Financial Policies and Internal Governance with Heterogeneous Risk Preferences *

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Abstract

We consider a group of investors with heterogeneous risk preferences that determines a firm's investment policy, and each investor's compensation function. The optimal investment policy is a time-varying weighted average of investors' optimal policies and converges to the policy of the least (most) risk averse investor in booms (busts), reconciling the diversification of opinions hypothesis and the group shift hypothesis. The most (least) risk averse investor has a strictly concave (convex) claim on the firm's net worth. For intermediate risk preferences investors' claim is S-shaped, resembling preferred stock. We derive investors' utility weights absent wealth distribution and under social optimization.

Keywords: group decisions, investment, payout, risk preference, governance

1 Introduction

This paper studies corporate policies and internal governance when decisions are made by a group with heterogeneous risk preferences. Standard corporate finance models assume that firms are run by an owner-manager, or by an agent (manager, CEO) who acts on behalf of the principal (owner). In reality, corporate policies are often decided by a group of managers or investors, all of which exert some influence on the decisions.

Experimental studies show that group decisions are significantly different from individuals when faced with identical information about uncertain outcomes. An experimental study by Shupp and Williams (2007) reveals that the variance of risk preferences is generally smaller for groups than individuals. Furthermore, the average group is more risk averse than the average individual in high-risk situations, but groups tend to be less risk averse in low-risk situations. An experimental study by Ambrus, Greiner, and Pathak (2015) finds that members have different impact on the group decision making process with the median member, and those closest to the median, having the most significant impact.

The literature offers two competing hypotheses for group decisions. The group shift hypothesis (e.g., Moscovici and Zavalloni (1969) ; Kerr (1992)) suggests that the opinion of team members shifts towards the opinion of the dominant person in a team. As that person typically holds very pronounced opinions, a team eventually gravitates towards extremes. Consequently, teams make more extreme decisions than individuals do. In contrast, the diversification of opinions hypothesis suggests that the team opinion is the average opinion of the team members (e.g., Sah and Stiglitz (1986, 1988)). Extreme opinions of members in a team are averaged out and teams eventually make less extreme decisions than individuals do. Although the diversifications of opinions hypothesis appears to enjoy strong empirical support (see Bar, Kempf and Ruenzi (2010)), it struggles to explain the puzzling observation that the average group is more (less) risk averse than the average individual in high (low) risk situations.

Chava and Purnanandam (2010) find that managerial risk-preferences influence corporate decisions in significant ways, with CFOs exerting a stronger influence on some finer aspects of corporate financial policy (such as the debt maturity choice and earnings smoothing through accrual management) than CEOs. Graham, Harvey and Puri (2013) report that more risk-tolerant CEOs make more acquisitions. They also find an empirical link between managerial risk-aversion with compensation structure, and pay-performance sensitivity.

Despite the clear evidence that corporate policies are the result of group decision making, there is no theoretical model that explains this decision making process for groups with heterogeneous risk preferences.[1] Such a theory is also relevant for our understanding of venture capital groups, partnerships, financing syndicates and fund management teams. As Rajan (2012) points out, internal governance is an under-researched area in finance, and the debate regarding the boundaries of the firm becomes more interesting when one considers the firm's activities involving many collaborators.

This paper develops a dynamic, open-horizon model of a firm's investment, financing and payout policies when these decisions are made by a group of investors (or managers) with heterogeneous risk preferences. Investment in risky assets (projects) and payouts are made in continuous time. Our model focuses on Pareto optimal policies, which implies that investors maximize their joint, weighted life-time utility from the payout they receive from the firm. The investors' utility weights are endogenously determined when they start up the firm, and depend on the capital contributed by each investor. The utility weights determine the firm's internal governance.

[1]Risk preferences are known to vary widely across the population (see for instance Robert B. Barsky et. al. (1997)). The most commonly accepted measures of the coefficient of relative risk aversion lie between 1 and 3, but there is a wide range of estimates in the literature, from as low as 0.2 to 10 and higher (See Chetty (2006) , Campo et al. (2011), among others).

We consider a frictionless environment where there are no operational synergies from combining assets (e.g. there are constant returns to scale), assets are perfectly divisible, and individuals have the same investment opportunity set as groups. We assume that investors must agree on one single investment policy and capital structure for the firm, but investors can agree to tailor payouts (or compensations) to the preferences of individual group members.

We show that the optimal investment policy for the group is a weighted average of each individual investor's optimal policy, consistent with the diversification of opinions hypothesis. However, the weights vary with the firm's net worth. As net worth increases (decreases) in good (bad) times increasingly more weight is put on the least (most) risk averse investor, consistent with the group shift hypothesis. Consequently, the fraction of the firm's net worth invested in risky projects is not constant, but increases (decreases) in good (bad) times. Leverage is procyclical as the firm's net debt ratio rises in good times, and falls in bad times. Firms dynamically delever towards lower net debt ratios as losses accumulate. Only when all investors have the same coefficient of risk aversion does the firm maintain a constant net debt ratio. By dynamically rebalancing assets and liabilities in response to income shocks, debt is kept safe at all times.

We find that the payout to the least (most) risk averse investor is a convex (concave) function of the firm's net worth. For investors with intermediate levels of risk aversion, payout is S-shaped in net worth (i.e. convex for low levels of net worth, and concave for high levels). Payouts converge to a fixed cap as net worth increases, except for the least risk averse investor whose payout becomes arbitrarily large as net worth goes to infinity. We show that the claim on the firm's assets of the least risk averse investor is similar to a common equity contract. The claims of investors with higher levels of risk aversion resemble preferred stock with more risk averse investors having a higher seniority but lower maximum payout. Only when all investors have the same coefficient of risk aversion do we obtain payouts and claims that are linear in the firms assets and net worth, reflecting equity contracts in which each investor gets a fixed proportion of the firm's total payouts and owns a constant fraction of the firm's net worth. Our model shows that an investor's payout and claim value depend not only on her own risk preferences, but also on the risk preferences of her co-investors.

Our model implies that less risk averse investors (managers) opt for riskier income streams (compensation packages) with higher upside potential but more downside risk. Furthermore, less risk averse investors (or managers) exert more influence over corporate decision making when the firm is doing well, whereas more risk averse investors (or managers) assume more control when the firm is in trouble.

We show there exists a unique set of utility weights for which all investors are indifferent between operating independently or as part of a group. Any other combination of weights makes at least one investor worse off.[2] Similar to the Negishi (1960) welfare weights, these are the only weights that do not lead to redistribution. Under the chosen utility weights, investors within the group receive the same payouts and accumulate the same wealth as if they were operating on their own. This is an example of Coase theorem at work. Given there are no frictions nor operational synergies from combining assets, it does not matter whether or not an investor is part of a group. We obtain a group irrelevance result.

We show that under voluntary participation an investor's utility weight increases with her capital contribution to the firm, and depends on her degree of risk aversion. Heterogeneous risk preferences generate skewed utility weighting as strongly risk averse investors are under-represented relative to their capital contribution. With homogenous risk preferences, increasing investors' risk aversion leads to more concentrated utility weights as measured by the Herfindahl index.

[2]The utility weights are unique only in relative terms, not in absolute terms. In other words the weight of one of the investors serves as a numeraire.

The utility weights under voluntary participation do not maximize the aggregate life-time utility of the group (i.e. the so-called Utiliarian social welfare function). Aggregate utility is maximized by giving all investors equal weight in the coalition. The increase in overall utility arises from redistribution among investors. Social welfare optimization is not achieved under voluntary participation, unless investors are endowed with the socially optimal level of capital to start with. Social optimization entails that more risk averse agents are endowed with a larger amount of capital.[3]

Why are the optimal claims and payouts for investors in a group (e.g. corporation) non-linear in the firm's net worth, whereas those for a sole proprietorship are linear in net worth? Claims issued by the group are written on the firm's *total* assets (or net worth). Once investors join the group, their capital is pooled together and they receive a claim on the firm as a whole. Given that group irrelevance applies, only non-linear contracts issued by corporations can replicate the cashflows individual investors would get if they each operated in a sole proprietorship. Since the net worth of standalone investors with heterogeneous risk preferences grows at different rates, it has to be the case that the relative share of less risk averse investors within the group increases (decreases) during good (bad) times, which in turn leads to non-linear payout functions.

In summary, our continuous-time, open-horizon model generates rich dynamics regarding the firm's investment, payout and financing policies, and investors' control. Moreover, we obtain tractable, closed-form solutions that capture the complex, dynamic nature of group decision making and corporate financial policies. These dynamics allow us to reconcile the diversification of opinions hypothesis and the group shift hypothesis.

[3]Capital need not be financial in nature. Investors can contribute human capital, or "sweat equity" to the startup by working below their opportunity wage.

2 Literature Review

Our paper relates to a growing literature in corporate finance that jointly models a firm's investment, payout, and borrowing decisions in a dynamic framework. Recent continuous-time papers in this strand include Gryglewicz (2011), Bolton, Chen, and Wang (2011), Décamps, Mariotti, et al. (2011), Décamps, Gryglewicz, et al. (2016), and Lambrecht and Myers (2017). In these papers the firm's financial policies are set by a single decision maker.

Dynamic models of group decision making in corporate finance are very rare. A notable exception is Garlappi, Giammarino and Lazrak (2017) who study a dynamic corporate investment problem by a group of agents holding heterogeneous beliefs and adopting a Utilitarian aggregation mechanism. They show that group decisions are dynamically inconsistent due to learning and that this may lead to underinvestment. Garlappi, Giammarino and Lazrak (2018) study a canonical real option model where the decisions to acquire a license and develop a project are made sequentially by a group of agents with heterogeneous beliefs who makes decisions based on majority voting.[4] Both papers do not consider optimal capital structure, payout structure, and internal governance, and all agents have the same utility function (the focus is on heterogeneous beliefs).

Our model derives the Pareto optimal payout (or compensation) for each group member. As such the paper is related to a growing literature within corporate finance on optimal dynamic security design. Existing models are cast within a standard principal agent setting, and focus on issues such as moral hazard and adverse selection. Dynamic continuous-time principal agent models include Grenadier and Wang (2005) , DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin and Rochet (2007), DeMarzo, Fishman, He and Wang (2012), DeMarzo and Sannikov (2016), Grenadier, Malenko and Malenko (2016), among others. These papers do

[4] There is also a large literature on decisions and voting behavior by corporate boards (see e.g. Levit and Malenko (2011) and Malenko (2014), among others).

not consider teams of investors or agents, and are usually set in a risk neutral setting to ensure tractability.

Our paper also relates to the literature on partnerships and syndicates that studies how risk should be shared between partners (see e.g. Wilson (1968), Eliashberg and Winkler (1981), Pratt and Zeckhauser (1989)). These static models solve cases where agents have homogeneous (i.e. identical) hyperbolic absolute risk aversion (HARA), and show that each partner's efficient consumption has a fixed component and a variable component that is linear in the aggregate outcome, with the more risk tolerant partners bearing higher shares of risk in the aggregate outcome. More recently, Mazzocco (2004) examines the savings behavior of couples with homogenous HARA preferences. He finds that couples' savings are a U-shaped function of individual risk aversion and prudence. The single-period model by Hara, Huang and Kuzmics (2007) shows that heterogeneity in the individual consumers' risk attitudes tends to make the representative consumer's absolute risk tolerance convex and relative risk aversion decreasing. The sharing rule for the least (most) risk averse agent is convex (concave). The sharing rules for the other agents are initially convex and eventually concave. Whereas existing papers focus on the consumption decision, our paper considers both payout and investment decisions and shows how these corporate decisions are interrelated. We contribute to this literature by showing that group decision making and group risk aversion are dynamic in nature. Payouts and investment dynamically depend on net worth and on the coefficient of risk aversion of all individuals. Investors' share of payout and their relative control over investment also vary over time, creating non-linearities in investors' claims. We show explicitly how group policies relate to the individuals' optimal policies and how they are aggregated.

Finally, a large body of papers studies the effect of heterogeneous agents on asset prices in a market equilibrium framework. Some of the seminal papers include Constantinides and Duffie (1996), Wang (1996), Dumas, Uppal and Wang (2000), and Chan and Kogan (2002), among others. For a review of this literature we refer to Duffie (2010). In the corporate finance tradition, our paper focuses on firm level decisions (i.e. investment, financing and payout policies), not on market equilibrium prices and interest rates. Furthermore, our paper does not restrict the types of contracts between investors, whereas market setting contracts are constrained by the available set of marketed securities.

3 The Model Setup

Consider a firm with n investors.[5] The firm invests an amount A_t in risky assets (projects) that generate an after-tax return given by the diffusion process:

$$\frac{dA_t}{A_t} = \mu dt + \sigma dB_t \tag{1}$$

where B_t is a Brownian motion. The drift and volatility of the process are μ and σ, respectively. We assume that assets are perfectly divisible, and subject to constant returns to scale. Without loss of generality, we assume that there is only one risky asset. Our results hold for a framework with multiple risky assets, provided that all investors have access to the same investment opportunity set. In that case the risky asset can always be thought of as a composite asset (mutual fund).[6]

The firm finances its assets with debt D_t and equity W_t. i.e $A_t = W_t + D_t$. W_t is the firm's net worth, and D_t is net debt. If D_t is negative then the firm has a net cash position. The firm can borrow and save at the risk free rate, and continuously role over its net debt position.[7]

[5] Alternatively, our model can be reformulated for a corporation run by a team of n managers.

[6] Merton (1971) shows that when asset prices are lognormally distributed all investors hold a combination of the risk-free asset and a mutual fund. The proportions of each asset held by the mutual fund only depend on the price distribution parameters and are independent of individual preferences and wealth distribution.

[7] It turns out that debt is risk free under the firm's optimal financial policies because asset returns follow a diffusion process (i.e. no jumps) and the amount of debt can be continuously rebalanced in response to economic shocks.

The after-tax cost (return) of debt (cash) is r. The firm's net debt position therefore evolves as follows:

$$dD_t = rD_t dt \tag{2}$$

We define $\xi \equiv \frac{\mu - r}{\sigma}$ as the Sharpe ratio associated with the risky assets.

At each instant t the investors receive payout at a rate c_{it} $(i = 1, ..., n)$, and invest an amount A_t in risky projects, given the firm's net worth W_t. The net worth process is therefore:

$$dW_t = dA_t - dD_t - \left(\sum_{i=1}^{n} c_{it}\right) dt = \left[(\mu - r) A_t + rW_t - \sum_{i=1}^{n} c_{it}\right] dt + \sigma A_t dB_t \tag{3}$$

Define $\omega_t \equiv \frac{A_t}{W_t}$ as the fraction of the firm's net worth invested in risky projects. If $\omega_t > 1$ then the firm invests all its net worth W_t in risky projects and borrows an amount $(\omega_t - 1)W_t$ that is also invested in risky assets. The process for W_t is now

$$dW_t = \left[(\omega_t(\mu - r) + r) W_t - \sum_{i=1}^{n} c_{it}\right] dt + \sigma\, \omega_t\, W_t\, dB_t \tag{4}$$

All investors have a power utility function and therefore constant relative risk aversion. Investors can, however, have a different coefficient of risk aversion γ_i, i.e. investor i's utility function is given by $u_i(c_{it}) = \frac{c_{it}^{1-\gamma_i} - 1}{1-\gamma_i} = \frac{c_{it}^{\eta_i} - 1}{\eta_i}$, where $\gamma_i > 0$ for $i = 1, ..., n$ and where $\lim_{\gamma_i \to 1} u_i(c_{it}) = ln(c_{it})$. Since $\gamma_i > 0$, all investors are strictly risk averse.[8] All investors have a subjective discount rate ρ.

Each investor wants to maximize the expected life-time utility from her payouts. Investors must, however, jointly agree on the payout policies c_{it} ($i = 1, ..., n$) and the investment policy ω_t. We focus on Pareto efficient payout policies among the n investors. That is, we identify policies that are not dominated by any other contract when considered from the perspective of

[8] Our model also holds if we are dealing with n classes of investors (rather than n single investors) with investors having the same risk preference within each class.

all the investors as individuals. We know (see Duffie (2010), chapter 10) that a necessary and sufficient condition for $(c_{1t}, ..., c_{nt})$ to be a Pareto efficient allocation is that there exist non-negative weights $(\lambda_1, \lambda_2, ..., \lambda_n)$ such that $(c_{1t}, ..., c_{nt})$ and ω_t are the solution to the following optimization problem:

$$J(W_t) = \max_{\{c_{1t},...c_{nt};\omega_t\}} E_t \left[\sum_{i=1}^{n} \int_t^{\infty} e^{-\rho(s-t)} \lambda_i u_i(c_{is})\, ds \right] \tag{5}$$

subject to the intertemporal constraint (3) and the transversality condition $\lim_{t\to\infty} [e^{-\rho t} J(W_t)] = 0$.

For the moment we take the utility weights λ_i as exogenously given. Apart from the weights being non-negative, we do not impose any other constraints on them. We endogenize the weights in section 5. Each investor's utility weight in the firm will depend on the net worth she contributes at the firm's start-up.

4 Optimal Financial Policies

The below proposition presents the solution to optimization problem (5).

Proposition 1 *The firm's optimal investment policy ω_t^* and payout c_{it}^* to investors are:*

$$\omega_t^* = \left(\frac{A_t}{W_t}\right)^* = \frac{\xi}{\sigma}\frac{W'(z)}{W(z)} = \sum_{i=1}^{n} \omega_{it}(z)\frac{\xi}{\sigma\gamma_i} = \sum_{i=1}^{n} \frac{c_{it}^*}{\frac{W_t}{a_i}}\frac{\xi}{\sigma\gamma_i} \equiv \sum_{i=1}^{n} \frac{c_{it}^*}{c_{it}^S}\omega_i^S \tag{6}$$

$$c_{it}^* = \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}} = \omega_{it}\frac{W_t}{a_i} \qquad for \quad i = 1..n \tag{7}$$

where z is the solution to

$$W(z) = \sum_{i=1}^{n} G_i e^{\frac{z}{\gamma_i}} \tag{8}$$

and where

$$\omega_{it}(z) = \frac{G_i e^{\frac{z}{\gamma_i}}}{\sum_{i=1}^{n} G_i e^{\frac{z}{\gamma_i}}} \quad and \quad \sum_{i=1}^{n} \omega_{it}(z) = 1 \quad and \quad \omega_i^S = \frac{\xi}{\sigma\gamma_i} \tag{9}$$

$$G_i = \frac{\gamma_i^2 \lambda_i^{\frac{1}{\gamma_i}}}{r\gamma_i^2 + (\rho + \frac{\xi^2}{2} - r)\gamma_i - \frac{\xi^2}{2}} \equiv a_i \lambda_i^{\frac{1}{\gamma_i}} \tag{10}$$

Investors' joint weighted life-time utility, $J(W)$, is given by:

$$J(W(z)) = \sum_{i=1}^{n} J_i(z) = \sum_{i=1}^{n} \left[\frac{G_i}{1 - \gamma_i} e^{\frac{(1-\gamma_i)z}{\gamma_i}} - \frac{1}{\rho} \frac{\lambda_i}{1 - \gamma_i} \right] \tag{11}$$

The investment and payout policies, and the firm's weighted claim value are not explicitly expressed as a function of its total net worth W_t but as a function of an auxiliary variable z_t. Net worth is, however, a continuous, monotonically increasing function of z as expressed by equation (8). As z ranges from $-\infty$ to $+\infty$, W varies from 0 to $+\infty$, i.e. $\lim_{z\to-\infty} W(z) = 0$ and $\lim_{z\to+\infty} W(z) = +\infty$.

For every value of z_t, there is a unique corresponding value for W_t. For expositional purposes it will be easier to express our result in terms of z, rather than W. z can be interpreted as some kind of "normalized" wealth variable.

4.1 Investment policy and control weights

From Merton (1969, 1971) we know that the optimal investment and payout policy of a single investor i with wealth W_t are, respectively, given by $\omega_i^S = \xi/(\sigma\gamma_i)$ and $c_{it}^S = W_t/a_i$. We find that the optimal investment policy ω_t^* of a coalition of investors is a weighted average of the optimal investment policies ω_i^S of the individual investors. The weight ω_{it} reflects investor i's influence or "control" over the investment policy. The optimal control weights ω_{it} are given by the ratio c_{it}^*/c_{it}^S of the individuals' payout c_{it}^* and her optimal payout c_{it}^S if she ran the whole firm alone. An increase in an investors' cashflow rights c^*_{it}, all else equal, coincides with an increase in her influence over the investment policy. Importantly, the control weights ω_{it} are time varying as they depend on the firm's net worth W through the auxiliary variable z.

Corollary 1 *The firm's risky asset to net worth ratio, ω^*_t, increases in the firm's net worth, i.e.* $\frac{\partial \omega^*_t}{\partial W_t} \geq 0$

The corollary implies that the firm invests more aggressively in risky assets (i.e. projects) in good times, and reduces its weight in risky assets during bad times.[9] Consider the n-investor case where agent 1 (n) is least (most) risk averse (i.e. $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$). In that case,

$$\lim_{z \to +\infty} \omega^*(z) = \frac{\xi}{\sigma \gamma_1} \quad \text{and} \quad \lim_{z \to -\infty} \omega^*(z) = \frac{\xi}{\sigma \gamma_n}$$

As the firm accumulates (loses) more wealth, its investment policy converges towards the one of the least (most) risk averse partner. $\omega^*(z)$ is an S-shaped function situated between the asymptotes $\frac{\xi}{\sigma\gamma_1}$ and $\frac{\xi}{\sigma\gamma_n}$. $\omega^*(z)$ is constant for the knife edge case of homogenous risk preferences.

Although investor i's control is increasing in her utility weight, i.e. $\frac{\partial \omega_{it}}{\partial \lambda_i} > 0$, the relation is highly non-linear as can be inferred from the ratio of investors' control weights:

$$\frac{\omega_{jt}}{\omega_{it}} = \frac{a_j \, \lambda_j^{\frac{1}{\gamma_j}}}{a_i \, \lambda_i^{\frac{1}{\gamma_i}}} \, e^{z\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} = \frac{a_j}{a_i} \frac{c^*_{jt}}{c^*_{it}} \tag{12}$$

More (less) risk averse investors exert relatively more control in bad (good) times as $z \to -\infty$ ($z \to +\infty$). More control goes together with a relatively higher payout as reflected by the ratio c^*_{jt}/c^*_{it} in equation (12). The following corollary results at once from equation (12).

Corollary 2 *Investors' balance of power is constant over time if and only if all investors have*

[9] The proofs of the corollaries are in Internet Appendix A.

the same risk preferences. Investors' control (ω_{it}) is strictly proportional to the utility weights (λ_i) if and only if all investors have logarithmic utility.

Finally, for the special case where all investors have the same degree of risk aversion γ, the investment policy becomes independent of time, and is given by the standard Merton (1969) ratio: $\omega = \xi/(\sigma\gamma)$.

4.2 Payout policy

The payout (in dollars) to each partner is increasing in the firm's net worth. Unlike Merton (1969), where payout is linear in wealth, payout is non-linear in total net worth if investors have different levels of risk aversion. Since $c^*_{it} = \omega_{it} W_t / a_i$ (see Equation (7)), the non-linearity is driven by the time-varying control weight ω_{it}. For the n-investor case one can prove the following proposition:

Proposition 2 *Payout is a convex (concave) increasing function of the firm's total net worth W for the least (most) risk averse investor. For investors with an intermediate level of risk aversion, there exists a critical wealth level W^*_i $(i = 2, 3, ..., n-1)$ such that payout is convex in wealth for low levels of wealth (i.e. $W \leq W^*_i$) and concave in wealth for high levels of wealth (i.e. $W \geq W^*_i$). Furthermore, $c_{it}(z = -\infty) = 0$ for $i = 1, ..., n$, and*

$$\lim_{z \to +\infty} \frac{\partial c_{1t}}{\partial W_t} = \frac{1}{a_1} \quad \textit{and} \quad \lim_{z \to +\infty} \frac{\partial c_{jt}}{\partial W_t} = 0 \quad \textit{for} \quad j = 2, 3, ..., n$$

Payout is linear in net worth if investors have homogeneous risk preferences:

$$\frac{\partial^2 c^*_{it}}{\partial W_t^2} = 0 \textit{ and } c_{it} = \frac{\lambda_i^{\frac{1}{\gamma}} W_t}{a_i \sum_{j=1}^n \lambda_j^{\frac{1}{\gamma}}} = \frac{\lambda_i^{\frac{1}{\gamma}} W_t}{a \sum_{j=1}^n \lambda_j^{\frac{1}{\gamma}}} \quad \textit{for } i=1,...,n \Leftrightarrow \gamma_1 = ... = \gamma_n = \gamma \quad (13)$$

It is Pareto optimal for the least (most) risk averse agent to have a convex (concave) payout function. Since all payouts are zero for $W_t = 0$, it follows that, among all investors, the most risk averse investor is paid the most for very low levels of net worth, whereas the least risk-averse investor gets paid the most for very high levels of net worth. The least risk averse investor is the residual claimant who gets the upside potential in good times, but is heavily exposed in bad times. All other risk averse investors are less exposed in downturns, but their payout in good times is capped. Each investor's payout is non-linear in total net worth, except if all investors have the same coefficient of risk aversion. These findings are also reflected in investors' relative cash flow rights:

$$\frac{c_{jt}}{c_{it}} = \frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} \tag{14}$$

Equations (13) and (14) lead to the following corollary.

Corollary 3 *Each investor receives a constant fraction of total payout if and only if all investors have the same risk preferences. An investor's payout is strictly proportional to her utility weight λ_i if and only if all investors have logarithmic utility.*

Consider next the firm's payout yield c_t^*/W_t. It follows from proposition 1 that:

$$\frac{c_t^*}{W_t} = \sum_{i=1}^{n} \frac{c_{it}^*}{W_t} = \sum_{i=1}^{n} \frac{\omega_{it}}{a_i} \tag{15}$$

The payout yield c_t^*/W_t is time-invarying if all investors have the same coefficient of risk aversion γ (and therefore $a_i = a$ for all i):

$$\frac{c_t^*}{W_t} = \frac{\sum_{i=1}^n \lambda_i^{\frac{1}{\gamma}} e^{\frac{z}{\gamma}}}{\sum_{i=1}^n a\lambda_i^{\frac{1}{\gamma}} e^{\frac{z}{\gamma}}} = \frac{1}{a} = \frac{r\gamma^2 + (\rho + \frac{\xi^2}{2} - r)\gamma - \frac{\xi^2}{2}}{\gamma^2} = \frac{\rho}{\gamma} - \frac{r(1-\gamma)}{\gamma} - \frac{\xi^2(1-\gamma)}{2\gamma^2} \tag{16}$$

This is the same average propensity to consume out of wealth as in Merton (1969, Eq. (40)).

Since $\frac{\partial \frac{c_t^*}{W_t}}{\partial W} \frac{\partial W}{\partial z} = \frac{\partial \frac{c_t^*}{W_t}}{\partial z}$ and since $\frac{\partial W}{\partial z} > 0$, it folows that $\frac{\partial \frac{c_t^*}{W_t}}{\partial W}$ and $\frac{\partial \frac{c_t^*}{W_t}}{\partial z}$ have the same sign. In general, the sign of $\frac{\partial \frac{c_t^*}{W_t}}{\partial W}$ is hard to determine as it depends on both the weights λ_i and the coefficient of risk aversion γ_i. However, one can show (proof available upon request) for the two-investors case that the payout yield increases in the firm's net worth if the least risk averse investor is sufficiently risk averse.

4.3 Debt policy

The firm's net debt level is given by $D_t = (\omega_t^* - 1)W_t$. If $D_t < 0$ then the firm's cash-holdings exceed its debt, so that the firm has a net cash surplus (or negative debt). The below corollary states the firm's net debt ratio under the optimal investment and payout policy. With negative debt, the traditional net debt ratio (NDR) is hard to interpret. Lambrecht and Pawlina (2013) provide an economic rationale for using NDR as defined below:

Proposition 3 *Define the firm's net debt ratio as* $NDR_t \equiv \frac{D_t}{W_t + \Psi D_t}$ *where* $\Psi = 1$ *if* $D_t \geq 0$ *and* $\Psi = 0$ *if* $D_t < 0$. *The NDR is time varying, procyclical, and given by*

$$\frac{\frac{\xi}{\sigma\gamma_n} - 1}{1 + (\frac{\xi}{\sigma\gamma_n} - 1)\Psi} \quad < \quad NDR_t = \frac{\omega_t^* - 1}{1 + (\omega_t^* - 1)\Psi} \quad < \quad \frac{\frac{\xi}{\sigma\gamma_1} - 1}{1 + (\frac{\xi}{\sigma\gamma_1} - 1)\Psi} \tag{17}$$

The firm's leverage is procyclical. In good times, the NDR rises and converges to the optimal NDR of the least risk averse investor. In bad times, as net worth shrinks, the firm rebalances by selling risky assets or shutting down risky projects, and using the proceeds to pay off debt. As $W \to 0$ (or $z \to -\infty$), the NDR converges towards the most risk-averse investor's optimal NDR.

As investor n becomes infinitely risk averse ($\gamma_n \to +\infty$), the lower bound for the NDR converges to -1, which represents an all equity-financed firm that invests in the safe asset (cash) only. As investor 1 moves towards risk neutrality ($\gamma_1 \to 0$), the upper bound for the NDR converges to $+1$, which represents a near 100% debt-financed firm that invests in risky assets or projects only. Therefore, if the most and least risk averse agents have highly divergent risk preferences, then the intertemporal variation in the firm's NDR across the business cycle could be large.

4.4 Group risk aversion and financial policies

Corollary 4 *The group's level of relative risk aversion is endogenous and can be defined in terms of the value function as:*

$$RRA_G \;\equiv\; -W(z)\,\frac{J_{WW}}{J_W} \;=\; \frac{W(z)}{W'(z)} \;=\; \frac{\xi}{\sigma\,\omega_t^*} \tag{18}$$

Using our comparative statics for ω_t^* (see section 4.1), it follows that:

$$\frac{\partial RRA_G}{\partial W_t} \;\leq\; 0 \quad \text{and} \quad \lim_{z\to+\infty} RRA_G \;=\; \gamma_1 \quad \text{and} \quad \lim_{z\to-\infty} RRA_G \;=\; \gamma_n \tag{19}$$

Hence, the group's level of relative risk aversion decreases in its net worth, and converges to the level of the least (most) risk averse investor as net worth goes to infinity (zero). This explains the earlier described behavior of the firm's investment and financing policy. More generally, it may also help explain why investors in financial markets are very bullish during stock market booms and extremely risk averse during market crashes. Finally, $RRA_G = \gamma$ if all investors have the same coefficient of risk aversion γ.

5 Claim Values and Utility Weights

In this section we endogenize investors' utility weights λ_i when the firm is set up. Next, we derive the monetary certainty equivalent value of each investor's claim. We conclude the section with a few examples and a discussion of the results.

5.1 Endogenous utility weights

Given that assets are perfectly divisible and subject to constant returns to scale, it is possible for each investor to run a firm on her own as a sole proprietorship. Assuming that investor i is endowed with wealth W_{i0}, the life-time utility she can generate from the sole proprietorship is given by the Merton (1969) solution.

$$I_i(W_{it}) = \max_{\{c_{it};\omega_t\}} E_t\left[\int_t^{\infty} e^{-\rho(s-t)}u_i(c_{is})ds\right] = a_i^{\gamma_i}\frac{W_{it}^{1-\gamma_i}}{1-\gamma_i} - \frac{1}{\rho}\frac{1}{1-\gamma_i}, \tag{20}$$

where a_i was defined in equation (10).

Investor i joins a group with multiple investors if doing so makes her at least as well off as operating on her own. The life-time utility investor i generates from joining the group is given by

$$\frac{J_i(W_{it})}{\lambda_i} = \frac{a_i}{1-\gamma_i}\lambda_i^{\frac{1-\gamma_i}{\gamma_i}} e^{\frac{z_t(1-\gamma_i)}{\gamma_i}} - \frac{1}{\rho}\frac{1}{1-\gamma_i} \tag{21}$$

Therefore, investor i's participation constraint is satisfied if:

$$\frac{J_i(W_{i0})}{\lambda_i} \geq I_i(W_{i0}) \quad \Leftrightarrow \quad a_i\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_0}{\gamma_i}} \geq W_{i0} \tag{22}$$

Combining investors' participation constraints gives:

$$\sum_{i=1}^{n} a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_0}{\gamma_i}} \geq \sum_{i=1}^{n} W_{i0} = W_0 \tag{23}$$

We know, however, from proposition 1 (Eq. 8) that the above constraint is always satisfied as an equality. Consequently, if all participation constraints are satisfied, then they must bind. In other words, each investor is indifferent between joining the group or going it alone. As a result:

$$a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_0}{\gamma_i}} = W_{i0} \quad \text{for} \quad i = 1, ..., n \tag{24}$$

This leads to the following corollary:

Corollary 5 *The utility weights λ_i that ensure each investor's voluntary participation to the group without redistributing initial wealth are:*

$$\lambda_i = \left(\frac{W_{i0}}{a_i}\right)^{\gamma_i} e^{-z_0} \quad with \quad \frac{u_i'(c_{i0}^*)}{\frac{dI_i(W_{i0})}{dW_{i0}}} = 1 \quad for \quad i = 1, ..., n \tag{25}$$

With these weights, joining the group is a zero NPV decision for each investor. The corollary only defines the weights in relative terms. E.g. for $n = 2$, we get that

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{W_{20}}{a_2}\right)^{\gamma_2}}{\left(\frac{W_{10}}{a_1}\right)^{\gamma_1}}$$

Under voluntary participation, investors' utility weights are clearly not proportional to the amounts they invest. Heterogeneity in risk preferences generates a skewed utility weighting as more risk averse agents tend to be underweighted (see section 6.3 for a numerical example).

From Equation (25) it follows that the weighting normalizes the utility function by the investor's shadow price of wealth so that each investor's marginal utility of consumption for the rescaled utility equals 1. As such, our endogenous utility weights are similar to the Negishi (1960) welfare weights which have the unique feature of preserving the initial wealth distribution.

For the case of homogenous risk preferences, we obtain the following simpler rule, and the subsequent corollary:

$$\frac{\lambda_i}{\lambda_1} = \left(\frac{W_{i0}}{W_{10}}\right)^{\gamma} \quad if \quad \gamma_1 = \gamma_2 = ... = \gamma_n \equiv \gamma$$

Corollary 6 *Investors' utility weights are proportional to the capital they invest if and only if all investors have logarithmic utility (i.e. $\gamma = 1$).*

The following proposition shows how risk aversion affects the concentration of the utility weights.

Proposition 4 *Suppose all investors have homogenous risk aversion and that the utility weights add up to 1. A higher (lower) level of risk aversion leads to more (less) concentrated utility weighting as measured by the Herfindahl index $H = \sum_{j=1}^{n} \lambda_j^2$. I.e. $\frac{\partial H}{\partial \gamma} > 0$ if $\gamma_1 = ... = \gamma_n = \gamma$. Furthermore $0 < H < 1$.*

As all investors approach risk neutrality, the utility weights become uniform, i.e. $\lim_{\gamma \to 0} H = 1/n$. On the other hand, as all investors become increasingly risk averse, small differences in capital contribution lead to large differences in utility weights. For extreme risk aversion ($\gamma \to \infty$), (almost) all utility weight is concentrated on the largest capital contributor, i.e. $\lim_{\gamma \to \infty} H = 1$.[10] Although the Herfindahl index is still between 0 and 1 (i.e. $0 < H < 1$) if investors have heterogeneous risk preferences, the comparative statics for H become ambiguous as they depend on the distribution of investors' initial capital contribution.

[10]If investors have homogenous risk preferences and contribute the same amount of capital (i.e. $W_{10} = W_{20} ... = W_{n0}$) then the concentration index equals $H = 1/n$ for all values of γ.

5.2 Investors' claim values

Using corollary 5, which pins down the utility weights λ_i, and equation (7) for c^*_{it}, we can now state the monetary certainty equivalent value W_{it} of each investor's claim.

Corollary 7 *At each moment in time t, investor i is indifferent between remaining in the firm, or quitting the firm in return for a dollar amount W_{it} given by:*

$$W_{it} \,=\, a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} \,=\, a_i\, c^*_{it} \;= \omega_{it}\, W_t \tag{26}$$

W_{it} corresponds to investor i's certainty equivalent wealth. It is the dollar amount required by investor i to give up her stake in the firm. This amount W_{it} may differ from W_{i0}, the amount invested at time 0. W_{it} will be larger (smaller) than W_{i0} if the firm has increased (decreased) its total net worth W_t. W_{it} depends on the investor's coefficient of risk aversion γ_i. From Equation (26) it follows that investors' control weights ω_{it} also correspond to their effective ownership share in the firm's net worth. Investors' effective ownership share is time-varying.

The following corollary describes investors' certainty equivalent value as a function of the firm's net worth.[11] Since this value is a constant multiple of payout (i.e. $W_{it} = a_i c^*_{it}(W(z))$) the corollary mirrors the results we obtained in proposition 2 for investors' payout policies c_{it}.

Corollary 8 *The claim value of the most (least) risk averse investor is a convex (concave) function of the firm's total net worth W_t. The claim of investors with intermediate levels of*

[11] The corollary can be reformulated to describe claim values as a function of the firm's holding in risky assets A_t. The results are very similar.

risk aversion is S-shaped in net worth. Moreover, $\lim_{z\to-\infty} W_{it} = 0$ *for* $i = 1, ..., n$, *and*

$$\lim_{z\to+\infty} \frac{\partial W_{1t}}{\partial W_t} = 1 \quad and \quad \lim_{z\to+\infty} \frac{\partial W_{it}}{\partial W_t} = 0 \quad for \quad i = 2, ..., n \tag{27}$$

$$\lim_{z\to-\infty} \frac{\partial W_{nt}}{\partial W_t} = 1 \quad and \quad \lim_{z\to-\infty} \frac{\partial W_{it}}{\partial W_t} = 0 \quad for \quad i = 1, ..., n-1 \tag{28}$$

Finally, $\exists W_i^*$ *such that* $\frac{\partial^2 W_{it}}{\partial W_t^2} \geq (\leq)\ 0 \quad \Leftrightarrow \quad W_t \leq (\geq) W_i^*$ *for* $i = 2, .., n-1$.

As the firm becomes very wealthy, an additional dollar of net worth is almost entirely to the benefit of the least risk averse investor (i.e $\lim_{z\to+\infty} \frac{\partial W_{1t}}{\partial W_t} = 1$), whereas the other investors' claims converge to a fixed value. Conversely, for a firm with little or no wealth, the marginal dollar accrues almost entirely to the most risk averse investor (i.e $\lim_{z\to-\infty} \frac{\partial W_{nt}}{\partial W_t} = 1$).

The strictly convex claim of the least risk averse investor ($i = 1$) is unbounded and resembles a common stock contract. The claims of investors 2 to $n-1$ are convex (concave) for low (high) values of net worth, with each claim converging towards a maximum value and resembling a preferred stock contract. Preferred stocks are widely used in venture capital financing. They have also became a more popular source of financing with corporations (especially financial institutions) over the past decade. Preferred stock contracts are senior to common stock in that dividends may not be paid to common stock, unless the dividend is paid on all preferred stock. The dividend rate on preferred stock is usually fixed at the time of issue. Unlike debt contracts, the firm is not in default when it suspends dividends to preferred stockholders.[12] The preferred stock is junior to the debt claim D_t. In simple terms, one could view preferred stock as a debt instrument when dividends are not in arrears, and as an equity instrument when dividends are getting significantly in arrears. This is consistent with the claim value being convex and concave for, respectively, low and high values of net worth W_t.

[12] With cumulative preferred stock, unpaid dividends accumulate and have to be paid before regular dividends can be resumed. With non-cumulative preferred stock, unpaid dividends do not accumulate.

Preferred stocks usually do not carry voting rights in corporations. However, a frequent provision allows preferred shareholders to vote as a separate class whenever dividends have been in arrears for a specific period of time (normally four quarters) and to elect a specific number of directors. Preferred stockholders almost always have significant voting and control rights in venture capital partnerships.

The claim of the most risk averse investor ($i = n$) is strictly concave in W_t, and resembles a preferred stock contract. For the lowest levels of net worth, an extra dollar for the firm accrues almost entirely to the most risk averse investor. As the firm accumulates more net worth, other preferred stock holders start sharing more in the payouts. The claims of the other investors resemble preferred stock contracts for which the level of seniority increases in the degree of risk aversion, and for which the face value of the contract (achieved as $W_t \to +\infty$) decreases in the degree of risk aversion. The maximum value of each preferred stock contract is determined by the maximum payout rate c_{it} as $W_t \to +\infty$.

As we pointed out before, in our model risk averse investors exert very little influence on the firm's investment and capital structure decision in good times. However, as the firm's net worth declines, their control increases. This is similar to what happens with some classes of preferred stock that grant increased control in bad times.

The variation in the shapes of the claim values is entirely driven by differences in investors' coefficient of risk aversion, as illustrated by the following corollary:

Corollary 9 *If all investors have the same coefficient of risk aversion ($\gamma_1 = \gamma_2 = ... = \gamma_n$), then the investors' claim values are linear in the firm's total net worth, i.e.*

$$W_{it} = \frac{\lambda_i^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} \lambda_i^{\frac{1}{\gamma}}} W_t \quad for \quad i = 1, 2, ..., n \tag{29}$$

If all investors have the same coefficient of risk aversion, then they share profits according to fixed proportions. This gives rise to common stock contracts that are linear in net worth. If all investors have log utility ($\gamma_i = 1$ for $i = 1, .., n$) and the utility weights sum to one then each investor's certainty equivalent claim value is proportional to her utility weight λ_i since $W_{it} = \lambda_i W_t$.

5.3 Examples and discussion

Figure 1 numerically evaluates the firm's optimal payout and financing policies. The firm has 4 investors (or 4 classes of investors) with coefficients of relative risk aversion $\gamma_1 = 0.5, \gamma_2 = 0.6, \gamma_3 = 3, \gamma_4 = 9$, and with initial endowments $W_{10} = 15, W_{20} = 25, W_{30} = 15$, and $W_{40} = 15$.[13]

Panel A of Figure 1 plots investors' payouts as a function of the firm's net worth. For very low (high) net worth levels payout is increasing (decreasing) in investors' coefficient of risk aversion. In bad times, the more risk averse agents enjoy a higher payout level at the expense of less risk averse agents who take the hit. In good times, the least risk averse agent enjoys the upside, whereas all other agents' payout is capped. The cap decreases with agents' coefficient of risk aversion. The little windows within the figure shows that the payout function for each investor, except for the least risk averse one (represented by the dotted line), converges to a cap as wealth becomes large. One can verify that the payout function for the least (most) risk averse agents is strictly convex (concave). For the other agents, the payout function is initially convex up to some inflection point, and concave thereafter.

Panel B plots the firm's NDR as a function of net worth (thick solid line). Recall that a negative NDR ($D_t < 0$) corresponds to a firm with a net cash position. The NDR in-

[13] Empirical estimates for the coefficient of RRA range from 0.2 to 10 and higher (see footnote 1). Our parameter values for γ_i fall within this range.

creases in the firm's net worth when investors have heterogeneous risk preferences. Leverage is pro-cyclical and converges towards 35.71% as $W_t \to +\infty$. The firm delevers in economic downturns, reduces its holdings in risky assets and, in doing so, keeps debt risk free. The NDR approaches -0.91 as $W_t \to 0$. A negative NDR of -0.9136 means that the firm's net worth is invested for 91.36% in cash and the remainder in risky assets. In line with proposition 3 the firm's NDR is bounded below and above by the optimal NDR for the most and least risk averse investor, respectively. Therefore, our model generates a large amount of inter-temporal variation in leverage if investors' risk preferences are widely dispersed.

The NDR is constant over time for a sole proprietorship (or when all investors have the same coefficient of risk aversion). For a single investor with coefficient of risk aversion equal to 0.5, 0.6, 3 or 9, the sole proprietorship adopts a constant net debt ratio of, respectively, 0.3571, 0.2286, -0.7407 and -0.9136.

Panel C plots the payout yields as a function of net worth. The optimal payout yield is constant when investors operate on their own (or all have the same coefficient of risk aversion). The four constant, horizontal lines represent the optimal payout yield for each sole proprietorship, which can either be calculated using the Merton (1969) model, or be obtained by calculating the ratio of the investor's payout c^*_{it} in the group and her certainty equivalent claim W_{it} in the firm (c^*_{it}/W_{it}). Both calculations yield the same result. The payout yield for sole proprietorships is not monotonic in the coefficient of risk aversion. The optimal payout yield is highest for investor 2 (i.e. $\gamma_2 = 0.6$), not for investor 1 ($\gamma_1 = 0.5$). The payout yield of the group (c_t/W_t is given by the thick solid line) increases in net worth because investors are sufficiently risk averse. As W_t goes to infinity the payout yield converges to the optimal yield of investor 1 ($\gamma_1 = 0.5$), which in this particular case is below the optimal yield for investor 2.

Panel D plots investors' certainty equivalent claim value as a function of total net worth. When the firm was set up ($t = 0$), three investors contributed 15 in net worth, whereas investor 2 contributed 25 (i.e $W_{20} = 25$). The claim value for the most risk averse agent (solid line; $\gamma_4 = 9$) is strictly concave, the value of which converges to 27.43 as total net worth becomes very large. The claim value of the least risk averse agent (dotted line; $\gamma_1 = 0.5$) is strictly convex. The claim value for investors with intermediate risk aversion (long dashed line for $\gamma_3 = 3$; short dashed lined for $\gamma_2 = 0.6$) are first convex and then concave. Except for the claim held by the least risk averse investor, all other claims converge to a bounded value as $W_t \to +\infty$. Even for investor 2, who is relatively less risk averse, the claim converges to a finite level (as can be noted from the little window furthest to the right that plots the claim values for very high levels of net worth). The bounded claims resemble preferred stock claims.[14] In our model, preferred stocks can have different payout rate caps stacked in an echelon style fashion: investors with a higher level of risk aversion have a lower payout rate cap that is reached for lower levels of net worth.

Panel E plots investors' control weights ω_{it}. As net worth increases less risk averse investors tend to exert more influence over the firm's investment and financing policy. This relation can, however, be non-monotonic as illustrated by investor 2's control weight which first increases and then decreases in the firm's net worth. This highlights the complicated and non-trivial interactions in group decision making. For intermediate net worth levels ("normal" times) the members around the median level of risk aversion (investors 2 and 3) exert most influence (consistent with the findings of the experimental study by Ambrus et al. (2013)).

Panel F plots the group's level of relative risk aversion, RRA_G. Group risk aversion is fairly low (around 0.51) for most levels of net worth, but spikes up dramatically once net worth drops below 50, converging to 9 as W_t goes to zero. During busts the group's risk aversion and financial policies are determined (almost) entirely by the most risk averse investor.

[14]For example, the preferred stock valuation model by Emanuel (1983) produces claim values for cumulative preferred stock and non-cumulative preferred stock that are graphically very similar to ours (see figures 3 and 4 on pages 1148-1149 in Emanuel (1983)), even though his valuation formulas are very different.

6 Investors' Welfare

In this section we study investors' welfare under voluntary participation and under social optimization. We explore whether groups make individual investors better off. We conclude with a numerical example.

6.1 Investors' welfare under voluntary participation

So far we took the time at which investors set up the firm as exogenously given. One might wonder whether it matters when investors join the firm. Consider therefore the scenario where investors do not join the firm at time 0, but at some future time t. Assume that during the interval $[0, t]$ each single investor operates on her own as a sole proprietorship. We know from Merton (1969) that the net worth of each single individual investor, denoted by W_{it}^{s}, evolves as:

$$\frac{dW_{it}^{s}}{W_{it}} = \left[\frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i}\right] dt + \frac{\xi}{\gamma_i} dB_t \equiv \alpha_i dt + \sigma_i dB_t \tag{30}$$

$$W_{it}^{s} = W_{i0}^{s} e^{\left[\left(\alpha_i - \frac{\sigma_i^2}{2}\right)t + \sigma_i B_t\right]} \tag{31}$$

One can then prove the following proposition:

Proposition 5 *Assume each investor can operate as a sole proprietorship during the interval* $[0, t]$*. Whether investors join the group at time zero, or at some later time* t*, they join under the same relative utility weights. In particular:*

$$\frac{\lambda_i}{\lambda_j} = \frac{\left(\frac{W_{i0}^{s}}{a_i}\right)^{\gamma_i}}{\left(\frac{W_{j0}^{s}}{a_j}\right)^{\gamma_j}} = \frac{\left(\frac{W_{it}^{s}}{a_i}\right)^{\gamma_i}}{\left(\frac{W_{jt}^{s}}{a_j}\right)^{\gamma_j}} \tag{32}$$

This result may come as a surprise. How come the relative utility weights remain the same even though the net worth of less risk averse sole proprietors grows on average at a faster rate than the net worth of their more risk averse counterparts? The reason is that the utility weights are adjusted for investors' degree of risk aversion as illustrated by the following corollary:

Corollary 10 *The evolution of investors' sole proprietors' net worth satisfies the following condition:*

$$\left(\frac{W_{it}^s}{W_{i0}^s}\right)^{\gamma_i} = \left(\frac{W_{jt}^s}{W_{j0}^s}\right)^{\gamma_j}, \quad \textit{or equivalently} \tag{33}$$

$$\left[\left(\alpha_i - \frac{\sigma_i^2}{2}\right)t + \sigma_i B_t\right]\gamma_i = \left[\left(\alpha_j - \frac{\sigma_j^2}{2}\right)t + \sigma_j B_t\right]\gamma_j = -\left(\rho - \frac{\xi^2}{2} - r\right)t + \xi B_t \tag{34}$$

Equation (34) shows, after simplifying, that the left and right side of equation (33) do not depend on γ_i and γ_j, respectively. Even if investor i is less risk averse ($\gamma_i < \gamma_j$) and her wealth grows on average at a faster rate than investor j's outside the group, the formula for the group's utility weights "penalizes" the less risk averse investor in such a way that both investors join the group under the same terms at time t as at time 0. No investor can therefore improve his or her utility weight in the group by joining later.

The question remains whether any investor enjoys a different payout flow while being outside the group compared to what she would get inside the group.

Proposition 6 *If the utility weights are set as in corollary 5, then the payout flow to each investor is the same whether she operates on her own, or whether she is part of a group of n investors with heterogeneous risk preferences, i.e. $c_{it} = c_{it}^s$ for all t. Each investor's net worth stake within the partnership, W_{it}, equals at all times the wealth W_{it}^s the investor would accumulate on her own, i.e. $W_{it} = W_{it}^s$ for all t.*

We obtain a group irrelevance result: in an ideal frictionless world, it does not matter whether investors operate on their own, or as part of a group of n investors. Since all investors are subject to the same source of uncertainty, there are no benefits from diversification. Neither are there (dis)economies of scale from running a larger firm. Should we conclude that groups are useless and cannot enhance welfare in this ideal environment? We show in section 6.2 that the answer is no. The irrelevance result only holds if all utility weights satisfy condition (32), or equivalently if there is no redistribution.

Let us consider next the aggregate utility of a group if investors voluntarily join. We know that if each investor i is part of a group of n investors then:

$$J(W_t) = \max_{\{c_{1t},\ldots c_{nt};\omega_t\}} E_t\left[\sum_{i=1}^{n}\lambda_i\int_t^{\infty} e^{-\rho(s-t)}u_i(c_{is})ds\right] = \sum_{i=1}^{n}J_i(z) \tag{35}$$

The life-time utility to investor i from joining the group is $V_i = J_i(z)/\lambda_i$. The aggregate utility for the group, V_G, is therefore

$$V_G = \sum_{i=1}^{n}V_i(z) = \sum_{i=1}^{n}\frac{J_i(z)}{\lambda_i} \tag{36}$$

In contrast, the aggregate utility generated by n sole proprietorships is given by:

$$I(W_{1t},...,W_{2t}) = \sum_{i=1}^{n}I_i(W_{it}) = \sum_{i=1}^{n}a_i^{\gamma_i}\frac{W_{it}^{1-\gamma_i}}{1-\gamma_i} - \frac{1}{\rho}\frac{1}{1-\gamma_i} \tag{37}$$

The following corollary gives the relation between the aggregate utility of the group V_G, and the aggregate utility $I(W_1,..,W_n)$ of the n sole proprietorships.

Corollary 11 *The aggregate life-time utility of n sole proprietorships with given endowments W_{i0} $(i = 1..n)$ equals the aggregate utility of the corresponding n-investor group with voluntary* *participation:*

$$
\begin{aligned}
V_G &= \sum_{i=1}^{n} V_i(z) = \sum_{i=1}^{n} \frac{a_i}{1 - \gamma_i} \lambda_i^{\frac{1-\gamma_i}{\gamma_i}} e^{\frac{z_0(1-\gamma_i)}{\gamma_i}} - \frac{1}{\rho}\frac{1}{1-\gamma_i} \\
&= \sum_{i=1}^{n} \frac{a_i^{\gamma_i}}{1 - \gamma_i} W_{i0}^{1-\gamma_i} - \frac{1}{\rho}\frac{1}{1-\gamma_i} = \sum_{i=1}^{n} I_i(W_{i0}) = I(W_{10}, ..., W_{n0})
\end{aligned}
\tag{38}
$$

The marginal utility (shadow price) to the group of an additional dollar contributed by member i *is given by:*

$$
\frac{\partial V_G}{\partial W_{i0}} = \left(\frac{a_i}{W_{i0}}\right)^{\gamma_i} = \frac{1}{\lambda_i e^{z_0}} \tag{39}
$$

The corollary states that the aggregate utility achieved by a group with n voluntary investors is the same as the aggregate utility by n sole proprietorships. This is a logical consequence of the group irrelevance result stated in proposition 6.

As to be expected, the shadow price of an extra dollar of net worth, declines with the group's total net worth already in place (as captured by the transformed net worth variable z). Less standard is the result that the shadow price depends on which investor contributes the extra dollar. In particular a dollar contributed by the investor with the smallest (largest) weight is most (least) valuable. The reason for this is that the utility weights λ_i agreed by the investors do not maximize the group's aggregate utility V_G. We show below that aggregate utility is maximized by allocating all investors equal weight (i.e. $\lambda_i = \lambda$ for all i). Therefore, contributions from the smallest (highest) investor are the most (least) valuable. Equal weights do not lead in general to voluntary participation because the λ_i that ensure investors' participation depend on their coefficient of risk aversion and wealth contribution, both of which may vary arbitrarily across investors.

6.2 Investors' welfare under social optimization

If investors' participation can be enforced by a social planner then aggregate utility can be improved through a group, but this will come at the expense of at least one investor being worse off. The socially optimal utility weights are the solution to the maximization problem:

$$V_G^o = \max_{\{\lambda_1,\ldots,\lambda_n\}} \sum_{i=1}^{n} \frac{J_i(z)}{\lambda_i} = \sum_{i=1}^{n} \left(\frac{a_i}{1-\gamma_i} \left(\lambda_i e^z\right)^{\frac{1-\gamma_i}{\gamma_i}} - \frac{1}{\rho} \frac{1}{1-\gamma_i} \right) \tag{40}$$

The first-order conditions are given by:

$$\frac{a_i}{\gamma_i} \lambda_i^{\frac{1-\gamma_i}{\gamma_i}-1} e^{\frac{1-\gamma_i}{\gamma_i}z} + \sum_{j=1}^{n} \frac{1}{\lambda_j} \frac{\partial J_j(z)}{\partial z} \frac{\partial z}{\partial \lambda_i} = 0 \quad \text{for} \quad i = 1, \ldots, n \tag{41}$$

Solving the system of first order conditions gives the following proposition:

Proposition 7 *It is socially optimal to give investors equal weight in the firm (i.e $\lambda_i = \lambda_i^o = \lambda$ for $i = 1, \ldots, n$).*

The socially optimal weighting scheme is not unique but, in absolute terms, only determined up to a scalar. Only λe^z is uniquely determined. This can be inferred from equation (40) for V_G^o. Setting $\lambda_i = \lambda$ (for all i) in equation (40), it follows that λ and e^z can no longer be separately identified since V_G is a function of the variable $y \equiv \lambda e^z$, which is the inverse of the shadow price of the group's net worth, i.e. $\delta = 1/y$. To pin down a unique value for λ, one can impose that the utility weights sum to 1, in which case $\lambda = 1/n$.

Corollary 12 *The aggregate utility of the group under the socially optimal weighting scheme is given by:*

$$V_G^o = \sum_{i=1}^{n} V_i^o = \sum_{i=1}^{n} \left(\frac{a_i}{1-\gamma_i} \left(\lambda e^z\right)^{\frac{1-\gamma_i}{\gamma_i}} - \frac{1}{\rho} \frac{1}{1-\gamma_i} \right) \tag{42}$$

At the social optimum, the marginal utility (shadow price) of an additional dollar does not *depend on which group member contributes the money, i.e.:*

$$\frac{\partial V_G^o}{\partial W_i} = \frac{\partial z}{\partial W}\sum_{j=1}^{n}\frac{a_j}{\gamma_j}\left(\lambda e^z\right)^{\frac{1-\gamma_j}{\gamma_j}} = \frac{\partial z}{\partial W}\sum_{j=1}^{n}\frac{a_j}{\gamma_j}y^{\frac{1-\gamma_j}{\gamma_j}} = \frac{\partial V_G^o}{\partial W_k} = \delta \quad for \quad i,k = 1,...,n \tag{43}$$

where $\frac{dz}{dW} = \left[\sum_{j=1}^{n}\frac{a_j}{\gamma_j}\left(\lambda e^z\right)^{\frac{1}{\gamma_j}}\right]^{-1}$, $\delta \equiv \frac{\sum_{j=1}^{n}\frac{1}{\lambda_j}\frac{\partial J_j}{\partial z}}{\frac{dW}{dz}} = 1/y$ *is the shadow price, and* $y \equiv \lambda e^z$ *is the solution to:* $\sum_{j=1}^{n} a_j y^{\frac{1}{\gamma_j}} = \sum_{j=1}^{n} W_j$.

The socially optimal utility weights are such that the marginal utility of consumption is equalized across investors. If that were not the case, then aggregate utility could be improved by increasing the weight of the investor with the highest marginal utility.

Unless investors agree to equal weights, utility will be lost compared to the outcome achieved by a social optimizer. The inefficiency of voluntary participation results from a "governance overhang" created when the group is set up. Differences in the investors' risk aversion γ_i and net worth contribution W_i require unequal utility weights to ensure that all investors participate. Once these weights are fixed, it is not in any group member's interest to reduce her weight relative to other members' even though this would increase the group's overall value. The value lost from this governance overhang is

$$V_G^o - V_G = \sum_{i=1}^{n}\frac{a_i}{1-\gamma_i}\left[\left(\lambda e^{z^o}\right)^{\frac{1-\gamma_i}{\gamma_i}} - \left(\lambda_i e^z\right)^{\frac{1-\gamma_i}{\gamma_i}}\right] = \sum_{i=1}^{n}\frac{a_i}{1-\gamma_i}\left[y^{\frac{1-\gamma_i}{\gamma_i}} - \left(\frac{W_i}{a_i}\right)^{1-\gamma_i}\right] \tag{44}$$

Finally, consider a social planner that optimally allocates funding to n sole proprietorships. The aggregate utility of the n sole proprietorships is given by: $\sum_{i=1}^{n} I_i(W_{it}) \equiv I(W_{1t},...,W_{2t})$. The social planner solves the following optimization problem:

$$I^o = \max_{W_{10},W_{20},...,W_{n0}} I(W_{10},...,W_{n0}) \quad subject\ to \quad \sum_{i=1}^{n} W_{i0} = W_0 \tag{45}$$

Collecting the first-order conditions gives the following proposition.

Proposition 8 *A social planner optimally allocates wealth across n sole proprietorships as follows:*

$$\left(\frac{W_{10}^o}{a_1}\right)^{\gamma_1} = \left(\frac{W_{20}^o}{a_2}\right)^{\gamma_2} = ... = \left(\frac{W_{n0}^o}{a_n}\right)^{\gamma_n} \quad \text{with} \sum_{i=1}^{n} W_{i0}^o = W_0 \tag{46}$$

This is also the wealth allocation that enables all investors voluntary to join a socially optimal group in which each investor has equal weight.

We know from corollary 5 that the utility weights under voluntary participation are given by equation (25). Combining equation (25) with equation (46), it follows that the socially optimal wealth allocation across investors also enables a group that is socially optimal under voluntary participation. Social welfare optimization is achieved by allocating more resources to more risk averse investors, as we show in the below numerical example.

6.3 A numerical example

We conclude this section by providing a numerical example that compares investors' life-time utility under social optimization with their life-time utility under voluntary participation (i.e. no redistribution) assuming in the latter case that all investors are endowed with the same amount of initial wealth.

Table 1 considers 10 investors with different coefficients of risk aversion ranging from $\gamma_1 = 0.5$ to $\gamma_{10} = 9$. Investors have a combined initial wealth of $W_0 = 100$. In the socially optimal firm (see case 2 in Table 1) all investors have equal utility weight (i.e. $\lambda_i = 0.1$ for $i = 1, ..., 10$), irrespective of their initial net worth contribution. Equivalently, the social planner can endow investors with the amount of initial wealth that induces the socially optimal utility weights under voluntary participation (see case 1 in Table 1). The socially optimal wealth allocation W_i^o follows from proposition 8 and is given in column 3. The social planner allocates the largest (lowest) amount of wealth 15.72 (2.13) to the most (least) risk averse agent in order to level the playing field in terms of utility weight. Under the socially optimal allocation investor 10 and 2 achieve the highest (-3.10) and lowest (-10.73) life-time utility I_i^o, respectively. The combined life-time utility for the ten investors, I^o, is -66.94 utils.[15]

Consider next the utility weights under voluntary participation (case 3 in Table 1), and assume all investors have the same initial endowment (i.e. $W_{i0} = 10$ for $i = 1, .., n$). This gives investor 10 ($\gamma_1 = 9$) and investor 2 ($\gamma_2 = 0.6$) the lowest (0.2%) and highest (23.6%) utility weight, respectively. More generally, the utility weights are highly skewed with the four least risk averse investors taking more than 80% of the total weight, leaving the other six investors with the remainder. Less risk averse investors clearly get more "bang for their buck". The disparity in life-time utility is as significant, with investor 10 and investor 1 achieving the lowest (-160.80) and highest (0.46) utility V_i. Investors' life-time utility ranking has been turned upside down compared to the socially optimal outcome. The potential welfare gain from social optimization to investor 10 is large (+157.69), whereas the welfare loss to investor 1 is relatively modest (-11.01). Total combined life-time utility for the firm has increased from -297.66 to -66.94, an aggregate gain in utility of 230.72.

7 Investors with Exponential Utility

Exponential utility has been the workhorse utility function in the existing literature on group decision making. Exponential utility implies constant absolute risk aversion (CARA) and leads to simple, tractable solutions. To compare and contrast the solutions in this paper with

[15] Since $U_i(c_i) = (c_i^{1-\gamma_i} - 1)/(1 - \gamma_i)$, coefficients of risk aversion above 1 generate negative utility levels. However, the utility function is well behaved in that it is monotonic, continuous and differentiable in γ_i for all $\gamma_i \geq 0$. Life-time time utility is negative for $\gamma_1 = 0.5$ and $\gamma_2 = 0.6$ due to the constant term $-1/(\rho(1 - \gamma_i))$ in the life-time utility function.

those in the existing literature, we briefly revisit our model for investors with heterogeneous CARA preferences. If $u_i(c_{it}) = -\frac{1}{\eta_i}\exp(-\eta_i c_{it})$, where $\eta_i > 0$ for all $i = 1..n$ then investors' payout and investment policies are (see Internet Appendix B for a proof):

$$c_{it}^* = \frac{\frac{1}{\eta_i}}{\sum_{j=1}^n \frac{1}{\eta_j}} rW + \frac{1}{r\eta_i}\left(\rho + \frac{1}{2}\xi^2 - r\right) + \left[\frac{1}{\eta_i}\ln\lambda_i - \frac{\frac{1}{\eta_i}}{\sum_{j=1}^n \frac{1}{\eta_j}}\left(\sum_{j=1}^n \frac{1}{\eta_j}\ln\lambda_j\right)\right] \tag{47}$$

$$\omega_t^* = \left(\frac{A_t}{W_t}\right)^* = \frac{\xi}{\sigma}\frac{1}{r}\sum_{i=1}^n \frac{1}{\eta_i}\frac{1}{W_t} \tag{48}$$

The solution for c_{it}^* and ω_t^* collapses to the Merton (1969, Equations (64) and (65)) solution for the single investor case ($n = 1$). The main addition to the multiple investor case is the term in square brackets in equation (47), which reflects side payments from more risk averse investors with low utility weight (λ_i) to relatively less risk averse investors with a larger utility weight. One can verify that these side payments net out across investors since

$$\sum_{i=1}^n c_i^* = rW + \frac{\rho + \frac{1}{2}\xi^2 - r}{r}\sum_{i=1}^n \frac{1}{\eta_i} \tag{49}$$

Payout is linear in net worth with CARA utility, whereas our model with CRRA utility can generate a rich variety of non-linear compensation schemes. The absolute dollar amount invested in risky assets (A_t) remains constant with CARA. Therefore, as net worth increases the proportion of net worth invested in risky assets falls. In contrast, the ratio of risky assets to net worth increases (is constant) in W_t for investors with heterogeneous (homogenous) CRRA preferences. We do not discuss the CARA solution in further detail as its properties are well known. Furthermore, this class of utility functions and its solutions are behaviorally less plausible.

8 Empirical Implications and Conclusions

Our paper provides empirical implications and conclusions that are relevant for a variety of literatures in financial economics.

8.1 Risk aversion and group decision making

The literature offers two competing hypotheses for group decisions. The group shift hypothesis (e.g., Moscovici and Zavalloni (1969) ; Kerr (1992)) and the diversification of opinions hypothesis (e.g., Sah and Stiglitz (1986, 1988)). Although the diversifications of opinions hypothesis appears to enjoy strong empirical support (see Bar, Kempf and Ruenzi (2010)), it struggles to explain the puzzling observation that the average group is more (less) risk averse than the average individual in high (low) risk situations. Our model provides a possible explanation by showing that group decisions are not merely a weighted average of individuals' preferred decisions but, crucially, the weights vary with the state of the economy. In our model, more weight shifts towards the least risk averse agent during booms, whereas in busts increased weight shifts towards the most risk averse agent. The group's risk aversion is determined primarily by the most risk averse agent when net worth falls to critically low levels. During "normal" times the member with the median level of risk aversion exerts most influence (consistent with the findings of the experimental study by Ambrus et al. (2013)). As such our model reconciles the diversification of opinions hypothesis with the group shift hypothesis.

8.2 Corporate capital structure

Adrian and Shin (2010), among others, provide empirical evidence that leverage is procyclical for banks and other financial intermediaries - that is, leverage is high during booms and low during busts. They argue that procyclical leverage can be seen as a consequence of the active management of Value-at-Risk (VaR) by financial intermediaries through adjustments of their balance sheets. Our model is the first to show that heterogeneity in managers' risk preferences can also lead to procyclical leverage, and be a significant source of intertemporal variation in a firm's debt ratio.

Chava and Purnanandam (2010) show that CEOs risk preferences affect leverage and cash-holding policies, whereas CFOs risk preferences are relatively more important in explaining debt maturity structure and accrual decisions. They conclude that "closer attention should be paid to the risk preferences and attitudes of managers to better understand the corporate financial decision making". Our model explicitly links a firm's investment and financing decisions to individual managers' risk preferences.

8.3 Payout and compensation

Our model predicts that executive compensation will be tailored to managers' risk preferences, with the less (more) risk averse executives having claims that tend to be relatively more convex (concave) in the firm's net worth. Importantly, a manager's optimal compensation contract not only depends on her own level of risk aversion, but also on the risk preferences of the other decision makers. An executive may opt for a contract that is relatively performance sensitive when matched with more risk averse executives, and choose a contract that is relatively performance insensitive when matched with less risk averse co-workers. These findings generate new empirical hypotheses regarding relative compensation within firms and internal governance more generally.

We show that contracts resembling preferred stock are an efficient way to compensate a group of investors with heterogeneous risk preferences. The contracts we derive cap the payout of more risk averse investors at lower levels in good times, but gives the more risk averse investors priority in terms of payout in bad times. This novel rationale for using preferred stock may help explain the huge variety of preferred stock contracts that we observe in the venture capital sector.

Graham et al. (2013) provide empirical evidence that pay performance sensitivity decreases with risk aversion. They find that risk averse CEOs are significantly more likely to be compensated by salary and less likely to be compensated with performance related packages such as stock, options and bonuses. They also find that female CEOs, on average, are less likely to accept riskier pay packages.

8.4 Governance and control

Our model shows that increasing heterogeneity in investors' risk preferences leads, all else equal, to higher utility weight concentration, with less risk averse investors having the larger utility weights. As a result, investors who start ex ante with equal wealth endowments may ex post end up with highly unequal say or influence on the firm's decision making. Utility weights, control rights and cash flow rights are proportional to the amount of capital invested if and only if all investors have logarithmic utility ($\gamma = 1$). With homogenous risk preferences, the Herfindahl index for the utility weights of n investors converges to $1/n$ or 1, if all investors become risk neutral or infinitely risk averse, respectively. In the latter case (almost) all utility weight is concentrated in the hands of the firm's largest capital contributor. Our model shows that an individual investor's control over the group's financial policies is time-varying and potentially non-monotonic in the firm's net worth.

The utility weights we obtain under voluntary participation are not socially optimal. They are, however, the only weights that preserve investors' initial wealth distribution. Aggregate utility can be increased by giving equal weight to all investors. Since investors cannot directly transfer utility, this would require a redistribution of investors' initial wealth.

Appendix

Proof of Proposition 1

The Hamilton–Jacobi–Bellman (HJB) equation for the optimization problem (5) derived by the method of dynamic programming described in Dixit and Pindyck (1994) is

$$\rho J(W_t) = \max_{\{c_{it},\omega_t\}} \sum_{i=1}^{n} \lambda_i u_i(c_{it}) + \left[(\omega_t(\mu - r) + r)\, W_t - \sum_{i=1}^{n} c_{it}\right] J^{'}(W_t) + \frac{1}{2}\sigma^2\omega_t^2 W_t^2 J^{''}(W_t) \tag{50}$$

The first order conditions are

$$\omega_t^* = -\frac{\xi}{\sigma}\frac{J^{'}(W_t)}{W_t J^{''}(W_t)} \qquad \text{and} \qquad c_{it}^* = \left(\frac{J^{'}(W_t)}{\lambda_i}\right)^{-\frac{1}{\gamma_i}} \tag{51}$$

Substituing the first order conditions into the HJB yields

$$\sum_{i=1}^{n}\left[\left(\frac{1}{1-\gamma_i} - 1\right)\lambda_i^{\frac{1}{\gamma_i}} J'(W_t)^{-\frac{1}{\gamma_i}+1} - \frac{\lambda_i}{1-\gamma_i}\right] + rW_t J'(W_t) - \frac{1}{2}\xi^2\frac{J'(W_t)^2}{J''(W_t)} - \rho J(W_t) = 0 \tag{52}$$

We omit the time index t from now. The non-linear ordinary differential equation cannot be solved analytically in terms of W. However, tractable results are made possible by introducing an auxiliary state variable $z \equiv -\ln(J'(W))$, which is similar to the method introduced by Sethi, Taksar and Presman (1992). By the definition of z, we have $J'(W) = e^{-z}$ and

$J''(W) = -\frac{e^{-z}}{W'(z)}$. Substituting into equation (52) shows

$$\sum_{i=1}^{n}\left[\left(\frac{1}{1-\gamma_i} - 1\right)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{1-\gamma_i}{\gamma_i}z} - \frac{\lambda_i}{1-\gamma_i}\right] + rW(z)e^{-z} + \frac{1}{2}\xi^2 W'(z)e^{-z} - \rho J(W(z)) = 0 \tag{53}$$

Differentiating the transformed ODE with respect to z, and then dividing both sides by e^{-z} yields a second-order linear inhomogeneous ODE in z

$$\sum_{i=1}^{n}\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}} + \left(r - \frac{1}{2}\xi^2 - \rho\right)W'(z) + \frac{1}{2}\xi^2 W''(z) - rW(z) = 0 \tag{54}$$

The solution has two components, a particular solution W_p and a complementary solution W_c, i.e $W = W_p + W_c$. One can verify that

$$W_p(z) = \sum_{i=1}^{n} G_i e^{\frac{z}{\gamma_i}} \quad \text{with} \qquad G_i = \frac{\gamma_i^2 \lambda_i^{\frac{1}{\gamma_i}}}{r\gamma_i^2 + (\rho + \frac{\xi^2}{2} - r)\gamma_i - \frac{\xi^2}{2}} \equiv a_i \lambda_i^{\frac{1}{\gamma_i}} \tag{55}$$

The complementary solution is of the form $W_c(z) = H_1 e^{b_+ z} + H_2 e^{b_- z}$, where H_1 and H_2 are constants to be determined and $b_\pm = \frac{\left(\rho + \frac{1}{2}\xi^2 - r\right) \pm \sqrt{(\rho + \frac{1}{2}\xi^2 - r)^2 + 2\xi^2 r}}{\xi^2}$. It can be shown that $b_+ > 0$ and $b_- < -1$.

The particular solution captures the expected lifetime utility under a particular payout and investment policy. The complementary solution captures the value from growth options and abandonment/outside options, which investors do not have. Therefore, the complementary part must be zero, i.e. $H_1 = H_2 = 0$, as otherwise a bubble component would be added that explodes for the negative root when $z \to -\infty$ ($W \to 0$), and for the positive root when $z \to +\infty$ ($W \to +\infty$). Therefore,

$$W(z) = \sum_{i=1}^{n} G_i e^{\frac{z}{\gamma_i}} \tag{56}$$

where G_i is defined as in (10). A solution to the optimization problem needs to satisfy the transversality condition and feasibility condition. These conditions are satisfied if

$$\gamma_i > \frac{-(\rho + \frac{1}{2}\xi^2 - r) + \sqrt{(\rho + \frac{1}{2}\xi^2 - r)^2 + 2r\xi^2}}{2r} \equiv \gamma^* \in (0,1) \tag{57}$$

This is an analog of the condition (equation(40)) shown in Merton (1969). Substituting $W(z)$ into equation (53) gives equation (11), while substituting into the FOCs (51) gives equation (6) and (7).

Define the right side of equation (50) as $\phi(W)$. One could then verify the second order conditions by showing that

$$\frac{\partial^2\phi}{\partial\omega^2}|_{c_i=c_i^*,\omega=\omega^*} = -\frac{\sigma^2 W(z)^2}{W'(z)}e^{-z} < 0 \qquad \frac{\partial^2\phi}{\partial c_i^2}|_{c_i=c_i^*,\omega=\omega^*} = -\lambda_i\gamma_i(c_i^*)^{-1-\gamma_i} < 0$$

As $\frac{\partial^2\phi}{\partial\omega\partial c_i} = 0$ and $\frac{\partial^2\phi}{\partial c_i\partial c_j} = 0$ for $i \neq j$, the determinants of the leading principle minors of the Hessian matrix are $\det(H_1) = \frac{\partial^2\phi}{\partial\omega^2}$ and $\det(H_k) = \frac{\partial^2\phi}{\partial\omega^2}\prod_{i=1}^{k-1}\frac{\partial^2\phi}{\partial c_i^2}$ for $k > 1$. The sequence does alternate in signs which completes the verification. ■

Proof of Proposition 2

Suppose $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$. One can show that $\lim_{z\to-\infty} c_{it} = \lim_{z\to-\infty} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}} = 0$, and

$$\lim_{z\to+\infty}\frac{\partial c_{it}}{\partial W_t} = \lim_{z\to+\infty}\frac{\frac{1}{\gamma_i}\left(\lambda_i e^z\right)^{\frac{1}{\gamma_i}}}{W'(z)} = \lim_{z\to+\infty}\left[a_i + \sum_{j=1}^{i-1}\frac{a_j\gamma_i}{\gamma_j}\frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}}e^{z\left(\frac{1}{\gamma_j}-\frac{1}{\gamma_i}\right)} + \sum_{k=i+1}^{n}\frac{a_k\gamma_i}{\gamma_k}\frac{\lambda_k^{\frac{1}{\gamma_k}}}{\lambda_i^{\frac{1}{\gamma_i}}}e^{z\left(\frac{1}{\gamma_k}-\frac{1}{\gamma_i}\right)}\right]^{-1} \tag{58}$$

If $\gamma_i > (<)\,\gamma_l$, $\lim_{z\to+\infty} e^{\left(\frac{1}{\gamma_l}-\frac{1}{\gamma_i}\right)z} = +\infty\,(0)$, and $\lim_{z\to-\infty} e^{\left(\frac{1}{\gamma_l}-\frac{1}{\gamma_i}\right)z} = 0\,(+\infty)$, which implies

$$\Longrightarrow \lim_{z\to+\infty}\frac{\partial c_{1t}}{\partial W_t} = \frac{1}{a_1}, \qquad \lim_{z\to+\infty}\frac{\partial c_{it}}{\partial W_t} = 0 \qquad \text{for} \qquad i = 2,3,...,n \tag{59}$$

The second derivative of the payout policy with respect to total net worth is

$$\frac{\partial^2 c_{it}}{\partial W_t^2} = \frac{(\lambda_i e^z)^{\frac{1}{\gamma_i}}}{\gamma_i W'(z)^3}\left\{\sum_{j=1}^{i-1}\frac{a_j}{\gamma_j}(\lambda_j e^z)^{\frac{1}{\gamma_j}}\left(\frac{1}{\gamma_i}-\frac{1}{\gamma_j}\right)+\sum_{k=i+1}^{n}\frac{a_k}{\gamma_k}(\lambda_k e^z)^{\frac{1}{\gamma_k}}\left(\frac{1}{\gamma_i}-\frac{1}{\gamma_k}\right)\right\} \tag{60}$$

For $i = 1$ and $i = n$ we have, respectively:

$$\frac{\partial^2 c_{1t}}{\partial W_t^2} = \frac{(\lambda_1 e^z)^{\frac{1}{\gamma_i}}}{\gamma_1 W'(z)^3}\sum_{k=2}^{n}\left[\frac{a_k}{\gamma_k}(\lambda_k e^z)^{\frac{1}{\gamma_k}}\left(\frac{1}{\gamma_1}-\frac{1}{\gamma_k}\right)\right] > 0 \tag{61}$$

$$\frac{\partial^2 c_{nt}}{\partial W_t^2} = \frac{(\lambda_n e^z)^{\frac{1}{\gamma_n}}}{\gamma_n W'(z)^3}\sum_{j=1}^{n-1}\left[\frac{a_j}{\gamma_j}(\lambda_j e^z)^{\frac{1}{\gamma_j}}\left(\frac{1}{\gamma_n}-\frac{1}{\gamma_j}\right)\right] < 0 \tag{62}$$

For $i = 2, ..., n-1$, the first term in the bracket of (60), which is negative, will be dominated by the positive second term initially. However, as z increases, the first term will start to dominate as $\gamma_j < \gamma_i < \gamma_k$, which makes the payout functions for intermediate risk averse investors first convex and then concave as total net worth increases. Moreover, the inflexion point decreases as the degree of risk aversion increases.

Lastly, setting $\gamma_1 = ... = \gamma_n = \gamma$ shows $\frac{\partial^2 c_{it}}{\partial W_t^2} = 0$ and $c_{it} = \frac{\lambda_i^{\frac{1}{\gamma}} W_t}{a\sum_{j=1}^{n}\lambda_j^{\frac{1}{\gamma}}}$. ■

Proof of Proposition 3

As shown in equation (6) in proposition 1,

$$\omega_t^* = \frac{\xi}{\sigma}\sum_{i=1}^{n}\frac{1}{\gamma_i}\omega_i(z) = \frac{\xi}{\sigma}\left[\frac{1}{\gamma_1}\omega_1(z) + \sum_{i=2}^{n-1}\frac{1}{\gamma_i}\omega_i(z) + \frac{1}{\gamma_n}\omega_n(z)\right] \tag{63}$$

and for each ω_i, we could rewrite it as

$$\omega_i = \frac{G_i e^{\frac{z}{\gamma_i}}}{\sum_{j=1}^{n} G_j e^{\frac{z}{\gamma_j}}} = \frac{1}{1+\sum_{j\neq i}\frac{a_j\lambda_j^{\frac{1}{\gamma_j}}}{a_i\lambda_i^{\frac{1}{\gamma_i}}}e^{z\left(\frac{1}{\gamma_j}-\frac{1}{\gamma_i}\right)}} \equiv \frac{1}{1+\sum_{j\neq i}k_j(z)} \tag{64}$$

If $\gamma_i > (<) \gamma_j$, $\lim_{z\to+\infty} k_j(z) = +\infty\,(0)$ and $\lim_{z\to-\infty} k_j(z) = 0\,(+\infty)$, which implies

$$\lim_{z\to+\infty} \omega_1(z) = 1; \quad \lim_{z\to-\infty} \omega_1(z) = 0 \tag{65}$$

$$\lim_{z\to+\infty} \omega_n(z) = 0; \quad \lim_{z\to-\infty} \omega_n(z) = 1 \tag{66}$$

$$\lim_{z\to+\infty} \omega_i(z) = 0; \quad \lim_{z\to-\infty} \omega_i(z) = 0 \quad \text{for} \quad i = 2, ..., n-1 \tag{67}$$

It then follows that:

$$\lim_{z\to+\infty} \omega(z) = \frac{\xi}{\sigma}\frac{1}{\gamma_1} \qquad \text{and} \qquad \lim_{z\to-\infty} \omega(z) = \frac{\xi}{\sigma}\frac{1}{\gamma_n} \tag{68}$$

The inequality (17) regarding NDR_t follows directly from (68), the monotonicity of ω_t^* w.r.t z (see corollary (1)) and the monotonicity of NDR_t w.r.t ω_t^*. ■

Proof of Proposition 4

Suppose that $\gamma_1 = ... = \gamma_n = \gamma$ and arrange W_{i0} such that $W_{10} \geq W_{20} \geq ..... \geq W_{n0}$. By the assumption that the utility weights add up to 1, we can show that

$$\lambda_i = \frac{\left(\frac{W_{i0}}{W_0}\right)^{\gamma}}{\sum_{j=1}^{n}\left(\frac{W_{j0}}{W_0}\right)^{\gamma}} \tag{69}$$

The level of concentration measured by the Herfindahl index can then written as

$$H = \sum_{i=1}^{n} \lambda_i^2 = \frac{\sum_{i=1}^{n}\left(\frac{W_{i0}}{W_0}\right)^{2\gamma}}{\left[\sum_{j=1}^{n}\left(\frac{W_{j0}}{W_0}\right)^{\gamma}\right]^2} \tag{70}$$

$0 < H < 1$ follows immediately from the Cauchy-Schwarz inequality. Moreover, $H = \frac{1}{n}$ if

$\gamma = 1$. We could further show that

$$\frac{\partial H}{\partial \gamma} = \frac{2}{\left(\sum_{j=1}^{n}\left(\frac{W_{j0}}{W_0}\right)^{\gamma}\right)^3}\left[\sum_{i=1}^{n}\sum_{j>i}\left(\frac{W_{i0}}{W_0}\right)^{\gamma}\left(\frac{W_{j0}}{W_0}\right)^{\gamma}\left(\left(\frac{W_{i0}}{W_0}\right)^{\gamma} - \left(\frac{W_{j0}}{W_0}\right)^{\gamma}\right)\ln\left(\frac{W_{i0}}{W_{j0}}\right)\right] \geq 0 \tag{71}$$

as $W_{i0} \geq W_{j0}$ for all $j > i$. ■

Proof of Proposition 5

The first equality follows directly from corollary 5. We prove the second equality by raising both sides of equation (31) by the power of γ_i, i.e. $\left(\frac{W_{it}^s}{W_{i0}^s}\right)^{\gamma_i} = e^{\gamma_i\left[(\alpha_i - \frac{\sigma_i^2}{2})t + \sigma_i B_t\right]}$ for $i = 1, ..., n$. Further simplification shows that the exponential power is indeed independent of risk aversion: $\gamma_i\left[(\alpha_i - \frac{\sigma_i^2}{2})t + \sigma_i B_t\right] = -\left(\rho - \frac{1}{2}\xi^2 - r\right)t + \xi B_t$, which yields

$$\frac{\left(\frac{W_{it}^s}{a_i}\right)^{\gamma_i}}{\left(\frac{W_{jt}^s}{a_j}\right)^{\gamma_j}} = \frac{\left(\frac{W_{i0}^s}{a_i}\right)^{\gamma_i}}{\left(\frac{W_{j0}^s}{a_j}\right)^{\gamma_j}}\frac{e^{\gamma_i\left[(\alpha_i - \frac{\sigma_i^2}{2})t + \sigma_i B_t\right]}}{e^{\gamma_j\left[(\alpha_j - \frac{\sigma_j^2}{2})t + \sigma_j B_t\right]}} = \frac{\left(\frac{W_{i0}^s}{a_i}\right)^{\gamma_i}}{\left(\frac{W_{j0}^s}{a_j}\right)^{\gamma_j}} = \frac{\lambda_i}{\lambda_j} \tag{72}$$

■

Proof of Proposition 6

Since $c_i = (\lambda_{it}e^z)^{\frac{1}{\gamma_i}} = \frac{W_{it}}{a_i}$ and $c_{it}^s = \frac{W_{it}^s}{a_i}$, it is enough to show that W_{it} and W_{it}^s are two identical processes. By substituting the optimal investment and payout policy (equation (6) and (7)) into equation (4), one can get

$$dW_t = \left[\xi^2 W_t'(z) + rW_t(z) - \sum_{i=1}^{n}(\lambda_i e^z)^{\frac{1}{\gamma_i}}\right]dt + \xi W_t'(z)dB \equiv \mu_W dt + \sigma_W dB \tag{73}$$

Applying Itô's Lemma to z gives

$$dz = \left[\frac{\xi^2 W_t'(z) + rW_t(z) - \sum_{i=1}^{n}(\lambda_i e^z)^{\frac{1}{\gamma_i}}}{W'(z)} - \frac{1}{2}\xi^2\frac{W_t''(z)}{W_t'(z)}\right]dt + \xi dB \equiv \mu_z dt + \sigma_z dB \tag{74}$$

In the sole proprietorship, the net worth process for each investor, W_{it}^s, evolves according to equation (30). We would like to compare the two claim value processes W_{it} and W_{it}^s to see whether joining a partnership with the weight $\lambda_i = \left(\frac{W_{i0}}{a_i}\right)^{\gamma_i} e^{-z_0}$ can create value for individuals. Applying Itô's lemma to $W_{it} = a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}}$ shows that

$$\frac{dW_{it}}{W_{it}} = \left[\frac{\mu_z}{\gamma_i} + \frac{1}{2}\frac{\sigma_z^2}{\gamma_i^2}\right] dt + \frac{\sigma_z}{\gamma_i} dB \tag{75}$$

All we need to check is the following: $\frac{\mu_z}{\gamma_i} + \frac{1}{2}\frac{\sigma_z^2}{\gamma_i^2} = \frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i}$ and $\frac{\sigma_z}{\gamma_i} = \frac{\xi}{\gamma_i}$. The second equality is satisfied as $\sigma_z = \xi$. The first equality also holds as

$$\begin{aligned}
& \frac{\xi^2}{\gamma_i} + \frac{rW(z) - \sum_{i=1}^n c_i}{\gamma_i W'(z)} - \frac{1}{2}\xi^2 \frac{W''(z)}{\gamma_i W'(z)} + \frac{1}{2}\frac{\xi^2}{\gamma_i^2} = \frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i} \\
\Leftrightarrow \quad & \sum_{i=1}^n \left(r - \frac{\xi^2 - 2(\rho + \frac{1}{2}\xi^2 - r)\gamma_i - 2r\gamma_i^2}{-2\gamma_i^2} + \frac{\rho + \frac{1}{2}\xi^2 - r}{\gamma_i} - \frac{\xi^2}{2\gamma_i^2} \right) W_i(z) = 0 \\
\Leftrightarrow \quad & \sum_{i=1}^n \left[-(\frac{\rho + \frac{1}{2}\xi^2 - r}{\gamma_i}) + -(\frac{\rho + \frac{1}{2}\xi^2 - r}{\gamma_i}) \right] W_i(z) = 0
\end{aligned}$$

Given that the initial values are the same, i.e. $W_{i0} = W_{i0}^s$, we can conclude that W_{it} and W_{it}^s are indeed two identical processes, that is, $W_{it} = W_{it}^s$ for all t and $i = 1, ..., n$. Similarly, we could also conclude that c_{it} and c_{it}^s are also two identical processes given $c_{i0} = c_{i0}^s$. ■

Proof of Proposition 7

The optimization consists of the objective function (40) and the intertemporal constraint (8), which gives

$$L^o = \max_{\{\lambda_1, \ldots, \lambda_n\}} \sum_{i=1}^n \frac{J_i(z)}{\lambda_i} + \delta \left(\sum_{i=1}^n W_i - \sum_{i=1}^n a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{1}{\gamma_i}} \right) \tag{76}$$

where δ is the Langrange multiplier (shadow price). Differentiating equation (8) with respect to λ_i gives $\frac{\partial z}{\partial \lambda_i} = -\frac{\frac{a_i}{\gamma_i}\lambda_i^{\frac{1-\gamma_i}{\gamma_i}} e^{\frac{z}{\gamma_i}}}{W'(z)}$. Substituting this into the first-order conditions (41) gives

$$\frac{1}{\lambda_i} = \frac{\sum_{j=1}^n \frac{a_j}{\gamma_j}\lambda_j^{\frac{1-\gamma_j}{\gamma_j}} e^{\frac{z}{\gamma_j}}}{W'(z)} \iff \frac{1}{\lambda_i e^z} = \frac{\sum_{j=1}^n \frac{a_j}{\gamma_j}(\lambda_j e^z)^{\frac{1-\gamma_j}{\gamma_j}}}{W'(z)} \equiv \delta \tag{77}$$

As a result, we have $\lambda_1 = \lambda_2 =, ..., = \lambda_n = \lambda$.

To check the second order conditions, note that $\frac{\partial^2 L^o}{\partial \delta^2} = 0$, and $\frac{\partial^2 L^o}{\partial \lambda_i \partial \lambda_j} = 0$ for $i \neq j$. With equal weighting scheme, we also have $\frac{\partial^2 L^o}{\partial^2 \lambda_i} = -\frac{a_i}{\gamma_i}\lambda_i^{-2}\delta^{-\frac{1-\gamma_i}{\gamma_i}} < 0$ and $\frac{\partial^2 L^o}{\partial \delta \partial \lambda_i} = -\frac{a_i}{\gamma_i}\delta^{-\frac{1}{\gamma_i}}\lambda_i^{-1} < 0$. These imply the determinants of the principle matries for the Bordered Hessian are of the following form: $\det(H_k^o) = -(\frac{\partial^2 L^o}{\partial \delta \partial \lambda_1})^2$ for $k = 1$ and $\det(H_k^o) = -\sum_{i=1}^k \left[\left(\frac{\partial^2 L^o}{\partial \delta \partial \lambda_i}\right)^2 \prod_{l \neq i} \frac{\partial^2 L^o}{\partial \lambda_l^2}\right]$ for $k \neq 1$. As $\frac{\partial^2 L^o}{\partial^2 \lambda_i^2} < 0$, the sequence of leading principle minors starts with $\det(H_1^o) < 0$ and alternates in signs afterwards, which indicates that the equal weighting scheme is indeed optimal. ■

Proof of Proposition 8

The optimization problem (45) has the following Lagrangian function:

$$L^o = \sum_{i=1}^n a_i^{\gamma_i} \frac{W_{i0}^{1-\gamma_i}}{1-\gamma_i} - \kappa\left(\sum_{i=1}^n W_{i0} - W_0\right) \tag{78}$$

where κ is the Langrange multiplier. The Euler conditions are $\left(\frac{a_i}{W_{i0}^o}\right)^{\gamma_i} = \kappa$ for all i which implies the equation (46). ■

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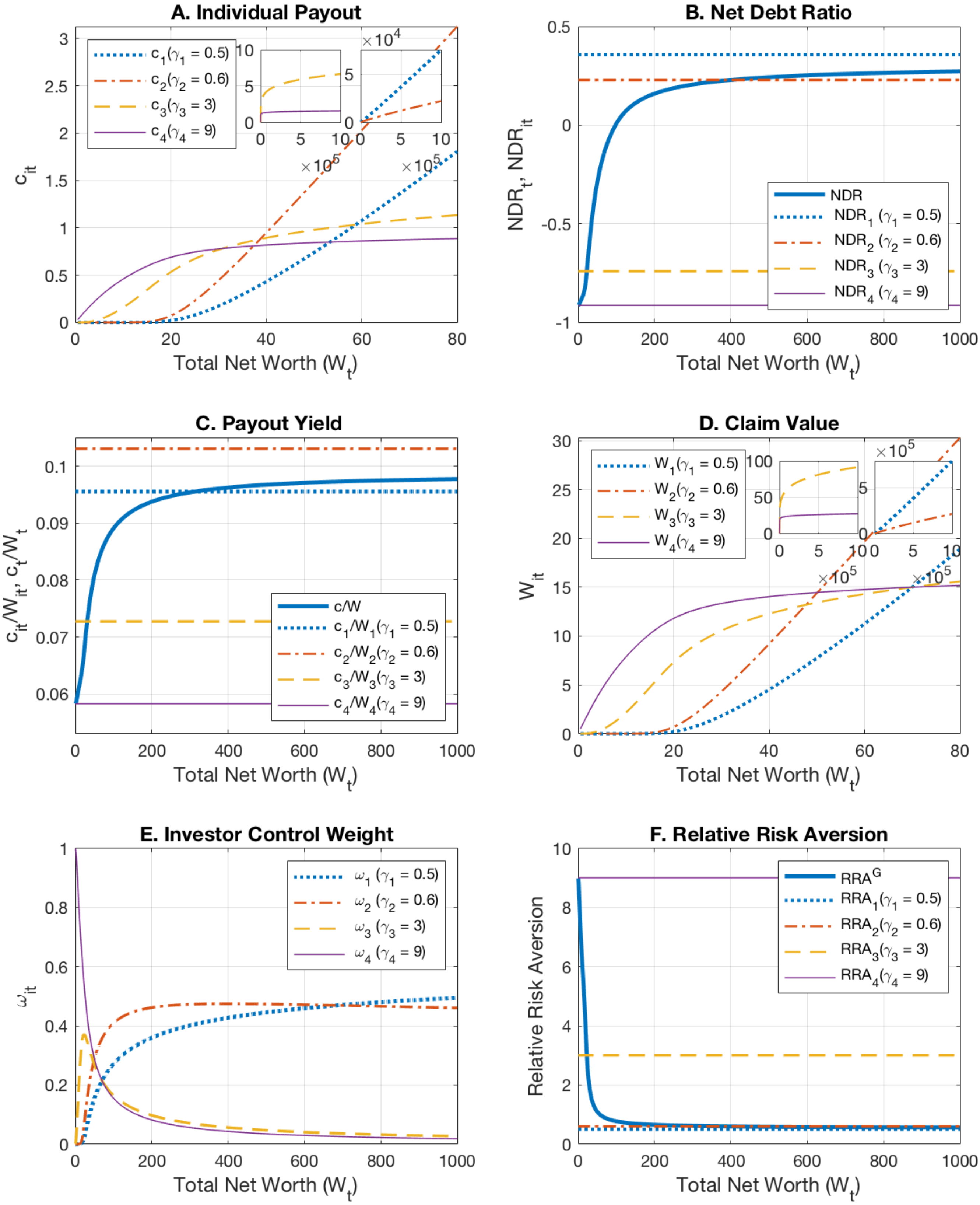

Figure 1: Corporate policies and claim values

The parameter values for the graphs are risk free rate $(r) = 0.05$, subjective discount rate $(\rho) = 0.1$, drift $(\mu) = 0.12$, volatility $(\sigma) = 0.3$ and the correponding Sharpe ratio $(\xi) = 0.233$. In this figure, we consider a group of 4 investors with heterogeneous risk aversion $(\gamma_1 = 0.5, \gamma_2 = 0.6, \gamma_3 = 3$, and $\gamma_4 = 9)$ and exogenous initial endowments $(W_{10} = 15, W_{20} = 25, W_{30} = 15, W_{40} = 15)$. Panels A, B, C, D, E and F plot, respectively investors' individual payout, the firm's Net Debt Ratio, the firm's payout yield, investors' certainty equivalent claim values, investor control weight and group's level of relative risk aversion as a function of the firm's net worth W_t. In panels B and C, the thick solid lines represent the NDR and payout yield based on group decision making, whereas the other lines represent an individual investor's optimal policy.

		Case 1			Case 2			Case 3					
Investor	**Risk Aversion**	**Socially Optimal Wealth Allocation**			**Socially Optimal Utility Weights**			**Utility Weights under Voluntary Participation (no redistribution)**					
	γ_i	W_i^o	I_i^o	I^o	λ_i^o	V_i^o	V_G^o	W_{i0}	λ_i	V_i	V_G	$V_i^o - V_i$	$V_G - I^o$
1	0.5	2.13	-10.55	-66.94	0.1	-10.55	-66.94	10	0.227	0.46	-297.66	-11.01	-230.72
2	0.6	2.58	-10.73	-66.94	0.1	-10.73	-66.94	10	0.236	-0.45	-297.66	-10.28	-230.72
3	1.5	6.58	-9.17	-66.94	0.1	-9.17	-66.94	10	0.196	-3.67	-297.66	-5.5	-230.72
4	2	8.21	-8.20	-66.94	0.1	-8.20	-66.94	10	0.155	-4.94	-297.66	-3.25	-230.72
5	3	10.55	-6.69	-66.94	0.1	-6.69	-66.94	10	0.089	-8.00	-297.66	1.32	-230.72
6	4	12.12	-5.62	-66.94	0.1	-5.62	-66.94	10	0.048	-12.62	-297.66	7.00	-230.72
7	5	13.25	-4.84	-66.94	0.1	-4.84	-66.94	10	0.026	-20.15	-297.66	15.31	-230.72
8	6	14.1	-4.25	-66.94	0.1	-4.25	-66.94	10	0.013	-32.82	-297.66	28.58	-230.72
9	7	14.76	-3.78	-66.94	0.1	-3.78	-66.94	10	0.007	-54.67	-297.66	50.88	-230.72
10	9	15.72	-3.10	-66.94	0.1	-3.1	-66.94	10	0.002	-160.80	-297.66	157.69	-230.72

Table 1: Investors' lifetime utility and aggregate group utility under social optimization and voluntary participation

The parameter values for this example are the risk free rate (r) = 0.05, subjective discount rate (ρ) = 0.1, drift (μ) = 0.12, volatility (σ) = 0.3 and the correponding Sharp ratio (ξ) = 0.233. In case 1 the social planner optimally allocates a total of 100 in funding to 10 sole proprietorships, where W_i^o is the optimal initial amount of wealth allocated to investor i, I_i^o is the life-time utility investor i gets from operating on her own, and I^o is the aggregate utility generated by the 10 sole proprietorships given the optimal initial allocation W_i^o. In case 2 the social planner endows one single firm with 100 and implements the socially optimal utility weights by giving equal weight to all investors ($\lambda_i^o = 0.1$ for all i). Given this weighting scheme (λ_i^o), V_i^o is investor i's life-time utility from joining the group and V_G^o is the group's aggregate utility. In case 3 each investor has an initial endowment W_{i0} equal to 10 and receives a utility weight λ_i that induces her voluntarily to join the group. V_i is investor i's life-time utility from joining the group, V_G is the aggregate utility, $V_i^o - V_i$ is the difference between investor i's life-time utility under social optimization and voluntarily participation, and $V_G - I^o$ is the aggregate welfare loss.

Internet Appendix for "Financial Policies and Internal Governance with Heterogeneous Risk Preferences"

A Appendix: Proofs of Corollaries

Proof of Corollary 1

Differentiate equation (6) with respect to W_t shows $\frac{\partial \omega_t^*}{\partial W_t} = \frac{\xi}{\sigma} \frac{W_t''(z)W_t(z) - W_t'(z)^2}{W_t(z)^2} \frac{\partial z}{\partial W_t}$. This implies

$$
\begin{aligned}
\operatorname{sign}\left(\frac{\partial \omega_t^*}{\partial W_t}\right) &= \operatorname{sign}\left[W_t''(z)W_t(z) - W_t'(z)^2\right] \\
&= \operatorname{sign}\left[\sum_{i=1}^{n} \frac{a_i}{\gamma_i^2} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}} \sum_{i=1}^{n} a_i \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}} - \left(\sum_{i=1}^{n} \frac{a_i}{\gamma_i} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}}\right)^2\right] \\
&= \operatorname{sign}\left[\sum_{i,j=1}^{n} a_i a_j \lambda_i^{\frac{1}{\gamma_i}} \lambda_j^{\frac{1}{\gamma_j}} e^{\frac{z}{\gamma_i} + \frac{z}{\gamma_j}} \frac{1}{\gamma_i} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j}\right)\right] \\
&= \operatorname{sign}\left[\sum_{i \neq j}^{n} a_i a_j \lambda_i^{\frac{1}{\gamma_i}} \lambda_j^{\frac{1}{1-\gamma_j}} e^{\frac{z}{\gamma_i} + \frac{z}{\gamma_j}} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j}\right)^2\right]
\end{aligned}
\tag{A.1}
$$

which means that if $\gamma_1 = ... = \gamma_n = \gamma$, $\frac{\partial \omega_t^*}{\partial W_t} = 0$. Otherwise $\frac{\partial \omega_t^*}{\partial W_t} > 0$. ■

Proof of Corollary 2

The first part of the corollary follows directly from equation (12) . For the second half, the control over investment defined in equation (9) suggests that strict proportional to λ_i implies $\gamma_i = 1$ for all i. When all investors have the same level of risk aversion, equation (8) becomes

$$
W_t = a e^{\frac{z}{\gamma}} \sum_{i=1}^{n} \lambda_i^{\frac{1}{\gamma}} \implies z = \gamma \ln \frac{W_t}{a \sum_{i=1}^{n} \lambda_i^{\frac{1}{\gamma}}}
\tag{A.2}
$$

Substituting the above into equation (9) shows that $\omega_{it} = \frac{\lambda_i^{\frac{1}{\gamma}}}{\sum_{j=1}^{n} \lambda_j^{\frac{1}{\gamma}}}$. Proportional control result follows if we further assume all investors have log utility ($\gamma = 1$). ■

Proof of Corollary 3

This follows directly from equation (13) and (14).

Proof of Corollary 4

The group's level of relative risk aversion is defined in (18) can be simplified as

$$\begin{aligned}
RRA_G &= -W\frac{J_{WW}}{J_W} = -W\frac{\left(\frac{\partial W}{\partial z}\right)^{-3}\left(\frac{\partial^2 J}{\partial z^2}\frac{\partial W}{\partial z} - \frac{\partial J}{\partial z}\frac{\partial^2 W}{\partial z^2}\right)}{\frac{\partial J}{\partial z}\frac{\partial z}{\partial W}} \\
&= -W\left[\left(\frac{\partial W}{\partial z}\right)^{-1}\frac{\partial^2 J}{\partial z^2}\left(\frac{\partial J}{\partial z}\right)^{-1} - \left(\frac{\partial W}{\partial z}\right)^{-2}\frac{\partial^2 W}{\partial z^2}\right] \\
&= \frac{W(z)}{W'(z)}\left(\frac{W''(z)}{W'(z)} - \frac{J''(z)}{J'(z)}\right) \\
&= \frac{\sum_i a_i\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}\left[\frac{\sum_i \frac{a_i}{\gamma_i^2}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}} - \frac{\sum_i \frac{a_i}{\gamma_i^2}(1-\gamma_i)\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{1-\gamma_i}{\gamma_i}z}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{1-\gamma_i}{\gamma_i}z}}\right] \\
&= \frac{\sum_i a_i\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}\left[\frac{\sum_i \frac{a_i}{\gamma_i^2}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}} - \frac{\sum_i \frac{a_i}{\gamma_i^2}(1-\gamma_i)\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{1}{\gamma_i}z}e^{-z}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{1}{\gamma_i}z}e^{-z}}\right] \\
&= \frac{\sum_i a_i\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}\left[\frac{1}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}\sum_i \frac{a_i}{\gamma_i^2}(\lambda_i e^z)^{\frac{1}{\gamma_i}}(1 - 1 + \gamma_i)\right] \\
&= \frac{\sum_i a_i\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}}{\sum_i \frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}e^{\frac{z}{\gamma_i}}} = \frac{W(z)}{W'(z)} = \frac{\sigma}{\xi}\frac{1}{\omega(z)}
\end{aligned} \tag{A.3}$$

■

Proof of Corollary 5

The first equality follows immediately from equation (24). Based on the utility function and equation (20), one could show that for $i = 1, ..., n$

$$u_i'(c_{i0}^*) = \left[(\lambda_i e^{z_0})^{\frac{1}{\gamma_i}}\right]^{-\gamma_i} = (\lambda_i e^{z_0})^{-1} = \left(\frac{a_i}{W_{i0}}\right)^{\gamma_i} = \frac{dI_i(W_{i0})}{dW_{i0}} = \left(\frac{a_i}{W_{i0}}\right)^{\gamma_i} \tag{A.4}$$

■

Proof of Corollary 6

Substituting equation (A.2) into (25) shows that

$$\lambda_i = \left(\frac{W_{i0}}{W_0}\right)^\gamma (\sum_{i=1}^n \lambda_i^{\frac{1}{\gamma}})^\gamma \implies \frac{\lambda_i}{\sum_{j=1}^n \lambda_j} = \frac{W_{i0}^\gamma}{\sum_{j=1}^n W_{j0}^\gamma} \tag{A.5}$$

Proportionality requires $\gamma = 1$. If $\gamma = 1$, we have $\frac{\lambda_i}{\sum_{j=1}^n \lambda_j} = \frac{W_{i0}}{W_0}$. ■

Proof of Corollary 7

Substituting equation (25) from corollary 5 into (8) shows the first equality. The second equality follows from equation (7). ■

Proof of Corollary 8

From corollary 7, we know that $W_{it} = a_i c_{it}$. Scaling by a_i does not change the curvature, i.e the concavity and convexity property. Similar to the proof of p proposition 2 and 3 one could show that

$$\frac{\partial W_{it}}{\partial W_t} = \frac{\frac{a_i}{\gamma_i}\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z}{\gamma_i}}}{\sum_{j=1}^n \frac{a_j}{\gamma_j}\lambda_j^{\frac{1}{\gamma_j}} e^{\frac{z}{\gamma_j}}} = \frac{1}{1 + \sum_{j\neq i} \frac{a_j}{a_i}\frac{\gamma_j}{\gamma_i}\frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)}} = \frac{1}{1 + \sum_{j\neq i} k_j(z)\frac{\gamma_j}{\gamma_i}} \tag{A.6}$$

and as a result,

$$\lim_{z\to-\infty} \frac{\partial W_{nt}}{\partial W_t} = 1 \quad \text{and} \quad \lim_{z\to-\infty} \frac{\partial W_{it}}{\partial W_t} = 0 \quad \text{for} \quad i = 1, ..., n-1 \tag{A.7}$$

$$\lim_{z\to+\infty} \frac{\partial W_{1t}}{\partial W_t} = 1 \quad \text{and} \quad \lim_{z\to+\infty} \frac{\partial W_{it}}{\partial W_t} = 0 \quad \text{for} \quad i = 2, ..., n \tag{A.8}$$

The remaining of the corollary follows directly from above and proposition 2. ■

Proof of Corollary 9

Suppose $\gamma_1 = ... = \gamma_n = \gamma$, then

$$\frac{W_{it}}{W_t} = \frac{a\lambda_i^{\frac{1}{\gamma}} e^{\frac{z}{\gamma}}}{\sum_{j=1}^n a\lambda_j^{\frac{1}{\gamma}} e^{\frac{z}{\gamma}}} = \frac{\lambda_i^{\frac{1}{\gamma}}}{\sum_{j=1}^n \lambda_j^{\frac{1}{\gamma}}} \tag{A.9}$$

which completes the proof. ■

Proof of Corollary 10

See the proof for proposition 5. ■

Proof of Corollary 11

Equation (38) is derived by substituting λ_i (25) into (36). Differentiating V_G with respect to W_{i0} gives the required shadow price. ■

Proof of Corollary 12

Equation (42) is derived by substituting $\lambda_1 = ... = \lambda_n = \lambda$ into Equation (40). For $i, k = 1, ..., n$, the shadow price follows immediately as

$$\frac{\partial V_G^o}{\partial W_{i0}} = \frac{\partial V_G^o}{\partial z}\frac{\partial z}{\partial W_{i0}} = \frac{\partial V_G^o}{\partial z}\frac{\partial z}{\partial W_0}\frac{\partial W_0}{\partial W_{i0}} = \frac{\partial V_G^o}{\partial z}\frac{\partial z}{\partial W_0} = \frac{\sum_{j=1}^n \frac{1}{\lambda}\frac{\partial J_j}{\partial z}}{\sum_{j=1}^n \frac{a_j}{\gamma_j}\left(\lambda e^z\right)^{\frac{1}{\gamma_j}}} = \frac{\partial V_G^o}{\partial W_{k0}} \tag{A.10}$$

■

B Appendix: CARA Utility

Suppose all investors have an exponential utility function and therefore constant absolute risk aversion. However, investors have different coefficient of absolute risk aversion η_i, i.e. investor i's utility function is given by $U_i(c_{it}) = -\frac{1}{\eta_i}\exp(-\eta_i c_{it})$, where $\eta_i > 0$ for all i.

The Hamilton-Jacobi-Bellman equation remains the same as shown in the proof of proposition 1, which is

$$\rho J(W_t) = \max_{\{c_{it},\omega_t\}} \sum_{i=1}^n \lambda_i u_i(c_{it}) + \left[(\omega_t(\mu - r) + r)\, W_t - \sum_{i=1}^n c_{it}\right] J'(W_t) + \frac{1}{2}\sigma^2\omega_t^2 W_t^2 J''(W_t) \tag{B.1}$$

with the first order conditions being

$$\omega_t^* = -\frac{\xi}{\sigma}\frac{J'(W_t)}{W_t J''(W_t)} \qquad \exp(-\eta_i c_{it}^*) = \frac{J'(W_t)}{\lambda_i} \tag{B.2}$$

Substitute the conjecture $J(W_t) = -\frac{b}{a}\exp(-aW_t)$ into the first order conditions above shows

$$\omega_t^* = \frac{\xi}{\sigma}\frac{1}{aW_t} \tag{B.3}$$

$$c_{it}^* = \frac{a}{\eta_i}W_t - \frac{1}{\eta_i}\ln\left(\frac{b}{\lambda_i}\right) \tag{B.4}$$

These implies that $u_i(c_{it}^*) = -\frac{1}{\eta_i}\frac{b}{\lambda_i}\exp(-aW_t)$, and substituting back to the HJB shows

$$\sum_{i=1}^{n}\frac{a}{\eta_i} - \frac{1}{2}\xi^2 - \rho - \sum_{i=1}^{n}\frac{a}{\eta_i}\ln\left(\frac{b}{\lambda_i}\right) + \left[\left(\sum_{i=1}^{n}\frac{a^2}{\eta_i}\right) - ra\right]W_t = 0$$

Therefore, one can derive

$$a = r\left(\sum_{i=1}^{n}\frac{1}{\eta_i}\right)^{-1} \tag{B.5}$$

$$\ln b = \left(\sum_{i=1}^{n}\frac{a}{\eta_i}\right)^{-1}\left[\left(\sum_{i=1}^{n}\frac{a}{\eta_i}(1 + \ln\lambda_i)\right) - \frac{1}{2}\xi^2 - \rho\right] \tag{B.6}$$

Further simplification pins down the constant b as

$$b = \exp\left(-\frac{1}{r}\left(\rho + \frac{1}{2}\xi^2 - r\right)\right)\prod_{i=1}^{n}\lambda_i^{\frac{1/\eta_i}{\sum_j 1/\eta_j}} \tag{B.7}$$

Substituting the results into the FOCs shows that

$$c_{it}^* = \frac{\frac{1}{\eta_i}}{\sum_{j=1}^{n}\frac{1}{\eta_j}}rW_t + \frac{1}{r\eta_i}\left(\rho + \frac{1}{2}\xi^2 - r\right) + \left[\frac{1}{\eta_i}\ln\lambda_i - \frac{\frac{1}{\eta_i}}{\sum_{j=1}^{n}\frac{1}{\eta_j}}\left(\sum_{j=1}^{n}\frac{1}{\eta_j}\ln\lambda_j\right)\right] \tag{B.8}$$

$$\omega_t^* = \left(\frac{A_t}{W_t}\right)^* = \frac{\xi}{\sigma}\frac{1}{rW_t}\sum_{i=1}^{n}\frac{1}{\eta_i} \tag{B.9}$$

$$J(W_t) = -\frac{b}{a}\exp(-aW_t) \tag{B.10}$$

where a and b are as in equation (B.5) and (B.7). Moreover, one could sum up all the payout

c_{it} and get

$$\sum_{i=1}^{n} c_{it}^{*} = rW_t + \frac{\rho + \frac{1}{2}\xi^2 - r}{r} \sum_{i=1}^{n} \frac{1}{\eta_i} \tag{B.11}$$

These suggest that the optimal payout c_{it}^{*} is linear in net worth, where the first term is the regular interest payment, the second term is compensation for risk aversion while the terms in the square bracket consist of the side payment among investors. The side payments will net out across investors when we sum up the payout over all individuals. The group will invest a constant absolute amount of net worth into the risky portfolio, i.e. $\omega_{it}W_{it}$ stays constant, which makes the representatitve agent having the same form of indirect value function.