The Nash Bargaining Solution and the Coco-value in the Real Options Games between Asymmetric Firms

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Abstract: To make an investment decision based on the classic real options approach, the value of an investment option should be compared with the benefits of an instantaneous investment. However, if the investment option is a shared one, a firm should take into account how its decision influences its competitor decision, and how it itself may be impacted by rival's reactions. Therefore, firms' strategic choices could be described as a (non-zero sum) real options game.

We present a general model of real options games for two (asymmetric) firms operating in the competitive market. The main goals and the unique contribution of this paper are to find when the game between competitors is a bargaining game type and to consider the Nash bargaining solution and the cooperative-competitive value (the coco value) as a solution of a real options bargaining game between competitors. We try to specify the optimal recommendation for firms in each bargaining case and to check whether both parties will always be consistent in choosing the solution approach. Sensitivity of the firm's strategy profile, the Nash bargaining solution and the coco value to changes in the important variables (especially project risk and market shares) are also provided. These considerations lead to very important conclusions, especially for companies that dominate the market.

Keywords: An investment option, real options games, a bargaining game, the Nash bargaining solution, the cooperative-competitive value, the coco value.

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1 Introduction

Operating on a competitive market is not simply. However, some companies achieve success and are able to keep it, others fall out of the market. We would like to analyse some cases.

Nokia – has had various industries in its history achieving huge successes. At the turn of the centauries Nokia was a leader on the mobile telephony market. But in the second half of the 21th century, everything changed. The global smartphone market share held by Nokia decreased from quarter to quarter since year 2007, and its competitors Apple and Samsung grew in importance. In the second quarter of 2007, Nokia's market share was 50.8%, by the second quarter of 2013 it had fallen to just 3.1%. (Fig. 1) In the following years, the situation did not improve. In Q4 17 Nokia smartphones captured only 1% of global market share. At the same time, Samsung's shares increased from 3.3% at the end of 2009, to over 30% in 2012-2013 and stabilize around 20% in the years 2015-2017. Why Nokia lost its market share?

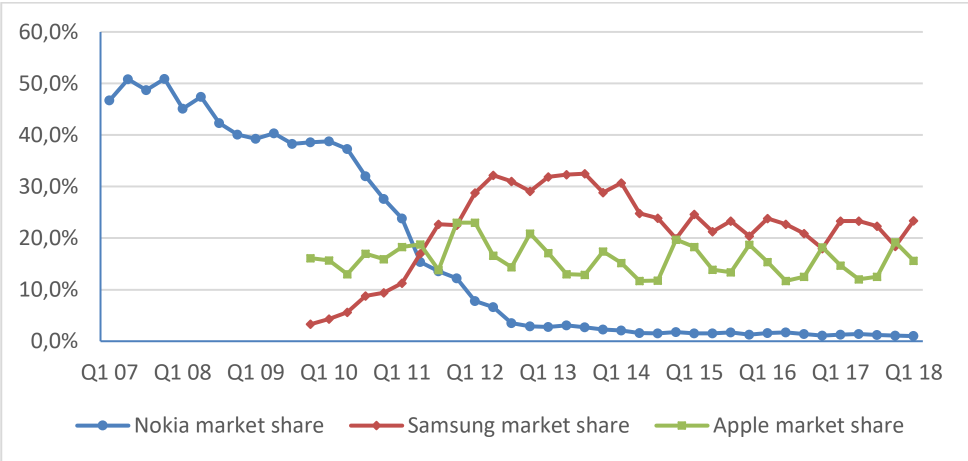

Fig. 1. Global market shares held by smartphones Nokia 2007-2017, Apple 2009-2017, Samsung 2009-2017
Source: Statista.com/statistics

Kodak – a photographic industry icon. One of the great companies in the 20th century; Kodak had no equal in new technologies in the field of photography. In the 20th century, almost 20,000 patents were awarded to Kodak engineers. In 1975, it was Kodak who created the first digital camera and developed this technology in superprofessional photographic equipment and technical devices for medicine or industry. In 1976, Kodak had 90% of the movie market and 85% of the camera market. But the beginning of the 21th century brought a series of growing problems: a drastic decline in profits, a drop in brand value, and a reduction in employment. And finally in 2012 Kodak declared bankruptcy and the annual Academy Award ceremony (Oscars) was held in the Dolby Theatre formerly known as the Kodak Theatre. Why did such a giant fall? You can ask a similar question by analyzing the stories of Xerox, Atari and other past giants.

The reasons for these falls are of course very complex, but one common feature can be indicated. Both Nokia and Kodak had an investment option to implement an innovative solution (a smartphone, a digital camera for a mass customer). But when strategic decisions were set they did not take into account movements of competitors or the fear of failure led them to suboptimal decisions. Nokia and Kodak saw the direction the market is developing, but the high risk of innovation made them waiting. In 2007 the first iPhone was released by Steve Jobs. Analogous solutions (under other operating systems) were also available for Nokia and Samsung (a shared option). Samsung immediately joined the game in this market, but Nokia was convinced that the risk of failure is too high and the mass market for smartphones is a matter of a distant future. Kodak developed its technology towards professional devices. The firm had the opportunity to release a digital camera to the mass market, but they considered the investment too risky. In the mid-1990s, Kodak's director predicted that analogue photography would not be surpass by digital photography for the next 20 years. In both cases, too long delay of the investment decision resulted in a loss of market shares.

We want to analyze these problems in the game theory approach. The game theory provides tools for describing and analysing the situation in which the effect of competitors' behaviors should be incorporated into firm's decision making process. Investment decision making is described as a game between two players, and the real options approach is used to find a value of an investment project; therefore the paper falls in the area of the real options games (Smit, Ankum 1993, Grenadier 2000, Smit, Trigeorgis 2004, Azevedo, Paxson 2010, Chevalier-Roignant, Trigeoris 2011, Trigeorgis, Baldi 2013, Azevedo, Paxson 2014). We will assume both competitors share the same investment opportunity – it is a shared option. Then we are going to discuss different games which can occur between competitors and formulate recommendations to them. For the most commonly used solution concept of a game – the Nash equilibrium may but not need to be the best solution for players we are going to consider bargaining solution concepts: the Nash bargaining solution and the cooperative-competitive value.

The rest of the paper is structured as follows. Section 2 presents basic assumptions of a model of interaction between firms and relations between model's parameters. The next section 3 contains the real options bargaining games analysis and their solutions as the Nash bargaining solution and the cooperative-competitive value. In the section 4 we discuss under what conditions does each game matter. Section 5 concludes.

2 The Model of Interaction Between Firms

Two risk-neutral firms (A and B) operate on a competitive market. Each of them face an investment opportunity in business innovation. Both competitors identify the same investment opportunity – it is a shared investment option (Smit, Trigeorgis 2004, 35). Each firm can adopt the innovation at an investment expenditure I, $I > 0$ and the lifetime of the investment project is infinite.

The investment project generates cash flows (Y_t), which evolve in accordance with the geometric Brownian motion, with drift α, $\alpha > 0$ and volatility σ, $\sigma > 0$ under the risk-neutral measure. A risk-free asset yields a constant rate of return *r*; δ is a convenience yield (δ>0) and it reflects an opportunity cost of delaying construction of the project and instead keeping the option to invest alive (Dixit, Pindyck 1994, 149). The present value of the project is determined by the discounting and accumulating of its future cash flows. It is equal to $V(Y_0) = \frac{Y_0}{\delta}$ (Dixit, Pindyck 1994, 181).

To analyze the model we should factor in links between model parameters. According to (Dixit, Pindyck 1994, 148-150, 178-179) we assume that the total expected rate of return from owning the completed project is a sum of the expected percentage rate of growth of Y_t (α) and the convenience yield (δ) and it is equal to the expected rate of return of a financial asset (non-dividend paying) perfectly correlated with Y_t (according to CAPM), so:

$$\alpha + \delta = r + (r_m - r) \cdot \beta \, ,$$

where additionally:

r_m is the expected return on the market and coefficient β indicates whether the asset is more ($\beta > 1$) or less ($0 < \beta < 1$) volatile than the market, $\beta = \frac{\sigma \cdot \rho_m}{\sigma_m}$ (where ρ_m is the correlation of the asset with the market portfolio and σ_m is the standard deviation of r_m).

In the face of an investment opportunity ($t = 0$) a firm has tree possible decisions. It can invest immediately, wait to get more information from the market and reduce the risk of failure, or it can abandon the investment project. The conventional concept of real options required a comparison of the value of the investment option with the net present value (the NPV) of the project to make an investment decision.

However, if the investment option is a shared option, the firm should take into account how its decision influences its competitor decision, and how it itself may be impacted by rival's reactions. Therefore, firms' strategic choices could be described as a non-zero sum game.

We assume that when firms are both active on a market there may be a market power asymmetry between them. Without loss of generality, we will assume that firm A has u market share dominance ($u \geq 0.5$); firm B is left with $(1 - u)$ share. While only the one firm decides to invest, it corners the whole market and the firm which is deferring investment losses its market share towards the investing firm.

Both competitors in the model make their decisions (*Wait* or *Invest*) at the moment t=0 so there are four possible combinations and functions of payments are as follows (Rychłowska-Musiał 2018):

1. Firms A and B invest immediately and simultaneously. They share the project's benefits in accordance with their market shares; the payment for each firm is the net present value of the project:

$$NPV_0^A := NPV(u \cdot Y_t)|_{t=0} = V(u \cdot Y_0) - I;$$
$$NPV_0^B := NPV((1-u) \cdot Y_t)|_{t=0} = V((1-u) \cdot Y_0) - I.$$

where:

u – firm A's market share.

2. Firms A and B defer and keep the investment option. The payment for each of them is the call option value from the Black-Scholes-Merton model (the underlying asset is the present value of the project determined with an appropriate part of project's benefits ($u \cdot Y_t$ for firm A or $(1-u) \cdot Y_t$ for firm B), and the exercise price is the investment expenditure I):

$$F_0^A := F(u \cdot Y_t)|_{t=0};$$
$$F_0^B := F((1-u) \cdot Y_t)|_{t=0}.$$

3. Firm A invests immediately and gains the whole market. Its payment is the net present value of the project:

$$NPV_0 := NPV(Y_t)|_{t=0} = V(Y_0) - I,$$

firm B defers and its payment is zero. It is forced to abandon the investment project.

4. Firm A defers and firm B invests immediately. Then their payments are corresponding to these of the case 3.

To visualize values of these payments and to analyze games, let us assume a basic set of parameters: investment expenditure $I = 6$ (in monetary unit), expiration date $T = 2$ (years), risk free rate $r = 1.96\%$ (*YTM* of treasury bonds with maturity date equal to expiration date of investment option), the expected percentage rate of change of project cash flows $\alpha = 1\%$ (expert prediction), volatility of the project's benefits $\sigma = 60\%$, the expected return on the market $r_m = 6.03\%$ (rate of return on market index WIG 2012-2016), the standard deviation of r_m, $\sigma_m = 14,65\%$, the correlation of the asset with the market portfolio $\rho_m = 0.5$ (expert calculations), so $\beta = 2.05$ – the project is more volatile than the market.

Assumptions about the parameters reflect a situation of a real company (a similar approach is used by authors of cited papers). Additionally we assume the firm A's market share is $u = 0.75$ (so 0.25 of the market pie is left for firm B).

We are going to consider different initial values of the cash flows generated by the project $Y_0 > 0$, which enables us to formulate games and find most advantageous strategies for each firm under different present values of the project $V_0 > 0$.

3 Real Options Games. The Nash bargaining solution and the coco value

For each firm we can identify four types of relationships between possible payments: the net present value of the project and the investment option value. These are:

I_X. $NPV_0^X < NPV_0 < F_0^X,\ F_0^X > 0$
II_X. $NPV_0^X < 0 < F_0^X < NPV_0$
III_X. $0 < NPV_0^X < F_0^X < NPV_0$
IV_X. $0 < F_0^X < NPV_0^X < NPV_0$
where $X = A$ or $X = B$.

Depending on which area the value of the project for the company belongs to, we observe different types of games between companies, but these games have one general normal form, which is presented in Table 1.

Table 1 Payoff matrix

		FIRM B	
		WAIT (W)	*INVEST (I)*
FIRM A	*WAIT (W)*	$(F_0^A; F_0^B)$	$(0;\ NPV_0)$
	INVEST (I)	$(NPV_0; 0)$	$(NPV_0^A;\ NPV_0^B)$

Source: own study.

We are going to identify what bargaining games occurs between competitors. We'd like to work out a solution concept of each game (a rule for predicting how a game would be played for rational players) and formulate recommendation to firms. The most commonly used solution concept is the Nash equilibrium. In the NE no player has anything to gain by changing only its own strategy. If the other player is rational, it is reasonable for each of them to expect its opponent to follow the recommendation of NE as well (Watson, 2013, 82).
So, we are going to determine a dominant strategy (if it exists) for each player in each game and indicate Nash equilibria. However it is well known that the NE may but not need to give players the highest possible payoffs. In these cases, firms could consider negotiations as a way to achieve better results and the game becomes a bargaining one.

Various concepts have been proposed to find a reasonable and fair solution of a bargaining game. We are going to analyze and compare two interesting approaches on how to determine a solution of a bargaining real options game: the Nash bargaining solution and the cooperative-competitive value.

Nash bargaining scheme:

The common approach in terms of arbitrating the game is the Nash arbitration scheme and the Nash bargaining solution. This is a unique solution to a two-person bargaining game that satisfies four axioms: rationality, linear invariance, symmetry and independence of irrelevant alternatives (Nash 1950). It should maximizes the product:

$$\max(x - x_{SQ})\,(y - y_{SQ}),$$

where $x \geq x_{SQ}$ and $y \geq y_{SQ}$ and $SQ(x_{SQ}, y_{SQ})$. SQ is the status quo point and that means payoffs obtained if one decides not to bargain with the other player. There are a few propositions of finding the SQ point. One possibility is to assume that in the case of lack of agreement among players they can assure at least their security levels (Straffin 1993, p.108). It is going to be our assumption.

Nonetheless, the Nash bargaining solution may turn out to be difficult to put into practice, especially in real options games, when firms have to choose only one strategy only one time. The Nash bargaining solution formulates recommendations as mixed strategies and payoffs as expected values. Besides there is no payoff transfer between players in this approach.

Coco-value:

The cooperative-competitive value (the coco value for short) may be an interesting alternative of the bargaining game's solution (Kalai and Kalai 2009 and 2013). The calculation of the coco value relies on a natural decomposition of a strategic game into two component games. If (X, Y) are payoff matrices for a (complete information) game, the decomposition has a form:

$$(X, Y) = \left(\frac{X+Y}{2}, \frac{X+Y}{2}\right) + \left(\frac{X-Y}{2}, -\frac{X-Y}{2}\right).$$

The first term is a cooperative component, the highest possible payoffs that the players can mutually arrange under the agreement to share their payoffs equally. The second term is a competitive component taking into account firms strategic positions.

The coco value is a sum of the *maxmax* payoff for cooperative team game (equal for both players) and the *minmax* one for the competitive game (the value of the zero-sum game) which is an adjustment compensating transfer from the strategically weaker player to the stronger one (Kalai and Kalai, 2009, 2).:

$$coco\text{–}\,value\ (X, Y) \equiv maxmax\left(\frac{X+Y}{2}, \frac{X+Y}{2}\right) + minmax\left(\frac{X-Y}{2}, -\frac{X-Y}{2}\right).$$

Firms' Decisions:

Let us notice that when the project value is very low and for both firms inequalities for ranges I_A (firm A) and I_B(firm B) are fulfilled, the solution of the game is very easy to find and there is no need to bargain. The investment option values exceed the net present values whatever there is the only one investor in the market or there are two. In this case waiting is a dominant strategy for both competitors, it is the optimal decision for them both.

Also when the project value is very high and inequalities for ranges IV_A (firm A) and IV_B (firm B) are fulfilled for both firms, the solution of the game is indisputable and firms can find it without bargaining. The net present values of the project (in a case of simultaneous investment) exceed the investment option values for both competitors, then instantaneous investment is a dominant strategy and optimal decision for both firms.

However for a wide range of project values between these extreme intervals we can observe interesting games.

Game 1: "Bargaining Chip of a Weaker Firm"

An interesting conditions to lead the negotiations open up in the following case: for the stronger firm A the value of the investment option is higher than the net present value of a whole project (for the only investor) ($NPV_0 < F_0^A$), but at the same time for the weaker firm B the value of the investment option is lower than the net present value of a whole project (for the only investor) ($F_0^B < NPV_0$) (the range I_A for firm A and II_B for firm B).
It is a very motivating situation for firm B and a thought-provoking one for firm A.
Sample payoffs in the game in this case are presented in Table 2.

Table 2 Example of payoff matrix of the game 1 ($V_0 = 6.25$, base case)

	WAIT (*W*)	*INVEST* (*I*)
WAIT (*W*)	$(0.80, 0.03)$	$(0, 0.25)$
INVEST (*I*)	$(0.25, 0)$	$(-1.32, -4.44)$

In this case firm A has a dominant strategy – *Wait*, so if it is a rational player it delays investment decision. Firm B has no dominant strategy, but under the assumption of common knowledge, it is seeking it's best response to the A' dominant strategy *Wait*. It turns to be the strategy *Invest* and the strategy profile $(W; I)$ is the Nash equilibrium. Therefore, firm B invests and corners the market.

The assumption of common knowledge means also, that the firm A anticipates firm B's decision. Hence, there arise a problem to find a way of arbitrating the game. The argument in these negotiations is the fact that if they both invest, they will both suffer losses. However A’s loss is smaller than B’ one. So firms ought to negotiate an investment delaying and the initiating party should be firm A.

It is worth to mention that the A’ dominant strategy occurs only if the net present value of the project for firm A when both firms invest is negative ($NPV_0^A < 0$). Otherwise there is no dominant strategy in the game 1. This does not, however, affect the form of solutions.

As was found above we are going to propose and compare two approaches to determine a reasonable and fair solution of a bargaining real option game: the Nash bargaining solution and the cooperative-competitive value.

The Nash bargaining solution

We start with the Nash bargaining scheme. The security levels of both players are $(0{,}0)$ so the status quo point is $SQ(0{,}0)$. Then we have to maximize:

$$\max(x-0)\,(y-0)=xy,$$

under condition: $y=-0.269x+0.25$, which gives us: $-0.269x^2+0.25x$, where $0\le x\le 0.80$ and $0\le y\le 0.25$.

The Nash arbitrated solution turns out to be $0.58\,(W,W)+0.42(W,I)$ and it provides payoffs: $(0.46{,}0.125)$. This means that we could recommend to the competitors to play strategy (W,W) with probability 0.58 and strategy (W,I) with probability 0.42. The outcomes are determined as expected values.

Obviously for different SQ points there will be another solutions and recommendations. It is worth to mention that if SQ is $(-1.32,-4.44)$, the lowest possible payments if both parties invest, the Nash arbitrated solution is (W,W) and it provides payoffs: $(0.80{,}0.03)$.

As we know, these both recommendations may turn out to be difficult to put into practice. It is hard to indicate an incentive to induce the weaker firm not to invest.

The coco-value

The coco-value calculation requires decomposition of game into two parts: cooperative component and competitive one.

Table 3 presents the decomposition of the game 1 from Table 2.

Table 3 The *coco* decomposition of game 1

Cooperative component

$(0.415, 0.415)$	$(0.125, 0.125)$
$(0.125, 0.125)$	$(-2.88, -2.88)$

+

Competitive component

$(0.385, -0.385)$	$(-0.125, 0.125)$
$(0.125, -0.125)$	$(1.56, -1.56)$

The cooperative payoffs are the maxmax solution $(0.415, 0.415)$ of the cooperative matrix. Obviously these equal payments have to be adjusted in order to take into account the strategic positions of firms. So we add the competitive payoffs $(0.315, -0.315)$ which are the min-max value of the competitive matrix, the classic solution of a zero-sum game. The coco-value of the game 1 is computed as:

$$coco\text{-}value = (0.415, 0.415) + (0.315, -0.315) = (0.73, 0.10).$$

The coco-value provides the firm A the payoff 0.73 and the firm B the payoff 0.10. The recommendation to the competitors is to play strategy (W, W) with payoff transfer of 0.07 from the firm A to the firm B.

At first sight it may seem strange that firm B would be willing to accept the payoff of 0.1 instead of the payoff 0.25 which it could get in the case of (W, I) – the B's best response to the A's dominant strategy. The reason is simple: $0.25 - 0.1 = 0.15$ is a price (an opportunity cost) to be paid by firm B to secure against a loss (-4.44) which could hit hard the firm B if both firms would play strategies Invest.

And the payoff transfer $0.80 - 0.73 = 0.07$ is a price (a real cost) to be paid by the firm A to the firm B to ensure that firm B will delay the investment decision.

Game 2: "Asymmetric Chicken Game"

Another case in which negotiations can be expected is as follows: the net present values of the project when both firms invest are negative ($NPV_0^A < 0$ and $NPV_0^B < 0$) and for both competitors the investment option values are lower than the net present value of a whole project (for the only investor) ($F_0^A < NPV_0$ and $F_0^B < NPV_0$) (the range II_A for firm A and II_B for firm B).

It is a difficult situation for both firms.

Sample payoffs in the game in this case are presented in Table 4.

Table 4 Example of payoff matrix of the game 2 ($V_0 = 7.70$, base case)

	WAIT (*W*)	*INVEST* (*I*)
WAIT (*W*)	$(1.29, 0.07)$	$(0, 1.70)$
INVEST (*I*)	$(1.70, 0)$	$(-0.22, -4.07)$

This game has no dominant strategy for any player. However there are two pure non-equivalent and non-interchangeable equilibria of the game: (W, I) and (I, W) and a mixed strategy equilibrium where each player waits with probability p $(p \cdot W, (1-p) \cdot I;\ p \cdot W, (1-p) \cdot I)$. Competitors' decisions without coordination may lead to strategy profile (I, I), which gives them both the worst possible payments.

This result indicates that negotiations can be a good idea in this case.

Support for negotiations can be: the Nash bargaining solution and the cooperative-competitive value.

The Nash bargaining solution

In this case the security levels of both players are $(0,0)$ and the status quo point is $SQ(0,0)$, as well as in the former case. Then we have to maximize:

$$\max(x-0)\,(y-0) = xy = -x^2 + 1.70x,$$

where $0 \leq x \leq 1.70$ and $0 \leq y \leq 1.70$.

The Nash arbitrated solution is the strategy profile $0.5\,(I, W) + 0.5(W, I)$ and it provides payoffs: $(0.85, 0.85)$. This means that we could recommend to both players to play strategy (I, W) with probability 0.5 and strategy (W, I) with probability 0.5. The outcomes are determined as expected values. It seems, however, that in this solution we lose the influence of the strategic positions of both firms.

For the SQ point $(-0.22, -4.07)$, the lowest possible payments if both parties invest, the Nash arbitrated solution is (I, W) and it provides payoffs: $(1.7, 0)$.

The coco-value

The coco-value decomposition of game 2 into two parts: cooperative component and competitive one is presented in Table 5.

Table 5 The *coco* decomposition of game 2

Cooperative component

$(0.68, 0.68)$	$(0.85, 0.85)$
$(0.85, 0.85)$	$(-2.145, -2.145)$

+

Competitive component

$(0.61, -0.61)$	$(-0.85, 0.85)$
$(0.85, -0.85)$	$(1.925, -1.925)$

The coco-value of the game 2 has a form:

$$coco\text{-}value = (0.85{,}0.85) + (0.85, -0.85) = (1.70{,}0).$$

The coco-value provides the firm A the payoff 1.7 and the firm B the payoff 0. It means that the recommendation to the competitors is to play strategy (I, W) without any payoff transfer.
This recommendation is similar to the Nash bargaining solution with SQ point $(-0.22, -4.07)$ – the worst possible payments.

It means that the strategic position of the firm A is stronger than the strategic position of the firm B. And in the case of negotiations on delaying the investment decision, firm B has nothing to offer. So in this case firm A should invest and firm B is forced to abandon the project.

Game 3: "Prisoner's Dilemma"

One more case in which negotiations are advisable in consideration of the interest of both competitors is: the net present values of the project when both firms invest are positive but lower than the investment options values (the range III_A for firm A and III_B for firm B). This type of game appears with less asymmetry between companies (e.g. $u = 0.55$, $1 - u = 0.45$).

This game has one dominant strategy – *Invest* and only one Nash equilibrium – strategy profile (I, I). In this case, the simultaneous investment does not lead to losses, but nonetheless, both companies would benefit from delaying investment decision and observing the market. The payoff amounts in the strategy profile $(W; W)$ are higher than in profile (I, I). The game in this case is the prisoner's dilemma type.
Sample payoffs in the game for this region are presented in Table 6.

Table 6 Example of payoff matrix of the game 3 ($V_0 = 15, u = 0.55, 1 - u = 0.45$)

	WAIT (*W*)	*INVEST* (*I*)
WAIT (*W*)	(2.65,1.79)	(0,9)
INVEST (*I*)	(9,0)	(2.25,0.75)

The Nash bargaining solution

In the case of prisoner's dilemma the security levels are $(2.25, 0.75)$ (simultaneously, they are the worst possible payments) and the status quo point is $SQ(2.25, 0.75)$. Then we have to maximize:

$$\max(x - 2.25)\,(y - 0.75) = -x^2 + 10.5x - 18.5625,$$

where $2.25 \leq x \leq 9$ and $0.75 \leq y \leq 9$.

The Nash arbitrated solution is the strategy profile $0.58\,(I, W) + 0.42(W, I)$ and it provides payoffs: $(5.25, 3.75)$. So we may recommend to both players to play strategy (I, W) with probability 0.58 and strategy (W, I) with probability 0.42. The outcomes are determined as expected values.

The coco-value

The coco-value decomposition of game 3 into two parts: cooperative component and competitive one is presented in Table 7.

Table 7 The *coco* decomposition of game 3

Cooperative component

$(2.22, 2.22)$	$(4.5, 4.5)$
$(4.5, 4.5)$	$(1.5, 1.5)$

+

Competitive component

$(0.43, -0.43)$	$(-4.5, 4.5)$
$(4.5, -4.5)$	$(0.75, -0.75)$

The coco-value of the game 2 has a form:

$$coco\text{–}\,value \;=\; (4.5, 4.5) + \;(0.75, -0.75) = (5.25, 3.75).$$

The coco-value provides the firm A the payoff 5.25 and the firm B the payoff 3.75. It means that the best recommendation to the competitors is to play the strategy (I, W) with payoff transfer of 3.75 from the firm A to the firm B or to play strategy (W, I) with payoff transfer of 5.25 from the firm B to the firm A (although it seems less likely).

Payoffs are the same as in Nash bargaining solution but the way they are obtained is different. This solution reflects the strategic difference between companies.

What is the conclusion? The simultaneous implementation of the project by both firms independently is not fortunate. Firms should cooperate in the implementation of the project. In the real economy payoff transfer might not necessarily mean a cash transfer, but for example share of the profits in return for cooperation.

Now we are going to make some preliminary comparisons of the Nash solution and the coco value.
Huynh (2016) has formulated a conclusion that the Nash solution favours the player who can assure the most whereas the coco-value favours the player who could ensure the most in the worst case compared to the other player.
In our study it seems that the Nash solution (if the status quo point is calculated by players' security levels) favours rather the player who has the stronger strategic position and whose decisions strongly affect payments for both players.

If as the status quo point we choose payments in the worst possible case, the Nash bargaining solution and the coco-value give players similar payments (although the ways of achieving them are different) and then the player who could ensure the most in the worst case compared to the other player is favoured.
Moreover the coco-value has the one main advantage over the Nash bargaining solutions - it is easier to apply.

What is the significance of the value of the investment (real) option in this analysis?
Let's look at the relationship between the value of immediate project implementation (NPV_0^X), and the value of the investment option (F_0^X) for each companies ($X = A, B$). For each of the companies, we observe $NPV_0^X < F_0^X$ (regardless of whether NPV_0^X is positive or negative) which would mean delaying of the project implementation and keeping the real option open when the influence of the competitor's decision would be neglected. However, it turns out that in these cases the highest possible payouts (for both players) are guaranteed only by analyzing the entire strategic situation as a bargaining game. The firm's strategy should be built on the solution of this game.

In my opinion, this argument emphasizes the importance of option games in the theory of decision making and strategic analysis of investment projects. The classic approach has recommended the use of the NPV analysis of the project in making investment decisions. The real options concept (ROA) enriched this approach by including the calculation of the value of potential opportunities inherent in the project, its flexibility. But it was only the concept of the real options games (ROG) that included the analysis of the competitors' behaviors and proposed their inclusion in the strategy making process.

Because a number of observations have been made on the basis of the selected set of parameters, it is important to examine how a change in the value of individual parameters of the model will affect the solution: the amount of payments to players and recommended strategies.
This will be discuss in a section 4.

4 The Impact of Model Parameters. Under what conditions does each game matter?

We have discussed three bargaining games that could occur between companies operating on a competitive market and their possible solutions. The analysis was carried out for the basic set of parameters. It is worth checking the impact of model parameters on the occurrence of games and their solutions. Which game is more likely in certain situations and what will be its suggested solution?
The sensitivity analysis will be done for: the market shares ($0.5 \leq u < 1$) and the risk of the project ($5\% \leq \sigma \leq 160\%$).

Game 1: "Bargaining Chip of a Weaker Firm"

It is obvious that for symmetric firms games of this type does not occur. As the degree of asymmetry between companies increases, the range of project values, for which we observe a game of this type, becomes wider. For u = 0.95 there is already a range of project values $V_0 \in (5.97, 8.60)$ (other parameters from the base set). It is worth mentioning that for a large asymmetry the game between competitors combines the features of games 1 and 2 (no dominant strategy for any players, but recommended solutions as in game 1). The impact of project risk is similar. You can check that this type of game does not occur with a low risk of the project. When the risk of a project increases, the game appears for a wider range of project values. (for $\sigma = 160\%$ there is already $V_0 \in (6.47, 8.95)$).

Let us now analyze the recommended solutions.

When the asymmetry between firms is negligible (see Table 8) the Nash solution of the game in this case is the strategy profile (W, W) regardless of whether as the SQ point we use the security levels or the worst payout possible. But the coco-value provides solution as the strategy profile (W, W) with payoff transfer from the firm B to the firm A! The value of the investment option of the (slightly) weaker firm (B) is so high that the firm should fight for its retaining, and have to convince the competitor (firm A) that it does not take a disadvantage (for both) strategy *Invest*.

As the asymmetry between companies increases the range of the project's values, where we observe the game between competitors, is widened and the direction of payoff transfer in the coco solution is reversed, from the firm A (the larger market share) to the firm B (the smaller market share).
The Nash bargaining solution (SQ – security levels) recommends the mixed strategy: $p \cdot (W, W) + (1 - p) \cdot (W, I)$. We can observe that when the degree of asymmetry in firms' market shares increases, the probability of choosing the strategy (W, I) increases too – the weaker company risks and implements the project.

Besides, in the coco solution, the larger the asymmetry between firms, the greater the transfer of payment from the firm A to the firm B. It means that in the large asymmetry case the strategic importance of the weaker firm's decision is greater. A company with larger shares is willing to pay more in order to ensure the execution of the optimal strategy.
It is a very strong signal to companies dominate the market that in the face of new investment opportunities, their market positions could be especially threatened by seemingly irrelevant players and they should be covered by a particularly insightful observation. (A drastic examples of a game of this type and the nasty consequences of choosing the wrong strategy by the dominant player are cases of Nokia and Kodak described in the section 1).
At the same time, however, a very high project risk can reduce the foolhardiness of the weaker firm. With a high risk of the project, the Nash arbitration solution recommends a strategy (W, W). However, in the coco concept, a high risk is also associated with a high payoff transfer, so that the strategy (W, W) could be implemented. It is related to the fact that the increase in the risk of the project causes the increase of the value of the investment option and increase the motivation to retain it.

In summary, the importance of the game is the greatest when the asymmetry between the companies' market shares is large and the risk of the project is high too (innovative projects). In particular, it should be considered especially by companies with dominant market shares.

Table 8 Recommended strategies for firms and their payments in the Nash bargaining solution (for two possible types of the status quo points) and in the Cooperative-competitive solution for various market shares (u) and project risk (σ) in the Game 1: "Bargaining Chip of a Weaker Firm". V_0 means the present value of the whole project.

V_0	u	σ	The Nash Bargaining Solution		The coco solution
			SQ – security levels	SQ – the worst possible payments	
The impact of market shares (u)					
6.25	0.55	60%	(W, W) $= (0.37{,}0.22)$	(W, W) $= (0.37{,}0.22)$	(W, W)+payoff transfer $(0.37{,}0.22)$ $+ (0.05, -0.05)$ $= (0.42{,}0.17)$
6.25	0.65	60%	$0.85\ (W, W) +$ $0.15\ (W, I)$ $=(0.48{,}0.125)$	(W, W) $= (\ 0.57{,}0.10)$	(W, W)+payoff transfer $(0.57{,}0.10)$ $+ (-0.03, 0.03)$ $= (0.54{,}0.13)$
6.25	0.75 (base case)	60%	$0.58\ (W, W) +$ $0.42\ (W, I)$ $=(0.465{,}0.125)$	(W, W) $= (0.80{,}0.03)$	(W, W)+payoff transfer $(0.80{,}0.03)$ $+ (-0.07, 0.07)$ $= (0.73{,}0.102)$

6.25	0.85	60%	0.51 $(W,W)+$ 0.49 (W,I) =(0.547,0.125)	(W,W) $=(1.07{,}0.005)$	(W,W)+payoff transfer $(1.07{,}0.005)$ $+(-0.097,0.097)$ $=(0.973{,}0.102)$
The impact of project risk (σ)					
6.02	0.75	20%	0.5 $(W,W)+$ 0.5 (W,I) =(0.045,0.01)	(W,W) $=(0.09,0^{+})$	(W,W)+payoff transfer $(0.09,0^{+})$ $+(-0.001,0.001)$ $=(0.089{,}0.001)$
6.06	0.75	40%	0.52 $(W,W)+$ 0.48 (W,I) =(0.212,0.03)	(W,W) $=(0.41{,}0.002)$	(W,W)+payoff transfer $(0.41{,}0.002)$ $+(-0.024,0.024)$ $=(0.386{,}0.026)$
6.25	0.75 (base case)	60%	0.58 $(W,W)+$ 0.42 (W,I) =(0.465,0.125)	(W,W) $=(0.80{,}0.03)$	(W,W)+payoff transfer $(0.80{,}0.03)$ $+(-0.07,0.07)$ $=(0.73{,}0.102)$
6.25	0.75	80%	0.89 $(W,W)+$ 0.11 (W,I) =(1.0,0.125)	(W,W) $=(1.12{,}0.11)$	(W,W)+payoff transfer $(1.12{,}0.11)$ $+(-0.116,0.116)$ $=(1.004{,}0.226)$
6.25	0.75	100%	(W,W) =(1.39,0.2)	(W,W) $=(1.39{,}02)$	(W,W)+payoff transfer $(1.39{,}02)$ $+(-0.157,0.157)$ $=(1.233{,}0.357)$

Game 2: "Asymmetric Chicken Game"

Game 2 occurs between competitors with similar market shares or not very strong asymmetry. For $u = 0.5$, game 2 is played when the project values $V_0 \in (6.29{,}11.98)$; and the range of the project values for which game 2 occurs and its strategies are recommended to players, shrinks with the increase of asymmetry between the market shares of firms. If $u \geq 0.8$, game 2 does not appear in its pure form.
The impact of project risk is similar. For $\sigma = 5\%$, game 2 appears when the project values $V_0 \in (5.98{,}7.98)$; but if the project risk $\sigma \geq 100\%$, game 2 does not appear between competitors.

Let us now analyze the recommended solutions.

Let's first notice that in this type of game, regardless of the size of the company's market share, the Nash arbitrated solution (the SQ point – security levels) turns out to be the mixed strategy profile $0.5\ (I,W) + 0.5(W,I)$ and it provides equal payoffs for both players. This solution loses the influence of the strategic positions of both firms.
When the SQ point is the worst possible payments the Nash solution is the strategy (I,W). This solution (with the SQ point as the worst possible payments) we observe for any market shares. Similarly, with greater asymmetry between market shares, the coco-value recommends to the competitors playing the strategy (I,W) without any payoff

transfer. This is the madman's strategy described in the literature: the firm A should make a commitment convincing to the firm B that it is going to choose strategy *Invest*.
If a firm has a large share of the market it can afford this strategy. With a lower asymmetry, the coco concept recommends cooperation and implementation (I, W) strategy with proportional payoff transfer (strategy (W, I) with proportional payoff transfer is also acceptable, although unlikely). Of course, the greater the asymmetry of market shares, the smaller the payoff transfer.
In the game 2, the change of the project risk level does not affect the recommended solutions.

To sum up, the importance of the game is the greatest when the asymmetry between the companies' market shares is small and the risk of the project is not high (standard projects).

Game 3: "Prisoner's Dilemma"

Game 3 occurs between competitors with similar market shares or very small asymmetry. For $u = 0.5$, game 3 appears when the project values $V_0 \in (11.98, 18.65)$; and the range of the project values for which game 2 occurs and its strategies are recommended to players, shrinks with the increase of asymmetry between the market shares of firms. If $u > 0.6$, game 3 does not appear.
The impact of project risk is quite opposite. For low risk projects, the game 3 does not occur between competitors. When the risk of the project $\sigma = 20\%$ the game is played for a very small range of project values, but when the risk grows, the range of project values the game is played expands. For $\sigma = 160\%$ it is the range of $V_0 \in (13.32, 21.18)$ (base case, $u = 0.55$).
Does changing the degree of asymmetry between competitors or changing the risk of a project affect the recommended solutions?
It turns out that only to a small extent.

When firms are symmetric then the Nash arbitrated solution turns out to be the strategy profile $0.5\ (I, W) + 0.5(W, I)$ and it provides equal payoffs for both players. In this type of game, the security levels are identical with the worst possible payments.
The coco-value provides the same equal payoffs for both players as in NS but these payoffs are the consequence of playing the strategy (I, W) or (W, I) with transfer of 0.5 payment from the investing player to the abstaining one.
When the asymmetry between companies increases, the profile of the recommended strategy does not change, only the amounts of payments and possible payoff transfers change. In the Nash bargaining solution the probability of choosing the strategy (I, W) increases (see Table 9).

In game 3, the risk of the project does not affect the strategies and amounts of payment because in this game both the NS and the coco concept recommend cooperation in the project execution and the distribution of profits (possible payoff transfer) depends on firms' market shares.

Table 9 Recommended strategies for firms and their payments in the Nash bargaining solution (for two possible types of the status quo points) and in the Cooperative-competitive solution for various market shares (u) in the Game 3: "Prisoner's Dilemma". V_0 means the present value of the whole project.

V_0	u	σ	The Nash Bargaining Solution SQ – security levels = the worst possible payments	The coco solution
The impact of market shares (u)				
15	0.50	60%	$0.5\ (I,W) + 0.5\ (W,I)$ $=(4.5,4.5)$	(I,W)+payoff transfer $(9,0) + (-4.5, 4.5)$ $= (4.5,4.5)$ Or (W,I)+payoff transfer $(0,9) + (4.5, -4.5)$ $= (4.5,4.5)$
15	0.55 (base case)	60%	$0.58\ (I,W) + 0.42\ (W,I)$ $=(5.25,3.75)$	(I,W)+payoff transfer $(9,0) + (-3.75, 3.75)$ $= (5.25,3.75)$ Or (less likely) (W,I)+payoff transfer $(0,9) + (5.25, -5.25)$ $= (5.25,3.75)$
15	0.58	60%	$0.63\ (I,W) + 0.37\ (W,I)$ $=(5.7,3.3)$	(I,W)+payoff transfer $(9,0) + (-3.3,3.3)$ $= (5.7,3,3)$ Or (less likely) (W,I)+payoff transfer $(0,9) + (5.7, -5.7)$ $= (5.7,3.3)$

To sum up, the importance of the game is the greatest when the companies' market shares are almost equal and the risk of the project is high (innovative projects).

5 Final Remarks

The real options approach (ROA) takes an important place among decision-support tools. There are known conditions under which this concept can be used, usually on a monopolistic market. However, as shown in the paper, on a competitive market, where most identified real options are shared ones, decision making solely on the basis of ROA can lead to catastrophic consequences.

A company with such an investment (shared) option must take into account the impact exerted by its investment decision on its competitor as well as the impact exerted by the rival's reactions. This situation can be described as a real options game. In some cases (especially for very low or very high present values of the project) there is no problem with finding the solution of the game which is fair and satisfying for both players. However for "medium" present values of the project, when its net present values for both competitors are slightly below or above zero the game between competitors is the bargaining one.

The decision to launch negotiations should be taken by the dominant party, case-by-case, based on the Nash bargaining solution or cooperative-competitive value of the real option game. The dominant party should pay particular attention to the behaviour of a competitor when the value of the investment option suggests delaying an investment decision or when the risk of a project is high. If an investment (shared) option concerns an innovative project (of high risk) and there is a great disproportion between firms market shares then game "Bargaining Chip of a Weaker Firm" could be played between competitors; when firms' market shares are almost equal, game "Prisoner's Dilemma" can occur.

The analysis presented in the paper provides important conclusions especially for firms that dominate their markets. The correct valuation of the project is important, the real options approach (ROA) can be helpful, but only the real options games (ROG) gives us a complete picture of the situation and enable us to devise comprehensive strategies. This may be one of the reasons why despite of the initial enthusiasm, the real options approach (ROA) has proven difficult to put into practice and why its implementation is slow.

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