

Is “IPO valuation premium puzzle” really a puzzle?

Entrepreneur’s exit choice with Game Theoretic Real Options Approach

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Abstract

“IPO valuation premium puzzle” is an intriguing issue for the entrepreneurial exit strategy. This refers to a situation where many private firms choose to be acquired rather than to go public at higher valuations by the market participants. The objective of this paper is to explain this “puzzle” from the viewpoint of the interaction between an entrepreneur and a venture capitalist. The theoretical analysis of the “private benefits of control” (Bayar and Chemmanur, 2011, 2012) with the game theoretic real options approach shows that the “puzzle” is not really a puzzle. In addition, a new exit choice criterion is provided. The results of the numerical simulation show that even when the start-up business is highly evaluated by the market, acquisition and IPO is indifferent. This also suggests that the “puzzle” is not really a puzzle.

Keywords: IPO, Acquisition, Real Options, Game theory

1. Introduction

Planning exit strategies is one of the central issues not only for investors such as the venture capitalist but also the entrepreneurs who have created their start-ups. IPO and acquisition are two well-recognised exit strategies for the entrepreneurial businesses. It seems often to be thought that achieving IPO is no doubt favourable for both entrepreneur and venture capitalist because huge returns could be predicted. Unfortunately, it is not uncommon at the later stage of the venture projects that the business turns out not to be successful and the desired return cannot be expected even if IPO were implemented. In such a situation, acquisition appears to be thought as the inferior exit strategy to IPO.

Is IPO always expected to bring about the huge return? It is necessary to point out the underpricing of the share of IPO firms. Underpricing means the phenomenon that the first trading price on the secondary market becomes lower than the issue price of a new share. For example, the empirical study by Allen and Faulhaber (1989) shows that in certain periods and in certain industries, new issues (initial public offerings) are underpriced. Jain and Kini (1994) find post-issue declines in the market-to-book ratio, price/earnings ratio, and earnings per share. Ljungqvist and Wilhelm Jr. (2003) argue that following the so-called “dot-com bubble” (which occurred around 1997 to 2001), IPO underpricing reached astronomical levels during 1999 and 2000. Despite of the underpricing issue, however, IPO may still be superior to acquisition as long as entrepreneurs and venture capitalists are expecting to obtain greater financial returns from IPO than that from acquisition. Carter et al. (1998) found an interesting result about the underperformance of IPO stocks, showing that IPOs managed by more reputable underwriters are associated with less short-run underpricing.

In addition, the financial market characteristics should be taken into account in order to assess whether the huge return can be expected by IPO. On this point, the American IPO markets is the largest and most active in the world (see for instance “Global IPO trends: Q3 2017” released by Ernest & Young), while the European IPO market remains fragmented (Andrieu 2013). Andrieu also points out that bank plays a great role in the start-up financing in Europe, and bank-affiliated venture capital (VC) firms dominate the VC market in continental Europe. In other words, these countries rely on bank-centred capital markets, whereas the United States is stock market-

oriented. As for another aspect of market characteristics, Santana Félix et al. (2013) argue that the size of the M&A market is relevant in explaining VC investment, and the VC market may grow in countries with vibrant M&A markets even if their IPO market is not very developed. Bertoni and Groh (2014) give an interesting suggestion about the exit strategy for young high-tech companies backed by VC in seven European countries, saying that the impact of cross-border investors on the exit mode also depends, more specifically, on the exit opportunities available there (local exit condition). This “local exit condition” is related to the M&A market in Europe, and the authors also point out that the mechanism is stronger for trade sales than for IPOs. These evidences suggest that the priority of IPO to acquisition as an exit option might not be conclusive when taking the local market conditions for IPO and M&A market conditions into consideration. The exit strategy choice might not be an easy task especially in Europe. If so, then one question arises: “which exit option should we choose, IPO or acquisition?”

Bayar and Chemmanur (2011) are the first researchers who answer this question and provide a theoretical model. The most important point to be noticed in their work is that they are trying to address an intriguing issue, named “IPO valuation premium puzzle” (Bayar and Chemmanur, 2011, 2012). According to their definition, “IPO valuation premium puzzle” refers to a situation where many private firms choose to be acquired rather than to go public at higher valuations. As mentioned above, if we think of IPO being a superior exit strategy to acquisition, it is true that, from the view point of “homo economicus”, choosing acquisition rather than IPO is not rational even when their business is highly valued and they can expect high economical return. Thus, they call this situation “puzzle.” It is a quite interesting point and worth scrutinising because this “puzzle” situation might be directly linked to the answer for the question above.

Then, the next question arises: Holding the assumption of rational homo economicus, why does this puzzle occur? Before answering this question, Bayar and Chemmanur (2011) propose the following assumptions, in their theoretical model building, that the entrepreneur, being a long-term investor, may be concerned about the sustainability of high valuation, and the VC, being a short-term investor, may be less affected by such concern. Based on this idea, they insist that entrepreneurs choose acquisition over IPO when the long-term expected pay-off will be lower in the case of an IPO compared to its acquisition value. That is to say, the choice of an exit strategy, IPO or acquisition, by an entrepreneur should be determined by the market value, and in some

cases at least theoretically, the value obtained by choosing IPO can be lower than the one by acquisition. They describe this condition as “IPO valuation premium disappears.” They have already designed an empirical research, and proved the existence of this condition (Bayar and Chemmanur, 2012), which relates to the fact that IPO valuation premium vanishes even for larger non-venture capital backed firms and shrinks substantially for smaller firms as well.

Their theoretical model can answer the dilemma between IPO and acquisition, by solving the maximisation problem. This model’s answer results mainly from market conditions. As IPO valuation premia disappears, acquisition should be chosen. As mentioned above, there is also empirical evidence. Thus, their model can no doubt be a ground breaking toward the practical application for the exit strategy planning. However, one variable named as “private benefits of control, B ” that should be handled with care due to its intrinsic property, which does not come from the market conditions. It is just simply added into their theoretical model in the manner with the other variables of the Expected NPV, the sell fraction of shares and the probability of choosing IPO by entrepreneur. On the other hand, they explain one of their empirical findings (Bayar and Chemmanur, 2012) as follows; firms which are harder to value by IPO market investors, more capital-intensive firms, and those operating in industries characterised by greater private benefits of control, are more likely to go public rather than to be acquired. Their theoretical model assumes that the determining factor for choosing either IPO or acquisition is mainly the market condition. If their empirical finding is correct, “private benefits of control” is also an important determining factor for that choice. This suggests that we should not put too much weight only on the market condition. Therefore, the importance of this “private benefits of control” must have been clearly explained, and its property must be much further emphasised, not as it is simply put in the theoretical model.

The main objective of this paper is to analyse the property of “private benefits of control” by the game theoretic real options approach and to reveal the importance of its role as a criterion of choosing the exit option. In addition, this paper is also trying to understand what the “IPO valuation premium puzzle” actually is. It could be quite helpful for setting up the exit strategy effectively more than ever.

The contribution of this paper is to analyse the “IPO valuation premium puzzle” from the different perspective proposed by Bayar and Chemmanur (2011, 2012). It is true that

understanding the exit strategy criterion from the market viewpoint is essential. At the same time, the perspective of behaviours of the players should not be ignored. In other words, both macro and micro viewpoints should be combined. This paper is mainly written from this viewpoint. In this sense, this paper is complementary with the work by Bayar and Chemmanur (2011, 2012). The topic about the start-up exit choice is relatively new, and few researches have been done so far. Thus, this paper contributes to the development of the exit strategy planning method in a scientific way.

The rest of this paper is organised as follows; Section 2 presents the methods for analysing and modelling of the exit strategy choice. Section 3 explains “IPO valuation premium puzzle.” Section 4 presents the numerical simulation and its results. Finally, Section 5 concludes.

2. Analysing and modelling of the exit strategy choice

1) *The original choice model by Bayar and Chemmanur*

Bayar and Chemmanur (2011) have provided the model for choosing IPO or acquisition as solution of the following maximisation problem;

$$\max_{a \in \{0,1\}} a \cdot [\delta_E(1 - \gamma)(\alpha_E P_{IPO}^E + (1 - \alpha_E)(I + V_q) + B)] + (1 - a) \cdot \delta_E \rho V_A$$

Where a : the exit choice ($a = 0$: acquisition, $a = 1$: IPO), α_E : the entrepreneur's the sell fraction, δ_E : the entrepreneur's initial holding fraction, γ : the fraction of shares sold to new shareholders, P_{IPO}^E : the IPO valuation, I : the investment by acquiring firm, V_q : the expected NPV when the firm goes public, ρV_A : the acquired firm's project NPV, B : the private benefits of control.

As mentioned before, this model assumes that the primary determining factor for choosing either IPO or acquisition is the market condition. However, comparing with the other variables in the equation, it is obvious that the property of "private benefits of control, B " is different, and must be handled with care because it is not determined by the market condition. The property of this variable is analysed precisely in the section 3.

2) *Real options analysis*

In order to grow the business and arrive at the exit phase, entrepreneur needs the funds by external investors in most cases. Equity investors, such as venture capitalists, have many opportunities to control the business of the start-up they invest while debt investors, such as banks, have less. Thus, the entrepreneurs' financing choice has a great influence not only on the success of the business but also on the exit strategy.

Many researches show that the support by venture capitalists increase the probability of success. Colea et al. (2016) compare the effect of two main sources of entrepreneurial finance, which are banks versus venture capital (VC), on small firm formation and growth. They find the effect of VC to be both economically and statistically significant in stimulating new firms, and do not find

similar evidence for banks. Andrieu and Groh (2012) say that independent VC firms provide better support quality than by bank-affiliated VC firm, though the latter have access to very large financial resources. However, even if entrepreneur could obtain the support of VC, the success of the business is not sure and uncertainty remains. Uncertainty cannot be completely eliminated, and it is no exception in the exit strategy phase. Thus, the important thing is to focus on how to manage it.

Real options analysis (ROA) is the well-known method to manage the uncertainty related to the strategy planning. The original idea of ROA was proposed by Myers (1977). He assumed that the concepts of the financial options for the marketable securities could be applied to the corporate finance issues as a remedy of the deficiency of the net present value (NPV) calculation. In many cases, NPV becomes negative when it is applied to the start-up valuation because the larger discount rate is used due to great uncertainty. Nevertheless, especially in the start-up financing, it is not always the case that investment should be refrained. Venture capitalists usually adapt the staged financing, for example. This method allows to invest flexibly with the market condition and to resolve uncertainty (e.g. Copeland and Antikarov 2003, Dixit and Pindyck 1994). ROA can capture the value derived from such flexibility in the strategic planning, and thus, it has been nowadays widely applied not only to the similar situations. The choice of an exit option is also affected directly by the market expectation, and thinking its flexibility is essential for both venture capitalist and entrepreneur. Therefore, ROA is quite a suitable method to grasp the properties of the exit option choice.

3) Game theoretic real options approach

In addition to the issue of managing the uncertainty of market condition, entrepreneur must pay attention to the relationship with VC when deciding the exit strategy option. Even if the probability of the success could be increased by the support of VC, the entrepreneur's equity share decreases when the fund is provided as equity. For the purpose of analysing "private benefits of control", this issue should be seriously considered. There are several ways that explain the relationship among economic players. In particular, game theory is one of the well-known methods. According to Rasmusen (2007), game theory is concerned with actions of decision

makers who are conscious that their actions affect each other. This fits right in with the exit choice situation with the interaction of entrepreneur and venture capital.

Although game theory itself is not the new method, in recent years, the theoretical combination of game theory and real options has been developed (e.g. Smit and Ankum 1993, Smit and Trigeorigs 2006). The exit option choice issue contains the two main factors, the market condition and the interaction between entrepreneur and venture capitalist, therefore, it is possible to say that the game theoretic real options approach is a quite suitable analytical tool. In the following section, this game theoretic real options approach is employed for modelling the exit strategy choice.

4) 2-period binomial tree model

For simplicity and understandability, it is usual to assume that both entrepreneur (Ent) and venture capitalist (VC) are risk-neutral, and the expected values of the venture business constitute the binomial tree. In general, both entrepreneur and venture capitalists will have to set up strategies under uncertain situations where they do not have sufficient information. Thus, they try to defer their decisions until the situation become realised and obtain certain information. This can be applied to the exit strategy setting up. It is clearly more favourable to avoid fixing their exit strategy at the early stage, and to make decisions after waiting and seeing how their business goes. Related to this point, DeTienne et al. (2015) say that while actual exits are important, the early stage and founders' ongoing actions and decisions are often based upon intended exit strategies. In this discussion, entrepreneur and venture capitalist are thus assumed to make their exit strategy decisions not at the beginning but at some later period. In this sense, the 2-period binomial tree model is quite suitable to be applied as shown in Fig 1. It is assumed that they can set up their exit strategies at $T=t_1$ with the predictions at $T=t_2$.

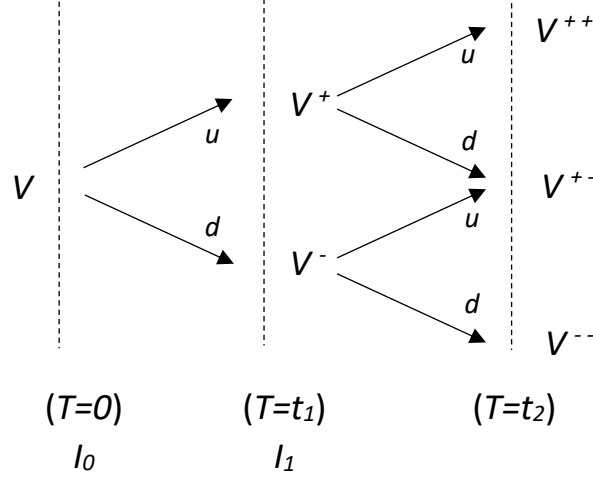


Fig.1: The binomial tree value model

The 2-period binomial tree model is also quite compatible with the start-up financing scheme. The value of the venture business that venture capitalists assess before investment ($T=0$), V , derives from the commitment of I_0 by entrepreneur. In the case of the business being successful, the value at $T=t_1$ will be $V^+ = uV$, and $V^- = dV$ otherwise. In this timing, entrepreneur asks for the additional equity capital to venture capitalists, and they provide the amount of I_1 . The value at $T=t_2$ are expected to be $V^{++} = uuV$, $V^{+-} = udV$, or $V^{--} = ddV$. Let π be the risk neutral probability, it can be calculated as $\pi = (e^{rT} - d)/(u - d)$ where r represents the risk-free rate. This assumption is in line with the discussion by Faria and Barbosa (2014). They found that only the later-stage VC capital is promoting innovation. They also insist that their result is consistent with the view that the VC role is more to help the commercialization of innovation rather than to foster its creation.

In the case that the business being expected to be successful, the value outcome at $T=t_1$ for entrepreneur, V_{Ent}^+ , and for venture capitalist, V_{VC}^+ , can be calculated as shown below, respectively;

$$V_{Ent}^+ = \left[\pi \times \frac{I_0}{I_0 + I_1} \times V^{++} + (1 - \pi) \times \frac{I_0}{I_0 + I_1} \times V^{+-} \right] \times e^{-r(t_2 - t_1)}$$

$$V_{VC}^+ = \left[\pi \times \left(\frac{I_1}{I_0 + I_1} \times V^{++} - I_1 \right) + (1 - \pi) \times \left(\frac{I_1}{I_0 + I_1} \times V^{+-} - I_1 \right) \right] \times e^{-r(t_2 - t_1)}$$

In the same manner, at $T=t_1$ when the business being expected to be unsuccessful, the proportion of the value can be calculated as follows:

$$V_{Ent}^- = \left[\pi \times \frac{I_0}{I_0+I_1} \times V^{+-} + (1 - \pi) \times \frac{I_0}{I_0+I_1} \times V^{--} \right] \times e^{-r(t_2-t_1)}$$

$$V_{VC}^- = \left[\pi \times \left(\frac{I_1}{I_0+I_1} \times V^{+-} - I_1 \right) + (1 - \pi) \times \left(\frac{I_1}{I_0+I_1} \times V^{--} - I_1 \right) \right] \times e^{-r(t_2-t_1)}$$

5) Nash Equilibrium and private benefits of control

Regardless of the choice of exit, either IPO or acquisition, entrepreneurs and venture capitalists are not able to obtain the whole value calculated above. They must let some proportion of their equity share off, and hold only the remaining fraction. Defining this remaining holding fraction as below, the value outcomes at $T=t_1$ for both entrepreneur and venture capitalist can be calculated.

$\alpha_{Ent}^A, \alpha_{VC}^A$: the fraction when acquisition is chosen as exit strategy

$\alpha_{Ent}^I, \alpha_{VC}^I$: the fraction when IPO is chosen as exit strategy

Based on these results derived from the market condition, entrepreneur and venture capitalist make an exit decision. As noted before, however, not only the market condition is a determining factor. It is necessary to consider the interaction between them, and game theoretic framework can be adopted. In this point, although it is not uncommon to predict that there is some relationship between the exit choice and the support by VC, there seems to be no consensus about it at this moment. For example, the empirical study by Bayar and Chemmanur (2012) shows that the likelihood of an IPO over an acquisition is greater for venture backed firms and those characterized by higher pre-exit sales growth. On the contrary, the empirical research by Cumming (2012) shows that ex ante, stronger VC control rights increase the likelihood that an entrepreneurial firm will exit by an acquisition, rather than through a write-off or an IPO. Therefore, this modelling adopts the game theoretic situation that both entrepreneur and venture

capitalist can make decisions independently and these decisions do not affect each other. Considering the game theoretic framework, 2×2 matrix normal form representation of the game can be drawn in Table 1 below. This framework is inspired by the one proposed by Smit and Trigeorigs (2006).

Table 1: Value outcome (at $T=t_1$)

		VC	
		Acquisition	IPO
Entrepreneur	Acquisition	(i) $\alpha_{Ent}^A \cdot V_{Ent}$ $\alpha_{VC}^A \cdot V_{VC}$	(ii) $\alpha_{Ent}^A \cdot V_{Ent}$ $\alpha_{VC}^I \cdot V_{VC}$
	IPO	(iii) $\alpha_{Ent}^I \cdot V_{Ent}$ $\alpha_{VC}^A \cdot V_{VC}$	(iv) $\alpha_{Ent}^I \cdot V_{Ent}$ $\alpha_{VC}^I \cdot V_{VC}$

($V_{Ent}: V_{Ent}^+$ or V_{Ent}^- and $V_{VC}: V_{VC}^+$ or V_{VC}^-)

In general, they give up a majority of their share (or all of their shares) when they choose acquisition and loose the control on that business, while they sell off only a small portion of it. The values should be $0 < \alpha^A \ll \alpha^I < 1$, and it would be possible to put the number as $\alpha_{Ent}^A = 0.1$, $\alpha_{VC}^A = 0.1$, $\alpha_{Ent}^I = 0.9$, $\alpha_{VC}^I = 0.9$, for example.

From Table 1 above, the case (iv) can be the unique (pure-strategy) Nash Equilibrium. This makes sense in practice because they would usually hope the great success of the business and set IPO as an intended goal when making a financing contract at $T=t_1$.

Although it is normal that both entrepreneur and venture capitalist intend IPO, it is also possible that they choose acquisition as the agreed exit strategy. From the game theoretic scheme, this contradictory phenomenon is able to be described as the existence of two Nash Equilibria. This means that the case (iv) in Table 1 is not always a unique Nash Equilibrium, but the case (i) can be another Nash Equilibrium. For realising this situation, at least theoretically, some positive

value(s) must be added to the original one. In regard to this point, Bayar and Chemmanur (2011, 2012) mention the variable which is named as “private benefits of control.” They insist that this value should be taken into consideration when choosing either IPO or acquisition (as solving the maximization problem). Although they do not define it clearly, it can be interpreted that the property of this variable should be grasped as the proportion of holding equity share. Therefore, this paper adopts the control power on the business obtained by the proportion of the equity share as the definition of “Private benefits of control.” Then, it is possible to explain that the larger the proportion is, the more the private benefits of control increase.

Introducing the new values, B_{Ent} , B_{VC} , Table 1 can be rewritten as Table 2 below. Let B_{Ent} , B_{VC} be the value of private benefits of control for entrepreneur and venture capitalist, respectively.

Table 2: Value outcome with Private Benefits of control variable (at $T=t_1$)

		VC	
		Acquisition	IPO
Entrepreneur	Acquisition	(i') $\alpha_{Ent}^A \cdot V_{Ent} + B_{Ent}$ $\alpha_{VC}^A \cdot V_{VC} + B_{VC}$	(ii') $\alpha_{Ent}^A \cdot V_{Ent}$ $\alpha_{VC}^I \cdot V_{VC}$
	IPO	(iii') $\alpha_{Ent}^I \cdot V_{Ent}$ $\alpha_{VC}^A \cdot V_{VC}$	(iv') $\alpha_{Ent}^I \cdot V_{Ent}$ $\alpha_{VC}^I \cdot V_{VC}$

Comparing with Table 1, not only the case (iv') but also the case (i') can be Nash Equilibrium. In effect, these are the two Nash Equilibria if:

$$\alpha_{Ent}^A \cdot V_{Ent} + B_{Ent} > \alpha_{Ent}^I \cdot V_{Ent} \quad \text{and} \quad \alpha_{VC}^A \cdot V_{VC} + B_{VC} > \alpha_{VC}^I \cdot V_{VC}.$$

These inequalities can be summarised as below:

$$B > (\alpha^I - \alpha^A) \cdot V$$

The right side of the inequality is composed of two parts, $(\alpha^I - \alpha^A)$ and V . Quantitatively, the greater the difference of $(\alpha^I - \alpha^A)$ is, the higher the level of private benefits of control should be required for choosing acquisition rather than IPO as an exit strategy. For example, the probability of acquisition being chosen would be larger when $\alpha^I = 0.9, \alpha^A = 0.3$, rather than when $\alpha^I = 0.9, \alpha^A = 0.1$, because the lower level of private benefits of control is required. The situation when they choose IPO and give off their share is the same. The probability of IPO being chosen would be larger when $\alpha^I = 0.9, \alpha^A = 0.1$, rather than when $\alpha^I = 0.7, \alpha^A = 0.1$, for another example, because the higher level of private benefits of control is required for acquisition. These could be intuitively understandable in practice, considering the trade-off whether they choose to hold their equity or to require another control benefits instead. Acquisition is chosen only if entrepreneur and venture capitalist can expect to enjoy the higher level of private benefits of control.

In the same way, in order to let acquisition be a dominating strategy over IPO, the high level of private benefits of control should be required when the market value of the business is estimated to be high, such as V^+ at $T=t_I$. Under this circumstance, the case (iv') can often be the unique Nash Equilibrium, and IPO will become a favourable choice. This makes sense in the context of the real business world because both entrepreneur and venture capitalist hope to success of the business, and there is almost no reason not to intend to achieve IPO in such a situation.

In contrast, we must handle with care the situation where the business is not going well and the market value of the business is estimated not to be high, rather low, such as V^- at $T=t_I$. In this situation, the high level of private benefits of control is no more required for choosing acquisition according to the inequality and it leads to the consequence that both cases (i') and (iv') can be Nash Equilibria. Therefore, it is not often the case that IPO becomes the unique dominating exit strategy, and instead, acquisition enhances its presence as an alternative one in the game theoretic framework.

6) *Choice of exit option*

Considering the value outcome for entrepreneur at $T=0$, Table 2 should be rewritten as Table 3 and 4. Although showing up $4 \times 4 = 16$ outcomes is mathematically sound, it might not be efficient in the context of practical business. As mentioned above, both entrepreneur and venture capitalist usually desire the big success of their venture project and realise IPO as an exit strategy. In the process of the business development (at $T=t_1$), acquisition could sometimes turn out to be the alternative exit strategy. Therefore, it would make sense in general that considering the following two cases: one is the case (Case A) that hoping to achieve IPO and going forward to IPO. This can be represented as the combination of (iv') from Table 3 and (iv') from Table 4. The other one (Case B) is that hoping to achieve IPO but shift to acquisition. This can be represented as the combination of (iv') from Table 3 and (i') from Table 4.

Table 3: Value outcome (V^+ at $T=t_1$)

		VC	
		Acquisition	IPO
Entrepreneur	Acquisition	(i') $\alpha_{Ent}^A \cdot V_{Ent}^+ + B_{Ent}$ $\alpha_{VC}^A \cdot V_{VC}^+ + B_{VC}$	(ii') $\alpha_{Ent}^A \cdot V_{Ent}^+$ $\alpha_{VC}^I \cdot V_{VC}^+$
	IPO	(iii') $\alpha_{Ent}^I \cdot V_{Ent}^+$ $\alpha_{VC}^A \cdot V_{VC}^+$	(iv') $\alpha_{Ent}^I \cdot V_{Ent}^+$ $\alpha_{VC}^I \cdot V_{VC}^+$

Table 4: Value outcome (V^- at $T=t_1$)

		VC	
		Acquisition	IPO
Entrepreneur	Acquisition	(i') $\alpha_{Ent}^A \cdot V_{Ent}^- + B_{Ent}$ $\alpha_{VC}^A \cdot V_{VC}^- + B_{VC}$	(ii') $\alpha_{Ent}^A \cdot V_{Ent}^-$ $\alpha_{VC}^I \cdot V_{VC}^-$
	IPO	(iii') $\alpha_{Ent}^I \cdot V_{Ent}^-$ $\alpha_{VC}^A \cdot V_{VC}^-$	(iv') $\alpha_{Ent}^I \cdot V_{Ent}^-$ $\alpha_{VC}^I \cdot V_{VC}^-$

In Case A, the value outcome at $T=0$ can be calculated as follows:

$$V_{Ent,0}^{I,I} = [\pi \times \alpha_{Ent}^I \cdot V_{Ent}^+ + (1 - \pi) \times \alpha_{Ent}^I \cdot V_{Ent}^-] \times e^{-rt_1} - I_0$$

In Case B, the value outcome at $T=0$ can be also calculated as follows:

$$V_{Ent,0}^{I,A} = [\pi \times \alpha_{Ent}^I \cdot V_{Ent}^+ + (1 - \pi) \times (\alpha_{Ent}^A \cdot V_{Ent}^- + B_{Ent})] \times e^{-rt_1} - I_0$$

$$\text{Where } B_{Ent} > (\alpha_{Ent}^I - \alpha_{Ent}^A) \cdot V_{Ent}^-$$

The difference of these two value outcomes, $V_{Ent,0}^{I,A} - V_{Ent,0}^{I,I}$, becomes positive when acquisition is chosen over IPO:

$$V_{Ent,0}^{I,A} - V_{Ent,0}^{I,I} = [(1 - \pi) \times (B_{Ent} - (\alpha_{Ent}^I - \alpha_{Ent}^A) \cdot V_{Ent}^-)] \times e^{-rt_1} > 0$$

3. Explaining “IPO valuation premium puzzle”

1) *The minimum value of the private benefits of control*

It is obvious that the condition of $B_{Ent} > (\alpha_{Ent}^I - \alpha_{Ent}^A) \cdot V_{Ent}^-$ is crucial for choosing either IPO or acquisition. In order to choose acquisition rather than IPO, the private benefits of control for entrepreneur must be greater than the minimum value, B_{Ent}^{min} .

$$B_{Ent}^{min} = (\alpha_{Ent}^I - \alpha_{Ent}^A) \cdot V_{Ent}^- = (\alpha_{Ent}^I - \alpha_{Ent}^A) \times \frac{1}{1+(I_1/I_0)} \times (\pi ud + (1 - \pi)dd)V \cdot e^{-r(t_2-t_1)}$$

The third term in the equation represents the net present value (NPV) of the market value of the business.

The minimum value is the negative function of α_{Ent}^A . If entrepreneur could obtain the larger remaining holding fraction for entrepreneur through the negotiation with acquirer, they would require the lower level of minimum control benefits. In contrast, the minimum value is the positive function of the equity proportion of venture capital I_0 and the NPV of the market value of the business. If venture capitalists assess the business is expected to be successful, then they invest more and the equity proportion of venture capitalist I_1/I_0 increases and the one of entrepreneur decreases. This would encourage entrepreneur to increase the minimum level control benefit because entrepreneur and venture capitalist must divide the benefit they could obtain by selling the business to acquirer. High NPV means that the success is highly expected, and merely keeping the equity share can be the source of economic benefit. At the same time, this action leads to the creation of benefits of control. As we can see, the minimum value of the private benefits of control, B_{Ent}^{min} , incorporates not only the market condition but also the contract aspect.

B_{Ent}^{min} , which can be a critical index is derived from the difference of the values between the two exit options of IPO and acquisition, especially when the market expectation for the start-up business is not high. In other words, it can only be measured indirectly rather than directly measured or observed, but it can unveil that there must be something valuable. In this sense, it

would be possible to say that it has a similar property of the ‘Goodwill’ in the financial accounting item, which is recognised as the benchmark of the expectation of the firm’s business at the timing of the M&A being implemented. Therefore, it could become a new benchmark for the exit strategy planning.

2) *Is “IPO valuation premium puzzle” really a “puzzle”?*

As mentioned before, “IPO valuation premium puzzle” can be seen in the real business world. This game theoretic approach can be the very core explanation for “IPO valuation premium puzzle”, and the part that has not yet explained explicitly by Bayar and Chemmanur (2011, 2012). It is sometimes thought in practice that IPO is the primary exit strategy to be chosen when the venture business becomes successful and acquisition can be the secondary strategy as some kind of risk hedge when failed. This thought may not be wrong but not right at least. In fact, acquisition seems to be favoured by venture capitalists as a method of collecting the cash invested as much as possible when the business turns out not to be successful as they have desired. Nevertheless, the main purpose of acquisition must never be to recover the failure of the venture project. Acquisition itself has its own practical benefits. As Bayar and Chemmanur (2012) say, “the benefit of an acquisition over an IPO is that the acquiring firm can provide support to the acquired firm in product market competition by increasing its probability of success in the product market while a stand-alone firm has to fend for itself after an IPO.” If entrepreneurs are offered some managing position inside the acquiring firm, such as technical chief of the product they invent, it could be said that they still have the control benefit because they have an opportunity to achieve their original goal through the acquiring firm’s distribution channel (Roizen 2016).

Furthermore, there is even a research by Rosenbusch et. al (2013), saying that performance effects, which are mainly related to firm growth, are reduced when the funded firms are very young or very mature. They also say that VC funding seems to lose value after the funded firm goes public.

When the value of the business is expected to be high, choosing acquisition over IPO may be a “puzzle” from market viewpoint. However, as this theoretical analysis above shows, the exit

option choice can be determined by evaluating whether the private benefits of control is greater than the minimum level of control benefit, B_{Ent}^{min} , or not. It is drawn not from the market condition, rather from its inherent property that can be explained by the game theoretic framework. Thus, IPO and acquisition should be compared equivalently especially when the market expectation for the business is not high. This view is different from the original one by Bayar and Chemmanur. IPO contains intrinsically the high level of benefits of control because the holding fraction is usually quite large. In contrast, the fraction when acquisition is chosen tends to be small, though it depends on the bargain power balance between entrepreneur and acquirer. Therefore, by using the value of B_{Ent}^{min} , if it turns out for entrepreneur to be able to expect to enjoy more benefits of control, acquisition is not the inferior exit choice to IPO. “IPO valuation premium puzzle” is not really a “puzzle” when looking through the lens of the inherent property of private benefits of control.

4. Numerical simulations

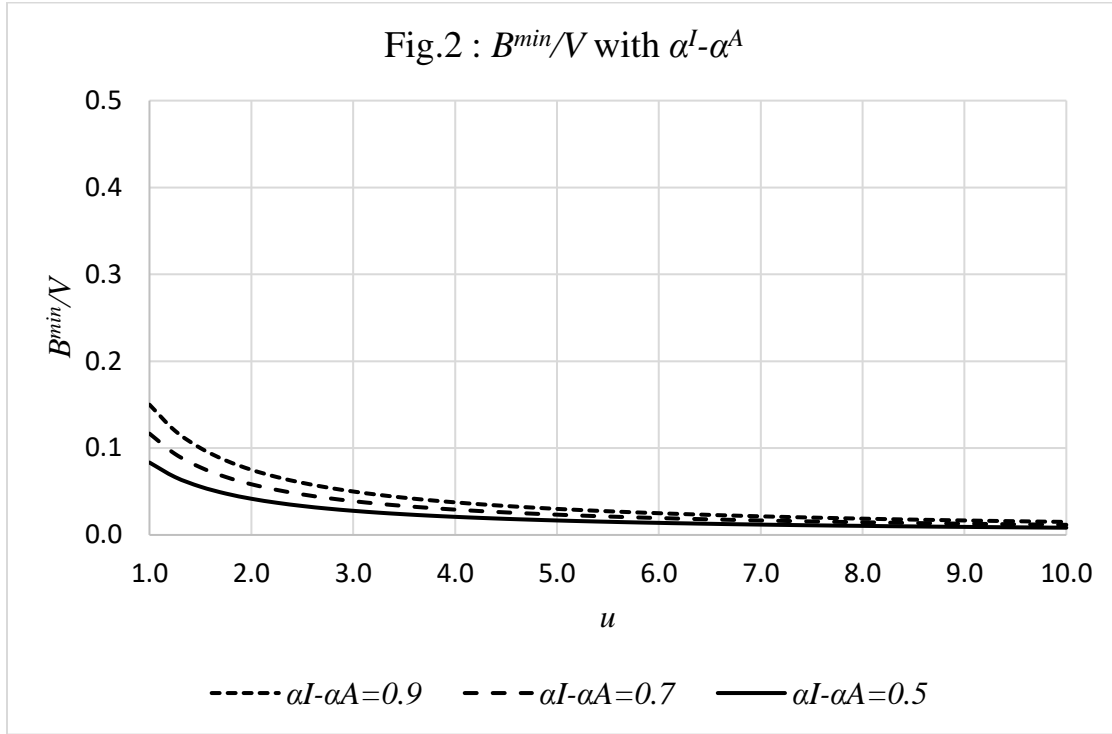
Although the origin of B_{Ent}^{min} has been revealed, the characteristics of B_{Ent}^{min} can become understandable more precisely by considering the multiplier of B_{Ent}^{min}/V which represents the ratio of B_{Ent}^{min} to the value of the start-up business that venture capital assesses before investment ($T=0$). Furthermore, the numerical simulation is useful in order to capture the behaviour of the multiplier. In order to implement the simulation, the following assumptions, $d = 1/u$, $t_2 = 2$, $t_1 = 1$, $r = 0.05$ are added. Then, the multiplier can be calculated as below:

$$\frac{B_{Ent}^{min}}{V} = (\alpha_{Ent}^I - \alpha_{Ent}^A) \times \frac{1}{1+(I_1/I_0)} \times \frac{1}{u}$$

When considering the exit strategy, how the business value would be predicted is the primary concern. Thus, it is logical that the multiplier should be compared with the variable u , which represents the amount of upper movement. The range of this variable is theoretically from 1.0 to infinite. In fact, the price of the securities can move up more than 100 times at IPO in the case of biotech ventures or IT start-ups. However, in this simulation, the variable u is assumed to range from 1.0 to 10.0. The two types of numerical simulations are possible: how the multiplier behaves with the changes of $(\alpha_{Ent}^I - \alpha_{Ent}^A)$, or with the ones of (I_1/I_0) . The following sections explain the simulation results.

1) Simulation with changing $\alpha_{Ent}^I - \alpha_{Ent}^A$

The difference of the remaining holding fraction between IPO and acquisition is assumed to range from 0.9 (i.e. $\alpha_{Ent}^I = 0.9, \alpha_{Ent}^A = 0.0$) to 0.5 (i.e. $\alpha_{Ent}^I = 0.8, \alpha_{Ent}^A = 0.3$). The case of $\alpha_{Ent}^A = 0.0$ is possible when the acquirer does not allow the original entrepreneurs to participate in the newly operating business. In this simulation, I_1/I_0 , which represents the ratio of equity, is fixed to be 5.0 (times). The result of the simulation with changing the value of $\alpha_{Ent}^I - \alpha_{Ent}^A$ is shown in Fig.2.



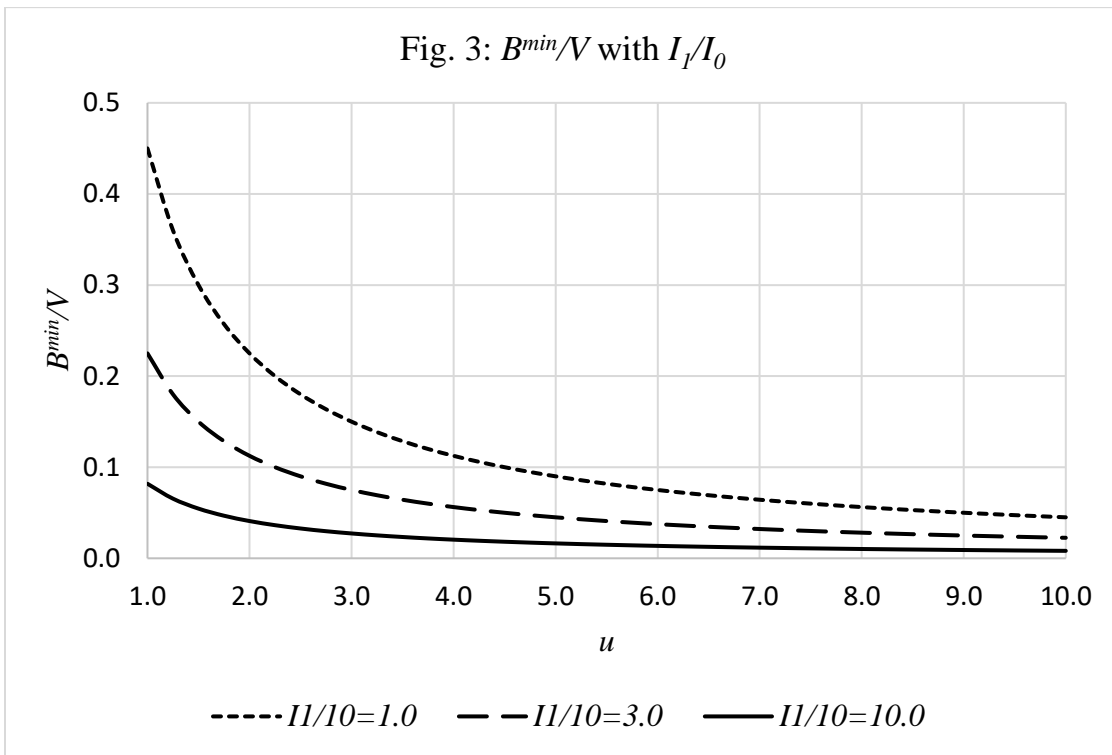
We can observe two characteristics from this result. First, the multiplier varies little according to the value of $\alpha_{Ent}^I - \alpha_{Ent}^A$. This difference of remaining holding fractions is closely related to the bargaining power of entrepreneur in the acquisition contract negotiation. When the power is strong, α_{Ent}^A would become high and $\alpha_{Ent}^I - \alpha_{Ent}^A$ becomes small. In this sense, $\alpha_{Ent}^I - \alpha_{Ent}^A$ can be predicted to influence the minimum value of the private benefits of control, B_{Ent}^{min} , or the multiplier. However, the result does not support that prediction. This gives us an interesting suggestion that the entrepreneur's bargaining power in the acquisition contract negotiation with acquirer would have little effect on the exit strategy planning.

Second, the multiplier stays in the low level regardless of the upper movement u . This means that acquisition and IPO would be indifferent options for entrepreneur no matter what the market expectation is. In this simulation, I_1/I_0 is assumed to be 5.0, which represents several venture capitalists have already provided funds and have a relatively large equity share. In the situation where entrepreneur has relatively small share of equity, the exit option choice would be irrelevant to the entrepreneur's private benefits of control level, and there would be less incentive for

entrepreneur to actively choose exit options. In the next simulation, the condition of the equity share is varied.

2) *Simulation with changing I_1/I_0*

The result of the simulation which has been conducted with the changes of I_1/I_0 is shown in Fig.3. $\alpha_{Ent}^I - \alpha_{Ent}^A$ is fixed as 0.9, and I_1/I_0 is assumed to range from 1.0 to 10.0.



In this result, we can find that the multiplier becomes increased sharply as the upper movement u becomes lower, especially in the case of the lower ratio ($I_1/I_0 = 1.0$), though the result is the same with the one of the previous simulation in the high ratio case ($I_1/I_0 = 10.0$). This means that the ratio influences significantly the minimum value of the private benefits of control, B_{Ent}^{min} , or the multiplier. In contrast to the previous result, this suggests that the incentive for entrepreneur to actively choose exit options becomes increased as entrepreneur obtains the larger

equity share. In particular, the choice of exit option becomes critical for entrepreneur in the range of lower level of the upper movement u , where the market expectation for the start-up business is low. In this situation, the value or benefit expected to be obtained from the market is quite low, and almost no benefit would remain for entrepreneur if the equity ratio becomes high. Therefore, if acquisition is chosen as an exit option, it is logical for entrepreneur to require the private benefits of control in exchange for giving up its majority of equity share. This could be reflected to the sharp increase of the multiplier in this range of u .

Based on the results of both simulations above, it is also worth mentioning that the multiplier, or the minimum value of private benefits of control stays quite low as the upper movement u becomes large. Even when the equity ratio is low, this trend is hold. Therefore, when the start-up business is highly evaluated by the market, acquisition and IPO would be indifferent regardless of the equity share and the entrepreneur's bargaining power with acquirer. As explained above, when entrepreneur chooses acquisition as an exit options rather than IPO in such a situation, it is called "IPO valuation premium puzzle." However, according to the results, is not uncommon for entrepreneur to choose acquisition over IPO even when the business is highly evaluated. "Puzzle" means incomprehensibility. Nevertheless, the results of these simulations also show that this phenomenon is comprehensible.

5. Conclusion

Planning exit strategies is one of the central issues not only for investors such as the venture capitalists but also the entrepreneurs who have created their start-ups. Firstly, this paper is analysing the property of “private benefits of control” as a criterion of choosing the exit option, either IPO or acquisition. For the exit choice, there is an intriguing issue. There is a situation where many private firms choose to be acquired rather than to go public at higher valuations. Bayar and Chemmanur (2011, 2012) called this situation as “IPO valuation premium puzzle.” This paper is also trying to understand what the “IPO valuation premium puzzle” actually is, and scrutinizing whether “IPO valuation premium puzzle” is really a “puzzle”.

As the start-up exit strategy, IPO is often thought to be superior to acquisition from the viewpoint of the market expectation. However, from the game theoretic real options approach, IPO or acquisition should be treated equivalently, and the choice criterion can be explained with the concept of Nash Equilibrium. Consequently, “Private benefits of control” can be explained as the condition for holding the state of Nash Equilibrium between entrepreneur and venture capital. Moreover, it seems that the phenomenon of “IPO valuation premium puzzle” is not really a “puzzle” when looking through the lens of the inherent property of private benefits of control.

In the course of the analysis, the minimum value of the “Private benefits of control”, B_{Ent}^{min} , is derived from the difference between the expected values of the two exit options, IPO and acquisition, especially when the market expectation for the start-up business is not high. This critical index has two important components: One is the difference of remaining holding fractions, $\alpha_{Ent}^I - \alpha_{Ent}^A$, which is closely related to the bargaining power of entrepreneur in the acquisition contract negotiation. The other one is the equity ratio, I_1/I_0 , which represents the entrepreneur’s equity share.

Based on these components, the two types of numerical simulations against the multiplier of B_{Ent}^{min}/V have been implemented. The results give us an interesting suggestion that the entrepreneur’s bargaining power in the acquisition contract negotiation with acquirer would have little effect on the exit strategy planning. However, the results also show that the equity share influences significantly on that. The results suggest that there would be less incentive for

entrepreneur to actively choose exit options when the entrepreneur's equity share is relatively low. In contrast, the incentive for entrepreneur to actively choose exit options would become increased as entrepreneur obtains the larger equity share. In addition, the results suggest that the phenomenon so called "IPO valuation premium puzzle" is neither incomprehensible, nor uncommon for entrepreneur.

The variable of "Private benefits of control", B_{Ent}^{min} , has a great potential to become a criterion for choosing the exit options, acquisition and IPO. It is similar to goodwill in the financial accounting item, and it could become a new benchmark for the exit strategy planning. Although the consensus of this issue has not yet been obtained, many approaches are the ones from the market viewpoint. These assume that the market condition would give the exit option criteria. Nevertheless, it is often the case that the interactions or internal relationship between the players, such as entrepreneur and venture capitalists have not been taken into consideration. Needless to say, the viewpoints about the players' interactions should not be ignored.

This paper focuses on the relationship between entrepreneur and venture capitalists. However, there are other players, such as banks or bank-affiliate venture capitalists, in the issue of venture financing. When they would provide funds as debt, especially as convertible bonds, the capital structure will be changed. Furthermore, it is not uncommon that the investment policies of individual venture capitalists are different. As a result, the control benefits would be affected. These issues should be incorporated in the exit option choice process, thus further study and research are needed.

6. References

- Ana Paula Faria, Natália Barbosa, “Does venture capital really foster innovation?”, *Economics Letters*, 122, 2014, pp.129–131
- Alexander Ljungqvist, William J. Wilhelm Jr., “IPO Pricing in the Dot-com Bubble”, *The Journal of Finance*, Volume 58, Issue 2, April 2003, pp.723–752
- Avinash K. Dixit, Robert S. Pindyck, “Investment under uncertainty”, *Princeton University Press*, 1994
- Bharat. A. Jain, Omesh Kini, “The Post-Issue Operating Performance of IPO Firms”, *The Journal of Finance*, Volume 49, Issue 5, 1994, pp.1699–1726
- Dawn R. DeTienne, Alexander McKelvie, Gaylen N. Chandler, “Making sense of entrepreneurial exit strategies: A typology and test”, *Journal of Business Venturing*, 30, 2015, pp.255–272
- Douglas Cumming, “Contracts and Exits in Venture Capital Finance”, *The Review of Financial Studies*, Vol. 21, No. 5, 2008, pp.1947-1982
- Elisabete Gomes Santana Félix, Cesaltina Pacheco Pires, Mohamed Azzim Gulamhussen, “The Determinants of Venture Capital in Europe—Evidence Across Countries”, *Journal of Financial Services Research*, 44, 2013, pp.259–279
- Eric Rasmusen, “Games and information: An introduction to game theory”, *Blackwell Publishing*, 2007
- Ezra Roizen, “Magic Box Paradigm”, Advsr, LLC, 2016
- Fabio Bertoni and Alexander Peter Groh, “Cross-Border Investments and Venture Capital Exits in Europe”, *Corporate Governance: An International Review*, 22(2), 2014, pp.84–99
- Franklin Allen, Gerald R. Faulhaber, “Signalling by underpricing in the IPO market”, *The Journal of Financial Economics*, Volume 23, Issue 2, 1989, pp.303-323
- Guillaume Andrieu, “The Impact of the affiliation of venture capital firms: A survey”, *Journal of Economic Surveys*, Vol. 27, No. 2, 2013, pp. 234–246
- Guillaume Andrieu, Alexander Peter Groh, “Entrepreneurs' financing choice between independent and bank-affiliated venture capital firms”, *Journal of Corporate Finance*, 18, 2012, pp.1143–1167

- Han T.J. Smit, L.A. Ankum, “A Real Options and Game-Theoretic approach to Corporate Investment Strategy Under Competition”, *Financial Management*, Autumn 1993, pp.241-250
- Han T.J. Smit, Leons Trigeorgis, “Real Options and games: Competition, alliances and other applications of valuation and strategy”, *Review of Financial Economics*, 15, 2006, pp. 95-112
- Nina Rosenbusch, Jan Brinckmann, Verena Müller, “Does acquiring venture capital pay off for the funded firms? A meta-analysis on the relationship between venture capital investment and funded firm financial performance”, *Journal of Business Venturing*, 28, 2013, pp.335–353
- Onur Bayar, Thomas J. Chemmanur, “What drives the valuation premium in IPOs versus acquisitions? An empirical analysis”, *Journal of Corporate Finance*, 18, 2012, pp.451–475
- Onur Bayar, Thomas J. Chemmanur, “IPOs versus Acquisition and the Valuation Premium Puzzle: A Theory of Exit choice by Entrepreneurs and Venture Capitals”, *Journal of Finance and Quantitative Analysis*, Vol.46, No.6, 2011, pp.1755–1793
- Rebel Colea, Douglas Cumming, Dan Lic, “Do banks or VCs spur small firm growth?”, *Journal of International Financial Markets, Institutions & Money*, 41, 2016, pp.60–72
- Richard B. Carter, Frederick H. Dark, Ajai K. Singh, “Underwriter Reputation, Initial Returns, and the Long-Run Performance of IPO Stocks”, *The Journal of Finance*, Volume 53, Issue 1, February 1998, pp.285–311
- Stewart C. Myers, “Determinants of corporate borrowing”, *Journal of Financial Economics*, Volume 5, Issue 2, 1977, pp.147-175
- Tom Copeland, Vladimir Antikarov, “Real Options: A practitioner’s guide”, *CENGAGE Learning*, 2003