Option Value of Renewable Identification Numbers (RINs)

Abstract

We model the dynamics of equilibrium prices in the renewable identification number (RIN) market. Our modeling framework is different than the usual practice which prices RINs in a static model. Using a continuous-time stochastic control formulation, we explicitly model the option value inherent in the RINs prices as an American spread option, given the institutional constraints of the market. To this end, we utilize two different processes for the underlying prices, namely, geometric Brownian motion and geometric mean-reversion. The former enables deriving a closed-form solution of the price, and the latter allows for a numerical but yet more realistic approximation. Among other results, we show that the price of RINs has a positive relationship with the volatility of ethanol and crude oil prices and a *negative* relationship with the correlation between the two price processes. We also show that the choice of time-series model for gasoline and ethanol prices has a significant impact on the value of RINs. In particular, we solve the model for GBM and mean-reverting price processes and simulate both specifications using real-world calibrated parameters.

Keywords: Biofuels Mandates, Valuation, Real Options, Strangle Option

1. Introduction

Biofuels have become an integral part of renewable energy portfolios in many countries around the world. The United States and EU countries have specific mandates for the minimum share of biofuels in the transportation (and in some cases electricity) sectors' energy inputs. One key question in this regard is how to enforce the minimum share of biofuels in practice. In this paper we focus on modeling the dynamics of Renewable Identification Numbers (RINs), as one successful market-based mechanism.

In simple terms, the renewable identification number (RIN) is a floor and trade mechanism (as opposed to cap and trade) to ensure that the minimum quantity of biofuels (first generation ethanol, second generation ethanol, and biodiesel) has been blended with fossil fuels (mainly gasoline and diesel). In this paper, we are focusing on the specific case of

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the biofuels markets. Our modeling framework however, can be extended to other markets in which a firm can either purchase a traded certificate from the market or produce the mandated commodity.

The Environmental Protection Agency (EPA) in the US imposes Renewable Fuel Standards (RFS) using biofuel blending mandates. The mandates which are set on an annual basis should be met by final motor gasoline producers and importers (hereinafter referred to as blenders). Meeting this mandate compliance is verified by the blender submitting sufficient quantity of Renewable Identification Numbers (RINs) to EPA. These RINs, generated through blending biofuels, can be bought and sold in the market. (See section 3 for further description and detail). In this paper we model the dynamic of equilibrium prices of RINs. Particularly, first we address the option value inherent in the RIN prices. Second, under this framework, we model the price of RIN under two scenarios. One in which the price of gasoline and the biofuel follows a Geometric Brownian Motion (GBM) process and another case where the prices are explained by the exponential of two mean-reverting Ornstein-Uhlenbeck processes. The option value associated with RINs in both cases are modeled and analyzed. As will be shown later, each scenario has certain advantages and limitations. Under the GBM setup, the value of RINs can be obtained using a closed-form formula. This formula may also be used to approximate the value of RIN once the possibility of diffusion jump in the underlying prices is taken into account. The GMR setup, does not yield to a closed-form formula for the value, but is considered more realistic when one models commodity market prices.

Our paper adds a novel aspect to the extant literature on the economics of renewable energy and also the option pricing literature. The majority of papers that analyzed the value of RINs have assumed its price to reflect the *static* difference between ethanol supply and demand price as the basis of their work, e.g., McPhail et al. (2011), Whistance and Thompson (2014), Pouliot and Babcock (2015), Lade and Lin (2015), and Markel et al.

(2016). In a recent study, Korting and Just (2017) use a partial equilibrium model and show that the core value of a RIN in equilibrium reflects the marginal cost of employing one additional ethanol-equivalent unit of biofuel for the blender. This is in accordance with how price of permits in the cap and trade literature is considered to reflect the marginal cost of abatement (Carmona and Hinz (2011), Seifert et al. (2008), and Hitzemann and Uhrig-Homburg (2014)).

In the conventional framework the price of RINs will be zero as soon as the cost of blending ethanol is lower than the price of gasoline. However, we are showing that this formulation misses the option value component of price. If the option value is properly accounted, the price of RINs will never become zero, even if mandates are not statically binding (i.e., it is cheaper to blend than produce pure gasoline). We will show that for a reasonable range of volatility estimates, the option value of RINs can be considerable.

Our work is closely related to the price dynamics of traded emission certificates such as European carbon emission certification (ETS). Besides contextual differences, our work differs from the existing literate in two key mathematical aspects. First, the underlying process for ETSs is typically a single stochastic process; whereas, the value of RINs is driven by the difference of two stochastic process. The "spread option" feature of RINs makes the problem substantially more complicated. Second, we solve the problem for both GBM and mean-reverting price processes and compare the impact of time-series dynamics of underlying processes on the price dynamics of certificates.

The rest of this paper is organized as follows. We provide a summary of relevant literature in Section 2. Section 3 presents some basic institutional details on RFS and RINs, and further surveys the related literature. Section 4 explains the setup of the model for the price dynamics of RIN. Section 5 offers numerical analysis of the developed framework for the RIN prices. Section 6 discusses the implications of the results. Section 7 concludes and proposes future research.

2. Literature Review

Previous studies have used a number of models to explain what the core value of RIN reflects. Aside from the studies mentioned in the previous section, Zhou and Babcock (2017) use the competitive storage model to estimate the impact of ethanol and fueling investment on corn prices. They find that increase of mandate level increases biofuel consumption, reduces petroleum imports, and will probably reduce emission greenhouse levels by substituting gasoline with ethanol. In a well received study, under a deterministic continuous-time setting, Rubin (1996) investigates the banking dynamics of emission trading in a finite horizon. Using optimal control theory, the paper argues that permit prices must equal the marginal cost of abatement and grow with risk-less interest rate. McPhail (2010) applied this setup in the RINs market under RFS2 to find the optimum level of banking without putting any constraint on banking/borrowing. Thompson et al. (2009) is another paper to model RFS2 in a dynamic setting. They conduct a stochastic analysis to asses the shocks of corn and oil markets on ethanol price with and without the existence of RFS2. However, these dynamic models do not reflect the option value associated with RINs. The dynamic programming approach is also incapable of illustrating the sensitivity of RINs on the dynamics of the underlying prices.

Applying real option framework has only recently gained attention in the renewable energy literature. Santibañez-Aguilar et al. (2016) analyze optimal planning of bio-refineries with respect to risk in the supply chain. In a similar paper, McCarty and Sesmero (2015) uses this framework to study gasoline prices that trigger entry for investing in a corn stover-based cellulosic biofuel plant. Ghoddusi (2017) examined the short profits of a bio-fuel plant by postulating its value as a strangle option.

A strand of literature compares the effects of carbon intensity standards and renewable fuel share mandates. With a Pigouvian tax approach, Holland et al. (2015) show that achieving the same level of carbon emission reduction is far less costly under a cap and trade program as compared to RFS2. Their analysis is based on the result that under the current RFS, ethanol prices reflect an implicit subsidy, while gasoline is priced as if it were taxed, a result reported by a number of early works including Lapan and Moschini (2012), De Gorter and Just (2009), and Lade and Lin (2013). Holland (2009) on the other hand shows that in the presence of market power, renewable share mandates are more efficient than cap and trade systems. Rajagopal et al. (2011) perform a multi-criteria comparison of RFS, LCFS, and carbon tax to analyze the effect of these policies on fuel price and emission reductions. They find that if fuel policy is applied regionally and the economy is open to trade, RFS increases the share of renewable to conventional fuels, and may decrease domestic fuel price and carbon emissions.

Another dimension of the RFS literature, explores the impact of policy uncertainty on the incentive for investment in new technologies, for example Lade et al. (2016) estimate the effect of 'policy shocks' imposed by EPA by reducing RFS mandates on compliance costs, commodity markets, and the market value of publicly traded biofuel firms. They argue that these shocks reduced the incentive to invest in the technologies required to meet the future objectives of the RFS. Other studies in this dimension include Miao et al. (2012), and Clancy and Moschini (2015).

In addition, more empirical papers have explored other aspects of RFS. Lade and Bushnell (2016), and Knittel and Ben Meiselman (2015) study impacts of the RIN taxes and subsidies on downstream wholesale and retail fuel markets. Pouliot and Babcock (2014) estimate demand for high ethanol blends (E85), which is later integrated in a short term partial equilibrium model by Pouliot and Babcock (2015). Drabik et al. (2014) use consumption data from Brazil to examine the switching option between using high and low ethanol fuels with logistic curve. de Gorter et al. (2015) perform a similar analysis in the US market. Schmit et al. (2011) incorporate two stochastic variables to model revenues and costs to find optimal entry and exit trigger prices for an ethanol producing firm. In a similar study but with a single stochastic variable, McCarty and Sesmero (2015) analyze the effect of hysteresis in the investment of second-generation biofuel production. Some other papers have examined the effect of RFS mandates on commodity prices (Roberts and Schlenkera (2013), Wright (2014), Carter et al. (2016)).

3. Biofuels Markets and Renewable Identification Numbers (RINs)

The Energy Policy Act of 2005 established Renewable Fuel Standards (RFS) to necessitate the use of biofuels (such as ethanol) in the automative fuel supply in the US. RFS were later amended and broadened under the Energy Independence and Security Act of 2007 and referred to as RFS2. RFS2 requires blenders to meet four Renewable Volume Requirements (RVOs) that are announced based on ratio percentage of renewable to non-renewable fuel production. Blenders prove meeting their share of mandates by using a tracking system known as renewable identification numbers (RINs). A RIN generates as a 38-character code upon the use of one gallon of renewable fuel produced or imported and is designed to make meeting the required mandates more flexible. After the renewable fuel is blended, the blender can detach the RIN and either store it for compliance or sell it at the market as a commodity. The possibility of trade gives some blenders an incentive to blend biofuels beyond their share of mandate and encourages some to meet the mandate by obtaining RINs rather than directly blending biofuels in the product. To lower the cost of compliance, the Environmental Protection Agency (EPA) also allows RINS to be banked and borrowed up to 20% of the projected production of the next year. However, in 2013, following concerns over high compliance costs and lower than expectation investment in advanced biofuel, EPA was led to reduce the RVOs for the next three consecutive years.

Similar policies have also been adopted in Europe. The 2009 EU Renewable Energy Directive sets a 10% minimum target for renewable energy consumption in the transportation sector for the member states by 2020. In addition, Fuel Quality Directive requires the road transport fuel mix in the EU to be 6% less carbon intensive than a fossil diesel and gasoline baseline by 2020. Since, the countries have been pursuing these goals mainly by setting minimum biofuel (bio-ethanol and bio-diesel) use mandate levels. To allow for adaption, the 28 countries use target increase policies of the minimum mandate level on a yearly basis. These targets are different across different countries. For example, though some such as Finland have already surpassed the 10% and are undertaking more ambitious goals, other members such as Greece are still far below the target level. A detailed description of the current policies by the EU countries is provided by Lieberz (2017). Similar to the US, the availability of a market mechanism known as certificate trading has offered flexibility to smoothing the transition.

4. Model Setup and Valuation

The producer (blender) is assumed to produce a fixed amount of fuel $\bar{Q}_i$ during each year i, making it obliged to obtain $\bar{R}_i$ of RINs by the end of that period. To obtain this amount of RINs, the representative blender has the option of either producing the required ethanol or obtaining the RINs directly from the market. Similar to other EPA regulations, during each year, the blender is also allowed to bank or borrow a certain percentage of its RINs.

The price of RIN reflects the difference between the adjusted ethanol[1] and gasoline prices plus the option value associated with RINs. This option value comes from the fact that RINs can be stored and exercised instead of ethanol upon necessity and when optimal. It is also assumed that no cost is associated with RIN storage. We examine two scenarios. A case where prices follow a Geometric Brownian Motion (GBM) process and one where they are governed by the exponential of an Ornstein-Uhlenbeck (also known as Geometric Mean-Reversion) process. Under both scenarios, and under any other price dynamics the value of

[1]Note that across the paper, the price of ethanol should also be interpreted as the adjusted price of ethanol to control for the difference between the energy content of ethanol and gasoline.

a RIN can be described by definition 1.

Definition 1. *The value of a generated RIN at time t (V_t), under ξ% banking and borrowing constraint is defined as*

$$V_t(P_E, P_G, R) = \begin{cases} (1-\xi)\sup_{\tau_1} e^{-r(\tau_1 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_1}[(P_E - P_G)^+] + \xi \sup_{\tau_2} e^{-r(\tau_2 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_2}[(P_E - P_G)^+] & R \leq \bar{R}_i \\ \frac{\bar{R}_i}{R}\left((1-\xi)\sup_{\tau_1} e^{-r(\tau_1 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_1}[(P_E - P_G)^+] + \xi \sup_{\tau_2} e^{-r(\tau_2 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_2}[(P_E - P_G)^+]\right) & R > \bar{R}_i \end{cases} \quad (1)$$

*where $\mathbb{Q}$ is risk-neutral measure, ξ is the borrowing/banking cap. R represents the total number of RINs obtained from the first day of the current compliance year up to time t. $\tau_i = T_i - t$, $0 \leq t \leq \tau_1 \leq T_1 \leq \tau_2 \leq T_2$, and T_1 and T_2 represent the compliance date in the current and consecutive year, respectively. Further a RIN is said to be **exercised** at time t if the blender decides to use an equivalent volume of gasoline instead of ethanol at that time.*

Note that whether or not the required level of compliance has been met, the value of a RIN of last year's vintage, i.e., that expires in the current year mandate, lacks the second term in definition 1 in its value. Further, if c_u would show the number of RINs obtained in day u ($0 \leq u < t$), then $\int_0^t c(u)du = R$, subject to $0 \leq c \leq \chi$. The last constraint is interpreted as follows: The minimum amount of RIN obtained in a single day for a firm is a number great than or equal zero. The maximum amount however, depends on various factors such as different kinds of biofuel the blender produces and the maximum amount of ethanol that can be blended in each product (these products are further discussed in later sections), and production capacity. Regardless of the specific features and nature of these constraints, the model assumes one can not generate an infinite amount of RIN in a single day.

Looking at definition 1, it can immediately be noticed that under both conditions, each of the terms in the definition is identical to the value of an American exchange option (i.e., a spread option with strike price equal to zero).[2] With the difference between the terms being the time to maturity. To better illustrate the exchange option feature of RIN, recall that by the end of the compliance period, the blender has to meet the obligation of that year. This is done through all periods where $t \leq T$ by either producing ethanol with cost $P_E(t)$ or by exercising RINs. But the decision between the two choices has to be made at time t. Obtaining RINs gives its holder the *option* to choose when to exercise the RIN. This in essence is the fundamental *option value* associated with RINs. Another feature of the valuation that needs to be addressed is the effect of meeting the compliance level or in other words obtaining the required number of RINs, R_i, *before* the compliance date. As long as the economy has still not produced enough RINs, RINs are valued similar to an American option as described. Once the required RINs are generated, not all RINs would be equal. In fact, after meeting the threshold R_i, each RIN has only a probability (less than one) of being submitted to EPA. Since RINs of the same vintage are assumed identical in nature, this probability is simply defined as $\frac{\bar{R}_i}{R}$. If RIN is not submitted to EPA and not banked for the next year, it would become worthless. Therefore, after the threshold, the value of RIN in the remaining days would be multiplied by their chance of being submitted to EPA.

The option value embedded in RINs makes them also similar to another class of contracts known as swing options. Swing options are a family of contracts in the commodity market that enable the holder to choose the volume of deliveries within a delivery period in the future. In other words, swing options allow the owner to receive variable amounts of a commodity within a time period at fixed prices. These contracts are especially popular in hydro-power production planning and electricity markets in general, where physical transfer

[2]To be more precise, RINs are in fact Bermudan options as they can only be *exercised* in a discrete set of time. We address this distinction when the underlying prices are assumed to be mean-reverting.

of the underlying commodity is subject to volume restrictions. By providing flexibility in terms of delivery time and volume, swing options provide protection against price fluctuations. Numerous studies in the literature have examined the value of swing contracts based on different specifications and assumptions. Examples include Jaillet et al. (2004), Carmona and Touzi (2008), Fleten and Wallace (2009), Wahab et al. (2010).

In this respect, RINs are comparable to the class of swing options with one exercise right. The value of the contract in this case is equivalent to that of a American (Bermudan) option. But the core value of RINs is also different from a swing contract due to certain features. Conventional swing contracts which are written on a single underlying commodity; whereas, as mentioned above, RIN value reflects the spread between the price of two commodities. The compliance level discussed above and the possibility of banking and borrowing between years are among other characteristics that distinguish RINs from swing contracts (and for that matter exchange options).

Before discussing the RIN value under different scenarios, it is useful to briefly discuss the effects of banking and borrowing constraint. In the absence of banking/borrowing the blender has to meet the mandate level in each period and once the compliance date passes, any excess RIN remaining would be worthless. The introduction of provisions allows the blender to minimize costs by obtaining more or less than the mandate level based on their expectation of future. The industry as a whole would also become able to carry net deficit/surplus. This possibility combined with the option value associated with RINs may lead to instances where future vintage RINs trade at a premium as compared to current vintages. As pointed out by Lade et al. (2015) this demonstrates that the market expects the banking constraints to bind. Consequently, if there is no cap on banking/borrowing restrictions, the constraints do not bind and there is no price difference for same RINs with different vintages. This price wedge can also be easily observed in definition 1. The price of a generated RIN transferable to next year, includes an additional term compared to a RIN that expires by the end of the current year.

4.1. Geometric Brownian Motion Processes

In this setup, prices of ethanol (P_E) and gasoline (P_G) are assumed to be unit-root geometric Brownian motion processes under real world probability measure $\mathbb{P}$ following:

$$dP_i = P_i \mu_i dt + P_i \sigma_i dB_i(t) \tag{2}$$

where $i \in \{E, G\}$, μ_i and σ_i are constant, and correlation between the two processes is through the driving Brownian motions. i.e., $\mathbb{E}\{dB_E(t)dB_G(t)\} = \rho dt$. It has been proven (e.g., Broadie and Detemple (1997) and Bjerksund and Stensland (1993)) that in this setting, as there is no dividend paid to the owner of the option, the problem of valuing an American Exchange option can be reduced to that of a European Exchange option. i.e., in our terminology, at all times before final compliance date, the blender always decides to produce ethanol rather than exercise the RIN. If at the end of the compliance period, ethanol is cheaper than gasoline, the blender meets the mandate by producing ethanol, thus wasting the premium paid (the RIN price). However, if gasoline ends up being the more expensive product, the blender gets the price difference by exercising the option. Thus, to anaylze the value of a RIN, one can first derive the value of each term separately and describe the total value as the sum of two terms. Margrabe (1978) proves a closed form price solution exists for this setting.

Proposition 1. *If the underlying processes are given by GBMs with correlation ρ, the price* *of RIN at time t is given by:*

$$V_t = \begin{cases} (1-\xi)\big(P_E(t)\Phi(d_{1+}) - P_G(t)\Phi(d_{1-})\big) + \xi\big(P_E(t)\Phi(d_{2+}) - P_G(t)\Phi(d_{2-})\big) & R \le \bar{R}_i \\ \frac{\bar{R}_i}{R}\Bigg((1-\xi)\big(P_E(t)\Phi(d_{1+}) - P_G(t)\Phi(d_{1-})\big) + \xi\big(P_E(t)\Phi(d_{2+}) - P_G(t)\Phi(d_{2-})\big)\Bigg) & R > \bar{R}_i \end{cases} \tag{3}$$

where

$$d_{1\pm} = \frac{\ln(P_E(t)/P_G(t))}{\sigma\sqrt{T_1 - t}} \pm \frac{1}{2}\sigma\sqrt{T_1 - t} \quad d_{2\pm} = \frac{\ln(P_E(t)/P_G(t))}{\sigma\sqrt{T_2 - t}} \pm \frac{1}{2}\sigma\sqrt{T_2 - t} \tag{4}$$

and:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-u^2/2}du \quad \sigma^2 = \sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G \tag{5}$$

with T_1 and T_2 being the last day in the current and consecutive year, respectively.

proof:. See Appendix A.

It is interesting to note that the formulae are independent of the risk free interest rate, r. This is because in a risk-neutral world, both prices of ethanol and gasoline increase at the same discount rate, offsetting each other in the computation appearing the in the definition of RIN.

4.1.1. Extension to Jump-Diffusion Models

Proposition 1 provides a simple and useful closed-form valuation of RIN under GBM setup. This valuation can be adjusted to approximate the price of RINs when one allows for jumps in the risk-neutral dynamics of the underlying prices. The inclusion of jumps is particularly useful in modeling commodity prices and has been studies extensively in the previous literature (e.g., Hilliard and Reis (1998), Deng (2000), Geman (2009)). This section examines this application when gasoline and ethanol price dynamics are introduced to Merton's jump. In mathematical terms, the neutral-risk price dynamics are assumed to follow:

$$\frac{dP_i(t)}{P_i(t-)} = (r - \lambda_i \mu_i)dt + \sigma_i dB_i t(t) + (e^{J_i(t)} - 1)dN_i(t) \tag{6}$$

where again $i \in \{E, G\}$. $J_i(k)_{k\geq 1}$ are two independent sequences of Gaussian random variables with distribution $N(m_i, s_i^2)$ and $\mu_i = e^{m_i + s_i^2/2} - 1$. $\mathbb{E}\{dB_E(t)dB_G(t)\} = \rho dt$. N_E and N_G are two Poisson processes with intensities λ_E and λ_G, respectively, that are independent from each other and from B_E and B_G. A Merton jump process is interpreted as follows: at the time the Poisson process $N_i(t)$ jumps for the $k-$th time, the price $P_i(t)$ jumps by an amount of $P_i(t-)(e^{J_i(k)} - 1)$. The spread option pricing shown and proved by Carmona and Durrleman (2003) can be used to approximate the value of RINs in this setup.

Proposition 2. *The price of RIN when the underlying prices follow equation 6 can be approximated by:*

$$\begin{aligned}\hat{V}_t = (1-\xi)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-(\lambda_E+\lambda_G)T_1} \frac{(\lambda_E T_1)^j (\lambda_G T_1)^i}{i!j!} P_1 \\ +\xi \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-(\lambda_E+\lambda_G)T_2} \frac{(\lambda_E T_2)^j (\lambda_G T_2)^i}{i!j!} P_2\end{aligned} \tag{7}$$

When $R \leq \bar{R}_i$, and $\frac{\bar{R}_i}{R}\hat{V}_t$ when $R > \bar{R}_i$ In this equation P_1 and P_2 are derived by a slight adjustment of the components in the RIN value under GBM setup. For $\iota \in \{1, 2\}$

$$P_\iota = x_{E,\iota}\Phi(d_{\iota,+}) - x_{G,\iota}\Phi(d_{\iota,-})$$

where

$$
\begin{aligned}
x_{E,\iota} &= P_E(t)\exp\left(-\lambda_E\mu_E T_\iota + j(m_E + s_E^2/2)\right) \\
x_{G,\iota} &= P_G(t)\exp\left(-\lambda_G\mu_G T_\iota + i(m_G + s_G^2/2)\right) \\
d_{\iota,\pm} &= \frac{\ln(x_{E,\iota}/x_{G,\iota})}{\bar{\sigma}_\iota\sqrt{T_\iota - t}} \pm \frac{\bar{\sigma}_\iota}{T_\iota - t} \\
\bar{\sigma}_\iota{}^2 &= \bar{\sigma}_{E,\iota}^2 + \bar{\sigma}_{G,\iota}^2 - 2\bar{\rho}_\iota\bar{\sigma}_{E,\iota}\bar{\sigma}_{G,\iota} \\
\bar{\sigma}_{E,\iota} &= \sqrt{\sigma_E^2 + j s_E^2/T_\iota} \\
\bar{\sigma}_{G,\iota} &= \sqrt{\sigma_G^2 + i s_G^2/T_\iota} \\
\bar{\rho}_\iota &= \frac{\rho\sigma_E\sigma_G}{\bar{\sigma}_{E,\iota}\bar{\sigma}_{G,\iota}}
\end{aligned}
$$

proof:. See Carmona and Durrleman (2003).

The formula consists of the summation of infinite series, and this is the main source of approximation. Choosing what number of terms to include is based on the prescribed error threshold.

4.2. Geometric Mean-Reverting Processes

The fundamental differences between energy commodities and traditional financial securities have been extensively addressed in the literature. These differences include higher storage costs, lower liquidity level, higher volatilities, and reversion of prices to long-run equilibrium levels. These empirical properties have been reported by Clewlow and Strickland (2000), Carmona and Durrleman (2003), and Eydeland and Wolyniec (2003) among other studies. Therefore, the literature tends to favor mean-reverting models to GBM model to represent energy prices. Schwartz (1997), and Gibson and Schwartz (1990) are among the first studies to utilize Ornstein-Uhlenbeck process to describe the price behavior of commodities. Following these studies *Mean-reversion* has remained a highly argued property of the energy prices which historically tend to return to a local or asymptotic mean level despite their random evolution. To control for this feature, the dynamics of the underlying prices

(P_E, P_G) are assumed to be given by the exponential of two Ornstein-Uhlenbeck processes instead of geometric Brownian motions. i.e., we assume the risk-neutral dynamics of the prices are given by stochastic differential equations of the form:

$$\begin{aligned} P_i(t) &= \exp\left(\theta_i + X_i(t)\right) \\ dX_i(t) &= -\alpha_i X_i(t)dt + \sigma_i dB_i(t) \end{aligned} \tag{8}$$

where $i \in \{E, G\}$, α_is are the constant mean reversion coefficients. $B_E(t)$ and $B_G(t)$ are standard Brownian motions with correlation ρ under the risk neutral measure $\mathbb{Q}$. $\exp(\theta_i)$ is known as the asymptotic mean reversion level and σ_i is the volatility. Appendix Appendix B shows how the risk-neutral dynamic is derived from real world measure $\mathbb{P}$.

Unlike the case of GBM, one cannot ignore the possibility of early exercise under GMR assumption. In fact, with the mean-reversion property acting similar to a dividend yield, the early exercise premium is expected to be positive. Intuitively, this can be demonstrated by considering instances where one of the prices reaches a very high deviation from its mean.

The American option valuation can thus be solved by framing the valuation as a free boundary problem and solving for the resulting partial differential equation (PDE). Setting $x = \ln P_E - \theta_E$ and $y = \ln P_G - \theta_G$ this value function, $V(t, x, y)$, must satisfy the following PDE:

$$\frac{\partial V}{\partial t} + \alpha_E(\theta_E - x)\frac{\partial V}{\partial x} + \alpha_G(\theta_G - y)\frac{\partial V}{\partial y} + \frac{1}{2}\left(\sigma_E^2 \frac{\partial^2 V}{\partial x^2} + 2\rho\sigma_E\sigma_G \frac{\partial^2 V}{\partial x \partial y} + \sigma_G^2 \frac{\partial^2 V}{\partial y^2}\right) - rV = 0 \tag{9}$$

So far we have assumed that firms must comply with the mandates regardless of underlying prices. However, in reality, EPA enforces the mandates by penalizing firms that fail to meet the mandate levels. The effect of this penalty can be captured in the boundary conditions of the PDE. In fact, let us assume the blender faces a penalty of Π_i for each unit of shortage in RINs. Consequently, once the price of ethanol becomes extremely higher than gasoline, an upper bound is imposed on the value of RIN which is the penalty itself. In this situation the firm is better off paying the penalty, rather than obtaining RIN at very high prices. Therefore, the PDE is governed with these boundary conditions

$$\begin{aligned} &\lim_{x\to\infty} V(t,x,y) = \Pi_i \quad \lim_{x\to 0} V(t,x,y) = 0, \\ &\lim_{y\to\infty} V(t,x,y) = 0 \quad \lim_{y\to 0} V(t,x,y) = P_E(t), \end{aligned} \tag{10}$$

satisfying the value matching

$$V(t,x^*(t,y^*),y^*) = e^{\theta_E + x^*(t,y^*)} - e^{\theta_G + y*} \tag{11}$$

and the smooth pasting conditions:

$$\frac{\partial V(t,x^*(t,y^*),y^*)}{\partial x} = e^{\theta_E + x^*(t,y^*)} \quad \frac{\partial V(t,x^*(t,y^*),y^*)}{\partial y} = -e^{\theta_G + y*} \tag{12}$$

where $(x^*(t,y^*),y^*)$ is the trigger curve in the (x,y) plane as function of time.

4.2.1. Pricing

To date, no analytical solution for pricing American options with mean-reverting underlying process(es) has been developed. Therefore, to value RINs that satisfy the PDE in equation 9, one can utilize implicit, explicit or Crank-Nicolson finite difference methods similar to other derivative pricing problems. Another common pricing approach in this setting is tree approximations or Fast Fourier Transformations (e.g., Jaimungal and Surkov (2011), and further tailored by Jaimungal et al. (2013)). In this paper we primarily take advantage of the least square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001). Note that one may assume choosing between blending or using RIN for a blender is a decision made on a discrete time basis (daily, weekly or monthly). Consequently pricing RINs can be treated similar to a Bermudan option with finite exercise dates. This is in accordance with the LSMC method for pricing American options in which the option is approximated in the form of a Bermudan option with a set of exercise opportunity dates with the time between them being small. LSMC provides a *lower bound* for the value of the RIN. Using the primal-dual representation, one can calculate and *upper bound* for the price through a minimization problem over a class of (super) martingales (established by Andersen and Broadie (2004), and Haugh and Kogan (2004)).

The logic behind LSMC is similar to any other simulation approach to price American options. At each exercise opportunity, the value of immediate exercise is compared to the expected payoff from continuation,i.e., holding on to the option. This set of decisions constructs the value of the option. LSMC combines least-square linear regression and backward induction to approximate the value of holding the option. After generating a set of paths for the underlying price processes, by using backward induction, at each exercise date the continuation value is estimated by regressing the discounted subsequent realized cash flows from continuation on a set of basis functions. The regression is only fitted on paths where the option is *in-the-money*. Finally, price of the option is the discounted cash flows.

To formulate this procedure in pricing RINs, setting time of expiry as $T = t_n$, the value of RIN can be priced recursively on the exercise dates $m \in \{1, 2, ..., n\}$ as follows:

$$v_{t_m}(x, y) = \begin{cases} (e^{\theta_E + x} - e^{\theta_G + y})^+, & m = n \\ \max\left\{(e^{\theta_E + x} - e^{\theta_G + y})^+, C_{t_m}\right\}, & m \in \{1, 2, ..., n-1\} \end{cases} \tag{13}$$

where $C_{t_m} = e^{-r\Delta t_m}\mathbb{E}[v_{t_{m+1}}(X_{t_{m+1}}, Y_{t_{m+1}})|X_{t_m} = x, Y_{t_m} = y]$ is the value of holding the RIN and not exercising it at time t_m, which can be calculated as:

$$C_{t_m} = \sum_{r=1}^{d} \beta_{mr}\psi_r(x, y) \tag{14}$$

for some basis functions ψ_r and coefficients β_{mr}. We choose the basis functions based on the algorithm's ability to replicate the results of Haug (2007) which uses 3D binomial trees, and Jaimungal et al. (2013) which uses Fourier Transform method. This leads to eight basis functions:

$$\begin{aligned} &\psi_1 = x, \quad \psi_2 = y, \quad \psi_3 = xy, \quad \psi_4 = x^2, \\ &\psi_5 = y^2, \quad \psi_6 = x^2 y, \quad \psi_7 = xy^2, \quad \psi_8 = x^2 y^2 \end{aligned} \tag{15}$$

Finally, to reduce the variance of simulations, we use the exact price of a European exchange option as the control variate. Li et al. (2008) provide a closed-form expression of the European exchange option price when the underlying processes are mean-reverting.

Proposition 3. *for d basis functions, at each of the $m-1$ early exercise dates afte time t, let $LSMC(x, y, d, m)$ denote the discounted cash flows resulting from following the comparison rule of equation 13 when the immediate exercise value is greater than or equal to C_{t_m} defined in equation 14. Then the lower bound value of RIN can be approximated as:*

$$V_t = (1 - \xi) \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} LSMC(x, y, d, m_1) + \xi(\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} LSMC(x, y, d, m_2)) \tag{16}$$

for $Rleq\bar{R}_i$ and $\frac{\bar{R}_i}{R} V_t$ for $R > \bar{R}_i$. In the above equation, N represents the number of generated paths that are in-the-money. Further m_1 and m_2 represent the number of early exercise dates remaining in the current and next consecutive year.

The upper bound of RIN price can be computed with the dual representation of definition 1. Andersen and Broadie (2004) provide a detailed description on deriving the upper bound value of options. Here, we use those results to briefly describe the application process in this setup. Let H represent the space of adapted martingales π for which $\sup_\tau |\pi_t| < \infty$ and $\pi_0 = 0$. For any martingale and using the Optional Sampling theorem, at time 0, the following relation holds:

$$v_0 \leq \pi_0 + \mathbb{E}_0^{\mathbb{Q}} [\max_t \frac{h_t}{e^{rt}} - \pi_t]$$

where $h_t = \max(P_E(t) - P_G(t))^+$. With slight modification of the above inequality and using the fact that π was arbitrary (meaning, the inequality should hold after taking infimum) an upper bound for the ethanol and gasoline spread is calculated as

$$\inf_{\pi \in H} \left(\pi_0 + \mathbb{E}^{\mathbb{Q}}[\max_{\tau} e^{-r\tau}((P_E - P_G)^+ - \pi_t]\right) \tag{17}$$

To achieve a "tight" upper bound,π_t is defined as $\pi_0 = L_0$, $\pi_1 = e^{-rt}L_1$, and for $2 \leq m \leq n$ with n being the number of exercise times

$$\pi_m = \pi_{m-1} + e^{-rt}L_m - e^{-r(t-1)}L_{m-1} - l_{t_m-1}\mathbb{E}[e^{-rt}L_m - e^{-r(t-1)}L_{m-1}] \tag{18}$$

with l_{t_m-1} indicating the decision at $t_m - 1$ (= 0 for continuation and = 1 for exercise), and L_t is the value of an option that is newly issued at time t. Then the upper bound is given by:

$$V^{up} = L_0 + \mathbb{E}^{\mathbb{Q}}[\max_{\tau} e^{-r\tau}((P_E - P_G)^+ - \pi_t] \tag{19}$$

The above valuation of the spread can then be used to estimate each of the terms in definition 1 to calculate the upper bound value of RINs individually.

4.2.2. Exercise boundary

To analyze the exercise boundary or the trigger curve with respect to time to maturity it should first be noted that in the case of an American call option with dividend, the limiting exercise level does not always approach the strike price. Under GBM assumptions, achieving limiting trigger level equal to strike requires large dividends. In the same fashion, in our setup the limiting trigger curve is not simply $P_E^* = P_G^*$. This may be explained by the argument that the mean-reversion property acts similar to a dividend yield, though through a very different role.

Proposition 4. *Under mean-reversion, as time to expiry approaches, the limiting boundary* *of the trigger region follows a curve in the* (P_E, P_G) *plane satisfying the constraint*

$$\frac{P_E^*}{P_G^*} = \max\left(1, \frac{\alpha_G(\ln(P_G - \theta_G) + (r - \frac{1}{2}\sigma_G^2)}{\alpha_E(\ln(P_E - \theta_E) + (r - \frac{1}{2}\sigma_E^2)}\right) \tag{20}$$

proof:. See Jaimungal et al. (2013).

5. Numerical Simulations

In this section, we represent the results of implementing the models and numerical methods. To this end, first the GBM and GMR models are calibrated to spot price data of gasoline and ethanol. Weekly spot prices between Jan 1^{st} 2010 and Dec 1^{st} 2017 are obtained from the EIA website. Prices are then fitted to the models by the method of maximum pseudo-likelihood of one dimensional stochastic differential equation. Results are reported in table 1. [3]

Table 1: Parameter Estimates

Model	Parameter	Estimate
GBM	r	2.55%
	μ_E	0.00
	μ_G	0.00
	σ_E	0.045
	σ_G	0.035
	ρ	0.85
GMR	θ_E	0.65
	θ_G	0.81
	α_E	0.12
	α_G	0.10
	σ_E	0.046
	σ_G	0.037

Parameters used in numerical simulations. Coefficients are estimated based on weekly gasoline and ethanol spot prices between Jan 1^{st} 2010 and Dec 1^{st} 2017.

Figure 1 compares the various sensitivity of the price of RIN to the underlying parameters

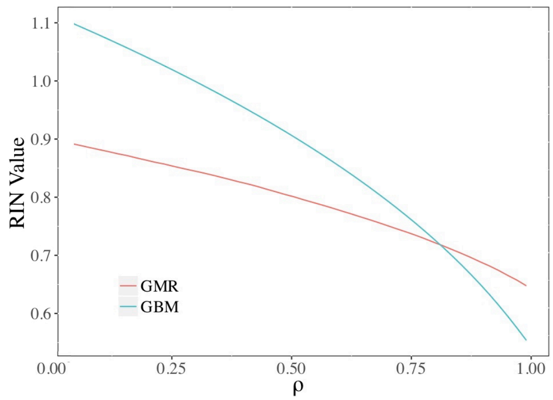
[3] The calibration is done using `Sim.DiffProc` package provided by Guidoum and Boukhetala (2017).

under two assumptions of GBM and GMR. As the graphs illustrate, with both GBM and GMR underlyings, the value of RIN has a negative relation with the correlation between the spot price of ethanol and gasoline. As one expects, the value of RIN decreases as the compliance date approaches, though the value drops more rapidly in the case of GMR. A major reason for this difference is the possibility of early exercise under GMR. But the most interesting observation in figure 1 is the sensitivity of RIN value to the volatility of the underlyings. First, under GBM, volatility of ethanol and gasoline both have the same effect on the price. Second, a U-shape relation exists between volatility and price under both scenarios and for both underlyings. The third noticeable remark which can be better observed with the help of figure 2 is that the price of RIN converges to the same value as the volatility of one of the underlyings increases. What makes this observation noteworthy is the fact that the convergence holds even if the penalty assumption is relaxed and firms are expected to comply regardless. This means that even if the first boundary condition in equation 10 is changed to $\lim_{x\to\infty} V(t,x,y) = \infty$, RIN values converge. While the first mentioned feature can be directly inferred from the closed-form valuation, the two other features may be discussed in more detail.

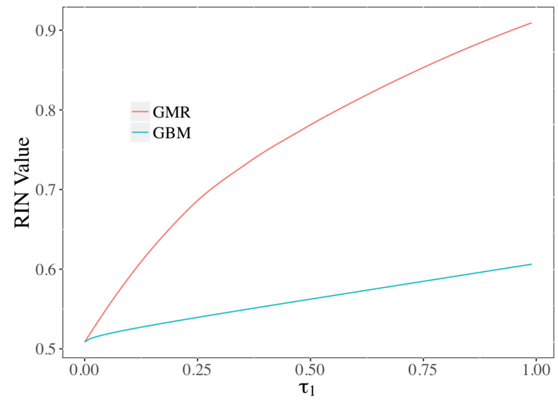
Figure 2 examines the role volatility plays on RIN prices in more detail. In particular, the four graphs show the relation between volatility of one of the underlyings with the RIN value for various levels of volatility of the other underlying. Using this graph, let us seek explanation to the previously raised questions: Why is the relation between volatility and RIN value negative at first and positive after a certain threshold? And when the volatility of one of the underlyings is at extreme levels, why do the RIN values converge to the same level for all volatility levels of the other underlying? In the case of GBM, the answer to both questions can also be inferred from the closed form formula. But to better address these questions for both scenarios, let us first focus on the negative part of the relationship. Consider a set of synthetic parameters for the underlying, volatility of zero for one of the underlyings (in this example ethanol) and a positive volatility for the other underlying (here, gasoline). Also assume that the initial price level of ethanol in the case of GBM and its long term level (in the case of GMR) is higher than that of gasoline. In this setup, the variance of ethanol price is zero but there is a certain probability that the realized gasoline price would be greater than that of ethanol. Next, assume all parameters are held constant, except for the ethanol volatility which is increased to a small positive value. Here, the value of RIN also decreases slightly. The reason is the because of the increased variance, the probability of price difference ($P_E - P_G$) being negative (and thus the RIN being worthless) increases. But this negative realtion holds only up to a certain threshold of ethanol volatility. After this level, even though the instances where the RIN value becomes zero increases, the probability of ethanol price achieving extremely high values also increases. In more details, in instances where the ethanol price is lower than gasoline price, it is not important how much this difference is. The RIN's value becomes zero regardless. In the very high ethanol price realizations however, the magnitude of difference has an impact on RIN value. The same logic explains why at very extreme levels of volatility for one underlying, the RIN value tends to converge for various levels of volatility of the other underlying.

Finally, the effect of the level of banking/borrowing allowance (ξ)on the prices is also illustrated in figure 1. As the formulation in definition 1 shows, banking/borrowing allowance level has a linear effect on the price of RINs. The intuition behind this effect is that a RIN of this year vintage, has $(1 - \xi)$ probability to be exercised before this year's compliance date and $\xi\%$ chance to be transferred to the next year.

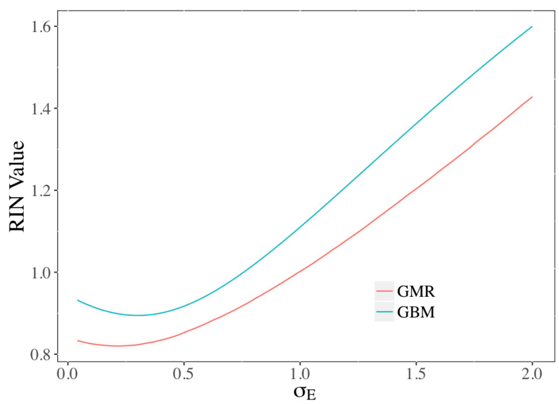
Figure 3 shows the effect of the initial underlying prices on RIN value in the case of GBM. As can be seen, *ceteris paribus* the value of RIN decreases for higher levels of initial gasoline price. Though the level of decrease for a unit increase in the price of gasoline, gradually decreases until it converges to zero. The reasoning given the setup is evident. As the price of gasoline (and subsequently, the difference between the price of ethanol and



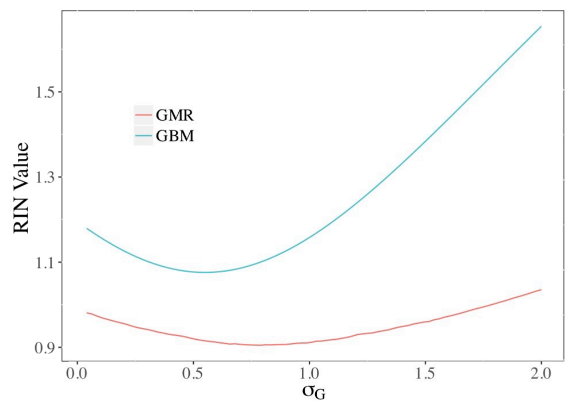
(a) Correlation



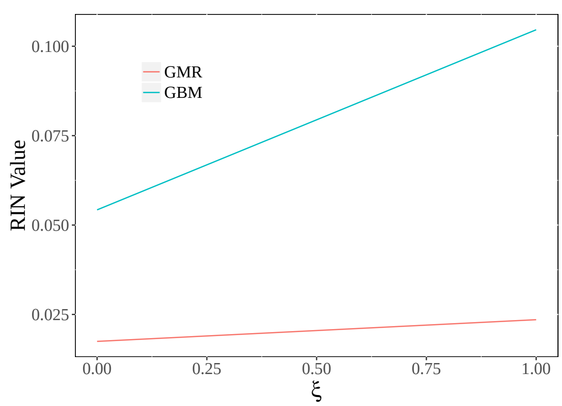
(b) Time to expiry



(c) Volatility of ethanol

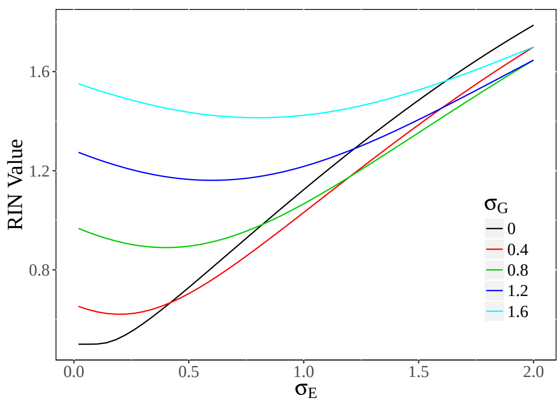


(d) Volatility of gasoline

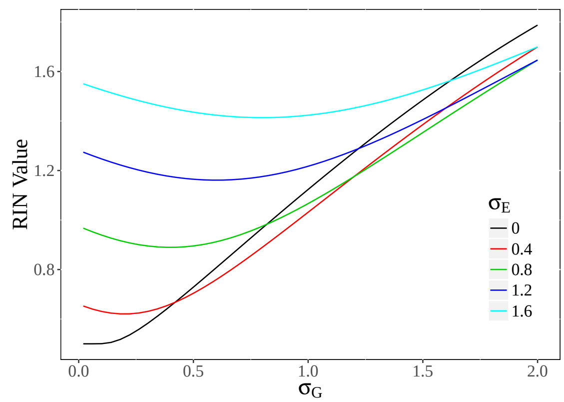


(e) Borrow/Banking allowance

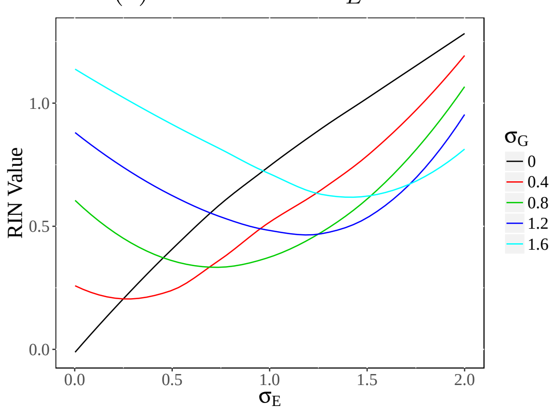
Figure 1: Various sensitivities of RIN under two different underlying behaviors. The initial price values are set as $P_E = 2.5$, $P_G = 2.0$. The remaining factors but one are held constant and are as reported in table 1.



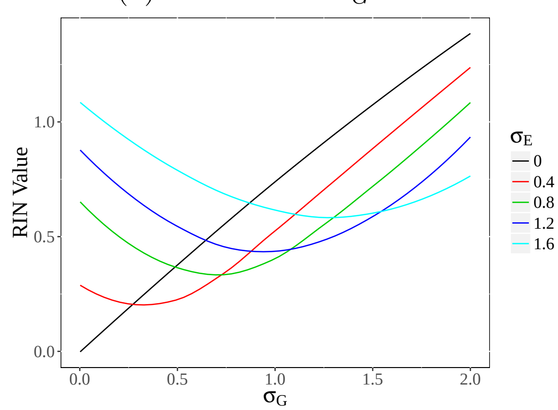
(a) The effect of σ_E for GBM



(b) The effect of σ_G for GBM



(c) The effect of σ_E for GMR



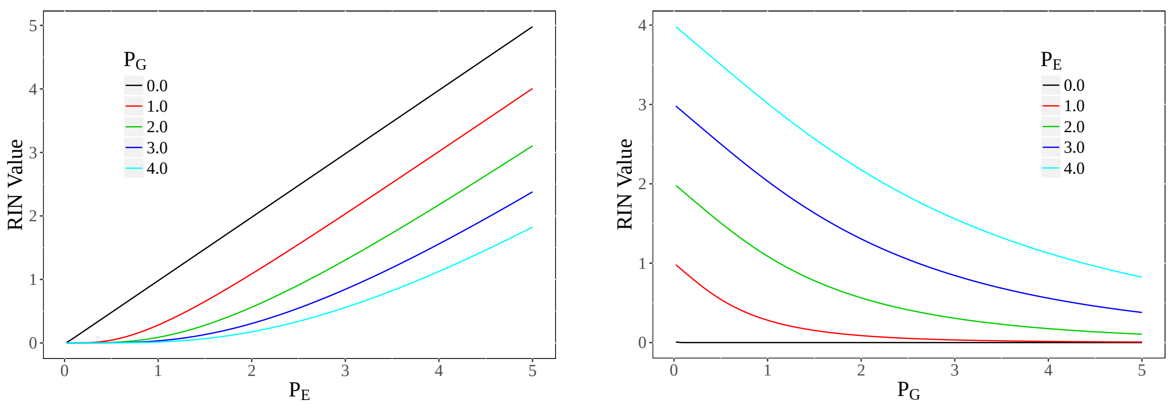
(d) The effect of σ_G for GMR

Figure 2: The effect of volatility on RIN value under both scenarios. The models' parameters are that of figure 1.

gasoline) increases, the chances of ethanol price being above gasoline by the expiry date decreases until it is almost surely zero.

The effect of volatility for various initial prices (under GBM) and long term price levels (under GMR) is further studied in figure 4. Here again a U-shape relation is observed between volatility and RIN value. Part *a* shows as the chance of P_E being above P_G increases (either due to increased volatility or higher initial value), the value of RIN increases. The amount of this increase has a positive relation with the price of ethanol. (or to be more exact, the difference between the price of ethanol and gasoline.) Figure 4-b shows clearly how the price of RIN converges the same value once the volatility of gasoline increases to very high levels. To see the reason, assume the initial price is 2.5 for ethanol and 1 for gasoline. If the volatility of ethanol is a positive number and volatility of gasoline is zero, discounted ethanol prices have the same probability to be above 2.5 by the compliance date. The expected RIN price would therefore remain very close to the static difference. Once the positive volatility of ethanol and the initial prices are held constant and one increases the volatility of gasoline, the probability of the gasoline price being above the ethanol price increases. A possibility that decreases the value of RIN. This is where were are observing the downward slope of the U-shape graph. However, once the volatility of gasoline passes a certain threshold, the relation changes to positive. The intuition in again similar to that of figures 1 and 2. Above this threshold of volatility, though it is possible to have very high and very low levels of gasoline values, the only values that effect the price of RIN are those that still keep the price difference positive. In other words, for extreme levels of volatility, gasoline prices have a non-negligible probability of achieving levels (much) higher than ethanol, but at the same time they also have a greater chance to be at very low levels. While for large values the only fact that effects RIN value is that the gasoline price is above ethanol (and not how much it is above ethanol), in low values, the price difference matters.

A similar behavior is observed in the GMR scenario. The relation is negative for low levels



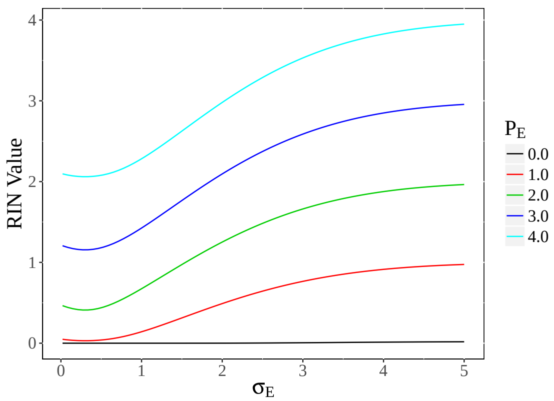
(a) The effect of P_E (b) The effect of P_G

Figure 3: The effect of initial gasoline and ethanol price on RIN value in the case of GBM. All remaining models' parameters are that of figure 1.

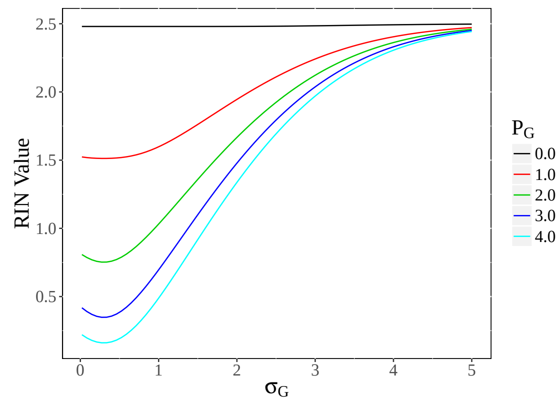
of volatility and then becomes positive when the volatility is above a certain threshold. Regardless of long-run equilibrium price levels, prices tend to converge for high levels of gasoline volatility. Though the speed of convergence is considerably slower as compared to the case of GBM. RIN values also diverge faster for higher long-run equilibrium price levels with the increase of σ_E. This is attributed to the possibility of early exercise.

6. Implications and Interpretations

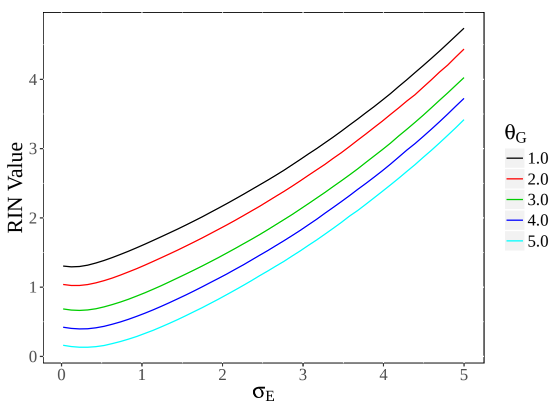
The first evident distinction between results of the two setups is that unlike GBM, it may be optimal to *exercise* a RIN early when prices follow GMR. To see the intuition behind this feature, let us first assume the case of GBM, where it is never optimal to early-exercise. For the sake of simplification first assume there are only two days in a compliance period and banking/borrowing is not allowed. The blender is endowed with one RIN and based on her production projection, she needs to have two RINs by the end of the second day. In this situation, the blender always obtains the required RIN by producing ethanol in the first day. In the second day, she only blends gasoline (in other words, she exercises the RIN), which would be worthless the next day. This holds regardless of the absolute or relative price of ethanol and gasoline in the two days. In a more general framework, when there are n days



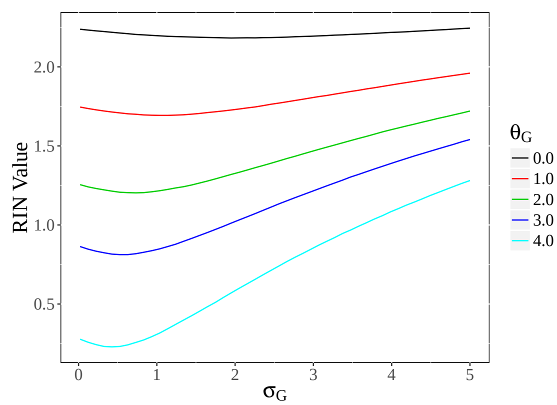
(a) The effect of σ_E for GBM



(b) The effect of σ_G for GBM



(c) The effect of σ_E for GMR



(d) The effect of σ_G for GMR

Figure 4: The effect of volatility for different initial prices of ethanol and gasoline. The models' parameters are that of figure 1.

in the compliance period, m RINs need to be obtained, and production level is the same in all days. The blender obtains RINs by producing ethanol or buying in the market, but only in the last day does she use the excess RIN. She is indifferent with respect to the time when those RINs are obtained.

On the contrary, in the GMR case the blender may find it optimal at times to *exercise* her RINs at certain times. To *exercise* RINs however, does not necessarily require owning the excess RINs. The blender may in fact choose to use gasoline instead of ethanol in the optimum dates by borrowing the RINs from the future. In other words, the blender obtains the required RINs (by either producing ethanol or buying RINs at market) in those subsequent days when it is not optimal to exercise.

The effect of borrow/banking constraint on the value of RIN can be seen in figure 1. Ceteris paribus the value of RIN is at its lowest when banking and borrowing is not allowed. As can be inferred from definition 1, this is because allowing for bank/borrow adds a second non-negative term to the value of RIN. The increase in the allowance level makes RINs more valuable as the probability that a RIN would be transferable to the next compliance period increases. The reason being that under both scenarios as the time of expiry is extended, the option value increases.

Finding the optimal number of years to allow banking/borrowing remains a policy question to be more analyzed. On one hand, short-time horizons make it harder for all firms to comply. On the other hand, the main drawback of allowing long time horizons of borrowing is obvious: Perhaps in the hope of future changes or amendments in the policies, blenders find the incentive to move toward borrowing RINs from the future indefinitely, making the policy effectively useless.

Further, exercising RINs is a *bang-bang* type control. Meaning once it is optimal to exercise, the blender uses pure gasoline, and once it is not, the blender obtains the maximum amount of RIN possible; either through producing ethanol or buying RINs at the market.

Currently, there are four distinct ethanol-gasoline blends available in the US: E0 (with no ethanol), E10 (with up to 10% ethanol), E15 (containing up to 15% ethanol) and E85, which is only designated for flexible-fuel vehicles and contains between 51-83% of ethanol (average of 74%). Of these, E10 is by far the two dominant types of fuels in the market, and E85 is the fuel that has been introduced as the more sustainable alternative. Therefore, the only way blenders can produce more than a certain level of RIN would be through using the produced ethanol in E85. This in fact has been one of the incentives behind EPA's efforts in increasing mandate levels (EIA (2016)). Our model assumes blenders are able to perform this switch with no cost.

7. Conclusion

The United States and EU countries have forms of mandates for the minimum use of bio and renewable fuels. One of these mandates in the US, which is part of the current Renewable Fuel Standards is the requirement for the gasoline producers (blenders) to use about 10% biofuels (ethanol) in their transportation fuel production. Similar policies have been adapted in different European countries. While these policies have differences in design, execution and flexibility for adaption, they all allow for the existence of a market for trading renewable fuel certificates such as Renewable Identification Numbers. A core feature common in the design of these markets is that they give an option value to the certificate being traded. In this paper, we focus on RINs in the US market to examine the value of these certificates, their relation with parameters that explain the price of gasoline and ethanol, and their affect on production.

This paper models the price of RINs as an American spread option. We use two different models for the price of ethanol and gasoline as the underlying processes of this option, namely Geometric Brownian motion, and Geometric mean-reverting process. In this *real option* framework, the value of RIN reflects the supremum of the expected difference between the price of ethanol and gasoline. As the option is American, it is possible for the owner to find exercising the option (i.e., RIN) optimal. In this setup, RIN is exercised when the blender decides to use gasoline instead of the required number of RINs in that day of production. Using RINs is a bang-bang type control (that is, the control only takes values from a finite set). In more details, at times when it is optimal to exercise the RIN, the blender decides to blend pure gasoline, and use her previously acquired RINs instead of ethanol, buy RINs at the market, or borrow the RINs from future. When it is not optimal to exercise RIN, the blender decides to obtain the maximum amount of RIN possible, either through production (and blend of ethanol) or buying RINs in the market.

The GBM setup leads to a closed-form solution for the price of RINs. Under this setup, the blender never finds it optimal to use RINs instead of ethanol. Though the commodity market literature does not favor GBM setup for explaining the price of commodities (for various reasons the most important of which are mentioned in the text), the resulting closed-form solution allows for simple and rapid computation of prices. We also extend the GBM setup for the underlying and introduce the GBM dynamics to Merton's jump. The resulting model leads to an approximation of the price of RINs.

Next, we consider the prices to follow the more advocated dynamic of Gemoetric mean-reverting processes and model the value of RIN. Under this assumption, the value of RIN can be obtained with a lower and an upper bound. With the derived valuations, we perform a set of numerical simulation to examine the sensitivity of RIN prices to various underlying parameters. In particular, we study the effect of the correlation between gasoline and ethanol prices, time left until the compliance date, banking/borrowing allowance level, initial price of gasoline and ethanol (under GBM setup), long-run equilibrium price levels (under GMR setup), and and volatility. Among other results, we observe the linear effect of banking/borrowing level on the price of RIN and the U-shape relation between both underlying volatilities and RIN value.

The results of this paper can be used to examine renewable certificate markets from various aspects. The framework setup and the resulting valuation models can be used in designing similar markets in other countries or in amending the current markets. This is particularly useful when the blenders are failing to comply with the policy mandates or the set targets have proven to be too ambitious. An example of these instances is in December 2015 when the EPA decided to lower the mandate requirement, which was an acknowledgment of the difficulty of compliance with the original goals. The results can also be used to study the affect of mandates on production plan of blenders.

The framework used in this study can be applied to examine floor-and-trade mechanisms in other markets. In the RINs market in particular, this work can be further extended to better capture the market dynamics. The price of fuel, and especially ethanol is different in different regions of the US. A possible future step of this work may be incorporating a framework similar to Litzenberger and Rabinowitz (1995) to differentiate between producers. As previously explained, there are a number of fundamental questions with regards to policy design in markets similar to RINs that still need to be better addressed. These include blended biofuel share of the fuel, optimal level of allowance for borrowing and banking, optimal size of compliance period, penalty per shortage, and subsidies. For the purpose of simplification, we have not taken into account nested mandates. Future studies may incorporate nested mandates, which can be useful especially in designing and amending the policies and its affects on the trade market.

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Appendix A. Proposition 1 Proof

Assuming no banking constraint, the value of RIN under the risk-neutral measure $\mathbb{Q}$ (which will be derived later) is

$$p = e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(P_E(T) - P_G(T)), 0] \tag{A.1}$$

Corollary A.1. Assuming $dB_E(t)dB_G(t) = \rho dt$, processes $W_1(t)$ and $W_2(t)$, defined as follows are independent Brownian motions.

$$\begin{aligned} B_E(t) &= W_1(t) \\ B_G(t) &= \int_0^t \rho dW_1(s) + \int_0^t \sqrt{1-\rho^2}dW_2(s) \end{aligned} \tag{A.2}$$

proof. First note that $dW_1(t) = dB_E(t)$ and:

$$dW_2(t) = \frac{-\rho}{\sqrt{1-\rho^2}}dB_E(t) + \frac{1}{\sqrt{1-\rho^2}}dB_G(t) \tag{A.3}$$

Which leads to:

$$W_2(t) = \int_0^t \frac{-\rho}{\sqrt{1-\rho^2}}dB_E(u) + \int_0^t \frac{1}{\sqrt{1-\rho^2}}dB_G(u) \tag{A.4}$$

$W_1(t)$ is obviously a Brownian motion (BM). Also $W_2(0) = 0$. From equation A.4 and the fact that B_E and B_G are both BM and thus martingales (i.e., the stochastic integral is also a martingale), we conclude $\mathbb{E}[W_2(t)] = 0$. Thus $W_2(t)$ is a continuous martingale process.

The following also holds:

$$
\begin{aligned}
dW_2(t).dW_2(t) = & \\
& \frac{\rho^2}{1-\rho^2}dB_E dB_E - \frac{\rho}{1-\rho^2}dB_E dB_G - \frac{\rho}{1-\rho^2}dB_G dB_E + \frac{1}{1-\rho^2}dB_G dB_G \quad \text{(A.5)} \\
& = \frac{\rho^2}{1-\rho^2}dt - \frac{\rho}{1-\rho^2}\rho dt - \frac{\rho}{1-\rho^2}\rho dt + \frac{1}{1-\rho^2}dt = dt
\end{aligned}
$$

Meaning $[W_2, W_2]_{(t)} = t$. Using Levy theorem, one can conclude $W_2(t)$ is also a BM. It remains to show W_1 and W_2 are independent.

$$
\begin{aligned}
dW_1(t)dW_2(t) &= dB_E(t)\left(\frac{-\rho}{\sqrt{1-\rho^2}}dB_E(t) + \frac{1}{\sqrt{1-\rho^2}}dB_G(t)\right) \\
&= -\frac{\rho}{1-\rho^2}dt + \frac{\rho}{1-\rho^2}dt = 0
\end{aligned} \tag{A.6}
$$

Again, using Levy theorem, it is concluded that W_1 and W_2 are independent Brownian motions.

Using corollary A.1, one can describe the price dynamics as:

$$
\begin{aligned}
\frac{dP_G(t)}{P_G(t)} &= \mu_E dt + \sigma_E dW_1(t) \\
\frac{dP_E(t)}{P_E(t)} &= \mu_G dt + \sigma_G(\rho dW_1(t) + \sqrt{1-\rho^2}dW_2(t))
\end{aligned}
$$

where $W(t) = (W_1(t), W_2(t))$ is a two-dimensional Brownian motion.

Corollary A.2. There is no arbitrage in the given dynamics and a risk-neutral measure exists.

proof. There are 2 equations and 2 unknowns for defining the market price of risks processes Θ_1, Θ_2. According to Shreve (2004), these equations are:

$$
\mu_i - r = \sum_{j=1}^{2} \sigma_{ij}\Theta_j \tag{A.7}
$$

where $\sigma_{11} = \sigma_E, \sigma_{12} = 0, \sigma_{21} = \rho\sigma_G, \sigma_{22} = \sqrt{1-\rho^2}\sigma_G$ This leads to:

$$\begin{aligned} \Theta_1 &= \frac{\mu_E - r}{\sigma_E} \\ \Theta_2 &= \frac{(\mu_G - r)\sigma_E - (\mu_E - r)\rho\sigma_G}{\sigma_E\sigma_G\sqrt{1-\rho^2}} \end{aligned} \tag{A.8}$$

Also, it is easy to verify that:

$$\mathbb{E}\Big(\exp(\frac{1}{2}\int_0^T (\Theta_1^2 + \Theta_2^2)dt)\Big) < \infty$$

Therefore, based on the first fundamental theorem of asset pricing, no arbitrage exists in the market and a risk-neutral measure (equivalent to the original measure) exists, which will be subsequently derived.

Corollary A.3. The risk neutral dynamics when the solution to market price of risk follows equation A.8 is given by:

$$\begin{aligned} dP_G(t) &= rP_G(t)dt + \sigma_E d\tilde{B}_E(t) \\ dP_E(t) &= rP_E(t)dt + \sigma_G d\tilde{B}_G(t) \end{aligned} \tag{A.9}$$

where $\tilde{B}_i = B_i + \int_0^t \gamma_i(u)du$ and $\gamma_i(t) = \sum_{j=1}^2 \frac{\sigma_{ij}}{\sigma_i}\Theta_j$. Further $d\tilde{B}_E d\tilde{B}_G = \rho dt$

proof. Z is a Radon-Nykodym derivate such that:

$$Z(t) = \exp\{-\int_0^t \Theta(u).dW(u) - \frac{1}{2}\int_0^t ||\Theta(u)||^2 du\} \tag{A.10}$$

Setting $Z = Z(T)$, then $\mathbb{E}Z = 1$, and under this probability measure $\mathbb{Q}$ given by

$$\mathbb{Q}(A) = \int_A Z(\omega)d\mathbb{P}(\omega)$$

it can be stated that

$$\tilde{W}_j(t) = W_j(t) + \int_0^t \Theta_j du$$

are independent Brownian motions, according to Girsanov's theorem. Note that $\mathbb{P}$ is the original probability measure. First it is shown that $\tilde{B}_i$ is a Brownian motion under $\mathbb{Q}$. Note that by definition of $B_i(t)$:

$$\begin{aligned}\tilde{B}_i(t) &= \sum_{j=1}^{2} \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i} dW_j(u) + \gamma_i(u)\Big) \\ &= \sum_{j=1}^{2} \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i} (dW_j(u) + \Theta_i(u)du)\Big) \\ &= \sum_{j=1}^{2} \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i} d\tilde{W}_j(t)\Big)\end{aligned} \tag{A.11}$$

Using Levy Theorem, one can show $\tilde{B}_i(t)$ is a Brownian motion under the original probability measure.[$\tilde{B}_i(t) = 0$. Also $\mathbb{E}[\tilde{B}_i] = 0$, thus $\tilde{B}_i(t)$ is a continuous martingale and $d\tilde{B}_E(t)d\tilde{B}_G(t) = dt$] By theorem 4.3.1 each of the Ito integrals in the definition of $\tilde{B}_i(t)$ is a continuous martingale (starting at zero). The quadratic variation is

$$d\tilde{B}_i.d\tilde{B}_i = \sum_{j=1}^{2} \frac{\sigma_{ij}^2}{\sigma_i^2} dt = dt$$

. Therefore, again by using Levy Theorem, $\tilde{B}_i(t)$ is a martingale under $\mathbb{Q}$. Given the dynamics in the question and the relation between B_i and $\tilde{B}_i$ we can show:

$$\begin{aligned}\frac{dP_i}{P_i} &= \mu_i dt + \sigma_i dB_t \\ &= \mu_i dt + \sigma_i \sum_{j=1}^{2} \frac{\sigma_{ij}}{\sigma_i} dW_j(t) \\ &= \mu_i dt + \sigma_i \sum_{j=1}^{2} \frac{\sigma_{ij}}{\sigma_i} (d\tilde{W}_j(t) - \Theta_j dt) \\ &= \mu_i dt + \sigma_i (d\tilde{B}_i(t) - \sum_{j=1}^{2} \frac{\sigma_{ij}\Theta_j}{\sigma_i} dt) \\ &= (\mu_i - \sum_{j=1}^{2} \sigma_{ij}\Theta_j) dt + \sigma_i d\tilde{B}_i(t) \\ &= r dt + \sigma_i d\tilde{B}_i(t)\end{aligned} \tag{A.12}$$

Now, to show the correlation between the new Brownian motions:

$$\begin{aligned}d\tilde{B}_E d\tilde{B}_G &= (dB_E(t) + \gamma_1(t)dt)(dB_G(t) + \gamma_2(t)dt) \\ &= dB_E(t) dB_G(t) = \rho dt\end{aligned} \tag{A.13}$$

Corollary A.4. Risk-neutral measure $\mathbb{Q}$ is unique.

proof. As the driving Brownian motions are the only source of uncertainty, the only way multiple risk-neutral measures can arise is via multiple solutions to the market price of risk equations. Equation A.8 reveals the market prices of risk Θ_1, Θ_2, have unique solutions. It is concluded that the risk-neutral measure is also unique.

The value of RIN under the risk-neutral measure is

$$\begin{aligned}p &= e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(P_E(T) - P_G(T)), 0] \\ &= e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0) P_G(T)]\end{aligned} \tag{A.14}$$

or in other words, the price of P_E can be explained in terms of P_G and still remain a GBM. This is the basic idea behind solving the question when the strike price K is equal zero. Further, the risk-neutral dynamics can be explained by:

$$
\begin{aligned}
\frac{dP_G}{P_G} &= rdt + \sigma_E d\tilde{W}_1(t) \\
\frac{dP_G}{P_G} &= rdt + \sigma_G(\rho d\tilde{W}_1(t) + \sqrt{1-\rho^2} d\tilde{W}_2(t))
\end{aligned}
\tag{A.15}
$$

Where $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are independent Brownian motions defined in corollary A.3. Proof is provided in corollary A.1. Note that by applying Ito's lemma:

$$
\begin{aligned}
\frac{d(P_E(t)/P_G(t))}{P_E(t)/P_G(t)} =& rdt + \sigma_G[\rho d\tilde{W}_1(t) + \sqrt{1-\rho^2} d\tilde{W}_2(t)] - [rdt + \sigma_E d\tilde{W}_1(t)] \\
& + \sigma_E^2 dt + [rdt + \sigma_E d\tilde{W}_1(t)][rdt + \sigma_G \rho d\tilde{W}_1(t) + \sqrt{1-\rho^2} d\tilde{W}_2(t)] \\
=& (\rho\sigma_G - \sigma_E) d\tilde{W}_1(t) + \sqrt{1-\rho^2}\sigma_G d\tilde{W}_2(t) + \sigma_E(\sigma_E - \rho\sigma_G)dt
\end{aligned}
\tag{A.16}
$$

Now, defining a new probability measure, $\mathbb{C}$, with Radon-Nikodym derivative can be done using Girsanov theorem:

$$
\frac{d\mathbb{C}}{d\mathbb{Q}} = \frac{1}{P_G(0)} P_G(T) = \exp(-\frac{1}{2}\sigma_E^2 T + \sigma_E dW_1(T)) \tag{A.17}
$$

Under this new measure, according to Grisanov theorem, $\hat{W}_1(t) = \tilde{W}_1(t) - \sigma_E t$ is a Brownian motion (so is $\hat{W}_2(t) = \tilde{W}_2(t)$). Therefore:

$$
\begin{aligned}
p =& e^{-rT}\mathbb{E}_{\mathbb{P}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0) P_G(T) \\
& * \exp(\frac{1}{2}\sigma_E^2(T) - \sigma_E W_1(T) + (r - \frac{1}{2}\sigma_E^2)T + \sigma_E W_1(T))] \\
=& P_G(0)\mathbb{E}_{\mathbb{P}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0)]
\end{aligned}
\tag{A.18}
$$

Therefore, using the result above, one can describe $P_E(t)/P_G(t)$ under $\mathbb{C}$ as a GBM with volatility σ as:

$$\frac{d(P_E(t)/P_G(t))}{P_E(t)/P_G(t)} = \sigma dW_3(t) \tag{A.19}$$

In the above equation,

$$\begin{cases} \sigma^2 = \sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G \\ dW_3(t) = \frac{\rho\sigma_G - \sigma_E}{\sigma} d\hat{W}_1(t) + \frac{\sqrt{1-\rho^2}\sigma_G}{\sigma} d\hat{W}_2(t) \end{cases} \tag{A.20}$$

where the volatility σ equality holds because:

$$[(\rho\sigma_G - \sigma_E)d\hat{W}_1(t) + (\sqrt{1-\rho^2}\sigma_G)d\hat{W}_2(t)]^2 = (\sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G)dt$$

Also, $W_3(t)$ is a BM under $\mathbb{C}$ because i) $\hat{W}_1(t)$ and $\hat{W}_2(t)$ are BMs, independent and thus martingales. Subsequently, their stochastic integral is also a martingale. ii) $\mathbb{E}W_3(0) = 0$. Therefore, $W_3(t)$ is a continuous martingale. iii)

$$\begin{aligned} dW_3(t).dW_3(t) &= [\frac{(\rho\sigma_G - \sigma_E)}{\sigma} d\hat{W}_1(t) + \frac{(\sqrt{1-\rho^2}\sigma_G)}{\sigma} d\hat{W}_2(t)]^2 \\ &= (\frac{(\rho^2\sigma_G^2 + \sigma_E^2 - 2\rho\sigma_E\sigma_G)}{\sigma^2} + \frac{(1-\rho^2)\sigma_G^2}{\sigma^2}) = dt \end{aligned} \tag{A.21}$$

From here on, the solution is relatively straightforward. Using the BSM formula for a European Call Option, the derivative security can be priced as:

$$p = P_E(0)N(d_1) - P_G(0)N(d_2) \tag{A.22}$$

where

$$d_1 = \frac{\ln(P_E(0)/P_G(0))}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \quad d_2 = \frac{\ln(P_E(0)/P_G(0))}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}$$

and σ is defined as in equation A.20. This is equivalent to the well-known Margrabe (1978) equation.

Appendix B. Deriving risk-neutral dynamics of mean-reverting processes

Suppose real world dynamics of the prices follow

$$\begin{aligned} P_i(t) &= \exp(\tilde{\theta}_i + X_i(t)) \\ dX_i(t) &= -\tilde{\alpha}_i X_i(t)dt + \sigma_i d\tilde{B}_i(t) \end{aligned} \tag{B.1}$$

where $i \in \{E, G\}$, $\tilde{\alpha}_i$s are the constant mean reversion coefficients, estimated using regressing historical data. $\tilde{B_E}(t)$ and $\tilde{B_G}(t)$ are standard Brownian motions with correlation ρ under real world measure $\mathbb{P}$. $\exp(\tilde{\theta}_i)$ is known as the asymptotic mean reversion level and σ_i is the volatility.

In order to provide the risk-neutral measure required for valuation, we use the market price of risk. The market prices of risk $\lambda_i(t)$ associated with the Brownian motions, $\tilde{B_E}(t)$ and $\tilde{B_G}(t)$, are assumed to be affine in $X_i(t)$, more specifically:

$$\lambda_i(t) = \frac{a_i + b_i X_i(t)}{\sigma_i} \tag{B.2}$$

with a_i and b_i as constants. Using Girsanov's theorem one can show that under the equivalent risk-neutral measure $\mathbb{Q}$, the drift adjusted processes, $B_i(t) = \int_0^t \lambda_i(s)ds + \tilde{B}_i(t)$ are also standard Brownian motions with correlation ρ. Consequently, the price dynamics under this new measure are:

$$\begin{aligned} P_i(t) &= \exp(\theta_i + X_i(t)) \\ dX_i(t) &= -\alpha_i X_i(t)dt + \sigma_i dB_i(t) \end{aligned} \tag{B.3}$$

where $\alpha_i = \tilde{\alpha}_i - b_i$, and $\theta_i = \tilde{\theta}_i + a_i/\alpha_i$.