Toehold M&A Dynamic Games*

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Abstract

In this paper we show how a toehold held by a bidder can be an effective strategy for inducing a target to accept a hostile bid. We suggest a new explanation for the benefits of using a toehold. By considering both friendly mergers and hostile takeovers as alternative M&A strategies, we show that a toehold enhances the bargaining power of the bidder, inducing the target to be more prone to accept a hostile bid. We show that toehold hostile takeovers require sufficiently large synergies to become preferable over toehold friendly mergers. Uncertainty may have an ambiguous effect on the strategy choice. In general, a higher toehold lowers the required synergies. Larger bidders tend also to be more prone to enter in hostile deals. Accordingly, toeholds can be used to overcome possible size disadvantages of bidders, suggesting that larger firms need less to use toeholds to succeed in hostile takeovers.

Keywords: M&A; Real Options; Hostile Takeovers; Friendly Mergers; Toeholds

JEL codes: G34; C73; D81.

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1 Introduction

Since the turn of the millennium, roughly 800000 Merger and Acquisitions (M&A) transactions worth over 57 trillion USD in total have been announced worldwide. This corresponds to approximately 120 transactions per day. Despite this seemingly experienced process, designing M&A right is among the most prominent challenges for national and international bidders in several respects. First, estimating ex ante the synergies expected from a M&A suffers from significant information asymmetry and raise fears of paying too much. Second, proposing a public bid to a target sends a signal to other rivals which might result in a takeover battle causing the initial bidder either to loose the contest or to pay too much for the target. Third, the internalization of the target's assets is further complicated by cultural, legal, and accounting peculiarities.

Various M&A payment mechanisms that allow to mitigate these risks have been studied in the literature (Huang and Walkling (1987); Reuer et al. (2004); Mantecon (2009), among others). Apart from structuring the payment by means of cash or the use of contingent earn-out payments, toeholds are a common strategy to strengthen the bidder's position when negotiating full acquisitions. In particular, toeholds in M&As have great similarities with joint ventures. Bidders establish a toehold by buying a non-controlling but significant equity stake in a target. Hence, a toehold allows the bidder to profit from future M&A activity the target might, in several ways, be involved in. First, the buyer might consider to take over the target at a later date and thus a toehold reduces the number of shares he has to acquire at a larger premium (Eckbo and Langohr (1989); Betton and Eckbo (2000)). Second, should the target be fully acquired by a rival bidder in a bidding contest, then these shares can be sold at a considerable profit (Burkart (1995); Singh (1998)). Third, a large enough toehold can discourage rivals to enter into a takeover battle later on thereby increasing the probability of a successful single bid contest while at the same time reducing the chances of suffering from the "winner's curse" (Betton and Eckbo (2000); Ettinger (2009)). Finally, a toehold reduces information asymmetry between the bidder and the

target thereby mitigating the target managements private benefits of control (Bulow et al. (2009); Ouimet (2013); Povel and Sertsios (2014)). In particular, partial ownership allows the bidder to learn about the target during the transitional period towards full acquisition.

Despite the considerable advantages toeholds provide, there has been a steady decrease in the occurrence of toeholds in recent years which has given rise to an extensive research on toeholds and to shed light on this toehold puzzle.[1] While previous empirical and analytical literature on toeholds have predominantly looked on how toeholds affect the bidding behavior of potential buyers in takeover contest less attention, however, has been paid to the strategic effects toeholds unfold in single bid contest between one bidder and one target. For example, (Strickland et al., 2010, p. 60) highlight that "*if a bidder concludes that target management is entrenched and is unlikely to negotiate a mutual beneficial merger price, a toehold should be employed, as the toehold increases the likelihood of bid success in a difficult economic climate*". Indeed, as reported by Betton et al. (2009) the majority of toehold investments lead to hostile takeovers rather than friendly mergers. This raises the question: Why? The purpose of this paper is to looker closer at the choice between hostile takeovers and friendly mergers and on how toeholds affect this decision under uncertainty. We thereby want to answer several important questions. How does the design of toeholds impact the choice between friendly and hostile takeovers? Do large toeholds induce the bidder to fully acquire the target sooner?

To answer this question we draw upon the literature on investment under uncertainty which puts a special emphasis on contingent dynamic decision making and game theory; important features we find most suited for our analysis. Within this strain of literature analyzing the option features inherent in M&A has a long tradition. While past papers have predominantly looked at the flexibility M&As provide and at the growth option they generate from an empirical perspective (Folta and OBrien, 2007; Bekkum et al., 2011; Bonaime et al., 2018) recently papers explicitly focus on modelling the negotiation process and the dynamic contract design of M&As ((Lambrecht, 2004; Morellec and Zhdanov, 2005; Alvarez and Stenbacka, 2006; Lambrecht and Myers, 2007; Thijssen, 2008; Lukas

[1]For example, Betton et al. (2009) report that only two percent of more that 12.000 bidders chose to install a toehold in a target prior to full acquisition.

et al., 2012)). The results have provided answers with respect to how hostile takeover negotiation and merger negotiation, respectively, have an impact on takeover timing and sharing of the surplus under uncertainty and how contingent payments mitigate moral hazard. However, the explicit modelling of toeholds in M&A by means of option games has gained less attention so far. That does, however, not mean that modelling toeholds has been neglected in the corporate finance literature. Rather, quite a few paper look at the link between toeholds and takeover contest from a microeconomic perspective (Shleifer and Vishny, 1986; Hirshleifer and Titman, 1990; Chowdhry and Jegadeesh, 1994; Burkart, 1995; Singh, 1998; Betton and Eckbo, 2000; Povel and Sertsios, 2014). While the results support e.g. the view that toeholds lead to agressive bidding behavior of the toehold holder, that increase a bidder's chance of winning a takeover battle, or that the reduce the winner's curse these models do not focus on the choice between hostile takeover and friendly merger neither do they investigate the impact of uncertainty on the M&A terms. To the best of our knowledge, this paper is one of the first that aims to model the link between toehold and the choice of M&A strategy, i.e. hostile takeover versus friendly merger.

We model a friendly merger as a cooperative game. Departing from previous literature (Alvarez and Stenbacka, 2006; Thijssen, 2008) we consider a possible hostile takeover as a credible outside option. Hostile takeovers are modeled as non-cooperative games (Lambrecht, 2004; Lukas and Welling, 2012). Allowing for different synergies and costs, hostile takeover may become optimal, being the choice based on the option values. We derive analytically the level of synergies that induce the choice of each strategy. We show that hostile takeovers require sufficiently large synergies, which varies with some key factors.

When a bidder holds a toehold in the target, hostile takeovers become more likely. This is the result of two main effects: (i) a toehold reduces the threshold for a hostile takeover, because the bidder saves part of the premium needed to induce the target to accept the offer; (ii) it gives the bidder a credible threat to takeover the firm in a hostile manner, and enhances its bargaining power in a friendly merger, inducing the target to

be more willing to accept the hostile takeover.

Additionally, we show that uncertainty may have an ambiguous effect on the strategy choice and that larger bidders tend to be more prone to enter in a hostile deal, producing a similar effect as that of the toehold. Accordingly, toeholds can be used to overcome possible size disadvantages of bidders, suggesting that larger firms need less to use toeholds to succeed in hostile takeovers.

The paper unfolds as follows. The following Section 2 presents the derivation of the model. Section 3 performs a comparative statics of the main drives of toehold M&As, and Section 4 concludes.

2 The Model

Consider two firms active in the market labeled as B and T. In the merging process B stands for bidder and T for target. For the sake of simplicity, we will assume that each firm is endowed with a capital stock $K_{B,T}$ and subject to an industry wide shock modeled by means of a geometric Brownian motion, i.e.:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dW \tag{1}$$

where $\alpha \in \mathbb{R}$ denotes the instantaneous drift, $\sigma \in \mathbb{R}^+$ denotes the instantaneous variance and dW denotes the standard Wiener increment. Under risk-neutrality we set $\alpha = r - \delta$, where r is the risk-free rate and $\delta > 0$ is a return shortfall. Additionally, we will assume that:

$$V_{B,T}(t) = K_{B,T}x(t) \tag{2}$$

were $V_{B,T}(t)$ approximates the firms' individual stand-alone values.

In addition to its own assets in place, firm B also holds a toehold in T, which corresponds to a minority stake θ. This minority stake is strategically held by B with the purpose of influencing the final outcome of a merging process. For instance, in case of a friendly merger, θ may impact the relative bargaining power of each firm, affecting the way the surplus is shared between them. Additionally, the toehold position creates a credible

threat in case of a hostile acquisition, as the firm holding θ assumes the role of a credible bidder in the event of takeover (e.g., Strickland et al., 2010).

Upon merging, the value of the new entity M will depend on the type of deal (hostile or friendly), and is given by:

$$V_{Mh,f}(t) = K_{Mh,f}x(t) = (\omega_{h,f} + K_B + K_T)x(t) \tag{3}$$

where $\omega_{h,f} > 0$ denotes synergies arising from the transaction. Our model is sufficiently flexible to assume different synergy levels depending on the type of deal (ω_h for hostile takeovers, and ω_f for the friendly mergers). One may argue that the synergies produced in a hostile takeover are not smaller than those of a friendly merger (i.e., $\omega_h \geqslant \omega_f$), due to the acquirer's ability to substitute target's managers for a more efficient management team. These changes typically do not occur in friendly mergers, where the incumbent managers normally stay in office, along with new managers designated by the acquirer.[2]

The merger game considers two possible strategies. One strategy is the hostile takeover, modeled by means of a non-cooperative game, where each entity maximizes its own utility by integrating the reaction-function of the other party. As previously presented in the literature (e.g., Lambrecht, 2004; Lukas and Welling, 2012), one player decides upon the premium to be offered (or to be demanded) and the other reacts by accepting the deal at the timing that maximizes its own utility. In our case, we assume that firm B offers a given premium to the shareholder of firm T (i.e., the bidder is willing to share some of the merging synergies with the target), which, in turn, accepts the deal at a given optimal timing.

The other strategy available for the players is to negotiate a friendly merger, where their stake on the new entity M is cooperatively determined. The game is modeled herein by means of a Nash bargaining solution. However, the way we model the bargaining game significantly differs from what can be found in the related dynamic M&A literature. In our setting, each player takes into consideration the alternative he has (or that he will end-up

[2]The literature on M&A refers to such takeovers as disciplinary takeovers where bidder opt to replace the targets management because of their incapacity of maximizing shareholder wealth (Jensen and Ruback (1983); Morck et al. (1989)).

in) in the case of a negotiation failure. In fact, each firm has an outside option, acting as a latent alternative, which they realistically consider in the cooperative bargaining game. In our setting, B credibly threats to take the role of a bidder (due to its strategic toehold position), while T may end-up as a target, in a hostile takeover that may happen if no agreement is achieved in the friendly negotiations. By considering, realistically, the existence of hostile outside options for both parties, we depart from the existing related literature, where no threat values are assumed (Thijssen, 2008).

2.1 Hostile takeover

Let us start by modeling the hostile takeover. As mentioned, we rely on a non-cooperative game where each party maximizes its own position, one by offering (or demanding) a premium on top of firm's assets in place, i.e., sharing (or asking for) some of the merger synergies, while the other party optimally accepts the deal (Lambrecht, 2004; Lukas and Welling, 2012). Herein, we follow a setting where firm B proposes the premium $\psi - 1 > 0$, and the shareholders of firm T accept the deal at their value maximizing timing.

Let $\epsilon_B Y_h$ and $\epsilon_T Y_h = (1 - \epsilon_B) Y_h$ denote the transaction costs assigned to each party where $\epsilon_B \in (0, 1)$ indicates the fraction of the irreversible transaction costs of the hostile takeover, Y_h, assigned to the bidder.

At the moment the game begins, firm B has been already endowed with a strategic toehold position on the target firm, corresponding to the fraction θ in T's equity. The toehold is a minority stake, so that $\theta < 0.5$, and the remaining $(1-\theta)$ is held the shareholders of firm T.

When the takeover takes place, the target shareholders receive a net premium of $(\psi - 1)(1-\theta)K_T x(t)$ in exchange for their stake on the firm, and have to bear their fraction in the transaction costs $(1-\theta)(1-\epsilon_B)Y_h$. For any given premium level, shareholders of T decide to accept the offer at the timing that solves the following optimization problem:

$$f(x) = \max_{\tau} \left[\mathbf{E} \left[\left((\psi - 1)(1-\theta) K_T x(t) - (1-\theta)(1-\epsilon_B) Y_h \right) e^{-r\tau} \right] \right] \tag{4}$$

where $\mathbf{E}(.)$ is an expectation operator. Standard real options arguments allow us to present

the optimization problem as follows:

$$f(x) = \max_{x_h^*(\psi)} \left[((\psi - 1)(1-\theta)K_T x_h^*(\psi) - (1-\theta)(1-\epsilon_B)Y_h) \left(\frac{x(t)}{x_h^*(\psi)} \right)^{\beta_1} \right] \tag{5}$$

where $x_h^*(\psi)$ is the takeover threshold for any given ψ, and $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}$ is the positive root of the standard fundamental quadratic equation (see Dixit and Pindyck, 1994).

On the other side, the bidder anticipates the target shareholders reaction function and offers an optimal premium such that it maximizes his objective function, i.e.:

$$\max_{\psi} \left[\left((K_{Mh} - K_B - \theta K_T - \psi(1-\theta)K_T) \, x_h^*(\psi) - (\epsilon_B + \theta(1-\epsilon_B))Y_h \right) \left(\frac{x(t)}{x_h^*(\psi)} \right)^{\beta_1} \right] \tag{6}$$

where $K_{Mh} = \omega_h + K_B + K_T$. At the threshold, the bidder's payoff has the following intuition: he receives the merged firm, $K_{Mh} x_h^*(\psi)$, whose value includes the proper synergy value $\omega_h x_h^*(\psi)$, loses the stand alone positions, $K_B x_h^*(\psi)$ and $\theta K_T x_h^*(\psi)$, and pays, with a premium, the acquisition of the assets held by the target shareholders, $\psi(1-\theta)K_T x_h^*(\psi)$. In addition, the bidder pays the corresponding transaction costs: both its own fraction, $\epsilon_B Y_h$, and, indirectly, the fraction related to the toehold position, $\theta(1-\epsilon_B)Y_h$.

Solving objective functions (5) and (6) recursively leads to the following results for the threshold, premium and firms' option values:

Proposition 1. *The hostile takeover takes place at the threshold:*

$$x_h^* \equiv x_h^*(\psi^*) = \frac{\beta_1}{(\beta_1 - 1)^2} \frac{(\beta_1 - \epsilon_B(1-\theta) - \theta)Y_h}{\omega_h} \tag{7}$$

as a result of the optimal premium offered by B:

$$\psi^* = 1 + \frac{(\beta_1 - 1)(1-\epsilon_B)}{\beta_1 - \epsilon_B - \theta(1-\epsilon_B)} \frac{\omega_h}{K_T} \tag{8}$$

The takeover option values for B and T are:

$$Bx^{\beta_1} = \frac{(\beta_1 - \epsilon_B - \theta(1-\epsilon_B))Y_h}{(\beta_1 - 1)^2}\left(\frac{x}{x_h^*}\right)^{\beta_1} \tag{9}$$

$$Tx^{\beta_1} = \frac{(1-\theta)(1-\epsilon_B)Y_h}{\beta_1 - 1}\left(\frac{x}{x_h^*}\right)^{\beta_1} \tag{10}$$

Proof. See Appendix. □

From (7) and (8) we see that a higher toehold, θ, induces the bidder to offer a higher premium ($\partial\psi^*/\partial\theta > 0$), leading the hostile takeover to occur for a lower threshold ($\partial x_h^*/\partial\theta < 0$). In addition, standard results apply for uncertainty as a higher σ (i.e., a lower β_1), induces the bidder to offer a lower premium deterring the takeover ($\partial\psi^*/\partial\sigma < 0$, $\partial x_h^*/\partial\sigma > 0$), whereas for higher synergies, the deal occurs sooner with a higher premium ($\partial\psi^*/\partial\omega_h > 0$, $\partial x_h^*/\partial\omega_h < 0$).

2.2 Friendly merger

Let us now consider a friendly merger between firms B and T. In particular, let us assume that after the merger, each firm shareholders are granted with an equity stake (γ_B for B and $\gamma_T = 1 - \gamma_B$ for T) in the new entity M, giving up their stand-alone values $V_{B,T} = K_{B,T}x(t)$. Both B and T shareholders benefit from the synergies ω_f but pay their fraction (respectively, ϵ_B and $1 - \epsilon_B$) of the transactions costs Y_f. Hence, each party has the following net gain in the merger:

$$\left(\gamma_B K_{Mf} - K_B - \theta K_T\right) x(t) - \left(\epsilon_B + \theta\left(1 - \epsilon_B\right)\right) Y_f \tag{11}$$

for firm B shareholders, and

$$\left((1-\gamma_B)K_{Mf} - (1-\theta)K_T\right) x(t) - (1-\theta)(1-\epsilon_B)Y_f \tag{12}$$

for those of firm T, where $K_{Mf} = \omega_f + K_B + K_T$ denotes the size of M after incorporating the proper synergies arising from the friendly merger.

Assume that each firm possesses a certain baseline level of bargaining power, η_B for firm B and $\eta_T = 1 - \eta_B$ for firm T. In our setting, this baseline bargaining power is enhanced by the firm holding the toehold, firm B, as this strategic minority stake on T's capital endows B with a greater negotiating power. For the sake of simplicity, we assume that this enhanced bargaining power is proportional to the toehold, meaning that B extracts from T a part of his η_T propositional to θ.

Solving the cooperative bargaining game by means of the Nash-Bargaining solution, the optimal share that each firm will capture in the new venture solves the following optimization problem:

$$\max_{\gamma_i} \Bigg[\Big(\gamma_B K_{Mf} - K_B - \theta K_T)x(t) - \epsilon_B Y_f - \theta(1-\epsilon_B)Y_f - A_B x^{\beta_1} \Big)^{\eta_B + \theta(1-\eta_B)} \\ \Big(((1-\gamma_B)K_{Mf} - (1-\theta)K_T)x(t) - (1-\theta)(1-\epsilon_B)Y_f - A_T x^{\beta_1} \Big)^{(1-\theta)(1-\eta_B)} \Bigg] \quad (13)$$

where the power terms $\eta_B + \theta(1-\eta_B)$ and $(1-\theta)(1-\eta_B)$ represent, respectively, the bargaining power of firm B (enhanced by the toehold) and T (diminished by the toehold). We can easily see that, as $\theta \to 0$ the bargaining power of both firms tend to their baseline values.

The generic terms $A_B x^{\beta_1}$ and $A_T x^{\beta_1}$ represent each firm's threat value, which correspond to the outside options available to B and T, respectively. As previously stated, these outside options act as latent alternatives that both firms will face if the cooperative game brakes down: firm B, due to the strategic toehold position, threatens to be a (credible) bidder in a hostile takeover, and, consequently, T ends-up as a hostile target.

From the previous section, we see that the constants A_B and A_T are as follows:

$$A_B = \frac{(\beta_1 - \epsilon_B - \theta(1-\epsilon_B))Y_h}{(\beta_1 - 1)^2} \left(\frac{1}{x_h^*(\psi^*)} \right)^{\beta_1} \quad (14)$$

$$A_T = \frac{(1-\theta)(1-\epsilon_B)Y_h}{\beta_1 - 1} \left(\frac{1}{x_h^*(\psi^*)} \right)^{\beta_1} \quad (15)$$

The solution to the maximization problem leads to the following proposition:

Proposition 2. *Both firms will agree to merge when $x(t)$ hits the optimal timing threshold*

x_f^* from below:

$$x_f^* = \frac{\beta_1}{\beta_1 - 1} \frac{Y_f}{\omega_f} \tag{16}$$

Firm B's optimal stake γ_B^* in the merger amounts to:

$$\gamma_B^* = \frac{K_B + \theta K_T}{K_{Mf}} + \frac{\omega_f}{K_{Mf}} \left(\frac{\beta_1 - 1}{\beta_1} \left(\epsilon_B + \theta(1 - \epsilon_B) + \eta_B + \theta(1 - \eta_B) \right) + \right.$$
$$\left. \frac{\beta_1 - 1}{\beta_1} \frac{(1 - \eta_B - \theta(1 - \eta_B))A_B - (\eta_B + \theta(1 - \eta_B))A_T}{Y_f} \right) \left(\frac{\beta_1}{\beta_1 - 1} \frac{Y_f}{\omega_f} \right)^{\beta_1} \tag{17}$$

and, naturally, $\gamma_T^* = 1 - \gamma_B^*$.

Proof. See Appendix. □

After the derivation the optimal policy for merging, we can deduce each firm's ex-ante option value. For firm B is:

$$F_B(x) = \left((\gamma_B^* K_{Mf} - K_B - \theta K_T)\, x(t) - (\epsilon_B + \theta\,(1 - \epsilon_B))\, Y_f \right) \left(\frac{x(t)}{x_f^*} \right)^{\beta_1}, \qquad x(t) < x_f^* \tag{18}$$

and for firm T:

$$F_T(x) = \left(((1 - \gamma_B^*) K_{Mf} - (1 - \theta) K_T)\, x(t) - (1 - \theta)(1 - \epsilon_B) Y_f \right) \left(\frac{x(t)}{x_f^*} \right)^{\beta_1}, \qquad x(t) < x_f^* \tag{19}$$

As it becomes apparent, the threat values do not impact the timing of the merger, but, importantly, they do affect the sharing rule ($\gamma_{B,T}^*$) and, therefore, the option value $F_{B,T}(x)$.

Since we are on a cooperative game, it is possible to show that the optimal investment threshold equals the one of the central planner. The central planner's objective function equals:

$$\begin{aligned} G(x, \omega_f, Y_f) &= \max_{\tau} \left[\mathbf{E} \left[(\omega_f x(t) - Y_f)\, e^{-r\tau} \right] \right] \\ &= \max_{x_f^*(\omega_f, Y_f)} \left[\left(\omega_f x_f^*(\omega_f, Y_f) - Y_f \right) \left(\frac{x(t)}{x_f^*(\omega_f, Y_f)} \right)^{\beta_1} \right] \end{aligned} \tag{20}$$

Proposition 3. *The threshold for merging of the individual firms, given by Equation* (16)*, is the same as that of a central planner maximizing the overall payoff* $\omega_f x(t) - Y_f$*.*

Proof. See Appendix. □

Given the two alternatives available for the firms (to engage in a friendly merger or to go for a hostile takeover), is it very important to study the conditions under which one alternative is preferable to the other. This analysis is particularly relevant for studying the effect of the toehold on the definition of the dominant strategy. That is what we perform in the next section. Also, the relative magnitude of synergies arising from strategy in relation to the other (along with the specific costs) play an important role.

2.3 When do hostile bids take place?

Let us start by considering the extreme situation where $\omega_h = \omega_f = \omega$ and $Y_h = Y_f = Y$. Under this setting it is straightforward to show that friendly mergers occurs sooner than the hostile takeovers, i.e., $x_f^*(\omega, Y) < x_h^*(\omega, Y)$, and reveals the timing inefficiency of hostile bids, as already discussed in the literature. However, as we will show, and depending on the level of synergies, hostiles takeovers can be optimally accepted earlier or later that the timing of an friendly merger.

Early takeovers. If the synergies of the takeover are sufficiently larger than those of the friendly mergers ($\omega_h >> \omega_f$), a reverse order on the thresholds may occur, i.e., $x_h^* < x_f^*$. Typically, the hostile takeovers are more expensive than friendly mergers, and so the difference in the synergies needs to compensate for that. Accordingly:

Proposition 4. *The condition for a hostile takeover to occur before a friendly merger can be simply derived from Equations (7) and (16)*

$$\omega_h > \omega_t = \frac{\beta_1 - \epsilon_B - \theta(1-\epsilon_B)}{\beta_1 - 1} \frac{Y_h}{Y_f} \omega_f \tag{21}$$

where $Y_h \gtrless Y_f$ *ensures that* $\omega_t > \omega_f$*.*

However, firm B, pursuing its value maximization, will take the decision to move towards T by means of a takeover if, at the threshold, the intrinsic value of the hostile bid is larger than that of the friendly merger. Accordingly, a similar condition for ω_h can be set.

Proposition 5. *The hostile bid effectively takes place if:*

$$\omega_h > \omega_{e_B} = \left(\frac{(\beta_1 - 1)(\eta_B + \theta(1 - \eta_B))}{\beta_1 - \epsilon_B - \theta(1 - \epsilon_B)} \frac{Y_f}{Y_h} \right)^{1/\beta_1} \omega_t \tag{22}$$

When x hits $x_h^*|_{\omega_h > \omega_{e_B}}$, the bidder firm offers the optimal premium $\psi^*(\omega_h)$ (see Equation (8)) to the shareholders of the target firm, which, in turn, will decide on their own best interest. Two decisions can be taken: either they accept or reject the offer. By accepting the bid, they benefit from the premium paid by the bidder, as a result of a higher synergy (higher than that of a friendly merger, $\omega_h > \omega_f$). If the hostile bid is refused, firm B and firm T have the chance to negotiate, later on, a friendly merger. This friendly merger occurs as soon as $x = x_f^*$.

By deciding based on value, the shareholders of target firm will only accept the bid if the intrinsic value of the hostile takeover reveals to be larger than the option value of a friendly merger, i.e. if $T > F_T(x)|_{x<x_f^*}$.

The condition for the takeover to be accepted by the shareholders of T is as follows:

$$\omega_{e_T} > \left(\frac{1 - \eta_B}{1 - \epsilon_B} \frac{Y_f}{Y_h} \right)^{1/\beta_1} \omega_t \tag{23}$$

Notice that if the synergies are not sufficiently large to produce enough premium to be accepted by the shareholders of firm T (i.e., if $\omega_{e_B} < \omega_h < \omega_{e_T}$), the only possibility for the bidder firm is to wait and agree on a friendly merger based on a lower synergy ω_f, which will happen in a later moment (at $x = x_f^* > x_h^*|_{\omega_f < \omega_{e_B} < \omega_h < \omega_{e_T}}$). For modeling purposes, no threat values are considered in the case of this subsequent friendly negotiation, as the threshold for the hostile takeover has already been achieved.

Late takeovers. Finally, we need to consider a last possible situation, where the threshold for the friendly merger is achieved but, for the parties, the continuation value of the hostile takeover reveals to be more valuable. Accordingly:

Proposition 6. *For a level of synergies larger than ω_l i.e.,:*

$$\omega_h > \omega_l = \left(\frac{\beta_1 - 1}{(\beta_1 - 1) + \beta_1(1-\theta)(1-\epsilon_B)} \frac{Y_f}{Y_h} \right)^{1/\beta_1} \omega_t < \omega_t \tag{24}$$

the deal will occur later by means of a hostile takeover.

It is possible to show that the condition ω_l applies both for bidder and target firms, as natural result of the Nash bargaining solution.

3 Comparative statics

The choice of the best strategy is done by firms based on the option values, but if the goal is to study which strategy is more likely to occur, then the probability of reaching the thresholds becomes relevant. Therefore, in this section, we present a comparative statics of the main drivers of the timing and options values of toehold friendly mergers and toehold takeovers.

In the previous section we show that friendly mergers timing is unaffected by the threat values used by firms, occurring always at x_f^* (Equation (16)). Hostile takeovers occur at x_h^* (Equation (5)) and earlier than friendly mergers if synergies are sufficiently large, i.e. $w_h > w_t > w_f$ (Equation (21)). The timing of both strategies is independent of the sequence in which they become optimal (i.e., their magnitude). Therefore, hostile takeovers are more likely to occur the smaller its threshold is, relatively to the friendly merger threshold:

$$\phi = \frac{x_h^*}{x_f^*} = \frac{\beta_1 - \epsilon_B - \theta(1-\epsilon_B)}{\beta_1 - 1} \frac{Y_h}{Y_f} \frac{\omega_f}{\omega_h} \tag{25}$$

When we look at both thresholds (Equations (5) and (16)), the only common parameter appearing is uncertainty (β_1). For the remaining parameters, whenever they reduce the strategy threshold, it becomes more likely.

The choice of the best strategy is ultimately done using option values. From the previous section, we have two cases: (i) late hostile takeovers, occurring later than friendly mergers, and (ii) early hostile takeovers, occurring sooner than friendly mergers. Early takeovers are more likely to occur but require higher synergies, which has the opposite effect. In both cases, hostile takeovers and friendly mergers are possible. For the first case, where the friendly merger accounts for the hostile threat value, the option value of the friendly merger $F_{B,T}$ may not be higher than the hostile threat values (B and T), in which case late hostile takeovers become optimal. Late hostile takeovers occur if synergies are higher that ω_l (Equation (24)) For the second case, in which (early) hostile takeovers occur sooner than friendly mergers, synergies need to be higher than ω_{eT} (Equation (23)). Hostile takeovers that require lower synergies are more likely to occur. Therefore, the sensitivity of ω_l and ω_{eT} to the parameters allows us to infer how these parameters affect the likelihood of each strategy.

3.1 The effect of toeholds

When a bidder holds a toehold in the target, the takeover timing is affected by its size:

Corollary 1. *A higher toehold induces the shareholders of the bidder to offer higher premiums* ($\partial\psi^*/\partial\theta > 0$)*, hastening hostile takeovers* ($\partial x_h^*/\partial\theta < 0$) *and, therefore, increasing the likelihood of occurring hostile takeovers* ($\partial\phi/\partial\theta < 0$)*.*

The bidder shareholders are willing to offer higher premiums (ψ^*) because they benefit of the same synergies while holding a stake in the target. Looking more carefully to the amount of the premium paid ($\Psi^* = (1-\theta)(\psi^* - 1)K_T$), it is possible to conclude that the effect of the toehold is the opposite: a higher toehold decreases the amount of the premium that the bidder needs to pay ($\partial\Psi^*/\partial\theta < 0$).

Additionally to the effect of a reduction of the takeover threshold (Corollary 1), hostile takeovers are more likely to occur because they require lower synergies as the toehold increases:

Corollary 2. *Hostile takeovers require lower synergies to become optimal as the toehold increases* ($\partial\omega_l/\partial\theta < 0$ *and* $\partial\omega_e/\partial\theta < 0$) *and are, therefore, more likely to occur the greater*

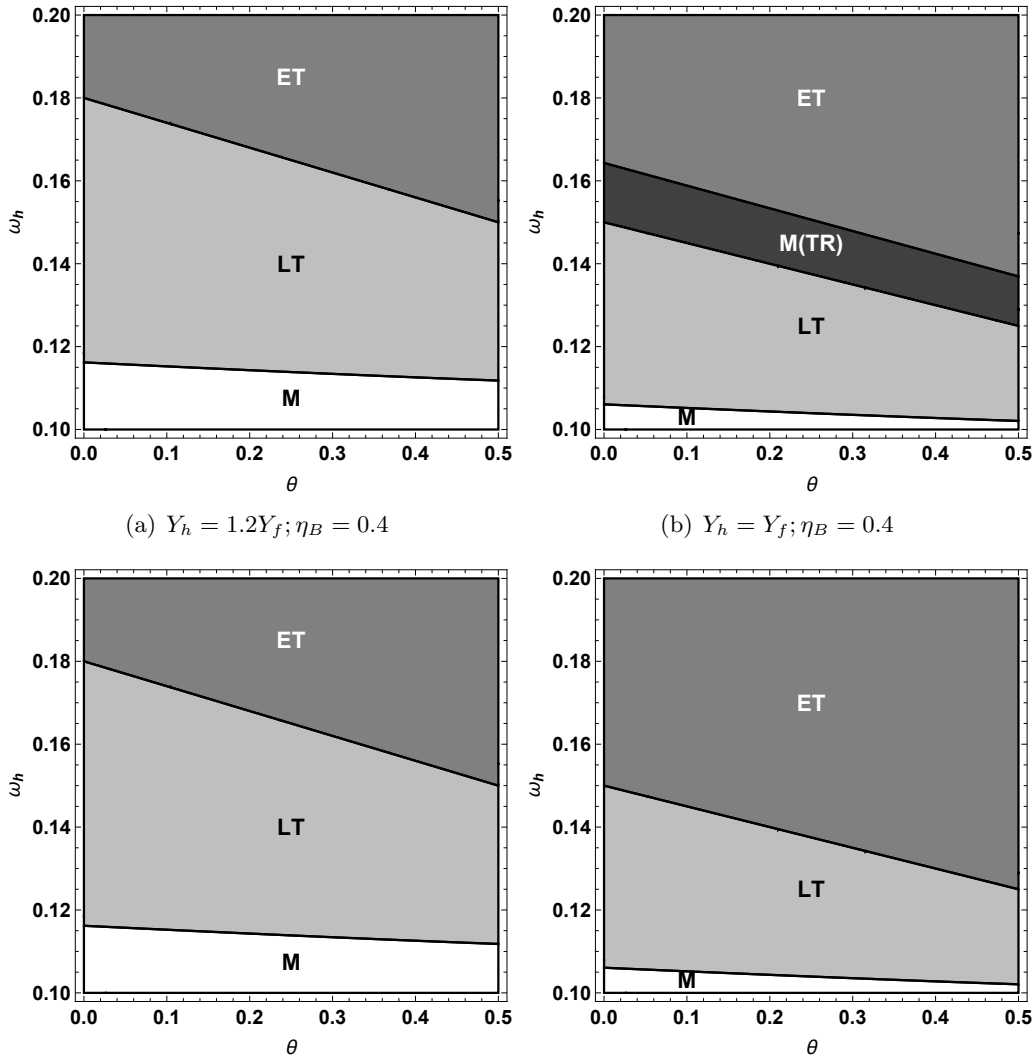
the toehold is.

Figure 1 illustrates the effect of the toehold size for different sets of parameters, showing the optimal strategy for combinations of the toehold and the hostile takeover synergies. In general, we can see how friendly mergers become less likely as the toehold increases, particularly early hostile takeovers that seem to be more sensitive to the toehold size, requiring less synergies to become optimal. Please note that even when, for a certain level of synergies (for instance $\omega_h = 1.4$ in Figure 1(a)), the optimal (hostile takeover) strategy does not change, that does not mean that it has the same probability of occurrence. A higher toehold reduces the threshold for the takeover, increasing the probability of reaching it. As previously referred, friendly mergers that occur in equilibrium later the hostile takeovers may become optimal because the target is better off rejecting a possible hostile bid. Figure 1(b) shows that case: for intermediate synergies, that strategy becomes optimal (M(TR)). For those synergy levels, friendly mergers become more likely as the toehold initially increases, and both a small and big toehold make hostile takeovers more likely. This exception to the effect of the toehold, that only occurs for intermediate levels of synergies, disappears as the sunk hostile takeover cost increases (Figures 1(a)), the bargaining power of the bidder increases (Figure 1(d)), or both (Figure 1(c)), which are plausible assumptions.

3.2 The effect of uncertainty

A higher uncertainty may have an ambiguous effect on the probability of exercising real options (Sarkar, 2000). On the one hand, it increases the threshold and, on the other, it also increases the probability of reaching them (through the stochastic process). In our case, the latter effect is irrelevant, since we are comparing two strategies affected by the same stochastic process, meaning that the strategy with a higher threshold is always the less likely strategy. Regarding the effect of uncertainty, it is possible to state the following corollary:

Corollary 3. *A higher uncertainty deters mergers and acquisitions* ($\partial x_f^*/\partial\sigma > 0, \partial x_h^*/\partial\sigma > 0$) *and reduces the likelihood of occurring hostile takeovers* ($\partial\phi/\partial\sigma > 0$).



(a) $Y_h = 1.2Y_f; \eta_B = 0.4$ (b) $Y_h = Y_f; \eta_B = 0.4$

(c) $Y_h = 1.2Y_f; \eta_B = 0.5$ (d) $Y_h = Y_f; \eta_B = 0.5$

$K_B = 1$, $K_T = 1$, $\omega_f = 0.1$, $Y_f = 0.1$, $\epsilon_B = K_B/(K_B + K_T) = 0.5$, $\beta_1 = 2$.
M = Friendly merger; LT = Late hostile takeovers; ET = Early hostile takeovers; M(TR) = Friendly merger (after a takeover that is rejected by the target).

Figure 1: Optimal M&A strategy: sensitivity to the toehold size

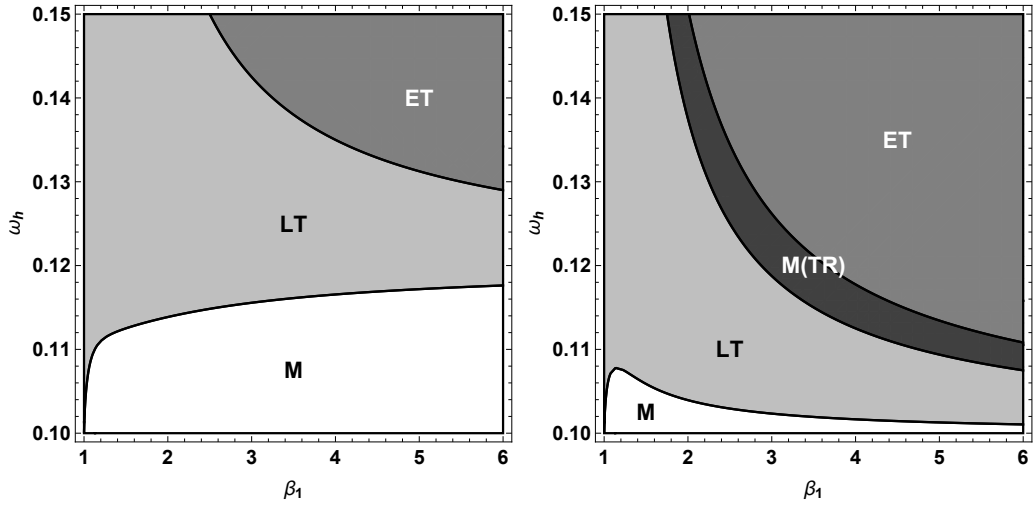
Although the effect of uncertainty on the probability of reaching both thresholds may be ambiguous, it is always true that hostile takeovers become more likely, because the hostile threshold is more sensitivity to uncertainty than the friendly threshold. In fact, for a sufficiently high uncertainty, a hostile takeover may occur sooner than a friendly merger (whenever $\omega_h > \omega_t$).

While uncertainty has an unequivocal effect of deterring mergers and acquisitions and,

simultaneously, making the hostile takeovers more likely through their timing (Corollary 3), the effect on the probability of the strategy choice is less clear:

Corollary 4. *Uncertainty has ambiguous effect on the synergies required for late takeovers to become more likely* ($\partial\omega_l/\partial\sigma \gtrless 0$)*. Early takeovers are less likely to occur as uncertainty increases* ($\partial\omega_{eT}/\partial\sigma > 0$)*.*

The ambiguous effect of uncertainty on late takeovers is shown in Figure 2. When the costs of mergers and takeovers are the same (Figure 2(b)) a lower uncertainty (higher β_1) makes hostile takeovers more likely, but that also occurs for extremely high levels of uncertainty for the late takeovers ($\beta_1 \to 1$). In contrast, when the costs of the takeover are higher than those of the friendly merger (Figure 2(a)), late hostile takeovers become more likely as uncertainty decreases. In any case, early takeovers are more likely to occur as uncertainty decreases. Firms operating in more volatile industries or periods will tend to enter less in early hostile takeovers (preferring friendly mergers).



(a) $Y_h = 1.2Y_f$ (b) $Y_h = Y_f$

$K_B = 1$, $K_T = 1$, $\omega_f = 0.1$, $Y_f = 0.1$, $\epsilon_B = K_B/(K_B + K_T) = 0.5$, $\eta_B = 0.4$, $\beta_1 = 2$.
M = Friendly merger; LT = Late hostile takeovers; ET = Early hostile takeovers; M(TR) = Friendly merger (after a takeover that is rejected by the target).

Figure 2: Optimal M&A strategy: sensitivity to uncertainty

3.3 The effect of firms' size

Firms' bargaining power in a friendly merger does not affect the timing of friendly mergers. Similarly, firms' size also does not affect the timing of mergers and acquisitions, unless the fraction of the transaction costs in a hostile takeover is determined by the firms' size. For that case, the following corollary holds:

Corollary 5. *The higher the fraction of the transaction cost paid by the bidder (the bigger it is), the sooner hostile takeovers occur* ($\partial x_h^*/\partial \epsilon_B < 0$)*, i.e. the more likely they become* ($\partial \phi/\partial \epsilon_B < 0$).

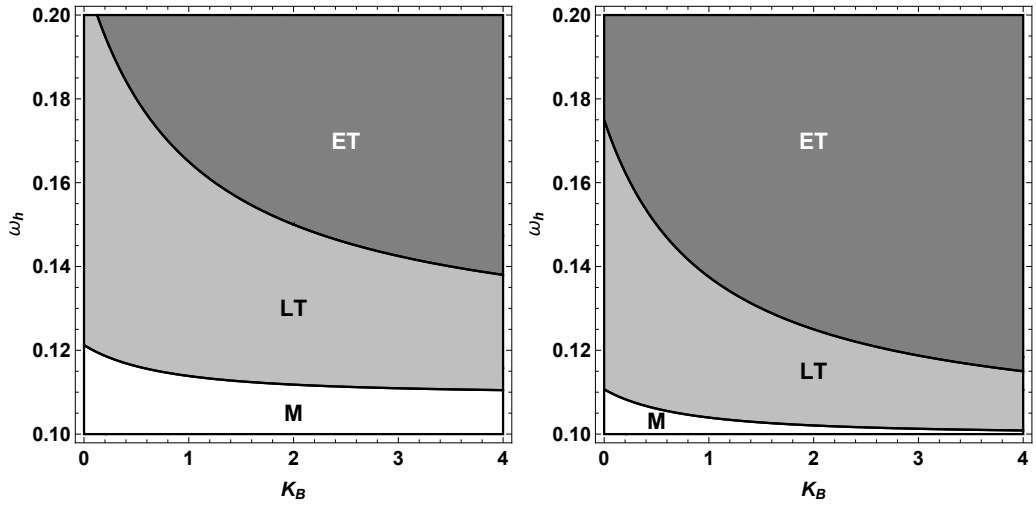
A higher asymmetry in the transactions costs in favor of the bidder, possibility determined by the firms' size, increases the likelihood of occurring hostile takeovers. In such case, the sunk costs paid by the target are smaller, which reduce the threshold, inducing the bidder to offer a smaller premium (ψ^*), which has the opposite effect. The overall effect is an anticipation of the hostile takeover.

The synergies thresholds are not affected directly by the individual firms' size (K_B and K_T). However, it is reasonable to assume that both the fraction of the transaction costs (ϵ_B) and the bargaining power (η_B) may be related to the relative size of firms. Assuming that both are proportional the firm's size, it is possible to state the following corollary:

Corollary 6. *Bigger bidders are more prone to engage in hostile toehold takeovers as they require lower synergies to chose that strategy over a friendly merger.*

Figure 3 illustrates this corollary, showing that both early and late takeovers require lower synergies as the size of the bidder increases. A small target prefers the power of choosing the hostile timing over a small bargaining power in a friendly merger. Interestingly, for highly asymmetric firms and identical transaction costs a hostile takeover may not require a significant different synergy to become preferable to a friendly merger (Figure 3(b)).

Given that a toehold and the size of the bidder may produce similar impacts on the likelihood of choosing a hostile takeover, toeholds can be used to overcome a bidder size disadvantage, which suggests the following corollary:



(a) $Y_h = 1.2Y_f$ (b) $Y_h = Y_f$

$K_T = 1$, $\omega_f = 0.1$, $Y_f = 0.1$, $\epsilon_B = K_B/(K_B + K_T)$, $\epsilon_B = K_B/(K_B + K_T)$, $\beta_1 = 2$.
M = Friendly merger; LT = Late hostile takeovers; ET = Early hostile takeovers.

Figure 3: Optimal M&A strategy: sensitivity to firm size

Corollary 7. *Larger firms need less to use toeholds to succeed in hostile takeovers.*

4 Conclusion

This paper builds on recent advances in the domain of option games under uncertainty takes a closer look at the determinants that drive the choice between toehold friendly mergers and toehold hostile takeovers. Given that the outcome of the M&A is uncertain, each firm calculates its payoff resulting from either a friendly merger or hostile takeover thereby taking into account that a friendly merger may be followed by a hostile takeover and vice-versa. Consequently, the paper advances recent literature by explicitly considering both takeover strategies simultaneously, and accounting for the associated hostile threat values during negotiation.

We show that hostile takeovers require sufficiently large synergies to become preferable over friendly mergers. However, the additional synergies needed vary with some key factors. In general, a higher toehold lowers the required synergies.

The fact that the bidder holds a toehold in the target has two main effects: (i) it reduces the threshold for a hostile takeover, because the bidder saves part of the premium

needed to induce the target to accept the offer; (ii) it gives the bidder a credible threat to takeover the firm in a hostile manner, and enhances its bargaining power in a friendly merger, inducing the target to be more willing to accept the hostile takeover.

Additionally, uncertainty may have an ambiguous effect on the strategy choice. Nevertheless, it always hastens hostile takeovers if the synergies are sufficiently larger than in a friendly merger. Bigger bidders may be more prone to engage in hostile toehold takeovers as they require lower synergies to chose that strategy over a friendly merger. Given that a toehold is capable of producing a similar effect, it can be used to overcome a size disadvantage of the bidder, which suggests that larger firms need less to use toeholds to succeed in hostile takeovers.

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Appendix

A Proofs of propositions

To be added.

B Proofs of corollaries

To be added.