Investment Timing and Capacity Decisions with Time-to-Build in a Duopoly Market*

Haejun JEON[†]
Osaka University

Abstract

We investigate firms' optimal decisions of investment timing and capacity in the presence of time-to-build and competition. Because of uncertainty in time-to-build, the product of the leader which makes the investment first might enter the market later than the follower's one. We show that a firm dominated in terms of investment lags can become a leader and that the leader's optimal capacity increases in the size of the dominated firm's lags, even when the dominated one becomes the leader. This result is consistent with electric vehicles market in which a relatively new firm lacking in the experience of mass production makes aggressive investment, while the biggest car makers capable of mass production with shorter lags are timing their investment. With welfare-maximizing policy, however, the dominant firm always becomes the leader. Comparing with investments chosen by welfare-maximizing policy, those of the leader and the follower chosen in the market are made inefficiently late and early, respectively. The welfare-maximizing capacities of both the leader and the follower are much higher than those determined in the market but the difference is more pronounced in the leader's capacity. There is a significant loss of social welfare resulting from the dominated firm becoming the leader, and the loss increases as the dominated firm's time-to-build gets longer.

JEL Classifications Code: D25, G32, L13

Keywords: Time-to-build, investment lags, investment capacity, duopoly market, real options

*First version: January 25, 2019.

†Corresponding Author. Center for Mathematical Modeling and Data Science, Osaka University, 1-7 Machikaneyama-cho, Toyonaka, Osaka 560-0043, Japan; E-mail: jeon@econ.osaka-u.ac.jp; Phone: 81-6-6850-5222, Fax: 81-6-6850-5277. This work was supported by the JSPS KAKENHI (grant number 17K13728) and the Ishii Memorial Securities Research Promotion Foundation.

1 Introduction

Rome was not built in a day and neither were products in the market. It usually takes time to make products available for sale from the initial investment, and the lags between the timing of investment and that of making profits are inherently uncertain, especially when they are of a large scale or based on state-of-the-art technology. The investment lags become more important when there is competition between firms because they are directly linked to the leader-follower relationship in the market.

In this study, we investigate firms' optimal decisions of investment timing and capacity in a duopoly market in the presence of time-to-build. The uncertainty in time-to-build changes the leader-follower relationship significantly; the product of the leader which makes the investment first can enter the market later than the follower's one. Time-to-build affects the firms' capacity decisions as well, which are associated with the price of the product and consumer surplus. We examine how time-to-build changes firms' optimal investment strategies in a duopoly market and the level of social welfare based on a continuous-time dynamic framework.

First of all, we show that a firm dominated in terms of time-to-build can become a leader by making investment first in a market equilibrium. Given significant asymmetry of investment lags, a dominant firm has less incentive to invest first because its product can enter the market first even if it makes the investment later. That is, a relative advantage of time-to-build weakens the incentive of preemption significantly, and the dominated firm with longer lags gets a chance to become a leader in the market, although there is no guarantee that the leader's product enters the market first. This is a novel result in that existing literature on asymmetric duopoly market has shown that a dominant firm with cost advantage always becomes the leader in the market (e.g., Huisman (2001), Pawlina and Kort (2006), Kong and Kwok (2007)). The novelty can also be found from the fact that we have shown a firm's incentive to invest late without positive externalities or availability of new technology, which is in sharp contrast with the argument of a second-mover advantage from existing literature (e.g., Hoppe (2000), Femminis and Martini (2011), Dutta et al. (1995), Hoppe and Lehmann-Grube (2001)).

We also show that the leader's investment capacity increases in the size of the dominated firm's time-to-build, even when the dominated one becomes the leader. Namely, a dominated firm makes more aggressive investment not only by the timing but also by the capacity as its time-to-build gets longer. This finding, combined with the previous one, can explain car companies' investment behavior in electric vehicles market. A relatively new firm with a lack of mass production experience such as Tesla has been making aggressive investment in electric cars including the construction of gigantic battery factories and the installation of charging stations all over the world. However, the biggest car makers capable of mass production with shorter lags such as Toyota and Volkswagen have not made preemptive investment in electric cars, timing their investments. These investment behaviors can be explained by significant asymmetry of time-to-build between the firms.

Furthermore, we derive the welfare-maximizing investment policy in a duopoly market. This policy assigns the roles of the leader and the follower to the dominant and the dominated firms in terms of time-to-build, respectively. Namely, the firm with shorter investment lags always becomes the leader in this policy. The results also show that the welfare-maximizing investments of the leader and the follower are made earlier and later than those chosen in the market, respectively. In other words, the leader and the follower's investments in the market are made inefficiently late and early, respectively. This is in contrast with Huisman and Kort (2015) in which the investments of both the leader and the follower are made inefficiently early in the market. Socially optimal capacities for both the leader and the follower are much higher than those chosen in the market and the difference is more significant for the leader's capacity. It is socially efficient to have products in the market as early as possible, and thus, the social planner exploits the dominant firm's shorter lags to maximize social welfare by putting more emphasis on the dominant firm's investment in terms of both its timing and capacity.

Comparing with a monopoly market, time-to-build makes more significant impacts on firms' investment policies and social welfare in a duopoly market. A monopolistic firm's investment timing, which is delayed as time-to-build increases, coincides with the welfare-maximizing one, and the capacity choices of both the monopolistic firm and the social planner are irrelevant to investment lags. In contrast, the presence of time-to-build can switch the roles of the leader and the follower in a duopoly market, and investment timing of them are significantly different from the socially optimal ones; as mentioned earlier, the investments of the leader and the follower are made inefficiently late and early, respectively. The capacities chosen by the firms and the social planner are associated with the size of investment lags as well. In this sense, we clarify that time-to-build makes different impacts in a different market structure.

Many papers have been devoted to investigate the effects of time-to-build. Majd and Pindyck (1987) pioneered this field of research by considering an investment project that requires a series of investment. Bar-Ilan and Strange (1996) elucidated this problem by assuming that a certain period of time has to be passed to yield revenue and showed that the lags can lead to an earlier investment by reducing the value of wait to invest. Bar-Ilan and Strange (1998) extended this research to the decisions on a two-stage investment. Some studies even considered debt financing for an investment project with time-to-build. Tsyplakov (2008) and Agliardi and Koussis (2013) studied the effects of time-to-build on investment and debt dynamics, and Sarkar and Zhang (2015) examined a two-stage investment with debt financing. Jeon (2018) endogenized capital structure and the timing of investment and default in the presence of time-to-build. These studies, however, were limited to an analysis on a monopolistic firm's decision and did not discuss the effects of time-to-build in the presence of competition.

Grenadier (1996) examined a duopolistic real estate market in which development requires time-to-build, but there was no uncertainty in the construction lags and the paper put more emphasis on the investment triggered by the decrease of demand shock rather than the effects of time-to-build. Pacheco-de-Almeida and Zemsky (2003) studied a multi-stage investment in a duopoly market with time-to-build and showed that the lags can lead to an equilibrium in which firms make incremental investment, but there was no uncertainty in time-to-build in their model as well. Weeds (2002) studied the effects of uncertain investment lags in a duopoly market from the perspective of R&D competition, but it only examined a timing decision in a winner-take-all scenario. None of these studies discussed asymmetry in time-to-build and its effects on the firms' investment decisions.

Putting time-to-build aside, we can find much more studies on investment decision with competition. Nielsen (2002) examined a duopoly market not only with negative externalities but also with positive externalities. Huisman and Kort (2004) studied investment strategy in a duopoly market with future availability of new technology. Bouis et al. (2009) considered competition between three firms instead of two and found the accordion effect of the investment triggers. Mason and Weeds (2010) showed that the leader's investment threshold might not monotonically increase with uncertainty in the presence of preemption. Thijssen et al. (2012) adopted a general setup for examining a duopoly market and showed that the probability that both firms invest simultaneously in spite of their lower values in the case of sequential investments is non-zero. Huisman and Kort (2015) considered not only the decision of timing but also that of capacity in a duopoly market and evaluated the effects of competition on social welfare. These studies, however, assumed homogeneity of the firms in the market.

There are a number of studies that introduced asymmetry between the firms. Huisman (2001) provides a comprehensive analysis on duopolistic firms' investment strategies for a wide range of setup. Lambrecht and Perraudin (2003) studied a preemption of a duopoly market in the presence of incomplete information regarding investment costs. Pawlina and Kort (2006) allowed cost asymmetry and showed that there can be an equilibrium in which one of the firm has no preemption incentive and that the identical firms can result in a socially less desirable outcome than if one of them has a cost advantage. Kong and Kwok (2007) incorporated asymmetry in both cost structure and revenue flow and presented a fuller characterization of equilibrium. Casadesus-Masanell and Ghemawat (2006) studied a mixed duopoly in which a profit-maximizing firm is competiting with a not-for-profit competitor. Thijssen (2010) incorporated player-specific uncertainty in a duopoly market and showed that preemption does not necessarily occur.

There is a growing body of literature on a second-mover advantage in an oligopolistic market which can lead to an attrition game rather than a preemption game. Dutta et al. (1995) investigated quality competition in a duopoly market and addressed a second-mover advantage as an option to adopt new technology. Hoppe and Lehmann-Grube (2001) extended their work by incorporating R&D costs and showed that a second-mover advantage is greater when R&D costs are high. Hoppe (2000) and Femminis and Martini (2011) elucidated the second-mover advantage resulted from spillover of innovation. Décamps and Mariotti (2004) studied the effects of positive externalities taking private information on investment costs into account. Amir and Stepanova (2006) provided a fuller characterization of equilibrium under a general setup including asymmetric costs and various types of demands.

The remainder of this paper is organized as follows. To facilitate understanding, we examine a monopoly market first in Section 2. Section 2.1 discusses a firm's optimal decisions on investment timing and capacity, and Section 2.2 derives the investment policy that maximizes social welfare. Based on this argument, we proceed to a duopoly market in Section 3. In Section 3.1, we assume that the firms' roles as a leader and a follower are exogenously given and investigate their optimal investment strategies. Section 3.2 endogenizes the firms' roles in the market taking preemption incentives into account. In Section 3.3, we derive the welfare-maximizing investment policy of a duopoly market. Given these arguments, we present the results of comparative statics regarding time-to-build and discuss their economic implications in Section 4. Sections 4.1 and 4.2 are dedicated to discussion of the firms' optimal investment strategies and welfare-maximizing policy, respectively. Section 5 provides a brief summary of the paper and discusses possible future works.

2 Monopoly market

To facilitate the understanding of the framework, we consider a monopoly market first. A firm makes the decisions of investment timing and its capacity. The price of the product at time t is given by

$$P(t) = X(t)(1 - \eta Q(t)), \tag{1}$$

where $Q(t)$ is total market output, $\eta > 0$ is a constant, and $X(t)$ is a demand shock. The demand shock follows a geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}W(t), \tag{2}$$

where μ and σ are positive constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion on a filtered space $(\Omega, \mathcal{F}, \mathbb{F} := (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions. The discount rate is given by a constant $r(> \mu)$.

The investment incurs costs δ per unit, which amounts to the total costs of δQ. The manufacturing process, however, takes an uncertain amount of time. Namely, it takes exponential time with an intensity parameter λ to succeed in manufacturing products and yield revenue. The investment lags are assumed to be independent of the demand shock.

2.1 Investment timing and capacity decisions

The firm makes the decisions of investment timing and capacity taking the investment lags into account. Thus, the monopoly firm's value function can be written as follows:

$$V_M(X) = \max_{T_M\geq 0, Q_M\geq 0} \mathbb{E}\left[\int_{\bar{T}_M}^{\infty} e^{-rt} Q_M X(t)(1-\eta Q_M)\mathrm{d}t - e^{-rT_M}\delta Q_M \middle| X(0) = X\right], \tag{3}$$

where $\bar{T}_M := T_M + \tau$ denotes the timing of manufacturing and τ follows an exponential distribution with an intensity parameter λ.

In this dynamic framework, the optimal investment decision is comprised of two parts: its timing and capacity, and the timing can be represented by the level of demand shock at which the firm makes the investment. Namely, the investment timing can be described as $T_M := \inf\{t > 0 | X(t) \geq X_M\}$. Following the standard argument of real options literature, the optimal investment timing and capacity decisions and the firm value are derived as follows:

Proposition 1 (Monopoly firm's investment) *The monopoly firm's optimal investment capacity given the demand shock X is*

$$Q^*_M(X) = \frac{1}{2\eta}\Big(1 - \frac{\delta(r-\mu)(r+\lambda-\mu)}{\lambda X}\Big). \tag{4}$$

The value of the monopoly firm is

$$V_M(X) = \begin{cases} A_M X^\beta, & \text{if } X < X^*_M, \\ \frac{\big(\lambda X - \delta(r-\mu)(r+\lambda-\mu)\big)^2}{4\lambda X\eta(r-\mu)(r+\lambda-\mu)}, & \text{if } X \geq X^*_M, \end{cases} \tag{5}$$

where

$$A_M = \frac{(\beta-1)^{\beta-1}\lambda^\beta}{\eta\delta^{\beta-1}(\beta+1)^{\beta+1}(r-\mu)^\beta(r+\lambda-\mu)^\beta}, \tag{6}$$

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2r}{\sigma^2}} \ (> 1). \tag{7}$$

The optimal investment threshold and the corresponding optimal capacity are

$$X^*_M = \frac{(\beta+1)\delta(r-\mu)(r+\lambda-\mu)}{(\beta-1)\lambda}, \tag{8}$$

$$Q^*_M := Q^*_M(X^*_M) = \frac{1}{(\beta+1)\eta}. \tag{9}$$

PROOF See Appendix A.1.

Note that the firm's capacity decision is irrelevant to investment lags (i.e., Q^*_M is independent of λ), while the investment timing decision is affected by the lags (i.e., X^*_M depends on λ). Regarding the effects of time-to-build on the timing decision, we have the following straightforward results:

Corollary 1 *As the investment lags converge to 0 (i.e., $\lambda \to \infty$), the investment trigger decreases and converges to the one in the absence of time-to-build:*

$$\frac{\partial X^*_M}{\partial \lambda} < 0, \qquad \frac{\partial^2 X^*_M}{\partial \lambda^2} > 0, \tag{10}$$

$$\lim_{\lambda\to\infty} X^*_M = \frac{(\beta+1)\delta(r-\mu)}{\beta-1}. \tag{11}$$

2.2 Consumer surplus and social welfare

To investigate the impacts of investment lags on social welfare, we evaluate consumer surplus first. Given the demand shock X and capacity Q, the instantaneous consumer surplus is

$$\int_P^X \frac{1}{\eta}\Big(1 - \frac{P}{X}\Big)\mathrm{d}P = \frac{\eta X Q^2}{2}, \tag{12}$$

where the price P is given by (1). Since there is no entry of other firms, the total expected consumer surplus given the investment strategies of X_M and Q_M and the demand shock X is

$$\begin{aligned} CS_M(X, X_M, Q_M) &= \mathbb{E}\Big[\int_{\bar{T}_M}^\infty e^{-rt}\frac{\eta X(t) Q_M^2}{2}\mathrm{d}t\Big| X(0) = X\Big] \\ &= \Big(\frac{X}{X_M}\Big)^\beta \frac{\lambda\eta X_M Q_M^2}{2(r-\mu)(r+\lambda-\mu)}, \end{aligned} \tag{13}$$

and the expected producer surplus corresponds to the monopoly firm's value:

$$PS_M(X, X_M, Q_M) = \Big(\frac{X}{X_M}\Big)^{\beta}\Big[\frac{\lambda Q_M X_M(1-\eta Q_M)}{(r-\mu)(r+\lambda-\mu)} - \delta Q_M\Big]. \tag{14}$$

We can evaluate the expected social welfare as the sum of them:

$$\begin{aligned} SW_M(X, X_M, Q_M) &= CS_M(X, X_M, Q_M) + PS_M(X, X_M, Q_M) \\ &= \Big(\frac{X}{X_M}\Big)^{\beta}\Big[\frac{\lambda Q_M X_M(2-\eta Q_M)}{2(r-\mu)(r+\lambda-\mu)} - \delta Q_M\Big]. \end{aligned} \tag{15}$$

Now we examine the investment decision of a social planner who takes both consumer and producer surpluses into account. Following the same argument, the welfare-maximizing investment policy and the loss of welfare in a monopoly market can be derived as follows:

Proposition 2 (Welfare-maximizing policy and welfare loss in a monopoly market) *The investment threshold and capacity that maximize social welfare in a monopoly market are*

$$X_M^{**} = \frac{(\beta+1)\delta(r-\mu)(r+\lambda-\mu)}{(\beta-1)\lambda}, \tag{16}$$

$$Q_M^{**} = \frac{2}{(\beta+1)\eta}, \tag{17}$$

and the expected welfare loss in the monopoly market given the demand shock X is

$$WL_M(X) = SW_M^{**}(X) - SW_M^{*}(X) \tag{18}$$

$$= \frac{(\beta-1)^{\beta-1}\lambda^{\beta}X^{\beta}}{2\eta(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^{\beta}(r+\lambda-\mu)^{\beta}}, \tag{19}$$

where

$$SW_M^{**}(X) := SW_M(X, X_M^{**}, Q_M^{**}), \tag{20}$$

$$SW_M^{*}(X) := SW_M(X, X_M^{*}, Q_M^{*}). \tag{21}$$

Proof See Appendix A.2.

Note that the welfare-maximizing investment threshold coincides with the monopoly firm's optimal investment threshold (i.e., $X_M^{**} = X_M^{*}$), while the welfare-maximizing capacity is twice of the firm's optimal capacity (i.e., $Q_M^{**} = 2Q_M^{*}$), which are consistent with Huisman and Kort (2015). That is, the relationship between a firm's optimal strategy and a welfare-maximizing policy in a monopoly market holds regardless of time-to-build. Regarding the effects of time-to-build on the loss of social welfare, we can easily derive the following results:

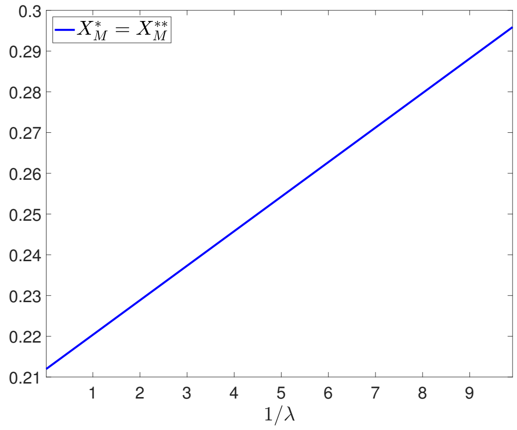
Corollary 2 *The loss of welfare in a monopoly market decreases in the size of investment lags (i.e., $1/\lambda$):*

$$\frac{\partial WL_M}{\partial \lambda} > 0, \tag{22}$$

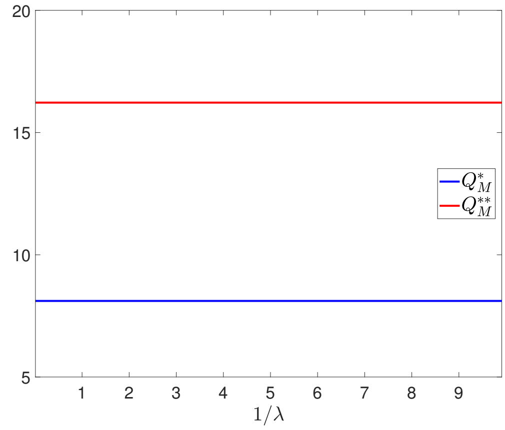
$$\lim_{\lambda\to 0} WL_M = 0, \tag{23}$$

$$\lim_{\lambda\to\infty} WL_M = \frac{(\beta-1)^{\beta-1}X^{\beta}}{2\eta(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^{\beta}}. \tag{24}$$

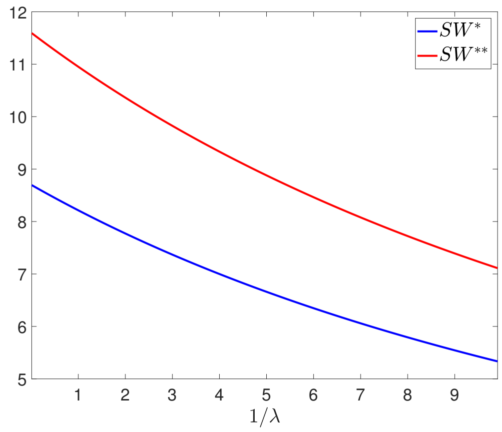
This result implies that the shorter the investment lags are, the more severe the loss of social welfare is (Figures 1c and 1d). This is because the investment timing is advanced as investment lags shorten (Figure 1a), while the difference between capacity decisions (Figure 1b) is irrelevant to the lags.



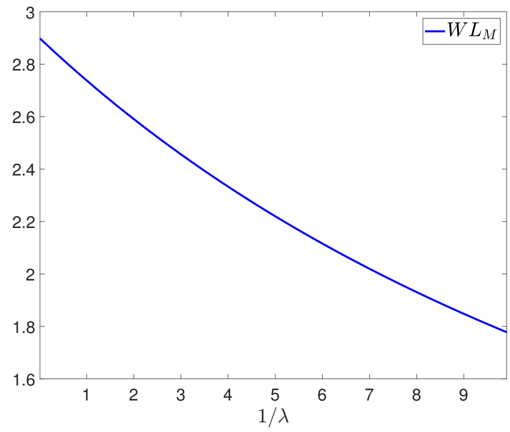
(a) Optimal investment threshold



(b) Optimal investment capacity



(c) Social welfare



(d) Welfare loss

Figure 1: Comparative statics regarding the expected time-to-build in a monopoly market

3 Duopoly market

Now we proceed to a duopoly market in which two firms compete with homogeneous goods. First, we assume the firms' roles in the market are exogenously given in Section 3.1. Given these arguments, we endogenize the firms' roles taking preemption incentives into account in Section 3.2. In Section 3.3, we derive the welfare-maximizing policy and discuss how time-to-build and competition in the market affect the level of social welfare.

3.1 Exogenous role

There are two firms in the market, type A and B firms, and they are asymmetric in terms of time-to-build. Namely, type i firm's investment lags follow an exponential distribution with an intensity parameter λ_i for $i \in \{A, B\}$. We designate the firms' roles in a duopoly market as a leader and a follower based on their investment timing. Note that because of the uncertain investment lags, the follower's products can be manufactured prior to that of the leader. To be more specific, there are three possible scenarios regarding the order of the firms' products entering the market:

- *Case 1*: The leader's product enters the market even before the follower makes the investment, and the follower's product enters the market afterwards.
- *Case 2*: The leader's product enters the market after the follower's investment yet before its manufacturing, and the follower's product enters the market afterwards.
- *Case 3*: The follower's product enters the market before the leader's one in spite of their investment timing, and the leader's product enters the market afterwards.

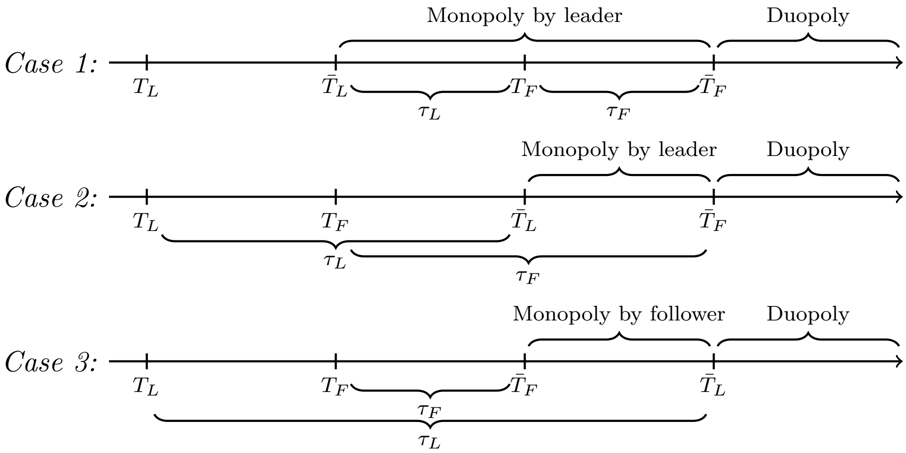


Figure 2: Timeline of three scenarios

For the ease of exposition, let us suppose the firms' roles in the market are exogenously given for now. Namely, we assume type i and j firms are a leader and a follower, respectively, for $i, j \in \{A, B\}$ and $i \neq j$, and we derive type i and j firms' optimal investment strategies as a leader and as a follower, respectively.

By backward induction, we investigate the follower's investment decision first. Suppose the leader's product has already entered the market. That is, we examine the follower's investment strategy in *Case 1*. Given type i leader's product in the market, type j follower chooses its investment timing and capacity taking the leader's capacity Q_L^i into account, and the follower's value can be evaluated as follows:

$$V_{F1}^j(X, Q_L^i) = \max_{T_{F1}^j \geq 0, Q_{F1}^j \geq 0} \mathbb{E}\left[\int_{\bar{T}_{F1}^j}^{\infty} e^{-rt} Q_{F1}^j X(t)(1 - \eta(Q_L^i + Q_{F1}^j))\mathrm{d}t - e^{-rT_{F1}^j} \delta Q_{F1}^j \middle| X(0) = X \right], \quad (25)$$

where $\bar{T}_{F1}^j := T_{F1}^j + \tau_j$ denotes type j follower's manufacturing timing for the investment made after the leader's product enters the market and τ_j follows an exponential distribution with a parameter λ_j. Type j follower's investment timing can be described as $T_{F1}^j := \inf\{t \geq \bar{T}_L^i | X(t) \geq X_{F1}^j\}$ where $\bar{T}_L^i$ denotes type i leader's manufacturing timing. Following the same argument, the follower's investment decisions after the completion of the leader's project are derived as follows:

Proposition 3 (Follower's investment after leader's product enters the market) *Provided that type i leader's product has entered the market, type j follower's optimal capacity corresponding to the demand shock X and the leader's capacity Q_L^i is*

$$Q_{F1}^{j*}(X, Q_L^i) = \frac{1}{2\eta}\Big(1 - \eta Q_L^i - \frac{\delta(r-\mu)(r+\lambda_j-\mu)}{\lambda_j X}\Big). \tag{26}$$

The value function of the follower after the leader's product enters the market is

$$V_{F1}^j(X, Q_L^i) = \begin{cases} A_{F1}^j(Q_L^i)X^\beta, & \text{if } X < X_{F1}^{j*}(Q_L^i), \\ \frac{\{(1-\eta Q_L^i)\lambda_j X - \delta(r-\mu)(r+\lambda_j-\mu)\}^2}{4\lambda_j X\eta(r-\mu)(r+\lambda_j-\mu)}, & \text{if } X \geq X_{F1}^{j*}(Q_L^i), \end{cases} \tag{27}$$

where

$$A_{F1}^j(Q_L^i) = \frac{(1-\eta Q_L^i)^\beta(\beta-1)^{\beta-1}\lambda_j^\beta}{\eta\delta^{\beta-1}(\beta+1)^{\beta+1}(r-\mu)^\beta(r+\lambda_j-\mu)^\beta}. \tag{28}$$

The optimal investment threshold and the corresponding optimal capacity are

$$X_{F1}^{j*}(Q_L^i) = \frac{(\beta+1)\delta(r-\mu)(r+\lambda_j-\mu)}{(\beta-1)(1-\eta Q_L^i)\lambda_j}, \tag{29}$$

$$Q_{F1}^{j*}(Q_L^i) := Q_{F1}^{j*}(X_{F1}^{j*}(Q_L^i), Q_L^i) = \frac{1-\eta Q_L^i}{(\beta+1)\eta}. \tag{30}$$

Proof See Appendix A.3.

Now we suppose that type i leader invested in capacity Q_L^i but its product has not been produced due to the investment lags. Namely, we investigate the follower's investment strategy regarding *Case 2* and *Case 3*. When making the investment decisions, the follower takes into account not only the leader's capacity but also the leader's expected manufacturing timing, and its value can be evaluated as follows:

$$\begin{aligned} V_{F0}^j(X, Q_L^i) = \max_{T_{F0}^j \geq 0, Q_{F0}^j \geq 0} \mathbb{E}\Big[&1_{\{T_{F0}^j < \bar{T}_L^i \leq \bar{T}_{F0}^j\}}\Big\{\int_{\bar{T}_{F0}^j}^\infty e^{-rt}Q_{F0}^j X(t)(1-\eta(Q_L^i + Q_{F0}^j))\mathrm{d}t - e^{-rT_{F0}^j}\delta Q_{F0}^j\Big\} \\ &+ 1_{\{\bar{T}_{F0}^j < \bar{T}_L^i\}}\Big\{\int_{\bar{T}_{F0}^j}^{\bar{T}_L^i} e^{-rt}Q_{F0}^j X(t)(1-\eta Q_{F0}^j)\mathrm{d}t + \int_{\bar{T}_L^i}^\infty e^{-rt}Q_{F0}^j X(t)(1-\eta(Q_L^i+Q_{F0}^j))\mathrm{d}t - e^{-rT_{F0}^j}\delta Q_{F0}^j\Big\} \\ &+ 1_{\{\bar{T}_L^i \leq T_{F0}^j\}} e^{-r\bar{T}_L^i} V_{F1}^j(X_{\bar{T}_L^i}, Q_L^i)\Big| X(0) = X\Big] \end{aligned} \tag{31}$$

where $\bar{T}_{F0}^j := T_{F0}^j + \tau_j$ denotes type j follower's manufacturing timing for the investment made before the leader's product enters the market. The follower's investment timing can be described as $T_{F0}^j := \inf\{t \in (0, \bar{T}_L^i) | X(t) \geq X_{F0}^j\}$. Note that the first row of (31) corresponds to *Case 2* in which the leader's product starts to be made before the follower's one, while the second row represents *Case 3* in which the follower's product enters the market before the leader's one. The third row is *Case 1* in which the leader's product enters the market even before the follower makes the investment, which corresponds to (25).

Following similar arguments, we can derive the follower's investment strategies before the leader's product is produced as follows:

Proposition 4 (Follower's investment before leader's project enters the market) *Provided that type i leader's product has not entered the market yet, type j follower's optimal capacity corresponding to the demand shock X and the leader's capacity Q_L^i is*

$$Q_{F0}^{j*}(X, Q_L^i) = \frac{1}{2\eta}\Bigg[\frac{(1-\eta Q_L^i)\lambda_i + (r+\lambda_j-\mu)(1-\frac{\eta\lambda_i Q_L^i}{r+\lambda_i-\mu})}{r+\lambda_i+\lambda_j-\mu} - \frac{\delta(r-\mu)(r+\lambda_j-\mu)}{\lambda_j X}\Bigg]. \tag{32}$$

The value function of the follower before the leader's project enters the market is

$$\begin{aligned} &V_{F0}^j(X, Q_L^i) \\ &= \begin{cases} A_{F1}^j(Q_L^i)X^{\beta} + A_{F0}^j(Q_L^i)X^{\beta_i}, & \text{if } X < X_{F0}^{j*}(Q_L^i), \\ \frac{\{(1-\eta Q_L^i)\lambda_i\lambda_j X(r+\lambda_i-\mu)+\lambda_j X(r+\lambda_j-\mu)(r+\lambda_i-\mu-\eta\lambda_i Q_L^i)-\delta(r-\mu)(r+\lambda_i-\mu)(r+\lambda_j-\mu)(r+\lambda_i+\lambda_j-\mu)\}^2}{4\eta\lambda_j X(r-\mu)(r+\lambda_j-\mu)\{(r+\lambda_i-\mu)(r+\lambda_i+\lambda_j-\mu)\}^2}, & \\ & \text{if } X \geq X_{F0}^{j*}(Q_L^i), \end{cases} \end{aligned} \tag{33}$$

where

$$\begin{aligned} A_{F0}^j(Q_L^i) = \Bigg[&\frac{Q_{F0}^{j*}(Q_L^i)\lambda_j X_{F0}^{j*}(Q_L^i)}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big\{1-\eta Q_{F0}^{j*}(Q_L^i) - \frac{\eta\lambda_i Q_L^i}{r+\lambda_i-\mu} + \frac{\{1-\eta(Q_L^i+Q_{F0}^{j*}(Q_L^i))\}\lambda_i}{r+\lambda_j-\mu}\Big\} \\ &- A_{F1}^j(Q_L^i)\{X_{F0}^{j*}(Q_L^i)\}^{\beta} - \delta Q_{F0}^{j*}(Q_L^i)\Bigg]\Big(\frac{1}{X_{F0}^{j*}(Q_L^i)}\Big)^{\beta_i}, \end{aligned} \tag{34}$$

and

$$\beta_i = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\Big(\frac{1}{2}-\frac{\mu}{\sigma^2}\Big)^2 + \frac{2(r+\lambda_i)}{\sigma^2}} \ (>1). \tag{35}$$

The optimal investment threshold and capacity, $X_{F0}^{j*}(Q_L^i)$ and $Q_{F0}^{j*}(Q_L^i) := Q_{F0}^{j*}(X_{F0}^{j*}(Q_L^i), Q_L^i)$, are implicitly derived from (32) *and*

$$\begin{aligned} &\frac{\delta(1-\eta Q_L^i)(\beta_i-\beta)}{\eta(\beta+1)(\beta-1)}\Big(\frac{X_{F0}^{j*}(Q_L^i)}{X_{F1}^{j*}(Q_L^i)}\Big)^{\beta} + \beta_i\delta Q_{F0}^{j*}(Q_L^i) \\ &= \frac{(\beta_i-1)Q_{F0}^{j*}(Q_L^i)\lambda_j X_{F0}^{j*}(Q_L^i)}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big[1-\eta Q_{F0}^{j*}(Q_L^i) - \frac{\eta\lambda_i Q_L^i}{r+\lambda_i-\mu} + \frac{\{1-\eta(Q_L^i+Q_{F0}^{j*}(Q_L^i))\}\lambda_i}{r+\lambda_j-\mu}\Big]. \end{aligned} \tag{36}$$

Proof See Appendix A.4.

From the follower's optimal investment strategies in (26) and (32), we can easily obtain the following result:

Corollary 3 *For a given demand shock X and a leader's capacity Q_L^i, a follower's optimal capacity before the completion of the leader's project always dominates than that after the completion of the leader's project:*

$$Q_{F0}^{j*}(X, Q_L^i) > Q_{F1}^{j*}(X, Q_L^i). \tag{37}$$

This is a natural result considering that there is a chance for the follower to preempt the market before the leader's product enters the market. However, this does not imply that the follower's optimal capacity before the completion of the leader's project (i.e., $Q_{F0}^{j*}(Q_L^i)$) always dominates the one after the completion of the leader's project (i.e., $Q_{F1}^{j*}(Q_L^i)$) because they depend on the investment timing $X_{F0}^{j*}(Q_L^i)$ and $X_{F1}^{j*}(Q_L^i)$, respectively, and $X_{F0}^{j*}(Q_L^i)$ can only be derived implicitly from (32) and (36).

In this subsection, we assume that the firms' roles are exogenously given, and thus, the follower can only invest after the leader's investment. Namely, the follower's value in Proposition 4, $V_{F0}^j(X, Q_L^i)$, assumes that the leader has already made the investment (i.e., $X \geq X_L^i$). Before the leader's investment (i.e., $X < X_L^i$), the follower's value depends on the leader's investment timing and can be written as follows:

$$V_F^j(X, X_L^i, Q_L^i) = \Big(\frac{X}{X_L^i}\Big)^{\beta} V_{F0}^j(X_L^i, Q_L^i). \tag{38}$$

Lastly, we proceed to the analysis on type i leader's investment decision. The leader takes the follower's investment timing and capacity decisions into account, and the firm value can be expressed as follows:

$$\begin{aligned} V_L^i(X) = \max_{T_L^i \geq 0, Q_L^i \geq 0} \mathbb{E}\Big[& 1_{\{\bar{T}_L^i < T_{F0}^{j*}\}} \Big\{ \int_{\bar{T}_L^i}^{\bar{T}_{F1}^{j*}} e^{-rt} Q_L^i X(t)(1-\eta Q_L^i)\mathrm{d}t + \int_{\bar{T}_{F1}^{j*}}^{\infty} e^{-rt} Q_L^i X(t)(1-\eta(Q_L^i + Q_{F1}^{j*}))\mathrm{d}t \Big\} \\ & + 1_{\{T_{F0}^{j*} \leq \bar{T}_L^i < \bar{T}_{F0}^{j*}\}} \Big\{ \int_{\bar{T}_L^i}^{\bar{T}_{F0}^{j*}} e^{-rt} Q_L^i X(t)(1-\eta Q_L^i)\mathrm{d}t + \int_{\bar{T}_{F0}^{j*}}^{\infty} e^{-rt} Q_L^i X(t)(1-\eta(Q_L^i + Q_{F0}^{j*}))\mathrm{d}t \Big\} \\ & + 1_{\{\bar{T}_{F0}^{j*} \leq \bar{T}_L^i\}} \int_{\bar{T}_L^i}^{\infty} e^{-rt} Q_L^i X(t)(1-\eta(Q_L^i + Q_{F0}^{j*}))\mathrm{d}t - e^{rT_L^i} \delta Q_L^i \Big] \end{aligned} \tag{39}$$

where $\bar{T}_L^i := T_L^i + \tau_i$ denotes type i leader's manufacturing timing and $\bar{T}_{F0}^*$ and $\bar{T}_{F1}^*$ denote type j follower's manufacturing timing given the optimal investment in Propositions 3 and 4. As before, the leader's investment timing can be expressed as $T_L^i := \inf\{t \in (0, T_{F0}^j) | X(t) \geq X_L^i\}$. The first row of (39) corresponds to *Case 1* in which the leader's product enters the market even before the follower's investment. The second row of (39) is associated with *Case 2* in which the leader's product enters the market after the follower's investment yet before its manufacturing, while the third row of (39) is *Case 3* in which the follower's product enters the market before the leader's one in spite of their investment timing.

Following the similar arguments, we can derive the leader's optimal investment strategies and its value as follows:

Proposition 5 (Leader's investment) *Type i leader's optimal capacity corresponding to demand shock X is*

$$Q_L^{i*}(X) = \underset{Q_L^i \geq 0}{\arg\max} A_L^i(X, Q_L^i), \tag{40}$$

where

$$\begin{aligned} A_L^i(X, Q_L^i) = & \frac{Q_L^i(1-\eta Q_L^i)\lambda_i X}{(r-\mu)(r+\lambda_i-\mu)} - \Big(\frac{X}{X_{F0}^{j*}(Q_L^i)}\Big)^{\beta_i} \frac{\eta Q_L^i Q_{F0}^{j*}(Q_L^i)\lambda_i\lambda_j X_{F0}^{j*}(Q_L^i)}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)} \Big(\frac{1}{r+\lambda_i-\mu} + \frac{1}{r+\lambda_j-\mu}\Big) \\ & - \Big\{\Big(\frac{X}{X_{F1}^{j*}(Q_L^i)}\Big)^{\beta} - \Big(\frac{X}{X_{F0}^{j*}(Q_L^i)}\Big)^{\beta_i}\Big(\frac{X_{F0}^{j*}(Q_L^i)}{X_{F1}^{j*}(Q_L^i)}\Big)^{\beta}\Big\} \frac{\eta Q_L^i Q_{F1}^{j*}(Q_L^i)\lambda_j X_{F1}^{j*}(Q_L^i)}{(r-\mu)(r+\lambda_j-\mu)} - \delta Q_L^i. \end{aligned} \tag{41}$$

The value function of the leader is

$$V_L^i(X) = \begin{cases} A_L^i(X_L^{i*}, Q_L^{i*})\left(\frac{X}{X_L^{i*}}\right)^{\beta}, & \text{if } X < X_L^{i*}, \\ A_L^i(X, Q_L^{i*}(X)), & \text{if } X \geq X_L^{i*}, \end{cases} \tag{42}$$

where the leader's optimal investment threshold and capacity are

$$X_L^{i*} = \underset{X_L^i \geq 0}{\arg\max} A_L^i(X_L^i, Q_L^{i*}(X_L^i)), \tag{43}$$

and $Q_L^{i*} := Q_L^{i*}(X_L^{i*})$, *respectively.*

Proof See Appendix A.5.

Recall that the follower's optimal investment strategies are directly linked to the leader's capacity choice Q_L^i. Thus, the leader's optimal investment strategies are solved numerically because the follower's decisions of $Q_{F0}^{j*}(Q_L^i)$ and $X_{F0}^{j*}(Q_L^i)$ in Proposition 4 can only be derived implicitly.

If we characterize type i and j firms' roles as a leader and a follower and their investment strategies by the following tuple:

$$\mathcal{S}_{ij} = (i, j, X_L^i, X_{F0}^j, X_{F1}^j, Q_L^i, Q_{F0}^j, Q_{F1}^j), \tag{44}$$

we can immediately obtain the following straightforward result:

Corollary 4 *Given the exogenous roles of type i and j firms as a leader and a follower, respectively, the optimal investment strategy can be summarized as follows:*

$$\mathcal{S}_{ij}^* = (i, j, X_L^{i*}, X_{F0}^{j*}(Q_L^{i*}), X_{F1}^{j*}(Q_L^{i*}), Q_L^{i*}, Q_{F0}^{j*}(Q_L^{i*}), Q_{F1}^{j*}(Q_L^{i*})). \tag{45}$$

From the arguments derived in Proposition 5, we can easily obtain the state price of each investment scenario at the timing of the leader's investment as follows:

Corollary 5 *Given investment strategy $\mathcal{S}_{ij}$, the state price of Case 1 in which the leader's product enters the market even before the follower makes the investment and the follower's product enters the market afterwards is*

$$\Phi_1(\mathcal{S}_{ij}) := \mathbb{E}\Big[1_{\{\bar{T}_L^i < T_{F0}^j\}} e^{-r(\bar{T}_{F1}^j - T_L^i)}\Big] = \left\{\left(\frac{X_L^i}{X_{F1}^j}\right)^{\beta} - \left(\frac{X_L^i}{X_{F0}^j}\right)^{\beta_i}\left(\frac{X_{F0}^j}{X_{F1}^j}\right)^{\beta}\right\}\frac{\lambda_j}{r+\lambda_j}. \tag{46}$$

The state price of Case 2 in which the leader's product enters the market after the follower's investment yet before its manufacturing and the follower's product enters the market afterwards is

$$\Phi_2(\mathcal{S}_{ij}) := \mathbb{E}\Big[1_{\{T_{F0}^j \leq \bar{T}_L^i < \bar{T}_{F0}^j\}} e^{-r(\bar{T}_{F0}^j - T_L^i)}\Big] = \left(\frac{X_L^i}{X_{F0}^j}\right)^{\beta_i}\frac{\lambda_i\lambda_j}{(r+\lambda_j)(r+\lambda_i+\lambda_j)}, \tag{47}$$

and that of Case 3 in which the follower's product enters the market before the leader's one and the leader's product enters the market afterwards is

$$\Phi_3(\mathcal{S}_{ij}) := \mathbb{E}\Big[1_{\{\bar{T}_{F0}^j \leq \bar{T}_L^i\}} e^{-r(\bar{T}_L^i - T_L^i)}\Big] = \left(\frac{X_L^i}{X_{F0}^j}\right)^{\beta_i}\frac{\lambda_i\lambda_j}{(r+\lambda_i)(r+\lambda_i+\lambda_j)}. \tag{48}$$

3.2 Endogenous role

So far, we have assumed that the firms' roles are predetermined. Now we relax the assumption and endogenize their roles in the market. Given competition in a duopoly market, the firms might have incentives to preempt the market and we have to take the preemption incentives into account analyzing their roles in the market and investment strategies.

If type i firm chooses to invest after type j firm's investment in Q_L^j, its value as a follower can be represented as $V_{F0}^i(X, Q_L^j)$ given by (33). Meanwhile, if type i firm makes the investment given demand shock X to preempt the market, its value as a leader with preemption investment can be evaluated as follows:

$$V_P^i(X) = \begin{cases} A_L^i(X, Q_L^{i*}(X)), & \text{if } X < X_{F0}^{j*}(Q_L^{i*}(X)), \\ \frac{Q_L^{i*}(X)\lambda_i X}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big[1 - \eta Q_L^{i*}(X) - \frac{\eta Q_{F0}^{j*}\lambda_j}{r+\lambda_j-\mu} \\ \qquad + \frac{(1-\eta(Q_L^{i*}(X)+Q_{F0}^{j*}))\lambda_j}{r+\lambda_i-\mu}\Big] - \delta_i Q_L^{i*}(X), & \text{if } X \geq X_{F0}^{j*}(Q_L^{i*}(X)). \end{cases} \tag{49}$$

Note that the upper case of (49) corresponds to the lower case of (42). The lower case of (49) implies that if type i firm makes the investment after the demand shock exceeds type j's investment threshold, both firms invest simultaneously.

It is straightforward that a firm has an incentive to preempt the market only if its value as a leader exceeds that as a follower. We can conjecture that there is a level of demand shock at which type i firm is indifferent between being a leader and being a follower denoted by $X_P^i := \inf\{X > 0 | V_P^i(X) \geq V_{F0}^i(X, Q_L^j)\}$. Since a leader is more likely to appreciate monopoly profits than a follower does, it is natural to presume $X_P^i \cup X_P^j \neq \emptyset$ for $i \neq j$; at least one of the firms has an incentive to preempt the market.[1]

Based on the arguments discussed in the previous subsections, we can summarize the firms' endogenous roles in the market and their investment strategies taking preemption incentives into account as follows:

Proposition 6 (Optimal investment strategy with preemption incentive) *The level of demand shock at which type i firm is indifferent between being a leader and being a follower is derived from*

$$V_P^i(X_P^{i*}) = V_{F0}^i(X_P^{i*}, Q_P^{j*}), \tag{50}$$

where $Q_P^{j*} := Q_L^{j*}(X_P^{i*})$ *for* $i, j \in \{A, B\}$ *and* $i \neq j$.

A) If $X_P^{i*} \neq \emptyset$ *for* $i \in \{A, B\}$ *and* $X_P^{i*} \leq X_P^{j*}$ *for* $i \neq j$, *and*

i) if $X_P^{j*} \leq X_L^{i*}$, *the optimal investment strategy is*

$$\mathcal{S}^* = (i, j, X_P^{j*}, X_{F0}^{j*}(Q_P^{i*}), X_{F1}^{j*}(Q_P^{i*}), Q_P^{i*}, Q_{F0}^{j*}(Q_P^{i*}), Q_{F1}^{j*}(Q_P^{i*})), \tag{51}$$

where $Q_P^{i*} := Q_L^{i*}(X_P^{j*})$.

[1] We cannot explicitly prove that at least one of the firms has preemption incentive because the leader's investment strategies can only be obtained by numerical calculation.

ii) if $X_P^{j*} > X_L^{i*}$, the optimal investment strategy is

$$\mathcal{S}^* = \mathcal{S}_{ij}^* = (i, j, X_L^{i*}, X_{F0}^{j*}(Q_L^{i*}), X_{F1}^{j*}(Q_L^{i*}), Q_L^{i*}, Q_{F0}^{j*}(Q_L^{i*}), Q_{F1}^{j*}(Q_L^{i*})), \tag{52}$$

where $Q_L^{i*} := Q_L^{i*}(X_L^{i*})$.

B) If $X_P^{i*} \neq \emptyset$ and $X_P^{j*} = \emptyset$ for $i \neq j$, the optimal investment strategy is (52).

PROOF See Appendix A.6.

In case (A), both firms have incentive to preempt the market, and the firm willing to make the investment at lower demand shock becomes the leader, bearing the cost of earlier investment. In case (B), type j firm's value as a follower dominates that as a leader with preemptive investment, while type i firm is willing to burden the cost of earlier investment for the preemption.

Note that in case (A-i), type i firm does not invest at X_P^{i*} at which it is indifferent from being a leader and being a follower; it makes the investment at X_P^{j*} at which its competitor is indifferent between being a leader and being a follower. With $X_P^{i*} \leq X_P^{j*} \leq X_L^{i*}$, there is no reason for type i firm to invest at X_P^{i*} because it can still be a leader by investing at $X_P^{j*} - \epsilon$, which is closer to its optimal investment threshold X_L^{i*} than X_P^{i*} is. It is needless to say that in cases (A-ii) and (B), type i firm has no reason to deviate from its optimal investment strategy in the absence of preemption incentive.

3.3 Welfare analysis

Now we proceed to the welfare analysis in a duopoly market. By the argument discussed in Section 2.2, consumer surplus in a duopoly market given the demand shock X and investment strategy $\mathcal{S}_{ij}$ are expressed as follows:

$$\begin{aligned} CS_D(X, \mathcal{S}_{ij}) = \mathbb{E}\Bigg[& 1_{\{\bar{T}_L^i < T_{F0}^j\}} \Big\{ \int_{\bar{T}_L^i}^{\bar{T}_{F1}^j} e^{-rt} \frac{\eta X(t)(Q_L^i)^2}{2} \mathrm{d}t + \int_{\bar{T}_{F1}^j}^{\infty} e^{-rt} \frac{\eta X(t)(Q_L^i + Q_{F1}^j)^2}{2} \mathrm{d}t \Big\} \\ & + 1_{\{T_{F0}^j \leq \bar{T}_L^i < \bar{T}_{F0}^j\}} \Big\{ \int_{\bar{T}_L^i}^{\bar{T}_{F0}^j} e^{-rt} \frac{\eta X(t)(Q_L^i)^2}{2} \mathrm{d}t + \int_{\bar{T}_{F0}^j}^{\infty} e^{-rt} \frac{\eta X(t)(Q_L^i + Q_{F0}^j)^2}{2} \mathrm{d}t \Big\} \\ & + 1_{\{\bar{T}_{F0}^j \leq \bar{T}_L^i\}} \Big\{ \int_{\bar{T}_{F0}^j}^{\bar{T}_L^i} e^{-rt} \frac{\eta X(t)(Q_{F0}^j)^2}{2} \mathrm{d}t + \int_{\bar{T}_L^i}^{\infty} e^{-rt} \frac{\eta X(t)(Q_L^i + Q_{F0}^j)^2}{2} \mathrm{d}t \Big\} \Big| X(0) = X \Bigg] \end{aligned} \tag{53}$$

Following the same argument in Appendix A.5, we can rearrange (53) as follows:

$$\begin{aligned} CS_D(X, \mathcal{S}_{ij}) = & \frac{\eta}{2(r-\mu)} \Big(\frac{X}{X_L^i}\Big)^{\beta} \Bigg[\frac{\lambda_i X_L^i (Q_L^i)^2}{r + \lambda_i - \mu} + \Big(\frac{X_L^i}{X_{F0}^j}\Big)^{\beta_i} X_{F0}^j \Big\{ - \frac{\lambda_i (Q_L^i)^2}{r + \lambda_i - \mu} \\ & + \frac{\lambda_i}{r + \lambda_i + \lambda_j - \mu} \Big((Q_L^i)^2 + \frac{\lambda_j \{2Q_L^i Q_{F0}^j + (Q_{F0}^j)^2\}}{r + \lambda_j - \mu} \Big) + \frac{\lambda_j}{r + \lambda_i + \lambda_j - \mu} \Big((Q_{F0}^j)^2 + \frac{\lambda_i \{2Q_L^i Q_{F0}^j + (Q_L^i)^2\}}{r + \lambda_i - \mu} \Big) \Big\} \\ & + \Big\{ \Big(\frac{X_L^i}{X_{F1}^j}\Big)^{\beta} - \Big(\frac{X_L^i}{X_{F0}^j}\Big)^{\beta_i} \Big(\frac{X_{F0}^j}{X_{F1}^j}\Big)^{\beta} \Big\} \frac{\lambda_j X_{F1}^j \{2Q_L^i Q_{F1}^j + (Q_{F1}^j)^2\}}{r + \lambda_j - \mu} \Bigg]. \end{aligned} \tag{54}$$

Meanwhile, there are two producers in a duopoly market, and thus, producer surplus is evaluated as the sum of the two firms' values:

$$PS_D(X, \mathcal{S}_{ij}) = V_L^i(X, \mathcal{S}_{ij}) + V_F^j(X, \mathcal{S}_{ij}), \tag{55}$$

where the two terms on the right-hand side of (55) correspond to the values of the leader and the follower for given investment strategies, respectively. Thus, the level of social welfare in a duopoly market is evaluated as follows:

$$
\begin{aligned}
&SW_D(X,\mathcal{S}_{ij}) = CS_D(X,\mathcal{S}_{ij}) + PS_D(X,\mathcal{S}_{ij}) \\
&= \Big(\frac{X}{X_L^i}\Big)^{\beta}\Big[\frac{Q_L^i(2-\eta Q_L^i)\lambda_i X_L^i}{2(r-\mu)(r+\lambda_i-\mu)} + \Big(\frac{X_L^i}{X_{F0}^j}\Big)^{\beta_i}\Big[\frac{X_{F0}^j}{2(r-\mu)}\Big\{-\frac{Q_L^i(2-\eta Q_L^i)\lambda_i}{r+\lambda_i-\mu} \\
&+ \frac{\lambda_i}{r+\lambda_i+\lambda_j-\mu}\Big(Q_L^i(2-\eta Q_L^i) + \frac{Q_{F0}^j(2-\eta Q_{F0}^j - 2\eta Q_L^i)\lambda_j}{r+\lambda_j-\mu}\Big) \\
&+ \frac{\lambda_j}{r+\lambda_i+\lambda_j-\mu}\Big(Q_{F0}^j(2-\eta Q_{F0}^j) + \frac{Q_L^i(2-\eta Q_L^i - 2\eta Q_{F0}^j)\lambda_i}{r+\lambda_i-\mu}\Big)\Big\} - \delta Q_{F0}^j\Big] \\
&+ \Big\{\Big(\frac{X_L^i}{X_{F1}^j}\Big)^{\beta} - \Big(\frac{X_L^i}{X_{F0}^j}\Big)^{\beta_i}\Big(\frac{X_{F0}^j}{X_{F1}^j}\Big)^{\beta}\Big\}\Big(\frac{Q_{F1}^j(2-\eta Q_{F1}^j - 2\eta Q_L^i)\lambda_j X_{F1}^j}{2(r-\mu)(r+\lambda_j-\mu)} - \delta Q_{F1}^j\Big) - \delta Q_L^i\Big]. \quad (56)
\end{aligned}
$$

Given these arguments, we can derive the welfare-maximizing investment policy of type i and j firms for exogenously given roles as a leader and a follower, respectively, as follows:

Proposition 7 (Welfare-maximizing policy for given roles in the market) *Type j follower's welfare-maximizing investment strategies after type i leader's product enters the market are characterized as follows:*

$$X_{F1}^{j**}(Q_L^i) = \frac{(\beta+1)\delta_j(r-\mu)(r+\lambda_j-\mu)}{(\beta-1)(1-\eta Q_L^i)\lambda_j}, \quad (57)$$

$$Q_{F1}^{j**}(Q_L^i) = \frac{2(1-\eta Q_L^i)}{(\beta+1)\eta}. \quad (58)$$

Before type i leader's product enters the market, type j follower's welfare-maximizing investment strategies, $Q_{F0}^{j**}(Q_L^i)$ and $X_{F0}^{j**}(Q_L^i)$, are implicitly derived from the following equations:

$$Q_{F0}^{j**}(Q_L^i) = \frac{1}{\eta}\Big[1 - \frac{(r+\lambda_j-\mu)\eta Q_L^i\lambda_i}{r+\lambda_i+\lambda_j-\mu}\Big(\frac{1}{r+\lambda_i-\mu} + \frac{1}{r+\lambda_j-\mu}\Big) - \frac{(r-\mu)(r+\lambda_j-\mu)\delta}{\lambda_j X_{F0}^{j**}(Q_L^i)}\Big], \quad (59)$$

$$
\begin{aligned}
&\frac{2(\beta_i-\beta)(1-\eta Q_L^i)\delta}{\eta(\beta+1)(\beta-1)}\Big(\frac{X_{F0}^{j**}(Q_L^i)}{X_{F1}^{j**}(Q_L^i)}\Big)^{\beta} \\
&= \frac{(\beta_i-1)X_{F0}^{j**}(Q_L^i)}{2(r-\mu)}\Big[\frac{\lambda_i}{r+\lambda_i+\lambda_j-\mu}\Big\{Q_L^i(2-\eta Q_L^i) + \frac{Q_{F0}^{j**}(Q_L^i)\{2-\eta Q_{F0}^{j**}(Q_L^i) - 2\eta Q_L^i\}\lambda_j}{r+\lambda_j-\mu}\Big\} \\
&+ \frac{\lambda_j}{r+\lambda_i+\lambda_j-\mu}\Big\{Q_{F0}^{j**}(Q_L^i)\{2-\eta Q_{F0}^{j**}(Q_L^i)\} + \frac{Q_L^i\{2-\eta Q_L^i - 2\eta Q_{F0}^{j**}(Q_L^i)\}\lambda_i}{r+\lambda_i-\mu}\Big\}\Big] - \beta_i\delta Q_{F0}^{j**}(Q_L^i). \quad (60)
\end{aligned}
$$

Type i leader's welfare-maximizing capacity corresponding to demand shock X is

$$Q_L^{i**}(X) = \underset{Q_L^i \geq 0}{\arg\max} A_W^i(X,X,Q_L^i), \quad (61)$$

where

$$
\begin{aligned}
A_W^i(X, X_L^i, Q_L^i) &= \frac{Q_L^i(2-\eta Q_L^i)\lambda_i X_L^i}{2(r-\mu)(r+\lambda_i-\mu)} + \Big(\frac{X_L^i}{X_{F0}^{j**}(Q_L^i)}\Big)^{\beta_i}\Big[\frac{X_{F0}^{j**}(Q_L^i)}{2(r-\mu)}\Big\{-\frac{Q_L^i(2-\eta Q_L^i)\lambda_i}{r+\lambda_i-\mu} \\
&+ \frac{\lambda_i}{r+\lambda_i+\lambda_j-\mu}\Big(Q_L^i(2-\eta Q_L^i) + \frac{Q_{F0}^{j**}(Q_L^i)\{2-\eta Q_{F0}^{j**}(Q_L^i)-2\eta Q_L^i\}\lambda_j}{r+\lambda_j-\mu}\Big) \\
&+ \frac{\lambda_j}{r+\lambda_i+\lambda_j-\mu}\Big(Q_{F0}^{j**}(Q_L^i)\{2-\eta Q_{F0}^{j**}(Q_L^i)\} + \frac{Q_L^i\{2-\eta Q_L^i-2\eta Q_{F0}^{j}(Q_L^i)\}\lambda_i}{r+\lambda_i-\mu}\Big)\Big\} - \delta Q_{F0}^{j**}(Q_L^i)\Big] \\
&+ \Big\{\Big(\frac{X_L^i}{X_{F1}^{j**}(Q_L^i)}\Big)^{\beta} - \Big(\frac{X_L^i}{X_{F0}^{j**}(Q_L^i)}\Big)^{\beta_i}\Big(\frac{X_{F0}^{j**}(Q_L^i)}{X_{F1}^{j**}(Q_L^i)}\Big)^{\beta}\Big\} \\
&\times \Big(\frac{Q_{F1}^{j**}(Q_L^i)\{2-\eta Q_{F1}^{j**}(Q_L^i)-2\eta Q_L^i\}\lambda_j X_{F1}^{j**}(Q_L^i)}{2(r-\mu)(r+\lambda_j-\mu)} - \delta Q_{F1}^{j**}(Q_L^i)\Big) - \delta Q_L^i. \qquad (62)
\end{aligned}
$$

Type i leader's welfare-maximizing investment threshold and capacity are

$$X_L^{i**} = \underset{X_L^i \ge 0}{\arg\max} A_W^i(X, X_L^i, Q_L^{i**}(X_L^i)), \qquad (63)$$

and $Q_L^{i**} := Q_L^{i**}(X_L^{i**})$*, respectively. The welfare-maximizing investment strategy for type i and j firms as a leader and a follower, respectively, is*

$$\mathcal{S}_{ij}^{**} = (i, j, X_L^{i**}, X_{F0}^{j**}, X_{F1}^{j**}, Q_L^{i**}, Q_{F0}^{j**}, Q_{F1}^{j**}). \qquad (64)$$

Proof See Appendix A.7.

Given these arguments, we can immediately obtain the following results:

Corollary 6 (Welfare-maximizing policy and welfare loss in a duopoly market) *The welfare-maximizing policy is*

$$\mathcal{S}^{**} = \underset{\mathcal{S}_{ij}^{**}}{\arg\max} SW_D(X, \mathcal{S}_{ij}^{**}), \qquad (65)$$

for $i, j \in \{A, B\}$ *and* $i \neq j$*, and the expected loss of welfare in a duopoly market is*

$$WL_D(X) = SW_D^{**}(X) - SW_D^{*}(X), \qquad (66)$$

where $SW_D^{**}(X) := SW_D(X, \mathcal{S}^{**})$ *and* $SW_D^{*}(X) := SW_D(X, \mathcal{S}^{*})$*.*

In a nutshell, a social planner assigns the roles of the leader and the follower such that their investment strategies given by (64) maximize the level of social welfare.

4 Comparative statics and discussion

In this section, we present the results of comparative statics regarding time-to-build and discuss their economic implications. The results of the firms' optimal investment strategies and those with welfare-maximizing policy are given in Sections 4.1 and 4.2, respectively. For numetical calculation, we used the parameters given in Table 1:

Notation	Value	Description
r	0.06	Discount rate
μ	0.02	Expected growth rate of demand shock
σ	0.3	Volatility of demand shock
$1/\lambda_A$	2	Type A firm's expected time-to-build
$1/\lambda_B$	[2, 7]	Type B firm's expected time-to-build
η	0.05	Coefficient of inverse demand function
δ	1	Investment costs per unit
$X(0)$	0.1	Initial demand shock

Table 1: Parameters for the numerical calculation

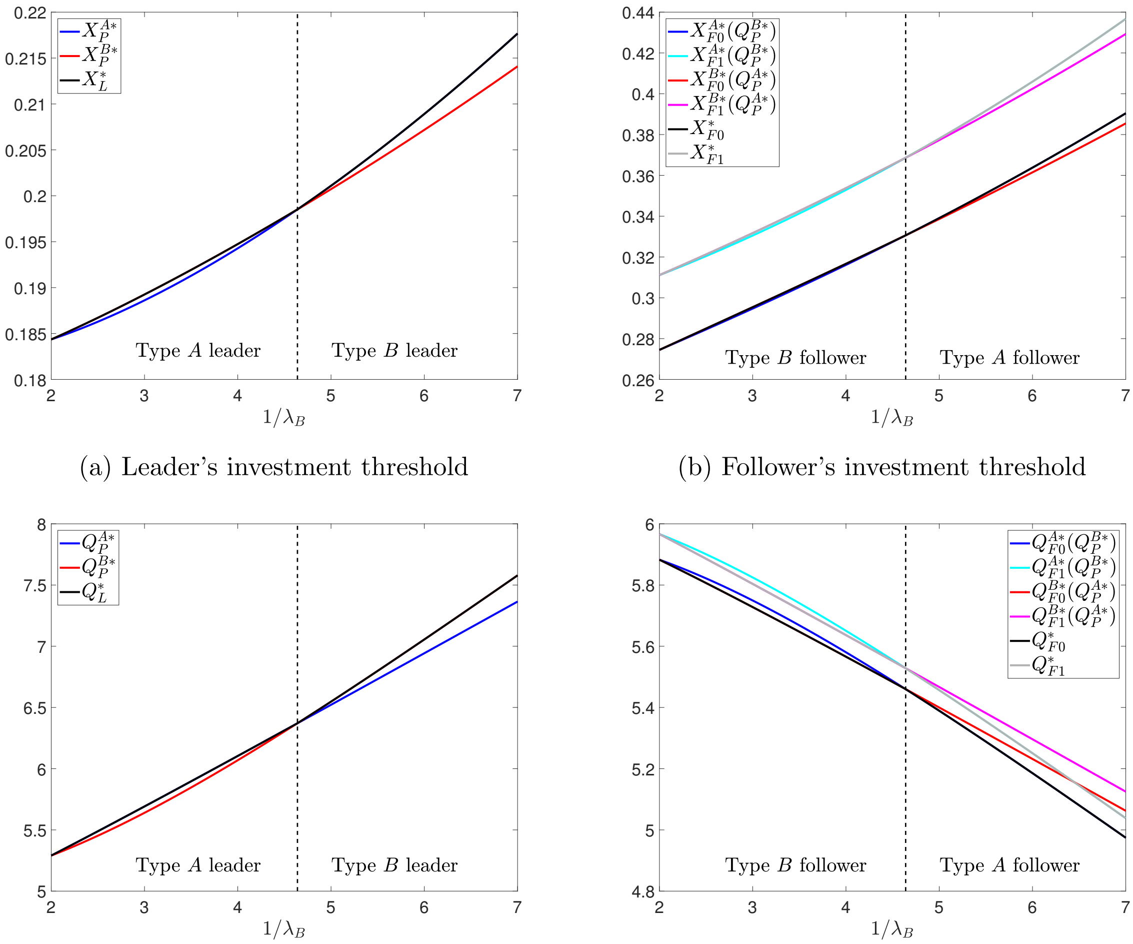
Note that without loss of generality, we fix type A firm's expected time-to-build and see how the results change as type B firm's expected time-to-build varies. Namely, we designate type A and B firms as a dominant and a dominated firms, respectively, in terms of time-to-build.

4.1 Firms' optimal investment strategies

First of all, we can see from Figure 3a that when the difference between the firms' time-to-build is insignificant, the dominant firm (i.e., type A firm) becomes the leader, whereas the dominated firm (i.e., type B firm) becomes the leader given a significant difference between time-to-build. To be more specific, when $1/\lambda_B$ is not high enough, the level of demand shock at which type A firm is indifferent between being a leader and being a follower is lower than that of type B firm (i.e., $X_P^{A*} < X_P^{B*}$), and type A becomes the leader by making the investment at the threshold X_P^{B*}. Note that type A firm makes the investment at X_P^{B*}, not at X_P^{A*}, because it is closer to its optimal investment threshold in the absence of preemption (i.e., X_L^{A*}). When $1/\lambda_B$ is significantly higher than $1/\lambda_A$, the inverse relation holds regarding the thresholds (i.e., $X_P^{B*} \leq X_P^{A*}$), and type B firm, which is expected to take more time to manufacture, invests at the threshold X_P^{A*} and becomes the leader in the market. Figure 3b shows that in the former case, the dominated firm (i.e., type B firm) invests as a follower at $X_{F0}^{B*}(Q_P^{A*})$ or $X_{F1}^{B*}(Q_P^{A*})$ depending on the leader's manufacturing process, whereas in the latter case the dominant firm (i.e., type A firm) becomes the follower by making the investment at the thresholds of $X_{F0}^{A*}(Q_P^{B*})$ or $X_{F1}^{A*}(Q_P^{B*})$.

Figures 3c and 3d present the optimal capacities of the leader and the follower, respectively, and we can see that the leader's capacity strictly increases in the size of the dominated firm's time-to-build, while the follower's capacity strictly decreases in it. This result is in constrast with that from a monopoly market in which the firm's optimal capacity is irrelevant to time-to-build (Figure 1b). Namely, the existence of time-to-build makes a different impact on the firm's capacity decision in a different market structure. Note that the tendency of increase in the leader's capacity lasts even after the dominated firm becomes the leader. That is, the dominated firm makes more aggressive investment as a leader when its time-to-build gets longer.

These results are novel in that we have shown that the dominated firm can become a leader in



(a) Leader's investment threshold

(b) Follower's investment threshold

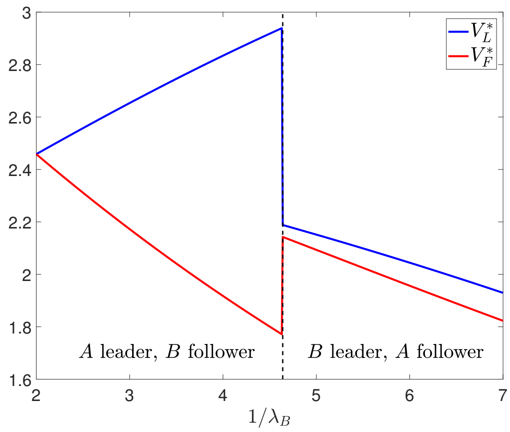
(c) Leader's investment capacity

(d) Follower's investment capacity

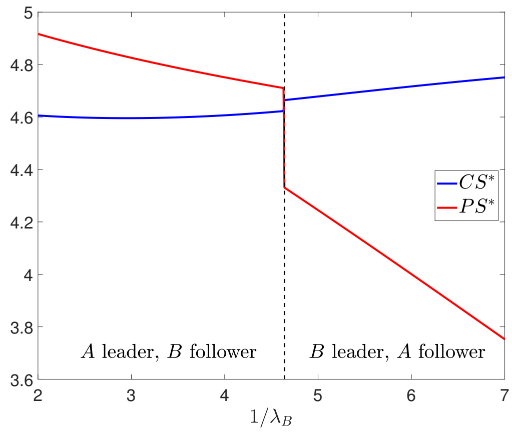
Figure 3: Comparative statics regarding the expected time-to-build in a duopoly market with the firms' optimal investment strategies

the market, which is in sharp contrast with the arguments from existing literature on asymmetric duopoly market. For instance, Pawlina and Kort (2006) investigated a duopoly market with asymmetric investment costs and showed that a firm with relative cost advantage becomes the leader in the market. Chapter 8 of Huisman (2001) discusses a more generalized setup including positive externalities and various levels of initial demand shock, but with negative externalities from competition and sufficiently low initial demand shock, the dominant firm makes the investment first. Kong and Kwok (2007) introduced asymmetry in both investment costs and revenue flow, but the firm with advantage of cost normalized by revenue becomes the leader in the market. In our model, however, a dominated firm can choose to make the investment first to become a leader, while a dominant firm voluntarily becomes a follower of the market.

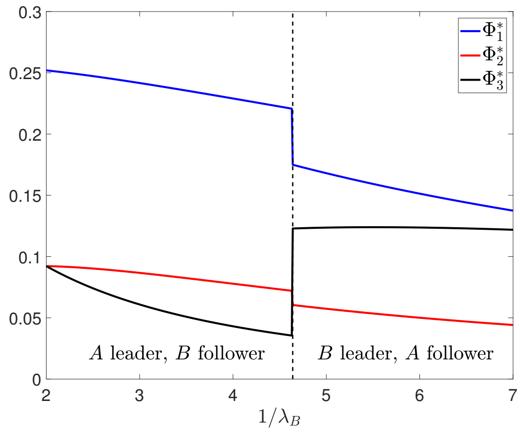
This novelty comes from the disparity of the timing of investment and that of manufacturing. Because of time-to-build and its inherent uncertainty, the product of the follower that makes the investment later can enter the market earlier than the leader's one. A significant advantage of time-to-build weakens the dominant firm's preemptive incentive because it can still expect



(e) Firm values



(f) Consumer and producer surpluses



(g) State prices of *Cases 1, 2, and 3*

Figure 3: Comparative statics regarding the expected time-to-build in a duopoly market with the firms' optimal investment strategies

its product to enter the market first even if it makes the investment later. This can be seen from Figure 3g which presents the state prices of *Cases 1, 2, and 3*. After the dominant firm becomes the follower, there is a significant increase of the state price of *Case 3* in which the follower's product enters earlier than the leader's one. Given the weak preemption incentive of the dominant firm, the dominated firm makes the investment first, hoping that its earlier investment leads to earlier entry of its product in the market. It is natural that after the dominated firm becomes the leader, there is a significant decrease of the state prices of *Cases 1 and 2* in which the leader's product enters market prior to the follower's one .

Recent years have witnessed a number of studies that focus on a firm's incentive to invest late. These arguments are mostly based on the availability of new technology that enables a second-mover to differentiate goods (e.g., Dutta et al. (1995), Hoppe and Lehmann-Grube (2001)) or spillover of innovation that allows a second-mover to save investment costs (e.g., Hoppe (2000), Femminis and Martini (2011)). Namely, the traditional argument involves either product innovation or process innovation. In contrast, our model shows a firm's incentive to invest late in the presence of competition without any innovation or positive externalities. To be more specific, the dominant firm (i.e., type A firm) has an incentive to invest late is not because there is a second-mover advantage in the market but because it has a relative advantage of investment lags which allows a second-mover to enjoy a first-mover advantage.

This result is of special interests not only from the theoretical perspective but also from empirical aspects because it is consistent with what we observe in electric vehicles market. In general, it is said that an electric car is much easier to build than a conventional car with an internal combustion engine.[2] However, the biggest car makers in the industry, which have much experience of mass production, have not put much efforts to enter the electric car market. There are a number of electric cars in the market now, but they have to be capable of 100km/h and have a range of more than 300km for a single charge with affordable price to penetrate mass markets. There are only two electric cars produced by top 5 car makers in the industry that satisfy these conditions,[3] and the major car companies have announced their plans to produce more electric cars in years to come.[4] In this sense, we can say that the dominant car makers which have a relatively shorter time-to-build have not made a preemptive investment in electric cars, timing for their optimal investment in electric car market.

While the major car companies were timing their investments, Tesla, a relatively new player in car business, has made a huge amount of investment to preempt the market, including the construction of gigantic battery factories and the installation of charging stations all over the world. In spite of its ambitious vision and preemptive investment, Tesla has been struggling with the mass production of Model 3, which is their first model targeted toward the mass market.[5] In general, it is said that this was mostly because of their lack of experience in mass production, and it will take more time to mass produce their cars as they initially planned. This corresponds to a relatively longer time-to-build of the dominated firm in our model.

[2] The first practical production electric car with high-capacity rechargeable batteries was invented by Thomas Parker in 1884, which was two years earlier than the invention of the first commercial production of motor vehicles with internal combustion engines by Carl Benz. The first mass-produced electric vehicle was General Motors' EV1 introduced in 1996, but they ceased the production in 1999.

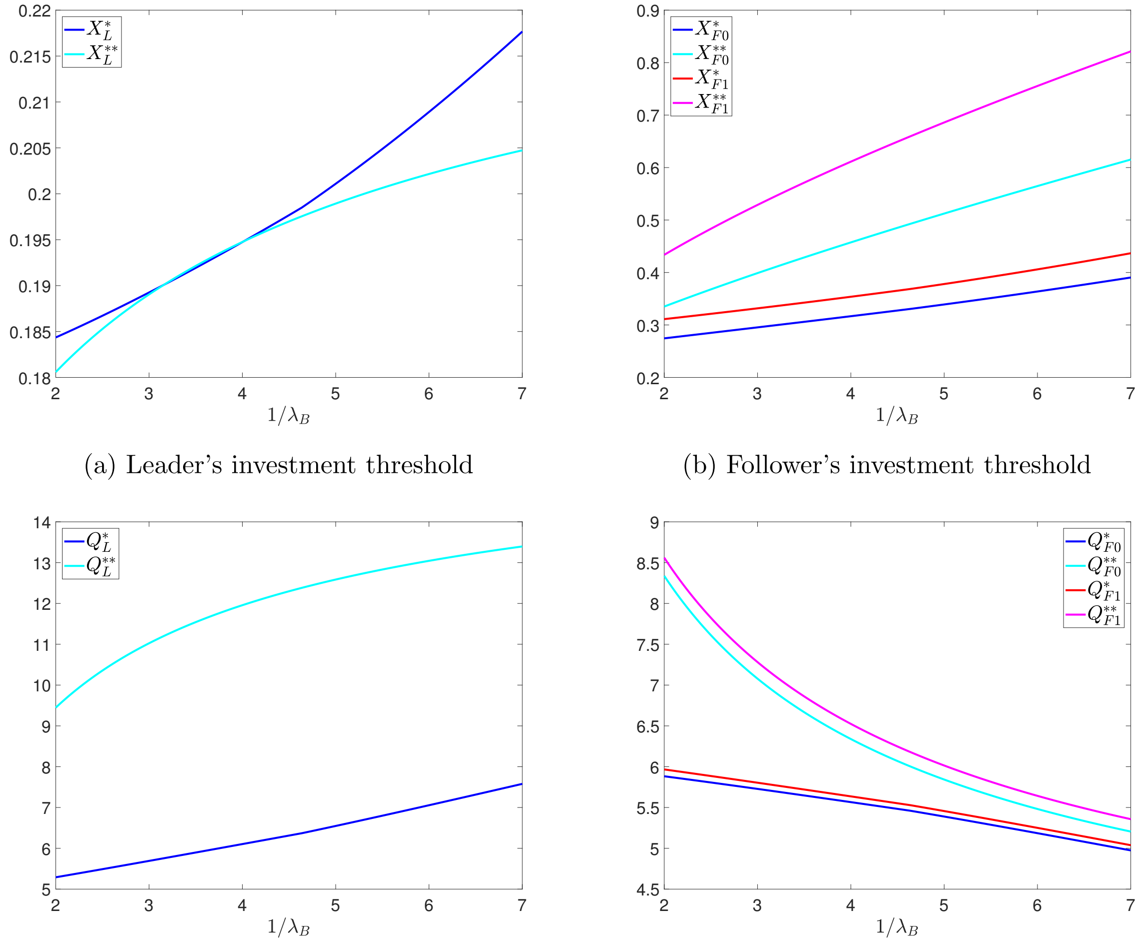
[3] According to Organisation Internationale des Constructeurs d'Automobiles (OICA), the top 10 car manufacturers in 2016 by their production volumes are as follows: 1. Toyota, 2. Volkswagen group, 3. Hyundai/Kia, 4. General Motors, 5. Ford, 6. Nissan, 7. Honda, 8. Fiat Chrysler Automobiles, 9. Renault, 10. Groupe PSA. Among the cars produced by the top 5 companies, Kona Electric by Hyundai and Chevrolet Bolt EV by General Moters are the only ones that satisfy the conditions. If we consider the ones from the top 10 makers, we have two more cars that meet the conditions; Leaf by Nissan and Zoe by Renault.

[4] Toyota announced a plan to completely phase out vehicles that function with internal combustion engine alone by 2025 and to sell more than 5.5 million electrified vehicles by 2030, but less than 20% of them are planned to be fully-electric zero-emission vehicles. Volkswagen group also announced a plan to sell more than 3 million electric vehicles a year in 2025.

[5] Tesla's goal was to produce 1,500 cars of Model 3 in the third quarter of 2017, but only 260 cars were made at that period of time. They were able to meet the original production plan from the third quarter of 2018, but there have always been issues of safety and quality control regarding the cars produced by Tesla. For instance, it turned out that they skipped "brake-and-roll" test to meet the production goal of Model 3, which led to more than a 7% decrease of stock price on July 3, 2018.

4.2 Welfare-maximizing investment policy

In this subsection, we present the results of comparative statics regarding time-to-build with welfare-maximizing investment policy and compare them with those determined in the market provided in Section 4.1.



(a) Leader's investment threshold

(b) Follower's investment threshold

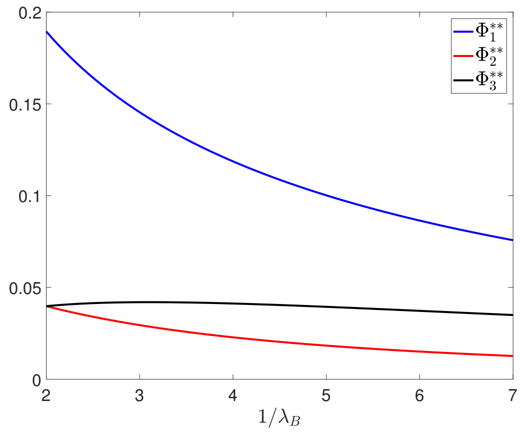
(c) Leader's investment capacity

(d) Follower's investment capacity

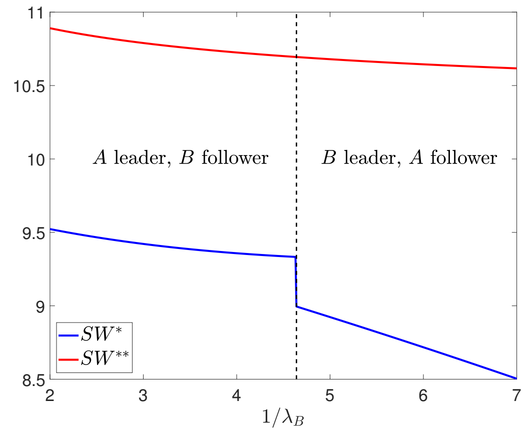
Figure 4: Comparative statics regarding the expected time-to-build in a duopoly market with welfare-maximizing policy

First of all, the welfare-maximizing investment policy always assigns the roles of the leader and the follower in the market to the dominant firm (i.e., type A firm) and the dominated firm (i.e., type B firm), respectively, regardless of the degree of asymmetry in time-to-build. There is a distinct difference between this result and that from the previous subsection in which the firms' endogenous roles change depending on their relative time-to-build. This result is intuitive because it is socially efficient to have products in the market as early as possible.

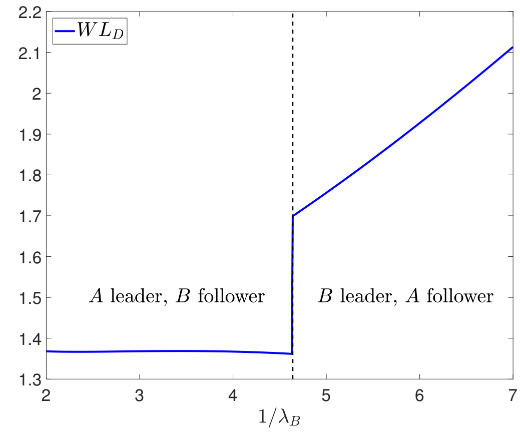
We can see from Figures 4a and 4b that the leader's socially optimal investment is made earlier than that endogenously determined in the market (i.e., $X_L^{**} \leq X_L^*$), while the follower's socially optimal investment is made later than that endogenously determined in the market regardless of the leader's manufacturing process (i.e., $X_{F0}^{**} > X_{F0}^*$ and $X_{F1}^{**} > X_{F1}^*$). In other words, the investments of the leader and the follower in the market are made inefficiently late



(e) State prices of *Cases 1, 2, and 3*



(f) Social welfare



(g) Welfare loss

Figure 4: Comparative statics regarding the expected time-to-build in a duopoly market with welfare-maximizing policy

and early, respectively.

This is in sharp contrast with Huisman and Kort (2015) in which the investments of both the leader and the follower in the market are made inefficiently early comparing with those maximizing social welfare. This difference comes from the existence of time-to-build. As mentioned earlier, it is socially efficient to have the products in the market as early as possible, and thus, the social planner decides to make the investment earlier taking the investment lags into account. This result also stands in opposition to that from a monopoly market discussed in Section 2.2 in which the firm's choice of investment timing coincides with that maximizing social welfare. Recall that this coincidence (i.e., $X_M^* = X_M^{**}$) holds in the absence of time-to-build as well (e.g., Huisman and Kort (2015)).

Figures 4c and 4d show that socially optimal capacities of both the leader and the follower are much higher than that determined in the market (i.e., $Q_L^{**} > Q_L^*$, $Q_{F0}^{**} > Q_{F0}^*$, and $Q_{F1}^{**} > Q_{F1}^*$). That is, both firms underproduce their products from the perspective of social welfare, and this is in line with Huisman and Kort (2015). This is an intuitive result, considering that overproduction lowers the price of the products, which harms the surplus of producers. As discussed in Section 2.2, we also have the issue of underproduction in a monopoly market, but the degree of underproduction is quite different. In a monopoly market, the socially optimal capacity is twice of the firm's choice (i.e., $Q_M^{**} = 2Q_M^*$). In a duopoly market, the leader's socially optimal capacity is around twice of that determined in the market (Figure 4c), whereas the difference between the follower's socially optimal capacity and that determined in the market significantly decreases as the dominated firm's time-to-build increases (Figure 4d).

Combining the results given in Figures 4a through 4d, we can conclude that the social planner exploits the relative advantage of the dominant firm to maximize social welfare by putting more emphasis on its investment in terms of both the timing and the capacity, while making the dominated firm's investment play an auxiliary role in the market.

Figure 4f presents the level of social welfare with the firms' optimal investment strategies and the welfare-maximizing policy, and Figure 4g describes the loss of welfare evaluated as the difference between them. We can see that there is a sharp increase in the loss of welfare after the dominated firm (i.e., type B firm) becomes the leader in the market and that the loss increases further as its time-to-build gets longer, which is a natural result.

5 Conclusion

In this paper, we examined how uncertain time-to-build affects the decisions of investment timing and capacity in a duopoly market. We showed that a dominated firm with longer investment lags can get a chance to be a leader in a market equilibrium and that the leader's capacity increases as the dominated firm's time-to-build gets longer, even when the dominated one becomes the leader. This is a novel result in that we showed that a firm with relative disadvantage can become a leader in the market, whereas existing literature has shown that a relative advantage becomes the leader. The novelty can also be found from the fact that we showed a firm's incentive to invest late in the absence of positive externalities or availability of new technology. Our result can explain car companies' investment behavior in electric cars market in which new car makers with longer time-to-build are making aggressive investment, while dominant firms with shorter lags are timing their investment. It is socially efficient to have products in the market as soon as possible, and thus, the social planner exploits shorter investment lags of the dominant firm; the welfare-maximizing policy always assigns the leader's role to the dominant firm and puts more emphasis on its investment in terms of both the timing and the capacity. Comparing with the investments chosen by welfare-maximizing policy, those of the leader and the follower determined in the market are inefficiently late and early, respectively. Last but not least, we clarified that time-to-build makes a more pronounced impact on investment strategies and social welfare in a duopoly market than in a monopoly market.

There remains a number of problems to be explored. We only considered homogeneous goods for competition in a duopoly market, but a competition between differentiated goods in the presence of time-to-build should be tackled in future works. For instance, the adoption of new technology for making products can be considered. Information asymmetry on time-to-build can also change the firms' investment strategies significantly. We only considered the case of all-equity firm in this study for tractability, but introducing debt financing can enrich the discussion by endogenizing the firms' choices of capital structure and default timing. It is to be hoped that this study will serve as a platform to investigate these problems in future works.

A Proofs

A.1 Proof of Proposition 1

Suppose the monopoly firm invests in capacity Q_M when the demand shock is X. The firm value at the timing of investment, denoted by $V_M(X, Q_M)$, can be evaluated as follows:

$$\begin{aligned} V_M(X, Q_M) &= \mathbb{E}\Big[\int_\tau^\infty e^{-rt} Q_M X(t)(1-\eta Q_M)\mathrm{d}t - \delta Q_M \Big| X(0) = X\Big] \\ &= \frac{\lambda Q_M X(1-\eta Q_M)}{(r-\mu)(r+\lambda-\mu)} - \delta Q_M. \end{aligned} \tag{67}$$

Maximizing (67) with respect to Q_M yields the optimal capacity rule in (4).

Meanwhile, the value of the monopoly firm's option to invest, denoted by $U_M(X)$, should satisfy

$$rU_M(X) = \mu X \frac{\mathrm{d}U_M(X)}{\mathrm{d}X} + \frac{1}{2}\sigma^2 X^2 \frac{\mathrm{d}^2 U_M(X)}{\mathrm{d}X^2}, \tag{68}$$

subject to

$$U_M(X_M^*) = V_M(X_M^*, Q_M), \tag{69}$$

$$\frac{\partial U_M(X)}{\partial X}\Big|_{X=X_M^*} = \frac{\partial V_M(X, Q_M)}{\partial X}\Big|_{X=X_M^*}, \tag{70}$$

$$U_M(0) = 0, \tag{71}$$

and a general solution of (68) with the boundary condition (71) is

$$U_M(X) = A_M X^\beta, \tag{72}$$

where β is given by (7). Substituting (4) and (67) into (69) and (70) yields

$$X_M^*(Q_M) = \frac{\beta\delta(r-\mu)(r+\lambda-\mu)}{(\beta-1)\lambda(1-\eta Q_M)}. \tag{73}$$

From (4) and (73), we can obtain the results in Proposition 1.

A.2 Proof of Proposition 2

From (15), we can obtain the value of social welfare in a monopoly market at the timing of investment in Q_M at X, denoted by $SW_M(X, Q_M)$, as follows:

$$SW_M(X, Q_M) = \frac{\lambda Q_M X(2-\eta Q_M)}{2(r-\mu)(r+\lambda-\mu)} - \delta Q_M. \tag{74}$$

Meanwhile, the value of a social planner's option to invest, denoted by $U_S(X)$, needs to satisfy

$$rU_S(X) = \mu X \frac{\mathrm{d}U_S(X)}{\mathrm{d}X} + \frac{1}{2}\sigma^2 X^2 \frac{\mathrm{d}^2 U_S(X)}{\mathrm{d}X^2}, \tag{75}$$

subject to

$$U_S(X_M^{**}) = SW_M(X_M^{**}, Q), \tag{76}$$

$$\frac{\partial U_S(X)}{\partial X}\Big|_{X=X_M^{**}} = \frac{\partial SW_M(X, Q)}{\partial X}\Big|_{X=X_M^{**}}, \tag{77}$$

$$U_S(0) = 0. \tag{78}$$

A straightforward calculation yields the welfare-maximizing policy given by (16) and (17). Given these results, the expected social welfare with welfare-maximizing policy at the initial period is

$$\begin{aligned} SW_M^{**}(X) :=& SW_M(X, X_M^{**}, Q_M^{**}) = \Big(\frac{X}{X_M^{**}}\Big)^{\beta} \frac{2\delta}{\eta(\beta+1)(\beta-1)} \\ =& \frac{2(\beta-1)^{\beta-1}\lambda^{\beta}X^{\beta}}{\eta(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^{\beta}(r+\lambda-\mu)^{\beta}}. \end{aligned} \tag{79}$$

Meanwhile, the expected social welfare with the firms' optimal investment strategies at the initial period is

$$\begin{aligned} SW_M^{*}(X) :=& SW_M(X, X_M^{*}, Q_M^{*}) = \Big(\frac{X}{X_M^{*}}\Big)^{\beta} \frac{3\delta}{2\eta(\beta+1)(\beta-1)} \\ =& \frac{3(\beta-1)^{\beta-1}\lambda^{\beta}X^{\beta}}{2\eta(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^{\beta}(r+\lambda-\mu)^{\beta}}. \end{aligned} \tag{80}$$

Thus, the expected welfare loss at the initial period is

$$\begin{aligned} WL_M(X) &= SW_M^{**}(X) - SW_M^{*}(X) \\ &= \frac{(\beta-1)^{\beta-1}\lambda^{\beta}X^{\beta}}{2\eta(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^{\beta}(r+\lambda-\mu)^{\beta}}. \end{aligned} \tag{81}$$

A.3 Proof of Proposition 3

Suppose type j follower invests in capacity Q_{F1}^j when the demand shock is X. It is natural that the follower's value at the timing of investment depends on X, Q_L^i, and Q_{F1}^j. Denoting the value by $V_{F1}^j(X, Q_L^i, Q_{F1}^j)$, the following holds:

$$\begin{aligned} V_{F1}^j(X, Q_L^i, Q_{F1}^j) &= \mathbb{E}\Big[\int_{\tau_j}^{\infty} e^{-rt} Q_{F1}^j X(t)(1-\eta(Q_L^i + Q_{F1}^j))\mathrm{d}t - \delta_j Q_{F1}^j \Big| X(0) = X\Big] \\ &= \frac{\lambda_j Q_{F1}^j X(1-\eta(Q_L^i + Q_{F1}^j))}{(r-\mu)(r+\lambda_j-\mu)} - \delta Q_{F1}^j. \end{aligned} \tag{82}$$

Maximizing (82) with respect to Q_{F1}^j yields (26).

Meanwhile, the value of the follower's option to invest after the leader's product enters the market, denoted by $U_{F1}^j(X)$, should satisfy

$$rU_{F1}^j(X) = \mu X \frac{\mathrm{d}U_{F1}^j(X)}{\mathrm{d}X} + \frac{1}{2}\sigma^2 X^2 \frac{\mathrm{d}^2 U_{F1}^j(X)}{\mathrm{d}X^2}, \tag{83}$$

subject to

$$U_{F1}^j(X_{F1}^{j*}) = V_{F1}^j(X_{F1}^{j*}, Q_L^i, Q_{F1}^j), \tag{84}$$

$$\frac{\partial U_{F1}^j(X)}{\partial X}\bigg|_{X=X_{F1}^{j*}} = \frac{\partial V_{F1}^j(X, Q_L^i, Q_{F1}^j)}{\partial X}\bigg|_{X=X_{F1}^{j*}}, \tag{85}$$

$$U_{F1}^j(0) = 0. \tag{86}$$

Following the same argument in Proposition 1, we can easily derive the results in Proposition 3.

A.4 Proof of Proposition 4

Suppose type j follower invests in capacity Q^j_{F0} when the demand shock is X. The follower's value at the timing of investment depends on X, Q^i_L, and Q^j_{F0}. Denoting the value by $V^j_{F0}(X, Q^i_L, Q^j_{F0})$, the following holds:

$$\begin{aligned} V^j_{F0}(X, Q^i_L, Q^j_{F0}) &= \mathbb{E}\Big[1_{\{\bar{T}^i_L \le \bar{T}^j_{F0}\}}\Big\{\int_{\bar{T}^j_{F0}}^\infty e^{-rt} Q^j_{F0} X(t)(1-\eta(Q^i_L + Q^j_{F0}))\mathrm{d}t\Big\} \\ &+ 1_{\{\bar{T}^j_{F0} < \bar{T}^i_L\}}\Big\{\int_{\bar{T}^j_{F0}}^{\bar{T}^i_L} e^{-rt} Q^j_{F0} X(t)(1-\eta Q^j_{F0})\mathrm{d}t + \int_{\bar{T}^i_L}^\infty e^{-rt} Q^j_{F0} X(t)(1-\eta(Q^i_L + Q^j_{F0}))\mathrm{d}t\Big\} \\ &- \delta Q^j_{F0}\Big| X(0) = X\Big]. \end{aligned} \tag{87}$$

The first row of (87) is evaluated as follows:

$$\begin{aligned} &\int_0^\infty \Big[\int_{\tau_i}^\infty \Big\{\int_{\tau_j}^\infty e^{-rt} Q^j_{F0} X(t)(1-\eta(Q^i_L + Q^j_{F0}))\mathrm{d}t\Big\}\lambda_j e^{-\lambda_j \tau_j}\mathrm{d}\tau_j\Big]\lambda_i e^{-\lambda_i \tau_i}\mathrm{d}\tau_i \\ &= \int_0^\infty \Big[\frac{Q^j_{F0}(1-\eta(Q^i_L + Q^j_{F0}))\lambda_j X}{r-\mu}\int_{\tau_i}^\infty e^{-(r+\lambda_j-\mu)\tau_j}\mathrm{d}\tau_j\Big]\lambda_i e^{-\lambda_i \tau_i}\mathrm{d}\tau_i \\ &= \frac{Q^j_{F0}(1-\eta(Q^i_L + Q^j_{F0}))\lambda_i\lambda_j X}{(r-\mu)(r+\lambda_j-\mu)}\int_0^\infty e^{-(r+\lambda_i+\lambda_j-\mu)}\mathrm{d}\tau_i \\ &= \frac{Q^j_{F0}(1-\eta(Q^i_L + Q^j_{F0}))\lambda_i\lambda_j X}{(r-\mu)(r+\lambda_j-\mu)(r+\lambda_i+\lambda_j-\mu)}. \end{aligned} \tag{88}$$

Likewise, the second row of (87) is calculated as follows:

$$\frac{Q^j_{F0}\lambda_j X}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big(1-\eta Q^j_{F0} - \frac{\eta\lambda_i Q^i_L}{r+\lambda_i-\mu}\Big). \tag{89}$$

Thus, (87) can be written as follows:

$$\begin{aligned} V^j_{F0}(X, Q^i_L, Q^j_{F0}) = &\frac{Q^j_{F0}(1-\eta(Q^i_L + Q^j_{F0}))\lambda_i\lambda_j X}{(r-\mu)(r+\lambda_j-\mu)(r+\lambda_i+\lambda_j-\mu)} \\ &+ \frac{Q^j_{F0}\lambda_j X}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big(1-\eta Q^j_{F0} - \frac{\eta\lambda_i Q^i_L}{r+\lambda_i-\mu}\Big) - \delta Q^j_{F0}. \end{aligned} \tag{90}$$

Maximizing (90) with respect to Q^j_{F0} yields $Q^{j*}_{F0}(X, Q^i_L)$, the follower's optimal capacity given demand shock X and the leader's capacity Q^i_L, in (32).

Meanwhile, the value of the follower's option to invest before the leader's product enters the market, denoted by $U^j_{F0}(X)$, should satisfy

$$rU^j_{F0}(X) = \mu X \frac{\mathrm{d}U^j_{F0}(X)}{\mathrm{d}X} + \frac{1}{2}\sigma^2 X^2 \frac{\mathrm{d}^2 U^j_{F0}(X)}{\mathrm{d}X^2} + \lambda_i\{U^j_{F1}(X) - U^j_{F0}(X)\}, \tag{91}$$

subject to

$$U^j_{F0}(X^{j*}_{F0}) = V^j_{F0}(X^{j*}_{F0}, Q^i_L, Q^j_{F0}), \tag{92}$$

$$\frac{\partial U^j_{F0}(X)}{\partial X}\Big|_{X=X^{j*}_{F0}} = \frac{\partial V^j_{F0}(X, Q^i_L, Q^j_{F0})}{\partial X}\Big|_{X=X^{j*}_{F0}}, \tag{93}$$

$$U^j_{F0}(0) = 0. \tag{94}$$

A general solution of (91) with the boundary condition (94) is

$$U^j_{F0}(X) = U^j_{F1}(X) + A^j_{F0} X^{\beta_i}. \tag{95}$$

By substituting (32) and (90) into (92) and (93), we can obtain the results given in Proposition 4.

A.5 Proof of Proposition 5

Suppose the follower's investment strategies are given by Q^j_{F0}, Q^j_{F1}, X^j_{F0}, and X^j_{F1}. Denoting the demand shock at the timing of the leader's investment by X^i_L, the first row of (39) is calculated as follows:

$$\begin{aligned}
&\int_0^\infty \Big[\int_0^{T^j_{F0}-T^i_L} \Big\{\int_{\tau_i}^{T^j_{F1}+\tau_j-T^i_L} e^{-rt} Q^i_L X(t)(1-\eta Q^i_L)\mathrm{d}t \\
&\quad + \int_{T^j_{F1}+\tau_j-T^i_L}^\infty e^{-rt} Q^i_L X(t)(1-\eta(Q^i_L+Q^j_{F1}))\mathrm{d}t\Big\}\lambda_i e^{-\lambda_i\tau_i}\mathrm{d}\tau_i\Big]\lambda_j e^{-\lambda_j\tau_j}\mathrm{d}\tau_j \\
&= \int_0^\infty \Big[\int_0^{T^j_{F0}-T^i_L} \Big\{\frac{Q^i_L(1-\eta Q^i_L)X^i_L}{r-\mu} e^{-(r-\mu)\tau_i} - e^{-r(T^j_{F1}-T^i_L)}\frac{\eta Q^i_L Q^j_{F1} X^j_{F1}}{r-\mu} e^{-(r-\mu)\tau_j}\Big\} \\
&\quad \times \lambda_i e^{-\lambda_i\tau_i}\mathrm{d}\tau_i\Big]\lambda_j e^{-\lambda_j\tau_j}\mathrm{d}\tau_j \\
&= \int_0^\infty \Big[\frac{Q^i_L(1-\eta Q^i_L)\lambda_i X^i_L}{(r-\mu)(r+\lambda_i-\mu)}\Big(1-e^{-(r+\lambda_i-\mu)(T^j_{F0}-T^i_L)}\Big) \\
&\quad - e^{-r(T^j_{F1}-T^i_L)}\frac{\eta Q^i_L Q^j_{F1} X^j_{F1}}{r-\mu} e^{-(r-\mu)\tau_j}\Big(1-e^{-\lambda_i(T^j_{F0}-T^i_L)}\Big)\Big]\lambda_j e^{-\lambda_j\tau_j}\mathrm{d}\tau_j \\
&= \frac{Q^i_L(1-\eta Q^i_L)\lambda_i}{(r-\mu)(r+\lambda_i-\mu)}\Big(X^i_L - e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)} X^j_{F0}\Big) \\
&\quad - \frac{\eta Q^i_L Q^j_{F1}\lambda_j X^j_{F1}}{(r-\mu)(r+\lambda_j-\mu)}\Big(e^{-r(T^j_{F1}-T^i_L)} - e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)} e^{-r(T^j_{F1}-T^j_{F0})}\Big).
\end{aligned} \tag{96}$$

Likewise, the second row of (39) can be expressed as follows:

$$\frac{Q^i_L(1-\eta Q^i_L)\lambda_i X^j_{F0}}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)} e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)} - \frac{\eta Q^i_L Q^j_{F0}\lambda_i\lambda_j X^j_{F0}}{(r-\mu)(r+\lambda_j-\mu)(r+\lambda_i+\lambda_j-\mu)} e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)}, \tag{97}$$

and the third row of (39) is

$$\frac{Q^i_L(1-\eta(Q^i_L+Q^j_{F0}))\lambda_i\lambda_j X^j_{F0}}{(r-\mu)(r+\lambda_i-\mu)(r+\lambda_i+\lambda_j-\mu)} e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)}. \tag{98}$$

The sum of (96), (97), and (98) is

$$\begin{aligned}
&\frac{Q^i_L(1-\eta Q^i_L)\lambda_i X^i_L}{(r-\mu)(r+\lambda_i-\mu)} - e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)}\frac{\eta Q^i_L Q^j_{F0}\lambda_i\lambda_j X^j_{F0}}{(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big(\frac{1}{r+\lambda_i-\mu}+\frac{1}{r+\lambda_j-\mu}\Big) \\
&\quad - \Big(e^{-r(T^j_{F1}-T^i_L)} - e^{-(r+\lambda_i)(T^j_{F0}-T^i_L)} e^{-r(T^j_{F1}-T^j_{F0})}\Big)\frac{\eta Q^i_L Q^j_{F1}\lambda_j X^j_{F1}}{(r-\mu)(r+\lambda_j-\mu)},
\end{aligned} \tag{99}$$

which amounts to the optimization problems of (40) and (43) with (41).

A.6 Proof of Proposition 6

$X_P^i \neq \emptyset$ for $i \in \{A, B\}$ implies that $V_P^i(X) \geq V_{F0}^i(X, Q_L^{j*}(X_P^j))$ for some $X > 0$. Namely, there exist the levels of demand shock at which each firm prefers to be a leader in spite of earlier investment.

Suppose $X_P^{i*} \leq X_P^{j*} \leq X_L^{i*}$ for $i \neq j$. When the demand shock reaches X_P^{i*}, type i firm's value as a leader exceeds that as a follower, while it is not sufficient enough for type j firm to bear the cost of earlier investment. Type i firm, however, does not invest at X_P^{i*} at which it is indifferent between being a leader and being a follower; its competitor, type j firm, will not invest until the demand shock reaches X_P^{j*}, and thus, type i firm makes the investment at $X_P^{j*} - \epsilon$, which is closer to its optimal investment threshold X_L^{i*} than X_P^{i*} is. It is needless to say that the investment is made at X_P^{j*} as ϵ approaches 0. Following the investment rule of (40), the capacity is chosen as $Q_P^{i*} := Q_L^{i*}(X_P^{j*})$. Given type i firm's investment in Q_P^{i*} at X_P^{j*}, type j firm has no choice but to be a follower, and it makes the investment following Propositions 3 and 4.

If $X_P^{i*} \leq X_P^{j*}$ and $X_L^{i*} \leq X_P^{j*}$, there is no reason for type i firm to deviate from its optimal investment strategies because type j firm will not make the preemptive investment until the demand shock reaches X_P^{j*}, which is higher than X_L^{i*}. Thus, the firms' investment strategies follow the ones in Propositions 3 through 5.

$X_P^i \neq \emptyset$ and $X_P^j = \emptyset$ for $i \neq j$ imply that $V_P^i(X) \geq V_{F0}^i(X, Q_L^{j*}(X_L^{j*}))$ for some $X > 0$, while $V_P^j(X) < V_{F0}^j(X, Q_L^{i*}(X_P^i))$ for all $X > 0$. Namely, only type i firm has an incentive to preempt the market; type j firm has no incentive to become a leader since its value as a follower is always higher than that as a leader. Thus, type i does not deviate from its optimal investment strategies given in Proposition 5, and type j firm, as a follower, makes the investment following Propositions 3 and 4.

A.7 Proof of Proposition 7

Suppose that type i and j firms' roles are given as a leader and a follower, respectively. Given the leader's products in the market, the social planner solves the following problem:

$$SW_{D,F1}^{ij}(X) = \max_{T_{F1}^j \geq 0, Q_{F1}^j \geq 0} \mathbb{E}\Big[\int_0^{\bar{T}_{F1}^j} e^{-rt} \frac{Q_L^i(2 - \eta Q_L^i)X(t)}{2} \mathrm{d}t$$
$$+ \int_{\bar{T}_{F1}^j}^{\infty} e^{-rt} \frac{(Q_L^i + Q_{F1}^j)\big(2 - \eta(Q_L^i + Q_{F1}^j)\big)X(t)}{2} \mathrm{d}t - e^{-rT_{F1}^j}\delta Q_{F1}^j \Big| X(0) = X\Big]. \tag{100}$$

At the timing of investment given the demand shock X, (100) is evaluated as follows:

$$\frac{Q_L^i(2 - \eta Q_L^i)X}{2(r + \lambda_j - \mu)} + \frac{(Q_L^i + Q_{F1}^j)\big(2 - \eta(Q_L^i + Q_{F1}^j)\big)\lambda_j X}{2(r - \mu)(r + \lambda_j - \mu)} - \delta Q_{F1}^j. \tag{101}$$

The first-order condition yields type j follower's welfare-maximizing capacity corresponding to the demand shock X and type i leader's capacity Q_L^i as follows:

$$Q_{F1}^{j**}(X, Q_L^i) = \frac{1}{\eta}\Big(1 - \eta Q_L^i - \frac{\delta(r - \mu)(r + \lambda_j - \mu)}{\lambda_j X}\Big). \tag{102}$$

Following the standard argument of value-matching and smooth-pasting conditions, we can easily derive the welfare-maximizing threshold $X_{F1}^{j**}(Q_L^i)$ as (57), and by substituting (57) to (102), we can obtain the welfare-maximizing capacity $Q_{F1}^{j**}(Q_L^i)$ in (58).

If the leader's product has not entered the market yet, the social planner's optimization problem is written as follows:

$$\begin{aligned}
SW_{D,F0}^{ij}(X) = &\max_{T_{F0}^j \geq 0, Q_{F0}^j \geq 0} \mathbb{E}\Bigg[1_{\{T_{F0}^j < \bar{T}_L^i \leq \bar{T}_{F0}^j\}}\Big\{\int_{\bar{T}_L^i}^{\bar{T}_{F0}^j} e^{-rt}\frac{Q_L^i(2-\eta Q_L^i)X(t)}{2}\mathrm{d}t \\
&+ \int_{\bar{T}_{F0}^j}^{\infty} e^{-rt}\frac{(Q_L^i+Q_{F0}^j)\big(2-\eta(Q_L^i+Q_{F0}^j)\big)X(t)}{2}\mathrm{d}t - e^{-rT_{F0}^j}\delta Q_{F0}^j\Big\} \\
&+ 1_{\{\bar{T}_{F0}^j < \bar{T}_L^i\}}\Big\{\int_{\bar{T}_{F0}^j}^{\bar{T}_L^i} e^{-rt}\frac{Q_{F0}^j(2-\eta Q_{F0}^j)X(t)}{2}\mathrm{d}t + \int_{\bar{T}_L}^{\infty}\frac{(Q_L^i+Q_{F0}^j)\big(2-\eta(Q_L^i+Q_{F0}^j)\big)X(t)}{2}\mathrm{d}t \\
&- e^{-rT_{F0}^j}\delta Q_{F0}^j\Big\} + 1_{\{\bar{T}_L^i \leq T_{F0}^j\}} e^{-r\bar{T}_L^i} SW_{D,F1}^{ij}(X_{\bar{T}_L^i})\Big| X(0)=X\Bigg] \qquad (103)
\end{aligned}$$

A tedious algebra similar with the one in Appendix A.4 shows that at the timing of investment with the demand shock X, (103) can be written as follows:

$$\begin{aligned}
&\frac{\lambda_i X}{2(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big(Q_L^i(2-\eta Q_L^i) + \frac{Q_{F0}^j(2-\eta Q_{F0}^j - 2\eta Q_L^i)\lambda_j}{r+\lambda_j-\mu}\Big) \\
&\quad + \frac{\lambda_j X}{2(r-\mu)(r+\lambda_i+\lambda_j-\mu)}\Big(Q_{F0}^j(2-\eta Q_{F0}^j) + \frac{Q_L^i(2-\eta Q_L^i - 2\eta Q_{F0}^j)\lambda_i}{r+\lambda_i-\mu}\Big) - \delta Q_{F0}^j. \qquad (104)
\end{aligned}$$

By first-order condition of (104) and value-matching and smooth-pasting conditions, we can derive (59) and (60) from which we can obtain type j follower's welfare-maximizing investment strategies before the leader's product enters the market: $X_{F0}^{j**}(Q_L^i)$ and $Q_{F0}^{j**}(Q_L^i)$.

Lastly, the social planner solves the following problem with regard to the leader's investment:

$$\begin{aligned}
SW_{D,L}^{ij}(X) = &\max_{T_L^i \geq 0, Q_L^i \geq 0} \mathbb{E}\Bigg[1_{\{\bar{T}_L^i < T_{F0}^{j**}\}}\Big\{\int_{\bar{T}_L^i}^{\bar{T}_{F1}^{j**}} e^{-rt}\frac{Q_L^i(2-\eta Q_L^i)X(t)}{2}\mathrm{d}t \\
&+ \int_{\bar{T}_{F1}^{j**}}^{\infty} e^{-rt}\frac{(Q_L^i+Q_{F1}^{j**}(Q_L^i))\{2-\eta(Q_L^i+Q_{F1}^{j**}(Q_L^i))\}X(t)}{2}\mathrm{d}t - e^{-rT_{F1}^{j**}}\delta Q_{F1}^{j**}(Q_L^i)\Big\} \\
&+ 1_{\{T_{F0}^{j**} \leq \bar{T}_L^i < \bar{T}_{F0}^{j**}\}}\Big\{\int_{\bar{T}_L^i}^{\bar{T}_{F0}^{j**}} e^{-rt}\frac{Q_L^i(2-\eta Q_L^i)X(t)}{2}\mathrm{d}t \\
&+ \int_{\bar{T}_{F0}^{j**}}^{\infty} e^{-rt}\frac{(Q_L^i+Q_{F0}^{j**}(Q_L^i))\{2-\eta(Q_L^i+Q_{F0}^{j**}(Q_L^i))\}X(t)}{2}\mathrm{d}t - e^{-rT_{F0}^{**}}\delta Q_{F0}^{j**}(Q_L^i)\Big\} \\
&+ 1_{\{\bar{T}_{F0}^{j**} \leq \bar{T}_L^i\}}\Big\{\int_{\bar{T}_{F0}^{j**}}^{\bar{T}_L^i} e^{-rt}\frac{Q_{F0}^{j**}(Q_L^i)(2-\eta Q_{F0}^{j**}(Q_L^i))X(t)}{2}\mathrm{d}t \\
&+ \int_{\bar{T}_L^i}^{\infty} e^{-rt}\frac{(Q_L^i+Q_{F0}^{j**}(Q_L^i))\{2-\eta(Q_L^i+Q_{F0}^{j**}(Q_L^i))\}X(t)}{2}\mathrm{d}t - e^{-rT_{F0}^{j**}}\delta Q_{F0}^{j**}\Big\} - e^{-rT_L^i}\delta Q_L^i\Big| X(0)=X\Bigg].
\end{aligned}$$

(105)

A tedious algebra shows that (105) can be written as follows:

$$
\begin{aligned}
SW_{D,L}^{ij}(X) &= \max_{X_L^i \geq 0, Q_L^i \geq 0} \Big(\frac{X}{X_L^i}\Big)^{\beta}\Big[\frac{Q_L^i(2-\eta Q_L^i)\lambda_i X_L^i}{2(r-\mu)(r+\lambda_i-\mu)} + \Big(\frac{X_L^i}{X_{F0}^{j**}(Q_L^i)}\Big)^{\beta_i}\Big[\frac{X_{F0}^{j**}(Q_L^i)}{2(r-\mu)}\Big\{-\frac{Q_L^i(2-\eta Q_L^i)\lambda_i}{r+\lambda_i-\mu} \\
&+ \frac{\lambda_i}{r+\lambda_i+\lambda_j-\mu}\Big(Q_L^i(2-\eta Q_L^i) + \frac{Q_{F0}^{j**}(Q_L^i)\{2-\eta Q_{F0}^{j**}(Q_L^i) - 2\eta Q_L^i\}\lambda_j}{r+\lambda_j-\mu}\Big) \\
&+ \frac{\lambda_j}{r+\lambda_i+\lambda_j-\mu}\Big(Q_{F0}^{j**}(Q_L^i)(2-\eta Q_{F0}^{j**}(Q_L^i)) + \frac{Q_L^i\{2-\eta Q_L^i - 2\eta Q_{F0}^{j**}(Q_L^i)\}\lambda_i}{r+\lambda_i-\mu}\Big)\Big\} - \delta Q_{F0}^{j**}(Q_L^i)\Big] \\
&+ \Big\{\Big(\frac{X_L^i}{X_{F1}^{j**}(Q_L^i)}\Big)^{\beta} - \Big(\frac{X_L^i}{X_{F0}^{j**}(Q_L^i)}\Big)^{\beta_i}\Big(\frac{X_{F0}^{j**}(Q_L^i)}{X_{F1}^{j**}(Q_L^i)}\Big)^{\beta}\Big\} \\
&\times \Big(\frac{Q_{F1}^{j**}(Q_L^i)\{2-\eta Q_{F1}^{j**}(Q_L^i) - 2\eta Q_L^i\}\lambda_j X_{F1}^{j**}(Q_L^i)}{2(r-\mu)(r+\lambda_j-\mu)} - \delta Q_{F1}^{j**}(Q_L^i)\Big) - \delta Q_L^i\Big] \qquad (106) \\
&= \max_{X_L^i \geq 0, Q_L^i \geq 0} \Big(\frac{X}{X_L^i}\Big)^{\beta}\Big[SW_{D,F0}^{ij}(X_L^i) - \delta Q_L^i\Big]. \qquad (107)
\end{aligned}
$$

Thus, type i leader's investment strategies that maximize social welfare are X_L^{i**} and Q_L^{i**} which can be derived from (61) and (63) with (62).

References

Agliardi, E. and Koussis, N. (2013), 'Optimal capital structure and the impact of time-to-build', *Finance Research Letters* **10**, 124–130.

Amir, R. and Stepanova, A. (2006), 'Second-mover advantage and price leadership in Bertrand duopoly', *Games and Economic Behavior* **55**, 1–20.

Bar-Ilan, A. and Strange, W. (1996), 'Investment lags', *American Economic Review* **86**, 610–622.

Bar-Ilan, A. and Strange, W. (1998), 'A model of sequential investment', *Journal of Economic Dynamics and Control* **22**, 437–463.

Bouis, R., Huisman, K. and Kort, P. (2009), 'Investment in oligopoly under uncertainty: The accordion effect', *International Journal of Industrial Organization* **27**, 320–331.

Casadesus-Masanell, R. and Ghemawat, P. (2006), 'Dynamic mixed duopoly: A model motivated by Linux vs. Windows', *Management Science* **52**, 1072–1084.

Décamps, J. and Mariotti, T. (2004), 'Investment timing and learning externalities', *Journal of Economic Theory* **118**, 80–102.

Dutta, P., Lach, S. and Rustichini, A. (1995), 'Better late than early: vertical differentiation in the adoption of a new technology', *Journal of Economics and Management Strategy* **4**, 563–589.

Femminis, G. and Martini, G. (2011), 'Irreversible investment and R&D spillovers in a dynamic duopoly', *Journal of Economic Dynamics and Control* **35**, 1061–1090.

Grenadier, S. (1996), 'The strategic exercise of options: Development cascades and overbuilding in real estate markets', *Journal of Finance* **51**, 1653–1679.

Hoppe, H. (2000), 'Second-mover advantages in the strategic adoption of new technology under uncertainty', *International Journal of Industrial Organization* **18**, 315–338.

Hoppe, H. and Lehmann-Grube, U. (2001), 'Second-mover advantages in dynamic quality competition', *Journal of Economics and Management Strategy* **10**, 419–433.

Huisman, K. (2001), *Technology investment: a game theoretic real options approach*, Kluwer Academic Publishers, Dordrecht.

Huisman, K. and Kort, P. (2004), 'Strategic technology adoption taking into account future technological improvement: A real options approach', *European Journal of Operational Research* **159**, 705–728.

Huisman, K. and Kort, P. (2015), 'Strategic capacity investment under uncertainty', *RAND Journal of Economics* **46**, 376–408.

Jeon, H. (2018), Investment and financing decisions in the presence of time-to-build. Working paper.

Kong, J. and Kwok, Y. (2007), 'Real options in strategic investment games between two asymmetric firms', *European Journal of Operational Research* **181**, 967–985.

Lambrecht, B. and Perraudin, W. (2003), 'Real options and preemption under incomplete information', *Journal of Economic Dynamics and Control* **27**, 619–643.

Majd, S. and Pindyck, R. (1987), 'Time to build, option value, and investment decisions', *Journal of Financial Economics* **18**, 7–27.

Mason, R. and Weeds, H. (2010), 'Investment, uncertainty and pre-emption', *International Journal of Industrial Organization* **28**, 278–287.

Nielsen, M. (2002), 'Competition and irreversible investments', *International Journal of Industrial Organization* **20**, 731–743.

Pacheco-de-Almeida, G. and Zemsky, P. (2003), 'The effect of time-to-build on strategic investment under uncertainty', *RAND Journal of Economics* **34**, 166–182.

Pawlina, G. and Kort, P. (2006), 'Real options in an asymmetry duopoly: Who benefits from your competitive disadvantage?', *Journal of Economics and Management Strategy* **15**, 1–35.

Sarkar, S. and Zhang, C. (2015), 'Investment policy with time-to-build', *Journal of Banking and Finance* **55**, 142–156.

Thijssen, J. (2010), 'Preemption in a real option game with a first mover advantage and player-specific uncertainty', *Journal of Economic Theory* **145**, 2448–2462.

Thijssen, J., Huisman, K. and Kort, P. (2012), 'Symmetric equilibrium strategies in game theoretic real option models', *Journal of Mathematical Economics* **48**, 219–225.

Tsyplakov, S. (2008), 'Investment frictions and leverage dynamics', *Journal of Financial Economics* **89**, 423–443.

Weeds, H. (2002), 'Strategic delay in a real options model of R&D competition', *Review of Economic Studies* **69**, 729–747.