companies for the purpose of exploration and production. Independent drilling contractors owning these units often are faced with capacity utilization problems and since the oil price crash in 2014, almost half of the available capacity in the sector has been temporarily idled. Once idled, or "laid up", these assets will only be re-activated in case of a sufficient market uptick, or will be eventually abandoned by way of scrapping otherwise. Such operational decisions as well as asset valuation problem are analyzed under the real options framework in a standard four-stage entry-exit model with stochastic daily revenues. On the basis of the numerical simulations, we conclude that recent bids for asset acquisitions are under-valued. Furthermore, the level of day rates does neither support a fresh investment nor re-activation of mothballed units. Re-activation costs and liquidation value justify maintaining the units in suspension rather than exercising the option to exit.

Key words

Real options, optimal switches, offshore drilling, mothballing, entry-exit decisions.

1. Introduction

Oil and gas companies and rig operators lease offshore drilling rigs for the purpose of utilizing them in their exploration and production activities. The revenues from these rigs, measured in terms of day rates to a large extent on term basis, can face dramatic changes: while a large drillship earned approximately $620.000 per day at the end of 2012, this amount fell to $200.000 by the end of 2016. Thus, offshore drilling companies must adapt to these rapid changes which are driven by oil price fluctuations, and which affect the capital expenditure decisions of oil companies for offshore projects. Capacity optimization in the segment has become increasingly important. Rig owners can idle some units to save on operational costs, which is known as “lay-up”, and reactivate these assets at a later time on at a one-off cost when the market prospects improve. Furthermore, the owners can decide to completely abandon the asset and scrap it in order to obtain one-off revenue, which is a function of scrap steel prices. There is an increasing concern on what to do with the large number of offshore drilling rigs which are currently idle and awaiting employment in the offshore centers around the world. Large investment requirements for entry necessitate a diligent valuation of the units. This study investigates investment and operational decisions in the offshore drilling sector by using real options.

The model presented in this paper is closely related to A. K. Dixit and Pindyck (1994), where a firm has the infinite number of options to switch between four states: invest, mothball, re-activate and exit. As such, the firm is able to end its mothballing state and return back to production, rather than choose to irreversible abandon. This is also more applicable to our case than the most other models available in the literature. Mossin (1968) was the earliest work on entry and exit decisions, which points out that an analysis of lay-up decisions must take into account the stochastic mechanism of day rates explicitly. The two-switch-model in Mossin (1968) defines the critical values for where the ship is to be laid up and when this decision is tobe reversed.

A. Dixit (1989) considers the entry and exit decisions under uncertain output price, which follows a random walk. An idle firm and an active firm are regarded as assets which are call options one on another. Trigger prices with respect to entry and exit showed a hysteresis effect implying higher and lower thresholds than simply covering the costs/ revenues, respectively. Shirakawa (1997) provides an analytical solution to the entry-exit problem in A. Dixit (1989).

A mathematical model by Duckworth and Zervos (2001) estimates the optimal time sequence at which entry or exit can take place. The firm is assumed to be either in an active or a passive mode: In the active mode the firm generates revenues following a stochastic process. In the passive mode it incurs costs. Guerra, Kort, Nunes, and Oliveira (2018) developed a similar model and observed a hysteresis effect.

In most of the studies in the literature, mothballing is an irreversible decision. Brennan and Schwartz (1985) is the earliest work of this type. The authors analyse a mining firm´s decision to operate or mothball (permanently) under the uncertainty of output prices. The value of the 'timing option" is among the key considerations by the authors. Zervos, Oliveira, and Duckworth (2018) is one of the most recent studies where an exit decision is irreversible. The objective is to maximize the payoff of the project while optimally exercising the mothballing and exit options. This stochastic control problem encompasses impulse control, i.e. sequence of optimal stopping problems, and discretionary times to stop.

As specific applications to maritime and related sectors, Gonçalves (1992) uses contingent claims methods to examine optimal investment and operations in shipping segment for ocean transportation services of bulk commodities. The model follows an arbitration approach by comparing the asset values implied by day rate futures rates versus those by considering a Geometric Brownian Motion (GBM) of day rates. The trigger values and optimal timing for ship investments are driven by stochastic optimal control theory. Parallel to the observation by A. Dixit (1989), optimal policies demonstrate an hysteresis effect, i.e. ship will only exit the market when the revenues are well below costs and re-enter the market when the day rates are well above the costs.

Another major application to shipping, Sødal, Koekebakker, and Aadland (2008) use a real options model to value a combination carrier. The optionality involved here is the switch between the dry bulk market and wet bulk market for. Their model is a mean-reverting Ornstein-Uhlenbeck version of a standard entry-exit model with stochastic prices to observe whether the dynamics would imply new entries into the market. A closed form solution for the option value is obtained.

Other than the optimal switches, there are also several empirical studies in the literature on the determinants of lay-up and scrapping decisions. Corts (2008) studies the stacking decisions of offshore drilling rigs and examines these decisions over the cycle 1998–2000 to investigate the firm level and rig-specific characteristics incentivizing idle capacity. Alizadeh, Strandenes, and Thanopoulou (2016) investigate the probability of scrapping compared to age, rig size and revenues of the rigs.

This paper contributes to the understanding of investment, temporary idling and exit of offshore drilling rigs within the context of real options. Most studies in the literature with regard to operational switches focus on the permanent closing of operations or deferral of investments.

Studies on temporary closures are relatively scarce. To the best of our knowledge, this paper is the first to analyze the switching dynamics of drilling rigs subsequent to the 2014 oil price crash in a real options setting.

The following sections are presented in the paper: After this introduction, in section two, an overview of the global offshore drilling market is provided. In section three, we introduce the model and in the next section we provide some numerical results. Finally in section five we conclude.

2. The Market for Offshore Drilling Rigs

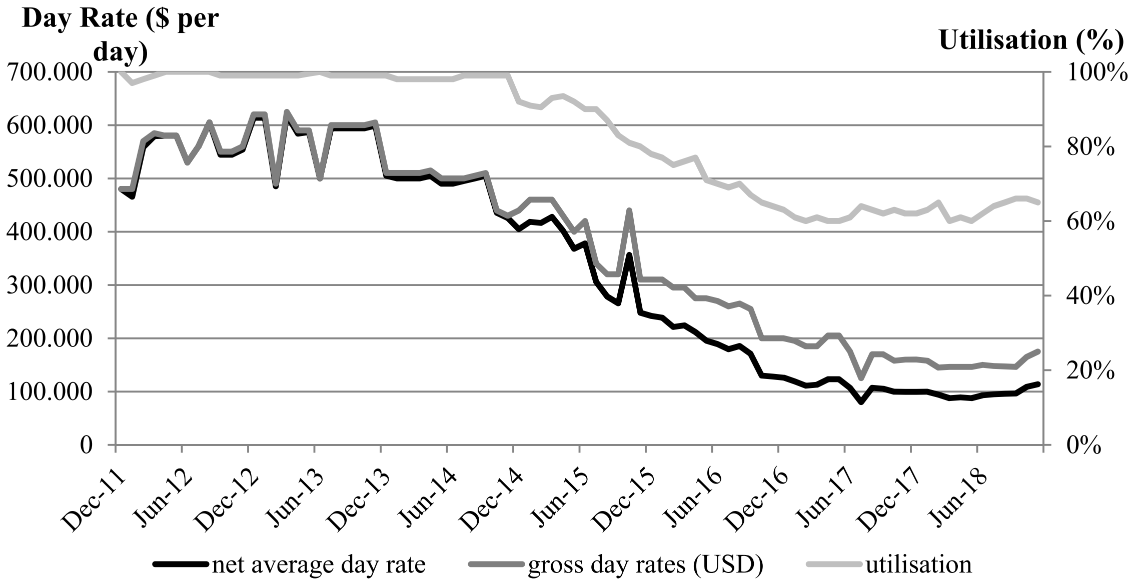
Offshore drilling rigs are classified in terms of the water depth they are employed in: shallow water rigs and deep water rigs. The legs of shallow water units reach to the ocean floor to support the drilling deck. Deep-water rigs are grouped into semi-submersibles and drillships. They support the decks by flotation Corts (2008).

The fundamental demand drivers for offshore drilling rigs are the combination of energy demand and oil in general and exploration and production capital expenditures and offshore activity in particular. Supply side is driven by the number of operational units.

2.1 Response to Oil Price Crash

The Brent oil prices were within the range of $90-$110 bbl in the period between 2011 and the first half 2014 -the production, capital and operating costs increased steadily which created a vulnerability for decreasing oil price levels. The oil-price crash (2014) therefore forced the oil majors to initiate cost saving measures and led, in the aftermath, to sluggish exploration and production activities. The offshore, representing about one quarter of global oil production (OECD, shipbuilding and the offshore industry, 2015), was similarly impacted. As a consequence, offshore drilling day rates went down by as much as 57% from their 2013 levelsFigure 1 displays the day rate and utilisation data for the large drillships (larger than 7,500 feet), which are the subject of our study.

. Figure 1 – Worldwide Drillships (larger than 7,500 feet) Average Day Rates vs. Total Contracted Utilization[1]



As the situation turned out to be persistent and severe, practically all players in the offshore drilling segment had to restructure the debt facilities. Some companies sought legal protection via chapter 11 or other bankruptcy proceedings (e.g. Seadrill, Pacific Drilling, Ultra Petroleum Corp.). Some companies joined forces to save costs and use synergies (e.g. Atwood acquired by Ensco, Ocean rig and Songa Offshore taken over by Transocean).

The owners of the offshore drilling rigs had to cut costs, by especially laying-off and centralizing their activities by way of so-called "fleet rationalization".

2.2 Gradual Recovery

The offshore sector exhibited some signs of improvement in 2017 and 2018. Oil prices remained above $50 for most of 2017 and even tested the $80 threshold in the first half of 2018. Nevertheless; the increases in the exploration and development spending have not been able to offset declines over the past few years.

Sector consolidation has been advocated since the outset of the oil crisis. So far this has not happened at the pace which was anticipated by the sector players. One possible reason is that the specialisation of the operators in terms of specific segment and area of operations. The market is characterised as an oligopoly, rather than perfect competition, principally due to relatively high barriers to entry.

[1] Source: IHS Markit

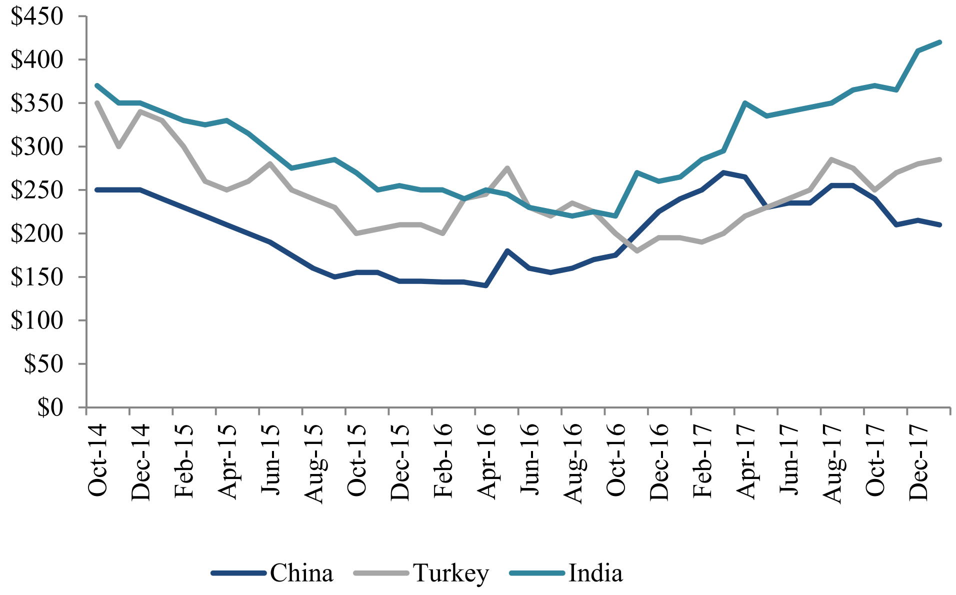
2.3 Capacity Optimization

The capacity reduction in shipping can be in three different ways: 1) In the short term, speed reductions are undertaken; 2) In the medium-term, firms traditionally place rigs in lay-up and 3) In the long-term; firms resort to scrapping of rigs (Alizadeh et al., 2016). Speed reduction is not of any relevance for offshore drilling rigs. The second option, lay-up or stacking, is used as a mechanism which temporarily suspends the unit from the market as a wait-and-see behavior. As such, it directly affects the supply side of the market. The unit can be put back in the market by paying a reactivation cost when there is an upturn. Lay-up can be in various forms, warm, cold etc. and in each case the costs involved are different and it is a reflection of the owner's anticipation of the future developments. Warm-stacking is characterized typically as at least some crew is laid-off and the rig is no longer placed in the market for a bid to take up a new task. In the case of cold stacking, the entire crew is laid off, and operations are ceased (Corts, 2008). At the time of writing (as of December-2018), 52 units were in lay-up (34 warm-stacked and 18 cold-stacked), representing 47.7% of the total, out of a total of a 109 units in the world fleet excluding 20 units which are still under construction[2].

Albeit demand and drilling dynamics of the offshore drilling market may suggest that some units have to leave the market, scrapping seems to be considered at a limited extent by the contractors. The main reasons for this are high transport costs as well as relatively low steel content of the rigs implying a relatively low scrap price. The proceeds from scrapping are per dwt (dead weight tonnage) of the unit and depend -among others- on the location where the unit is scrapped. Figure 2 presents the scrap steel price developments.

[2] Source: Bassoe Offshore, as in https://seekingalpha.com/article/4228025-offshore-drilling-end-2018-drillships.

Figure 2 –Scrap Steel Price Development (USD per ton)3



3. Empirical Analysis and Numerical Results

We considered day rates (revenues) time series data for the period 2011 – 2018 on a monthly basis as presented in figure 1.

The day rates for offshore drilling rigs have been quite volatile over the observed period. The monthly percentage changes in the day rates have been in the range of -30.4% and 34.2% over the eight years of data.

We analyse a stock-listed company owning and operating offshore drilling rigs. The company is an international company with access to debt and equity financing internationally. The value of the firm is a function of the annual revenues of the unit, calculated by considering the day rates multiplied by average number of days of utilisation in a given year.

3.1 Stationarity Test

We investigated whether GBM assumption is applicable to our day rate data. Following the approach by Sødal et al. (2008), we consider the version of the Augmented Dickey Fuller (ADF) Test suggested by Said and Dickey (1984), as given in.

$$\Delta P_t = d_t + \beta_0 P_{t-1} + \sum_{j=1}^{k} \beta_j \Delta P_{t-j} + \varepsilon_t. \quad (1)$$

where dt is related to deterministic components and Δ is the lag operator. In relation to dt, a

[3] Source: Bassoe Offshore Recycling Report 2018

usual consideration is to set it to its initial value at time zero and/or include a drift term. Following Sødal et al. (2008), we don´t include a time trend since this would imply that the day rate would increase or decrease additionally with a function of time. ADF test uses the null hypothesis that the parameter β_0=0 and therefore the series has a unit root. ADF test solves the problem with the autocorrelation in the error terms by "augmenting" the number of lags of the dependent variable (k). The earlier version of the ADF test was Dickey and Fuller (DF) test, as in Dickey and Fuller (1979) which did not include any lags of the dependent variable (k=0). To account for the choice of optimal number of lags (k), various information criterion are calculated and compared against. The model with the lowest information criteria is selected. The test statistics for the ADF test:

$$t - stat = \frac{\hat{\beta}_0}{SE(\hat{\beta}_0)}. \tag{2}$$

The critical values for the t-statistics are derived from Monte Carlo experiments (Brooks, 2014). $\hat{\beta}_0$ is an estimate derived from the Ordinary Least Squares (OLS) regression whereas $SE(\hat{\beta}_0)$ is its standard deviation.

ADF result on the day rate data in a monthly frequency implied that the null hypothesis of the day rate series has a unit root cannot be rejected. The test statistic is calculated as $\hat{\tau}_{ADF} = -0.22$ with a critical value of -2.89 at 5% confidence level. This indicates that a GBM process is a candidate for the consideration for the development of the day rate revenues and hence the same process for the project value given the assumption that the project has only one uncertainty parameter, being the revenues.

We define the discrete version of the GBM equation is by:

$$Ln\left[P(t)\right] = Ln\left[P(t-\Delta t)\right] + (\alpha - \sigma^2/2)\Delta t + \sigma N(0,1)\sqrt{\Delta t}. \tag{3}$$

Linear regression equation with time increments $\Delta t = 1$ can be described as an autoregressive process with one lag, AR(1). Dickey Fuller tests whether the hypothesis that b=1 can be rejected:

$$Ln(P_t) = a + bLn(P_{t-1}) + \varepsilon_t. \tag{4}$$

where ε_t is the error term of the regression which is normally distributed $\varepsilon \sim N(0,\sigma^2)$.

The estimated parameters from the regression equation (4) by using OLS are illustrated in the figure below:

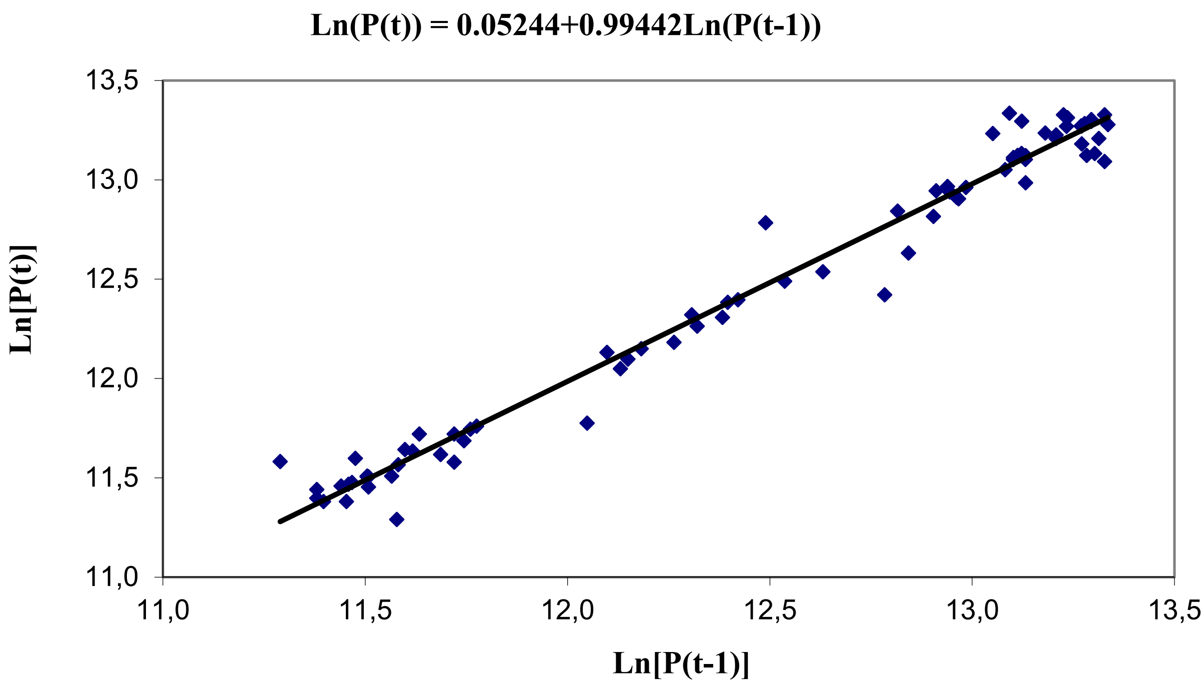
Table 1- Estimated Parameters from a discrete time model

	a	b	S	Number of Observations
Sample 2011-2018	0.05244 *(0.2029)*	0.994426 *(0.01618)*	0.7255	83

**t-statistics are shown in parentheses.*

The statistical test for the regression equation demonstrates that the constant b different than zero cannot be rejected. The autoregressive parameter b is statistically significant from zero. Its value (0.994426) is close to 1, implying that day rate depicts random walk behaviour.

Figure 3- Regression of Day Rates Drillships (>7,500 feet)



As shown by Pindyck (1999), the null hypothesis of existence of unit root under ADF may not easily be rejected. Albeit the fact that parameter b in the regression equation (4) is close to 1, but not equal to 1; there might still be dynamics of MRM with strong persistence. Accordingly, we perform complementary tests KPSS as proposed by Shin and Schmidt (1992) and variance ratio test by Pindyck (1999) and compare the outcomes. As opposed to ADF, KPSS uses the null hypothesis that the time series is stationary. KPSS test concludes that the null hypothesis of stationarity can be rejected at 1% significance level (test statistic: 1.0664 versus asymptotic critical value at 1% of 0.739). Variance ratio test verifies whether the shocks are permanent or temporary. With the probability of 0.2539, we cannot reject the hypothesis that the series is a martingale. In conclusion, the assumption that the day rate series follow a GBM is justified by all three tests performed.

3.2 Project Drift and Volatility Estimation for GBM

We apply a similar approach to Sødal et al. (2008). A detailed description can also be found in (Dias, 2015, pp. 117-128).

Then, the drift parameter α and the volatility parameter σ^2 is given by

$$\alpha = E[Ln(P_t) - Ln(P_{t-1})] + \frac{\sigma^2}{2} \tag{5}$$

$$\sigma^2 = Var[Ln(P_t) - Ln(P_{t-1})]$$

where α and σ^2 is on annual basis. Since our data is on monthly sequence, we let $N = 12$ as number of periods in a year. The error of the regression is $\varepsilon \sim N(0, \sigma^2 / \mathrm{N})$. The parameters of the GBM can be estimated from the equations:

$$\alpha = N\{E[Ln(P_t) - Ln(P_{t-1})] + \frac{\sigma^2}{2N}\} \tag{6}$$

$$\sigma^2 = NVar[Ln[(P_t)/(P_{t-1})]].$$

The estimated continuous time parameters for the GBM equation are $\hat{\alpha} = -14.18\%$ for the drift per annum and $\hat{\sigma} = 36.44\%$ volatility per annum.

Table 2 summarizes the model parameters.

Table 2 – Model Parameters[4]

Parameter Name	Description	Base Case Values
I	Average cost of fresh investment in a drilling rig	$ 346.5 million
E_M	One time cost of mothballing	$ 2.0 million
E_S	Cost of scrapping when already in mothball	$ 26.66 million ($430 per / light weight tons)
E_L	$E_L = E_M + E_S$, E: total cost of abandoning	-$ 2.66 million
R	Sunk cost for reactivation when mothballed	$ 60.0 million
M	Annual cost of maintenance when in mothball	$ 5.475 million ($15K /day x365)
C	Annual operating expenses when operational (fuel and labor costs, incl. regular maintenance)	$45. 625 million ($125 K/ day x 365)
P	Annual revenues (net adjusted day rate x 365)	$ 41.519 million ($ 113.75 K/day x 365)
α	Drift of P	-14.18% per annum
σ	Volatility of P	36.44% per annum
r	Risk-free rate	5% per annum
δ	Dividend yield	5% per annum

E_L<0 since the residual value is positive.

4. The Valuation of an Offshore Drilling Rig

4.1 Stochastic Process of the Price (P)

First of all, we present the optimal strategy path suggested by A. K. Dixit and Pindyck (1994), section 7.2. A. K. Dixit and Pindyck (1994) state that an operator with an active unit has an option to suspend or continue its operations at all times. A rig can be laid-up, and later reactivated should the price rates rise. Laying-up and reactivation involve one-off sunk costs.

[4] Source for costs parameters: Riglogix and Ensco. Average cost of fresh investment is based on an average estimate by Bassoe for the 7th generation drillships: https://www.offshoreenergytoday.com/bassoe-ultra-deepwater-drillship-deepsea-metro-i-sold-for-262-5-m/

Other than that; the owner of the unit has to incur operational costs constantly (flag requirements, maintenance, minimum level of crew on board etc.) during the lay-up albeit at a lesser extent than when the unit is operational. In their model the firm has a monopoly right to invest and accordingly, the possibility that other firms enter in competition is ignored. The firm maintains its market power throughout and keeps its investment option as it is at all times. Accordingly, the project is a perpetuity as long as the exit option is not exercised. The latter is a simplifying assumption to our study here. In practice, albeit barriers-to-entry the drilling industry might not qualify as a monopoly. Moreover, the model investigates an industry-wide uncertainty rather than a firm-wide one. This is more applicable to our study, since the principal uncertainty is related to the drilling industry demand and supply dynamics. We assume a risk-free firm. No arbitrage opportunity necessitates that $I + E > 0.$ We assume that the stochastic process of the day rate (price) follows a Geometric Brownian Motion (GBM) in the form:

$$dP = \alpha P dt + \sigma P dz. \tag{7}$$

where α is the drift or growth rate of the stochastic process and σ is the volatility parameter, dt is the time increment and dz is the increment of a standard Wiener process.

GBM is the most common used process in the real options related applications, and is adopted by all the references listed in the literature review, except for Sødal et al. (2008), which employs a Ornstein-Uhlenbeck process. Bhattacharya (1978) argues that mean-reverting processes would possibly be more relevant than a pure random walk process for investment projects. In the long run the authors anticipate the project cash flows to get back to the indifference levels to enter into new investments. Simulated values under GBM tend to significantly diverge over time over a long horizon Metcalf and Hasset (1995). Schwartz (1997), Tsekrekos (2010) and Sarkar (2003) provide further insights into this subject. We test for stationarity of the uncertainty parameter, day rates, to see whether the GBM assumption is valid.

4.2 Operational Switches between Lay-up/ Re-activate and Scrap

We investigate the operational switches. The company has the option to lay-up or scrap exercisable at any time. For the purpose of this paper, we only analyse the "cold stacking" case as a specific form of lay-up, since such a decision is economically more challenging than a "warm stacking" case. It requires larger sunk costs to reverse. After having entered into lay-up, the ship can be reactivated or scrapped. As these switches are conditional on each other, this creates a compound option problem, which requires the options to be valued simultaneously. We consider 4 switches based on the definitions shown in Table 2. In order to make economic sense $P_L \le P_M \le P_R \le P_H$.

Table 3: Definitions

Switch	**Definition**
Idle-to-invest	Invest when $P \ge P_H$
Operational to Lay-up	Suspend when $P \le P_M$
Lay-up to Re-activate	Re-activate when $P \ge P_R$
Lay-up to Scrap	Abandon when $P \le P_L$

Other than these four switches, there are other two which we ignored for simplification purposes or given limited economic sense. Direct switch from invest to scrap, for instance, would not make much of an economic sense since the high sunk costs would not easily justify such an action. Switch between reactivate to scrap has not been taken into account for simplification purposes and given the fact that the reactivation costs would not readily justify such an immediate action.

Value of an idle firm F(P), active firm V(P) and mothballed firm V_m(P) is given by the differential equations below as detailed in A. K. Dixit and Pindyck (1994, pp. 231-234) as well as in A. Dixit (1989):

$$\frac{1}{2}\sigma^2 P^2\, \mathrm{F}''(\mathrm{P}) + (\mathrm{r}-\delta)\,\mathrm{PF}'(\mathrm{P}) - \mathrm{rF}(\mathrm{P}) = 0. \quad (8)$$

$$\frac{1}{2}\sigma^2 P^2 V''(\mathrm{P}) + (\mathrm{r}-\delta)\,\mathrm{PV}'(\mathrm{P}) - \mathrm{rV}(\mathrm{P}) + \mathrm{P} - \mathrm{C} = 0 \quad (9)$$

$$\frac{1}{2}\sigma^2 P^2 V_m''(\mathrm{P}) + (\mathrm{r}-\delta)\,\mathrm{PV_m}'(\mathrm{P}) - \mathrm{rV_m}(\mathrm{P}) - M = 0 \quad (10)$$

We calculate the value of the firm in every state considering the available options going forward.

Equations (8), (9) and (10) have a general solution of the form

$$\mathrm{F}(P) = \mathrm{A}_1 \mathrm{P}^{\beta_1}, \quad (11)$$

$$V(P) = B_2 \mathrm{P}^{\beta 2} + \frac{P}{\delta} - \frac{C}{r}, \quad (12)$$

$$V_m(P) = D_1 \mathrm{P}^{\beta_1} + D_2 \mathrm{P}^{\beta_2} - \frac{M}{r}, \tag{13}$$

where A_1, B_2, D_1, D_2 are constants to be determined.

β_1 and β_2 refer to the roots of the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1)+(\rho-\delta)\beta-\rho=0.$$

The two roots are:

$$\beta_1=\frac{1}{2}-\frac{(\rho-\delta)}{\sigma^2}+\sqrt{\left[(\rho-\delta)/\sigma^2-\frac{1}{2}\right]+2\rho/\sigma^2}>1,$$

and

$$\beta_2=\frac{1}{2}-\frac{(\rho-\delta)}{\sigma^2}-\sqrt{\left[(\rho-\delta)/\sigma^2-\frac{1}{2}\right]+2\rho/\sigma^2}<0.$$

At each switching point, value-matching and smooth-pasting conditions are applicable.

(14)

$$\mathrm{F}(P_H)=V(P_H)-I, \qquad \mathrm{F}`(P_H)=V`(P_H),$$

$$V_m(P_R)=V(P_R)-\mathrm{R}, \qquad V_m`(P_R)=V`(P_R).$$

$$V_m(P_L)=F(P_L)-E_L, \qquad V_m`(P_L)=F`(P_L).$$

The respective equations in (11)-(14) will derive the values for P_H, P_M, P_R, P_L and the coefficients A_1, B_2, D_1, D_2.

Firstly, starting with the interaction between mothballing and reactivation, at the two threshold values P_M and P_R, we derive

$$-D_1P_R^{\beta_1}+(B_2-D_2)P_R^{\beta_2}+\frac{P_R}{\delta}-\frac{(C-M)}{r}=R, \tag{15}$$

$$-\beta_1D_1P_R^{\beta_1-1}+\beta_2(B_2-D_2)P_R^{\beta_2-1}+\frac{1}{\delta}=0, \tag{16}$$

$$-D_1P_M^{\beta_1}+(B_2-D_2)P_M^{\beta_2}+\frac{P_M}{\delta}-\frac{(C-M)}{r}=-E_M, \tag{17}$$

$$-\beta_1D_1P_M^{\beta_1-1}+\beta_2(B_2-D_2)P_M^{\beta_2-1}+\frac{1}{\delta}=0. \tag{18}$$

Above equations are now four equations and four unknowns D_1, $(B_2 - D_2)$, P_R and P_M.

Secondly, we consider the value-matching and smooth-pasting conditions for new investment:

$$-A_1 P_H^{\beta_1} + B_2 P_H^{\beta_2} + \frac{P_H}{\delta} - \frac{C}{r} = I, \tag{19}$$

$$-\beta_1 A_1 P_H^{\beta_1 - 1} + \beta_2 B_2 P_H^{\beta_2 - 1} + \frac{1}{\delta} = 0. \tag{20}$$

These conditions at the scrapping threshold P_L become

$$(D_2 - A_2) P_L^{\beta_1} + D_2 P_L^{\beta_2} - \frac{M}{r} = -E_L, \tag{21}$$

$$\beta_1 (D_1 - A_1) P_L^{\beta_1 - 1} + \beta_2 D_2 P_L^{\beta_2 - 1} = 0. \tag{22}$$

The solutions to this system is outlined in detail in (Dias, 2015, pp. 381-383). We obtain the equations with regard to the constants D_1, B_3, A_1, D_2 as follows:

(23)

$$D_1 = \frac{(\delta \beta_2 B_3 P_R^{\beta_2 - 1}) + 1}{\beta_1 P_R^{\beta 1 - 1} \delta},$$

$$B_3 = B_2 - D_2 = \frac{(P_R^{\beta_1 - 1} - P_M^{\beta_1 - 1})}{\beta_2 \delta \left[(P_R^{\beta_2 - 1} P_M^{\beta_1 - 1}) - (P_R^{\beta_1 - 1} P_M^{\beta_2 - 1}) \right]},$$

$$A_1 = \frac{(\mathrm{r D_1}\, P_L^{\beta_1}) + (rDP_L^{\beta_2}) - M + (rE_L)}{\mathrm{r}\, P_L^{\beta_1}},$$

$$D_2 = \frac{\mathrm{r}\, P_L^{\beta_1} \left[1 + (\beta_2 P_H^{\beta_2 - 1} - \delta B_3) - (\beta_1 P_H^{\beta_1 - 1} \delta D_1) \right] + \left[\beta_1 P_H^{\beta_1 - 1} \delta (M - rE_L) \right]}{\delta\, \mathrm{r} \left[(P_L^{\beta_2} \beta_1 P_H^{\beta_1 - 1}) - (P_L^{\beta_1} \beta_2 P_H^{\beta_2 - 1} \right]}.$$

Substituting these into (18)-(20) we derive

(24)

$$P_R^{(2\beta_1-2+\beta_2)} r\beta_2 - P_R^{(2\beta_1-1)} r\beta_2 P_M^{\beta_2-1} + P_R^{(\beta_1+\beta_2-1)} r\beta_1 P_M^{\beta_1-1} -$$
$$-P_R^{(2\beta_1-2+\beta_2)} r\beta_1 - P_R^{(\beta_1+\beta_2-1)} r\beta_1\beta_2 P_M^{\beta_1-1} +$$
$$+P_R^{(2\beta_1-1)} r\beta_1\beta_2 P_M^{\beta_2-1} + P_R^{(\beta_1+\beta_2-2)} \delta C\beta_1\beta_2 P_M^{\beta_1-1} -$$
$$-P_R^{(2\beta_1-2)} \delta C\beta_1\beta_2 P_M^{\beta_2-1} - P_R^{(\beta_1+\beta_2-2)} \delta M\beta_1\beta_2 P_M^{\beta_1-1} +$$
$$+P_R^{(2\beta_1-2)} \delta M\beta_1\beta_2 P_M^{\beta_2-1} - P_R^{(\beta_1+\beta_2-2)} R\delta r\beta_1\beta_2 P_M^{\beta_1-1} -$$
$$-P_R^{(2\beta_1-2)} R\delta r\beta_1\beta_2 P_M^{\beta_2-1} = 0.$$

(25)

$$P_R^{(\beta_1-2+\beta_2)} P_M^{\beta_1} r\beta_2 - P_R^{(\beta_1-1)} r\beta_2 P_M^{(\beta_1+\beta_2-1)} + P_R^{(\beta_1+\beta_2-1)} r\beta_1 P_R^{\beta_1-1} -$$
$$-P_R^{(2\beta_1-2)} P_M^{\beta_2} r\beta_1 - P_R^{(\beta_1+\beta_2-2)} r\beta_1\beta_2 P_M^{\beta_1} + P_R^{(2\beta_1-2)} r\beta_1\beta_2 P_M^{\beta_2} +$$
$$+P_R^{(\beta_1+\beta_2-2)} \delta C\beta_1\beta_2 P_M^{\beta_1-1} - P_R^{(2\beta_1-2)} \delta C\beta_1\beta_2 P_M^{\beta_2-1} -$$
$$-P_R^{(\beta_1+\beta_2-2)} \delta M\beta_1\beta_2 P_M^{\beta_1-1} + P_R^{(2\beta_1-2)} \delta M\beta_1\beta_2 P_M^{\beta_2-1} +$$
$$+P_R^{(\beta_1+\beta_2-2)} \delta rE_M\beta_1\beta_2 P_M^{\beta_1-1} + P_R^{(2\beta_1-2)} E_M\delta r\beta_1\beta_2 P_M^{\beta_2-1} = 0.$$

(26)

$$-P_L^{(2\beta_1-1+\beta_2)} rD_1\beta_1 P_H^{\beta_1-1}\delta\beta_2 + P_L^{(2\beta_1+\beta_2-1)} r\beta_2^2 P_H^{\beta_2-1}\delta B_3 +$$
$$+P_L^{(2\beta_1-1+\beta_2)} r\beta_2 - P_L^{(2\beta_1+\beta_2-1)} r\beta_1^2 P_H^{\beta_1-1} D_1\delta -$$
$$+P_L^{(2\beta_1+\beta_2-1)} r\beta_1\beta_2 P_H^{\beta_2-1}\delta B_3 - P_L^{(2\beta_1+\beta_2-1)} r\beta_1 -$$
$$-M\beta_1\beta_2\delta P_L^{(2\beta_1-1)} P_H^{\beta_2-1} - P_L^{(\beta_1+\beta_2-1)} M\beta_1 P_H^{\beta_1-1}\delta\beta_2 +$$
$$+E_L rP_L^{(2\beta_1-1)}\delta\beta_1\beta_2 P_H^{\beta_2-1} - P_L^{(\beta_1+\beta_2-1)} E_L r\beta_1\beta_2\delta P_H^{\beta_1-1} = 0.$$

(27)

$$\beta_1 r^2\delta P_H^{\beta_1}(D_1 P_L^{\beta_1} + D_2 P_L^{\beta_2} + E_L) - \beta_1 r\delta P_H^{\beta_1} M - \beta_2 r^2\delta P_H^{\beta_1}(D_1 P_L^{\beta_1} +$$
$$+D_2 P_L^{\beta_2} + E_L) + \beta_2 r\delta P_H^{\beta_1} M - r^2 P_L^{\beta_1} P_H(1-\beta_2) -$$
$$-rP_L^{\beta_1}\beta_2 C\delta - P_L^{\beta_1}\beta_2 Ir^2\delta = 0.$$

Since it is too complex to obtain an analytical solution, we will provide some numerical solutions and their sensitivities to the change of variables. From Mathlab software, we obtain such solutions to the equations (24)-(27) and derive the thresholds P_M, P_R, $P_L$and P_H,

4.3 Numerical Results of Four-switches-model and Sensitivity Analysis

Figure 5 outlines the main model results with respect to various levels of volatility. It can be seen that P_H and P_R as well as F, V_m and V are highly sensitive to the changes in volatility. The value of the option to invest in a drillship is positive at all times in the analysed volatility range 10%-40%. Under the base case we derive F, the value of the drillship in operation, as $ 362.956 million as well as the value when mothballed of $ 341.49 million. We compare this to $

262.5 million-$ 296.0 million acquisition price of modern drillships (6^{th} and 7^{th} generation) concluded recently (in October-2018 and May-2018 respectively)[5]. This implies that the drillship is under-valued in the market by at least 18.45.%. Furthermore, the day rate thresholds for operational switches in the base case are calculated by dividing the annual revenue thresholds P_H, P_R, P_M and P_L. The respective threshold day rates are P_H=$ 384.065, P_R=$ 202.802, P_M=$ 69.025 and P_L=$ 61.233. At the time of writing the average adjusted day rate (incorporating utilisation) in the market stood at $ 113.750. From our model we conclude that at the current level of day rates it is optimal for a drillship to be maintained in the lay-up mode. As implied by our model, the day rates did not pick up sufficiently yet to support a fresh investment in a drillship which requires a trigger level of $ 384.065 per day.

Table 4- Model Results in $ Million vs. Different Levels of Volatility (σ) in %

Volatility (%)	**P_H**	**P_R**	**P_M**	**P_L**	**F**	**V_M**	**V**
10	83,155	53,087	32,908	39,886	32,89588	62,684	88,469
15	93,269	57,135	30,876	35,319	76,67516	113,901	138,716
20	103,586	61,074	29,213	31,386	126,2832	170,065	194,006
25	114,227	64,984	27,799	28,072	177,1759	225,617	248,738
30	#NV	68,908	26,57	#NV	#NV	#NV	#NV
36,44*	**140,184**	**74,023**	**25,194**	**22,350**	**287,405**	**341,490**	**362,956**
40	148,788	76,891	24,514	20,977	318,5705	373,491	394,490

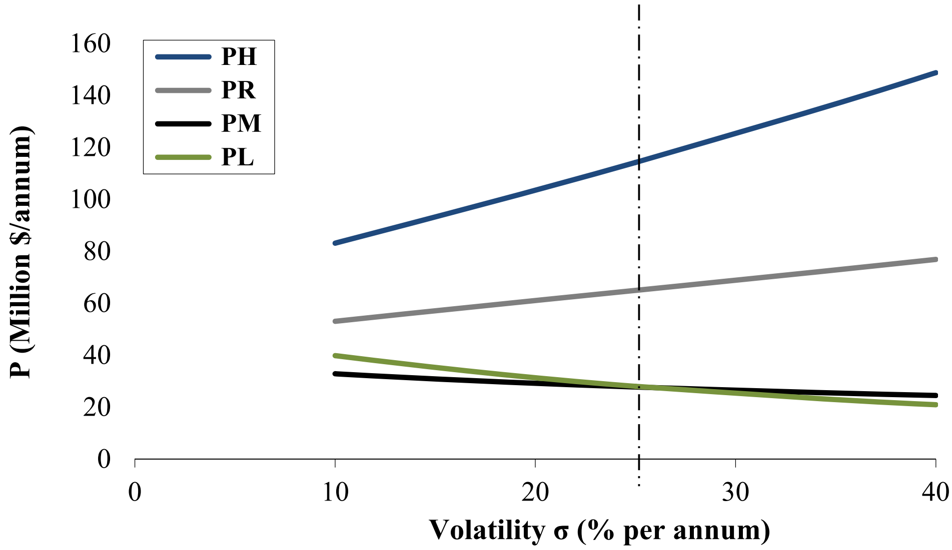
base case. #NV: not valid.

Figure 4 demonstrates that mothballing is solely optimal when the volatility is greater than approximately 30%. As in A. K. Dixit and Pindyck (1994), uncertainty over future demand conditions increases the firm's zone of inaction; that is, it causes the optimal investment and abandonment thresholds to be spread apart.

[5]Sources: https://www.offshoreenergytoday.com/bassoe-ultra-deepwater-drillship-deepsea-metro-i-sold-for-262-5-m/

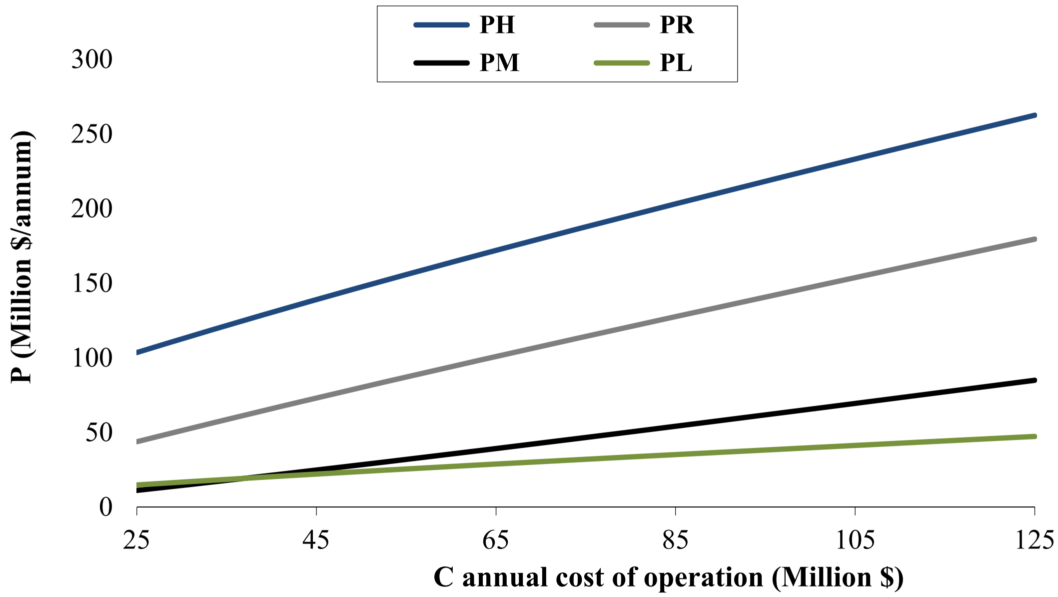
https://www.offshoreenergytoday.com/this-is-how-you-play-the-game-northern-drilling-takes-first-mover-advantage-in-the-deepwater-market-bassoe/

Figure 4- Critical Thresholds as Functions of Volatility



We can observe from figure 5 that the trigger points P_H and P_R are highly responsive to fluctuations in annual cost incurred when the unit is operational. P_M and P_L also have significant level of sensitivity to operational costs. High C implies a high threshold value to enter into an investment. For the trigger values below $ 32 million it is not economically viable to mothball or abandon the unit.

Figure 5- Critical Thresholds as Functions of Annual Cost of Operation C



In figure 6, as anticipated, the critical thresholds P_H and P_R do not show a high sensitivity to the varying values of E_L, total liquidation value obtained from abandoning the unit. When the total revenue from abandoning reaches close to \$55 million, it is economically more viable to liquidate the unit rather than temporarily suspending it.

Figure 6- Critical Thresholds as Functions of total Cost of Abandoning E_L

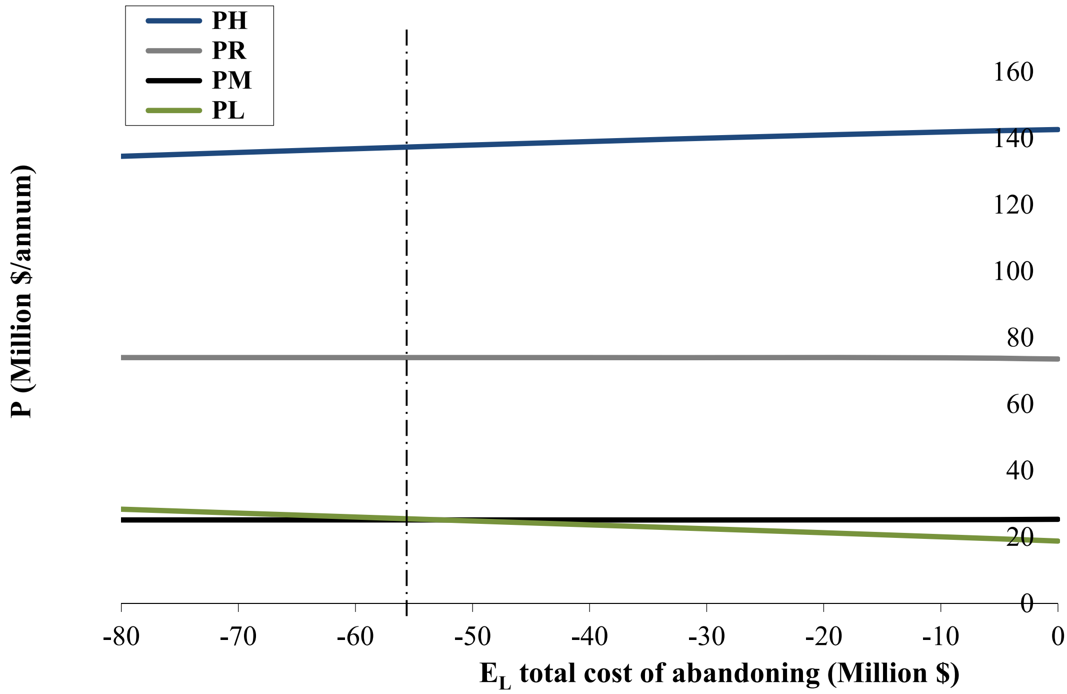
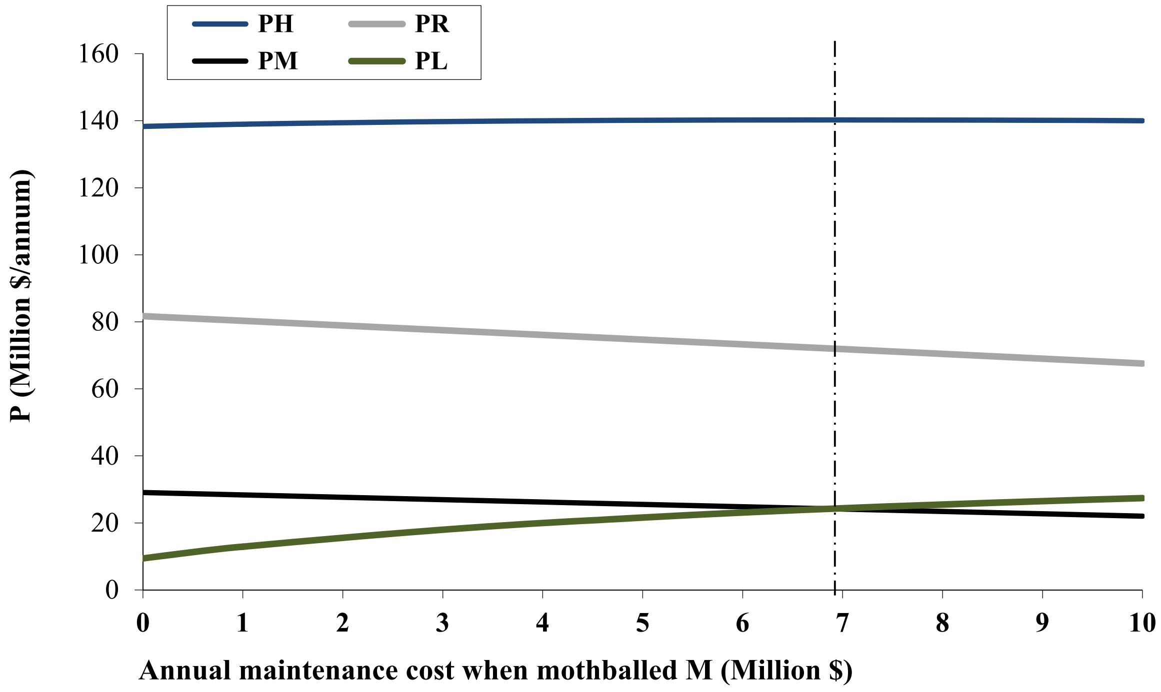


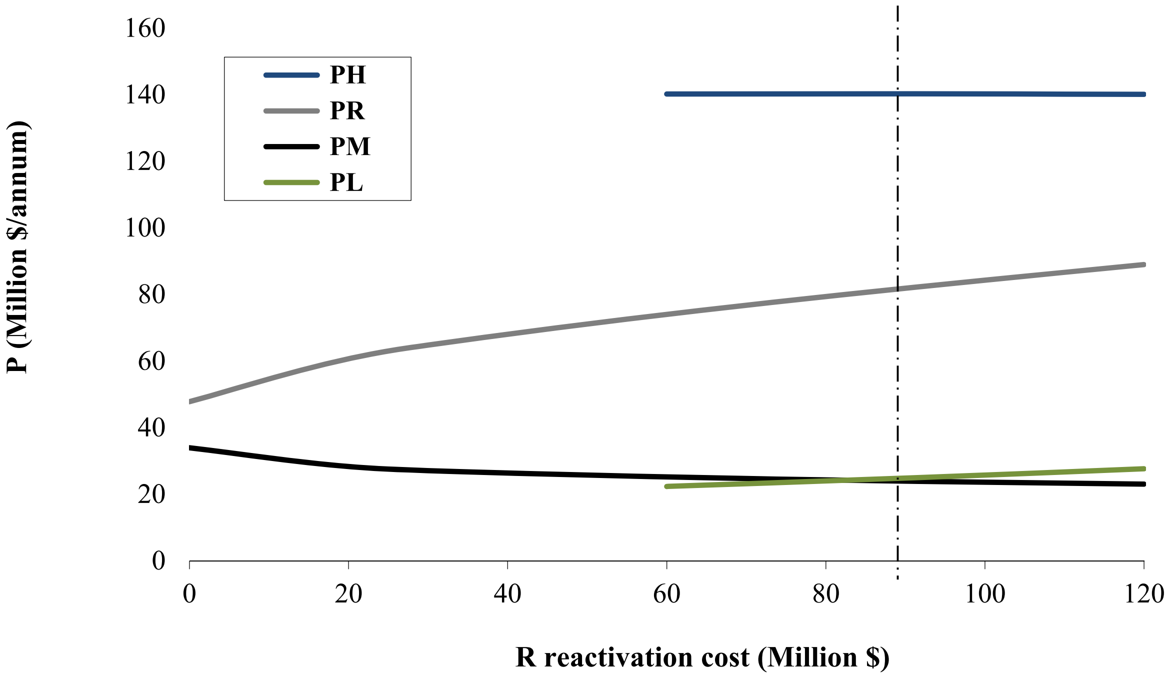
Figure 7 analyses the annual revenues in relation to changes in annual maintenance cost in the mothball mode of operations, M. The thresholds P_M, P_R and P_L depict some sensitivity to changes in M. When M becomes greater than approximately \$7.0 million, it does not make economic sense to lay-up the unit. Instead, abandoning the unit would be preferred. As observed in (Dias, 2015, p. 389) P_H depicts a limited sensitivity to the fluctuations in the value of M. This is explained by the fact that when it is optimal to invest, the probability of entering into a lay-up is remote. Lay-up decision is largely influenced by the values of M. Furthermore, P_H is anticipated to be much larger than P_M.

Figure 7- Critical Thresholds as Functions of Annual Maintenance Cost M



From figure 8, we can observe that above a reactivation cost level of approximately $85 million, it is preferable to scrap the unit directly instead of mothballing it. Especially P_R and P_M are responsive to changes in the reactivation cost.

Figure 8- Critical Thresholds as Functions of Reactivation Cost R



5. Conclusion

Since the offshore drilling sector has been distressed after the oil price crash in the second half of 2014, the companies increasingly focused on fleet rationalisation and capacity optimisation. In this paper we examined a valuation of a large drillship (greater than 7,500 feet) by considering its operational flexibilities by switching into temporary suspension or abandonment by way of scrapping. We constructed a standard entry-exit model with four-switches as in (A. K. Dixit & Pindyck, 1994) where a Geometric Brownian Motion assumption for the day rates (revenues) is tested and applied. Numerical solutions are provided regarding the optimal policies for investing, operating, laying-up and exiting as well as value of the investment opportunity, and value when the drillship is operational or in temporary suspension.

Major findings of this study are as follows: In our base case scenario we obtained the value of the drillship in operation as $ 362.956 million. Its value when in temporary suspension is $ 341.49 million. Market value of recent transactions (concluded in October-2018 and May-2018) to acquire drillships stood at $ 262.5 million and $ 296.0 million. When we compare this to the implied valuation from our model, we conclude that the drillship is under-valued in the market by at least 18.45%. In the longer run, we can expect the developments with regard to the energy transition into more sustainable energy resources to play a significant role. Another conclusion we draw from the model is that at the current level of day rates (as of December-2018), it is optimal for a drillship to be kept in lay-up mode. Moreover, albeit some recovery of the day rates the current levels are not high enough to enter into a new investment in a drillship as pointed out by the model. In addition, the actual level of sunk costs with regard to reactivation justify maintaining the units in suspension rather than direct exit at this moment. Finally, we have shown that the critical entry thresholds are highly sensitive to volatility and operational expenses. The optimal operational trigger points are largely responsive to the maintenance costs during mothballing and reactivation costs.

Furthermore, our model can guide the managers to optimize operations of a drillship. As of September-2018, out of 109 drillships operated globally, solely 57 are operational while the remaining is in lay-up. Given the significant sunk costs involved,decisions to place the unit in or take it of the market are crucial for the market players for survival and efficiency purposes. To the best of our knowledge, this paper is the first to model the investment and operational decisions in relation to drilling rigs after the 2014 oil price crash.

There are some limitations of this paper and next studies can examine the problem by incorporating the subsequent points: 1) Monopoly assumption is applied in the model which ignores the competition element when new players enter into the market. We note, however, that the offshore drilling related sector companies are generally considered as price-takers. Despite the fact that several players operate in the market, the market might not qualify as a perfect competition either principally due to barriers to entry. 2) Uncertainty in the scrap prices which affects the value of the abandonment option has not been considered. Instead, we applied a constant liquidation value when abandoning the investment. Implementation of this would bring in some complexities, since one has to deal with four-switches along with two uncertainties. 3) In practice, there are two different phases of lay-up: warm stacking and cold stacking. The former is when the unit kept running and ready to start to operate. The latter is when a rig is stored at a minimum expense. Warm stacking involves higher operational costs, but lower costs to reactivate compared to the cold stacking alternative. The warm version is usually for a short period of time and the cold version is what is referred to as “mothballing”. One might consider extending the four-phase-model into a five-phase-model including the warm-stacking stage.

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