# Capacity Expansion and Term Extension in Public Infrastructure Concessions

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**Abstract**

Government Concessions as well as Public Private Partnerships (PPPs), are usually bound by a time term defined in the contract, during which the private party must earn its payback and return on invested capital. When well designed, the contract rules should guarantee the smooth development of the concession, bringing about a win-win partnership. Yet, capacity expansion, especially in infrastructure concessions, should be driven by the existing demand and not mandated by contract. We argue in this research that in order to optimize such a financial decision, an investment in capacity expansion should not only be decided by the concessionaire, evidently under previously designed contract rules, but also allow for an extension of the concession period in order to made it financially viable for the investor, even if this expansion is to occur close to the end of the contract. In this sense, the purpose of this article is to model, under the real options approach, two different public policies for capacity expansion clauses in government concession projects. We apply our model to a road concession project and find that both policies add value to the concessionaire and to the government when compared to the traditional capacity expansion clauses.

**Keywords:** Capacity Expansion; Real Options; Term Extension; Concessions.

## 1. Introduction

Infrastructure projects have as characteristics long maturation period and high capital investments, which generates significant risks. Given the limited capacity of public funding and the worldwide tendency to grant to private initiative projects that can provide an adequate return to investors, a process of granting this class of projects to the private sector in Brazil began in the 1990s. Government Grants or Concessions, as well as Public Private Partnerships (PPPs), are usually bound by a time term defined in the contract, during which the private party must earn its payback and return on invested capital. In order to decide its capital investment in a such a venture, the private investor considers the expected return and risk of the sector, government and project, as well as revenues, costs, fiscal conditions and the uncertainties involved. On the other hand, the Government or Agency granting the concession may have different objectives, such as the quality of services rendered and low toll prices.

Although the objectives of both parties, public and private, may be different, they need not be antagonistic as both can profit from each other in the project involved: the government aims to increase the public benefits from the private investment, while the latter at the return it will obtain from the service rendered. Evidently, the private party will seek to maximize its profitability bounded by the limits of the contract, while the government must in some way control and verify the service rendered as well as investment made, with a limit to the tariff that can be charged by the private party. All these rules and boundaries must be fixed and clearly defined in the grant contract at the start of the concession.

When well designed, the contract rules should guarantee the smooth development of the concession, bringing about a win-win partnership. But even with clear rules and limits, concessions frequently run into problems which may occur due to events or variables that were not be forecasted at the time of the grant. Due to the risks involved, many of these projects are not attractive to private capital. In this sense, some mechanism of risk mitigation by the granting authority is required to make these investments viable, such as the collar option. The collar option mechanisms are relevant to attract private initiative and mitigate government spending. The advantage of this type of mechanism is that it is more efficient for risk mitigation than non-recourse assets or fixed counter-payments (Brandão et al., 2012). On the other hand, these guarantees have the potential to generate a contingent liability for the public agent, which can be extremely costly if not properly priced and modeled.

Also, since one of the major uncertainties that may affect a concession project is the demand and, consequently, has a probability of underestimating in the long term, it is understood that the concessionaire may have an interest in expanding the capacity of the project. However, the existence of collar options as a risk mitigation mechanism generates a disincentive for the concessionaire to expand the marginal cash flows would no longer be captured exclusively by the private investor to be shared with the government.

Capacity expansion, especially in infrastructure concessions such as roads, ports, trains, airports, among others, should be an option driven by demand since it is an intensive capital decision and is subject to a time frame limit. We argue in this research that in order to optimize such a financial decision, which should be beneficial to both parties involved, an investment in capacity expansion should permit extension of the time grant in order to turn it financially viable for the investor, even if close to the end of the contract term. In this sense, the purpose of this article is to model, through the real options approach, different policies of capacity expansion option with term extension in governmental concession grants.

This article is organized in as follows. After this introduction, we present a review of the related literature in the field. Next, in section 3 we present the background of highway concession projects and in section 4 we propose a model to evaluate different policies for capacity expansion with term extension in concession projects. In section 5 we present an application in a road concession project to verify the validity of our model, and in section 6 we conclude.

## 2. Literature Review

The Discounted Cash Flow (DCF) method is often used for the valuation of investment projects. This traditional method, although very robust, does not consider the managerial flexibilities and prices the project based only on information that is known at the initial moment, ignoring possible changes the may result from dynamically managing the project in an uncertain environment.

The Real Options Approach (ROA) arose as a response to the limitations of the traditional DCF model. This approach adapts the financial options pricing models originally developed by Black and Scholes (1973) and Merton (1973), allowing the treatment of investment under uncertainty and flexibility. Tourinho (1979) was the first to use this method to assess an oil reservoir project under price uncertainty. In the following years, McDonald and Siegel (1985), Titman (1985), Majd and Pindyck (1987) and Triantis and Hodder (1990) further developed the field by provided solutions to particular applications. A few years later, Dixit and Pindyck (1994) and Trigeorgis (1996) synthesize the main concepts and the possible applications of this methodology.

ROA has found several applications in infrastructure projects, such as transportation, roads, airports and highways. Bowe and Lee (2004), for example, analyze the construction of the Taiwan High-Speed Rail project, considering the expansion, postponement, abandonment, and contraction options. Their results suggest that management flexibility in the face of unexpected market developments is important to determine the economic feasibility of the project and confirm the theoretical result that the multiple options values will be non-additive if exercised in the presence of one another. Huang and Chou (2006) complement the analysis performed by Bowe and Lee (2004), considering the minimum revenue guarantee and the option to abandon during the project pre-construction phase.

Brandão et al. (2012) analyze the Line 4 concession of the São Paulo Subway System and compare the results of the project valuation under the traditional DCF method with the real options approach. The authors study the impact that the capacity limitation of the line has on the project risk value and propose a model that considers different guarantees levels for each traffic band in order to minimize the risk of not realizing the estimated revenues. Cheah and Liu (2006) also use ROA to study a BOT (Build – Operate – Transfer) concession project which had traffic guarantees involved. In this article, a model to evaluate government guarantees as an option is proposed. The authors suggest that the option value should be incorporated into the negotiation framework and conclude that the guarantee value could indeed be significant relative to the initial net present value.

Brandao and Saraiva (2008) propose a model to estimate the value of an MRG for traffic volume in highway projects when this variable follows a GBM (Geometric Brownian Motion) process. Unlike the previous literature, the authors use market data to determine stochastic project parameters to estimate the guarantee value. Feng, Zhang, and Gao (2015) develop a model to evaluate the minimum revenue guarantee, the minimum traffic guarantee, and the price compensation guarantee, and thereby determined the optimal toll price on highway projects. In addition, the authors verify the impact of government guarantees on toll collection, highway capacity and road quality.

Attarzadeh, Chua, Beer, and Abbott (2017) develop a different approach by proposing a model to evaluate a revenue guarantee on PPP projects of plants with equitable limits. For this, the authors use the fuzzy technique to deal with the uncertainties involved in estimating cash flow. On the other hand, instead of proposing a revenue guarantee put option, which has a limitation due to an upfront premium payment requirement, Shan, Garvin, and Kumar (2010) suggest a collar option to improve the effectiveness of risk management in a real toll project and to redistribute downside losses and upside profits to fulfil stakeholders' needs.

Buyukyoran and Gundes (2018) propose a model to evaluate the MRG in a BOT toll road project, considering that future traffic demand is perceived as the most critical risk factor that affects the financial viability of the project. In this sense, they combine an optimization approach with MCS (Monte Carlo Simulation) to identify the optimum upper and lower boundaries of options with the difference that they were modelled as European call and put options. Takashima, Yagi and Takamori (2010) present a real options model for analyzing the interaction between a private firm and a government over the timing of the decision to invest in a PPP project, considering the degree of sharing in the cost of the investment and the operation risk. The results show that when the government guarantee is large and/or the cost sharing rate for the private firm is low, then the private firm-maximizing policy exercises the investment option earlier than the project value-maximizing policy.

Carbonara and Pellegrino (2018) develop a model based on the ROA to calculate the optimal revenue floor and ceiling values in a way that creates a win-win condition for the concessionaire and the government, fairly sharing the risk between them. The authors apply this model to the Strait of Messina Bridge case and conclude that this mechanism can support the decision-making process of the government in assessing the values of public subsidies necessary to make the project attractive to private investors and assist public and private parties during the negotiation in pursuit of fair floor values and revenue ceiling.

Wang, Liu, Xiong and Zhu (2018) investigate how distinct types of government support can attract more private investment. To do this, the authors analyze 4,484 PPP projects across 130 developing countries and the results show that capital, revenue and in-kind subsidies attract more private capital, while indirect supports through government guarantee policies do not. Besides, statistical analyses indicate that increased institutional quality of a country can enhance the positive relationship between government supports and private investment, especially for government direct supports.

Aside from the importance and contributions of all these works, there is a standard feature in all of them: they analyze the guarantees mechanisms without considering its disadvantages, such as the potential to generate a contingent liability for the public agent and the disincentive for the concessionaire to expand the marginal cash flows. Therefore, the main contribution of this article is the discussion not only the advantages of using collar options but also the negative aspects of this type of mechanism and the impact of different capacity expansion policies in BOT projects.

## 3. Background: Highway Concession Projects

According to Yescombe (2013), a concession agreement is a contract between a public sector authority and a private company through which a project is constructed to provide a service directly to the public or that public authority in question. Concessions include services in several sectors, such as highways, railways, hospitals, airports and ports.

Regarding road concessions, developed and developing countries adopt this system in different ways. In France, the process began in 1955, where tariff revenues accounted for half of the financing needed to build, maintain and operate the road (Blank, 2008). On the other hand, only in the 1990s, England had its first highway granted under the DBFO (Design-Build-Finance-Operate) model (Machado, 2005). Other countries such as Mexico and Chile also only granted their highways in the 1990s. These two countries developed several mechanisms to reduce risk exposure, such as the minimum revenue mechanism.

In Brazil, the practice of granting highways became more common as of 1993, when the first contracts with the private sector were signed. As most of these contracts have a term of 25 years and the earlier ones will expire by 2021. One of these concessions refers to BR-040/MG/RJ highway which links Rio de Janeiro to Minas Gerais, two of the main states of Brazilian southeast. According to the National Land Transport Agency (ANTT, 2019), this highway was granted in March 1996 to CONCER concessionaire, which is responsible for 179.9 km and three toll plazas.

Despite being one of the oldest and most important Brazilian concessions, BR-040/MG/RJ highway presents a series of problems, which worsened after the construction of a new stretch was interrupted in 2016. Due to the precarious situation and poor conservation of the highway, ANTT has initiated a process that may lead to the cancellation of the concession. The government has also chosen to assume the costs of the construction instead of extending the concession to compensate for the payment of the project by the concessionaire.

Another road concession that is expected to end in 2021 is the BR-116/RJ/SP highway, which links São Paulo to Rio de Janeiro, the two largest metropolitan regions of the country. This highway was granted in March 1996 to CCR Nova Dutra concessionaire, which is responsible for 402 Km and six toll plazas (ANTT, 2019). Aside from being one of the main roads in the country, BR-116/RJ/SP is also the largest fully paved highway in Brazil.

In addition to the termination of these concession agreements, it is expected that there will be an increase in the number of concessions in the country, mainly due to the positions voiced by the new government that took office in January 2019. There are already studies of the concession of the Rio-Santos highway, which has almost 250 km. According to ANTT (2019), since the concession of the highway BR-116/RJ/SP will end in 2021, the initial idea is to unite the two main roads that connect São Paulo and Rio de Janeiro into a single concession agreement.

In this sense, the government, when renewing the concession contract with CCR Nova Dutra concessionaire, would require counterpart investments in the duplication of Rio-Santos. Studying the feasibility of these two highways in a single contract makes sense because both roads need urgent improvements. In the case of Rio-Santos, part of the BR-101, it is necessary to duplicate a major portion of the road; and in the case of BR-116/RJ/SP, the construction of additional lanes in somes stretches of the road is still pending.

Although the government is considering extending the term of these concessions, especially BR-116/RJ/SP, this decision has not yet been taken. There is still much discussion about what is best: extend the term of these concessions or grant them again through a new bid. If the government chooses to extend the term of these concessions, it may require that the concessionaire anticipate investments through contractual additions. On the other hand, if the government chooses to auction these highways again, it can achieve a reduction in the toll price, in addition to imposing new targets for the auction-winning concessionaires, which may or may not be the same ones that currently manage these roads.

In order to assist this decision-making process, the Brazilian government issued Law nº 13,448/2017 on June 5th of that year, which provides the general guidelines for extension and rebidding of concession contracts in the road, rail and airport sectors. Regarding the concession rebidding, this law establishes the friendly termination of the partnership agreement and the creation of a new negotiation adjustment for the project, in new contractual conditions and with new contractors, through a bid promoted for that purpose.

In contrast, regarding the extension of concession term, this law establishes that the request must be formally manifested by the concessionaire to the public agent at least 24 months before the end of the original contract; and, that the contract may be extended once only for a period equal to or shorter than the original contract period. Besides, this law establishes some rules for early extension of concession term, such as that the concessionaire must include investments not foreseen in the original contract and the extension can only be made between 50% and 90% of the original concession term and with more than 80% of the constructions foreseen in the original contract finalized.

Even with this law to guide the concession processes, the concessionaires and the government come up against legal issues, since they do not previously establish these options in their original contracts. In this sense, proposing in the concession contracts a capacity expansion options with term extension is interesting for both the public agents, since to bid again the same project would entail higher costs, limitations and bureaucracy; as well as the private agent, which may have the right to continue the project if demand proves to be growing. Given this, we develop a real options model to evaluate two different policies of capacity expansion option with term extension.

## 4. Model

We propose a model for investment decision in B.O.T. road concessions that considers the flexibility to expand the capacity offered and the optimal investment timing. In concessions, one of the main sources of uncertainty that can affect the private agent investment returns is the demand (*D*) for the service involved in the concession.

Particularly, in road concessions, in order to determine the total revenues in each year, it is common to assume that the traffic on the highway occurs 365 days a year and that the tool is charged in both directions. Also, it is usual to consider the traffic data in units, without distinguishing between cars and freight vehicles. But, as freight vehicles pay more toll than cars, a multiplying factor (*EVM* – Equivalent Vehicle Multiplier) is commonly applied to normalize the traffic data. Then, the total revenue in year *t* in road concession is defined by equation (1):

$$R_t = E[D_t] \times 365 \times T \times EVM \times 2 \tag{1}$$

where $R_t$ is the total revenue in year *t*; $E[D_t]$ is the expected demand in year *t*; *T* represents the toll price; and, *EVM* is the Equivalent Vehicle Multiplier Factor.

From these definitions, we can calculate the cash flow in each year $CF_t$ through equation (2):

$$CF_t = R_t \times [1 - c_t] \times [1 - \pi] - [Dep + C] \times [1 - \pi] + Dep \tag{2}$$

where $c_t$ represents variable costs; $\pi$ is the income tax; *Dep* means depreciation and *C* represents fixed costs.

To simplify the cash flow equation, we adopt the expression in equation (3):

$$CF_t = f\left(D_t\right) \tag{3}$$

To calculate the present value (*PV*) of concession projects that have the demand *D* as the main uncertainty, we use equation (4):

$$PV_0 = \int_{t=1}^{n} E\left[ f\left(D_t\right)\right] e^{-kt} dt \tag{4}$$

where *PV* is the present value of the concession project at time $t = 0$; $E[f(D_t)]$ is the expected value of the project's future cash flows, which are a demand function; *k* is the cost of capital (WACC); and, *n* is the concession term.

To evaluate the possible options in the project, we consider that the demand $\left(\tilde{D}\right)$ follows a Geometric Brownian Motion (GBM), as shown in equation (5):

$$dD_t = \mu D_t dt + \sigma D_t dz_t \tag{5}$$

where: $dD_t$ is the incremental variation of demand in the time interval $dt$; $\mu$ represents the expected growth rate of demand; $\sigma$ is the demand volatility; and, $dz_t = \varepsilon\sqrt{dt}$ represents the standard increment of Wiener, where $\varepsilon \approx N(0,1)$.

As the flexibilities studied in this paper can be modeled as American type real options, we adopt the discrete binomial tree model proposed by Cox, Ross and Rubinstein (1979) (CRR), adapted to the case studied. We initially model a demand *D* lattice with its volatility and the equivalent cash flow (FC) lattice using equation (4). The basic nodes of these lattices are shown in Figure 1.

The model parameters for the Cox, et. al, (1979) lattice model are presented in equation (6):

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = \frac{1}{u} \quad \text{and} \quad p = \frac{\left(1+r_f\right)^{\Delta t} - d}{u-d} \tag{6}$$

where *u* and *d* are, respectively, the upside and downside multiplying factors; *p* is the risk neutral probability; $\sigma$ is the demand volatility; and, $r_f$ is the risk-free rate.

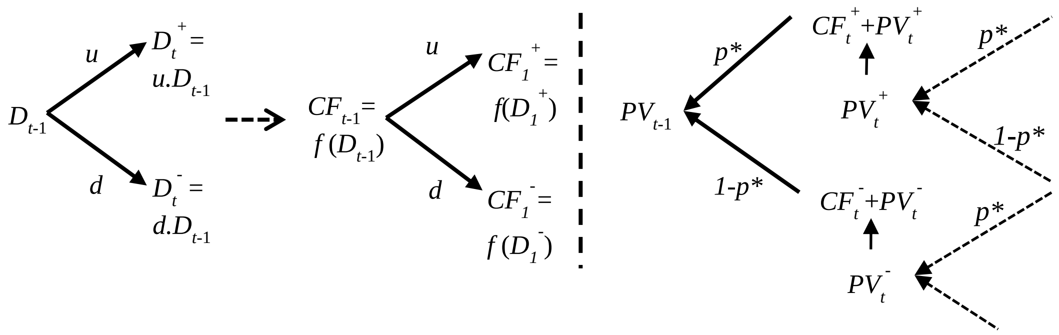


**Figure 1** – Demand and Cash Flow Lattice nodes and, on the right, node of Present Value (*PV*) lattice

The CRR model considers that, at each decision node, the stochastic variable $\tilde{D}$ should be modeled by equation (7).

$$D_t^+ = D_{t-1}e^{+\sigma\sqrt{\Delta t}}$$
$$D_t^- = D_{t-1}e^{-\sigma\sqrt{\Delta t}} \quad (7)$$

where $D^+$ and $D^-$, are the ascending and descending random values that are modeled within the cash flow (*CF*) tree.

Additionally, this option pricing model requires the use of the risk-neutral measure that can be determined by deducting the risk premium from the asset's rate of return and then discounting cash flows at the free rate of risk. Thus, the risk-neutral process of demand is defined by equation (8):

$$dD_t^R = (\mu - \zeta_D)D_t^R dt + \sigma D_t^R dz_t \quad (8)$$

where $\zeta_D$ represents the demand risk premium; $\mu$ is the rate of return of the demand; and, $dD_t^R$ is the incremental variation of the risk-neutral demand in the time interval *dt*.

As discussed by Freitas and Brandão (2009), the market risk premium can be observed directly or can be determined through CAPM (Capital Asset Pricing Model), where $\mu = r_f + \zeta$ and $\zeta = \beta\left(E\left[R_M\right] - r_f\right)$. On the other hand, the risk premium of incomplete market assets can only be calculated through indirect methods.

In order to evaluate the demand risk premium, we consider that the expected value of gains in the risk-neutral valuation, regardless of possible options, should be strictly equal to the expected value of gains in the traditional static valuation, as shows the equation (9):

$$\int_{t=1}^{n} f\left(\tilde{D}_t\right)e^{-\mu t}dt = \int_{t=1}^{n} f\left(\tilde{D}_t^R\right)e^{-(\mu-\zeta_D)t}dt \quad (9)$$

where *f (.)* represents the cash flows.

In this sense, this model will consider the following risk neutral probability, defined in equation (10):

$$p* = \frac{\left(1+\mu-\zeta_D\right)^{\Delta t} - d}{u-d} \quad (10)$$

And, equation (11) to calculate the risk-neutral present value of the project.

$$PV^R = \int_{t=1}^{n} E\left[f\left(D_t^R\right)\right]e^{-r_f t}dt \quad (11)$$

As the flexibilities to be modeled are American options and they will be exerted on the *PV* (present value) of the concession at each nod of the lattice thus creating a *PV* lattice which is the value of discounted future cash flows from the last step of the *CF* lattice up to the time considered. If the term concession time is: *n*, then the *PV* of the *n-1* step are calculated by equation (12).

$$PV_{n-1} = \left[CF_n^+ p* + CF_n^-\left(1-p*\right)\right] / (1+rf) \quad (12)$$

Generalizing this process we get equation (13).

$$PV_{i-1} = \left[\left(CF_i^+ + PV_i^+\right)p* + \left(CF_n^- + PV_i^-\right)\left(1-p*\right)\right] / (1+rf) \quad (13)$$

This process can be observed in Figure 1. As the *PV* lattice ends at time *n* with no continuation value, it will converge to zero at its end, in the case of a fixed time term grant.

Note that demand volatility can affect the private party investment return in both directions. If the demand reveals to be much less than expected, this will negatively affect the financial return of the subsidy, leading the private agent to abandon the concession, even with the financial consequences that may be imposed. On the other hand, if the demand reveals to be significantly above the expected trajectory will require capacity expansion from the concessionaire, with capital investments necessary in order to capture the upside and not cause demand bottleneck, which is an undesired effect for the agency grant.

Given that concessions generally expire after a time term specified in the grant contract, the closer the concession timing is to this term, the less time the investor will have to receive cash flows from any follow-up capacity expansion investments. Also, frequently, capacity expansions may be listed and included in the grant contract. But, it may prove useless to force an expansion either when there is no demand for such, or at a later time when the demand upside has already revealed itself in previous periods.

In order to verify the validity of this last statement, we compare three cases: (1) a traditional capacity expansion option, that is, without the concession extension term; (2) a capacity expansion option simultaneously with fixed term extension; and, (3) a capacity expansion option with additional fixed term extension. The last two cases are policies proposed in this article and are defined, respectively, in the following subsections.

### 4.1. Traditional Capacity Expansion Option

The traditional capacity expansion option can be seen as an American call option, since the concessionaire can make the decision to expand at any time throughout the life of the contract. Denoting the project value with expansion option by $PV_{exp1}$, the value of the real option is conditioned to the optimal exercise of the expansion option. In this sense, the expansion option value is expressed by equation (14):

$$PV_{t_{exp1}} = \max\left[PV_t^R; \lambda \times PV_t^R - I\right] \tag{14}$$

where $PV_t^R$ represents the risk-neutral present value of the project without considering any options; $\lambda$ is the expansion factor and *I* represents the expansion *CAPEX*.

### 4.2. Fixed Term Extension Simultaneous with Capacity Expansion Policy

The first policy proposed in this paper establishes that the term extension is fixed (equal to the grant term) and occurs at the time of the investment in capacity expansion. For model this policy, as in the previous item, we use the CRR binomial model and consider that this is an American call option. In order to evaluate this capacity expansion option with simultaneously fixed term extension, we consider equation (15):

$$PV_{t_{exp2}} = \max\left[PV_t^R; PV_{t \to n} - I\right] \tag{15}$$

where $PV_{t \to n}$ represents the present value of cash flows at time *t* until time *n*, which is the expanded time term of the policy considered, of the cash flows of the expansion capacity option.

Equation (16) shows how we calculate the perpetuity limited in *t* years:

$$PV_{\tau_{\exp 2}} = \int_{\tau}^{\tau+n} E\left[ f\left(\tilde{D}_t\right)\right] e^{-kt} dt \quad (16)$$

where $PV_\tau$ is the present value of the perpetuity limited in *n* years, which is the concession term.

Since we will use a discrete model, we consider equation (17) to calculate the present value of the perpetuity limited in *n* years:

$$PV_{t_{\exp 2}} = \frac{E\left[ f(\tilde{D}_{t+1})\right]}{k-\mu}\left(1-\left(\frac{1+\mu}{1+k}\right)^n\right) \quad (17)$$

where $PV_t$ is the present value of the perpetuity limited in *n* years, which is the concession term; and, $E\left[ f\left(\tilde{D}_{t+1}\right)\right]$ is the expected value of cash flows at time *t+1*.

### 4.3. Fixed Term Extension Additional to Concession Term Policy

On the other hand, the second policy proposed in this article establishes that the term extension is fixed (equal to the grant term) and additional to the pre-established term of the concession. We use the same binomial model and calculate the perpetuity limited in *n* years through equation (17). To evaluate this capacity expansion option with additional term extension, we consider the following equation (18):

$$PV_{t_{\exp 3}} = \max\left[ PV_t^R ; PV_{t\to N} - I\right] \quad (18)$$

where $PV_{t\to N}$ represents the present value of cash flows at time *t* until time *N*, which is the double expanded time term of the initial policy, of the cash flows of the expansion capacity option.

## 5. Numerical Application

### 5.1. Base Case

We apply our theoretical framework considering a road concession project with the following values listed in Table 1. It is important to emphasize that the data were based on a real project analyzed by Brandão (2002).

**Table 1** – Concession data and values

| | |
|---|---|
| Grant term | 25 years |
| CAPEX | 300 million $ |
| Fixed cost | 32 million $ (per year) |
| Variable cost | 35% of revenue |
| Tax rate | 34% (per year) |
| Depreciation | 25 years |
| Tariff | 8.80 $ (per vehicle) |
| Risk-free rate | 6.18% |
| Risk-adjusted rate | 9.42% |
| Maximum Road capacity (CAP) | 20,000 (AADT) |

Since the demand ($D$) is our stochastic variable, we model this uncertainty as a GBM, using the values and parameters given in Table 2.

**Table 2** – Stochastic Demand values and parameters

| | | |
|---|---|---|
| Initial demand (in $t$ = 0) | $D_0$ | 10,000 (AADT) |
| Equivalent Vehicle Multiplier | $EVM$ | 2.2 |
| Demand drift | $\mu$ | 6% (per year) |
| Demand Volatility | $\sigma$ | 15% (per year) |
| Lattice upside factor | $u$ | 1.1618 |
| Lattice downside factor | $d$ | 0.8607 |
| Demand risk premium | $\zeta_D$ | 2.22% |
| Upside move probability | $p$ | 0.5881 |
| Downside move probability | $(1-p)$ | 0.4119 |

Note: AADT means Average Annual Daily Traffic.

Based on equation (4), we calculate that the present value of this concession project at time $t$ = 0, without considering the road capacity limit, is $PV$ = 353,283,000 \$, yielding a Net Present Value (NPV) of 53,283,000\$. We also estimate this value through a lattice, in which we first model the demand as a GBM. From this, we estimate a cash flow lattice using equation (3), and finally, discounting this latter at the risk-free rate and using the risk neutral adjusted probabilities, we estimate the project value lattice, which gives the same present value ($PV$) calculated above. The project value ($PV$) lattice is displayed in Figure 2.

Note that this lattice is the discretization of a Brownian bridge since the concession has a term limit after which there is no cash flow, it will forcefully end in zero value at the expiration of the term (year 25) after paying the last cash flow of the project. Also, we can see that this lattice does not consider the maximum road capacity limit (CAP) as listed in Table 1.

Thus, when we consider this CAP on the road traffic, as an absorbing barrier, the lattice and the present value of this base case project changes ($PV$ = 283.065.000 \$), thus yielding a negative NPV of 16,935,000 \$. In Figure 2, we can see the lattice with and without capacity limit, as well as the present value in each case.

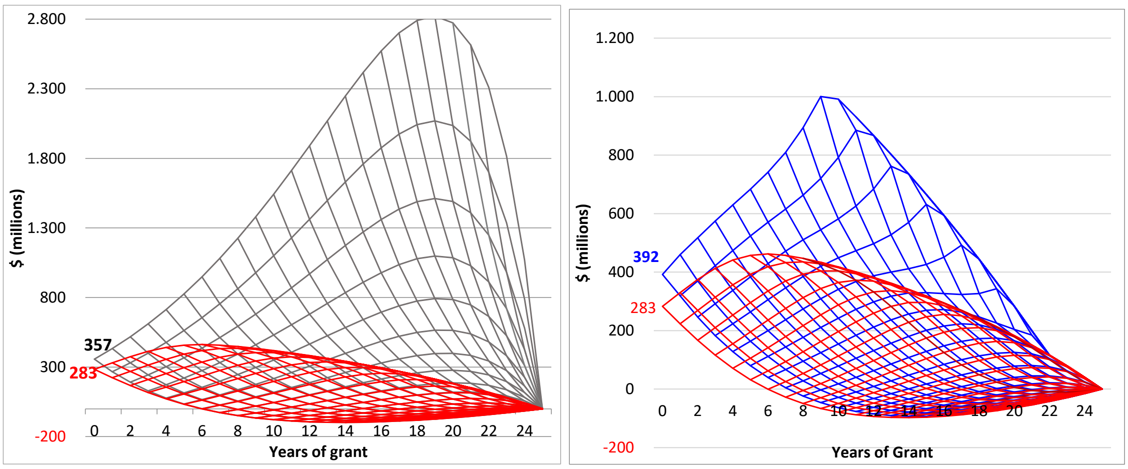


**Figure 2** – Project value lattice without and with demand capacity limit (left) (without demand capacity limit: – | on both figures with demand capacity limit :– | –: expected value of expansion) (right)

## 5.2. Capacity Expansion without Term Extension

We then analyze the case of a traditional capacity expansion option, that is, without any concession extension term. For this, we consider that the concessionaire has the option of investing $ 120 million (*Expansion CAPEX*) to have the right to increase the road capacity limit to 40,000 (AADT), therefore doubling the road capacity with in investment of 40% of the original CAPEX of the concession. Using the same approach with the demand lattice described in subsection 4.1 above, we plotted the lattice displaying both the project with traffic cap, and with the expansion option without term extension. It is also displayed in Figure 2 on the right side**Figure 3**.

We can observe that with the values used in this modeling, the expansion option is exerted a few times, increasing the projects *PV* as expected ($PV_{exp1}$ = 391,945,000 $), and even surpassing the original value without traffic demand limit, or cap, and bringing about a positive, NPV.

But it is also apparent that any capacity expansion, should it occur, will not happen close to the last years of the concession term, as the concessionaire will not have enough years of cash flow to have its payback on the *Expansion CAPEX*. To make the analysis more robust, we vary the values adopted for the expansion option to verify their impact on the option exercise, but we find that even at very low levels of *Expansion CAPEX*, the option will rarely be exercised close to the concession term end, which would be a desirable aspect for the granting party.

Therefore, these results show that this type of capacity expansion option is not optimal for both parties involved. In this sense, a possible solution for this is that the government, through the concession contract, allows the private agent to have the option of extending the concession term to encourage their investment in capacity expansion. It is important to note that as the term time is still fixed, this lattice still is the discretization of a Brownian Bridge, starting at *PV*, and converging to zero after 25 years, therefore limiting the value of any option exerted during the original term grant.

## 5.3. Crossing the Brownian Bridge: Capacity Expansion with Term Extension

The capacity expansion with term extension can be implemented through different policies. In this paper, we propose two approached or possible policies, for considering capacity expansion option with term extension in the grant contract. As the term extension consideration will not limit the model to the original term time limit, the lattice modeled will no longer forcefully converge to zero at this time, and therefore will no longer be a discretization of a Brownian Bridge. We will model two-term extension models considering the same expansion described in subsection 5.2

The first expansion policy model proposal extends the original term by the same time span: 25 years. But the time extension starts to count from the moment the concessionary declares the investment decision. This policy should tend to push the concessionaire to expand near the end of the original term since this will maximize the total time span of the contract. On the other hand, the second model proposal also extends the original term by the same time span, but always from the end of the first term. Therefore this should tend to drive the concessionaire in expanding as early as possible since it will always have two times the original term, conditional it invests the expansion CapEx. Both proposals are limiting frameworks for time extension, and are worth studying since any other model should fall between these two.

Applying the first policy in the concession project, we obtain the results presented in Figure 3. Note that, by adopting this first policy, the present value of the concession becomes equal to $PV_{exp2}$ = 638,125,000 $. In this sense, the implementation of this first policy generates an increase of 125.43% in the original *PV* of the concession project or an option value of 355,061,000 $.

On the other hand, when we apply the second capacity expansion policy in our numerical example, we obtain other results, which are presented in Figure 3. Observe that the present value of the concession becomes equal to $PV_{exp3}$ = 699,490,800 $. Thus, the implementation of the second policy generates an increase of 147.11% in the *PV* of the concession project or an option value of 416,425,000 $.

Comparing the two proposed policies, we can see that the latter, beyond increasing the present value also works as an incentive to anticipation the exercise of the expansion capacity option since the concessionaire will try to maximize the number of periods of his grant with increased cash flow. Note that the variable that guides the expansion decision is the present value of the concession project, but the variable that guides the option exercise is the demand.

Both policy models add substantial value to the concessionaire PV, and could also generate additional return to the conceding party as it can either regulate participation on this additional value or regulate a tariff reduction being this a conditional rule for permitting expansion and term extension for the concessionary.

It is worth noting that the high value of the conditional term extension derives from the fact that in Figure 3, the lattice is no longer a Brownian Bridge since it does not converge to a limiting value at its term. The model has crossed the Brownian Bridge.

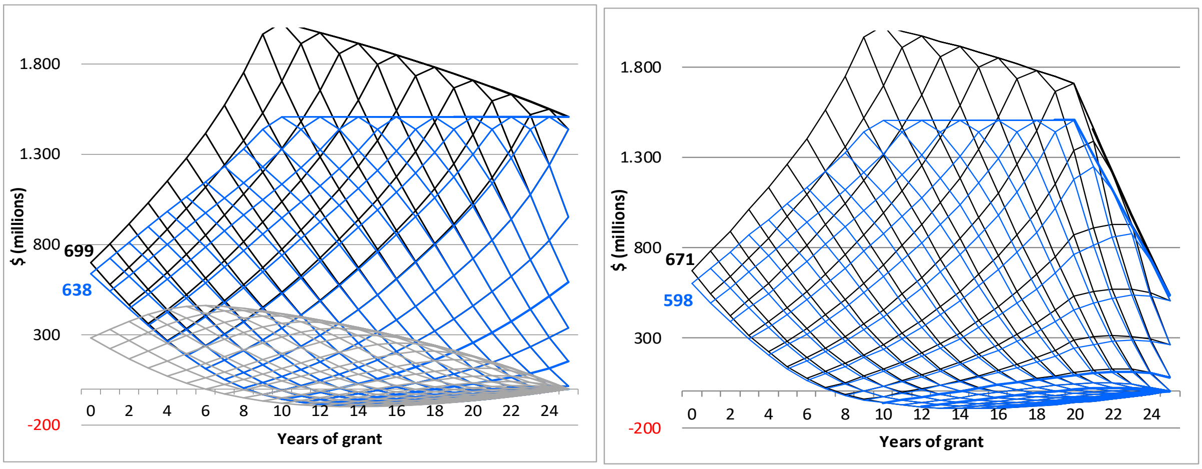


**Figure 3** – Project value lattice, with fixed term extension additional to grant term: — | with fixed term extension at the time of capacity expansion: — | with demand capacity limit, for comparison: —. On the left with no tariff reduction, and on the right with 10% tariff reduction on expansion from year 20 on until 50% tariff at year 25.

As the two policies described above are limiting models, we used them to input limits and tariff reduction associated with the term extension. We used the same model and expansion policies but forcing tariff reduction as the original term approaches its limit: after year 20, in case of expansion the concessionary will incur in a 10% reduction in tariff, and 10% more for each year of wait. The result is shown also in Figure 3 on the right graph. We can see that Present Value (PV) reduction for each policy is very small (from $ 699 M to $ 671 M in the case of the first polity and from $ 638 M to $ 598 M in case of the second policy). Therefore, it is apparent that a term extension policy can be flexibly applied by the conceding authority modeling several approaches of tariff reduction and, or, term extension limits, to the benefit of both the private party as well as the conceding authority,

## 6. Conclusions

In this paper, we propose two mechanisms that allow the concessionaire to exercise the capacity expansion option with term extension in the grant contract. The first policy establishes that the term extension is fixed and occurs at the time of the investment in capacity expansion. On the other hand, the second policy establishes that the term extension is fixed and additional to the pre-established term of the concession.

From this two proposed policies, we conclude preliminarily that both are extreme policies developed to promote the expansion capacity option, besides we find that they add value to the concessionaire and to the government when compared to the traditional capacity expansion option. Other policies can be proposed in this context and it is the authors' objective to continue developing this study.

Public policies, such as those presented in this article, help to minimize the uncertainties of an adverse economic environment and stimulate the infrastructure sector. Additionally, when well designed in the concession contract, this type of term extension granted by additional investments from the concessionaire can return a win-win partnership to a relation that might have been tilted by market conditions in which one part has felt impaired and looks for rebalancing of their contractual relation.

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