Investments under vertical relations and agency conflicts: A real options approach

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Abstract

This paper discusses how information asymmetries in a decentralized firm affect other firms in the same supply chain. Using the real options approach, we examine the case of a firm holding the option to make an uncertain and irreversible investment. The firm is decentralized and there is information asymmetry between the owner of the firm and the project manager regarding the price of a needed input (e.g. a key equipment) that needs to be purchased by an outside supplier with market power. We show that the total loss stemming from the information asymmetry is the sum of two terms: i) the loss in the decentralized firm itself and ii) the negative externality that the outside input supplier endures. We also show that the latter is likely not just a part, but rather the main component of the total loss. Last, we prove that the use of an audit technology, instead of a bonus-incentive mechanism, reduces the negative externality.

KEYWORDS: Real options, Vertical relations, Asymmetric information, Agency conflicts, Audit

JEL CLASSIFICATION: D82, L10.

1 Introduction

The real options approach is a standard framework for the analysis of investment opportunities. It builds on the idea that the option to undertake an investment project is analogous to an American call option on a real asset. Hence, when evaluating an investment option characterized by uncertainty and irreversibility, the potential investor needs to factor in that at the time of the investment s/he forgoes the option to reconsider the investment decision at some future time point when the uncertainty will be, naturally, partly resolved.1

The standard real options model does not account for agency conflicts and information asymmetries since the investment is assumed to be managed by the potential investor her/himself. However, in many modern corporations, investment decisions are delegated by the owner of the corporation (principal) to a manager (agent) who possesses a relevant skill set or piece of information.2 Of course the principal benefits from the expertise of the agent but, at the same time, s/he might be

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1See Dixit and Pindyck (1994) and Trigeorgis (1996) for an overview of the real options approach.

2Delegation is a standard practice when managing large enterprises (Amaral et al., 2006). For relevant examples from industries that have to do with textiles, construction, aeronautics, telecommunications, computers, automobiles, electronics and business services, see e.g. Agrell et al. (2004), Lee et al. (2004), Schieg (2008), Tang et al. (2009), Deshpande et al. (2011), Doorey (2011), Kayis et al. (2013), Bolandifar et al. (2016), Agrell and Bogetoft (2017) and Dietrich et al. (2008).
exposed to information asymmetries. If the agent has an informational advantage over the principal, then the latter must carefully consider the underlying motives when deciding the terms of the delegation. More precisely, the principal needs to develop an appropriate mechanism in order to incentivize the agent to share private information resolving the information asymmetry. The use of such a mechanism is costly for the principal but, without it, s/he is due to face further distortions stemming from the coordination failure.\(^3\)

As we will see in the next section, there is a growing body of papers that incorporate agency conflicts that stem from information asymmetries into the real options model. In spite of the differences in their analyses, what these papers share is the assumption that the investment cost is exogenous. As Billette de Villemeur et al. (2014) point out, this assumption is sensible when an investment is performed largely in-house as, for instance, in a research and development project. Nevertheless, this is not always true. Investment projects are often rather complex and specialized inputs might be needed.\(^4\) At the same time, the potential investor might lack the equipment and/or the expertise to manufacture the needed input. In these cases, the investment cost is endogenous since it is specified by the vertical relationship between the external input supplier and the potential investor. For instance, Billette de Villemeur et al. refer to investments in the vaccine industry where facilities are specifically designed for the production of a novel vaccine. In this case, the needed customized equipment is sourced on an intermediate market from input providers with market power. In the same vein, Pennings (2017) refers to large infrastructure projects as, e.g., a telecommunications network. In that case, an upstream firm (construction company) is responsible for the provision of an indispensable input (network), to a downstream firm (internet provider).

The key originality of this paper lies in the combination of the decentralized investment setting with the endogenous pricing of a necessary input. Using the real options approach, we examine the case of a potential investor who is contemplating making an uncertain investment. A prerequisite for the investment to take place is the provision of a specialized input by an external supplier with market power. Since the input is relationship-specific, i) the investment cost is sunk and ii) the principal delegates the investment decision to an agent with a relevant expertise.\(^5\)

Our findings suggest the following: Firstly, we verify that the presence of an external input supplier with market power, makes the investment more expensive, favoring its postponement while reducing the value of the option to invest. Secondly, we show that the total losses attributed to the information asymmetry and the corresponding agency conflict in the firm, can be decomposed into two components: i) the loss in the decentralized firm and ii) the negative externality that the input supplier endures when s/he fails to anticipate the agency conflict downstream. The first component of the total social loss is already analyzed in detail in the literature. However, the second component has attracted, to the best of our knowledge, no attention. The two components differ in nature in the sense that the first one captures the in-firm cost of using a mechanism to resolve the agency conflict whereas, the second one, captures the loss for another firm in the supply chain, in this case an input supplier, who cannot foresee the incentive misalignment downstream. We also derive a condition under which the second component is larger than the first one and we show that, unless certain parameters obtain extreme values, this condition will hold. Last, we show

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3For an overview of the literature on asymmetric information see e.g. Laffont and Martimort (2002).
4See e.g., Agrell and Bogetoft (2017), Hargadon and Sutton (2000) and Linder (2004).
5The agent might have specialized information related to the input (Mookherjee and Tsumagari, 2004), or s/he might be responsible for the solution of a matching problem if direct communication between the project originator and the input supplier is impossible, or prohibitively expensive (Faure-Grimaud and Martimort, 2001). Alternatively, the agent might just be what Hayek (1945) calls the "person on the spot". For instance, McAfee and McMillan (1995) assume that the principal opts for disintegration when the management of the project takes time and the principal’s time is limited. Similarly, Van Zandt (1999) argues that the need for delegation might stem from the fixed information processing capacity of the principal.
that when the principal resolves the agency conflict complementing the standard bonus-incentive mechanism with an audit technology, the negative externality is significantly reduced thanks to a reduction in the relevant timing distortion.

The remainder of the paper is organized as follows. In Section 2 we present an overview of the related literature. In Section 3 we present in detail the model set-up demonstrating the connections with previous work. In Section 4 we analyze the total loss that is attributable to the information asymmetry between the principal and the agent. In Section 5 we discuss the agency conflict when an audit technology is used. In Section 6 we present some numerical examples whereas in Section 7 we discuss the special case of supply chains characterized by traceability and transparency. Section 8 concludes.

2 Overview of the related literature

This work contributes to the research area that integrates the basic theory of irreversible investment under uncertainty as in Dixit and Pindyck (1994) and the literature on asymmetric information as in Laffont and Martimort (2002).

Grenadier and Wang (2005) analyze the timing and efficiency of an investment undertaken in a decentralized setting under the presence of information asymmetries and hidden action between the principal and the agent. They show that the principal can induce the agent both to extend effort and to reveal private information by using a bonus-incentive contract. Despite the fact that the use of such an instrument is suboptimal in the sense that the chosen investment timing differs from the timing in the setting with symmetry of information, the principal’s losses are reduced since further distortions are avoided. Shibata (2009) extends the analysis presented in Grenadier and Wang (2005) by complementing the bonus-incentive contract with an audit technology. Focusing on the adverse-selection-only case he shows that, by using both auditing and a bonus-incentive, the timing inefficiency is reduced, the principal’s value is larger whereas the agent’s value is smaller. Nevertheless, the audit technology does not necessarily lead to an increase in the aggregate value of the opportunity to invest.

Contributing to the same body of work, Shibata (2008) focuses on the impact of uncertainty on the timing and the value of the project whereas Shibata and Nishihara (2010), Grenadier and Malenko (2011), Morellec and Schürhoff (2011), Hori and Osano (2014) and Cardoso and Pereira (2015) among others examine the effect of capital structure and financing of the investment. Cong (2013) and Bouvard (2014) examine the implications of endogenous learning and experimentation respectively, whereas Mæland (2010) and Koskinen and Mæland (2016) approach the agency conflict assuming that the agent is the winner of an auction in which a number of potential delegates participate. Last, Broer and Zwart (2013) examine the optimal regulation of an investment undertaken by a monopolist who has private information on the investment cost whereas Arve and Zwart (2014) examine the case where the information asymmetry between the principal and the agent has to do with the starting point of the process that is used to capture the fluctuations of the stochastic parameter.

Despite the differences in the adopted framework, what all these papers have in common is the assumption that the investment cost is exogenous. However, as highlighted by Billette de Villemeur et al. (2014), the cost of an investment does not always reflect the project’s economic fundamentals.

6 In Grenadier and Wang (2005) the management effort is assumed to be exogenous. Shibata and Nishihara (2011) approach the same problem using a two-stage optimization problem that allows investment timing and management effort endogenously decided. The numerical examples that they present suggest that the management effort is greater under asymmetric, than under symmetric, information. This in turn implies that there are trade-offs between investment efficiency and management effort under asymmetric information.
In this work, we apply the endogenous pricing of the input à la Billette de Villemeur et al., in the adverse-selection-only version of the model presented by Grenadier and Wang (2005) and the model presented by Shibata (2009), and we discuss the effect of the agency conflict downstream on the input supplier upstream.

3 The model

In this section, we begin with a description of the basic set-up of the model. We then present the integrated case which corresponds to the problem with in-house production of the input, and the separated case, which corresponds to the case where the input is produced by the upstream input supplier. Finally, we present the case with delegation of the investment decision which is our original contribution.

3.1 The basic set-up

The owner of a firm, \( P \), holds the option to undertake an investment. In order to do so, \( P \) needs a specialized input (e.g., a key equipment) that can be produced in-house (integration) or by an upstream firm \( U \) (separation). The production cost of the input is equal to \( I \) and is assumed to be completely sunk. We assume that \( I \) can be "low" \((I_l)\) with probability \( q \in [0, 1] \) or "high" \((I_h)\) with probability \( 1 - q \) where \( I_h > I_l > 0 \) and \( \Delta I = I_h - I_l \).

**Assumption 1:** The probability distribution of \( I \) is common knowledge whereas the input producer is the only party who observes the true \( I \) as soon as this is chosen by Nature. This is a reasonable assumption since the individual with the best information on the production cost is usually the producer her/himself (see e.g., Celik, 2009; Broer and Zwart, 2013).

The value of the project is represented by \( X_t \) which is assumed to be fluctuating over time according to the following geometric Brownian motion:

\[
\frac{dX_t}{X_t} = \mu dt + \sigma dz_t
\]  
\( (1) \)

The parameter \( \mu \) stands for the positive constant drift, \( \sigma \) is the positive constant volatility and \( dz_t \) is the increment of a Wiener process. A lower case \( x \) is used to denote the current level of \( X_t \) \( x = X_0 \). We assume that \( P \), as the holder of the investment option, can continuously and verifiably observe the realization of \( X_t \) over time. On the contrary, \( U \) knows the structural parameters of process (1) but cannot observe the realizations of \( X_t \) at any point in time.

According to the real options literature, when a potential investor contemplates undertaking an investment characterized by uncertainty and irreversibility, the ability to delay the investment for some future time point is a source of flexibility that profoundly affects the decision to invest (see e.g., McDonald and Siegel, 1986). The investment takes place only as soon as the project’s expected payoff exceeds the cost of the investment by a margin equal to the option value of further postponing the completion of the project into the future. Keeping this in mind, it is assumed throughout the paper that \( x \) is sufficiently low so that future, rather than immediate, investment is preferred.

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7 In Section A.2 of the Appendix we extend our analysis considering a continuous \( I \).
8 We will analyze the importance of this assumption in subsection 3.3 below.
9 O’Brien et al. (2003), Leahy and Whited (1996) and Guiso and Parigi (1999) present strong empirical evidence supporting this argument.
10 Otherwise the problem reduces to the mere maximization of the net present value.
All the parties are assumed to be risk neutral with the risk-free interest rate denoted by $r$. For convergence we assume $r > \mu$.$^{11}$

### 3.2 Integrated case

In the integrated case, $P$ produces the needed input in house. S/he is first observing the magnitude of $I$ chosen by Nature ($I_h$ or $I_l$) and is then deciding when to invest. If $F(x; I)$ is the value of the option to invest, then the optimal investment time point $\tau$ is derived through the solution of the following maximization problem:

$$F(x; I) = \max_{\tau} E_x \left[ e^{-r \tau} \left( X^* - I \right) \right], \quad (2.1)$$

which, under $X^* > x$, can be rearranged as

$$F(x; I) = \max_X (X - I) \left( \frac{x}{X} \right)^{\beta}, \quad (2.2)$$

where:

- $\tau = \inf \{t > 0 | X_t = X^* \}$ is the random first time point that $X_t$ hits the barrier $X^*$ which is the project value that triggers the investment and,
- $\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left( \frac{1}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2\mu}{\sigma^2}} > 1$ is the positive root of the characteristic equation $\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu \beta - r = 0$.

The expressions for $F(x; I)$ and $\beta$ are standard in the real options literature. Solving the maximization problem from Eq. (2.2) we obtain the value maximizing investment trigger $X^*(I) = \frac{\beta}{\beta - 1} I$ and the ex-post value of the option to invest $F(x; I) = \frac{I}{\beta - 1} \left( \frac{x}{X^*(I)} \right)^{\beta}$ where $I \in \{I_l, I_h\}$. The ex-ante value of the investment option is $f(x; I_l, I_h) = qF(x; I_l) + (1 - q)F(x; I_h)$.

Since $\beta > 1$, we have $\beta/\beta - 1 > 1$ and $X^*(I) > I$. The difference between the optimal investment threshold $X^*(I)$ and the sunk investment cost $I$ is exactly the margin that we described in the end of the previous subsection and is attributed to the uncertainty and the irreversibility characterizing the investment. Note that $\beta$ is decreasing in $\sigma$ and $\mu$ whereas it is increasing in $r$. Consequently, the wedge $\beta/\beta - 1$ is increasing in $\sigma$ and $\mu$ whereas it is decreasing in $r$. In words, the greater is the amount of uncertainty and/or the expected rate of return over the future values of $X_t$, the larger is the excess return that the firm will demand before it is willing to make the irreversible investment. On the contrary, an increase in the risk-free interest rate $r$ makes waiting costlier for the potential investor.

A summary of the timing stages of the integrated case is given in Figure 1.

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11 This is a standard assumption. See e.g. Dixit and Pindyck (1994, pp. 138).

12 For the calculation of expected present values, see Dixit and Pindyck (1994, pp. 315-316).
3.3 Separated case

Suppose now that the upstream firm \( U \) produces the input. Following the presentation of Billette de Villemeur et al. (2014), we assume for simplicity that the potential investor \( P \) is a price-taker in the input market. In this case \( U \) chooses the input price \( p \) and a vertical distortion between \( P \) and \( U \) arises.

In order to derive the optimal input price and the optimal investment trigger under separation, we begin with the optimization problem of \( P \).

The value of the option to invest for \( P \) is

\[
F_s (x; p) = \max_{X(p)} (X(p) - p) \left( \frac{x}{X(p)} \right)^{\beta}
\]

where \( X(p) \geq x \). Solving, we obtain \( X_s (p) = \frac{\beta}{\beta-1} p \).\(^{13}\) Of course this is reminiscent of the optimal investment threshold in the integrated case, \( X^* (I) = \frac{\beta}{\beta-1} I \). Actually, the only difference between the two is the sunk investment cost that \( P \) needs to pay for the investment to take place. In the integrated case, the investment cost is equal to the input production cost as this is chosen by Nature, whereas in the separated case the investment cost is the price \( p \) chosen by the input manufacturer \( U \).

Now, keeping in mind that \( P \) will invest as soon as \( X_s (p) \) is reached, \( U \) chooses the optimal price \( p \) solving:

\[
K (x; \bar{p}) = \max_{p} (\bar{p} - I) \left( \frac{x}{\bar{X}(\bar{p})} \right)^{\beta}
\]

The optimal price is then equal to \( p = \frac{\beta}{\beta-1} I, I \in \{I_l, I_h\} \) which in turn implies \( X_s = \frac{\beta}{\beta-1} X^* \).\(^{14}\)

In this case, the ex-post value of the option to invest for \( P \) is \( F_s (x; I) = \left( \frac{\beta-1}{\beta} \right)^{\beta-1} F (x; I) \) whereas for \( U \) we have \( K (x; I) = \left( \frac{\beta}{\beta-1} \right)^{\beta-1} F (x; I) \) where, again, \( I \in \{I_l, I_h\} \). Similarly, the ex-ante option values for \( P \) and \( U \) are \( f_s (x; I_l, I_h) = \left( \frac{\beta-1}{\beta} \right)^{\beta-1} f (x; I_l, I_h) \) and \( k (x; I_l, I_h) = \left( \frac{\beta-1}{\beta} \right)^{\beta} f (x; I_l, I_h) \) respectively. Note that \( f_s (x; I_l, I_h) = \frac{\beta}{\beta-1} k (x; I_l, I_h) \) and that \( \lim_{\beta \to \infty} f_s (x; I_l, I_h) = k (x; I_l, I_h) \). In words, the value of the option to invest is shared in the separated case between \( P \) and \( U \), with the lion’s share going always to the holder of the investment option \( P \). The difference between \( f_s (x; p_l, p_h) \) and \( k (x; p_l, p_h) \) becomes negligible only when the value of the option to postpone the investment reaches its minimum, i.e., when \( \beta \) becomes very large and, consequently, when the wedge \( \beta/\beta - 1 \) tends to unity.

A summary of the timing stages of the separated case is given in Figure 2.

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\(^{13}\)The subscript "s" stands for "separated".

\(^{14}\)As already stressed in subsection 3.1, \( U \) is assumed to know only the structural parameters of Eq. (1). Note that if we relax this assumption allowing for an upstream supplier who can continuously and verifiably observe the state of \( X_t \), then \( U \) can choose the input price so that s/he appropriates all the benefits above a reservation value chosen for the potential investor. We present this case in subsection 7.2 below.
There are two important points to be made here.

First, the vertical distortion results in a more expensive investment \((p > I)\) which is realized, in expected terms, later \((X_s > X^*)\). This change in the sunk cost of the investment is also reflected in the aggregate ex-ante value of the investment option which is lower than the corresponding value in the integrated case: \(f_s(x; I_t, I_h) + k(x; I_t, I_h) < f(x; I_t, I_h)\).

Second, due to Assumption 1, in the separated case it is \(U\), not \(P\), the party that is observing the true magnitude of \(I\). However, the buyer of the input \((P\) in the separated case) can infer the true magnitude of \(I\) \((I_h\) or \(I_t)\) even if s/he cannot observe it directly. More precisely, when \(U\) chooses \(p_t = \frac{\beta}{\beta-1} I_t\), the buyer of the input can infer that \(I\) is \(I_t\) whereas when \(U\) chooses \(p_h = \frac{\beta}{\beta-1} I_h\), the buyer of the input can infer that \(I\) is \(I_h\).

The following remark summarizes this point:

**Remark 1** Keeping in mind the distribution of \(I\) and knowing that \(p = \frac{\beta}{\beta-1} I\), the buyer of the needed input can infer the (unobservable) true magnitude of \(I\) as soon as s/he observes the magnitude of \(p\).

The importance of Remark 1 will become obvious in the next section.\(^{15}\)

### 3.4 Delegation

Up to now, we assumed that the owner of the firm, \(P\), is responsible for the completion of the investment. Departing now from the analysis presented in Grenadier and Wang (2005), we suppose that \(P\) delegates the investment decision to an agent \(A\) who can make the right timing decision, given that \(P\) provides her/him with all the needed resources.\(^{16}\) The following assumptions describe exactly the relationship between the principal \(P\) and the agent \(A\):

**Assumption 2:** \(A\) is delegated with the investment decision, that is, i) the purchase of the discrete input from the intermediate market and ii) the choice of the investment timing.

**Assumption 3:** \(A\) and \(P\) share the same information about \(X_t\).\(^{17}\)

Note that since \(P\) is not the producer (Assumption 1) or the buyer (Remark 1) of the input, s/he cannot infer if \(p\) (and consequently \(I\)) turns out to be high or low. Actually, the only piece of information that \(P\) has about the magnitude of the input price is what \(A\) reports. Apparently, there is an information asymmetry between \(A\), who knows the true \(p\), and \(P\), who does not. The information asymmetry between the principal and the agent results in an agency conflict since the agent \(A\) has an incentive to report \(p_h\) no matter if this is true or not, in an attempt to appropriate the positive difference \(\Delta p = p_h - p_t = \frac{\beta}{\beta-1} \Delta I > 0\), when the price of the input turns out to be \(p_t\).

The principal \(P\) might not be able to observe the true \(p\) verifying the agent’s (dis)honesty and the truthfulness of her/his report, but s/he can induce \(A\) to reveal the true magnitude of the input price by giving a bonus-incentive. In order to do so, \(P\) designs a menu of contracts contingent on the observable \(X_t\). We assume that \(P\) submits the menu of contracts to \(A\) at time zero and that the chosen contract commits the actions of the two parties at the time of the investment.\(^{18}\)

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\(^{15}\)Note that even if the buyer of the input is subject to a certain level of noise, s/he will still be able to infer correctly the magnitude of the investment cost as long as the difference between \(I_h\) and \(I_t\) is large enough.

\(^{16}\)Contrary to the analysis presented here, Grenadier and Wang (2005) assume that the needed input is produced in-house or, alternatively, is purchased by a competitive input market. The presence of the input supplier \(U\) is what distinguishes our analysis from theirs.

\(^{17}\)Note that this is information that the agent either possesses ex-ante thanks to a certain expertise, or, information that the principal is sharing voluntarily in order to facilitate coordination in the firm.

\(^{18}\)Renegotiation of the contract terms is not allowed. This assumption is justified if the contract is enforceable and if the market of the agent is competitive. For a similar treatment see Grenadier and Wang (2005).
the menu of contracts is submitted, A observes the true \( p \) and chooses the corresponding contract. Given that \( p \) can take one of two possible values, "high" \( (p_h) \) or "low" \( (p_l) \), this menu is comprised by two contracts consisting of one information rent \( (w) \) and one investment threshold \( (X_D) \) each.\(^{19}\)

A summary of the timing stages of this case is given in Figure 3.

![Figure 3: Delegation](image)

The principal’s objective is to maximize her/his ex-ante value of the investment option through the choice of the contract terms \( \{ X_D^i, w_i \} , i \in \{ l, h \} \). More precisely, the problem is formulated as:

\[
\max q \left( X_D^l - w_l - p_l \right) \left( \frac{x_{l}}{X_D^l} \right)^\beta + (1 - q) \left( X_D^h - w_h - p_h \right) \left( \frac{x_{h}}{X_D^h} \right)^\beta \tag{5}
\]

Subject to:

\[
w_l \left( \frac{x_{l}}{X_D^l} \right)^\beta \geq (w_h + \Delta p) \left( \frac{x_{h}}{X_D^h} \right)^\beta \tag{6}
\]

\[
w_h \left( \frac{x_{h}}{X_D^h} \right)^\beta \geq (w_l - \Delta p) \left( \frac{x_{l}}{X_D^l} \right)^\beta \tag{7}
\]

\[
w_l \geq 0 \tag{8}
\]

\[
w_h \geq 0 \tag{9}
\]

\[
qw_l \left( \frac{x_{l}}{X_D^l} \right)^\beta + (1 - q) w_h \left( \frac{x_{h}}{X_D^h} \right)^\beta \geq 0 \tag{10}
\]

The inequalities (6) and (7) are the incentive compatibility constraints. They guarantee that if agent A observes \( p_i \), s/he will (weakly) prefer contract \( \{ X_D^i, w_i \} \) to contract \( \{ X_D^j, w_j \} \) where \( i, j \in \{ l, h \} \) and \( i \neq j \). In other words, constraints (6) and (7) guarantee that, at the time of the investment, the reported \( p \) is the true one. As one can see, an incentive compatible scheme eliminates potential incentive misalignments since both the principal and the agent are better-off when following the decision rules that are optimal for the system as a whole.

The inequalities (8) and (9) are the limited liability constraints and they are necessary to provide an incentive for the agent to get involved in the project. Finally, inequality (10) is the agent’s ex-ante participation constraint which ensures that A’s total value of accepting to abide by P’s menu of contracts is non-negative.

Solving the problem (5)-(10) we obtain the following:

**Proposition 2** The optimal menu of contracts is as follows:

\[
\{ X_D^l (p_l), w_l (p_l, p_h) \} = \left\{ X_s (p_l), \left( \frac{X_D^l (p_l)}{X_D^h (p_l, p_h)} \right)^\beta \Delta p \right\} \tag{11.1}
\]

\[
\{ X_D^h (p_l, p_h), w_h (p_l, p_h) \} = \left\{ X_s (p_h) + \frac{\beta}{\beta - 1} q \Delta p, 0 \right\} \tag{11.2}
\]

\(^{19}\)The subscript "D" stands for "delegation".
Proof. Available in Section A.1 of Appendix A. ■

Note that, on one hand, \(X_D^I(p_l) = X_s(p_l)\) and that, on the other, \(X_D^h(p_l, p_h) > X_s(p_h)\). At the same time, we have \(w_l(p_l, p_h) > 0\) and \(w_h(p_l, p_h) = 0\). In words, the attempt of \(P\) to solve the agency conflict through the choice of the optimal menu of contracts, represents a trade-off between timing efficiency and the information rent that must be paid to \(A\). When \(p_l\) happens to be the true input price, the principal is willing to pay an information rent equal to \(w_l(p_l, p_h)\) to the agent in order to make sure that the investment will take place as soon as the optimal investment threshold \((X_s(p_l))\) is reached. On the contrary, when \(p_h\) turns out to be the true input price, the principal is better off by allowing a time distortion \((X_D^h(p_l, p_h) > X_s(p_h))\) instead of paying a positive information rent. Given the menu of contracts from Proposition 2, the expected investment option values for \(P\), \(A\) and \(U\) are respectively equal to:

\[
f_D(x; p_l, p_h) = q \left( X_D^I(p_l) - p_l - w_l(p_l, p_h) \right) \left( \frac{x}{X_D^I(p_l)} \right) \beta + (1-q) \left( X_D^h(p_l, p_h) - p_h \right) \left( \frac{x}{X_D^h(p_l, p_h)} \right) \beta
\]

\[
z_D(x; p_l, p_h) = q w_l(p_l, p_h) \left( \frac{x}{X_D^I(p_l)} \right) \beta
\]

\[
k_D(x; p_l, p_h) = q (p_l - I_h) \left( \frac{x}{X_D^I(p_l)} \right) \beta + (1-q) (p_h - I_h) \left( \frac{x}{X_D^h(p_l, p_h)} \right) \beta
\]

4 Losses attributed to the agency conflict

Focusing on the value of the option to invest for \(P\) and \(A\), we define the firm’s loss stemming from agency issues as \(L = f_s - (f_D + z_D)\):

\[
L = (1-q) \left[ (X_s(p_h) - p_h) \left( \frac{x}{X_s(p_h)} \right) \beta - (X_D^h(p_l, p_h) - p_h) \left( \frac{x}{X_D^h(p_l, p_h)} \right) \beta \right]
\]

As expected, despite the fact that \(P\) chooses the optimal menu of contracts eventually solving the information asymmetry between her/him and \(A\), the agency conflict still proves costly both for \(P\) and for the firm as a whole. This result is actually driven by two opposing forces. On one hand, because of the agency conflict, the firm cashes the larger payout \((X_D^h(p_l, p_h) - p_h)\), instead of the lower \((X_s(p_h) - p_h)\). However, at the same time, the discount factor is lower \((x/X_s(p_h))^{\beta} > (x/X_D^h(p_l, p_h))^{\beta}\) which in turn increases \(L\). Obviously, the second effect dominates the first one since \((X_D^h(p_l, p_h) > X_s(p_h))\) and \(X_s(p_h) = \text{arg max} [(X - p_h)(x/X)^{\beta}]\).

\(L\) qualifies as a measure of inefficiency but it does not capture all the losses that can be attributed to the agency conflict since it does not account for the effect on the input supplier \(U\). Defining \(U\)’s loss as \(\Lambda = k - k_D\) we have:

\[
\Lambda = (1-q) (p_h - I_h) \left[ \left( \frac{x}{X_s(p_h)} \right)^{\beta} - \left( \frac{x}{X_D^h(p_l, p_h)} \right)^{\beta} \right]
\]

Note that \(L\) and \(\Lambda\) are different in their nature. On one hand, the positive difference \(L\) captures the cost of employing a bonus-incentive mechanism that is guaranteeing information symmetry between
Contrary to L, the term $\Lambda$ constitutes a deadweight loss since $A$ does not benefit from it in any way. $\Lambda$ is actually reflecting the inability of $U$ to anticipate the agency conflict between $P$ and $A$ and, eventually, the use of the bonus-incentive mechanism. Note in fact that $U$ does not recalibrate $p_h$ to account for the difference between $X^h_D(p_l,p_h)$ and $X^s(p_h)$. This is actually why the suboptimal trigger $X^h_D(p_l,p_h) > X^s(p_h)$ is affecting the term $\Lambda$ just through the lower discount factor $(x/X^h_D(p_l,p_h))^{\beta}$.

The following proposition summarizes this point:

**Proposition 3** The agency conflict between $P$ and $A$, results in a negative externality that affects the input supplier $U$. This adverse effect, is captured by the positive difference $\Lambda$.

A comparison between $\Lambda$ and $L$ suggests the following:

**Proposition 4** The inequality $I_h \geq \frac{\Delta}{1-q} \beta I$ is a sufficient condition for $\Lambda > L$.

**Proof.** Available in Section A.3 of the Appendix.

Obviously under certain conditions, and definitely when $I_h \geq \frac{\Delta}{1-q} \beta I$, the negative externality reflected in $\Lambda$ is not merely a part, but rather the main component, of the aggregate loss that stems from the agency conflict. Note that this condition captures the severity of the agency conflict between $P$ and $A$ and, consequently, the importance of the use of the bonus-incentive mechanism for $P$. For instance, a high $q$ suggests that it is highly probable that $p_l$ is the true input price which in turn suggests that it is highly probable that $A$ will choose contract (11.1). Since in this case there is no timing distortion, $U$ will likely be unaffected by the incentive misalignment between $P$ and $A$. As a result, high values of $q$ suggest that $P$ stands to lose more than $U$ from the agency conflict. The term $\Delta I$ affects the inequality $I_h \geq \frac{\Delta}{1-q} \beta I$ in a similar way. If the difference $\Delta I$ is very small, the principal can use the menu of contracts from Proposition 2 and effectively hedge almost completely against a possible misreport from $A$. However, since the upstream firm fails to anticipate the agency conflict downstream, even a small $\Delta I$ will still result in a substantially large negative externality. Last, note that an increase in $\beta$ tightens the aforementioned inequality. As we discussed in Section 3, the term $\beta$ captures the importance of the uncertainty and the irreversibility characterizing the investment. A comparison between $\Lambda$ and $L$ makes sense only as soon as the postponement of the investment is valuable for $P$ and $A$, i.e., as soon as $\beta$ is relatively small.

## 5 Delegation with auditing

Shibata (2009) extends the agency model developed by Grenadier and Wang (2005) introducing an audit technology. In this section, we do the same extending the analysis presented in Section 3 by examining how the employment of an audit technology affects the total loss attributed to the agency conflict.

We assume that if $P$ incurs a cost $c(\rho)$ then s/he can observe the true input price with probability $\rho \in [0, 1]$. As is standard in the literature, we assume $c(0) = 0$, $c_0 > 0$, $c_0'' > 0$ and $\lim_{\rho \to 1} c(\rho) = \infty$. The first assumption suggests that the technology is costly only when it is used. The second and the third assumption imply that the cost function is strictly increasing and convex in the probability of auditing. The final assumption suggests that complete auditing is prohibitively expensive.

As before, $P$ designs a menu of contracts contingent on the observable $X_t$. Each contract includes now an investment trigger $X$, a bonus-incentive $w$, a probability of auditing $\rho$ and a

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20 For simplicity, we assume that the probability of auditing is equal to the probability of detecting.
penalty $\omega$ which is paid by $A$ in case s/he is detected to be misreporting. Let subscript $\Delta$ refer to the optimum in the agency problem with auditing.

The agency problem with auditing is to maximize $P$’s ex-ante option value through the proper choice of investment triggers, information rents, auditing probabilities and penalties. The optimization problem is as follows:

$$\max q \left( X^l_\Delta - w_{\Delta l} - p_l - c(p_l) \right) \left( \frac{x}{X^l_\Delta} \right)^\beta + (1 - q) \left( X^h_\Delta - w_{\Delta h} - p_h - c(p_h) \right) \left( \frac{x}{X^h_\Delta} \right)^\beta \quad (17)$$

Subject to:

$$w_{\Delta l} \left( \frac{x}{X^l_\Delta} \right)^\beta \geq \left( w_{\Delta h} + \Delta p - p_h \omega_t \right) \left( \frac{x}{X^h_\Delta} \right)^\beta \quad (18)$$

$$w_{\Delta h} \left( \frac{x}{X^h_\Delta} \right)^\beta \geq \left( w_{\Delta l} - \Delta p - p_l \omega_h \right) \left( \frac{x}{X^l_\Delta} \right)^\beta \quad (19)$$

$$w_{\Delta l} \geq 0 \quad (20)$$

$$w_{\Delta h} \geq 0 \quad (21)$$

$$qw_{\Delta l} \left( \frac{x}{X^l_\Delta} \right)^\beta + (1 - q)w_{\Delta h} \left( \frac{x}{X^h_\Delta} \right)^\beta \geq 0 \quad (22)$$

$$\omega_t \leq w_{\Delta h} + \Delta p \quad (23)$$

$$\omega_h \leq w_{\Delta l} - \Delta p \quad (24)$$

$$1 \geq p_l \geq 0 \quad (25)$$

$$1 \geq p_h \geq 0 \quad (26)$$

$\omega_i$ is the penalty that $A$ is paying when detected to be wrongfully announcing $p_j$ and $\rho_i$ is the probability of auditing $A$ when s/he announces $p_i$, where $i, j \in \{l, h\}$ and $i \neq j$.

Constraints (18) and (19) are the ex-post incentive compatibility constraints and, similarly to constraints (6) and (7), they guarantee that if $A$ observes $p_i$, s/he will (weakly) prefer contract $\{X^i_\Delta, w_{\Delta i}, p_i, \omega_i\}$ to contract $\{X^j_\Delta, w_{\Delta j}, p_j, \omega_j\}$ where again $i, j \in \{l, h\}$ and $i \neq j$. In other words, constraints (18) and (19) guarantee that, at the time of the investment, the reported $p$ is the true one.

Constraints (20) and (21) are the limited liability constraints and, similarly to constraints (8) and (9), they are necessary to provide an incentive for $A$ to get involved in the project. Constraint (22) is $A$’s ex-ante participation constraint. Constraints (23) and (24) are the ex-post penalty constraints and they guarantee that the penalty cannot be greater than the benefit from misreporting. Last, constraints (25) and (26) need to hold as soon as $p_l$ and $p_h$ are probabilities.

Solving the problem (17)-(26) we obtain the following:

**Proposition 5** If $c'(0) \leq \frac{q}{1 - q} \Delta p$, the optimal menu of contracts under auditing is as follows:

$$\left\{ X^l_\Delta \left( p_l \right) , w_{\Delta l} \left( p_l, p_h \right) , p_l, \omega_l \right\} = \left\{ \frac{\beta}{\beta - 1} p_l \Delta p \left( 1 - p_h \right) \left( \frac{X^l_\Delta}{X^h_\Delta} \right)^\beta , 0, \Delta p \right\} \quad (27.1)$$

$$\left\{ X^h_\Delta \left( p_l, p_h \right) , w_{\Delta h} \left( p_l, p_h \right) , p_h, \omega_h \right\} = \left\{ \frac{\beta}{\beta - 1} \left[ p_h + c(p_h) + \frac{q}{1 - q} \Delta p \left( 1 - p_h \right) \right], 0, c' \left( \frac{q}{1 - q} \Delta p \right) \right\} \quad (27.2)$$
Proof. Available in Section B of the Appendix.

The condition \( c'(0) \leq \frac{q}{1-q} \Delta p \) guarantees that \( P \) is better off by using the audit technology. Otherwise, if \( c'(0) > \frac{q}{1-q} \Delta p \), auditing is too expensive and \( P \) uses the menu of contracts presented in Proposition 2. Note that the analysis presented in this section, collapses to the bonus-incentive-only case as this is presented in subsection 3.4 when \( c'(0) > \frac{q}{1-q} \Delta p \) and consequently, \( \rho_h = 0 \).

A comparison of the investment triggers gives \( X^I_{\Delta} (p_l) = X^I_D (p_l) = X_s (p_l) \) and \( X_s (p_h) < X^h_{\Delta} (p_l, p_h) < X^h_D (p_l, p_h) \). In words, the use of the audit technology does not affect the optimal timing when \( p_l \) turns out to be the true input price, but it does when we have \( p_h \). Actually the audit technology reduces the timing distortion when \( p_h \) turns out to be the input price since \( X^h_{\Delta} (p_l, p_h) < X^h_D (p_l, p_h) \). Obviously, there is a trade-off for the principal who is willing to pay the cost of auditing, \( c(\rho_p) > 0 \), in order to reduce this timing distortion.

The expected investment option values for \( P, A \) and \( U \) are respectively equal to:

\[
f_{\Delta} (x; p_l, p_h) = q \left( X^I_{\Delta} (p_l) - w_D - p_l \right) \left( \frac{x}{X^I_{\Delta} (p_l)} \right)^{\beta} + (1 - q) \left( X^h_{\Delta} (p_l, p_h) - p_h - c(\rho_h) \right) \left( \frac{x}{X^h_{\Delta} (p_l, p_h)} \right)^{\beta}
\]

\[
z_{\Delta} (x; p_l, p_h) = q w_D \left( \frac{x}{X^I_{\Delta} (p_l)} \right)^{\beta}
\]

\[
k_{\Delta} (x; p_l, p_h) = q (p_l - I_l) \left( \frac{x}{X^I_{\Delta} (p_l)} \right)^{\beta} + (1 - q) (p_h - I_h) \left( \frac{x}{X^h_{\Delta} (p_l, p_h)} \right)^{\beta}
\]

By construction, and as soon as \( c'(0) \leq \frac{q}{1-q} \Delta p \), we have \( f_{\Delta} (x; p_l, p_h) > f_D (x; p_l, p_h) \) and \( z_{\Delta} (x; p_l, p_h) < z_D (x; p_l, p_h) \). Obviously the audit technology always increases the principal’s ex-ante option value and decreases the manager’s ex-ante option value. As far as the ex-ante option value of the upstream firm is concerned, and thanks to \( X_s (p_h) < X^h_{\Delta} (p_l, p_h) < X^h_D (p_l, p_h) \), we have \( k (x; p_l, p_h) > k_{\Delta} (x; p_l, p_h) > k_D (x; p_l, p_h) \). Consequently, \( \Lambda_{\Delta} < \Lambda \) where \( \Lambda_{\Delta} = k - k_{\Delta} \). Apparently, the mechanism that the principal uses to resolve the agency conflict (i.e., bonus-incentive mechanism or audit technology) dictates how harmful the negative externality that \( U \) endures because of the agency conflict downstream really is. Since the use of an audit technology reduces the timing distortion, the upstream firm is better off when the principal employs such a technology. The following proposition summarizes this point.

Proposition 6 The negative externality that is stemming from the agency conflict between the principal and the agent is weaker when an audit technology complements the bonus-incentive mechanism.

Summing up, the use of an audit technology has an advantage over the use of a bonus-incentive mechanism since it reduces the timing distortion attributed to the agency conflict and, through that, the negative externality that stems from the agency conflict.

\( ^{21} \)The results are in line with what Shibata (2009) finds for the case where \( U \) is absent.
6 Numerical examples

6.1 Delegation with the use of a bonus-incentive mechanism

Suppose that the parameters are as follows: \( q = 0.5, \quad \sigma = 0.2, \quad r = 0.07, \quad \mu = 0.03, \quad I_l = 50 \) and \( I_h = 80 \). Table 1 below shows the results for the integrated case, the separated case and the case with delegation when a bonus-incentive mechanism is used. Note that for \( x = 100 \), the optimal strategy is to delay the investment until the state variable reaches the optimal trigger.

<table>
<thead>
<tr>
<th></th>
<th>Integrated Case</th>
<th>Separated Case</th>
<th>Delegation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_l )</td>
<td>128.44</td>
<td>329.91</td>
<td>329.91</td>
</tr>
<tr>
<td>( X_h )</td>
<td>205.49</td>
<td>527.86</td>
<td>725.82</td>
</tr>
<tr>
<td>( w_l )</td>
<td>-</td>
<td>-</td>
<td>21.19</td>
</tr>
<tr>
<td>( w_h )</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>( p_l )</td>
<td>-</td>
<td>128.43</td>
<td>128.43</td>
</tr>
<tr>
<td>( p_h )</td>
<td>-</td>
<td>205.49</td>
<td>205.49</td>
</tr>
<tr>
<td>( f )</td>
<td>45.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( f_s )</td>
<td>-</td>
<td>24.84</td>
<td>-</td>
</tr>
<tr>
<td>( k )</td>
<td>-</td>
<td>9.67</td>
<td>-</td>
</tr>
<tr>
<td>( f_D )</td>
<td>-</td>
<td>-</td>
<td>22.89</td>
</tr>
<tr>
<td>( z_D )</td>
<td>-</td>
<td>-</td>
<td>1.50</td>
</tr>
<tr>
<td>( k_D )</td>
<td>-</td>
<td>-</td>
<td>7.99</td>
</tr>
<tr>
<td>Aggr. Value</td>
<td>45.32</td>
<td>34.51</td>
<td>32.38</td>
</tr>
<tr>
<td>( L = f_s - (f_D + z_D) )</td>
<td>-</td>
<td>-</td>
<td>0.44</td>
</tr>
<tr>
<td>( \Lambda = k - k_D )</td>
<td>-</td>
<td>-</td>
<td>1.67</td>
</tr>
</tbody>
</table>

As expected, we find \( X_l^D(p_l) = X_s(p_l) > X^*(I_l) \) and \( X_h^D(p_l, p_h) > X_s(p_h) > X^*(I_h) \). The information rent when \( I_h \) realizes is equal to zero \( (w_h = 0) \) whereas it is positive and equal to \( w_l = 21.19 \) when \( I_l \) is the true investment cost. We also find that \( p_i > I_i, \) where \( i \in \{l, h\} \). As one can see, the difference is substantial since \( p_l/I_l \approx 2.56 \). As for the ex-ante value of the option to invest, we verify that the delegation of the investment decision to \( A \) makes both \( P \) and \( U \) worse off \((f > f_s > f_D \) and \( k > k_D)\), a result which is also reflected in the aggregate value of the option to invest. Last, we find that \( \Lambda \) is clearly larger than \( L \) which underlines the importance of the negative externality described in Section 4. Actually, since \( \beta = 1.63, \quad \frac{q}{1-q} = 1, \quad \Delta I = 30 \) and \( I_h = 80 \), we see that in this particular case the sufficient condition \( \frac{q}{1-q} \beta \Delta I \leq I_h \) is satisfied. As one can see in Figure 4, for \( q = 0.5, \beta = 1.63 \) and \( I_l = 50 \), the sufficient condition holds as soon as \( I_h \leq 128.43 \) which is out of the range \([50, 80]\) that we consider above.\(^{22}\) If instead we have \( q = 0.9 \), we obtain Figure 5. The sufficient condition in this case holds for any \( I_h \leq 53.64 \), but \( \Lambda > L \) for any \( I_h \leq 68.4 \).

\(^{22}\) We use the same values as Shibata (2009).

\(^{23}\) Actually even if \( I_h > 129.36 \), i.e., if the sufficient condition does not hold anymore, we still have \( \Lambda > L \) as soon as \( q = 0.5, \beta = 1.63 \) and \( I_l = 50 \).
6.2 Delegation with the use of an audit technology

Suppose that the principal can use an audit technology and that the relevant cost function is \( c(\rho_i) = 5 \left( 1 - \rho_i^2 \right), \ i = \{l, h\} \).\textsuperscript{24} Given that \( q = 0.5, \sigma = 0.2, \ r = 0.07, \mu = 0.03, \ I_l = 50, \ I_h = 80 \) and \( x = 100 \) we have \( \{X^l, w^l, \rho^l, \omega^l\} = \{329.91, 4.73, 0, 30\} \) and \( \{X^h, w^h, \rho^h, \omega^h\} = \{569.10, 0, 0.85, 0\} \). As expected, we have \( X^l = X^l_D = X_s \) and \( X_s < X^h < X^h_D \). From Eq. (28)-(30) we also have: \( f^\Delta = 24.35, \ z^\Delta = 0.34 \) and \( k^\Delta = 9.19 \). Note that as expected, \( f^\Delta > f_D, \ z^\Delta < z_D \) and \( k^\Delta > k_D \). The upstream firm is clearly better off when the bonus-incentive mechanism is complemented by an audit technology. Last, we have \( \Lambda^\Delta = 0.48 \) which of course is smaller than \( \Lambda = 1.67 \). Of course, this difference in the measure of the negative externality is attributed to the reduced timing distortion \( X^h < X^h_D \).

\textsuperscript{24}We use the cost function presented by Shibata (2009). Note that this cost function satisfies \( c'(0) = 0 \) which means that we obtain an interior solution, i.e., \( \rho^l_h > 0 \).
7 Delegation under supply chain transparency

Our analysis up to now is based on the assumption that $U$ is pricing the discrete input à la Billette de Villemeur et al. (2014), that is, $U$ has no information about the structure of the downstream industry. In this section we relax this assumption and we discuss traceability and transparency in the supply chain.

7.1 The case of traceability

A supply chain is characterized by traceability when the names of the firms involved in the supply chain are disclosed to the other firms in the supply chain as well as to end-users (Doorey, 2011 and Laudal, 2010).

In our setting, traceability implies that:

i) $U$ knows the structure of the downstream industry, that is, s/he knows that $P$ is the owner (principal) whereas $A$ is the manager (agent) and,

ii) $P$ knows that $U$ is the supplier of the necessary input.

Reapproaching the problem from Section 4, we have the following. The input supplier $U$, observing the delegation of the investment decision downstream, anticipates that the agency conflict will result in a loss equal to $\Lambda_{\Delta}$ or even $\Lambda$, depending on whether $P$ can use an audit technology or not. Of course $U$ can prevent that from happening by sharing the true price of the input with $P$. This way, the input supplier makes sure that there is information symmetry downstream and that, consequently, the principal does not need to use a mechanism to resolve the agency conflict. Actually the analysis under traceability coincides with the analysis for the separated case as this was presented in subsection 3.3.

As one can notice, traceability in the supply chain has a dual effect: Firstly, it is by definition resolving any relevant information asymmetries and, secondly, it is internalizing the negative externality described in Proposition 3.
7.2 The case of full transparency

In order to underline the importance of transparency/opacity in the supply chain, we now discuss a supply chain characterized by traceability assuming also that $P$, $A$ and $U$ all share the same information about the stochastic parameter, that is, they can all continuously and verifiably observe its realizations.

$P$ would be willing to share private information about $X_t$ with $U$ under the condition that $s/he$ receives a reservation value not smaller than $\Omega_i \equiv F_s (x; I_i) = (\frac{\beta - 1}{\beta})^{\beta - 1} F (x; I_i), i \in \{l, h\}$. This way, and by dictating the investment threshold $X^*$, $A$ can appropriate all the benefits above $P$’s reservation value. Keeping this in mind, $U$ is choosing the input price solving

$$\max_{\phi_i} (\varphi_i - I_i) \left( \frac{x}{X^*(I_i)} \right)^{\beta}$$

subject to

$$(X^*(I_i) - \varphi_i) \left( \frac{x}{X^*(I_i)} \right)^{\beta} \geq \Omega_i, i \in \{l, h\}. \tag{32}$$

The term $\varphi_i$ stands for the (new) price of the input. Since the objective function in problem (31) is increasing in $\varphi_i$, the solution is derived from the constraint (32). A binding constraint (32) implies that, $\varphi_i$ is such that $P$ is indifferent between an investment that costs $\varphi_i$ and takes place when $X^*(I_i)$ is reached, and an investment that costs $p_i$ and takes place when $X_s (p_i) (> X^*(I_i))$ is reached. Solving we obtain

$$\varphi_i = \frac{\beta}{\beta - 1} I_i \left( 1 - \frac{(\beta - 1)^{\beta - 1}}{\beta^\beta} \right), i \in \{l, h\}. \tag{33}$$

The input supplier $U$, chooses $\varphi_i (> I_i)$ at $X^*(I_i)$ submitting a take-it-or-leave-it offer to the principal $P$. In this case, the principal’s ex-ante value of the investment opportunity is:

$$f_T (x; \varphi_l, \varphi_h) = q (X^*(I_l) - \varphi_l) \left( \frac{x}{X^*(I_l)} \right)^{\beta} + (1 - q) (X^*(I_h) - \varphi_h) \left( \frac{x}{X^*(I_h)} \right)^{\beta} \tag{34}$$

The ex-ante value of the investment opportunity for $U$ is given by:

$$k_T (x; \varphi_l, \varphi_h) = q (\varphi_l - I_l) \left( \frac{x}{X^*(I_l)} \right)^{\beta} + (1 - q) (\varphi_h - I_h) \left( \frac{x}{X^*(I_h)} \right)^{\beta} \tag{35}$$

Finally, combining the two, the aggregate ex-ante value is equal to $f_T (x; \varphi_l, \varphi_h) + k_T (x; \varphi_l, \varphi_h) = f (x; I_h, I_l)$. This, of course, is to be expected since $U$ dictates the investment thresholds that maximize the industry value.

8 Epilogue

This paper contributes to a growing research area that integrates the theory of irreversible investment under uncertainty and the literature on asymmetric information and agency conflicts. According to this body of papers, when an investment project that is characterized by uncertainty

\[25\text{See Billette de Villemeur et al. (2014) for a similar treatment.}\]

\[26\text{The subscript "T" stands for "transparency".}\]
and irreversibility is undertaken in a decentralized setting, the information asymmetry between the principal and the agent will lead to an agency conflict. This results in the postponement of the investment and in the reduction of the value of the investment opportunity.

In this paper we examine how the analysis changes if the investment is conditional on the provision of an indispensable input that is produced by an upstream firm with market power. We show that as soon as the supply chain is opaque, i.e., the input supplier cannot anticipate the agency conflict downstream, s/he is due to endure a deadweight loss attributed to the agency conflict between the principal and the agent downstream. In other words, we show that the total loss attributed to the agency conflict has two components. On one hand, the loss in the decentralized firm itself and, on the other, the negative externality for the input supplier. We prove that the latter is larger than the former under certain conditions and we show that the mechanism that the principal employs in order to resolve the agency conflict affects the magnitude of the negative externality. More precisely, we show that the use of an audit technology, instead of a bonus-incentive mechanism, reduces the timing distortion and, consequently, the negative externality. This paper has limitations that can be addressed in future work. For instance, it would be interesting to generalize this model so that it takes into consideration a larger supply chain with more firms both upstream and downstream. Such an analysis would provide a better approximation of the total loss that is to be attributable to agency conflicts.
A Appendix

A.1 The menu of contracts

Under information asymmetry and in-house production of the discrete input, $P$ solves the following problem:

$$\max_{\{(X^l_D, w_l), (X^h_D, w_h)\}} q \left( X^l_D - w_l - p_l \right) \left( \frac{x}{X^l_D} \right)^\beta + (1 - q) \left( X^h_D - w_h - p_h \right) \left( \frac{x}{X^h_D} \right)^\beta$$  \hspace{1cm} (A.1)

Subject to:

$$w_l \left( \frac{x}{X^l_D} \right)^\beta \geq (w_h + \Delta p) \left( \frac{x}{X^h_D} \right)^\beta$$  \hspace{1cm} (A.2)

$$w_h \left( \frac{x}{X^h_D} \right)^\beta \geq (w_l - \Delta p) \left( \frac{x}{X^l_D} \right)^\beta$$  \hspace{1cm} (A.3)

$$w_l \geq 0$$  \hspace{1cm} (A.4)

$$w_h \geq 0$$  \hspace{1cm} (A.5)

$$qw_l \left( \frac{x}{X^l_D} \right)^\beta + (1 - q) w_h \left( \frac{x}{X^h_D} \right)^\beta \geq 0$$  \hspace{1cm} (A.6)

Working with constraints (A.2) and (A.5) we have:

$$w_l \left( \frac{x}{X^l_D} \right)^\beta \geq (w_h + \Delta p) \left( \frac{x}{X^h_D} \right)^\beta \geq \Delta p \left( \frac{x}{X^h_D} \right)^\beta > 0$$

$$\rightarrow \quad w_l > 0$$

Consequently, constraint (A.4) and constraint (A.6) are slack. This allows us to solve problem (A.1) only subject to constraints (A.2), (A.3) and (A.5). Setting constraint (A.3) aside for now, the Lagrangian is

$$Z = \left( X^l_D - w_l - p_l \right) \left( \frac{x}{X^l_D} \right)^\beta + \frac{1 - q}{q} \left( X^h_D - w_h - p_h \right) \left( \frac{x}{X^h_D} \right)^\beta$$

$$+ \lambda_1 \left[ w_l \left( \frac{x}{X^l_D} \right)^\beta - (w_h + \Delta p) \left( \frac{x}{X^h_D} \right)^\beta \right]$$

$$+ \lambda_2 w_h,$$  \hspace{1cm} (A.7)

where $\lambda_1$ is the Lagrangian multiplier that corresponds to constraint (A.2) and $\lambda_2$ is the Lagrangian multiplier that corresponds to constraint (A.5).

Now, keeping in mind the complementary slackness conditions for the two constraints, we maximize the Lagrangian with respect to $X^l_D, X^h_D, w_l$ and $w_h$. The first-order conditions with respect to $w_l$ and $w_h$ give $\lambda_1 = 1$ and $\lambda_2 = \left( \frac{1 - q}{q} + \lambda_1 \right) \left( \frac{x}{X^h_D} \right)^\beta > 0$ respectively. This means that both the incentive compatibility condition (A.2) and the limited liability condition (A.5) are binding, i.e.,

$$w_h = 0$$  \hspace{1cm} (A.8)
and

\[ w_l = \left( \frac{X_D^l}{X_D^h} \right)^\beta \Delta p. \]  \hspace{1cm} (A.9)

Given these, the first-order conditions with respect to the investment thresholds \( X_D^l \) and \( X_D^h \) result in:

\[ X_D^l (p_l) = \frac{\beta}{\beta - 1} p_l \]  \hspace{1cm} (A.10)

\[ X_D^h (p_l, p_h) = \frac{\beta}{\beta - 1} \left( p_h + \frac{q}{1 - q} \Delta p \right) \]  \hspace{1cm} (A.11)

One can easily show that the derived solutions satisfy the constraint (A.3) comprising the menu of contracts that \( P \) submits to \( A \).

### A.2 The investment cost as a continuous variable

In the main body of the paper we use a two-point distribution for the in-house production cost \( I \). Here we generalize allowing for a continuum of different levels of \( I \) in the interval \([I_l, I_h]\). Let \( g(I) \) and \( G(I) \) be the density and the cumulative distribution of \( I \) respectively. The interval \([I_l, I_h]\) is the support and, consequently, \( G(I_l) = 0 \) and \( G(I_h) = 1 \). As in the main body of the paper, we first analyze the case with in-house production of the input, we then discuss the separated case and last we allow for delegation of the investment decision.

#### A.2.1 The integrated case

The optimization problem that \( P \) needs to solve is given by\(^{27}\)

\[ \max \left\{ \int_{I_l}^{I_h} (X^* - I) \left( \frac{x}{X^*} \right)^\beta dG(I) \right\}. \]  \hspace{1cm} (A.12)

Solving pointwise we obtain:

\[ X^* (I) = \frac{\beta}{\beta - 1} I, \text{ for any } I \in [I_l, I_h] \]  \hspace{1cm} (A.13)

#### A.2.2 The separated case

The optimization problem that \( P \) needs to solve in this case is given by\(^{28}\)

\[ \max \left\{ \int_{p_l}^{p_h} (X_s - p) \left( \frac{x}{X_s} \right)^\beta dG(p) \right\}. \]  \hspace{1cm} (A.14)

Solving again pointwise we obtain:

\[ X_s (p) = \frac{\beta}{\beta - 1} p, \text{ for any } p \in [p_l, p_h] \]  \hspace{1cm} (A.15)

The price of the input is derived as the solution of

\[ \max \int_{I_l}^{I_h} (p - I) \left( \frac{x}{X_s (p)} \right)^\beta dG(I), \]  \hspace{1cm} (A.16)

\(^{27}\)In line with the assumption according to which investing at time zero is not preferable, we assume \( x < X^* \).

\(^{28}\)Similarly to the previous footnote, we assume \( x < X_s \).
Solving, we obtain
\[ p = \frac{\beta}{\beta - 1}I, \text{ for any } I \in [I_l, I_h]. \]  
(A.17)

### A.2.3 Delegation

Following the analysis of subsection 3.4 in the main body of the paper, the principal designs a menu of contracts contingent on the observable component \( X_t \). This menu is comprised, not by two, but by a continuum of contracts, one for every \( p \in [p_l, p_h] \) (i.e., one for every \( I \in [I_l, I_h] \)). The problem that \( P \) solves is formulated as:\(^{29}\)

\[
\max \left\{ \int_{p_l}^{p_h} (X_D(p) - w(p) - p) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \right\}
\]  
(A.18)

Subject to:
\[
w(p) \left( \frac{x}{X_D(p)} \right)^\beta \geq (w(\bar{p}) + \bar{p} - p) \left( \frac{x}{X_D(\bar{p})} \right)^\beta
\]  
(A.19)
\[
\int_{p_l}^{p_h} w(p) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \geq 0, \text{ for any } \bar{p}, p \in [p_l, p_h]
\]  
(A.20)

The objective function in problem (A.18) is the ex-ante value of the opportunity to invest for the principal. The inequalities in (A.19) are the incentive compatibility constraints, the inequalities in (A.20) are the limited liability conditions and inequality (A.21) is the agent’s ex-ante participation constraint. Last, the term \( p \) stands for the true, whereas the term \( \bar{p} \) stands for the reported, price of the input.

Following the analysis from Section A.1 of the Appendix and using similar arguments we know that the constraint (A.21) is slack, whereas the constraint (A.20) gives \( w(p_h) = 0 \) and \( w(p) > 0 \) for every \( p \in [p_l, p_h] \). The problem that we need to solve is then reduced to:

\[
\max \left\{ \int_{p_l}^{p_h} (X_D(p) - w(p) - p) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \right\}
\]  
(A.18)

Subject to:
\[
w(p) \left( \frac{x}{X_D(p)} \right)^\beta \geq (w(\bar{p}) + \bar{p} - p) \left( \frac{x}{X_D(\bar{p})} \right)^\beta
\]  
(A.19)
\[
w(p_h) = 0, \text{ for any } \bar{p}, p \in [p_l, p_h]
\]  
(A.22)

Let’s now focus on the constraints in Ineq. (A.19). It is useful to recall that the information rent is defined as \( w(\bar{p}, p) = t(\bar{p}) - p, \forall \bar{p}, p \in [p_l, p_h] \) where \( t(\bar{p}) \) is the money transfer from the principal to an agent who reports \( \bar{p} \).\(^{30}\) Of course under incentive compatibility, i.e. when Ineq.(A.19) holds, we have \( \bar{p} = p \) which gives \( w(p, p) = t(p) - p \). By slightly abusing notation, \( w(p, p) \) reduces to \( w(p) \) which is the term appearing above as well as in the main body of the paper.

Now, according to Ineq. (A.19), the quantity \( (t(p) - p) \left( \frac{x}{X_D(p)} \right)^\beta \) needs to be larger than any quantity \( (t(\bar{p}) - p) \left( \frac{x}{X_D(\bar{p})} \right)^\beta, \bar{p} \neq p \). Let’s now write this using the first and the second order conditions:

\(^{29}\) As before, we assume \( x < X_D(p) \).

\(^{30}\) See e.g. Laffont and Martimort (2002).
FOC and SOC Note first that

\[
\frac{\partial}{\partial \bar{p}} \left( t(\bar{p}) - p \right) \left( \frac{x}{X_D(p)} \right)^\beta = \left( i(\bar{p}) - \beta \left( t(\bar{p}) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} \right) \left( \frac{x}{X_D(p)} \right)^\beta, \tag{A.23}
\]

where \( \frac{\partial t(\bar{p})}{\partial \bar{p}} = i(\bar{p}) \) and \( \frac{\partial X_D(\bar{p})}{\partial \bar{p}} = \dot{X}_D(\bar{p}) \). Now, given the first-order derivative from Eq. (A.23), the first-order condition gives:

\[
i(p) - \beta \left( t(p) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} = 0 \tag{A.24}
\]

where \( \left. \frac{\partial t(\bar{p})}{\partial \bar{p}} \right|_{\bar{p}=p} = i(p) \) and \( \left. \frac{\partial X_D(\bar{p})}{\partial \bar{p}} \right|_{\bar{p}=p} = \dot{X}_D(p) \). The second-order derivative is:

\[
\frac{\partial^2}{\partial \bar{p}^2} \left( t(\bar{p}) - p \right) \left( \frac{x}{X_D(\bar{p})} \right)^\beta = \left[ \left( i(\bar{p}) - \beta \left( t(\bar{p}) - p \right) \frac{\dot{X}_D(\bar{p})}{X_D(\bar{p})} + \left( t(\bar{p}) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} + \frac{\ddot{X}_D(p)}{X_D(p)^2} \right) \frac{\dot{X}_D(p)}{X_D(p)} \right] \left( \frac{x}{X_D(p)} \right)^\beta. \tag{A.25}
\]

From the second-order condition and keeping in mind Eq. (A.24) we have:

\[
i(\bar{p}) - \beta \left( i(\bar{p}) \frac{\dot{X}_D(\bar{p})}{X_D(\bar{p})} + \left( t(\bar{p}) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} - \frac{\ddot{X}_D(p)}{X_D(p)^2} \right) \leq 0 \tag{A.26}
\]

Last, from the first-order condition we have:

\[
\frac{\partial}{\partial p} \left( i(p) - \beta \left( t(p) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} \right) = 0 \tag{A.27}
\]

\[
i(p) - \beta i(p) \frac{\dot{X}_D(p)}{X_D(p)} - \beta \left( t(p) - p \right) \frac{\dot{X}_D(p)}{X_D(p)} = -\beta \frac{\dot{X}_D(p)}{X_D(p)} \frac{\ddot{X}_D(p)}{X_D(p)^2} = 0 \]

From Ineq. (A.26) and Eq. (A.27) we obtain:

\[
\dot{X}_D(p) \geq 0 \tag{A.28}
\]

This is a standard monotonicity constraint.\(^{31}\) Last, applying the envelope theorem we obtain:

\[
\frac{\partial}{\partial p} \left( t(p) - p \right) \left( \frac{x}{X_D(p)} \right)^\beta = - \left( \frac{x}{X_D(p)} \right)^\beta \tag{A.29}
\]

**Rewriting the problem** Using Ineq. (A.28) and Eq. (A.29) we can rewrite the problem in the following way:

\[
\max \left\{ \int_{p_l}^{p_h} \left( X_D(p) - w(p) - p \right) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \right\} \tag{A.18}
\]

\(^{31}\)See Chapter 2 from Laffont and Martimort (2002) for more details. One can easily check that the monotonicity holds also when \( p \) is a discrete random variable.
Subject to:

\[
\dot{X}_D (p) \geq 0 \tag{A.28}
\]

\[
\frac{\partial (t(p) - p)}{\partial p} (\frac{x}{X_D(p)})^\beta = - (\frac{x}{X_D(p)})^\beta \tag{A.29}
\]

\[
w(p_h) = 0, \text{ for any } p \in [p_l, p_h] \tag{A.22}
\]

Now, from Eq. (A.29) and Eq. (A.22) we have:

\[
(t(p) - p) (\frac{x}{X_D(p)})^\beta = \int_{p_l}^{p_h} (\frac{x}{X_D(v)})^\beta dv \tag{A.30a}
\]

\[
w(p) (\frac{x}{X_D(p)})^\beta = \int_{p_l}^{p_h} (\frac{x}{X_D(v)})^\beta dv \tag{A.30b}
\]

Using Eq. (A.30), the objective function from problem (A.18) becomes:

\[
\int_{p_l}^{p_h} \left( X_D (p) - p - \int_{p_l}^{p_h} \left( \frac{X_D(p)}{X_D(v)} \right)^\beta dv \right) \left( \frac{x}{X_D(p)} \right)^\beta dG(p)
\]

which by integration by parts gives:

\[
\int_{p_l}^{p_h} \left( X_D (p) - p - \frac{G(p)}{g(p)} \right) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \tag{A.31}
\]

Using this expression we can rewrite the problem as:

\[
\max \left\{ \int_{p_l}^{p_h} \left( X_D (p) - p - \frac{G(p)}{g(p)} \right) \left( \frac{x}{X_D(p)} \right)^\beta dG(p) \right\} \tag{A.32}
\]

subject to,

\[
\dot{X}_D (p) \geq 0 \tag{A.28}
\]

Momentarily ignoring the monotonicity constraint (A.28), we solve the maximization problem (A.32) pointwise and we obtain

\[
X_D (p) = \frac{\beta}{\beta - 1} \left( p + \frac{G(p)}{g(p)} \right), \text{ for any } p \in [p_l, p_h]. \tag{A.33}
\]

From Eq. (A.33) we see that there is no timing distortion when \( p \) takes its minimum value (since \( G(p_l) = 0 \)), whereas there is an upward distortion for any \( p \in (p_l, p_h] \).

The last thing that we need to check is under what conditions our solution respects the monotonicity constraint (A.28). From Eq. (A.33) we have

\[
\dot{X}_D (p) = \frac{\beta}{\beta - 1} \left( 1 + \frac{\partial}{\partial p} \left( \frac{G(p)}{g(p)} \right) \right). \tag{A.34}
\]

The monotone hazard rate property \( \frac{\partial}{\partial p} \left( \frac{G(p)}{g(p)} \right) \geq 0 \) is a sufficient condition for \( \dot{X}_D (p) \geq 0 \) to hold. This condition is satisfied by most parametric single-peak densities (see Bagnoli and Bergstrom, 2005).
Last, note that from Eq. (A.30) we can also derive the optimal information rent:

\[ w(p) = \int_{p}^{p_h} \left( \frac{X_D(p)}{X_D(v)} \right)^{\beta} dv, \text{ for any } p \in [p_l, p_h] \]  

(A.35)

In words, the menu of contracts designed by \( P \) is built in such a way that for any \( p \in [p_l, p_h] \) a positive information rent is to be paid. The information rent is equal to zero only when \( p \) takes its maximum value \( (w(p_h) = 0) \). As one can notice, this is symmetric to \( w_l > 0 \) and \( w_h = 0 \) from subsection 3.4 of the main body of the paper.

### A.3 The inequality \( \frac{q}{1-q} \beta \Delta I \leq I_h \) as a sufficient condition for \( \Lambda > L \)

The difference between \( \Lambda \) and \( L \) is:

\[ \Lambda - L = \left( X_D^h(p_l, p_h) - (2p_h - I_h) \right) \left( \frac{x}{X_D^h(p_l, p_h)} \right)^{\beta} - (X_s(p_h) - (2p_h - I_h)) \left( \frac{x}{X_s(p_h)} \right)^{\beta} \]

The argument that maximizes the generic term \( (X - (2p_h - I_h))(x/X)^{\beta} \) is \( \tilde{X} = \frac{\beta}{\beta - 1} \frac{\beta + 1}{\beta - 1} I_h \), which is obviously larger than \( X_s(p_h) = \frac{\beta}{\beta - 1} \frac{\beta + 1}{\beta - 1} I_h \). At the same time, one can check that \( \tilde{X} \) is also larger or equal to \( X_D^h(p_l, p_h) \) when \( \frac{q}{1-q} \beta \Delta I \leq I_h \). Since \( X_D^h(p_l, p_h) > X_s(p_h) \), the weak inequality \( \frac{q}{1-q} \beta \Delta I \leq I_h \) guarantees \( \Lambda > L \).

### B Delegation with auditing

The problem that \( P \) solves is:

\[ \max q \left( X_D^l - w \Delta_l - p_l - c(p_l) \right) \left( \frac{x}{X_D^l} \right)^{\beta} + (1 - q) \left( X_D^h - w \Delta_h - p_h - c(p_h) \right) \left( \frac{x}{X_D^h} \right)^{\beta} \]  

(B.1)

Subject to:

\[ w \Delta_l \left( \frac{x}{X_D^l} \right)^{\beta} \geq (w \Delta_h + \Delta p - \rho_l \omega_l) \left( \frac{x}{X_D^h} \right)^{\beta} \]  

(B.2)

\[ w \Delta_h \left( \frac{x}{X_D^h} \right)^{\beta} \geq (w \Delta_l - \Delta p - \rho_l \omega_h) \left( \frac{x}{X_D^l} \right)^{\beta} \]  

(B.3)

\[ w \Delta_l \geq 0 \]  

(B.4)

\[ w \Delta_h \geq 0 \]  

(B.5)

\[ qw \Delta_l \left( \frac{x}{X_D^l} \right)^{\beta} + (1 - q) w \Delta_h \left( \frac{x}{X_D^h} \right)^{\beta} \geq 0 \]  

(B.6)

\[ \omega_l \leq w \Delta_h + \Delta p \]  

(B.7)

\[ \omega_h \leq w \Delta_l - \Delta p \]  

(B.8)

\[ 1 \geq \rho_l \geq 0 \]  

(B.9)

\[ 1 \geq \rho_h \geq 0 \]  

(B.10)

---

\(^{32}\)Note that the solution presented here is totally symmetric to the one available in Shibata (2009). The only difference is that here the investment cost is the price chosen by \( U, p_h \) or \( p_l \), and not the input production cost, \( I_h \) or \( I_l \). Shibata (2009) also provides the solution for the case with a continuous distribution of \( I \).
We start with some simplifications. Constraint (B.7) is binding since, by raising the penalty \( \omega_l \) as much as possible, \( P \) can reduce the right-hand side of (B.2) making it easier to satisfy.\(^{33}\) Also since, according to (B.5), \( w_{\Delta h} \) is non-negative, and given that \( \Delta p \) is positive, we find that \( \omega_l \) is strictly positive.

By construction, when \( A \) observes \( p_l \), s/he has no incentive to misreport announcing \( p_l \). This means that constraint (B.3) and constraint (B.8) are not binding whereas at the same time we have \( \omega_h = 0 \). Note also that, since auditing comes at a cost, the principal is better off when not auditing an agent who reports \( p_l \), that is, \( \rho_l = 0 \). As for constraint (B.10), we have \( 1 > \rho_h \) since \( \lim_{\rho \to 1} \psi(\rho) = \infty \).

Constraint (B.6) is automatically satisfied from constraints (B.4) and (B.5). Suppose now that Constraint (B.5) is not binding (\( w_{\Delta h} > 0 \)). Then, \( P \) can decrease \( w_{\Delta h} \) with all other constraints, namely, (B.2) and (B.7), satisfied. Thus we have \( w_{\Delta h} = 0 \) at the optimum.

Thanks to \( w_{\Delta h} = 0 \) and \( \omega_l = \Delta p \) (binding constraint (B.7)), constraint (B.2) becomes: \( w_{\Delta l} \geq \Delta p (1 - \rho_h) \left( \frac{X^l_{\Delta}}{X^h_{\Delta}} \right)^\beta \). Suppose that this is not binding, that is, \( w_{\Delta l} > \Delta p (1 - \rho_h) \left( \frac{X^l_{\Delta}}{X^h_{\Delta}} \right)^\beta \). Since \( P \) can reduce \( w_{\Delta l} \) with all other constraints satisfied, and since the objective function in (B.1) is decreasing in \( w_{\Delta l} \) we have \( w_{\Delta l} = \Delta p (1 - \rho_h) \left( \frac{X^l_{\Delta}}{X^h_{\Delta}} \right)^\beta \), i.e., constraint (B.2) is binding. Note also that since \( \Delta p > 0 \) and \( \rho_h < 1 \), we have \( w_{\Delta l} > 0 \), that is, constraint (B.4) is not binding.

Given all this, the problem that \( P \) needs to solve is:

\[
\max q \left( X^l_{\Delta} - p_l \right) \left( \frac{x}{X^l_{\Delta}} \right)^\beta + (1 - q) \left[ \left( X^h_{\Delta} - p_h - c(\rho_h) \right) - \frac{q}{1-q} \Delta p (1 - \rho_h) \right] \left( \frac{x}{X^h_{\Delta}} \right)^\beta
\]

s.t.

\( \rho_h \geq 0 \)

The Lagrangian is:

\[
\mathcal{L} = q \left( X^l_{\Delta} - p_l \right) \left( \frac{x}{X^l_{\Delta}} \right)^\beta + (1 - q) \left[ \left( X^h_{\Delta} - p_h - c(\rho_h) \right) - \frac{q}{1-q} \Delta p (1 - \rho_h) \right] \left( \frac{x}{X^h_{\Delta}} \right)^\beta + \xi \rho_h
\]

where \( \xi \) is the multiplier of the constraint. The first-order condition for \( X^l_{\Delta} \) gives:

\[ X^l_{\Delta} (p_l) = \frac{\beta}{\beta - 1} p_l \quad (= X^l_s (p_l)) \]

The first-order condition with respect to \( X^h_{\Delta} \) gives:

\[ X^h_{\Delta} (p_l, p_h) = \frac{\beta}{\beta - 1} \left[ p_h + c(\rho_h) + \frac{q}{1-q} \Delta p (1 - \rho_h) \right] \]

The first-order condition with respect to \( \rho_h \) is:

\[ (1 - q) X^h_{\Delta} \left[ -c'(\rho_h) + \frac{q}{1-q} \Delta p \right] + \xi = 0 \]

The Kuhn-Tucker conditions suggest that for \( \rho_h > 0 \) we require \( \xi = 0 \) which gives:\(^{34}\)

\[ \rho_h = c^{-1} \left( \frac{q}{1-q} \Delta p \right) \]

\(^{33}\)This is the so-called Maximal Punishment Principle. For more details see Laflont and Martimort (2002).

\(^{34}\)We focus on the case where \( \rho_h > 0 \) and, consequently, \( \xi = 0 \). Of course if instead we have \( \rho_h = 0 \) and \( \xi > 0 \), we are back in the case without auditing as this was presented in subsection 3.4 of the main body of the paper.
Summing up, unless auditing is prohibitively expensive, i.e., as soon as \( c'(0) \leq \frac{q}{1-q} \Delta p \), the optimal menu of contracts is:

\[
\begin{align*}
\{ X^t_\Delta (p_t), w_{\Delta t}(p_t, p_h), \rho_t, \omega_t \} &= \left\{ \frac{\beta}{\beta - 1} p_t, \Delta p (1 - \rho_h) \left( \frac{X^t_\Delta}{X^h_\Delta} \right)^\beta, 0, \Delta p \right\} \\
\{ X^h_\Delta (p_t, p_h), w_{\Delta h}(p_t, p_h), \rho_h, \omega_h \} &= \left\{ \frac{\beta}{\beta - 1} \left[ p_h + c(\rho_h) + \frac{q}{1-q} \Delta p (1 - \rho_h) \right], 0, c^{-1} \left( \frac{q}{1-q} \Delta p \right), 0 \right\}
\end{align*}
\]

with \( \rho_h > 0 \). Note that \( X^h_\Delta (p_t, p_h) = \frac{\beta}{\beta - 1} \left[ p_h + c(\rho_h) + \frac{q}{1-q} \Delta p (1 - \rho_h) \right] \) can be rewritten as:

\[
X^h_\Delta (p_t, p_h) = X_s (p_h) + \frac{\beta}{\beta - 1} \left[ c(\rho_h) + \frac{q}{1-q} \Delta p (1 - \rho_h) \right]
\]

or

\[
X^h_\Delta (p_t, p_h) = X^b_\Delta (p_t, p_h) + \frac{\beta}{\beta - 1} \left( c(\rho_h) - \rho_h \frac{q}{1-q} \Delta p \right)
\]

Now, from Eq. (B.13) we have \( X^h_\Delta (p_t, p_h) > X_s (p_h) \). From Eq. (B.14) we have \( X^h_\Delta (p_t, p_h) < X^b_\Delta (p_t, p_h) \) since \( c \) is a convex function of \( \rho_h \) whereas the term \( \rho_h \frac{q}{1-q} \Delta p \) is linear in \( \rho_h \).
References


