Strategic Capacity Investment under Uncertainty with Volume Flexibility

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Abstract

This article considers investment decisions in an uncertain and competitive framework, with a first investor, the leader, always producing up to full capacity (dedicated) and a second investor, the follower, being able to adjust output levels within constraint of the installed capacity (flexible). Both firms need to decide on the investment timing and investment capacity size. The main findings are as follows. Compared to a situation where the follower always produces up to full capacity, the leader has a larger incentive to accommodate a flexible follower. This is because the leader also benefits from the follower's volume flexibility. Due to the first mover advantage, the leader's value is higher than the follower's value, despite the follower's technological advantage in flexibility. Moreover, this paper also tries to analyze the preemption between the flexible and the dedicated firm in a competitive framework.

Keywords: Investment under Uncertainty, Volume Flexibility, Entry Deterrence/Accommodation, Capacity Choice, Duopoly

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1 Introduction

Uncertainty is a main characteristic of the business environment nowadays. The technology advancement has shortened product life cycles, increased product variety, and indulged more demanding consumers. This

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contributes to the uncertainty in consumer demand and poses challenges on the manufacturing firms. The ability to produce to the least cost is no longer enough. The capability to absorb demand fluctuations has become an important competitive issue. Flexibility is considered an adaptive response to the environmental uncertainty (Gupta and Goyal, 1989). Browne et al. (1984) have defined eight different flexibility types, among which, the volume flexibility is described as "the ability to operate an FMS (Flexible Manufacturing Systems) profitably at different production volumes." Later on, Sethi and Sethi (1990) further describe volume flexibility as "the ability to be operated profitably at different overall output levels." According to Beach et al. (2000), utilizing flexibility presents performance-related benefits. Numerous studies have supported the importance of volume flexibility (Jack and Raturi, 2002). For instance, Goyal and Netessine (2011) show that volume flexibility may help the firm combat the product demand uncertainty. In a monopolistic market, Hagspiel et al. (2016) and Wen et al. (2017) analyze the volume flexibility's influences on a monopolistic investor's investment decision and show that it increases the value of the investment. In a competitive setting, an important question for the investors would be how the volume flexibility influences investment decisions and the investors' strategic interactions.

This article considers volume flexibility in a homogenous good market. Demand is linear and subject to stochastic shocks, which follow a geometric Brownian motion process. There are two firms that decide on entering the market by investing in a production plant. More specifically, they have to decide about the investment timing and the investment capacity. I take the firm roles as exogenous, and the first investor, the leader, has dedicated technology. The follower, i.e., the firm that invests secondly, has volume flexibility. The leader always produces up to capacity and has a first mover advantage. The follower can adjust output levels according to market demand. A surprising outcome is that, because the market price is affected by the follower's flexible output, the leader benefits from the follower's flexibility when market demand is low. This is because the follower reduces the output quantity in such a case. This provides insights for the equilibrium with endogenous firm roles where the two firms can choose to be volume flexible or to be dedicated. It turns out that the dedicated leader and flexible follower described above is very likely to be the equilibrium for endogenous firm roles. The intuition is that volume flexibility mitigates the fluctuations in market prices, which is attainable with one flexible firm. If one firm chooses to be flexible, the other firm would choose to be dedicated. By doing this, the other firm can occupy a constant market share on one hand, and benefit from the mitigated price fluctuations on the other hand. Besides, the leader will not choose to be flexible. Otherwise the leader would loose its first mover advantage, i.e., the competitive advantage, because the leader cannot commit to an output quantity. The follower would choose to be volume flexible due to the fact that it yields higher value. One can easily find both dedicated and flexible firms in the electricity market: a nuclear power station is dedicated and a fossil fuel power station is flexible. According to Goyal and Netessine (2007), a firm may find it difficult to produce below capacity due to fixed costs associated with, for example, labor, commitment to suppliers and production ramp-up[1].

[1] I will not model these issues explicitly in this article.

The analysis starts with a market where no firms are active. Then two intervals on market demand are identified for the leader, with one interval where it is optimal to deter the entry of the flexible follower and the other one where it is optimal to accommodate the entry. I find that compared to a dedicated follower, the leader is less likely to deter a flexible follower. This is because when there is demand uncertainty, both the leader and the flexible follower tend to wait for more information about the future market and invest later. For the entry deterrence strategy, the leader has an incentive to overinvest to deter the entry of the follower[2]. Being dedicated and unable to the instant market demand, the leader is more vulnerable to the negative demand shocks. For the follower, the volume flexibility yields higher values and thus motivates to invest earlier compared with a dedicated follower. This results in a shorter monopoly period for the leader and diminishes the attractiveness of entry deterrence compared to the case where the follower is dedicated. Furthermore, compared to a dedicated follower, it is more likely for the leader to accommodate a flexible follower. For the accommodation strategy, the two firms invest at the same time, so the incentive to overinvest in order to deter the follower's entry disappears. The market price reacts to the follower's output adjustment, and this diminishes the leader's vulnerability to demand uncertainty. The incentive to overinvest in order to reduce the capacity size of the flexible follower and to benefit from the follower's output adjustment is still strong. This makes accommodation of the flexible follower more attractive to the leader.

I also find that in a fast growing market, the flexible follower produces below capacity right after investment. While in a slowly growing or shrinking market, the flexible follower produces up to capacity right after investment. In the intermediate case, the flexible follower produces up to capacity right after investment when uncertainty is low and below capacity when uncertainty is high. These findings are the same as that for the flexible monopolist by Wen et al. (2017). The strategic interactions between the leader and the flexible follower do not influence these results. Moreover, there is "free riding"on the follower's flexibility since the volume flexibility affects market prices, and thus enlarges the profitability of the leader. So, the flexible follower cannot fully capture the innovative benefits from the technology advancement. However, this does not diminish the follower's incentive to invest in the volume flexibility technology, because it still generates a larger value for the follower regardless the leader chooses and entry deterrence or entry accommodation strategy.

The duopoly model with volume flexibility first contributes to the research stream of monopolistic volume flexibility investment combining investment timing and capacity determination, by Dangl (1999), Hagspiel et al. (2016), and Wen et al. (2017). The general result is that flexibility leads to an increase in the installed capacity and project value. The influence of flexibility on investment timing depends on two effects, with one effect that higher value motivates a flexible firm to invest earlier and the other effect that larger installed capacity motivates it to invest later. This article shows that flexibility affects the flexible follower in a similar way as it affects the flexible monopolist. Its influence on the leader depends on the leader's competition

[2]Overinvesting refers to that a firm invests more capacity as the first investor than when investing simultaneously with the other firm at a predetermined point of time.

strategy, and the dedicated leader also gets a higher value when implementing the accommodation strategy.

In this article, firms not only make decisions about capacities, but also about investment timings in the continuous time setting. It contributes to the literature of capacity choices with volume flexibility in a competitive framework using discrete time models. Gabszewicz and Poddar (1997) study a two-stage model with capacity choice in the first stage and capacity constrained quantity competition in the second stage, and show that the firms choose the certainty-equivalent Cournot capacity. If the second stage is a capacity-constrained price competition instead of quantity competition, Reynolds and Wilson (2000) find that symmetric equilibrium does not exist in pure strategies for capacity choices if demand is sufficient volatile. Besanko and Doraszelski (2004) consider two types of competition: quantity competition and price competition in each period of an infinite time horizon. Quantity competition results in an industry structure of equal-sized firms, while price competition results in unequal-sized firms.

Besides the economics literature, volume flexibility is also studied in operations management. For example, Anupindi and Jiang (2008) consider the volume flexibility in a three-stage framework: capacity choice in the first stage, production decisions in the second stage and pricing decisions in the third stage. Flexible firms can make production decisions when demand is observed. Under competition, they find that firms choose to be inflexible for multiplicative demand shocks, while flexible for additive demand shocks. In a two-product setting with demand uncertainty for both products, Goyal and Netessine (2011) introduce volume flexibility and find that volume flexibility combats aggregate demand uncertainty for the two products. Current research on volume flexibility focuses more on the capacity choices and adopts discrete time models in the analysis. For every dynamic period, the firm needs to decide whether and how much to invest conditional on the available information at the beginning of the period, see for instance Besanko and Doraszelski (2004) and Besanko et al. (2010). In these two papers, the purpose is to analyze the firm sizes in market equilibria. By using a continuous time model, this article analyzes the decision on both investment timing and investment capacity. More specifically, this research analyzes the influence of volume flexibility on the timing of market entry. In a competitive setting, the first investor has a larger incentive to accommodate than to deter the entry of the second investor, given the second investor has volume flexibility. This is due to the fact that volume flexibility combats demand uncertainty for both investors in the market, similarly as that proposed by Goyal and Netessine (2011) for two products.

The duopoly model with flexibility in this research also extends the literature on entry deterrence and entry accommodation investment. According to Lieberman and Montgomery (1988), the first investor's investment serves as a commitment to maintain a high level of production output, which is a price cut threat to decrease entrant's profit. Spence (1977) studies preemptive commitment by constructing static investment models and show that entry can be deterred by installing excess capacity to make a new entrant unprofitable. Maskin (1999) introduces uncertainty and obtains the same conclusion. Dixit (1980) proves that in a static setting, entry deterrence is largely ineffective if the leader cannot commit to producing at full capacity, because the leader facing irrevocable entry finds it optimal to make an accommodating output reduction. I prove that in a stochastic dynamic setting, deterring the entry of a flexible follower does not necessarily make the leader better off. Using discrete time models, Reynolds (1987) shows that the equilibrium capacity choice is a decreasing function of the current rival capacity. This article shows that in a continuous time setting, the second investor's optimal capacity decreases with the first investor's capacity. Besanko et al. (2010) argue that preemption is more likely when the products have low heterogeneity and there is uncertainty about the entrant's exact cost/benefit of capacity addition/withdrawal. In this article, the products are homogeneous, time is continuous, and there is uncertainty about the market demand. My results show that the accommodation is more likely due to the benefits from the influence of volume flexibility on market prices. The asymmetric firm roles of a dedicated leader and a flexible follower, are direct extensions of symmetric firm roles of dedicated leader and follower by Huisman and Kort (2015). In this paper, the influence of volume flexibility is addressed and the entry accommodation is more likely to be implemented by the leader. For the given incumbent's decisions, YANG and ZHOU (2007) show that it is impossible for the incumbent with excess capacity to deter the potential entrant who holds the option to entry forever. This result is supported also by Huisman and Kort (2015), who consider both the deterrence and the accommodation of the potential entrant. In this article, by comparing situation of volume flexibility with situation of no volume flexibility, I focus on the possibility to implement entry deterrence and accommodation, and show that the first investor has less incentive to deter the entry of the second investor if the second investor has volume flexibility.

This article is organized as follows. Section 2 describes the duopoly investment problem. Section 3 analyzes the flexible follower's optimal investment decision. The dedicated leader's optimal investment decision is in Section 4. In Section 5, the influence of flexibility on the leader and the follower is analyzed. Section 7 concludes.

2 Model Setup

Consider a framework where two firms can invest in production capacity to enter a market or serve a particular demand. Of the two firms, the follower (second investor) has volume flexibility technology and adjust output levels up to the installed capacity after the investment. The leader (first investor) has no such technology and can only produce at full capacity level. Denote by $K_D \geq 0$ and $K_F \geq 0$ the capacity of the dedicated leader and the flexible follower, respectively. For both firms, the unit cost for capacity investment is $\delta > 0$ and the unit cost for production is $c > 0$. The price at time $t \geq 0$ is $p(t)$, given by the inverse demand function

$$p(t) = X(t) \left[1 - \gamma \left(q_D(t) + q_F(t)\right)\right],$$

where $\gamma > 0$ is a constant, $q_D(t)$, equal to K_D, and $q_F(t)$, no larger than K_F, denote the production output for the dedicated and flexible firm at time t, respectively, and the uncertainty in demand, $\{X(t) | t \geq 0\}$, follows a geometric Brownian Motion (GBM) process

$$\mathrm{d}X(t) = \alpha X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}W_t,$$

in which $X(0) > 0$, α is the trend parameter, $\sigma > 0$ is the volatility parameter, and dW_t is the increment of a Wiener process. The inverse linear demand function has among others been adopted by Pindyck (1988) and Huisman and Kort (2015). Both firms are risk neutral and have a discount rate of r, which is assumed to be larger than α, the trend of GBM $X(t)$. This is to prevent that it is optimal for the firms to always delay the investment (see Dixit and Pindyck, 1994). From now on I drop the argument of time whenever there can be no misunderstanding.

3 Flexible Follower's Optimal Investment Decision

The leader is assumed to be already in the market when the flexible follower makes investment decisions. Given $X(t) = X$ and the leader's investment capacity K_D, denote $\pi_F(X, K_D, K_F)$ as the profit for the flexible follower after investing in capacity K_F. The follower is flexible and can adjust its output quantity between 0 and the invested capacity K_F. The output maximizes the follower's profit flow, which is equal to

$$\pi_F(X, K_D, K_F) = \max_{0 \le q_F \le K_F} \left\{X\left[1 - \gamma\left(K_D + q_F\right)\right] - c_F\right\} q_F.$$

Given $0 \le K_D < 1/\gamma$, the optimal output level for the follower is

$$q_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{X-c}{2\gamma X} - \frac{K_D}{2} & X \ge \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ K_F & X \ge \frac{c}{1-\gamma K_D} \text{ and } K_F \le \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{1}$$

The corresponding profit flow is given by

$$\pi_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{(X-c-\gamma X K_D)^2}{4\gamma X} & X \ge \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ (X - c - \gamma X K_D) K_F - K_F^2 \gamma X & X \ge \frac{c}{1-\gamma K_D} \text{ and } K_F \le \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{2}$$

The flexible follower's investment decision is solved as an optimal stopping problem and can be formalized as

$$\sup_{T \ge 0, K_F \ge 0} E\left[\int_T^\infty \pi_F(X(t), K_D, K_F)\exp(-rt)\mathrm{d}t - \delta K_F \exp(-rT) \middle| X(0)\right],$$

conditional on the available information at time 0, where T is the time when the flexible follower invests, and K_F is the acquired capacity at time T. Denote by $V_F(X, K_D, K_F)$ the value for the flexible follower, and it satisfies the Bellman equation

$$rV_F = \pi_F + \frac{1}{\mathrm{d}t}E[\mathrm{d}V_F]. \tag{3}$$

Applying Ito's Lemma, substituting and rewriting lead to the following differential equation (see also, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_F(X,K_D,K_F)}{\partial X^2} + \alpha X \frac{\partial V_F(X,K_D,K_F)}{\partial X} - rV_F(X,K_D,K_F) + \pi_F(X,K_D,K_F) = 0. \tag{4}$$

Substituting (2) into (4) and employing value matching and smooth pasting for $X = c/(1-\gamma K_D)$ and $X = c/(1-\gamma K_D - 2\gamma K_F)$ yield the follower's value after investment as given by

$$V_F(X,K_D,K_F) = \begin{cases} L(K_D,K_F)X^{\beta_1} & 0 < X < \frac{c}{1-\gamma K_D}, \\ M_1(K_D,K_F)X^{\beta_1} + M_2(K_D)X^{\beta_2} & \\ \quad + \frac{(1-\gamma K_D)^2 X}{4\gamma(r-\alpha)} - \frac{c(1-\gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r+\alpha-\sigma^2)} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ N(K_D,K_F)X^{\beta_2} - \frac{cK_F}{r} + \frac{XK_F(1-\gamma K_D - \gamma K_F)}{r-\alpha} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \end{cases} \tag{5}$$

in which

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{6}$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < -1. \tag{7}$$

The expressions of $L(K_D,K_F)$, $M_1(K_D,K_F)$, $M_2(K_D)$, $N(K_D,K_F)$ can be found in Appendix A.1. If $K_D = 0$, the model reduces to the monopoly case.

The follower does not produce right after the investment for $0 < X < c/(1-\gamma K_D)$. Thus, $L(K_D,K_F)X^{\beta_1}$ is positive and represents the option value to start producing in the future as soon as X reaches $c/(1-\gamma K_D)$. $M_1(K_D,K_F)X^{\beta_1}$ is negative and corrects for the fact that if X reaches $c/(1-\gamma K_D - 2\gamma K_F)$, the follower's output will be constrained by the installed capacity level. $M_2(K_D)X^{\beta_2}$ has both a negative and a positive effect. The negative effect corrects for the positive quadratic form of cash flows even when X drops below $c/(1-\gamma K_D)$. The positive effect comes from the option that the follower would temporarily suspend production for a too small market demand. When $\sigma^2 < r + \alpha$, the negative effect dominates the positive effect, and if $\sigma^2 > r + \alpha$ the positive effect dominates[3]. $N(K_D,K_F)X^{\beta_2}$ is positive and describes the option value that if demand decreases, i.e., X drops below $c/(1-\gamma K_D - 2\gamma K_D)$, the follower produces below full capacity. The optimal investment decision is found in two steps. First, given K_D and the level of X, the optimal value of K_F is found by maximizing $V_F(X,K_D,K_F) - \delta K_F$, which yields $K_F(X,K_D)$. Second, the optimal investment threshold $X_F^*(K_D)$ for the follower can be derived. The two steps are summarized in the following proposition, where

$$F(\beta) = \frac{2\beta}{r} - \frac{\beta-1}{r-\alpha} - \frac{\beta+1}{r+\alpha-\sigma^2}. \tag{8}$$

[3]Compared to Hagspiel et al. (2016), the dominance of positive and negative effect can be determined in this paper. This is probably due to the fact that I adopt a multiplicative inverse demand structure, and they study an additive inverse demand function.

and $\bar{\sigma}$ is such that

$$\bar{\sigma}^2 = \frac{-2\left(\Lambda - \alpha^2\right)(2r-\alpha) + 4\sqrt{r\Lambda\left(\Lambda-\alpha^2\right)(r-\alpha)}}{\Lambda - (2r-\alpha)^2}, \tag{9}$$

with $\Lambda = \left(\frac{2\delta r(r-\alpha)-\alpha c}{c}\right)^2$. $\bar{\sigma} > 0$ is a value of the drift parameter that determines if the follower produces below or up to capacity right after investment. $\bar{\sigma}$ is only defined for $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$.

Proposition 1 *Given that the dedicated firm has already invested capacity $K_D \in [0, 1/\gamma)$, there are two possibilities for the follower's investment decisions:*

1. *Suppose $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$. The follower produces below capacity right after investment. For any $X \geq c/(1-\gamma K_D)$, the optimal capacity $K_F(X, K_D)$ that maximizes $V(X, K_D, K_F) - \delta K_F$ is given by*

$$K_F\left(X, K_D\right) = \frac{1}{2\gamma}\left(1 - \gamma K_D - \frac{c}{X}\left[\frac{2\delta\left(\beta_1-\beta_2\right)}{c\left(1+\beta_1\right)F\left(\beta_2\right)}\right]^{\frac{1}{\beta_1}}\right), \tag{10}$$

and the optimal investment threshold $X_F^*(K_D)$ satisfies

$$\frac{c(1-\gamma K_D)F(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{2c(1-\gamma K_D)}{r} + \frac{\beta_1+1}{\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}\right] - \delta K_F(X, K_D) = 0. \tag{11}$$

If $X(0) < X_F^*(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F\left(X_F^*(K_D), K_D\right)$. If $X(0) \geq X_F^*(K_D)$, then the follower invests at $t = 0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.

2. *Suppose $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma \leq \bar{\sigma}$. Then the follower produces up to capacity right after investment. For any $X \geq c/(1-\gamma K_D)$, the optimal capacity $K_F(X, K_D)$ satisfies*

$$\frac{c\left(1+\beta_2\right)F\left(\beta_1\right)}{2\left(\beta_1-\beta_2\right)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_2} + \frac{X(1-2\gamma K_F-\gamma K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0, \tag{12}$$

and the optimal investment threshold $X_F^*(K_D)$ satisfies

$$\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X}{c}\right)^{\beta_2}\left((1-\gamma K_D)^{1+\beta_2} - (1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right) + \frac{(\beta_1-1)X}{\beta_1}\frac{K_F-\gamma K_D K_F-\gamma K_F^2}{r-\alpha} - \frac{cK_F}{r} - \delta K_F = 0, \tag{13}$$

with $K_F = K_F(X, K_D)$. If $X(0) < X_F^*(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F(X_F^*(K_D), K_D)$. If $X(0) \geq X_F^*(K_D)$, then the follower invests at $t = 0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.

From Proposition 1, the influence of the leader's investment capacity on the follower's investment decision is concluded in Corollary 1. Their proof can be found in Appendix A.2 and A.3.

Corollary 1 *The dedicated leader's capacity level K_D influences the follower's investment decision such that if the leader invests more, then the follower invests later and invests less.*

This result is intuitive because the leader always produces up to capacity after investment, and the more the leader invests, the smaller market share is left for the flexible follower. When deciding on the capacity, the follower takes the future market demand into consideration. Thus, a smaller market share decreases the follower's investment capacity. Moreover, given the current market demand level, the market price decreases if the leader invests more. This would lower the follower's potential profits and delay the follower's entry because the follower prefers to wait for a higher market price.

4 Dedicated Leader's Optimal Investment Decision

The leader also takes the follower's decisions into consideration when deciding on the market entry. Suppose the leader invests at t with capacity size K_D and $X(t) = X$. Corollary 1 shows that the leader's capacity influences the follower investment timing. Assume there exists a capacity size for the leader, $\hat{K}_D(X)$, such that the follower's optimal threshold satisfies $X_F^*(\hat{K}_D) = X$, and $\hat{K}_D(X)$ can be derived from (11) as to satisfy

$$\frac{c(1-\gamma K_D)F(\beta_1)}{2\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2} + \frac{\beta_1-1}{2\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{c(1-\gamma K_D)}{r} + \frac{\beta_1+1}{2\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}$$
$$-\delta(1-\gamma K_D) + \frac{c\delta}{X}\left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}} = 0. \quad (14)$$

From Corollary 1 it can be concluded that if $K_D \leq \hat{K}_D(X)$, then $X \geq X_F^*(K_D)$, implying that the follower invests at the same time with the leader. If $K_D > \hat{K}_D(X)$, then $X < X_F^*(K_D)$, implying that the follower invests later than the leader. The former corresponds to the leader's entry accommodation strategy and the latter corresponds to the entry deterrence strategy, as described by Huisman and Kort (2015). In the following analysis, the leader's entry accommodation and entry deterrence strategy are characterized as the local optimum for the leader's value maximization problem given by

$$\sup_{K_D\geq 0} E\Bigg[\int_0^T (K_D(1-\gamma K_D)X(t) - cK_D)\, e^{-rt}\mathrm{d}t$$
$$+\int_T^\infty (K_D(1-\gamma K_D - \gamma q_F(X, K_D, K_F))X(t) - cK_D)\, e^{-rt}\mathrm{d}t - \delta K_D \,\Bigg|\, X(0) = X\Bigg],$$

where T is the moment that the flexible follower invests. Note that $T > 0$ under the entry deterrence strategy and $T = 0$ under the entry accommodation strategy.

The leader's investment value is generated by the leader's profit flow. Before the follower's entry, the leader is the only producer in the market. After the follower's entry, both firms are active in the market. The follower might not produce, produce below, and produces up to capacity after investment. Thus there are three cases for the leader's profit flow. For the given GBM level X and the leader's capacity size K_D, the leader's profit flow $\pi_D(X, K_D)$ is given by

$$\pi_D(X, K_D) = \begin{cases} K_D(1-\gamma K_D)X - cK_D & \text{if } 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{K_D}{2}(X - c - \gamma X K_D) & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ XK_D[1-\gamma(K_D + K_F^*(K_D))] - cK_D & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases}$$

Applying Ito's Lemma, substituting and rewriting leads to the following differential equation (see, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_D(X, K_D)}{\partial X^2} + \alpha X \frac{\partial V_D(X, K_D)}{\partial X} - rV_D(X, K_D) + \pi_D(X, K_D) = 0.$$

Substituting π_D into this differential equation and employing value matching and smooth pasting at $X = c/(1-\gamma K_D)$ and $X = c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$ give the value of the leader after the follower's investment as

$$V_D(X, K_D) = \begin{cases} \mathcal{L}(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} & \text{if } 0 \leq X < \frac{c}{1-\gamma K_D}, \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} & \\ \quad + \frac{XK_D(1-\gamma K_D)}{2(r-\alpha)} - \frac{cK_D}{2r} & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ \mathcal{N}(K_D)X^{\beta_2} - \frac{cK_D}{r} & \\ \quad + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{15}$$

The derivation and expressions of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, $\mathcal{N}(K_D)$, and their signs can be found in Appendix A.4. For $0 \leq X < c/(1-\gamma K_D)$, the demand is so low that the follower's production is temporarily suspended. However, the dedicated leader still produces at full capacity. In the leader's value function, $\mathcal{L}(K_D)X^{\beta_1}$ measures the decrease in the leader's value when the follower resumes production in the future. This happens as soon as X becomes larger than $c/(1-\gamma K_D)$. For $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) > (X-c)/(2\gamma X) - K_D/2$, i.e., $c/(1-\gamma K_D) \leq X < c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces below capacity right after investment. $\mathcal{M}_1(K_D)X^{\beta_1}$ corrects for the fact that if X reaches $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, then the production of the follower is constrained by the installed capacity, hence the value of the leader increases. The term $\mathcal{M}_2(K_D)X^{\beta_2}$ denotes the decrease in the leader's option value, due to the fact that when X falls below $c/(1-\gamma K_D)$, the market demand becomes so small that the follower suspends production, whereas the leader still produces at full capacity, which results in negative profit. For $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) \leq (X-c)/(2\gamma X) - K_D/2$, i.e., $X \geq c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces up to capacity right after investment. The term $\mathcal{N}(K_D)X^{\beta_2}$ corrects for the fact that when X drops below $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces below capacity, and the value of the leader would increase.

The leader's strategies are analyzed for two cases, i.e., the follower produces below and up to capacity right after investment. This is because according to Wen et al. (2017), the flexible firm always produces right after investment. Before the follower invests, the leader's value function consists of two parts with one part from the monopolistic profit flow, and the other part correcting for the fact that the leader loses its monopoly privilege when the follower invests. Given that the leader invests at X, let the leader's value before the follower's entry be

$$V_D(X, K_D) = \mathcal{B}(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r},$$

where $\mathcal{B}(K_D)$ has different expressions and will be derived for the two cases[4]. The leader's value function after the follower's investment is shown in (15). Then in every case both the entry deterrence and the entry accommodation strategy are analyzed.

- The flexible follower produces below capacity right after investment when $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$.

Given that the leader invests at X, the value function before and after the follower's entry is as follows

$$V_D(X, K_D) = \begin{cases} \mathcal{B}_1(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} & X < X_F^*(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} & X \ge X_F^*(K_D), \end{cases} \tag{16}$$

with

$$\mathcal{B}_1(K_D) = \mathcal{M}_1(K_D) + \mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) + \frac{cK_D}{2r}X_F^{*-\beta_1}(K_D), \tag{17}$$

according to value matching condition at $X_F^*(K_D)$, which is defined by (11). Intuitively, $\mathcal{B}_1(K_D)$ is negative (see Appendix A.5). It corrects for the fact that when $X(t)$ reaches $X_F^*(K_D)$, the follower enters the market, putting an end to the leader's monopolistic privilege. The leader's entry deterrence and accommodation strategies, when the follower produces below capacity right after investment, are described in the following proposition (see also Appendix A.5 for the proof).

Proposition 2 *Suppose $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$.*

(a) Entry Deterrence Strategy

The entry deterrence strategy will be considered whenever $X \in (X_1^{det}, X_2^{det})$, where X_1^{det} satisfies

$$\left(\frac{X^{det}}{X_F^*(0)}\right)^{\beta_1}\left[-\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha} - \frac{\beta_2}{r}\right) + \frac{c^{1-\beta_2}X_F^{*\beta_2}(0)}{2(\beta_1-\beta_2))}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right) - \frac{X_F^*(0)}{2(r-\alpha)} + \frac{c}{2r}\right] + \frac{X^{det}}{r-\alpha} - \frac{c}{r} - \delta = 0, \tag{18}$$

[4] $\mathcal{B}(K_D)$ and $\mathcal{L}(K_D)$ are different. According to Dixit and Pindyck (1994), the fundamental component in the leader's value function, i.e., $\frac{K_D(1-\gamma K_D)}{r}X - \frac{K_D}{r}$, is generated by the profit flows. $\mathcal{L}(K_D)X^{\beta_1}$ describes the deviation of $V_D(X, K_D)$ from the fundamental component due to the possibility that X will move across the boundary $\frac{c}{1-\gamma K_D}$. $\mathcal{B}(K_D)X^{\beta_1}$ describes the deviation of $V_D(X, K_D)$ from the fundamental component due to the possibility that X will move across the follower's optimal investment threshold X_F^*.

where $X_F^*(0)$ *can be derived from* (10) *and* (11) *given that* $K_D = 0$, *and* X_2^{det} *together with* $K_D^{det}(X_2^{det})$ *satisfy* (14) *and*

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D)(X^{det})^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X^{det}-\frac{c}{r}-\delta=0. \tag{19}$$

The optimal investment threshold X_D^{det} *and investment capacity* K_D^{det} *are*

$$\begin{aligned} X_D^{det} &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right), \\ K_D^{det} &\equiv K_D^{det}(X_D^{det})=\frac{1}{(\beta_1+1)\gamma}, \end{aligned}$$

when $X < X_D^{det}$ *and* $X_D^{det} \in [X_1^{det}, X_2^{det}]$. *If* $X_D^{det} \leq X \leq X_2^{det}$, *in order to implement the entry deterrence strategy, the leader invests immediately at* X *with capacity* $K_D^{det}(X)$ *that satisfies* (19). *Then the value of the entry deterrence strategy is*

$$V_D^{det}(X)=\mathcal{B}_1(K_D^{det}(X))X^{\beta_1}+\frac{K_D^{det}(X)\left(1-\gamma K_D^{det}(X)\right)}{r-\alpha}X-\frac{cK_D^{det}(X)}{r}. \tag{20}$$

(b) *Entry Accommodation Strategy*

The entry accommodation strategy will be considered if $X \geq X_1^{acc}$, *where* X_1^{acc} *and the corresponding* $K_D^{acc}(X_1^{acc})$ *satisfy* (14) *and*

$$\begin{aligned} \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)(X^{acc})^{\beta_1}+\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)(X^{acc})^{\beta_2} \\ +\frac{1-2\gamma K_D}{2(r-\alpha)}X^{acc}-\frac{c}{2r}-\delta=0. \end{aligned} \tag{21}$$

The optimal investment threshold X_D^{acc} *satisfies*

$$\frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{\beta_1 X^{acc}}{c(\beta_1+1)}\right)^{\beta_2}+\frac{(\beta_1-1)X^{acc}}{2(r-\alpha)(\beta_1+1)}-\frac{c}{2r}-\delta=0, \tag{22}$$

when $X < X_D^{acc}$ *and* $X_D^{acc} \geq X_1^{acc}$. *The optimal investment capacity for the entry accommodation strategy is*

$$K_D^{acc} \equiv K_D^{acc}(X_D^{acc})=\frac{1}{(\beta_1+1)\gamma}.$$

If $X \geq X_D^{acc}$, *in order to implement the entry accommodation strategy, the leader invests immediately at* X *with capacity* $K_D^{acc}(X)$ *that satisfies* (21). *The value of the entry accommodation strategy is*

$$V_D^{acc}(X)=\mathcal{M}_1(K_D^{acc}(X))X^{\beta_1}+\mathcal{M}_2(K_D^{acc}(X))X^{\beta_2}+\frac{K_D^{acc}(X)\left(1-\gamma K_D^{acc}(X)\right)}{2(r-\alpha)}X-\frac{cK_D^{acc}(X)}{2r}. \tag{23}$$

- The flexible follower produces up to capacity right after the investment when $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma \leq \bar{\sigma}$.

Similar to where the follower produces below capacity right after investment, given that the leader invests at X, the value function before and after the follower's entry can be written as

$$V_D(X,K_D)=\begin{cases} \mathcal{B}_2(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{cK_D}{r} & X < X_F^*(K_D), \\ \mathcal{N}(K_D)X^{\beta_2}+\frac{K_D(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}X-\frac{cK_D}{r} & X \geq X_F^*(K_D), \end{cases} \tag{24}$$

with

$$\mathcal{B}_2(K_D) = \mathcal{N}(K_D) X_F^{*\beta_2-\beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r-\alpha} X_F^{*1-\beta_1}(K_D), \tag{25}$$

according to the value matching condition at the flexible follower's investment threshold $X_F^*(K_D)$, which is defined by (13).

Similar as $\mathcal{B}_1(K_D)$, $\mathcal{B}_2(K_D)$ corrects for the fact that when the follower enters the market, i.e. X reaches $X_F^*(K_D)$, it would put an end to the leader's monopoly privilege. Thus, $\mathcal{B}_2(K_D)$ is negative, shown in Appendix A.6. Because $X_F^*(K_D)$ increases with K_D according to Corollary 1, it is possible for the dedicated leader to delay the entry of flexible follower through the entry deterrence strategy by investing $K_D^{det}(X) > \hat{K}_D(X)$. Otherwise, the two firms invest at the same time, implying the leader applies the entry accommodation strategy by investing $K_D^{acc} \leq \hat{K}_D(X)$. This critical size for the leader's capacity, $\hat{K}_D(X)$, can be derived from (13) with the follower's optimal investment capacity $K_F^*(X) \equiv K_F^*(K_D(X))$ satisfying (12).

The leader's investment decision under entry deterrence and accommodation strategies, when the follower produces up to capacity right after investment, are summarized in the following proposition with the proof in Appendix A.6.

Proposition 3 *Suppose $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma \leq \bar{\sigma}$.*

(a) Entry Deterrence Strategy

The entry deterrence strategy is possible if $X \in \left(X_1^{det}, X_2^{\mathrm{det}}\right)$. X_1^{det} satisfies

$$\frac{c}{2(\beta_1-\beta_2)}\left(\frac{X^{det}}{X_F^*(0)}\right)^{\beta_1}\left(\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left[\left(\frac{X_F^*(0)}{c}\right)^{\beta_2}-\left(\frac{X_F^*(0)\left(1-2\gamma K_F^*(0)\right)}{c}\right)^{\beta_2}\right]\right.$$
$$\left.-\frac{\beta_1-\beta_2}{r-\alpha}\frac{2\gamma X_F^*(0)K_F^*(0)}{c}\right)+\frac{X^{det}}{r-\alpha}-\frac{c}{r}-\delta=0, \tag{26}$$

where $K_F^*(0)$ and $X_F^*(0)$ can be derived from (12) *and* (13) *given that $K_D = 0$. X_2^{det}, $K_D^{det}(X_2^{det})$ and $K_F^*(X_2^{det})$ satisfy* (12), (13), *and*

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_2(K_D)(X^{det})^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X^{det}-\frac{c}{r}-\delta=0. \tag{27}$$

The optimal investment threshold X_D^{det} and the corresponding optimal capacity K_D^{det} are equal to

$$\begin{aligned} X_D^{det} &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right), \\ K_D^{det} &\equiv K_D^{det}(X_D^{det}) = \frac{1}{(\beta_1+1)\gamma}, \end{aligned}$$

if $X < X_D^{det}$ and $X_D^{det} \in [X_1^{det}, X_2^{det}]$. If $X_D^{det} \leq X < X_2^{det}$, in order to implement the entry deterrence strategy, the leader invests immediately at X with capacity $K_D^{det}(X)$ that satisfies (27)*. The value of the entry deterrence strategy is*

$$V_D^{det}(X) = \mathcal{B}_2(K_D^{det}(X))X^{\beta_1} + \frac{K_D^{det}(X)\left(1-\gamma K_D^{det}(X)\right)}{r-\alpha}X - \frac{cK_D^{det}(X)}{r}. \tag{28}$$

(b) Entry Accommodation Strategy

The entry accommodation strategy is possible if $X > X_1^{acc}$. X_1^{acc}, $K_D^{acc}(X_1^{acc})$, and $K_F^*(X_1^{acc})$ satisfy (12), (13), *and*

$$\frac{(1-\gamma K_D-\beta_2\gamma K_D)(X^{acc})^{\beta_2}}{K_D(1-\gamma K_D)}\mathcal{N}(K_D)+\frac{X^{acc}(1-\gamma K_D-\gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)}-\frac{c}{r}-\delta=0. \quad (29)$$

The optimal investment threshold X_D^{acc} satisfies

$$\frac{c(X^{acc})^{\beta_2}}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\left(\frac{1-\gamma K_D^{acc}}{c}\right)^{\beta_2}-\left(\frac{1-\gamma K_D^{acc}-2\gamma K_F^*(K_D^{acc})}{c}\right)^{\beta_2}\right)$$
$$+\frac{(\beta_1-1)X^{acc}}{\beta_1(r-\alpha)}\left(1-\gamma K_D^{acc}-\gamma K_F^*(K_D^{acc})\right)-\frac{c}{r}-\delta=0, \quad (30)$$

if $X < X_D^{acc}$ and $X_D^{acc} \geq X_1^{acc}$. The optimal investment capacity for the entry accommodation strategy is

$$K_D^{acc} \equiv K_D^{acc}(X_D^{acc}) = \frac{1}{(\beta_1+1)\gamma}.$$

If $X \geq X_D^{acc}$, in order to implement the entry accommodation strategy, the leader invests immediately at X and the corresponding capacity $K_D^{acc}(X)$ satisfies (29). *The value of the entry accommodation strategy is*

$$V_D^{acc}(X)=\mathcal{N}(K_D^{acc}(X))X^{\beta_2}+\frac{K_D^{acc}(X)\left(1-\gamma K_D-\gamma K_F^*(K_D^{acc}(X))\right)}{r-\alpha}X-\frac{cK_D^{acc}(X)}{r}. \quad (31)$$

A numerical example is provided to illustrate the possibility for the entry deterrence and accommodation strategies in Figure 1 and 2. Note that in this example the follower produces below capacity right after investment. Similar analysis can be conducted for the follower producing up to capacity right after investment.

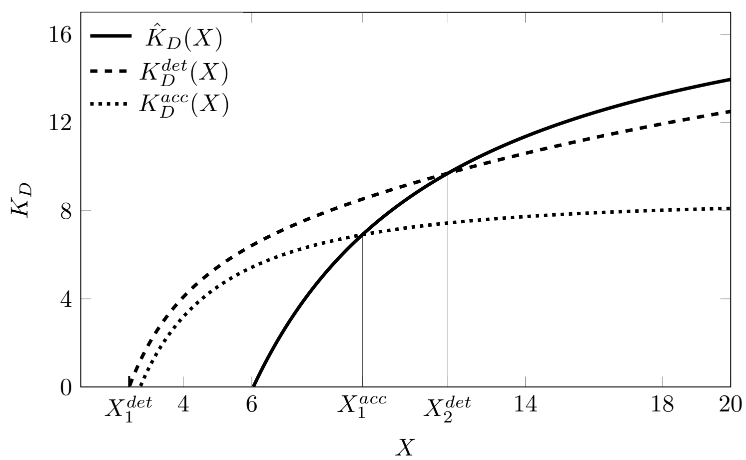

Figure 1: Illustration of $\hat{K}_D(X)$, $K_D^{det}(X)$, and $K_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 1 illustrates the capacity levels $\hat{K}_D$, K_D^{det}, and K_D^{acc} as functions of X. For the given parameter values, the leader implements the deterrence strategy for $X \in [X_1^{det}, X_2^{det}]$, and the accommodation strategy for $X \geq X_1^{acc}$. When both strategies are implementable, the leader chooses the strategy that generates higher values. More specifically, for the given parameter values in Figure 1, $X_1^{det} = 2.42$, $X_2^{det} = 11.73$. The optimal threshold for the entry deterrence strategy is $X_D^{det} = 6.30$. Suppose the current level of geometric Brownian motion is X. If $X < 6.30$, to delay the entry of the flexible follower, the leader waits until X reaches 6.30. For any X between 6.30 and 11.73, the leader needs to invest immediately to delay the flexible follower. For $X > 11.73$, the entry deterrence strategy is not possible because the market demand is large enough for both firms to be active. Moreover, $X_D^{acc} = 8.50 < X_1^{acc} = 9.23$, which makes X_D^{acc} have no meaning for the leader in this numerical example. This is because X has to reach X_1^{acc} to make the follower invest at the same time as the leader.

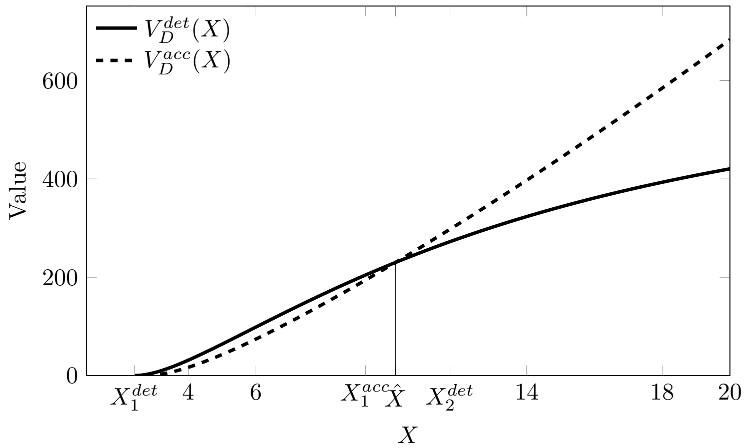

Figure 2: Illustration of $V_D^{det}(X)$ and $V_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 2 shows the value of the entry deterrence strategy V_D^{det} and accommodation strategy V_D^{acc} as functions of X, for the case that the flexible follower produces below capacity right after investment. Note that $V_D^{det} = V_D^{acc}$ at $X = \hat{X}$. For $X_1^{det} < X < \hat{X}$, the deterrence strategy is chosen and the leader invests at $X_D^{det} = 6.30$ with capacity $K_D(X_D^{det}) = 6.67$. For $X \geq \hat{X}$, the leader implements the accommodation strategy. Given that $\hat{X} > X_1^{acc}$, the leader invests immediately with capacity level $K_D^{acc}(X)$ if $X \geq \hat{X}$.

It can be concluded from Proposition 2 and 3 that the accommodation strategy is not possible if $X < X_1^{acc}$, and the deterrence strategy is not possible if $X > X_2^{det}$. When $X_1^{acc} < X < X_2^{det}$, the strategy that gives higher value will be chosen. Huisman and Kort (2015) have shown analytically that $X_1^{acc} < X_2^{det}$ when there is no volume flexibility. Figure 3 checks numerically whether this still holds for a flexible follower. Departing from the default parameter values $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, and $\gamma = 0.05$, when changing σ, α, r, c, δ and γ, X_2^{det} is always larger than X_1^{acc}. Thus, it can be assumed that $X_2^{det} > X_1^{acc}$ also holds for a flexible follower[5]. However, different from Huisman and Kort (2015), where $X_D^{acc} < X_1^{acc}$ always holds, the

[5]Given that $X_2^{det} > X_1^{acc}$, there is no boundary solution when analyzing the entry deterrence and entry accommodation strategies, which are two local optimum for the leader's investment problem.

numerical analysis in Figure 3 shows that for significantly small α or δ, $X_D^{acc} > X_1^{acc}$. Note that X_D^{acc} implies that the market demand should be large enough to accommodate both firms. When α is small or negative, and the follower produces up to full capacity right after investment, a larger market demand is required to accommodate two firms. This leads to $X_D^{acc} > X_1^{acc}$. When δ is small, i.e., investing is less costly, both firms are encouraged to install larger capacities and a larger X_D^{acc} results. The above analysis is summarized in the following proposition.

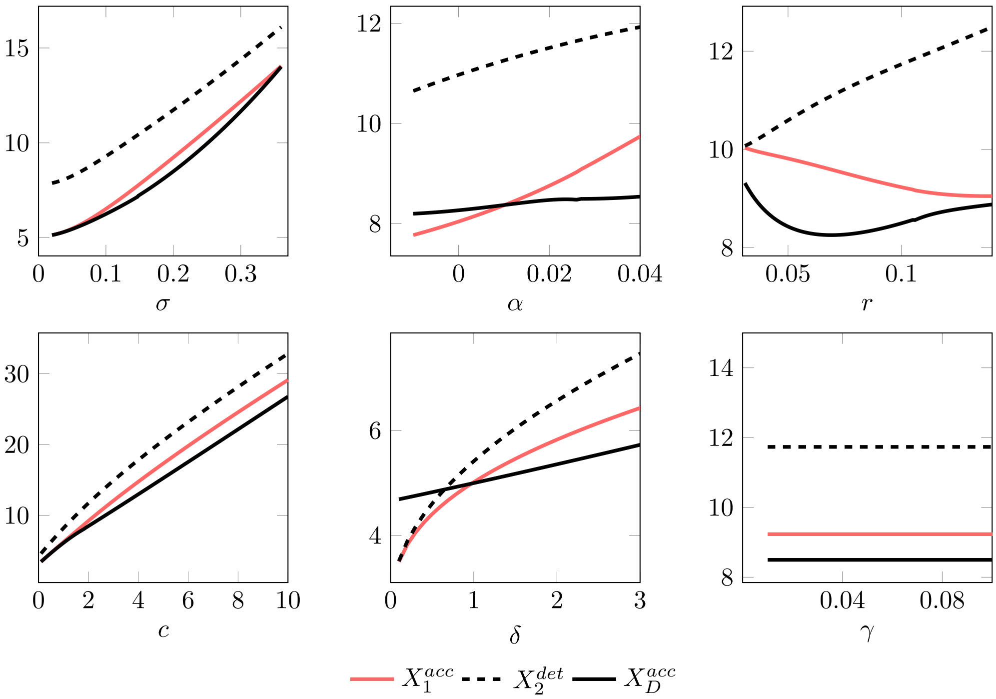

Figure 3: Illustration of X_1^{acc}, X_2^{det}, and X_D^{acc}. Default parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

Proposition 4 *Denote $\hat{X}$ as*

$$\hat{X} = \min\{X | X_1^{acc} < X < X_2^{det} \text{ and } V_D^{acc}(X) = V_D^{det}(X)\}.$$

Let $X(t) = X$, the optimal investment capacity for the leader is

$$K_D^*(X) = \begin{cases} K_D^{det}(X_D^{det}) & \text{if } 0 \le X < X_D^{det}, \\ K_D^{det}(X) & \text{if } X_D^{det} \le X < \hat{X}, \\ K_D^{acc}(X_D^{acc}) \text{ or } K_D^{det}(\hat{X}) & \text{if } \hat{X} \le X < X_D^{acc}, \\ K_D^{acc}(X) & \text{if } X \ge \max\{\hat{X}, X_D^{acc}\}. \end{cases} \tag{32}$$

The optimal investment threshold for the leader is

$$X_D^* = \begin{cases} X_D^{det} & \textit{if } 0 \leq X < X_D^{det}, \\ X & \textit{if } X_D^{det} \leq X < \hat{X}, \\ X_D^{acc} \textit{ or } \hat{X} & \textit{if } \hat{X} \leq X < X_D^{acc}, \\ X & \textit{if } X \geq \max\{\hat{X}, X_D^{acc}\}. \end{cases} \tag{33}$$

The leader's and the follower's optimal investment capacities, $K_D^*(X)$ and $K_F^*(X)$, are demonstrated in Figure 4 when the follower produces below capacity right after investment. For given parameter values $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, and $\delta = 0.5$, then $X_D^{det} = 4.3050$ and $\hat{X} = 4.4779$. If $X < X_D^{det}$, the leader waits until X reaches X_D^{det} to implement the entry deterrence strategy. If $X_D^{det} \leq X < \hat{X}$, the entry deterrence strategy is implemented immediately at X. When $X \geq \hat{X}$, the leader chooses entry accommodation strategy because it yields higher value. Different from Huisman and Kort (2015) that $X_D^{acc} < \hat{X}$, we have in this numerical example that $X_1^{acc} = 4.4072 < \hat{X} < X_D^{acc} = 4.8238$. For $\hat{X} \leq X < X_D^{acc}$, the leader waits until X reaches X_D^{acc}, i.e., the leader is holding an option to invest in the accommodation strategy. This is shown in Figure 4 as the void area for the interval $\hat{X} \leq X < X_D^{acc}$.

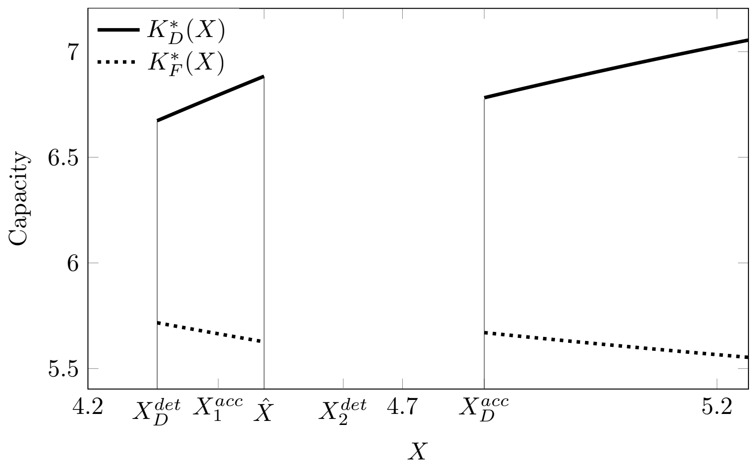

Figure 4: Illustration of $K_D^*(X)$ and $K_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 0.5$.

Figure 5 demonstrates the values of the leader and the follower as functions of X when the follower produces below capacity right after investment. If $X < X_D^{det}$, the leader waits to invest with the entry deterrence strategy capacity. The follower is also waiting to invest, and expects the leader to invest at X_D^{det} with capacity $K_D^{det}(X_D^{det})$. If $X_D^{det} \leq X < \hat{X}$, the leader invests immediately at level X with deterrence capacity $K_D^{det}(X)$. When $\hat{X} \leq X < X_D^{acc}$, the leader implements entry accommodation strategy and waits to invest at X_D^{acc} with capacity $K_D^{acc}(X_D^{acc})$. The follower invests at the same time but with capacity $K_F^*(K_D^{acc}(X_D^{acc}))$. Because of the switch from the entry deterrence to accommodation, the leader's value function has a kink and the follower's value function is shown to jump at $\hat{X}$. When $X \geq X_D^{acc}$, the leader invests immediately with the entry accommodation strategy capacity $K_D^{acc}(X)$. The follower also invests at the same time as the leader but with capacity $K_F^*(K_D^{acc}(X))$.

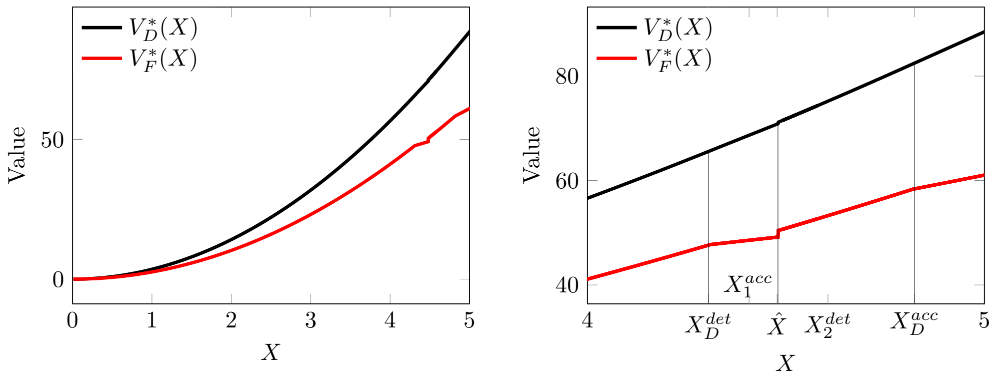

Figure 5: Illustration of $V_D^*(X)$ and $V_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

5 Influence of Flexibility

In order to analyze the influence of the follower's volume flexibility, the optimal investment decisions without flexibility are derived in Appendix B. By comparing the leader's investment decisions with a flexible and with a dedicated follower, I get the following proposition.

Proposition 5 *Volume flexibility does not influence the leader's investment decisions under entry deterrence strategy. Moreover, it also does not influence the leader's optimal capacity under entry accommodation strategy.*

In this section, numerical analysis is carried out to investigate how flexibility influences the leader's and the follower's investment decisions. More specifically, I consider the possibility of each strategy by comparing X_1^{det}, X_2^{det}, and $\hat{X}$, with and without the follower's volume flexibility. The analysis of $\hat{X}$ is because that the leader only switches to accommodation strategy when $X \geq \hat{X}$. I analyze how flexibility influences the leader's optimal capacity and option values at $\hat{X}$. Moreover, this section also considers the follower's optimal investment decisions, with and without volume flexibility, under the leader's deterrence and accommodation strategies. The influence of flexibility on the follower's values at the moment of investment is also analyzed.

5.1 Flexibility Influences Leader

This subsection analyzes numerically the dedicated leader's investment strategies. For the given parameter values in Figure 6, it is demonstrated in the left penal that $X_1^{acc} > X_D^{acc}$ when the follower is flexible, which makes the optimal threshold X_D^{acc} have no meaning as when the follower is not flexible by Huisman and

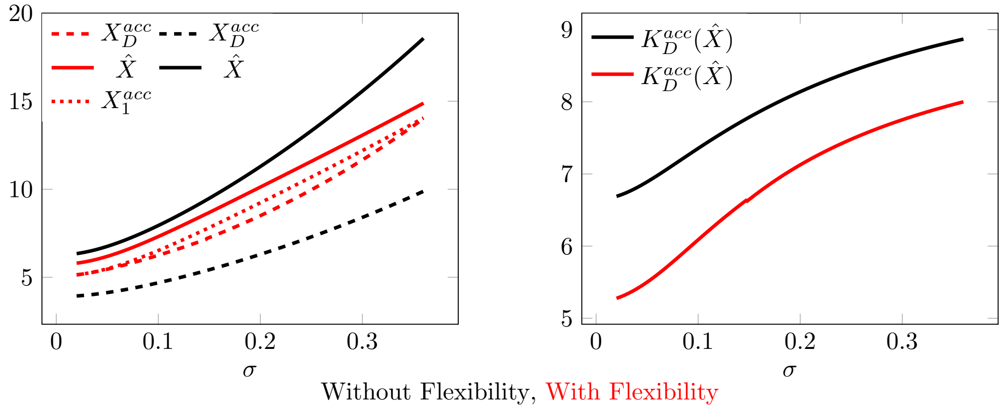

Figure 6: Illustration of X_D^{acc}, $\hat{X}$, and $K_D^{acc}(\hat{X})$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Kort (2015). From Proposition 4, the leader invests at $\hat{X}$ if $\hat{X} \geq X_D^{acc}$, because accommodation strategy generates higher value. In the left penal, it is also shown that $\hat{X} > X_1^{acc}$, implying that it is possible to implement at $\hat{X}$ the accommodation strategy. So I further analyze the influence of follower's flexibility on $\hat{X}$. $\hat{X}$ when follower is flexible is smaller than when follower is not flexible, implying that the leader switches to accommodation strategy earlier and the accommodation strategy is more likely, see Figure 7 for more.

Moreover, $\hat{X}$ increases with σ as shown in left penal of Figure 6. This means that the leader switches to accommodation strategy later in a more volatile market. The intuition is that both the leader and the follower invest more in case of upward demand shocks when there is more uncertainty, shown in the right panel. Furthermore, the right panel also shows that when switching to accommodation strategy, the leader invests less if the follower is flexible. This will be explained further in 5.2.

The follower's flexibility influences the possibility for the leader to implement two strategies. The analysis is carried out by considering the interval $[X_1^{det}, X_2^{det}]$, where the entry deterrence strategy is possible, and $X \geq \hat{X}$, where the accommodation strategy is considered. Figure 7 demonstrates that, the interval to implement deterrence strategy shrinks and the interval to implement accommodation strategy enlarges when the follower is flexible. The changes in the intervals reflect the tendency for the leader to implement the corresponding strategy. It holds that for the given parameters, the leader tends to delay the flexible follower's entry less and is more likely to implement the accommodation strategy.

The leader's tendency to implement different strategies depends on how the follower's flexibility influences its value. Figure 8 illustrates the leader's value under the entry deterrence strategy at threshold X_D^{det} and accommodation strategy at $\hat{X}$, with and without flexibility. For the entry deterrence strategy, it is shown that the leader's value at X_D^{det} under the follow's flexibility is no larger than that without flexibility. This is because the flexibility does not change the leader's investment threshold and capacity, but it makes the follower enter the market earlier, see Figure 8, implying an earlier end to the leader's monopoly privilege. Under the accommodation strategy, the leader invests earlier and less when the follower is flexible. However,

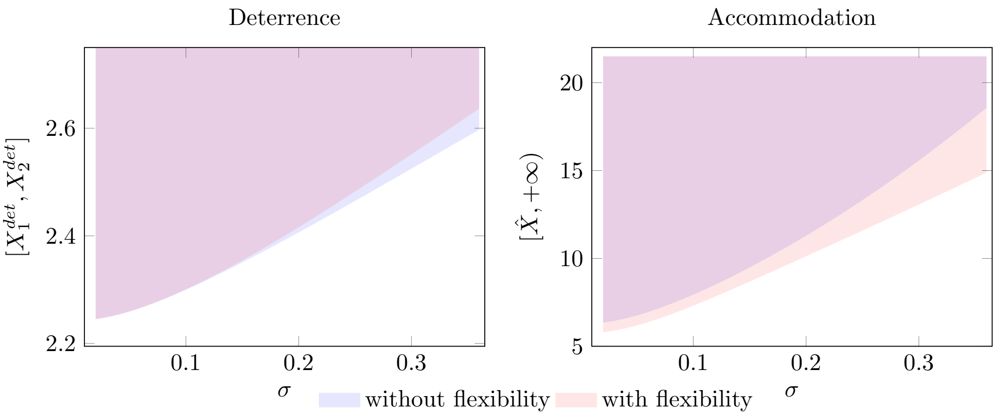

Figure 7: Illustration of X_1^{det}, X_2^{det}, and $\hat{X}$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

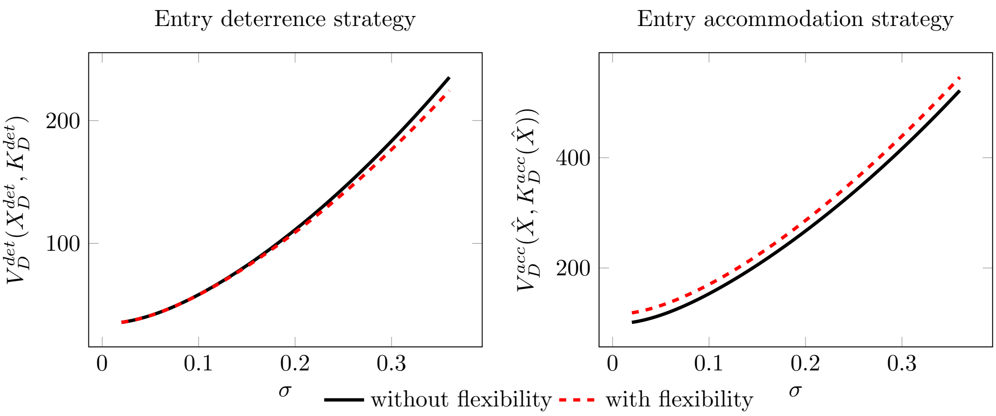

Figure 8: Illustration of $V_D^{det}(X_D^{det}, K_D^{det})$ when investing at the optimal threshold X_D^{det}, and $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ when investing at level $\hat{X}$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

as shown in Figure 8, the leader's values at the moment of investment is larger than that without flexibility[6]. This implies that when implementing the accommodation strategy, the leader also benefits from the follower's flexibility. However, if the leader deters follower's entry, then the follower's flexibility decreases its value.

5.2 Flexibility Influences Follower

In this subsection, I analyze how the volume flexibility influences the follower's investment threshold, capacity, and value under different leader strategies.

[6]Note that $\hat{X}$ is different, depending on whether the follower's flexible. When comparing investment values at different $\hat{X}$s, the comparison is made at a predetermined point of time, for instance, at $\hat{X}$ where the follower is flexible. The discount factor for the leader's value when the follower is flexible is equal to $\left(\hat{X}_{Li}/\hat{X}_{Lf}\right)^{\beta_1}$, where $\hat{X}_{Lf}$ stands for $\hat{X}$ in the flexible follower situation, and $\hat{X}_{Li}$ stands for $\hat{X}$ in the inflexible follower situation.

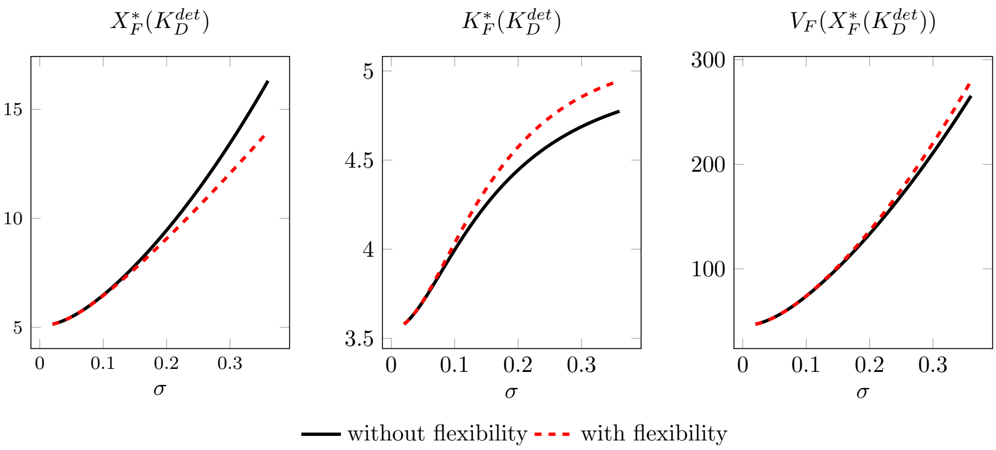

Figure 9: Illustration of $X_F^*(K_D^{det})$, $K_F^*(K_D^{det})$, and $V_F(X_F^*(K_D^{det}))$ under entry deterrence strategy with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

When the leader implements entry deterrence strategy, the flexible follower invests earlier with more capacity and has higher value, as shown in Figure 9. Given that the follower can adjust output levels to the market demand, and prefers to invest more in case the market demand increases in the future. Intuitively the firm would invest later so that the market demand is higher to compensate for the larger investment costs. However, as shown in Figure 9, this is not the case because of another effect that the technological advantage yields higher values for the follower (right panel) and motivates the follower to invest earlier. For the given parameter values, It is apparently that the latter effect dominates. Besides, the difference between with and without flexibility increases with σ for the follower. This is because for smaller σ, market uncertainty is low and the flexible follower produces up to capacity right after investment, so the differences in $X_F^*(K_D^{det})$ and $K_F^*(K_D^{det})$ are relatively small. However, with more market uncertainty, i.e., larger σ, the flexible follower produces below capacity right after investment and puts more capacity on hold for future positive demand shocks, so the differences are relatively large.

The dedicated leader switches from deterrence to accommodation strategy at $\hat{X}$. Note that for the accommodation strategy, the follower invests at the same time as the leader, thus $X_F^*(K_D^{acc}(\hat{X}))$ in Figure 10 is the same as $\hat{X}$ in Figure 6. Figure 10 shows that under leader's accommodation strategy, the flexible follower also invests earlier and more, and has higher value than an inflexible follower. The reason is similar as that in the deterrence strategy. Given that two firms invest at the same time, the leader also invests earlier than that when the follower is dedicated. For the leader, investing earlier implies smaller investment capacity because the leader is dedicated.

5.3 First Mover Advantage v.s. Technological Advantage

This subsection investigates whether the follower's technological advantage in volume flexibility can overcome the leader's first mover advantage.

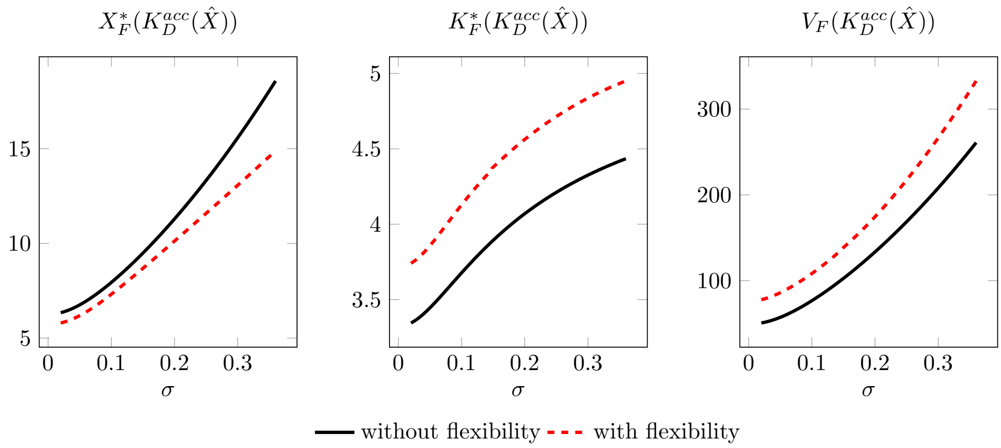

Figure 10: Illustration of $X_F^*(K_D^{acc}(\hat{X}))$, $K_F^*(K_D^{acc}(\hat{X}))$, and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

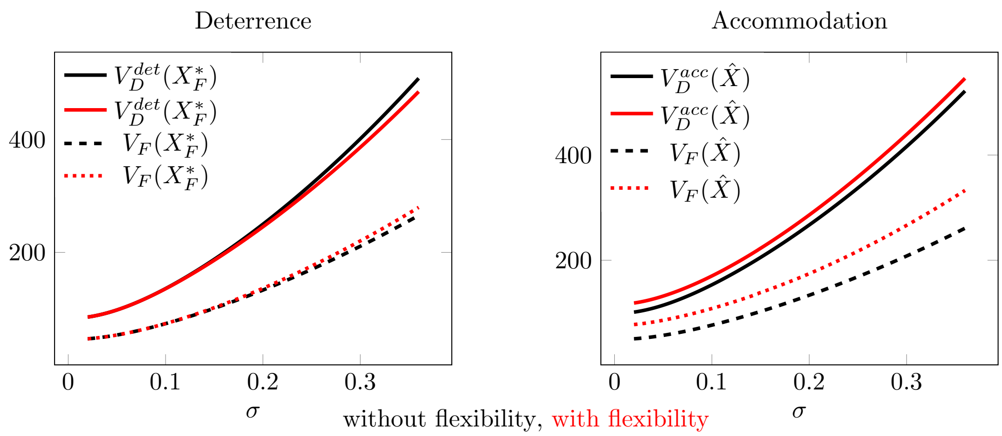

Figure 11: Comparison of $V_D^{det}(X_F^*, K_D^{det}(X_F^*))$ and $V_F(X_F^*, K_D^{det}(X_F^*))$ under the entry deterrence strategy, $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy, with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 11 compares the leader and the follower's values for the entry deterrence and accommodation strategies, with and without flexibility. The leader always has higher values than the follower, implying the first mover advantage cannot be leapfrogged by the volume flexibility advantage. Gal-Or (1985) has shown with symmetric players that the leader has larger profits compared to the follower if the follower's reaction function is downward-sloping. In my model with asymmetric firms and continuous time setting, the result is similar in that the optimal follower's optimal capacity decreases with the leader's installed capacity. Another possible reason is that the leader benefits from the follower's volume flexibility without sharing costs for these benefits.

6 Preemption Analysis between the Flexible Firm and the Dedicated Firm

7 Conclusion

This article introduces volume flexibility into the strategic capacity investment problem under uncertainty. In the duopoly framework, the follower has technological advantage over the leader in that the follower can adjust output quantity within the constraint of installed production capacity, and the leader always produces up to capacity. When making decisions about investment timing and investment capacity, the leader not only takes into account the incentives to preempt, but also the influence of the follower's volume flexibility on the market price. This is because the flexible follower competes against the dedicated leader on one hand, and on the other hand makes the market price fluctuate less when there is demand volatility. I show that compared to a dedicated follower, the dedicated leader is more likely to accommodate the entry of the flexible follower. This is due to that the entry deterrence strategy decreases the leader's value when the follower is flexible, and the accommodation strategy increases the leader's value. The leader does not like to deter because volume flexibility makes the follower to enter the market earlier and thus shortens the leader's monopoly period. Whereas when implementing the accommodation strategy, two firms enter the market later than that under the deterrence strategy, so the market demand is larger. In a way, the leader benefits more from the less fluctuating market prices due to follower's volume flexibility.

Several extensions are possible for this paper. For instance, under the multiplicative demand structure I find that the dedicated leader installs monopolistic capacity size for the entry deterrence and accommodation strategy. This is a strong result. It is worthwhile to do a robustness check with a different demand structure. Another extension is the firms only invest once in my model. If the firms can invest several times, i.e., have several investment options, it would be interesting to study their strategic interactions.

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Appendix

A Flexible Follower

A.1 Expression of $L_1(K_D, K_F)$, $M_1(K_D, K_F)$, $M_2(K_D)$, and $N(K_D, K_F)$

In the follower's value function $V_F(X, K_D, K_F)$, the lengthy expression for L_1, M_1, M_2 and N_2 are as follows,

$$\begin{aligned}
L(K_D, K_F) &= \frac{c^2 F(\beta_2)}{4\gamma(\beta_1-\beta_2)}\left(\left(\frac{1-\gamma K_D}{c}\right)^{\beta_1+1} - \left(\frac{1-2\gamma K_F-\gamma K_D}{c}\right)^{\beta_1+1}\right),\\
M_1(K_D, K_F) &= -\frac{c^2 F(\beta_2)}{4\gamma(\beta_1-\beta_2)}\left(\frac{1-2\gamma K_F-\gamma K_D}{c}\right)^{\beta_1+1},\\
M_2(K_D) &= \frac{c^2 F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{1-\gamma K_D}{c}\right)^{\beta_2+1},\\
N(K_D, K_F) &= \frac{c^2 F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\left(\frac{1-\gamma K_D}{c}\right)^{\beta_2+1} - \left(\frac{1-\gamma K_D-2\gamma K_F}{c}\right)^{\beta_2+1}\right).
\end{aligned}$$

In order to get more insight of the value function, I analyze the signs for these four expressions. Given that $r > \alpha$, it holds that $\beta_1 > 1$ and $F(\beta_2) > 0$. From Wen et al. (2017), it also holds that $\beta_2 < -1$, and $F(\beta_1) < 0$ when $\sigma^2 < r+\alpha$; $-1 < \beta_2 < 0$, and $F(\beta_1) > 0$ when $\sigma^2 > r+\alpha$. Thus, it can be concluded that $L(K_D, K_F) > 0$, $M_1(K_D, K_F) < 0$, and $N(K_D, K_F) > 0$. If $\sigma^2 < r+\alpha$, then $M_2(K_D) < 0$, and if $\sigma^2 > r+\alpha$, then $M_2(K_D) > 0$.

A.2 Proof of Proposition 1

The optimal investment capacity $K_F(X, K_D)$ of the follower maximizes $V_F(X, K_D, K_F) - \delta K_F$. The analysis is carried out for three different regions.

- Region 1: $0 < X < c/(1-\gamma K_D)$.

Given the expression of L_1, the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F gives

$$\frac{c(1+\beta_1)F(\beta_2)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_1} - \delta = 0. \tag{A.1}$$

Thus,

$$K_F(X, K_D) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{c}{X}\left[\frac{2\delta(\beta_1-\beta_2)}{cF(\beta_2)(1+\beta_1)}\right]^{\frac{1}{\beta_1}}\right). \tag{A.2}$$

- Region 2: $X \geq c/(1-\gamma K_D)$ and $K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}$.

Given the expression of M_1 and M_2, taking the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F yields

$$K_F(X, K_D) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{c}{X}\left[\frac{2\delta(\beta_1-\beta_2)}{cF(\beta_2)(1+\beta_1)}\right]^{\frac{1}{\beta_1}}\right). \tag{A.3}$$

- Region 3: $X \geq c/(1-\gamma K_D)$ and $K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}$.

Given the expression for N_2, the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F yields that $K_F(X, K_D)$ must satisfy

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_2} + \frac{X(1-2\gamma K_F-\gamma K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0. \tag{A.4}$$

The optimal investment threshold $X_F^*(K_D)$ in each region can be derived by the value matching and smooth pasting conditions at $X_F^*(K_D)$:

$$\begin{cases} AX_F^{*\beta_1}(K_D) & = V_F\left(X_F^*(K_D), K_D, K_F\left(X_F^*(K_D), K_D\right)\right) - \delta K_F\left(X_F^*(K_D), K_D\right), \\ \beta_1 AX_F^{*\beta_1-1}(K_D) & = \frac{\mathrm{d}}{\mathrm{d}X}\left[V_F\left(X_F^*(K_D), K_D, K_F\left(X_F^*(K_D), K_D\right)\right) - \delta K_F\left(X_F^*(K_D), K_D\right)\right]. \end{cases}$$

Thus, $X_F^*(K_D)$ satisfies the following implicit equation

$$\begin{aligned} & V_F\left(X_F, K_D, K_F\left(X_F, K_D\right)\right) - \delta K_F\left(X_F, K_D\right) \\ = \; & \frac{X_F(K_D)}{\beta_1}\frac{\mathrm{d}\left[V_F\left(X_F, K_D, K_F\left(X_F, K_D\right)\right) - \delta K_F\left(X_F, K_D\right)\right]}{\mathrm{d}X}. \end{aligned} \tag{A.5}$$

- Region 1

The implicit equation (A.5) implies that

$$\delta K_F = 0. \tag{A.6}$$

- Region 2

The optimal threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned} & -\frac{F(\beta_2)c^{1-\beta_1}\left(1-2\gamma K_F-\gamma K_D\right)^{1+\beta_1}X^{\beta_1}}{4\gamma\left(\beta_1-\beta_2\right)} + \frac{c^{1-\beta_2}(1-\gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1-\beta_2)}F(\beta_1)X^{\beta_2} + \frac{\left(1-\gamma K_D\right)^2 X}{4\gamma\left(r-\alpha\right)} \\ & \qquad - \frac{c\left(1-\gamma K_D\right)}{2\gamma r} + \frac{c^2}{4\gamma X\left(r+\alpha-\sigma^2\right)} - \delta K_F \\ = \; & \frac{X}{\beta_1}\left[\frac{\beta_2 F(\beta_1)c^{1-\beta_2}(1-\gamma K_D)^{1+\beta_2}X^{\beta_2-1}}{4\gamma(\beta_1-\beta_2)} + \frac{(1-\gamma K_D)^2}{4\gamma(r-\alpha)} - \frac{c^2}{4\gamma X^2(r+\alpha-\sigma^2)}\right] \\ & \qquad - \frac{F(\beta_2)c^{1-\beta_1}(1-\gamma K_D-2\gamma K_F)^{1+\beta_1}X^{\beta_1}}{4\gamma(\beta_1-\beta_2)}, \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \frac{c(1-\gamma K_D)F(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{2c(1-\gamma K_D)}{r}\right. \\ & \qquad \left. + \frac{\beta_1+1}{\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}\right] - \delta K_F = 0. \end{aligned} \tag{A.7}$$

- Region 3

The optimal investment threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned}&\frac{c\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{X}{c}\right)^{\beta_2}+K_F\left(\frac{(1-\gamma K_D-\gamma K_F)X}{r-\alpha}-\frac{c}{r}-\delta\right)\\ =\;&\frac{\beta_2}{\beta_1}\frac{c\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{X}{c}\right)^{\beta_2}+\frac{K_F}{\beta_1}\frac{X(1-\gamma K_D-\gamma K_F)}{r-\alpha}.\end{aligned}$$

Rearranging terms yields

$$\begin{aligned}&\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X}{c}\right)^{\beta_2}\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]\\ &\quad+\frac{(\beta_1-1)K_F}{\beta_1}\frac{X(1-\gamma K_D-\gamma K_F)}{r-\alpha}-\frac{cK_F}{r}-\delta K_F=0.\end{aligned}\tag{A.8}$$

Note that in the monopoly case by Wen et al. (2017), whether the flexible firm produces up to capacity depends on the economic setting. Similarly as for the follower in the duopoly situation, if the firm produces below capacity right after investment, then $q_F(X,K_D,K_F(X,K_D))<K_F(X,K_D)$, i.e.,

$$\frac{1}{2\gamma}\left(1-\gamma K_D-\frac{c}{X}\left[\frac{2\delta\left(\beta_1-\beta_2\right)}{cF\left(\beta_2\right)\left(1+\beta_1\right)}\right]^{\frac{1}{\beta_1}}\right)>\frac{X(1-\gamma K_D)-c}{2\gamma X}.$$

It is equivalent to

$$2\delta(\beta_1-\beta_2)<cF(\beta_2)(1+\beta_1),\tag{A.9}$$

which is the same as in the monopoly case. Furthermore, it can be deduced that

$$2\delta(\beta_1-\beta_2)\geq cF(\beta_2)(1+\beta_1)\tag{A.10}$$

defines Region 3, where the firm produces up to capacity right after investment. The definitions of Region 2, equation (A.9), and Region 3, equation (A.10), for the flexible follower firm are the same as that for the flexible monopoly firm in Wen et al. (2017).

A.3 Proof of Corollary 1

- Region 2

Derive $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ and check whether the leader's installed capacity level would delay the flexible follower's investment. Dividing (11) by $(1-\gamma K_D)$ yields that

$$\begin{aligned}&\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2}-\frac{\delta}{2\gamma}\left(1-\frac{c}{X(1-\gamma K_D)}\left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right]^{\frac{1}{\beta_1}}\right)\\ &\quad+\frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)}{r-\alpha}-\frac{2c}{r}+\frac{\beta_1+1}{\beta_1}\frac{c^2}{r+\alpha-\sigma^2}\frac{1}{X(1-\gamma K_D)}\right]=0.\end{aligned}\tag{A.11}$$

Comparing (A.11) with the implicit equation that determines the optimal investment threshold in the corresponding monopoly model, I find that $X(1-\gamma K_D)$ replaces X^* in the corresponding monopoly case. Apparently, $(1-\gamma K_D)X_F^{K_D}$ is a constant that solves (A.11). Thus, it can be concluded

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D}=\frac{\gamma X_F^*(K_D)}{1-\gamma K_D}>0,\tag{A.12}$$

implying that investing in more capacity by the dedicated leader would delay the investment of the flexible follower. According to (10), taking the derivative of $K_F^*(K_D)$ with respect to K_D, it follows that

$$\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D} \quad = \quad -\frac{\gamma K_F^*(K_D)}{1-\gamma K_D} \leq 0. \tag{A.13}$$

This implies that an increase in the inflexible leader's investment capacity decreases the flexible follower's optimal capacity to invest with.

- Region 3

The investment timing $X_F^*(K_D)$ and investment capacity $K_F^*(K_D)$ are determined by (12) and (13) when the follower produces up to capacity right after the investment. Rewriting these two equations yields

$$\frac{F(\beta_1)c^{1-\beta_2}(1+\beta_2)H^{\beta_2}(K_D)}{2(\beta_1-\beta_2)} + \frac{H(K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0, \tag{A.14}$$

and

$$\frac{c^{1-\beta_2}F(\beta_1)\left[W^{1+\beta_2}(K_D) - H^{1+\beta_2}(K_D)\right]}{4\gamma\beta_1 X_F^*(K_D)} - \left(\frac{c}{r}+\delta\right)K_F^*(K_D) + \frac{(\beta_1-1)\left(W^2(K_D)-H^2(K_D)\right)}{4\gamma\beta_1(r-\alpha)X_F^*(K_D)} = 0, \tag{A.15}$$

respectively, where

$$W(K_D) = X_F^*(K_D)(1-\gamma K_D),$$
$$H(K_D) = X_F^*(K_D)(1-\gamma K_D - 2\gamma K_F^*(K_D)).$$

Note that $H(K_D)$ is a constant and solves (A.14). From $\mathrm{d}H(K_D)/\mathrm{d}K_D = 0$, it follows that

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = \frac{\gamma X_F^*(K_D)}{1-\gamma K_D - 2\gamma K_F^*(K_D)}\left(2\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D}+1\right). \tag{A.16}$$

$W(K_D)$ solves equation (A.15). Taking the derivative of (A.15) with respect to K_D yields

$$\left(\frac{(1+\beta_2)c^{1-\beta_2}F(\beta_1)W^{\beta_2}(K_D)}{2\beta_1} + \frac{(\beta_1-1)W(K_D)}{\beta_1(r-\alpha)} - \left(\frac{c}{r}+\delta\right)\right)\frac{\gamma K_F^*(K_D) + (1-\gamma K_D)\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D}}{1-\gamma K_D - 2\gamma K_F^*(K_D)} = 0,$$

implying,

$$\frac{c(1+\beta_2)F(\beta_1)}{2\beta_1}\left(\frac{W(K_D)}{c}\right)^{\beta_2} + \frac{(\beta_1-1)W(K_D)}{\beta_1(r-\alpha)} = \frac{c}{r}+\delta. \tag{A.17}$$

(A.17) implies that $W(K_D)$ is also a constant and satisfies

$$\frac{\mathrm{d}W(K_D)}{\mathrm{d}K_D} = -\gamma X_F^*(K_D) + (1-\gamma K_D)\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = 0.$$

It can be further derived that

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = \frac{\gamma X_F^*(K_D)}{1-\gamma K_D} > 0. \tag{A.18}$$

Moreover, from (A.16) and (A.18), it follows that

$$\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D} = -\frac{\gamma K_F^*(K_D)}{1-\gamma K_D} < 0. \tag{A.19}$$

Thus, for the case that the flexible follower produces up to capacity right after the investment, the dedicated leader can delay and decrease the investment of the follower by investing in a larger capacity.

A.4 Expressions of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, $\mathcal{N}(K_D)$

Employing value matching and smooth pasting at $X_1 = c/(1-\gamma K_D)$ and $X_2 = c/\left(1-\gamma K_D - 2\gamma K_F^*(K_D)\right)$, then for a given K_D $(0 \le K_D < 1/\gamma)$, it can be derived that

$$\mathcal{M}_2(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_2}, \tag{A.20}$$

$$\mathcal{M}_1(K_D) = -\frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_1}, \tag{A.21}$$

$$\mathcal{L}(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left[\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_1}-\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_1}\right] \tag{A.22}$$

$$\mathcal{N}(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left[\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_2}-\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_2}\right] \tag{A.23}$$

In order to check the signs for $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, and $\mathcal{N}(K_D)$, first analyze the signs of $(\beta-1)/(r-\alpha)-\beta/r = \frac{\alpha\beta-r}{r(r-\alpha)}$ for $\beta=\beta_1$ and $\beta=\beta_2$.

If $\alpha \ge 0$, then $\alpha\beta_2 - r < 0$ because $\beta_2 < 0$. If $\alpha < 0$, then $\alpha\beta_2 - r = \alpha\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}-\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha} > 0$. From $\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}\right)^2-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2-\frac{2r}{\sigma^2} = -\frac{r}{\alpha}+\frac{r^2}{\alpha^2} > 0$, we get $\alpha\beta_2 - r < 0$. So, $\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r} < 0$.

If $\alpha \le 0$, then $\alpha\beta_1 - r < 0$. If $\alpha > 0$, then $\alpha\beta_1 - r = \alpha\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}+\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha} < 0$, because $r > \alpha$. From $\left(\frac{r}{\alpha}+\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2-\frac{2r}{\sigma^2} = \frac{r^2}{\alpha^2}-\frac{r}{\alpha} > 0$, it holds that $\alpha\beta_1 - r < 0$. So, $\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r} < 0$.

Thus, it can be concluded that when $0 \le K_D < 1/\gamma$, then $\mathcal{L}(K_D) < 0$, $\mathcal{M}_1(K_D) > 0$, $\mathcal{M}_2(K_D) < 0$, $\mathcal{N}(K_D) > 0$.

A.5 Proof of Proposition 2

A.5.1 Negative $\mathcal{B}_1(K_D)$

Before the derivation of the dedicated leader's optimal investment capacity in the entry deterrence and accommodation strategies, first check the sign of $\mathcal{B}_1(K_D)$.

$$\begin{aligned}\mathcal{B}_1(K_D) &= \mathcal{M}_1(K_D)+\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D)-\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D)+\frac{cK_D}{2r}X_F^{*-\beta_1}(K_D)\\ &= \frac{cK_D}{2X_F^{*\beta_1}(K_D)}\left[-\frac{1}{\beta_1-\beta_2}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left(\frac{X_F^*(K_D)}{X_2(K_D)}\right)^{\beta_1}-\frac{1}{r-\alpha}\frac{X_F^*(K_D)}{X_1(K_D)}+\frac{1}{r}\right.\\ &\qquad\left.+\frac{1}{\beta_1-\beta_2}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{X_F^*(K_D)}{X_1(K_D)}\right)^{\beta_2}\right].\end{aligned}$$

For $X_F^*(K_D)/X_i(K_D)$ with $i=\{1,2\}$, it holds that

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{X_F^*(K_D)}{X_i(K_D)} = \frac{1}{X_i^2(K_D)}\left(\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}-\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}\right) = 0.$$

This implies that $X_F^*(K_D)/X_i(K_D)$ is a constant and does not change with K_D. So I can set $K_D = 0$, then

$$\frac{X_F^*(K_D)}{X_1(K_D)} = \frac{X_F^*(0)}{c},$$

and

$$\frac{X_F^*(K_D)}{X_2(K_D)} = \frac{X_F^*(0)}{c}\left(1 - 2\gamma K_F^*(0)\right) = \left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}.$$

Equation (11) is the corresponding implicit equation to determine X^* in the monopoly case:

$$F(\beta_1)\left(\frac{X^*}{c}\right)^{\beta_2} + \frac{\beta_1-1}{r-\alpha}\frac{X^*}{c} - \frac{2\beta_1}{r} + \frac{\beta_1+1}{r+\alpha-\sigma^2}\frac{c}{X^*} - \frac{2\beta_1\delta}{c}\left(1 - \frac{c}{X^*}\left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right]^{\frac{1}{\beta_1}}\right) = 0. \quad \text{(A.24)}$$

Rewrite such that

$$\begin{aligned}
\mathcal{B}_1(K_D) &= \frac{cK_D}{2(\beta_1-\beta_2)X_F^{*\beta_1}(K_D)}\left[-\left(\frac{\beta_2-1}{r-\alpha} - \frac{\beta_2}{r}\right)\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right. \\
&\quad \left. + \left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{X^*}{c}\right)^{\beta_2} - (\beta_1-\beta_2)\left(\frac{1}{r-\alpha}\frac{X^*}{c} - \frac{1}{r}\right)\right] \\
&= \frac{cK_D}{2X_F^{*\beta_1}(K_D)}\mathcal{F}(X^*),
\end{aligned}$$

where X^* satisfies (A.24). Next, I show numerically that $\mathcal{F}(X^*)$ is negative. The demonstration is shown in Figure A.1. Note that γ does not influence $\mathcal{F}(X^*)$, so the numerical analysis is just about the influence of α, σ, r, c, and δ. The default parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$. Some combination of parameter values does not make the flexible follower produce below capacity right after investment. After ruling out these combinations, $\mathcal{F}(X^*)$ changing with parameters is illustrated in Figure A.1. The numerical analysis confirms the conjecture that $\mathcal{B}_1(K_D)$ is negative when the flexible follower produces below capacity right after investment. In the following analysis, I take $\mathcal{B}_1(K_D)$ as negative.

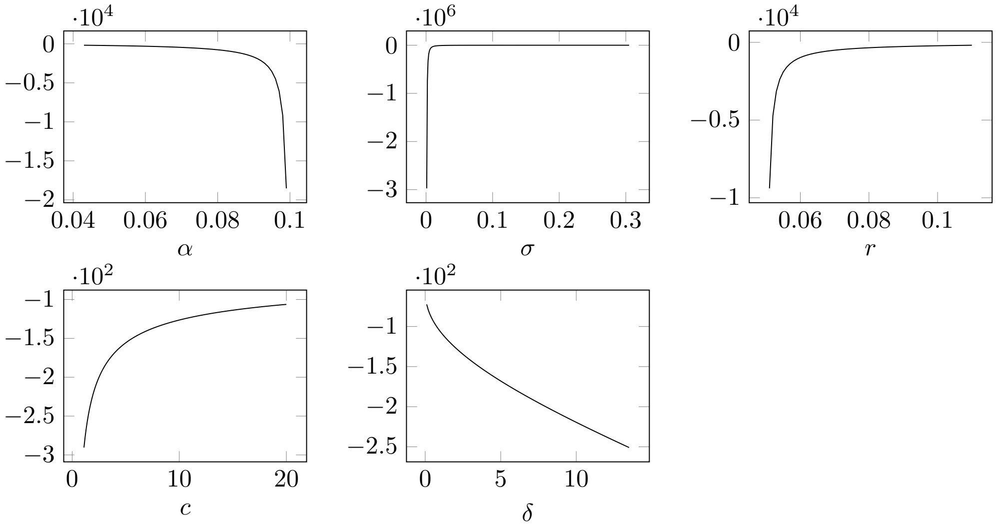

Figure A.1: Illustration of negative $\mathcal{F}(X^*)$ changing with α, σ, r, c, and δ. Default parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$.

A.5.2 Proof of Proposition 2

In order to get the optimal investment decisions for the dedicated leader, I first calculate the first derivative of $\mathcal{B}_1(K_D)$ with respect of K_D. First, $\mathcal{M}_1(K_D)$ can be rewritten as

$$\begin{aligned}\mathcal{M}_1(K_D) &= -\frac{c^{1-\beta_1}K_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left[\frac{c}{X_F^*(K_D)}\left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}\right]^{\beta_1}\\ &= -\frac{K_D}{X_F^{*\beta_1}(K_D)}\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right).\end{aligned}$$

With $\mathrm{d}K_F^*(K_D)/\mathrm{d}K_D$ and $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ given by (A.12) and (A.13), it can be calculated that

$$\frac{\mathrm{d}\mathcal{M}_1(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D).$$

Furthermore, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D).$$

Note also

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) = \frac{(1-\gamma K_D-\beta_1\gamma K_D)X_F^{*1-\beta_1}(K_D)}{2(r-\alpha)}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{cK_D}{2r}X_F^{*-\beta_1}(K_D) = \frac{cX_F^{*-\beta_1}(K_D)(1-\gamma K_D-\beta_1\gamma K_D)}{2r(1-\gamma K_D)},$$

then according to (17), it can be derived that

$$\frac{\mathrm{d}\mathcal{B}_1(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D).$$

Next, I analyze the entry deterrence and accommodation strategies for the dedicated leader, which include the optimal investment capacities and optimal investment thresholds.

1. Entry Deterrence Strategy

   The investment capacity $K_D^{det}(X)$ for a given level of X satisfies

   $$\frac{\partial V_D(X,K_D)-\delta K_D}{\partial K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D)X^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X-\frac{c}{r}-\delta=0. \tag{A.25}$$

   The entry deterrence strategy cannot happen when $K_D^{det}(X)<\hat{K}_D(X)$, which yields $X>X_2^{det}$ with X_2^{det} and $K_D^{det}(X_2^{det})$ satisfying (14) and (A.25). This is because the demand is high enough for the follower to invest immediately to enter the market. The entry deterrence strategy also does not happen when $K_D^{det}(X)<0$, yielding $X<X_1^{det}$ with X_1^{det} satisfying

   $$\left[-\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)+\frac{c^{1-\beta_2}X_F^{*\beta_2}(0)}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\right.$$

$$-\frac{X_F^*(0)}{2(r-\alpha)}+\frac{c}{2r}\Bigg]\left(\frac{X_1^{det}}{X_F^*(0)}\right)^{\beta_1}+\frac{X_1^{det}}{r-\alpha}-\frac{c}{r}-\delta=0, \tag{A.26}$$

where $X_F^*(0)$ can be derived from (10) and (11) given that $K_D = 0$. Thus, the entry deterrence strategy is only possible when $X \in (X_1^{det}, X_2^{det})$. Suppose the investment threshold of the dedicated leader is $X^{det}(K_D)$ if the follower invests with capacity K_D in the entry deterrence strategy. The leader's value function before and after the investment is as follows

$$V_D(X,K_D)=\begin{cases}\mathcal{A}(K_D)X^{\beta_1} & X<X^{det}(K_D),\\ \mathcal{B}_1(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{cK_D}{r} & X^{det}(K_D)\le X<X_F^*(K_D),\\ \mathcal{M}_1(K_D)X^{\beta_1}+\mathcal{M}_2(K_D)X^{\beta_2} & \\ +\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X-\frac{cK_D}{2r} & X\ge X_F^*(K_D).\end{cases} \tag{A.27}$$

The value matching and smooth pasting conditions to determine $X^{det}(K_D)$ are

$$\mathcal{A}(K_D)X^{\beta_1}=\mathcal{B}_1(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{cK_D}{r}-\delta K_D,$$
$$\beta_1\mathcal{A}(K_D)X^{\beta_1-1}=\beta_1\mathcal{B}(K_D)X^{\beta_1-1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}.$$

Thus, the threshold of the entry deterrence strategy $X^{det}(K_D)$ is

$$X^{det}(K_D)=\frac{\beta_1}{\beta_1-1}\frac{r-\alpha}{1-\gamma K_D}\left(\frac{c}{r}+\delta\right). \tag{A.28}$$

Substituting $X^{det}(K_D)$ into (A.25), the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ can be derived as

$$\begin{aligned}K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det}))=\frac{1}{(\beta_1+1)\gamma},\\ X^{det}(K_D^{det}) &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right).\end{aligned}$$

2. Entry Accommodation Strategy

Note that from (A.20), we can get

$$\frac{\partial\mathcal{M}_2(K_D)}{\partial K_D}=\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D).$$

The optimal capacity $K_D^{acc}(X)$ satisfies the following implicit equation

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)X^{\beta_1}+\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)X^{\beta_2}+\frac{1-2\gamma K_D}{2(r-\alpha)}X-\frac{c}{2r}-\delta=0. \tag{A.29}$$

The entry accommodation strategy only happens when $X \ge X_F^*(K_D)$, implying that the market demand is large enough to allow both the dedicated leader and the flexible follower to invest at the same time. Let X_1^{acc} be such that $X_1^{acc}=X_F^*(K_D^{acc}(X_1^{acc}))$, then X_1^{acc} and the corresponding $K_D^{acc}(X_1^{acc})$

satisfy (A.29) and (14). Suppose the dedicated leader invests at $X^{acc}(K_D)$ when the capacity level is K_D in the entry accommodation strategy, then the leader's value function before and after investment is

$$V_D(X, K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{acc}(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} & \\ +\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} & X \geq X_F^*(K_D) \geq X^{acc}(K_D). \end{cases} \tag{A.30}$$

The value matching and smooth pasting conditions to determine $X^{acc}(K_D)$ are

$$\mathcal{A}(K_D)X^{\beta_1} = \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} - \delta K_D,$$

$$\beta_1\mathcal{A}(K_D)X^{\beta_1-1} = \beta_1\mathcal{M}_1(K_D)X^{\beta_1-1} + \beta_2\mathcal{M}_2(K_D)X^{\beta_2-1} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}.$$

Thus, the investment capacity $K_D^{acc}(X^{acc})$ and investment threshold $X^{acc}(K_D^{acc})$ satisfy equation (A.29) and

$$(\beta_1 - \beta_2)\mathcal{M}_2(K_D)X^{\beta_2} + \frac{(\beta_1-1)K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{c\beta_1 K_D}{2r} - \beta_1\delta K_D = 0. \tag{A.31}$$

Rewrite these two equations, then $K_D^{acc}(X^{acc})$ and $X^{acc}(K_D^{acc})$ satisfy

$$\frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)X^{\beta_2} + \frac{1-\gamma K_D-\beta_2\gamma K_D}{1-\gamma K_D}\frac{c}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{X}{X_1}\right)^{\beta_2} + \frac{1-2\gamma K_D}{2(r-\alpha)}X - \frac{c}{2r} - \delta = 0,$$

and

$$\frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{X}{X_1}\right)^{\beta_2} + \frac{(\beta_1-1)(1-\gamma K_D)}{2\beta_1(r-\alpha)}X - \frac{c}{2r} - \delta = 0.$$

Solving these two equations yields

$$K_D^{acc} \equiv K_D^{acc}(X^{acc}(K_D^{acc})) = \frac{1}{(\beta_1+1)\gamma}.$$

The accommodation strategy threshold $X^{acc}(K_D)$ when follower is flexible can be compared to when follower is dedicated, where the threshold is $\frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right)$ according to Huisman and Kort (2015). To make the comparison, let

$$Z(X) = \frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{\beta_1}{(\beta_1+1)c}\right)^{\beta_2}X^{\beta_2} + \frac{\beta_1-1}{2(\beta_1+1)(r-\alpha)}X - \frac{c}{2r} - \delta.$$

Then

$$Z\left(\frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\right) = \frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{\beta_1(r-\alpha)}{(\beta_1-1)c}\left(\frac{c}{r}+\delta\right)\right)^{\beta_2} - \frac{\delta}{2} < 0.$$

Because

$$\frac{\mathrm{d}Z(X)}{\mathrm{d}X} = \frac{c\beta_2}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{\beta_1}{(\beta_1+1)c}\right)^{\beta_2}X^{\beta_2-1} + \frac{\beta_1-1}{2(\beta_1+1)(r-\alpha)} > 0,$$

it can be concluded that

$$X^{acc}(K_D^{acc}) > \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right),$$

implying the entry accommodation threshold when the follower is flexible is larger than that when the follower is not flexible.

A.6 Proof of Proposition 3

A.6.1 Negative $\mathcal{B}_2(K_D)$

When the flexible follower produces up to capacity right after investment, then

$$\begin{aligned}\mathcal{B}_2(K_D) &= \mathcal{N}(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r-\alpha}X_F^{*1-\beta_1}(K_D)\\ &= \frac{cK_D X_F^{*-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\left[\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\left(\frac{X_F^*(K_D)}{X_1(K_D)}\right)^{\beta_2}-\left(\frac{X_F^*(K_D)}{X_2(K_D)}\right)^{\beta_2}\right)\right.\\ &\qquad\left.+\frac{\beta_1-\beta_2}{r-\alpha}\left(\frac{X_F^*(K_D)}{X_2(K_D)}-\frac{X_F^*(K_D)}{X_1(K_D)}\right)\right].\end{aligned}$$

Note that

$$\begin{aligned}\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} &= \frac{\gamma X_F^*(K_D)}{1-\gamma K_D},\\ \frac{\mathrm{d}X_i(K_D)}{\mathrm{d}K_D} &= \frac{\gamma X_i(K_D)}{1-\gamma K_D},\ i\in\{1,2\}.\end{aligned}$$

Thus for the terms $X_F^*(K_D)/X_i(K_D)$ with $i\in\{1,2\}$ in $\mathcal{B}_2(K_D)$,

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{X_F^*(K_D)}{X_i(K_D)} = \frac{1}{X_i^2(K_D)}\left(\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}-\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}\right)=0.$$

Similar to the case that flexible follower produces below capacity right after investment, $X_F^*(K_D)/X_1(K_D)$ and $X_F^*(K_D)/X_2(K_D)$ are constants and do not change with K_D. Thus

$$\begin{aligned}\frac{X_F^*(K_D)}{X_1(K_D)} &= \frac{X_F^*(0)}{c},\\ \frac{X_F^*(K_D)}{X_2(K_D)} &= \frac{X_F^*(0)}{X_2(0)}.\end{aligned}$$

Let $X_F^*(0)=X^*$ and $X_2(0)=1-2\gamma K^*$, with X^* as the optimal investment threshold and K^* as the optimal capacity in the monopoly case where the firm produces up to capacity right after investment. I rewrite $\mathcal{B}_2(K_D)$ as

$$\mathcal{B}_2(K_D)=\frac{cK_D X_F^{*-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\mathcal{G}(X^*,K^*),$$

where X^* and K^* satisfy

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{(1-2\gamma K^*)X^*}{c}\right)^{\beta_2}+\frac{c}{r-\alpha}\frac{(1-2\gamma K^*)X^*}{c}-\frac{c}{r}-\delta=0$$

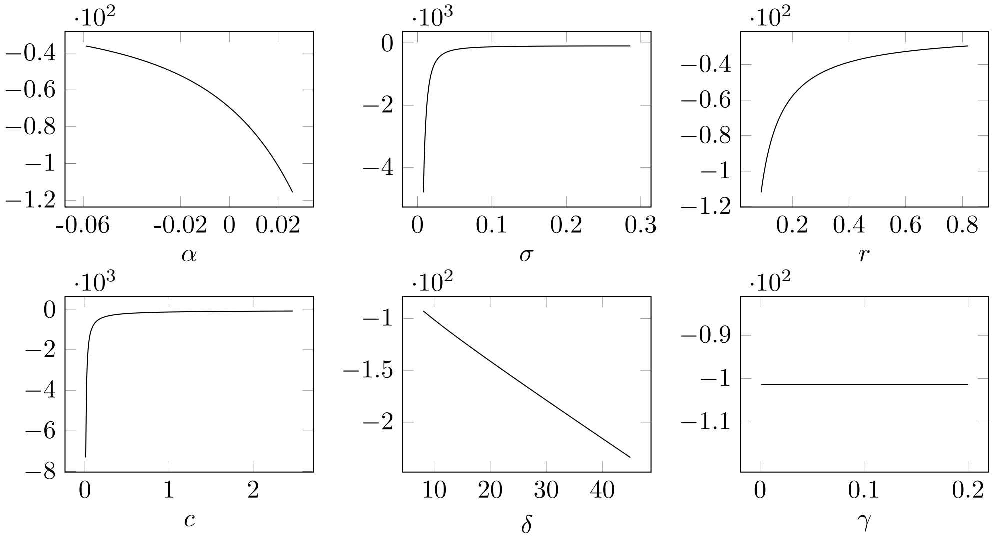

Figure A.2: Illustration of negative $\mathcal{G}(X^*, K^*)$ changing with α, σ, r, c, δ, and γ. Default parameter values are $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

and

$$\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\left(\frac{X^*}{c}\right)^{\beta_2} - (1-2\gamma K^*)\left(\frac{(1-2\gamma K^*)X^*}{c}\right)^{\beta_2}\right) + \frac{\beta_1 - 1}{\beta_1}\frac{(1-\gamma K^*)X^*K^*}{r-\alpha} - \frac{cK^*}{r} - \delta K^* = 0.$$

$\mathcal{B}_2(K_D)$ is intuitively negative. However, it is too complicated to show this analytically. So I try to show it is negative numerically to verify the conjecture. Figure A.2 demonstrates $\mathcal{G}(X^*, K^*)$ changing with parameters. The default parameter values are given as $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, and $\gamma = 0.05$. Some combination of parameter values does not define the case that the follower produces up to capacity right after investment. After ruling out such combinations, the negative $\mathcal{G}(X^*, K^*)$ is illustrated in Figure A.2. This confirms the conjecture that $\mathcal{B}_2(K_D)$ is negative. So in the following analysis, I assume negative $\mathcal{B}_2(K_D)$.

A.6.2 Proof of Proposition 3

I start with the derivative of $\mathcal{B}_2(K_D)$ with respect to K_D, where $K_F^*(K_D)$ and $X_F^*(K_D)$ are defined by (12) and (13), and $\mathrm{d}K_F^*(K_D)/\mathrm{d}K_D$ and $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ are defined by (A.18) and (A.19). It holds that

$$\frac{\mathrm{d}\mathcal{N}(K_D)}{\mathrm{d}K_D} = \frac{\mathcal{N}(K_D)(1-\gamma K_D - \beta_2\gamma K_D)}{K_D(1-\gamma K_D)},$$

$$\frac{\mathrm{d}\mathcal{N}(K_D)X_F^{*\beta_2-\beta_1}}{\mathrm{d}K_D} = \frac{(1-\gamma K_D - \beta_1\gamma K_D)\mathcal{N}(K_D)}{K_D(1-\gamma K_D)}X_F^{*\beta_2-\beta_1},$$

and

$$\frac{\mathrm{d}}{\mathrm{d}K_D}K_D K_F^* X_F^{*1-\beta_1} = \frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)}K_D K_F^* X_F^{*1-\beta_1}.$$

Thus,

$$\frac{\mathrm{d}\mathcal{B}_2(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_2(K_D).$$

1. Entry Deterrence Strategy

   The optimal capacity by the dedicated leader, $K_D^{det}(X)$, satisfies the first order condition

$$\begin{aligned}\frac{\partial V_D(X,K_D)-\delta K_D}{\partial K_D} &= \frac{\mathrm{d}\mathcal{B}_2(K_D)}{\mathrm{d}K_D}X^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X-\frac{c}{r}-\delta \\ &= \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_2(K_D)X^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X-\frac{c}{r}-\delta = 0. \quad \text{(A.32)}\end{aligned}$$

   The entry deterrence strategy cannot happen if $K_D^{det}(X) < \hat{K}_D(X)$. If the dedicated leader invests at X, then the deterrence strategy is only possible when $X < X_2^{det}$. X_2^{det}, $K_D^{det}(X_2^{det})$, and $K_F^*(K_D^{det})$ satisfy (12), (13), and (A.32), with $X_F^*(K_D^{det}) = X_2^{det}$. Similar to the case that the flexible follower produces below capacity right after investment, the deterrence strategy is not possible if $K_D^{det} < 0$, which results that $X > X_1^{det}$ with X_1^{det} satisfying

$$\frac{c}{2(\beta_1-\beta_2)}\left(\frac{X_1^{det}}{X_F^*(0)}\right)^{\beta_1}\left(\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left[\left(\frac{X_F^*(0)}{c}\right)^{\beta_2}-\left(\frac{X_F^*(0)\left(1-2\gamma K_F^*(0)\right)}{c}\right)^{\beta_2}\right]\right.$$
$$\left.-\frac{\beta_1-\beta_2}{r-\alpha}\frac{2\gamma X_F^*(0)K_F^*(0)}{c}\right)+\frac{1}{r-\alpha}-\frac{c}{r}-\delta=0, \quad \text{(A.33)}$$

   where $K_F^*(0)$ and $X_F^*(0)$ satisfy (12) and (13). Thus, the entry deterrence strategy is only possible if $X \in \left(X_1^{det}, X_2^{det}\right)$. If the leader applies the entry deterrence strategy and invests at $X^{det}(K_D)$ with capacity level K_D, then the value function before and after investment is

$$V_D(X,K_D) = \begin{cases}\mathcal{A}(K_D)X^{\beta_1} & X < X^{det}(K_D), \\ \mathcal{B}_2(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{cK_D}{r} & X^{det}(K_D) \le X < X_F^*(K_D), \\ \mathcal{N}(K_D)X^{\beta_2}+\frac{K_D(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}X-\frac{cK_D}{r} & X \ge X_F^*(K_D).\end{cases} \quad \text{(A.34)}$$

   For a given capacity level K_D, from value matching and smooth pasting at $X^{det}(K_D)$, $X^{det}(K_D)$ must satisfy

$$\begin{aligned}\mathcal{A}(K_D)X^{\beta_1} &= \mathcal{B}_2(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{cK_D}{r}-\delta K_D, \\ \beta_1\mathcal{A}(K_D)X^{\beta_1-1} &= \beta_1\mathcal{B}_2(K_D)X^{\beta_1-1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}.\end{aligned}$$

   It can be derived that

$$X^{det}(K_D) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right). \quad \text{(A.35)}$$

   K_D^{det} and $X^{det}(K_D^{det})$ satisfy (A.32), thus the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ are

$$\begin{aligned}K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det})) = \frac{1}{(\beta_1+1)\gamma}, \\ X^{det}(K_D^{det}) &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right).\end{aligned}$$

2. Entry Accommodation Strategy

The investment capacity by the dedicated leader $K_D^{acc}(X)$ for a given level of X satisfies the first order condition

$$\begin{aligned}\frac{\partial V_D(X,K_D)-\delta K_D}{\partial K_D} &= \frac{\mathrm{d}\mathcal{N}(K_D)}{\mathrm{d}K_D}X^{\beta_2}+\frac{X(1-\gamma K_D-\gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)}-\frac{c}{r}-\delta \\ &= \frac{(1-\gamma K_D-\beta_2\gamma K_D)\mathcal{N}(K_D)}{K_D(1-\gamma K_D)}X^{\beta_2}+\frac{X(1-\gamma K_D-\gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)} \\ &\quad -\frac{c}{r}-\delta=0. \end{aligned} \tag{A.36}$$

The entry accommodation strategy only happens when the market has grown large enough to hold the two firms, i.e., $X \geq X_F^*(K_D)$. Define $X_1^{acc}=X_F^*\left(K_D^{acc}(X_1^{acc})\right)$, then X_1^{acc}, $K_D^{acc}(X_1^{acc})$, and $K_F^*(K_D^{acc})$ satisfy (12), (13), and (A.36). Suppose the dedicated leader uses the entry accommodation strategy and invests at $X^{acc}(K_D)$ with capacity K_D, then the leader's value function is

$$V_D(X,K_D)=\begin{cases}\mathcal{A}(K_D)X^{\beta_1} & X<X^{acc}(K_D), \\ \mathcal{N}(K_D)X^{\beta_2}+\frac{K_D(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}X-\frac{cK_D}{r} & X\geq X_F^*(K_D)\geq X^{acc}(K_D).\end{cases} \tag{A.37}$$

From value matching and smooth pasting, I get that the investment threshold $X^{acc}(K_D)$ satisfies

$$\begin{aligned}\mathcal{A}(K_D)X^{\beta_1} &= \mathcal{N}(K_D)X^{\beta_2}+\frac{K_D(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}X-\frac{cK_D}{r}-\delta K_D, \\ \beta_1\mathcal{A}(K_D)X^{\beta_1-1} &= \beta_2\mathcal{N}(K_D)X^{\beta_2-1}+\frac{K_D(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}.\end{aligned}$$

Thus, it holds that $X^{acc}(K_D)$ must satisfy

$$\frac{\beta_1-\beta_2}{\beta_1}\mathcal{N}(K_D)X^{\beta_2}+\frac{\beta_1-1}{\beta_1(r-\alpha)}XK_D\left(1-\gamma K_D-\gamma K_F^*(K_D)\right)-\frac{cK_D}{r}-\delta K_D=0. \tag{A.38}$$

Rewrite (A.36) and (A.38), then $X^{acc}(K_D^{acc})$ and K_D^{acc} satisfy

$$\begin{aligned}&\frac{1-\gamma K_D-\beta_2\gamma K_D}{1-\gamma K_D}\frac{cX^{\beta_2}}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(X_1^{-\beta_2}-X_2^{-\beta_2}\right) \\ &\quad +\frac{X(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}\frac{1-2\gamma K_D}{1-\gamma K_D}-\frac{c}{r}-\delta=0,\end{aligned}$$

and

$$\frac{cX^{\beta_2}}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(X_1^{-\beta_2}-X_2^{-\beta_2}\right)+\frac{X(1-\gamma K_D-\gamma F^*(K_D))}{r-\alpha}\frac{\beta_1-1}{\beta_1}-\frac{c}{r}-\delta=0.$$

From

$$\frac{1-\gamma K_D-\beta_2\gamma K_D}{(\beta_1-\beta_2)(1-\gamma K_D)}=\frac{1}{\beta_1},$$

and

$$\frac{1-2\gamma K_D}{1-\gamma K_D}=\frac{\beta_1-1}{\beta_1},$$

it follows that the optimal investment capacity is

$$K_D^{acc}\equiv K_D^{acc}(X^{acc}(K_D^{acc}))=\frac{1}{(\beta_1+1)\gamma}.$$

A.7 Proof of Proposition 4

Given in the text.

A.8 Proof of Proposition 5

When the follower is flexible, from Proposition 2 and Proposition 3, the leader's entry deterrence strategy is the same regardless of whether the follower produces below or up to capacity right after investment. When there is no flexibility, the leader's entry deterrence (and entry accommodation strategy) can be found in Appendix B. The leader's entry deterrence strategy are the same regardless of with or without the follower flexibility. From Proposition 2 and Proposition 3, it also holds that the leader's investment capacity under entry accommodation strategy is $K_D^{acc} = \frac{1}{(\beta+1)\gamma}$, regardless of whether the follower produces below or up to capacity right after investment. This capacity level is the same as that when there is no follower flexibility.

B No Flexibility

This section analyzes what the follower and leader's decisions are when there is no flexibility. It means that both firms would always produce up to full capacity. For the follower, given that the leader invests and always produces K_D and the follower invests and always produces K_F, the profit flow at time t equals

$$\pi_F(t) = (X(t)(1-\gamma(K_D+K_F)) - c)\, K_F.$$

Here, I do not allow production suspension. So for a low level X, i.e., $X(1-\gamma(K_D+K_F)) < c$, the firms may have negative profit flows. Given the initial geometric Brownian motion level X, the value of the follower is

$$\begin{aligned} V_F(X, K_D, K_F) &= E\left[\int_{t=0}^{\infty} K_F\,(X(t)(1-\gamma(K_D+K_F)) - c)\exp(-rt)dt \mid X(0) = X\right] \\ &= \frac{XK_F(1-\gamma(K_D+K_F))}{r-\alpha} - \frac{cK_F}{r}. \end{aligned}$$

The follower's investment capacity maximizes

$$\max_{K_F>0} V_F(X, K_D, K_F) - \delta K_F,$$

thus, given X and K_D,

$$K_F(X, K_D) = \frac{1}{2\gamma}\left(1 - \gamma K_D - \frac{r-\alpha}{X}\left(\frac{c}{r}+\delta\right)\right). \tag{B.1}$$

Before the investment, the follower holds an option to invest. Suppose the option value is

$$V_F(X, K_D) = A_F(K_L)X^{\beta_1}.$$

According to value matching and smooth pasting, the investment threshold $X_F(K_D, K_F)$ when investing with K_F satisfies

$$A_F X_F^{\beta_1} = \frac{X_F^* K_F(1-\gamma(K_D+K_F))}{r-\alpha} - \frac{cK_F}{r} - \delta K_F,$$

$$\beta_1 A_F X_F^{\beta_1 - 1} \quad = \quad \frac{K_F(1-\gamma(K_D+K_F))}{r-\alpha}.$$

Thus,

$$X_F(K_D, K_F) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D - \gamma K_F)}\left(\frac{c}{r}+\delta\right). \tag{B.2}$$

Combining (B.1) and (B.2), the follower's optimal investment capacity and threshold are

$$K_F^*(K_D) \quad = \quad \frac{1-\gamma K_D}{(1+\beta_1)\gamma}, \tag{B.3}$$

$$X_F^*(K_D) \quad = \quad \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right). \tag{B.4}$$

If $X_F^*(K_D) \leq X(0)$, then the follower would invest immediately at $t=0$ with capacity $K_F^*(X(0), K_D)$.

For the leader, to deter or accommodate the entry of the follower would be dependent on the leader's critical capacity level

$$\hat{K}_D(X) = \frac{1}{\gamma}\left(1 - \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)X}\left(\frac{c}{r}+\delta\right)\right). \tag{B.5}$$

Entry Deterrence Strategy If the leader invests a capacity larger than $\hat{K}_D(X)$, then the follower invests later. However, if the leader invests a capacity not larger than $\hat{K}_D(X)$, then the follower invests at the same time with the leader. Suppose the investment threshold is $X_D^{det}(K_D)$ when investing capacity K_D, then the leader's value under entry deterrence strategy is assumed to be

$$V_D(X, K_D) = \begin{cases} A_D(K_D)X^{\beta_1} & \text{if } X < X_D^{det}(K_D), \\ B_D(K_D)X^{\beta_1} + \frac{XK_D(1-\gamma K_D)}{r-\alpha} - \frac{cK_D}{r} & \text{if } X_D^{\det}(K_D) \leq X < X_F^*(K_D), \\ \frac{\beta_1 X K_D(1-\gamma K_D)}{(1+\beta_1)(1-\alpha)} - \frac{cK_D}{r} & \text{if } X \geq X_F^*(K_D). \end{cases}$$

By value matching at $X_F^*(K_D)$, I get

$$B_D(K_D)X_F^{*\beta_1} + \frac{X_F^* K_D(1-\gamma K_D)}{r-\alpha} = \frac{\beta_1 X_F^* K_D(1-\gamma K_D)}{(\beta_1+1)(r-\alpha)}.$$

Thus,

$$B_D(K_D) = -\frac{K_D(1-\gamma K_D)X_F^*}{(\beta_1+1)(r-\alpha)}X_F^{*-\beta_1} = -\frac{K_D}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right)\right)^{-\beta_1}.$$

Suppose the leader invests at X, then the investment capacity under the deterrence strategy, $K_D^{det}(X)$, satisfies

$$-\frac{1-(\beta_1+1)\gamma K_D}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)(1-\gamma K_D)}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1} + \frac{X(1-2\gamma K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0. \tag{B.6}$$

The corresponding value for the leader's entry deterrence strategy is

$$V_D^{det}(X) \quad = \quad -\frac{K_D^{det}(X)}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)(1-\gamma K_D^{det}(X))}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1}$$

$$+\frac{XK_D^{det}(X)(1-\gamma K_D^{det}(X))}{r-\alpha}-\frac{cK_D^{det}(X)}{r}-\delta K_D^{det}(X). \tag{B.7}$$

If X is sufficiently small, then the optimal investment threshold is

$$X_D^{det} = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D^{det})}\left(\frac{c}{r}+\delta\right). \tag{B.8}$$

Substitute (B.8) into (B.6) gives

$$1-(\beta_1+1)\gamma K_D^{det}=\left(1-(\beta_1+1)\gamma K_D^{det}\right)\left(\frac{\beta_1}{\beta_1+1}\right)^{\beta_1}.$$

Thus,

$$\begin{aligned} K_D^{det} &= \frac{1}{(\beta_1+1)\gamma}, \\ X_D^{det} &\equiv X_D^{det}(K_D^{det})=\frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right). \end{aligned}$$

The corresponding follower's investment decisions are

$$\begin{aligned} K_F^*(K_D^{det}) &= \frac{\beta_1}{(\beta_1+1)^2\gamma}, \\ X_F^*(K_D^{det}) &= \frac{(\beta_1+1)^2(r-\alpha)}{\beta_1(\beta_1-1)}\left(\frac{c}{r}+\delta\right). \end{aligned}$$

Moreover, the entry deterrence strategy can not happen for

$$0\le \hat{K}_D(X)<K_D^{det},$$

i.e.,

$$X_1^{det}\le X\le X_2^{det},$$

where

$$X_2^{det}=\frac{2(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right)$$

and X_1^{det} satisfies

$$-\frac{1}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1}+\frac{X}{r-\alpha}-\frac{c}{r}-\delta=0. \tag{B.9}$$

If $X_D^{det}\le X$, then the deterrence strategy is implemented immediately with capacity $K_D^{det}(X)$ satisfying (B.6).

Entry Accommodation Strategy Under the entry accommodation strategy, the follower invests at the same time as the leader. Suppose the investment threshold is $X_D^{acc}(K_D)$ when investing capacity K_D, then the leader's value under entry accommodation strategy is assumed to be

$$V_D(X,K_D)=\begin{cases} A_D(K_D)X^{\beta_1} & \text{if } X<X_D^{acc}(K_D), \\ \frac{XK_D(1-\gamma K_D)}{2(r-\alpha)}-\frac{cK_D}{2r}+\frac{\delta K_D}{2} & \text{if } X\ge X_D^{acc}(K_D). \end{cases}$$

For a given level of X, the investment capacity under the entry accommodation strategy is

$$K_D^{acc}(X) = \frac{1}{2\gamma}\left(1 - \frac{r-\alpha}{X}\left(\frac{c}{r}+\delta\right)\right).$$

The accommodation strategy can only be chosen when $K_D^{acc}(X) \leq \hat{K}_D(X)$, which means that it is only possible when

$$X \geq X_1^{acc} = \frac{(\beta_1+3)(r-\alpha)}{\beta_1 - 1}\left(\frac{c}{r}+\delta\right).$$

Moreover, the value matching and smoothing pasting conditions yield that for the given capacity K_D, the investment threshold $X_D^{acc}(K_D)$ satisfies

$$\begin{aligned} A_D(K_D)X^{\beta_1} &= \frac{XK_D(1-\gamma K_D)}{2(r-\alpha)} - \frac{cK_D}{2r} - \frac{\delta K_D}{2}, \\ \beta_1 A_D(K_D)X^{\beta-1} &= \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}. \end{aligned}$$

Thus, it holds that

$$X_D^{acc}(K_D) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right).$$

Then the optimal investment capacity K_D^{acc} and the optimal investment threshold X_D^{acc} are

$$\begin{aligned} K_D^{acc} &= \frac{1}{(\beta_1+1)\gamma}, \\ X_D^{acc} &= \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)}\left(\frac{c}{r}+\delta\right). \end{aligned}$$

If $X_D^{acc} \leq X$, then the leader invests immediately at X with capacity

$$K_D(X) = \frac{1}{2\gamma}\left(1 - \frac{r-\alpha}{X}\left(\frac{c}{r}+\delta\right)\right).$$

Note that $X_1^{acc} > X_D^{acc}$. This means that the leader implements the accommodation strategy only when X reaches X_1^{acc}. Then the leader invests at X_1^{acc} with capacity

$$K_D(X_1^{acc}) = \frac{2}{(\beta_1+3)\gamma}.$$

The leader's value at X_1^{acc} is

$$V_D(X_1^{acc}, K_D(X_1^{acc})) = \frac{2}{(\beta_1-1)(\beta_1+3)\gamma}\left(\frac{c}{r}+\delta\right).$$

The corresponding follower's investment decisions under the leader's accommodation strategy are

$$\begin{aligned} K_F^*(K_1^{acc}) &= \frac{\beta_1+1}{(\beta_1+3)\gamma}, \\ X_F^*(K_1^{acc}) &= \frac{(\beta_1+3)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right). \end{aligned}$$