The Nash Bargaining Solution and the Coco-value in the Real Options Games between Asymmetric Firms

First, preliminary and incomplete version

Elżbieta Rychłowska-Musiał[1]

Abstract: To make an investment decision based on the classic real options approach, the value of an investment option should be compared with the benefits of an instantaneous investment. However, if the investment option is a shared one, a firm should take into account how its decision influences its competitor decision, and how it itself may be impacted by rival's reactions. Therefore, firms' strategic choices could be described as a (non-zero sum) real options game.

We present a general model of real options games for two (asymmetric) firms operating in the competitive market. The main goals and the unique contribution of this paper are to find when the game between competitors is a bargaining game type and to consider the Nash bargaining solution and the cooperative-competitive value (coco value) as a solution of a real options bargaining game between competitors. We try to specify the optimal recommendation for firms in each bargaining case and to check whether both parties will always be consistent in choosing the solution approach. Sensitivity of the firm's strategy profile, the Nash bargaining solution and the coco value to the changes in the important variables (especially project risk) are also provided. These considerations lead to interesting conclusions, especially for companies that dominate the market.

Keywords: Investment option, real options games, bargaining game, Nash solution, coco value.

[1] Elżbieta Rychłowska-Musiał
Poznań University of Economics and Business, Poland
elzbieta.musial@ue.poznan.pl

1 Introduction

2 The Model of Interaction Between Firms

We consider two risk-neutral firms (A and B) operating in a competitive market. Each of them can make a new investment. Both competitors share the same investment opportunity – it is a shared option (Smit, Trigeorgis 2004, 35). An investment expenditure I, $I > 0$ is the same for both competitors and the lifetime of the investment project is infinite.

The investment project generates cash flows (Y_t), which evolve in accordance with the geometric Brownian motion, with drift α, $\alpha > 0$ and volatility σ, $\sigma > 0$ under the risk-neutral measure. A risk-free asset yields a constant rate of return r; δ is a convenience yield (δ>0) and it reflects an opportunity cost of delaying construction of the project and instead keeping the option to invest alive (Dixit, Pindyck 1994, 149). The present value of the project is determined by the discounting and accumulating of its future cash flows. It is equal to $V(Y_0) = \frac{Y_0}{\delta}$ (Dixit, Pindyck 1994, 181).
To analyze the model we should factor in links between model parameters. According to (Dixit, Pindyck 1994, 148-150, 178-179) we will assume the following formula:

$$r + (r_m - r) \cdot \beta \ = \alpha + \delta,$$

where additionally:
r_m is the expected return on the market and coefficient β indicates whether the asset is more ($\beta > 1$) or less ($0 < \beta < 1$) volatile than the market, $\beta = \frac{\sigma \cdot \rho_m}{\sigma_m}$ (where ρ_m is the correlation of the asset with the market portfolio and σ_m is the standard deviation of r_m).

A firm has tree possible decisions: wait, invest or abandon the investment project. The conventional concept of real options required a comparison of the value of the investment option with the NPV of the project to make an investment decision. However, if the investment option is a shared option, the firm should take into account how its decision influences its competitor decision, and how it itself may be impacted by rival's reactions. Therefore, firms' strategic choices could be described as a non-zero sum game.

We assume that when firms are both active on a market there is an market power asymmetry between them. Without loss of generality, we will assume that firm A has u market share dominance; firm B is left with $(1 - u)$ share. While only the one firm decides to invest, it obtains the whole market and the firm which is deferring investment losses its market share towards the investing firm.

There are four possible cases under pure competition conditions. The decision to invest or to defer is made at time $t = 0$, therefore functions of payments are as follows (Rychłowska-Musiał 2018):

1. Firms A and B invest immediately and simultaneously. They share the project's benefits in accordance with their market shares; the payment for each firm is the net present value of the project:

$$NPV_0^A := NPV(u \cdot Y_t)|_{t=0} = V(u \cdot Y_0) - I;$$
$$NPV_0^B := NPV((1-u) \cdot Y_t)|_{t=0} = V((1-u) \cdot Y_0) - I.$$

where:
u – firm A's market share.
2. Firms A and B defer and keep the investment option. The payment for each of them is the call option value from the Black-Scholes-Merton model (the underlying asset is the present value of the project determined with a appropriate part of project's benefits ($u \cdot Y_t$ for firm A or $(1-u) \cdot Y_t$ for firm B), and the exercise price is the investment expenditure I):

$$F_0^A := F(u \cdot Y_t)|_{t=0};$$
$$F_0^B := F((1-u) \cdot Y_t)|_{t=0}.$$

3. Firm A invests immediately and gains the whole market. Its payment is the net present value of the project:

$$NPV_0 := NPV(Y_t)|_{t=0} = V(Y_0) - I,$$

firm B defers and its payment is zero. It is forced to abandon the investment project.
4. Firm A defers and firm B invests immediately. Then their payments are corresponding to these of the case 3.

To visualize values of these payments and to analyze games, let us assume a basic set of parameters: investment expenditure $I = 6$ (in monetary unit), expiration date $T = 2$ (years), risk free rate $r = 1{,}96\%$ (*YTM* of treasury bonds with maturity date equal to expiration date of investment option), the expected percentage rate of change of project cash flows $\alpha = 1\%$ (expert prediction), volatility of the project's benefits $\sigma = 60\%$, the expected return on the market $r_m = 6{,}03\%$ (rate of return on market index WIG 2012-2016), the standard deviation of r_m, $\sigma_m = 14{,}65\%$, the correlation of the asset with the market portfolio $\rho_m = 0{,}5$ (expert calculations), so $\beta = 2{,}05$ – the project is more volatile than the market. Assumptions about the parameters reflect a situation of a real company (a similar approach is used by authors of cited papers). Additionally we assume the firm A's market share is $u = 75\%$ (so 25% of the market pie is left for firm B).

We are going to consider different initial values of the cash flows generated by the project $Y_0 > 0$, which enables us to formulate games and find most advantageous strategies for each firm under different present values of the project $V_0 > 0$.

3 Real Options Games. The Nash bargaining solution and the coco value

For each firm we can identify four types of relationships between possible payments: the net present value of the project and the investment option value. These are:

I_X. $NPV_0^X < NPV_0 < F_0^X,\ F_0^X > 0$
II_X. $NPV_0^X < 0 < F_0^X < NPV_0$
III_X. $0 < NPV_0^X < F_0^X < NPV_0$
IV_X. $0 < F_0^X < NPV_0^X < NPV_0$
where $X = A$ or $X = B$.

Depending on which area the value of the project for the company belongs to, we observe different types of games between companies, but these games have one general normal form, which is presented in Table 1.

Table 1 Payoff matrix

		FIRM B	
		WAIT (W)	*INVEST (I)*
FIRM A	*WAIT (W)*	$(F_0^A; F_0^B)$	$(0;\ NPV_0)$
	INVEST (I)	$(NPV_0; 0)$	$(NPV_0^A;\ NPV_0^B)$

Source: own study.

We are going to determine a dominant strategy for each player in each game (if it exists) and indicate Nash Equilibria. In the NE no player has anything to gain by changing only its own strategy. If the other player is rational, it is reasonable for each of them to expect its opponent to follow the recommendation of NE as well (Watson, 2013, 82). So the Nash equilibrium is a kind of prediction of how the game will be played for rational players. However it is well known that the NE may but not need to give players the highest possible payoffs. In these cases, firms could consider negotiations as a way to achieve better results.

We are going to propose and compare two interesting approaches on how to determine a reasonable and fair solution of a bargaining (real options) game: the Nash bargaining solution and the coco value.

Nash bargaining scheme:
The common approach in terms of arbitrating the game is the Nash arbitration scheme and the Nash bargaining solution. This is a unique solution to a two-person bargaining

game that satisfies four axioms: rationality, linear invariance, symmetry and independence of irrelevant alternatives (Nash 1950). It should maximizes the product:

$$\max(x - x_{SQ})\,(y - y_{SQ}),$$

where $x \geq x_{SQ}$ and $y \geq y_{SQ}$ and $SQ(x_{SQ}, y_{SQ})$. SQ is the status quo point and that means payoffs obtained if one decides not to bargain with the other player. There are a few propositions of finding the SQ point. One possibility is to assume that in the case of lack of agreement among players they can assure at least their security levels (Straffin 1993, p.108). It is going to be our assumption.

Nonetheless, the Nash bargaining solution may turn out to be difficult to put into practice, especially in real options games, when firms have to choose only one strategy only one time. The Nash bargaining solution formulates recommendations as mixed strategies and payoffs as expected values. Besides there is no payoff transfer between players in this approach.

Coco-value:

The *cooperative-competitive value* (*coco value* for short) may be an interesting alternative of the bargaining game's solution (Kalai and Kalai 2009 and 2013). The calculation of the *coco value* relies on a natural decomposition of a strategic game into two component games. If (X, Y) are payoff matrices for a (complete information) game, the decomposition has a form:

$$(X, Y) = \left(\frac{X+Y}{2}, \frac{X+Y}{2}\right) + \left(\frac{X-Y}{2}, -\frac{X-Y}{2}\right).$$

The first term is a cooperative component, the highest possible payoffs that the players can mutually arrange under the agreement to share their payoffs equally. The second term is a competitive component taking into account firms strategic positions.

The *coco value* is a sum of the *maxmax* payoff for cooperative team game (equal for both players) and the *minmax* one for the competitive game (the value of the zero-sum game) which is an adjustment compensating transfer from the strategically weaker player to the stronger one (Kalai and Kalai, 2009, 2).:

$$coco\text{–}\,value\ (X, Y) \equiv \left(max_{i,j}\frac{X+Y}{2}, max_{i,j}\frac{X+Y}{2}\right) + minmax\left(\frac{X-Y}{2}, -\frac{X-Y}{2}\right).$$

Let us notice that when the project value is very low and for both firms inequalities for ranges I_A (firm A) and I_B(firm B) are fulfilled, the solution of the game is very easy to find and there is no need to bargain. The investment option values exceed the net present values whatever there is the one investor in the market or there are two. In this case waiting is a dominant strategy for both competitors, it is the optimal decision for them both.

Also when the project value is very high and inequalities for ranges IV_A (firm A) and IV_B (firm B) are fulfilled for both firms, the solution of the game is indisputable and firms can find it without bargaining. The net present values of the project (in a case of simultaneous investment) exceed the investment option values for both competitors, then instantaneous investment is a dominant strategy and optimal decision for both firms.

However for a wide range of project values between these extreme intervals we can observe interesting games.

Game 1: "a weaker firm has a strong bargaining chip"

An interesting conditions to lead the negotiations open up in the following case: the net present values of the project when both firms invest are negative ($NPV_0^A < 0$ and $NPV_0^B < 0$) but at the same time the value of the investment option is lower than the net present value of a whole project (for the only investor) but only for the weaker firm B ($F_0^B < NPV_0$) (the range I_A for firm A and II_B for firm B).
It is a very motivating situation for firm B and a thought-provoking one for firm A.
Sample payoffs in the game in this case are presented in Table 2.

Table 2 Example of payoff matrix of the game 1 ($V_0 = 6.14$, base case)

	WAIT (W)	*INVEST* (I)
WAIT (W)	$(0.77, 0.03)$	$(0, 0.14)$
INVEST (I)	$(0.14, 0)$	$(-1.39, -4.46)$

Firm A has a dominant strategy – *Wait*, so if it is a rational player it delays investment decision. There is no dominant strategy for firm B but, under the assumption of common knowledge, the B' best response to the A' strategy *Wait* is the strategy *Invest* and the strategy profile $(W; I)$ is the Nash equilibrium. Therefore, firm B should invest and take the whole market.

The firm A could obviously anticipate it, and there arise a problem to find a way of arbitrating the game which does take into account strategic inequalities but has a claim to fairness. There is also a circumstance which will strengthen A's bargaining position – it can threaten to invest immediately as well and to cause a loss for firm B. Actually, firm A also loses in this case, but its loss is smaller than B' one.
So the firm A ought to negotiate with firm B an investment delaying.

As was found above we are going to propose and compare two approaches to determine a reasonable and fair solution of a bargaining real option game: the Nash bargaining solution and the cooperative-competitive value.

The Nash bargaining solution

We start with Nash bargaining scheme. The security levels of both players are (0,0) so the status quo point is $SQ(0{,}0)$. Then we have to maximize:

$$\max(x-0)\,(y-0) = xy = -0.143x^2 + 0.140x,$$

where $0 \le x \le 0.77$ and $0 \le y \le 0.14$.

The Nash arbitrated solution turns out to be $0.65\,(W,W) + 0.35(W,I)$ and it provides payoffs: $(0.5{,}0.07)$. This means that we could recommend to the competitors to play strategy (W,W) with probability 0.65 and strategy (W,I) with probability 0.35. The outcomes are determined as expected values.

Obviously for different SQ points there will be another solutions and recommendations. It is worth to mention that if SQ is $(-1.39,-4.46)$, the lowest possible payments if both parties invest, the Nash arbitrated solution is (W,W) and it provides payoffs: $(0.77{,}0.03)$.

As we know, these both recommendations may turn out to be difficult to put into practice. It is hard to indicate an incentive to induce the weaker firm not to invest.

The coco-value

The coco-value calculation requires decomposition of game into two parts: cooperative component and competitive one.

Table 3 presents the decomposition of the game 1 from Table 2.

Table 3 The *coco* decomposition of game 1

Cooperative component

$(0.40{,}0.40)$	$(0.07{,}0.07)$
$(0.07{,}0.07)$	$(-2.925,-2.925)$

+

Competitive component

$(0.37,-0.37)$	$(-0.07{,}0.07)$
$(0.07,-0.07)$	$(1.535,-1.535)$

The cooperative payoffs are the maxmax solution (0.4,0.4) of the cooperative matrix. Obviously these equal payments have to be adjusted in order to take into account the strategic positions of firms. So we add the competitive payoffs (0.3,-0.3) which are the min-max value of the competitive matrix, the classic solution of a zero-sum game. The coco-value of the game 1 is computed as:

$$coco\text{–}value = (0.4{,}0.4) + (0.3,-0.3) = (0.7{,}0.1).$$

The coco-value provides the firm A the payoff 0.7 and the firm B the payoff 0.1. The recommendation to the competitors is to play strategy (W, W) with payoff transfer of 0.07 from the firm A to the firm B.
At first sight it may seem strange that firm B would be willing to accept the payoff of 0.1 instead of the payoff 0.14 which it could get in the case of (W, I) – the B's best response to the A's dominant strategy. The reason is simple: 0.14-0.1=0.04 is a price (an opportunity cost) to be paid by firm B to secure against a loss (-4.46) which could hit hard the firm B if both firms would play strategies Invest.
And the payoff transfer 0.77-0.7=0.07 is a price (a real cost) to be paid by the firm A to the firm B to ensure that firm B will delay investment decision.

In this base case it seems that the coco-value has an advantage over the Nash bargaining solutions: it is easier to apply and gives higher payments to players (in the case of SQ).

Therefore, an important question arises: will this advantage (in payments) be maintained also for other values of model parameters?
This will be discuss in a section 4.

Game 2: "without a dominant strategy"

Another case in which negotiations can be expected is as follows: the net present values of the project when both firms invest are negative ($NPV_0^A < 0$ and $NPV_0^B < 0$) and for both competitors the investment option values are lower than the net present value of a whole project (for the only investor) ($F_0^A < NPV_0$ and $F_0^B < NPV_0$) (the range II_A for firm A and II_B for firm B).
It is a difficult situation for both firms.
Sample payoffs in the game in this case are presented in Table 4.

Table 4 Example of payoff matrix of the game 2 ($V_0 = 7.70$, base case)

	WAIT (*W*)	*INVEST* (*I*)
WAIT (*W*)	$(1.29, 0.07)$	$(0, 1.70)$
INVEST (*I*)	$(1.70, 0)$	$(-0.22, -4.07)$

This game has no dominant strategy for any player. However there are two pure non-equivalent and non-interchangeable equilibria of the game: $(W; I)$ and $(I; W)$ and a mixed strategy equilibrium where each player waits with probability p $(p \cdot W, (1-p) \cdot I;\ p \cdot W, (1-p) \cdot I)$. Competitors decisions without coordination may lead to strategy profile $(I;\ I)$, which gives them both the worst possible payments.
This result indicates that negotiations can be a good idea in this case.

Support for negotiations can be: the Nash bargaining solution and the cooperative-competitive value.

The Nash bargaining solution

In this case the security levels of both players are $(0,0)$ and the status quo point is $SQ(0,0)$, as well as in the former case. Then we have to maximize:

$$\max(x-0)\,(y-0) = xy = -x^2 + 1.70x,$$

where $0 \leq x \leq 1.70$ and $0 \leq y \leq 1.70$.

The Nash arbitrated solution is the strategy profile $0.5\,(I,W) + 0.5(W,I)$ and it provides payoffs: $(0.85,0.85)$. This means that we could recommend to both players to play strategy (I,W) with probability 0.5 and strategy (W,I) with probability 0.5. The outcomes are determined as expected values. It seems, however, that in this solution we lose the influence of the strategic positions of both firms.

For SQ point $(-0.22,-4.07)$, the lowest possible payments if both parties invest, the Nash arbitrated solution is (I,W) and it provides payoffs: $(1.7,0)$.

The coco-value

The coco-value decomposition of game 2 into two parts: cooperative component and competitive one is presented in Table 5.

Table 5 The *coco* decomposition of game 2

Cooperative component

$(0.68,0.68)$	$(0.85,0.85)$
$(0.85,0.85)$	$(-2.145,-2.145)$

+

Competitive component

$(0.61,-0.61)$	$(-0.85,0.85)$
$(0.85,-0.85)$	$(1.925,-1.925)$

The coco-value of the game 2 is computed has a form:

$$coco\text{–}value = (0.85,0.85) + (0.85,-0.85) = (1.70,0).$$

The coco-value provides the firm A the payoff 1.7 and the firm B the payoff 0. It means that the recommendation to the competitors is to play strategy (I,W) without any payoff transfer.

This recommendation is similar to the Nash bargaining solution with SQ point $(-0.22,-4.07)$ – the worst possible payments.

It means that the strategic position of the firm A is stronger than the strategic position of the firm B. And in the case of negotiations on delaying the investment decision, firm B has nothing to offer. So in this case firm A should invest and firm B is forced to abandon the project.

We will discuss the impact of model parameters on these solutions in a section 4.

Game 3: "the prisoner's dilemma"

One more case in which negotiations are advisable in consideration of the interest of both competitors is: the net present values of the project when both firms invest are positive but lower than the investment options values (the range III_A for firm A and III_B for firm B). This type of game appears with less asymmetry between companies (e.g. $u = 55\%$, $1 - u = 45\%$).

This game has one dominant strategy – *Invest* and only one Nash equilibrium – strategy profile $(I; I)$. In this case, the simultaneous investment does not lead to losses, but nonetheless, both companies would benefit from delaying investment decision and observing the market. The payoff amounts in the strategy profile $(W; W)$ are higher than in profile $(I; I)$. The game in this region is the prisoner's dilemma type.
Sample payoffs in the game for this region are presented in Table 6.

Table 6 Example of payoff matrix of the game 3 ($V_0 = 15$, less asymmetric case: $u = 55\%$, $1 - u = 45\%$)

	WAIT (*W*)	*INVEST* (*I*)
WAIT (*W*)	(2.65,1.79)	(0,9)
INVEST (*I*)	(9,0)	(2.25,0.75)

The Nash bargaining solution

In the case of prisoner's dilemma the security levels are $(2.25, 0.75)$ and the status quo point is $SQ(2.25, 0.75)$. Then we have to maximize:

$$\max(x - 2.25)\,(y - 0.75) = -x^2 + 10.5x - 18.5625,$$

where $2.25 \leq x \leq 9$ and $0.75 \leq y \leq 9$.
The Nash arbitrated solution is the strategy profile $0.58\,(I, W) + 0.42(W, I)$ and it provides payoffs: $(5.25, 3.75)$. So we may recommend to both players to play strategy (I, W) with probability 0.58 and strategy (W, I) with probability 0.42. The outcomes are determined as expected values.

The coco-value

The coco-value decomposition of game 3 into two parts: cooperative component and competitive one is presented in Table 7.

Table 7 The *coco* decomposition of game 3

Cooperative component

$(2.22,2.22)$	$(4.5,4.5)$
$(4.5,4.5)$	$(1.5,1.5)$

+

Competitive component

$(0.43,-0.43)$	$(-4.5,4.5)$
$(4.5,-4.5)$	$(0.75,-0.75)$

The coco-value of the game 2 is computed has a form:

$$coco\text{–}\ value\ =\ (4.5,4.5) +\ (0.75,-0.75) = (5.25,3.75).$$

The coco-value provides the firm A the payoff 5.25 and the firm B the payoff 3.75. It means that the best recommendation to the competitors is to play strategy (I,W) with payoff transfer of 3.75 from the firm A to the firm B. Payoffs are the same as in Nash bargaining solution but the way they are obtained is different. This solution reflects the strategic difference between companies.

4 The Impact of Model Parameters

5 Final Remarks (*first, incomplete version*)

Most real options identified in business practice are shared options. Then a company with such an investment option must take into account the impact exerted by its investment decision on its competitor as well as of the impact exerted by the rival's reactions. This situation can be described by real options games. In some cases (especially for very low or very high present values of the project) there is no problem with finding the solution of game which is fair and satisfying for both players. However for "medium" present values of the project, when its net present values for both competitors are slightly below or above zero the game between competitors is the bargaining game.

The decision to launch negotiations should be taken by the dominant party, case-by-case, based on the Nash bargaining solution or cooperative-competitive value of the real option game.

The dominant party should pay particular attention to the behaviour of a competitor when the value of the investment option suggests delaying an investment decision. This awkward situation of the dominant firm may be one of the reasons why despite the initial enthusiasm, real options approach has proven difficult to put into practice and why its implementation is slow.

References

1. Chevalier-Roignant B, Trigeoris L (2011) Competitive Strategy: Options and Games. The MIT Press
2. Dixit AK, Pindyck RS (1994) Investment Under Uncertainty. Princeton University Press: Princeton, New Jersey
3. Grenadier SR (2000) Game Choices: The Intersection of Real Options and Game Theory, London: Risk Books
4. Huynh D (2016) Bargaining games: a comparison of Nash's solution with the Coco-value, working paper
5. Kalai A, Kalai E (2009) Engineering cooperation in two-player games, working paper
6. Kalai A, Kalai E (2013) Cooperation in strategic games revisited. The Quarterly Journal of Economics, 128: 917-966
7. Luehrman TA (1998) Strategy as a portfolio of real options, Harvard Business Review, No.76(5)
8. Nash JF (1950) The Bargaining Problem: Econometrica, Vol. 18 No. 2: 155-162
9. Rychłowska-Musiał E (2017) Value Creation in a Firm Through Coopetition. Real Options Games Approach: Contemporary Trends and Changes in Finance. Springer,
10. Rychłowska-Musiał E (2018) Real Options Games Between Asymmetric Firms on a Competitive Market: Advances in Panel Data Analysis in Applied Economic Research, Springer, forthcoming
11. Smit HTJ, Ankum LA (1993) A Real Options and Game-Theoretic Approach to Corporate Investment Strategy under Competition. Financial Management. Vol. 22 No.3: 241-250
12. Smit HTJ, Trigeorgis L (2004) Strategic Investment. Real Options and Games. Princeton and Oxford: Princeton University Press
13. Trigeogris L, Baldi F (2013) Patent strategies: Fight or cooperate?, Real options annual conference, Tokyo, 25-26 June 2013.
14. Watson J (2013) An Introduction to Game Theory, Norton W.W. & Company Inc., New York