

Portfolio of Oil Exploration Assets: Learning Options, Sequential Drilling Options and Defer Options

Marco Antonio Guimarães Dias¹

Luigi de Magalhães Detomi Calvette²

Abstract

In this article we consider the interaction of three different options: the learning option, the sequential option and the defer option for portfolios of two and three correlated prospects. We consider uncertainty about the petroleum existence (technical uncertainty) and long-term oil price uncertainty (market uncertainty). In contrast with the traditional exploratory economics, it could be optimal to drill a prospect with negative expected monetary value (EMV) if there are correlated prospects so that there are learning options embedded when drilling a prospect. In addition, we saw that the drilling order matters even when the prospects have the same EMV and the same NPV in case of success. Even more, may be optimal to prioritize (drilling first) prospects with lower EMV, so that the optimal exploratory prospects portfolio ranking can be very different of traditional exploration economics. Decisions as to drill the prospects immediately or to wait for a better price scenario were also addressed using the multi-period binomial to model the oil price uncertainty. With each prospect order being an alternative, we find disconnected exercise regions with intermediate waiting regions, resembling real options models of choice between discrete scale alternatives.

Keywords: real options, learning options, oil exploration, portfolio of real assets, optimal drilling sequence, defer options, interaction of technical and market uncertainties, disconnected exercise sets, intermediate waiting region.

1) Introduction and Literature Review

Real options theory is, nowadays, a consolidated approach to investment in real assets, more than 20 years after the first published textbooks (Dixit & Pindyck, 1994; Trigeorgis, 1996). Nonetheless, several challenges still remains for a more practical and applied approach. In particular, technical uncertainty and learning options interacting with market uncertainties and defer options is a theme that still

¹ Aggregated Professor, PUC-Rio, and Research Fellow, Applied Computational Intelligence Laboratory, Support Decision Methods Group, Department of Electrical Engineering, PUC-Rio. E-mail: marcoagd@ele.puc-rio.br

² Petrobras, Exploration & Production Department. E-mail: luigi.calvette@gmail.com

deserves new developments. This is the focus of this paper providing an application in oil exploration and development.

Dias (2005a and 2005b) presents an in depth analysis of learning options for oil exploratory prospects. We use several equations from there, especially for conditional chance factors after the information revelation provided by the first prospect drilling outcome. Smith (2005) and Smith & Thompson (2008) analyze a portfolio of petroleum exploration assets using real options approach. However, Smith (2005) focus on the specific optimal stopping problem of a drilling sequence of dry holes, not in the learning options we analyze neither in the optimal drilling sequence as we do. Smith & Thompson (2008) consider the optimal sequence and information revelation from the first drilling, but they don't consider the defer option, so that a change in the market (oil price) can invert the optimal order and can be optimal to wait and see instead exercising the current best sequence.

Dias (2006) and Calvette & Pacheco (2014) are even more related to this work. Dias (2006) discusses the portfolio of oil exploratory assets considering the learning options, synergy³ and defer options. But he does not address the problem of optimal drilling sequence, as we perform here, because he used totally symmetric prospects. In Dias (2006) the prospects were totally symmetric, so that the drilling order was irrelevant. Here we consider some asymmetries between the prospects.

Calvette & Pacheco (2014) discuss the optimal sequence of correlated exploratory prospects considering its embedded learning options and also synergy. They consider the learning intensity (correlation coefficient) as a function of the distance between prospects. However, they don't discuss defer options and some other aspects that we address here.

Dias & Teixeira (2009) also analyze learning and defer options for two oil exploration prospects. However, these prospects are owned by different firms, so that this paper focus on game theoretic aspects (war of attrition and bargain) combined with learning and defer options, not in exploratory assets portfolio optimization as done here.

A related literature is the interaction of scale option with defer option, as we'll see. Dias & Rocha & Teixeira (2003), Dias (2004), Décamps & Mariotti & Villeneuve (2006), and Dias (2015, ch. 27) using continuous-time approach, discuss the problem of discrete choice of investment scale alternatives, which reaches disconnected exercise sets with intermediate waiting regions. Dias (2014, ch. 8) analyzes the same problem using discrete-time approach. We will see similar features occurring in the discrete choice of different sequences of exploratory drilling. So, the literature of different investment scales is closely related with our article even not being a true problem of investment scale choice.

³ In Dias (2006) and in Calvette & Pacheco (2014) were considered the gain from the synergy in case of two success (NPV of joint development is higher than the sum of individual development NPVs) due to economy of scales (e.g., a larger platform instead two small platform, gains logistics, etc).

2) Learning and Sequential Options with Two Prospects

Consider the case of two oil exploration prospects, with chance factors (probability of existence of oil reserves) CF_1 and CF_2 , which are Bernoulli random variables (success or failure). These prospects are in the same geologic play (eg. both in Oligocene) and, hence, they are correlated with coefficient $\rho > 0$. We are interested in both the portfolio of prospects value and the optimal decision rule that underlies this portfolio value. Such decision rule provides the answers to strategic questions like: What is the optimal drilling sequence? Should we wait and see the oil price evolution or invest immediately? In this section we focus in the first question (suppose it is a *now or never decision*). The next section discusses the second one.

Let I_w be the exploratory well drilling investment (exercise price), net of fiscal benefits. In case of success (finding oil reserves) we can develop the oilfield obtaining the net present value from the oilfield development (NPV). For each prospect $i = 1$ or 2 , with chance factor CF_i , the payoff value from the exercise of the drilling option is called *expected monetary value* (EMV), given by:

$$EMV_i = -I_{wi} + [CF_i \cdot NPV_i] \quad (1)$$

The development NPV is a function of the oil price (P). However, after the oilfield discovery, typically the first oil production takes several years, especially for offshore oilfields (3 to 6 years is common). So, this kind of decision is based not in the *current* oil price but in the *long-term* oil price expectation. We use long-term *oil futures contract* as a proxy in order to estimate the current long-term oil price expectation, the volatility and the convenience yield⁴. The oilfield i development NPV is given in this article by the simple linear parametric equation named "Business Model"⁵:

$$NPV_i = V_i - I_{Di} = q_i B_i P - I_{Di} \quad (2)$$

Where q_i is the *economic quality of the reserve* (related to the productivity of such reserve), B_i is the *oil reserve volume* (number of oil barrels economically recoverable), P is the long-term oil price, and I_{Di} is the development investment. The development investment is positively correlated with the oil prices, so that when oil price rises the cost of platforms, oil rigs, and other investment input also increases. For this reason, in this paper, when performing a sensitivity analysis with the oil price, we will adjust the investment according to the following equation:

$$I_{Di}(P) = \left(\frac{P}{P_{base}} \right) I_{Di_base} \quad (3)$$

⁴ Good long-term oil price proxies are the 18th month WTI futures and 12th month Brent futures (sufficient liquidity). But the futures prices are *risk-neutral expectations*, so that we need to add a risk-premium to obtain the current long-term oil price expectation. The volatility of the long-term futures price is lower than the spot oil price (or than lower maturity futures price), an issue known as *Samuelson's Effect*.

⁵ See more details at http://marcoagd.usuarios.rdc.puc-rio.br/payoff_model.html or in Dias (2015).

Where I_{Di_base} is the development investment of oilfield i with estimate based in the oil price P_{base} . This is a simple way to perform a more realistic (than the case of investment independent of the oil market) sensitivity analysis, adjusting the investment when varying the long-term oil price. Because the exploratory investment I_{Wi} level does not depend on the long-term oil price changes (it is performed in the short-term), we will keep fix (without adjustment) in the long-term oil price sensitivity analysis.

Because the prospects have positive correlation, the drilling outcome of one prospect will change the chance factor of the other prospects (details in Dias, 2005a). If we drill the prospect 1, in case of success in this prospect 1, the chance factor of the second prospect (CF_2) must be revised upward (to CF_2^+) and in case of failure must be revised downward (to CF_2^-). The equations for updating the chance factors are:

$$CF_k^+ = CF_k + \sqrt{\frac{1 - CF_j}{CF_j}} \sqrt{CF_k (1 - CF_k)} \rho \quad (4)$$

$$CF_k^- = CF_k - \sqrt{\frac{CF_j}{1 - CF_j}} \sqrt{CF_k (1 - CF_k)} \rho \quad (5)$$

Where ρ is the correlation coefficient and $j, k = 1$ or $2, j \neq k$. However, it is not possible any correlation coefficient, the literature of multivariate distributions shows that are necessary limits of consistence for these distributions, named *Fréchet-Hoeffding limits*. For Bernoulli bivariate distribution the correlation coefficient has the following limits (Joe, 1997, p.210; or Dias, 2005a):

$$\text{Max} \left\{ -\sqrt{\frac{CF_k CF_j}{(1 - CF_k)(1 - CF_j)}}, -\sqrt{\frac{(1 - CF_k)(1 - CF_j)}{CF_k CF_j}} \right\} \leq \rho \leq \sqrt{\frac{\text{Min}\{CF_k, CF_j\} (1 - \text{Max}\{CF_k, CF_j\})}{\text{Max}\{CF_k, CF_j\} (1 - \text{Min}\{CF_k, CF_j\})}} \quad (6)$$

Consider the following numerical example, named "Case 1". There are two oil exploratory prospects named "Safe" (S) and "Risky" (R), because the first has higher chance factor than the second⁶. The correlation between the prospects is $\rho = 75\%$ ⁷. The other prospects characteristics are:

- Prospect Safe: $CF_S = 30\%$, $I_{WS} = 70$ million \$, $NPV_S = 200$ (conditional to success) because $P = 50$ \$/bbl, $q_S = 10\%$, $B_S = 500$ million bbl, $I_{DS} = 2,300$ million \$; and
- Prospect Risky: $CF_R = 20\%$, $I_{WR} = 50$ million \$, $NPV_R = 200$ (conditional to success, the same NPV of the Safe) because $P = 50$ \$/bbl, $q_R = 12\%$, $B_R = 700$ million bbl, $I_{DR} = 4,000$ million \$.

Using the eq. (1) the reader can confirm that the prospects have the same negative EMV:

$$EMV_S = EMV_R = -10 \text{ million } \$$$

⁶ This nomenclature was borrowed from Ball & Savage (1999). They illustrated the portfolio diversification effect.

⁷ A very high correlation for illustrative purpose, but it is consistent: by the eq. (6), the upper limit is $\rho_{max} = 76.4\%$.

The superficial analysis tells that we should not drill any of the prospects because both have negative EMV. However, the learning option was not considered: the expected negative value of the first drilling can be more than compensated by the *optional* drilling of the second prospect. The high correlation of 75% indicates that the learning option is very valuable. If we drill the Safe first, in case of success the chance factor of the other prospect (Risky) rises to $CF_R^+ = 65.8\%$, whereas in case of failure this chance factor drops to $CF_R^- = 0.4\%$. In case of failure, the EMV_R becomes even more negative ($EMV_R^- = -49.3$ million \$), but we have the option, not the obligation, to drill the other prospect, so that not drilling we keep the loss limited to the first drilling. But in case of success, with $CF_R^+ = 65.8\%$ the EMV_R becomes very positive ($EMV_R^+ = 81.65$ million \$) and we exercise the option capturing this expected positive value, which (as we'll see) more than compensates the expected negative value from the first drilling. We can think the first drilling as one investment in acquiring information, so that the negative EMV_S can be interpreted as the cost of information.

In this case, modeling the optionalities as $\text{Max}[0, \text{investment payoff}]$, the equation for the portfolio value is similar of Dias (2006)⁸. For the case of drilling the Safe prospect first (and then the Risky if it has positive value), the portfolio value drilling Safe first is given by:

$$\Pi_{SR} = EMV_S + [CF_S \text{Max}(0, EMV_R^+)] + [(1 - CF_S) \text{Max}(0, EMV_R^-)] \quad (7)$$

In words, the portfolio value if we drill first the Safe prospect is the EMV_S plus, in case of success (with probability CF_S) the value of the option to drill the prospect Risky with expected value EMV_R^+ , and in case of failure (with probability $1 - CF_S$) the value of the option to drill the prospect Risky, but now with lower (perhaps negative) expected value (EMV_R^-). Using the numerical values from the first example we get a positive value for this portfolio:

$$\Pi_{SR} = -10 + [30\% \text{Max}(0, 81.65)] + [70\% \text{Max}(0, -49.3)] = 14.50 \text{ million \$}$$

So, a very different portfolio value. The learning option indicates that we shall invest at least in one prospect. Without considering the learning option, we would reject this opportunity due to the negatives EMVs. But, drilling the Safe first is the best we can do? No!

Why not drill the Risky first? At first sight, the prospects have the same EMV and even the same development NPV in case of success, so that the drilling order appears not important. With the same EMVs, the superficial appearance points to drill the Safe first because has a higher chance factor for the same prior EMV. However, the Risky prospect has an important quality: *lower exploratory investment*,

⁸ Here we don't consider synergy effects in order to focus in the sequential option case. The extension to include synergy is straightforward and it is planned in a future work. Here we also left the option of not drill any prospect to eq. (9).

so a lower cost to gain this important information. So, in case of bad news (failure in the first drilling), we stop the investment with less losses than in the case of drilling the prospect Safe first.

The equation for the case of drilling the Risky prospect first (and then the Safe if it has positive value), the portfolio value drilling Safe first is similar the previous one, but permuting R and S:

$$\Pi_{RS} = EMV_R + [CF_R \text{Max}(0, EMV_S^+)] + [(1 - CF_R) \text{Max}(0, EMV_S^-)] \quad (8)$$

Using the numerical values from the first example we get a higher value for this portfolio than when drilling the Safe first:

$$\Pi_{RS} = -10 + [20\% \text{Max}(0, 127.5)] + [80\% \text{Max}(0, -44.4)] = 15.50 \text{ million } \$$$

Because we have the option to choose the drilling sequence and also the option of not drill any prospect, the two exploratory prospects portfolio value is:

$$\Pi_{S+R} = \text{Max}[0, \Pi_{SR}, \Pi_{RS}] \quad (9)$$

In case of different (prior) EMVs, the traditional portfolio prioritization/ranking of prospects uses the EMV for this job, prioritizing the prospect with higher EMV. But with learning options this simple criteria does not work! Let us see an example. Consider the same numerical example (case 1), but with oil price of 51 US\$/bbl (instead of 50). Using the eqs. (1), (2), and (3) for the two prospects, the reader can see that the development NPVs rise with P, but remains the same $NPV_S = NPV_R = 204$ million \$. However, now the EMVs are not equal anymore: $EMV_S = -8.8$ million \$ and $EMV_R = -9.2$ million \$ (so, Risky has a worse EMV). Hence, in this case if we use the higher EMV criteria, we drill the Safe first. However, drilling the Risky first remains the best choice: by using equations (7) and (8) with support of equations (1) to (5), we get: $\Pi_{SR} = 16.49$ million \$ and $\Pi_{RS} = 17.09$ million \$. So, in this numerical case, even with lower EMV (and lower chance factor) is better to start drilling the Risky prospect!

But the optimal sequence (Risky first in this case 1) could be not optimal anymore for other oil price. Figure 1 shows the portfolio value if we drill first one prospect (Risky or Safe) and then drill the other if it has positive EMV. Although for 50 US\$/bbl the alternative of drilling Risky first is more valuable than drilling Safe first, for oil prices $P > 52.6$ US\$/bbl the optimum changes to drill the Safe first.

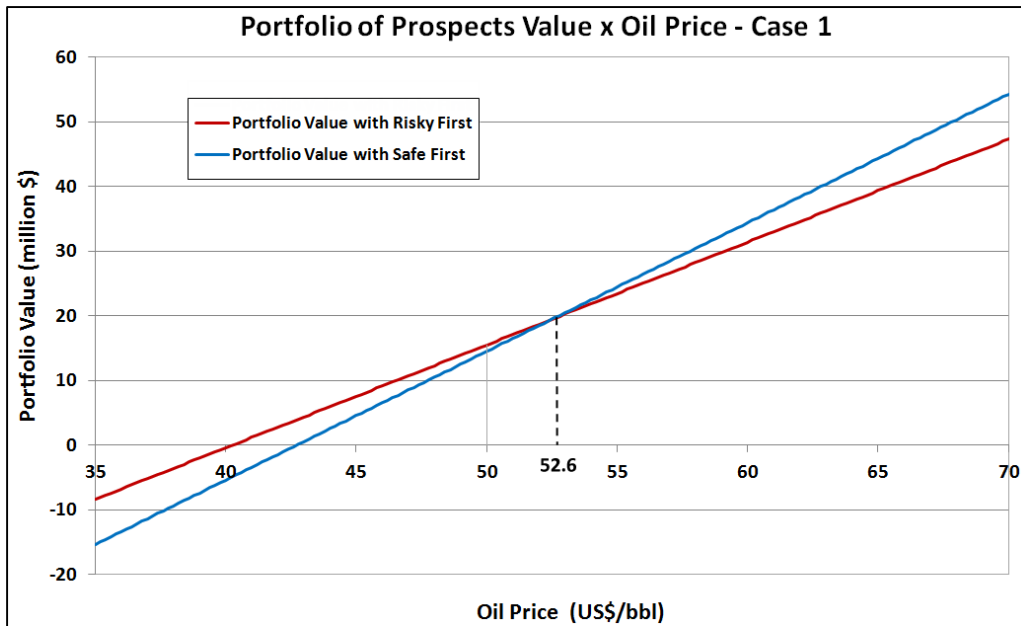


Figure 1 – Portfolio of Prospects Value x Oil Prices – Case 1

This is a now or never opportunity. But if the option is not expiring and we can wait, perhaps it would be better to wait due to the hope of higher oil price expectations, in which the optimal order invert to drill the Safe first. We will analyze the defer option in the next section.

We can represent this decision problem in a decision tree format. Figure 2 shows this decision tree to choose the optimal sequence of drilling if is optimal any drilling. In order to illustrate the learning effects in the chance factors, the Figure 2 shows the priors chance factors (CF_S and CF_R) and the posterior chance factors (CF_S^+ , CF_S^- , CF_R^+ , CF_R^-) for the example numerical we are working with correlation $\rho = 75\%$, which is very high (near of the upper limit).

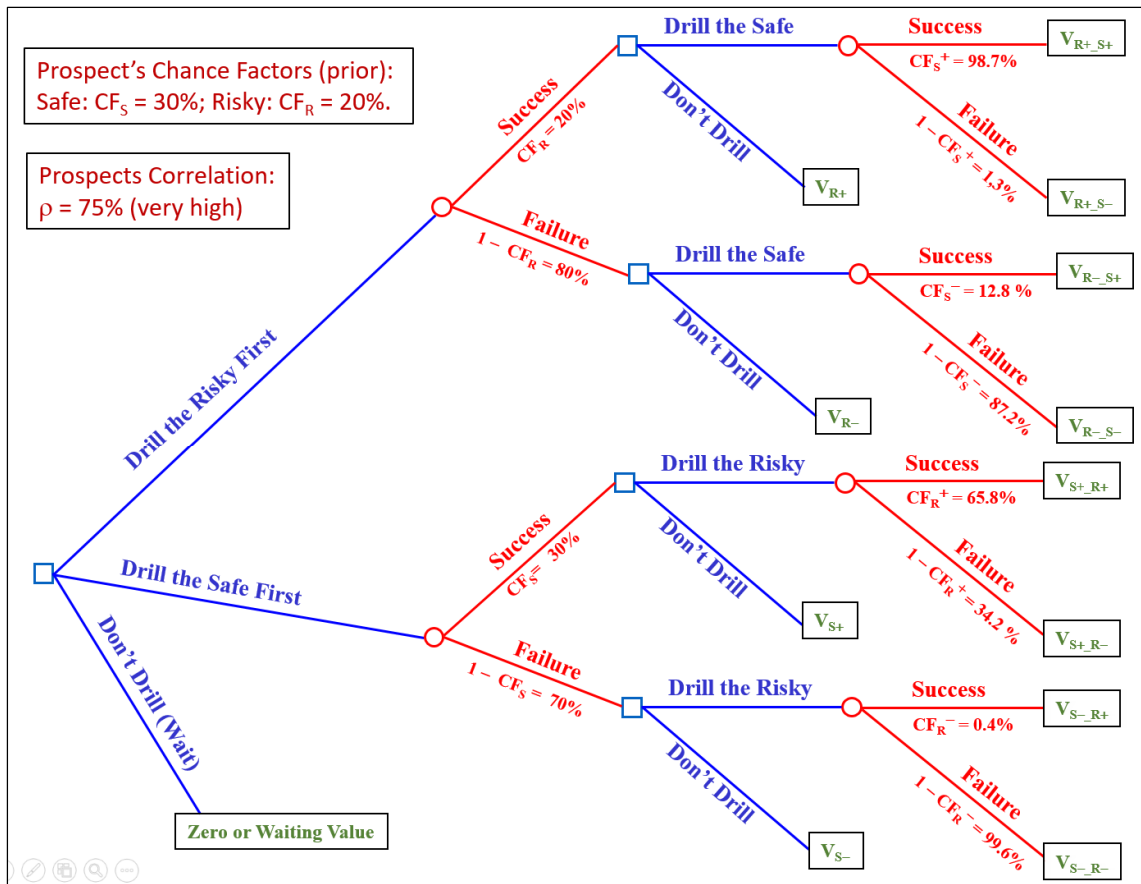


Figure 2 – Decision Tree for Oil Exploration Drilling Sequential Options

We used a very high correlation in this example. What if we use lower correlation? Consider the same numerical example, with the original $P = 50$ US\$/bbl, but varying the correlation coefficient. Figure 3 illustrate this sensitivity analysis.

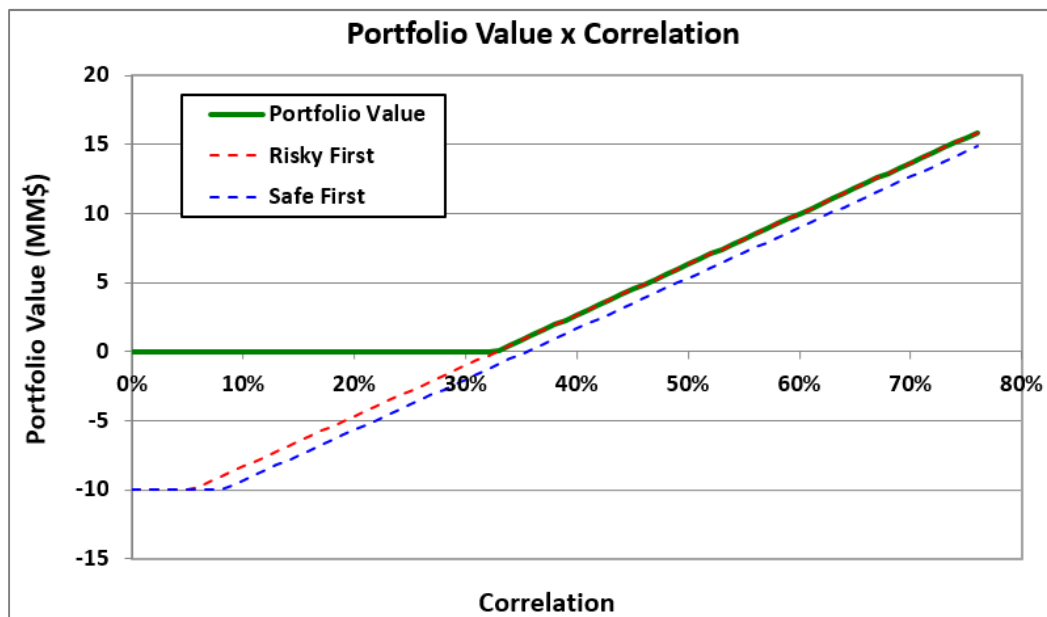


Figure 3 – Portfolio Value versus Correlation Coefficient

Figure 3 shows that even a much lower correlation like 35% the portfolio still has positive value (portfolio values zero if $\rho \leq 32.7\%$) even with negative prior EMVs. Note that for this case (oil price of 50, etc.) the value of drilling Risky first is higher than the value of drilling Safe first for almost range of correlation. Only for very low correlation ($< 6\%$) the value of Safe first is equal the Risky first (but < 0).

3) Learning, Sequential and Defer Options with Two Prospects

In this section we consider the defer option. The market uncertainty here is represented by the long term oil price expectation inferred from the futures market. We assume that the oil price follows a Geometric Brownian motion (GBM). The GBM in the risk-neutral measure for the long term oil price expectation P , has the following stochastic differential equation:

$$dP^Q = (r - \delta) P dt + \sigma P dz^Q \quad (10)$$

Given $P(t = 0) = P$. Where: r is the risk-free interest rate (continuously compound); δ is the convenience yield (measured in the futures market); σ is the volatility; and $dz (= N(0, 1)\sqrt{dt})$ is the standard Wiener increment. The superscript Q denotes the risk neutral measure for the stochastic terms (Q -measure).

In most countries, the exploration & production (E&P) of petroleum have phases of limited duration. So, we will consider finite expiration options. For finite lived American options are required numerical methods⁹. Here we use the binomial as numerical method to evaluate the defer option.

In the binomial method, after one period, the price P can go up to $P^+ = u P$ with risk-neutral probability q or can go down to $P^- = d P$ with probability $1 - q$. The Cox & Ross & Rubinstein (1979) binomial equations for the GBM, but considering the dividends (here the convenience yield), are given by¹⁰:

$$u = \text{Exp}[\sigma \sqrt{\Delta t}] \quad (11)$$

$$d = 1/u \quad (12)$$

$$q = \frac{e^{(r - \delta) \Delta t} - d}{u - d} \quad (13)$$

Here we are assuming the following data for the examples: $r = 3.92\%$ p.a. (continuous time, equivalent to 4% in discrete time), $\delta = 4.88\%$ p.a. (equivalent to 5% in discrete time), and $\sigma = 20\%$ p.a. Let us first consider $\Delta t = 1$ year. In this case, by using the eqs. (11) to (13), we get $P^+ = 61.07$ \$/bbl, $P^- = 40.94$ \$/bbl and $q = 42.65\%$. Consider also the numbers from the case 1. We shall compare the values of exercising

⁹ It is also interesting to develop the case of perpetual options with analytical solution. Although not realistic in most practical cases (exploration phase has finite maximal duration) the perpetual option value is an upper bound value. So, we can check the results from our numerical method: the finite lived option must have lower value than the perpetual option with other identical characteristics.

¹⁰ We can consider the dividends in the tree (in the up and down factors) or in the risk-neutral probability q , as done here and as in Back (2005, p. 93). See also Dias (2014) for a detailed discussion of dividends in binomial.

immediately (in this case, the Risky first is better than Safe first, as already seem: 15.495 > 14.495) with the waiting value. The present value of waiting (W) is given by:

$$W = e^{-r \Delta t} [q F_{t+1}^+ + (1 - q) F_{t+1}^-] \tag{14}$$

Where F_{t+1}^+ and F_{t+1}^- are the option values in the scenarios P^+ and P^- , respectively, that is, the maximum between exercising the Safe first, exercising the Risky first, and not exercising at all (here, one period, means abandon the opportunity and values zero), for the scenarios P^+ and P^- , respectively.

With these numbers, the reader can verify that the present value of the “wait and see” strategy is 15.577 million \$, which is higher than exercising immediately the option to drill. Figure 4 illustrate this one period defer option for this numerical case, showing also some intermediate numbers.

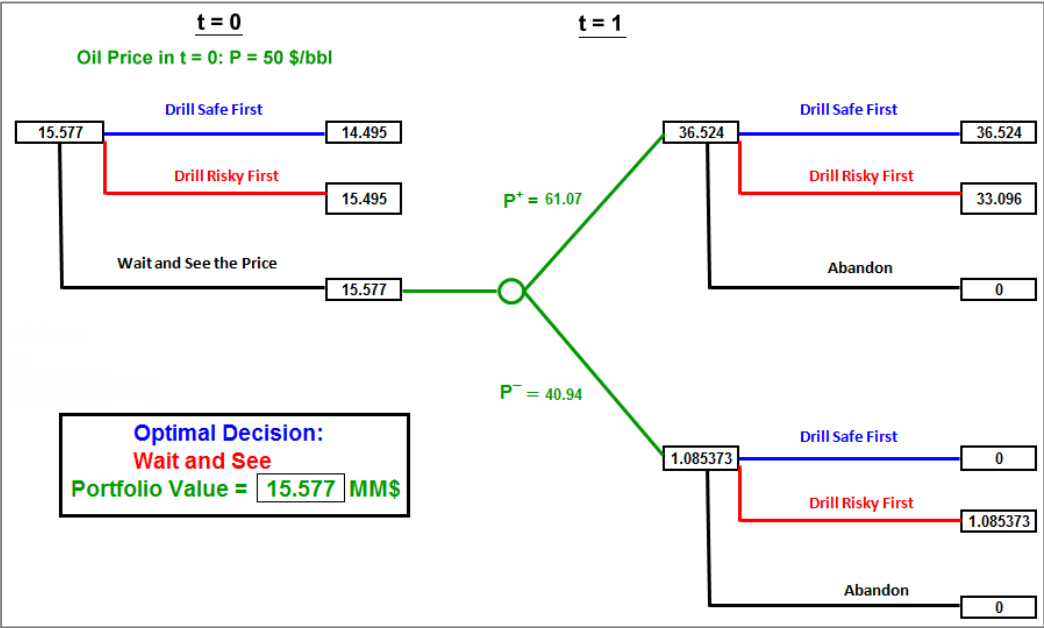


Figure 4 - Defer Option One Period for Two-Prospects Sequential Options

So, when we consider the defer option, the optimal decision changes from exercising the Risky first to “wait and see”. Higher oil prices than invert the best prospect to drill, as we commented when analyzing the Figure 1.

Now, we use the multi-period binomial and that we have three years before the expiration (T = 3 years). With longer time to wait, the waiting value is even higher and the optimal is “wait and see”: with T = 3 years and $\Delta t = 0.5$ month, the waiting value rises to W = 20.46 million \$, which is 32% higher than drilling immediately the best prospect option (Risky, with 15.5 million). So, neglecting the defer option can destroy a significant value.

Let us see a new numerical example (case 2). Again, there are two oil exploratory prospects named “Safe” (S) and “Risky” (R). The correlation between the prospects is $\rho = 65\%$ (again, near of the upper limit). For $P = 50$ \$/bbl, the other prospects characteristics are:

- Prospect Safe: $CF_S = 50\%$, $I_{WS} = 70$ million \$, $NPV_S = 100$ million \$ (conditional to success), $q_S = 10\%$, $B_S = 600$ million bbl, $I_{DS} = 2,900$ million \$; and
- Prospect Risky: $CF_R = 30\%$, $I_{WR} = 45$ million \$, $NPV_R = 200$ (conditional to success, now the NPVs are different), $q_R = 12\%$, $B_R = 500$ million bbl, $I_{DR} = 2,800$ million \$.

In this case, not only the NPVs are different as the EMVs: $EMV_S = -20$ million \$ and $EMV_R = 15$ million. For $P = 50$ US\$/bbl the value of drilling the Risky first is higher than the Safe first: $\Pi_{RS} = 23.89$ million \$, whereas $\Pi_{SR} = 17.29$ million \$. But, again, if the oil price is different it can be inverted. In reality, the reader can check that for oil prices higher than US\$ 66.61/bbl, is more valuable drill the Safe first. So, we call the break-even (BE) price the point of indifference and in this case $P_{BE} = 66.61$ US\$/bbl.

If we consider in this example (case 2) the defer option with $T = 3$ years to expiration and the same values for r , δ , and σ , again the optimal is “wait and see” because $W = 24.48$ million \$ is higher than the best exercise ($\Pi_{RS} = 23.89$) even here with the Risky much better than the Safe for $P = 50$ US\$/bbl.

Hence, for $P = 50$ US\$/bbl in $t = 0$, the optimal is “wait and see”. But, and for other initial oil prices? For what prices is the immediate exercise of the Safe (or the Risky) optimal? In more technical words, what are the oil prices threshold regions where is optimal drill the Safe first? Or to drill the Risky first?

Let us introduce some notation. Let P_{RS_L} be the lowest price in which we are indifferent between wait and drill the Risky first (and then perhaps the Safe); let P_{RS_H} be the highest price in which we are indifferent between wait and drill the Risky first; and let P_{SR} the indifference price between wait and drill the Safe first.

In order to investigate the regions of exercise and wait in the binomial, we change the initial price in the binomial tree and see the optimal action for a wide range of oil prices. If we perform this job for different time to expiration, we can draw the regions of exercise and wait. Figure 5 illustrates the case 2, showing the regions of exercise and regions of “wait and see”.



Figure 5 – Oil Price Thresholds for Two Prospects with Volatility of 20% p.a.

Note in Figure 5 that there are *disconnected regions of exercise*, with one *intermediate waiting region*. If the current oil price is exactly the break-even price (Risky first and Safe first with the same value), which is $P_{BE} = 66.61$ US\$/bbl, the optimal is “wait and see”. Figure 5 indicates even more: never is optimal to exercise any option in this break-even price except at the expiration. This is more general, it is not specific of this numerical example. Let us see the plot of portfolio value versus oil prices with the previous threshold nomenclature in the Figure 6 below, for some time before the expiration.

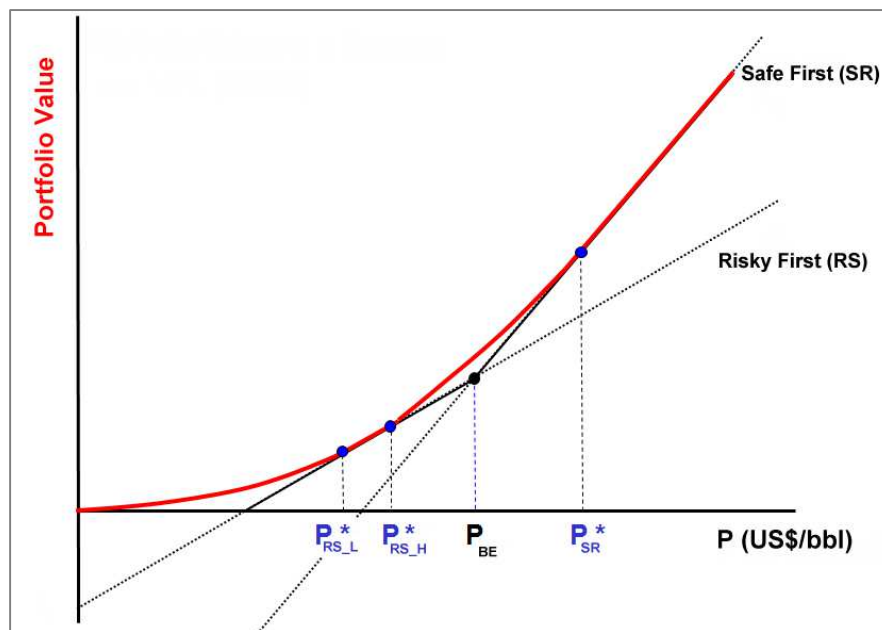


Figure 6 – Portfolio Value x Prices: Oil Price Thresholds

At the break-even price, the payoff function is not differentiable. Using the concept of *local time*, Décamps et al (2006) shows that (before expiration) is never optimal to exercise the option: “The reason is that the underlying payoff function of the decision-maker is not differentiable at the

indifference point... Using a local time argument, we show that this implies the optimality of delaying investment for values of the current output price in the neighborhood of this point". They prove for the case of perpetual options, but it is more general and valid for any time before the expiration.

In addition to local time argument used by Décamps et al (2006), we can use other argument. As pointed out by Dixit & Pindyck (1994, p.188) there is a Kac's theorem telling that if the option $F(P)$ is a continuous function and P follows a GBM, then $F(P)$ is doubly differentiable in relation to P . See the proof and details in Karatzas & Shreeve (1991, p.271) and see Dixit (1993, p.31) for an intuition. So, the function value (option before the expiration) must be smooth, without corner.

These disconnected exercise regions and intermediate inaction region also appear in other real options articles that focus discrete choice of scale of investment, as in Dias & Rocha & Teixeira (2003) and Dias (2004). See also Dias (2014, ch. 8) for an example in discrete time and Dias (2015, ch. 27) for a continuous time example with perpetual options (defer and scale) using *variational inequalities* approach for this American type option. However, all these cited articles works with scale (and defer) options. We show that these issues also appear with other kind of options.

For a higher volatility, the intermediate waiting region can disappear for sufficient longer time to expiration. Figure 7 shows the same case 2, but for a higher volatility ($\sigma = 30\%$ p.a.). Note that for $t = 0$ ($T = 3$ years), the intermediate inaction region does not exist. This region only appear for $t > 2.35$ years ($T < 0.75$ year). The reason is simply: when increasing the volatility we also increase the wait value and decrease the exercise regions, so the wait and see regions from Figure 5 ($\sigma = 20\%$) expand and merge for the higher volatility ($\sigma = 30\%$) case of Figure 7 for a range of time to expiration.

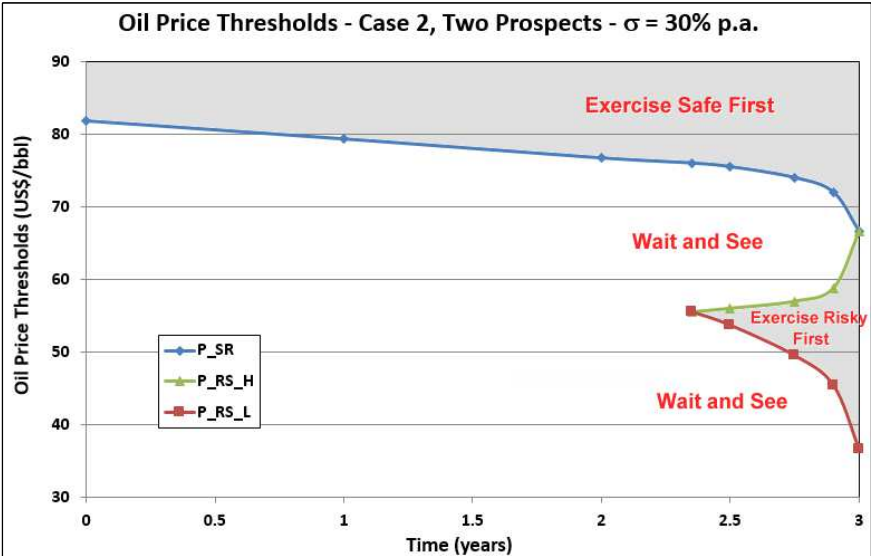


Figure 7 – Oil Price Thresholds for Two Prospects with Volatility of 30% p.a.

In the next sections we extend the methodology to the important practical case of three correlated prospects.

4) Learning and Sequential Options with Three Prospects

In this section we will extend the analysis of the Learning and Sequential Options will to a more complex scenario. Following the methodology set in the item 2, we'll consider a third prospect, namedly *Intermediate*. A three-prospects portfolio, holds six possible drilling sequences, as follows:

- Drilling Safe first, Intermediate then and Risky at last (SIR sequence);
- Drilling Safe first, Risky then and Intermediate at last (SRI sequence);
- Drilling Intermediate first, Safe then and Risky at last (ISR sequence);
- Drilling Intermediate first, Risky then and Safe at last (IRS sequence);
- Drilling Risky first, Intermediate then and Safe at last (RIS sequence) and
- Drilling Risky first, Safe then and Intermediate at last (RSI sequence).

Just as in the two-prospects portfolio, the three prospects are in the same geologic play and hence, they are correlated with coefficient $\rho > 0$. One difference between the prior model is that, here, whenever a first prospect is drilled, both chance factors of the others prospects are updated according to the information revealed by the wildcat. When drilling a second prospect the chance factor of the third prospect will be updated again. Evidently the more correlated the prospects are, the bigger is the potential to reveal information, and consequently the portfolio value, just as previously shown (see Figure 3).

Considering that drilling every prospect is optional, for the case of drilling the Safe prospect first (and then the Risky if it has positive value, and, at last drilling the Intermediate if it has positive value), the portfolio value for the SRI sequence is given by:

$$\begin{aligned} \Pi_{SRI} = & EMV_S + \{CF_S \text{Max}[0, EMV_R^+ + CF_R \text{Max}(0, EMV_I^{++}) + (1 - CF_R \text{Max}(0, EMV_I^+))] + \\ & + [(1 - CF_S) \text{Max}[0, EMV_R^+ + CF_R \text{Max}(0, EMV_I^+)] + (1 - CF_R \text{Max}(0, EMV_I^-))\} \end{aligned} \quad (15)$$

Formula (15) extends the formula (7) considering that the intermediate prospect will be the last one to be drilled (if it has positive value). Because we have the option to choose the drilling sequence and also the option to not drill any prospect, updating formula (9) to the three exploratory prospects portfolio follows as:

$$\Pi_{S+R+I} = \text{Max}[0, \Pi_{SRI}, \Pi_{SIR}, \Pi_{IRS}, \Pi_{ISR}, \Pi_{RSI}, \Pi_{RIS}] \quad (16)$$

Consider the following numerical example, named "Case 1 – Three Prospects". There are three oil exploratory prospects named "Safe" (S), "Intermediate" (I) and "Risky" (R), because the first has higher chance factor than the second and the last is one holds more uncertainty over the others. The

correlation between the prospects are $\rho_{SR} = 50\%$, $\rho_{SI} = 50\%$ $\rho_{RI} = 60\%$ ¹¹. The other prospects characteristics are:

- Prospect Safe: $CF_S = 30\%$, $I_{WS} = 70$ million \$, $NPV_S = 200$ (conditional to success) because $P = 50$ \$/bbl, $q_S = 10\%$, $B_S = 500$ million bbl, $I_{DS} = 2,300$ million \$; and
- Prospect Intermediate: $CF_I = 25\%$, $I_{WI} = 60$ million \$, $NPV_I = 200$ (conditional to success) because $P = 50$ \$/bbl, $q_I = 11\%$, $B_I = 600$ million bbl, $I_{DI} = 3,100$ million \$; and
- Prospect Risky: $CF_R = 20\%$, $I_{WR} = 50$ million \$, $NPV_R = 200$ (conditional to success, the same NPV of the Safe) because $P = 50$ \$/bbl, $q_R = 12\%$, $B_R = 700$ million bbl, $I_{DR} = 4,000$ million \$.

Just as in the two-prospects portfolio, using the eq. (1) the reader can confirm that the prospects have the same negative EMV:

$$EMV_S = EMV_R = EMV_I = - 10 \text{ million } \$$$

Conventional economic analysis would recommend to not drill any prospect. However, considering that the drilling is optional and that the drilling sequence may be optimized the general results are quite different. Figure 8 summarizes the portfolio value for every possible drilling sequence. Drilling first the Intermediate (and, then Risky and finally Safe, if their values are positive) is the option that brings more value (+ 29 million \$) to the analysis.

Possible Sequences			Portfolio Value (\$)
1st Prospect	2nd Prospect	3rd Prospect	
Safe	Risky	Intermediate	28,7
Safe	Intermediate	Risky	28,4
Risky	Safe	Intermediate	27,7
Risky	Intermediate	Safe	28,0
Intermediate	Safe	Risky	29,6
Intermediate	Risky	Safe	26,3

Figure 8 – Portfolio Value - Case 1 – Three Prospects

However the optimal sequence (Intermediate, Safe and Risky in this case 1) could not be the optimal anymore for a different oil price. Figure 9 shows the portfolio value if we drill first one prospect (Risky or Safe) and then drill the other if it has positive EMV. Although for 50 US\$/bbl the alternative of drilling Risky first is more valuable than drilling Safe first, for oil prices $P > 52.6$ US\$/bbl the optimum changes to drill the Safe first.

¹¹ A very high correlation for illustrative purpose, but it is consistent: by the eq. (6), the upper limit is $\rho_{max} = 76.4\%$.

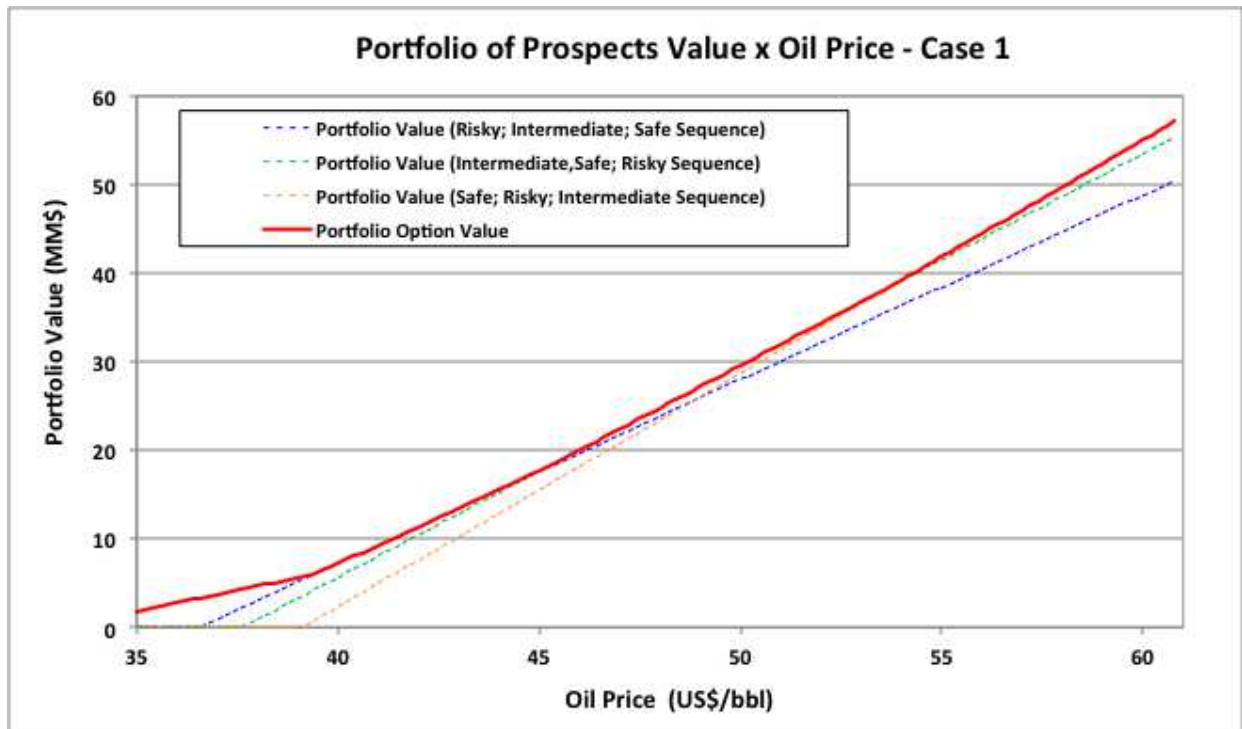


Figure 9 – Portfolio of Prospects Value X Oil Prices – Case 1 – Three Prospects

Although for 50 US\$/bbl the alternative of drilling Intermediate first is the more valuable one, than drilling Safe first, for oil prices $P < 44.8$ US\$/bbl the optimum changes to drill the Risky first. For $P > 53,8$ US\$/bbl it is optimal to drill the Safe first.

Consider another numerical example, named “Case 2 – Three Prospects”. Here we consider the correlation between the prospects as $\rho_{SR} = 50\%$, $\rho_{SI} = 5\%$ $\rho_{RI} = 45\%$. The other prospects characteristics are:

- Prospect Safe: $CF_S = 30\%$, $I_{WS} = 200$ million \$, $NPV_S = 1200$ million \$ (conditional to success);
- Prospect Intermediate: $CF_I = 25\%$, $I_{WI} = 200$ million \$, $NPV_I = 558$ million \$ (conditional to success); and
- Prospect Risky: $CF_R = 20\%$, $I_{WR} = 10$ million \$, $NPV_R = -40$ million \$ (conditional to success).

From the above and the eq. (1) follows that:

$$EMV_S = 160 \text{ million } \$ \quad EMV_R = -18 \text{ million } \$ \quad EMV_I = -61 \text{ million } \$$$

At a first look, the Risky Prospect seems to be the worst. Lower probability of success and negative NPV. However, the correlation of the prospects plays a major role. Due to its potential to reveal information the portfolio value is maximized when it is drilled first (Risky, Intermediate, Safe sequence results in an EMV of +174,2 million \$).

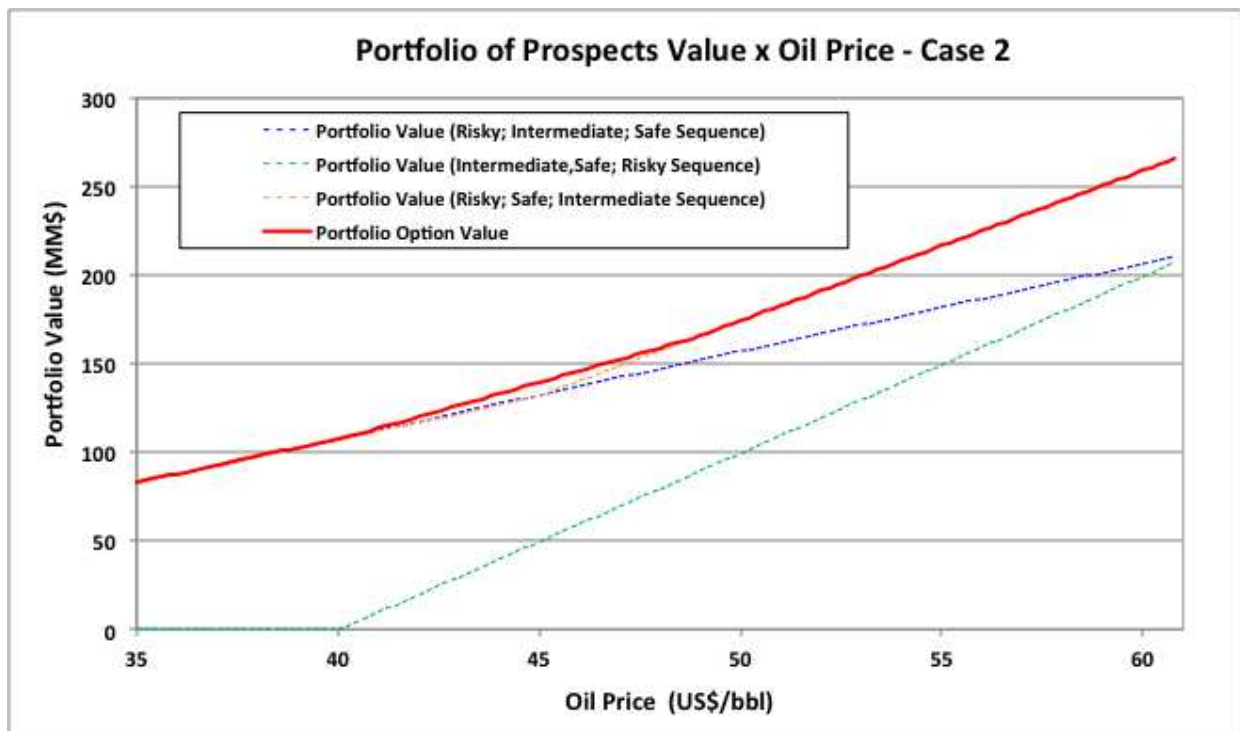


Figure 10 – Portfolio of Prospects Value X Oil Prices – Case 2 – Three Prospects

It is worth noticing that when we consider a different set of oil prices, still, the sequences that result in more profitable portfolio are the ones that start drilling the Risky Prospect.

5) Learning, Sequential and Defer Options with Three Prospects

In this section we will extend the analysis of the section 3 to a more complex scenario with three prospects. Here we are assuming the following data for the examples: $r = 3.92\% \text{ p.a.}$ (continuous time, equivalent to 4% in discrete time), $\delta = 4.88\% \text{ p.a.}$ (equivalent to 5% in discrete time), and $\lambda = 30\% \text{ p.a.}$ Let us first consider $\Delta t = 1 \text{ year}$. In this case, by using the eqs. (11) to (13), we get $P^+ = 67.49 \text{ \$/bbl}$, $P^- = 37.04 \text{ \$/bbl}$ and $q = 40.99\%$. Consider also the numbers from the case 1 – Three Prospects. We shall compare the values of exercising immediately (in this case, the Intermediate being drilled first, as already seen: 29.6 million \$) with the waiting value calculated using (14). With these numbers, the reader can verify that the present value of the “wait and see” strategy is 30.043 million \$, which is higher than exercising immediately the option to drill. Figure 4 illustrate this one period defer option for this numerical case, showing also some intermediate numbers.

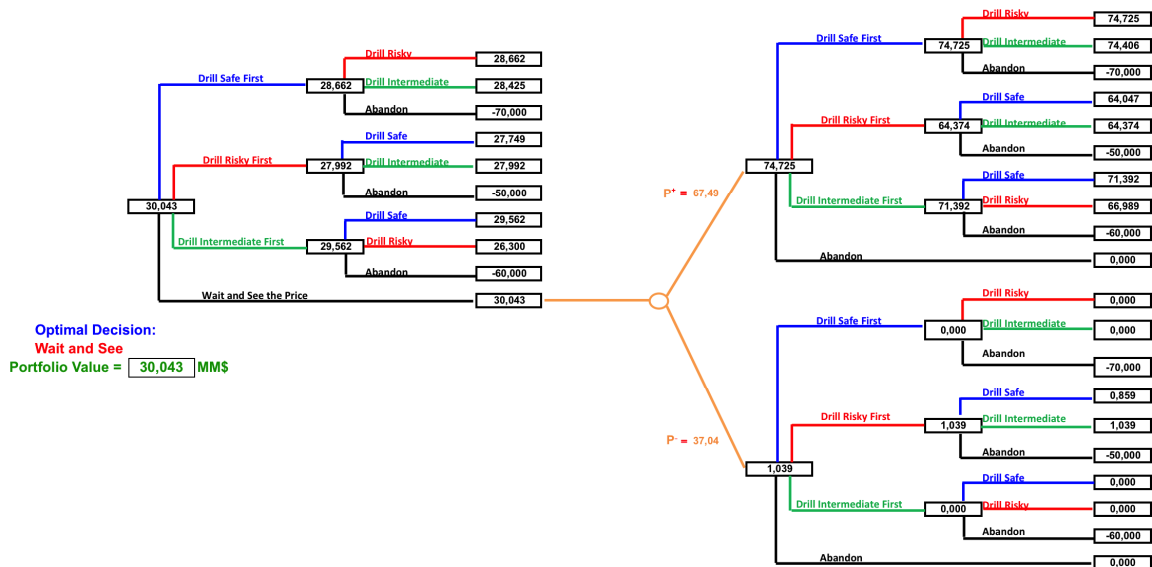


Figure 11 – Defer Option One Period for Case 1 – Three Prospects Sequential Options

Now, we use the multi-period binomial and that we have three years before the expiration ($T = 3$ years). With longer time to wait, the waiting value is even higher and the optimal is “wait and see”: with $T = 3$ years and $\Delta t = 0.5$ month, the waiting value rises to $W = 37.00$ million \$, which is 25% higher than drilling immediately the best prospect option (Intermediate, with 29.6 million). So, neglecting the defer option can destroy a significant value.

Proceeding with the methodology set in section 3, let us introduce some notation. Let $P_{R,L}$ be the lowest price in which we are indifferent between wait and drill the Risky first; let $P_{R,H}$ be the highest price in which we are indifferent between wait and drill the Risky first; let $P_{I,L}$ be the lowest price in which we are indifferent between wait and drill the Intermediate first; let $P_{I,H}$ be the highest price in which we are indifferent between wait and drill the Intermediate first and let P_S the indifference price between wait and drill the Safe first.

In order to investigate the regions of exercise and wait in the binomial, we change the initial price in the binomial tree and see the optimal action for a wide range of oil prices. If we perform this job for different time to expiration, we can draw the regions of exercise and wait. Figure 12 illustrates the case 1, showing the regions of exercise and regions of “wait and see”.

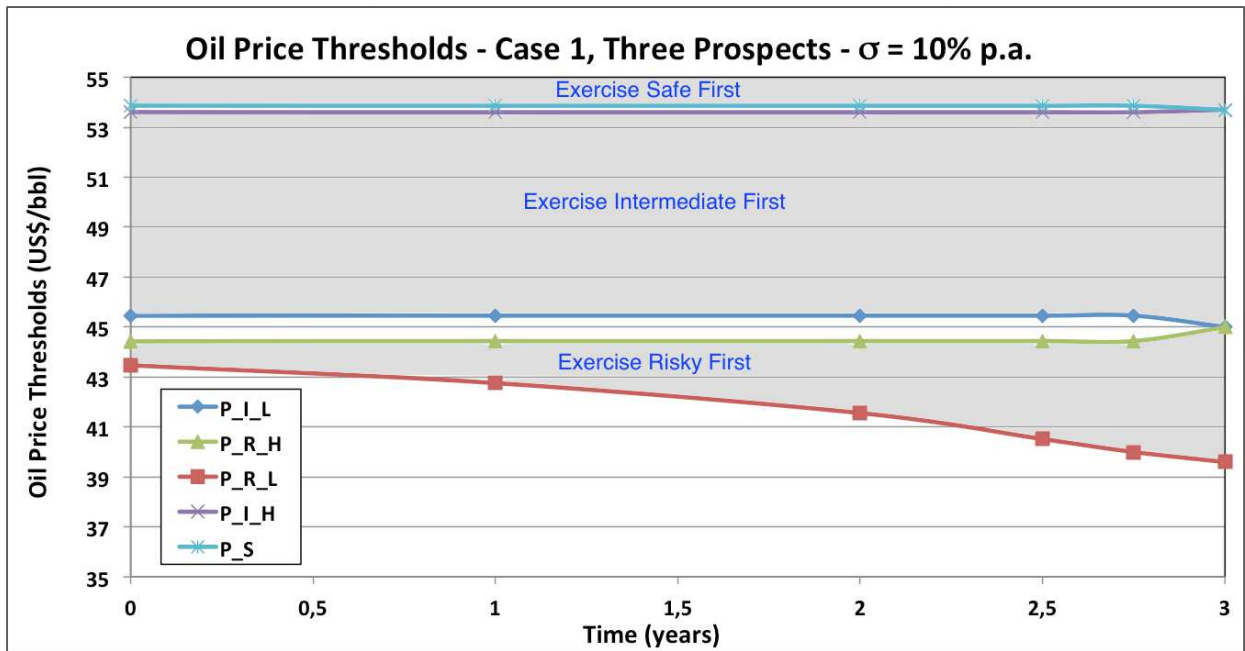


Figure 12 – Oil Price Thresholds for Three Prospects with Volatility of 10% p.a. ($\Delta t = 0.5$ month)

As expected, in Figure 12 there are 3 *disconnected regions of exercise*, with one *intermediate waiting region*. If the current oil price is exactly the break-even price the optimal is “wait and see”. Figure 5 indicates, as explained in section 3, that also, it is never optimal to exercise any option in this break-even price except at the expiration.

For a higher volatility, the intermediate waiting region can disappear for sufficient longer time to expiration. Figure 13 shows the same case 1, but for a higher volatility ($\sigma = 20\%$ p.a.) and for $\Delta t = 0.3$ month. Note that for $t = 0$ ($T = 3$ years), the intermediate inaction region does not exist. This region only appear for $t > 2.0$ years ($T < 1$ year).

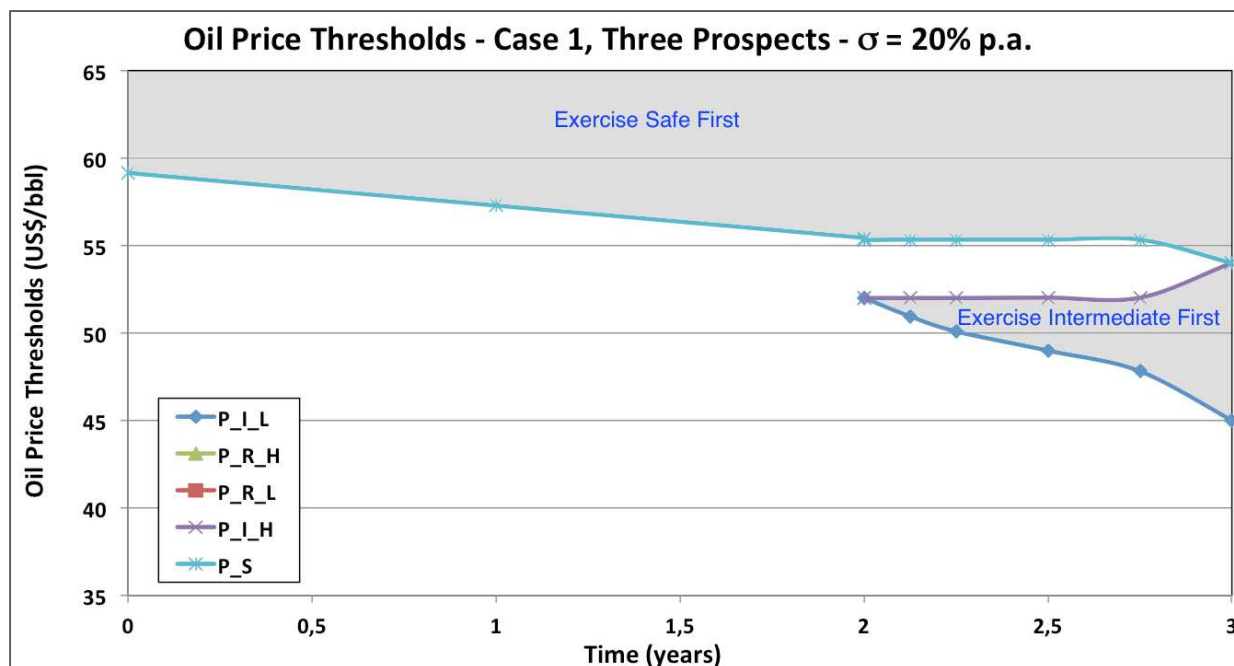


Figure 13 – Oil Price Thresholds for Three Prospects with Volatility of 20% p.a. ($\Delta t = 0.3$ month)

6) Conclusions and Suggestion of Extensions

In this article we approached the interaction of three different options: the learning option, the sequential option and the defer option. In contrast with the traditional exploratory economics, it could be optimal to drill a negative EMV prospect if there are correlated prospects so that there are learning options embedded when drilling a prospect. In addition, we saw that the drilling order matters even when the prospects have the same EMV and the same NPV in case of success. In this case of same EMV, we saw that could be optimal start drilling the riskier prospect (with lower chance factor). Also in contrast with the plain sight from traditional portfolio selection, it can be optimal to drill first the lower EMV prospect when it has a lower investment cost (so, cheaper to get information). We saw a case where, the prospect with the lower chance factor, the lower NPV and a negative EMV, should be drilled first due it's strong correlation with the others. Information revelation played a major role in managing the optimal exploratory sequence.

Introducing the defer option, we saw that even for positive exercise payoff could be better wait and see and neglecting the defer option can destroy significant value.

We found that in many cases we have disconnected exercise regions, an issue similar to what occurs in the literature of scale of investment options. This article shows that this issue is more general in the real options literature.

We can extend this work in several ways. One is to include synergy, as pointed in the text. Other planned research is to consider the perpetual option case that, although not realistic (there is an expiration in petroleum sector), is an upper bound for both the portfolio option value and for the highest threshold so that we can use this upper bounds to check the results from the numerical method for finite-lived case. Another extension possibility is to consider in more depth the delay option: in this work we assume that if it is optimal to drill one prospect, it is also optimal to develop the oilfield in case of exploratory success. For one prospect it is true because the oil price threshold to exercise the exploratory drilling option (P^{**}) is higher than the threshold P^* to develop the discovery oilfield ($P^{**} > P^*$), as shown by Dias (2006) and Dias (2015, ch. 27). However, especially for asymmetric prospects, we could consider the defer option after the first drilling even in case of success. This is left for a future research.

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