DISCOUNT RATES AND PRICE FORECASTS FOR UPSTREAM PETROLEUM VALUATIONS

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ABSTRACT

For their appraisals, most petroleum companies use discount rates that implicitly account for riskiness of projects. They draw this rate from their Weighted Average Cost of Capital (WACC) and then apply it to expected future cash flows. Yet, they forecast cash flows using expected prices that are sometimes at odds with the assumptions in WACC. More specifically, the risk-premiums within the price forecasts and the discount rate are of similar nature and should be compatible, but with the multitude of technical and market risks, it is not clear how to estimate these premiums. In this paper, we use the Schwartz and Smith (2000) two-factor price process and implied method of parameter estimation to discuss a consistent valuation framework. We determine the discount rate together with analysts' long-term prices forecasts. The suggested methodology is particularly useful in valuation of long-term capital investments.

1. INTRODUCTION

In a discussion paper published by the International Valuations Standard Council, participants from mining and petroleum industries commented on how they forecast prices and select discount rates[1]. Most responses revealed the inclination towards using market prices and WACC for valuation of capital investments, whereas there was disparity on how to actually apply the method.

Some respondents used forward commodity prices from the market and adjusted them with analysts' long-term views while others relied entirely on price forecasts based on macroeconomic fundamentals. Furthermore, most respondents agreed that the discount rate should reflect the riskiness of the investment, and a few indicated that risk-adjustment of project outcomes is more viable than using an all-purpose discount rate. All in all, the responses were illuminating. It showed the industry's concern of the critical valuation parameters: price forecasts and discount rates. In this paper, we discuss that this is essentially a single problem of determining risk premiums.

Project appraisals are about expressing uncertainty[2] in terms of value. Traditionally, this was done by discounting expected cash flows with a discount rate that reflects both the time value of money and risks. Using, for example, the Capital Asset Pricing Model (CAPM), we could calculate this risk premium and add it to the risk-free rate to make a risk-adjusted discount rate. An alternative approach to valuation is to calculate certainty-equivalent cash flows and discount them with the risk-free rate (Cox et al., 1985, Smith and Nau, 1995). In other words, we could risk-adjust the cash flows instead of the discount rate. If done consistently, both approaches to valuation yield identical results.

This reveals another important connection; the risk-premiums within the cash flows and the risk premiums of the discount rate account for identical risks. If we assume further that project risks are

[1] The discussion papers are available at https://www.ivsc.org/standards/international-valuation-standards
[2] Inaccurately labeled "risks" in the context of valuation, uncertainty creates value when the outcomes of a project are different from the expectation and managers change course to capitalize on those desirable surprises. It could also destruct value when the outcomes are different from the expectation and there are no remedies.

born of private and public risks[3], and that private risks are separately accounted for, then, we conclude that public risk premium—typically, price and interest rate risk-premium for petroleum projects—is related to the risk premium in the discount rate. In other words, forecasting prices and determining the discount rate are two sides of a coin. When analysts estimate prices for valuation and investment, they should also consult those that determine the discount rate.

In this paper, we use an implied method to calibrate parameters of the price process (introduced in Schwartz and Smith, 2000, and implemented by Jafarizadeh and Bratvold, 2012) and then discuss that the method is incapable of estimating price risk premiums. This is not a weakness of the implied calibration method per se, as the other approach to parameter estimation, using Kalman filter on historical prices, also provides statistically insignificant measurements of the risk premiums (Schwartz and Smith, 2000). As all calibration methods rely on hedged market prices (i.e. risk is removed from information about futures and options), it is natural to assume that we need additional sources of information to estimate the risk premiums. Hamilton and Wu (2014) used the information about interaction of hedgers and arbitrageurs in crude oil markets, and Cortazar et al. (2015) and Hahn et al (2018) utilized CAPM-like asset pricing model to estimate risk premia. More recently, Cortazar et al (2018) incorporated information from analysts' forecasts into estimation of risk premiums.

We suggest a consistent framework for joint estimation of price and discount rate risk premiums. We rely on analysts' insight and utilize learnings from Cortazar et al. (2015), Hahn et al. (2018), and Cortazar et al. (2018). We argue that the price risk is partly reflected in CAPM beta of petroleum companies and if we can filter out other effects, then we end up with a market-implied measure of commodity price risk premiums. This "project beta" is a measure of a project risks that come from unhedged prices. We follow Bernardo et al. (2007 and 2012) and Da et al. (2012) and use exogenous information about leverage and real options potentials of a firm to infer these CAPM estimates of project beta. We then use this measure to assess price risk premiums by comparing the results with those of the risk-neutral valuation. The two approaches converge to a single value if we correctly account for risks.

In the next section, we discuss implied parameter estimation using the market information. We discuss that, like Kalman filter method of parameter estimation on historical prices, this method is also incapable of accurately estimating price risk-premiums. Next in section 3, we discuss ways of estimating risk premiums using additional information about a company's market returns. This approach based on CAPM, quantifies the effect of market risk and tries to separate it from the effect of leverage and growth options. Section 4 provides discussions and section 5 concludes.

2. PARAMETER ESTIMATION

In this paper, we use the two-factor price model in Schwartz and Smith (2000) to discuss the nature of the problem, bearing in mind that other price models lead to similar discussions and conclusions. This price model is versatile yet simple; practitioners favour it because it fits the market observations and at the same time is intuitive for communication and discussion. Its core idea of considering prices as an aggregate of a short-term and long-term factor is appealing and practical. We then calibrate price parameters using market information within the implied method discussed in Jafarizadeh and Bratvold (2012).

[3] Those technical possibilities like encountering a larger than expected reservoir or blocked export pipelines, belong to the category of private risks whereas those market possibilities, such as lower than expected oil prices, belong to public risks. Smith and Nau (1995) have a stronger description; those risks that could be hedged with market instruments are market risks. Private risks could be correlated or independent of market risks. In this paper, we follow this stronger description.

2.1 Calibration of the Two-Factor Price Model

We denote S_t as spot price at time t, where $\ln S_t = \xi_t + \chi_t$. The short-term and long-term factors, χ_t and ξ_t, follow stochastic processes

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{1}$$

$$d\xi_t = \mu dt + \sigma_\xi dz_\xi \tag{2}$$

Here, dz_χ and dz_ξ, the increments of the Brownian motion process, are correlated with $dz_\chi dz_\xi = \rho_{\xi\chi} dt$.

We denote $F_{0,T}$ as the price of a futures contract for delivery at T. Under the risk-neutral measure, the expected spot prices at T will be equal to the futures price for delivery at T. Schwartz and Smith (2000) show that

$$\begin{aligned}\ln F_{0,T} = {} & e^{-\kappa T}\chi_0 + \xi_0 + (\mu - \lambda_\xi)T - (1 - e^{-\kappa T})\frac{\lambda_\chi}{\kappa} \\ & + ½\left((1 - e^{-2\kappa T})\frac{\sigma_\chi^2}{\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\end{aligned} \tag{3}$$

And the instantaneous variance of $\ln F_{0,T}$ is

$$Var(\ln F_{0,T}) = e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T}\rho_{\chi\xi}\sigma_\chi\sigma_\xi \tag{4}$$

This model elegantly matches the reality of price dynamics in the markets. For example, the forward curve and volatility term structure resulting from this model directly fit the observed forward and implied option volatilities in the market.

These relationships accommodate an implied method for estimating parameters of the model. Using the information in market prices of options and futures, we simply try to choose model parameters that best fit the actual futures prices observed in the market. In addition, we further use implied volatility of options on the futures to support our parameter calibration. Compared to the other commonly used method of calibration, using historical futures in a Kalman filter, our implied method is simpler and easier to implement.

The first step in implied parameter calibration is to construct a forward curve, forward prices with different maturities, using equation (3) and compare it with observed futures prices from the market. For additional support, we can also a construct the volatility term curve using equation (4) and compare it with implied option volatilities from the market. These option volatilities are not directly available, and we use the procedure below to obtain them from European options on futures.

The values of a European call option on a futures contract with expiry at T, denoted by c_T, and strike price K, is

$$c_T = e^{-rT}\left(F_{0,T}N(d) - KN\left(d - \sigma_\varphi(T)\right)\right) \tag{5}$$

And the value of a put option, p_T, is

$$p_T = e^{-rT}\left(KN(\sigma_\varphi(T) - d) - F_{0,T}N(d)\right) \tag{6}$$

Where $d = \frac{\ln F/K}{\sigma_\varphi(T)} + ½\sigma_\varphi(T)$, r is the risk-free discount rate, and $N(d)$ is the cumulative probability for the standard normal distribution. Furthermore, we assume the options expire at the same time as their underlying futures.

We observe option prices in the markets and we could "reverse" equations (5) and (6) to calculate the implied volatility of futures prices. In other words, the implied volatility is the volatility which, when used in equations (5) or (6), returns a theoretical value equal to the market price of the option. Because theoretical inverse functions are difficult, we developed computer code to perform numerical calculations as "goal seek" operations. Appendix A explains this code in the form of Excel VBA functions.

Returning to our task of parameter estimation, we have now all the information we need for calibration of the price model. We designed an optimization model that variates model parameters until the curves generated from equations (3) and (4) fit the market observations. Figure 1 shows the optimal result, were the sum of squared differences between model curves (dashed lines) and market information (solid lines) is minimum. These fitted curves represent the parameters in table 1.

Table 1 Parameters of the two-factor price process calibrated to market data

χ_0	0.300
ξ_0	3.960
κ	0.700
σ_χ	0.500
μ	-0.026
σ_ξ	0.200
ρ	0.192

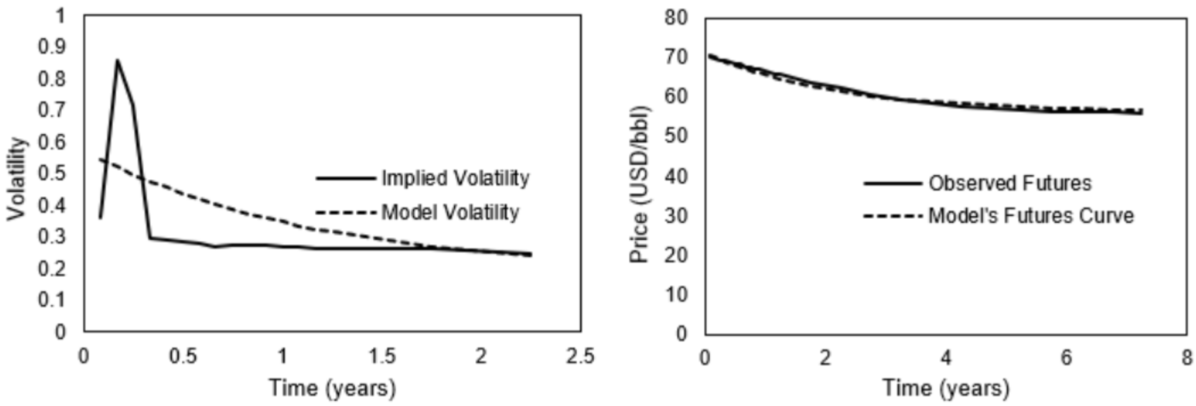

Figure 1 fitting the model's curves to market data

As this optimization may lead to a local, rather than the global, minimum, to help the users we included some provisions in the optimization program. Schwartz and Smith (2000) suggest weighting the errors to improve the fit, and Jafarizadeh and Bratvold (2012) suggest a step by step approach of first fixing some parameters to parts of the curves and then determining the rest. Our optimization model facilitates both these provisions by providing three steps of automated optimization, and user-defined error weights.

2.2 Empirical Data

Market data contain patterns and white noise not represented in the two-factor price process. For example, the SS model assumes the implied volatility for options with different strike prices but same maturity, will be flat; i.e. implied volatility does not change with strike prices. Yet, for a wide range of commodities and specifically for oil prices, this is not the case. In fact, the assumption of normal distribution for log of futures prices, through the expression $N(d)$ in equations (5) and (6), is questionable[4]. Perhaps this is because, with recent market crashes, the participants assign a higher probability of occurrence for extremely high or extremely low prices. This causes a pattern called "volatility smile" when we apply equations (5) and (6) to option prices with the same maturity but different strike prices.

Volatility smiles are the result of increasing implied volatility when the options are deep in the money or deep out of the money. At the same time, for at-the-money options (when strike price is close to the price of futures contract), the implied volatility tends to be the lowest. If we calculate implied volatilities for different strike prices across all available maturities, we get a volatility surface as in Figure 1. Here, volatility smiles are more prominent for options with near maturities. For our data set, the volatility smile tends to disappear for long-maturity options.

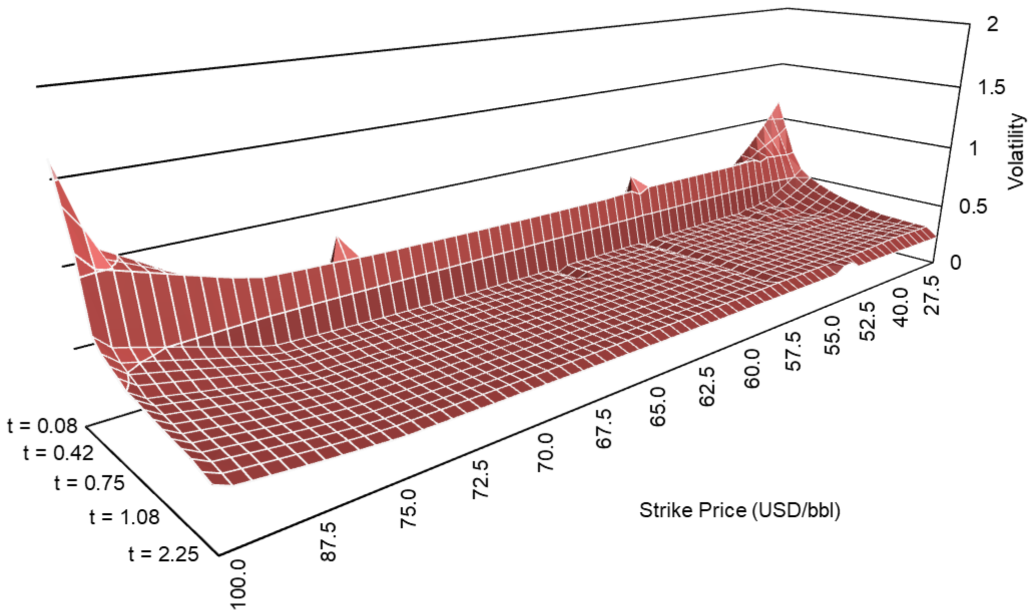

Figure 2 volatility surface (implied volatility across different maturities and strike prices) for European put options on futures, market data retrieved from NYMEX on 01/09/2018

This raises a few questions about data used in the implied parameter estimation. For example, in figure 1, we fit the model's volatility to implied volatility curve. We selected an implied volatility curve out of numerous curves of the volatility surface. Given the volatility smile, which curve better represent the volatility curve? Some authors suggest at-the-money curves, or those with highest traded volume (Geman, 2005). However, for oil markets, even though the strike price of 100 USD/bbl is highly traded, at current spot price of 75 USD/bbl, it is not at-the-money. For this reason, it is to analysts' discretion which curve to select for parameter estimation.

All in all, implied parameter estimation depends on the source data and the importance weights that we assign to each part of the curves. If the futures and the implied volatility curves are sufficiently informative and analysts' sound judgment selects the inputs, then we believe the parameter estimations will be reliable and useful for valuations.

[4] Jarque and Bera (1987) develop a statistic that tests the normality of time-series by combining tests for skewness and kurtosis. Recalling that a normally distributed data-set has no skewness and kurtosis, this statistic rejects normality when the test's value is less than the significance level. Applying this test to futures price usually reveals significant skewness at 5% level.

3. CALCULATING RISK-PREMIUMS

3.1 Calibrated Parameters Describe the Risk-Neutral Process

The calibrated model essentially describes the risk-neutral behaviour of prices. The model parameters are fitted to the forward prices and their implied volatility, i.e. the model describes the expected prices as if they were equal to forward prices.

$$\mathrm{E}^*(S_t) = F_{0,t} \tag{7}$$

This also means that the risk premiums λ_χ and λ_ξ are already embedded within the parameters of the model. In other words, during the process of calibration, the estimates of other parameters accommodated the effect of these premiums. How do we disentangle the effect of risk premiums from the price model? There is no single answer. Note that if we substitute λ_χ by $\lambda_\chi + \delta$, where δ could assume any amount, and in return substitute χ_0 and ξ_0 by $\chi_0 - \delta/\kappa$ and $\xi_0 + \delta/\kappa$, we still have the same risk-neutral process. Thus, with the current state of information, we cannot determine risk-premiums.

This difficulty is also prominent for methods that use Kalman filters on historical data. For example, Schwartz (1997), Schwartz and Smith (2000), and Cortazar and Naranjo (2006) mention large errors and statistically insignificant estimates for some parameters of the physical (true) process. Cortazar et al. (2015) suggest using external sources of information to estimate the risk premium parameters; an approach that we also adopt in this paper. Assuming we exogenously determined λ_χ and λ_ξ, we could use the equations below to transform the risk neutral process into the true process.

$$\mu = \mu^* + \lambda_\xi \tag{8}$$

$$\xi_0 = \xi_0^* + \frac{\lambda_\chi}{\kappa} \tag{9}$$

And the short-term factor is $\chi_0 = S_0 - \xi_0$.

3.2 Relationship Between Discount Rate and Price Forecasts

As a guiding principle, the valuations using risk-neutral and WACC approaches should lead to identical results. As we already have the inputs to risk-neutral valuation, perhaps we could adjust the risk-premiums in WACC approach so that both approaches yield the same valuation for a representative project. In other words, when we include premiums λ_χ and λ_ξ in the price process, we could also replace the risk-free rate r with $r + \epsilon$ to compensate for this effect. This appears to be simple and easy, but the question really is: how do we risk-adjust the discount rates so that the adjustment only represents the riskiness in prices?

Assume an upstream project produces Q barrels of oil in the future at time T. The cash flow received in the future is uncertain and will depend on the price of oil at time T, say S_T, but for convenience, the expected cash flow $\mathrm{E}(S_T) \times Q$ is used. The company uses $r + \epsilon$ to discount future cash flows so the value of the project $V(t = 0)$ is

$$V(0) = \frac{\mathrm{E}(S_T) \times Q}{(1 + \mathrm{r} + \epsilon)^T} \tag{10}$$

Alternatively, the company can use the same amount of cash as $V(0)$ to sell oil using forward contract $F_{0,T}$ with delivery at T. Note the delivery price is fixed and the cash flow is risk–free.

$$V(0) \times (1 + \mathrm{r})^T = F_{0,T} \times Q \tag{11}$$

By substituting $V(0)$ in equation (7) we get

$$\frac{\mathrm{E}(S_T)}{(1+\mathrm{r}+\epsilon)^T}=\frac{F_{0,T}}{(1+r)^T} \tag{12}$$

In other words, the forward price at time T discounted at risk–free rate is the present value of one barrel of oil sold at time T. Furthermore, by replacing terms in equation (12) we will have

$$\frac{\exp\left(e^{-\kappa T}\chi_0+\xi_0+B(T)\right)}{(1+\mathrm{r}+\epsilon)^T}=\frac{\exp\left(e^{-\kappa T}\chi_0+\xi_0+A(T)\right)}{(1+r)^T} \tag{13}$$

Where

$$B(T)=\mu T+½\left((1-e^{-2\kappa T})\frac{\sigma_\chi^2}{\kappa}+\sigma_\xi^2 T+2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)$$

$$A(T)=\left(\mu-\lambda_\xi\right)T-(1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa}\\+½\left((1-e^{-2\kappa T})\frac{\sigma_\chi^2}{\kappa}+\sigma_\xi^2 T+2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)$$

Equation (13) shows the relationship between risk premiums of price forecast and that of the discount rate. They should work in tandem for a coherent valuation framework. In other words, we could use to equation (13) to estimate price risk premiums if we know about the discount rate.

Yet, the discussion that led to equation (13) assumed we have a single cash flow purely affected by price risk; sales of Q barrels of oil sometime in the future. Project appraisals are more sophisticated than that. There are additional risks in a project's cash flows and the discount rate may reflect uncertainty in costs, tariffs, or exchange rates, as well as the embedded managerial and growth options. The price risk is arguably a significant portion of the total systematic risk of a project. With these complexities, we need a more versatile version of equation (13) for effective estimation of risk premiums.

We assume we could somehow separate the effect of various sources of uncertainty in a project and account for each of them individually. In this ideal environment, we could have a project that is merely affected by price risk and has a discount rate of $r+\epsilon$. In other words, we assume the costs, C_t, production, Q_t, and deductions, D_t, are deterministic and $t=0,\dots,T$. The value of this project, V_0, is the discounted sum of its cash flows and should remain the same whether we use risk-neutral prices and discount with risk-free rate or use expected spot prices and discount with project's discount rate.

$$V_0=\sum_{t=0}^{T}\frac{\mathrm{E}(S_t)\times Q_t-C_t-D_t}{(1+r+\epsilon)^t}=\sum_{t=0}^{T}\frac{F_{0,t}\times Q_t-C_t-D_t}{(1+r)^t} \tag{14}$$

Then the project-equivalent version of equation (13) becomes

$$\sum_{t=0}^{T}\frac{\exp\left(e^{-\kappa T}\chi_0+\xi_0+B(t)\right)\times Q_t-C_t-D_t}{(1+r+\epsilon)^t}=\sum_{t=0}^{T}\frac{\exp\left(e^{-\kappa T}\chi_0+\xi_0+A(t)\right)\times Q_t-C_t-D_t}{(1+r)^t} \tag{15}$$

Where $A(t)$ and $B(t)$ are the same as in equation (13).

With equation (15), we have an explicit relationship between price risk premiums λ_χ and λ_ξ and the discounting risk premium ϵ. If we know about ϵ, perhaps through the company-wide hurdle rate or an improved version of CAPM, then we could obtain a numerical solution for premiums λ_χ and λ_ξ. On the other hand, knowledge of λ_χ and λ_ξ, perhaps from analysts' forecast for the corporate planning price, will dictate the value for ϵ.

Our main contribution is the joint estimation of prices and discount rate; this would be straightforward using equations (14) and (15) for short-term investments as forward prices $F_{0,t}$ are available for the duration of the project, whereas, for long-term investments we still need to rely on analysts' estimate of the discount rate or expected price forecasts. In addition, equations (14) and (15) refer to projects with simple cash flows and devoid of any real options. Most projects have embedded managerial flexibilities and if blindly used in these equations will confuse the effect of price premiums. This seems like back to square one, as we still deal with the problem we laid out in the beginning of this manuscript. However, now armed with the knowledge of the relationship between risk premiums, we can use a version of CAPM that roughly filters out the effects of leverage and growth options from a company's beta. We could estimate ϵ for a "typical" project of any length and use it in equation (15) to forecast expected prices.

3.3 Estimating Project Risk and the Premium ϵ

To estimate ϵ, we could use the risk premium from CAPM. However, this measure of systematic risk reflects a myriad of factors including the embedded real options, financial leverage, and a mixture of macro-economic factors such as exchange rates and commodity prices. Although not an exact method, we use the following procedure to strip away other sources of risk from CAPM beta, until we are left with a measure for price risk. A detailed description of this approach is discussed in Jafarizadeh and Bratvold (2019).

First, there is financial leverage. Companies use debt to magnify their investing capabilities. Debt usually has a lower cost, but the commitment to pay its interests escalates the risk (and beta) of company's stock. In the end, the shares of a leveraged company are riskier although with no change to the nature of its projects.

Using Modigliani and Miller (1958) arguments, we distinguish between equity and business risks and account for the effect of leverage in beta. The asset beta (unlevered beta, β_{Asset}) compensates for the effect of leverage, and is a measure of riskiness in company's assets:

$$\beta_{\text{Asset}} = \frac{\beta}{\left(1 + (1 - Tax\ Rate)\frac{D}{E}\right)} \tag{16}$$

Where D and E are respectively the market values for company's debt and equity. Assuming corporate tax rate of 35%, figure 3 shows the asset beta in our sample of US domiciled petroleum companies[5].

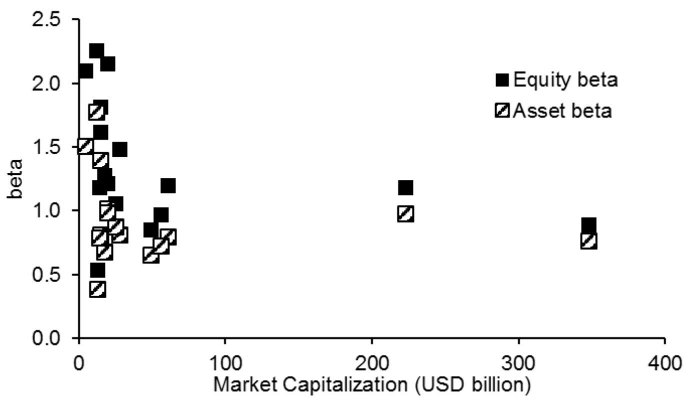

Figure 3 asset beta for our sample of upstream petroleum firms

[5] We selected a subgroup of upstream petroleum companies from COMPUSTAT that is large enough and, at the same time, does not introduce systematic risks such as uncertainty in exchange rate. Although this selection is not ideal, we could draw coherent conclusions about the companies' beta based on the public information about their portfolio of upstream projects.

Second, even though we cleared the effect of financial leverage in β_{Asset}, it still represents the mix of all assets, some of which have significant growth options. Using β_{Asset} in project valuations means we discount cash flows with a rate that reflects both the projects' systematic risk and growth options embedded in company's assets. If options are separately represented in a decision tree, using β_{Asset} means we are double discounting the options.

We can explain the above point using a simple example: a major oil and gas company considers acquiring the rights to an exploration tract. If they discover oil, the project will be a conventional mid-sized development. Here, the geological chance of discovery overshadows the risks associated with development project. Normally, the company's β_{Asset} reflects the systematic risk of existing assets and the growth opportunities from all tracts under consideration. For this specific valuation, using β_{Asset} (which already compensates for the risk of geologic success or failure) will distort the value.

In general, the equity returns of companies reflect the risk of existing operations together with embedded real options; these include the option to delay investments, contract/expand operations, and value of information. Furthermore, the options, at least from CAPM point of view, are riskier than the existing projects (as observed by Berk, Green, Naik, 1999, Dechow, Sloan, and Soliman, 2004, and Da, Guo, Jagannathan, 2012). Perhaps if we could separate the risk of options and projects, the valuation would be more straightforward. How can we strip the effect of embedded options from β_{Asset}?

If we assume the value of the firm V is composed of the value of its projects A and their embedded growth options G (Bernardo, Chowdhry, and Goyal, 2007) then

$$V = A + G \tag{17}$$

The firm's β_{Asset} is the weighted average of β_{Project} and β_{Option}, with weights corresponding to the ratio of A and G to the total value.

$$\beta_{\text{Asset}} = \frac{A}{V}\,\beta_{\text{Project}} + \left(1 - \frac{A}{V}\right)\beta_{\text{Option}} \tag{18}$$

$$\beta_{\text{Asset}} = \beta_{\text{Option}} - \left(\beta_{\text{Option}} - \beta_{\text{Project}}\right)\frac{A}{V} \tag{19}$$

Some authors, including Smith and Watts (1992) and Chen, Novy–Marx, Zhang (2010), argue that proxies such as book–to–market ratio or return on asset (ROA) can be useful in explaining the share of growth options in a firm's value. In other words, even without the knowledge of projects and operations, proxies could provide a rough measure of $\frac{A}{V}$ in equation (19). We assume "share book-to-price" ratio commonly reported by financial services estimates $\frac{A}{V}$. The "book" refers to the accounting valuation of a firm, while "price" is the market perception of the value. As accounting conventions do not recognize intangible assets and growth options (while market returns do) this ratio could reveal the potentials of value creation from embedded real options.

We also assumed (following Bernardo, Chowdhry, and Goyal, 2007) that β_{Project} is the same for all firms in the upstream petroleum industry. This is a convenient assumption and allows us to determine the parameters of equation (19), but at the same time sacrifices some realism by implying that a candidate project (stripped of its options) has the same risk across all companies within the sector. The variability in the companies' beta is then only caused by their embedded real options. Some may argue that there are cross–company variations in project risk, e.g. mature and declining fields are less risky compared to those in their ramp-up period, but we believe that these differences are small compared to the benefits of better decision making via an aggregate project beta.

With these assumptions, we could use information about β_{Asset} and book-to-market ratio in a regression analysis that estimates the parameters of equation (19). However, as this regression-based method is problematic (Bernardo, Chowdhry, and Goyal, 2007), we could split the data into two portfolios based on their market-to-book values. These portfolios represent the (equally weighted) means of β_{Asset} and market-to-book values of the stocks in them. A straight line that connects these two points yields the intercept and slope coefficients for equation (19). The detailed implementation of this approach is discussed in Jafarizadeh and Bratvold (2019).

Inherent variabilities aside, we observe that asset beta (and project beta) is slightly below one for most petroleum companies; as if petroleum projects, irrespective of their ownership, have similar level of risk across the industry. With these similarities, perhaps it is useful to establish standard valuation procedures using a single industry beta—from analysis above, we conclude $\beta_{\text{Project}} = 0.8$ is appropriate for upstream petroleum valuations.

4. APPLICATION AND DISCUSSION

In this section, we evaluate a petroleum exploration project using Bloomberg's oil price forecast and a discount rate of 9% commonly used in the oil and gas industry. We compare the results with our methodology of jointly estimating price and discount rate. We also point out where the two methodologies potentially diverge.

4.1 A Petroleum Exploration Example

Assume a company holds the drilling rights of an exploration tract. It costs USD 10 million to drill a well in this tract but the chance of success is only 30%. If successful, we expect the project to produce oil for eight years after one year of construction. We will later estimate the net present value of this development project but, for now, the company is facing an uncertain investment: receive $\text{NPV}_{\text{Develoment}}$ with 30% chance or bear the USD 10 million loss with 70% chance. These financial outcomes depend on the outcome of (unsystematic) geologic uncertainty. Instead of drilling, the company could sell the drilling rights altogether for USD 5 million and ask for an additional USD 5 million bonus in case of success. Which course of action, drilling or selling, is a value maximizing decision? The decision tree model is shown in figure 5.

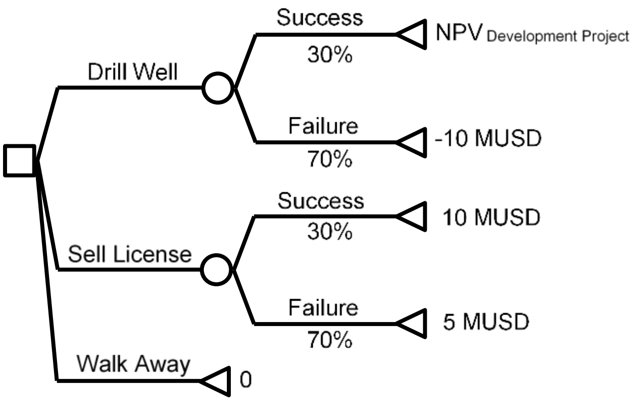

Figure 5 the decision tree model of the example

The decision depends on the net present value of the development project. Specifically, it is a function of the revenue of production, the costs, and the discounting. In table 2, we show that when the cash flows are estimated using the corporate planning price and discounted with 9% hurdle rate, the result would be different from when we use forward prices and discount with the risk-free rate of 2%. Here, the valuations deviate by USD 11 million.

Table 2 cash flow profile of the development project

Year	0	1	2	3	4	5	6	7	8
Corporate Price (USD/bbl)	68.0	67.0	67.0	67.0	67.0	67.0	67.0	67.0	67.0
Forward Price (USD/bbl)	70.3	66.6	63.0	61.0	58.0	56.8	56.2	56.0	56.0
Fitted Price (USD/bbl)	71.4	68.8	66.2	64.5	63.6	63.2	63.2	63.4	63.7
Production (Mbbl)		600	500	420	360	320	300	290	290
Cost (MUSD)	70	5	5	5	5	5	5	5	10
Cash Flow (Corporate)	-70	35.2	28.5	23.1	19.1	16.4	15.1	14.4	9.4
Cash Flow (Industry Beta)	-70	36.3	28.1	22.1	17.9	15.2	14.0	13.4	8.5
Cash Flow (Risk Neutral)	-70	35.0	26.5	20.6	15.9	13.2	11.9	11.2	6.2
NPV (with Hurdle Rate)	50.0	Discounting with Corporate Hurdle Rate: 9%							
NPV (with Industry Beta)	61.4	Discounting with WACC using Industry Beta: 5%							
NPV (Risk Neutral)	61.4	Discounting with Risk-Free Interest Rate: 2%							

For our improved valuation methodology, we used the procedure in section 3.3 to estimate the risk premium ϵ within the discount rate. If we assume this company has 50% debt in its capital structure at 4% interest, and the market risk premium is currently 5%, then using the beta of 0.8 we estimate WACC at 5%. This is considerably lower than the hurdle rates most oil and gas companies use.

Using the discount rate of 5%, i.e. $r + \epsilon = 5\%$, we use the equation (15) in a numerical optimization model to estimate the price risk premiums λ_ξ and λ_χ. The result is the fitted price curve in table 1. Figure 6 also compares this expected price curve with the observed forward curve, whereas the difference between these prices reflects the price risk premium.

We use these prices to calculate production revenue in this project and all those projects with similar risk levels. With these cash flows, discounting at 5% rate should essentially result in the same NPV as in the risk-neutral valuation method. In other words, whenever forward prices are available for the duration of a project, we could calculate the discount rate using industry beta and then estimate the price risk premiums. This makes price prediction straightforward.

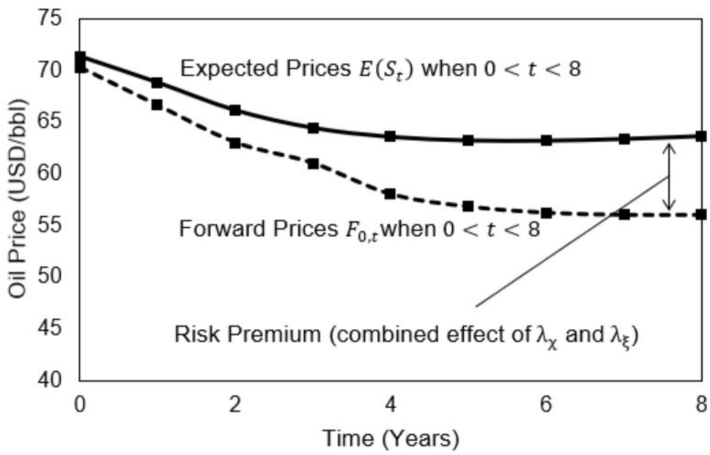

Figure 6 Forward curve and an estimate of expected spot prices through numerical estimation of risk premiums

Figure 7 shows the optimal decision for this exploration example. If we had used the 9% discount rate along with corporate planning prices, the optimal decision would have been to sell the license to the third party. However, our method of jointly estimating prices and discount rate leads to a different decision; to maximize shareholder value, the company itself should set bout drilling this prospect.

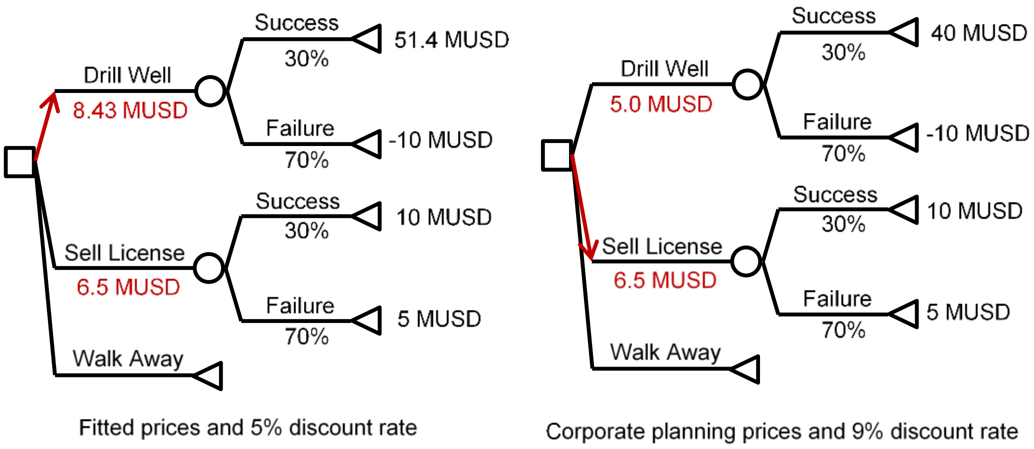

Figure 7 Optimal decisions when proposed valuation methodology is used (left) and when corporate planning prices and 9% discount rate used (right)

4.2 Price Predictions in the Long Run

If forward prices are available for the duration of a project, then our methodology is straightforward; we use a spreadsheet numerical optimization model to assess price risk premiums. However, most petroleum projects have time horizons much longer than the eight years of longest maturity forward contracts. For these projects, there are no forward prices to assess their cash flows, and no risk-neutral valuation to compare the results with. We discuss that even in the absence of market data, the analysts' long-term forecast of the prices is the additional source of information that we could utilize in our methodology and still generate consistent valuations.

The experts in crude oil markets, corporate analysts and institutions like Bloomberg, EIA, and IMF, assimilate additional information about risk into their forecasts. As Cortazar et al (2018) show in their calibration of three factor price model, combining analysts' forecast data with market's futures data, improves the consistency of expected price estimates. The result would be price curves that agree with the market in short-term and extend to long-term by relying on experts' data. We believe this price curve can be utilized in our methodology, it could be used to inform the decision makers of the proper discount rate, or if the discount rate is determined, could be used as an additional piece of information to calibrate the price curve.

5. CONCLUSIONS

Valuation is the basis of decision making in any corporation, yet, with the multitude of uncertainties and long lifespan of petroleum ventures, it is difficult to consistently account for their risks and opportunities. In this paper, we suggest a consistent valuation framework that jointly estimates the risk premium of prices and discount rates. We use CAPM to estimate the systematic risk of petroleum projects, and then clear the beta of the effects of leverage and growth options. What remains would be an indication of price risk. We then use this beta in an optimization model that estimates price risk premiums.

To formally demonstrate our methodology, we use the two-factor price model and calibrate its parameters to market data. This implied approach, implemented in the accompanying spreadsheet, estimates price parameters by fitting model curves to the curves of forward prices and implied volatility of options. The estimated parameters, devoid of risk, represent a risk-neutral model of prices. If we use these prices to estimate cash flows and discount with the risk-free rate, we get the risk-neutral value of a project. Next, in a cash flow model that discounts with a rate informed by project beta, a second optimization model estimates price risk premium by converging the project's value from varying expected prices to the value from risk-neutral valuation.

This methodology is simple and easy to implement, the optimization model is also versatile enough to accommodate a range of applications. In addition, we believe the benefits of this valuation framework, mainly the idea that price forecasts and discount rate are of the same nature, goes beyond its initial applications. For example, the concept of project beta could lead to long-term price forecasts in parallel with analysts' interpretations; serving as an extra source of information and helping analysts calibrate their long-term predictions. In other instances, we could use this method the other way around and compare implied discount rates from various institutions based on their price projections. These learnings could provide useful insights for planning and investment decision making.

The spreadsheet model of our methodology (including market data and the VBA code) is available in the link below

https://www.dropbox.com/s/sft4emhjgefm0ku/Price_and_Discount%20Rate.xlsm?dl=0

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APPENDIX A EXCEL VBA CODE

The accompanying spreadsheet contains the open source VBA code used for calibrating the parameters of the price process. In addition, the VBA code contains the optimization subroutine that estimates the risk premiums.

APPENDIX B DATA USED IN CAPM BETA ESTIMATES

We used publicly available data from NYMEX to calibrate our price process. The data covers futures prices across different maturities, as well as European call and put prices for a subset of these futures contracts, all observed on a specific date. The accompanying spreadsheet contains the data used in this paper.