Optimal Dual-Sourcing: A Real Options Approach

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1 Introduction

To maximize profit in the manufacturing sector, businesses turn to offshore production because of the low manufacturing costs. However, due to uncertain consumer demand and long lead times, it is difficult to determine an optimal offshoring strategy. In order effectively hedge the risk associated with offshoring, a dual-sourcing model is proposed. The dual-sourcing model enables companies to maximize their profit while eliminating risk by applying a real options approach to decision making.

Consider the case of the production and sale of winter boots. As a seasonal item, winter boots experience high demand during the winter months. However, it is difficult for retailers to accurately determine the expected demand at a future date due to the unknown weather. Offshore lead times can be up to six months or more and the costs of over or under production can be significant. The problem can be simplified as a single-period stochastic demand problem. Our approach, provides an optimal order quantity for the unknown demand as a dual-sourcing model. A portion of the expected demand would be purchased from the offshore manufacturer months in advance, accounting for the large lead time required. As time moves forward, the expected demand for boots becomes more certain based on the seasonal changes, and another order will be placed from an onshore (local) manufacturer with a much shorter lead time to meet this demand. This effectively hedges the offshore manufacturing option and allows for profit maximization.

The purpose of this paper is to provide a psuedo-analytical solution to a single-period dual-sourcing problem for perishable or seasonal goods under a general demand distribution. Future work would consider multiple demand and sourcing periods from the local manufacturer to determine a more realistic scenario. This more complicated approach, however, will no longer be psuedo-analytical and will require numerical methods to solve. Furthermore, we can consider the demand to be an observable process correlated to a traded, which can be hedged to reduce profit uncertainty.

2 Literature Review

Profit maximization in the retail industry can be summarized by optimal inventory control. (K.J. Arrow, 1951) developed what is now known as the Newsvendor model as a means of determining optimal inventory levels for a perishable product with fixed costs and unknown demand. This model has been expanded both as single-sourcing and multiple sourcing models. (Mahmut Parlar, 1996) considered a model where disruptions in product supply are prevalent and developed average cost models for single and multiple suppliers under constant demand, utilizing a Markov chain process for the case of two suppliers. Analytical closed-form solutions for single period dual-sourcing inventory control with supply disruptions for both channels were developed by (Anastasios Xanthopoulos, 2011).

Assuming a stochastic supply (yield) with a known demand, in special cases such as agricultural production, (Keren, 2009) considers additive and multiplicative yield risks within a single-period demand. Expanding on this, assuming an unlimited supply capacity with unreliable yield, (Lei Shu, 2015) shows that the value of the optimized expected utility becomes less sensitive to the initial inventory level as the degree of risk aversion decreases. Very detailed reviews of various supply chain risks and their management styles have been done by (Tang, 2006) and (William Ho, 2015).

Our model does not consider disruptions on the supplier-end, and instead treats the local supplier as a source of hedging. It is standard practice in industry for managers to have an "emergency" supplier that can provide needed service during times of peak demand when standard inventory management does not suffice. Early research from (Barankin, 1961) focused exactly on this issue - an instantaneous delivery problem with a regular lead time of one period. For the case when the lead-time difference between the two suppliers was exactly one period, (Fukuda, 1964) determined that the optimal order policy is a dual-index policy (DIP). A much more complex problem of when the lead time differences are more than one period between the suppliers, the optimal policy becomes difficult to compute and has a complex structure (Whittemore, 1977). This issue of various multiple sourcing models is reviewed in depth by (Minner, 2003).

For situations when the difference between lead times is very large, (Veeraraghavan, 2008) showed that these cases perform very closely to that of a DIP by separating the optimization of a two-dimensional DIP to two one-dimensional optimization problems with deterministic lead times. (Joachim Arts, 2011) then went on to use Markov chains to optimize the DIP as two one-dimensional problems with stochastic regular lead times.

3 Model Formulation

Our model formulation proceeds as follows. Consider an unknown demand X faced by the local company (LC) for a certain item is assumed to be a positive stochastic random variable with a probability density function $f_X(x)$ and a cumulative distribution function $F_X(x)$. Assume the case of a LC requiring product at time T. They have the ability to order M units of the product from an offshore manufacturer at time t_0 at a cost per item of C_M. Similarly, they have the option to order U units from an onshore (local) manufacturer at time τ, in the future where $t_0 < \tau < T$, at a cost per item of C_U. The offshore manufacturer has a significantly longer production lead time than the local manufacturer so order quantities at times t_0 have to be decided long before the selling period T. On the other hand, the offshore manufacturer has much lower production cost than the local manufacturer ($C_M < C_U$). The units are sold for a price P per item and have a salvage value of P_{Salv}. In the event that the demand is higher than the amount of stocked units available for sale, the local company faces lost sales or a strategic price of P_{Strat} per unit.

The expected profit function faced by the LC based on the order quantities M and U from the offshore and onshore manufacturers, respectively, is given by

$$\begin{aligned} \mathbb{E}[Profit(M,U)] = \mathbb{E}[&min(X, M+U)P + P_{Salv} \cdot (M+U-X)^+ \\ &- P_{Strat} \cdot (X-M-U)^+ - MC_M - UC_U] \end{aligned} \tag{1}$$

which can be expanded into nested expectations based on the order times t_0 and τ as shown in the following equation

$$\begin{aligned} &\mathbb{E}[Profit(M,U)] = \\ &\quad \mathbb{E}[-MC_M + \mathbb{E}_\tau[\underbrace{min(X, M+U)P + P_{Salv} \cdot (M+U-X)^+ - P_{Strat} \cdot (X-M-U)^+ - UC_U}_{G(X;M,U)}]] \\ &= \mathbb{E}[-MC_M + \mathbb{E}_\tau[G(X;M,U)]] \end{aligned} \tag{2}$$

where $G(X;M,U)$ is the profit function at time $t = \tau$.

It is important to note that the order quantities from each of the suppliers is time dependent. Equation 1 has regularization terms P_{Strat} and P_{Salv} which will affect the order quantities based on their weight and the observable demand at the order time. If $P_{Strat} < C_U - P_{Salv}$ then the order quantity from the onshore manufacturer at time τ will be less than the expected demand at time t_0 and what was already ordered from the offshore manufacturer M i.e. $U < \mathbb{E}_{t_0}[X] - M$. This can be explained by the fact that lost sales have less of an impact on the company's profit compared to the purchase price from the offshore supplier. Similarly, if $P_{Strat} > C_U - P_{Salv}$ then $U > \mathbb{E}_{t_0}[X] - M$

and shows that the cost of ordering extra units from the offshore supplier has less of an impact than the possibility of lost sales.

Working recursively, we can maximize the profit function $G(X; M, U)$ based on the conditional probability density function of X_T at time $t = \tau$ and $y = X_\tau$; $f_{X_T|X_\tau}(x|y)$. By taking the derivative of $G(X; M, U)$ with respect to U and setting it equal to zero, the optimal onshore order quantity U^* can be determined for any value of M.

We can rewrite $\mathbb{E}_\tau[G(X; M, U)]$ as

$$\begin{aligned}\mathbb{E}_\tau[G(X; M, U)] = \int_{-\infty}^{\infty} [(x, M + U)P + P_{Salv} \cdot (M + U - x)^+ \\ - P_{Strat} \cdot (x - M - U)^+] f_{X_T|X_\tau}(x|y)dx - UC_U\end{aligned} \tag{3}$$

and maximize the expected profit by taking the derivative with respect to U and setting the equation equal to zero, the details of which are provided in Appendix A. This allows us to determine the optimal onshore order quantity U^* which maximizes the overall profit at time $t = \tau$

$$U^* = \underbrace{F^{-1}_{X_T|X_\tau}\left(\frac{P + P_{Strat} - C_U}{P + P_{Strat} - P_{Salv}}\middle| y\right)}_{\gamma(y)} - M \equiv \gamma(y) - M. \tag{4}$$

For convenience the term $\gamma(y)$ is introduced to represent the inverse cumulative distribution function of $\frac{P+P_{Strat}-C_U}{P+P_{Strat}-P_{Salv}}$ at time $t = \tau$. The derived $\gamma(y)$ resembles a modified newsvendor model, and it will be used subsequently to solve for the optimal offshore order quantity M.

As expected, U^* is largely dependent on the offshore order quantity M. With that in mind, it is important to take into account that U^* in Equation 4 can be negative under large values of M. This can be explained under the circumstance where the LC is short-selling the local units, since the assumption was previously made that the offshore unit cost is always less than onshore unit ($C_M < C_U$). This strategy is unacceptable and a more accurate representation of U^* is

$$U^*(y) = max(\gamma(y) - M, 0) = (\gamma(y) - M)^+. \tag{5}$$

It is now possible to substitute the expression for U^* into Equation 1 to determine the profit function at time $t = t_0$ based on the optimal local order quantity U^*,

$$\mathbb{E}[Profit(M, U^*)] = \int_{-\infty}^{\infty}\left(-MC_M + \int_{-\infty}^{\infty} G(x; M, U^*(y)) f_{X_T|X_\tau}(x|y)dx\right) f_{X_\tau}(y)dy \tag{6}$$

Equation 6 can be reduced to (see Appendix B)

$$\begin{aligned}\mathbb{E}[Profit] = \int_{\gamma^{-1}(M)}^{\infty} &\big[\gamma(y) \cdot (P + P_{Strat} - C_U) + \gamma(y) \cdot F_{X_T|X_\tau}(\gamma(y)|y) \cdot (P_{Salv} - P - P_{Strat}) \\ &- P_{Strat} \cdot G(y; \gamma(y), \infty) + (P - P_{Salv}) \cdot G(y; 0, \gamma(y)) \\ &+ M \cdot (C_U - C_M)\big] f_{X_\tau}(y)dy + \\ \int_0^{\gamma^{-1}(M)} &\big[M \cdot (P + P_{Strat} - C_M) + M \cdot F_{X_T|X_\tau}(M|y) \cdot (P_{Salv} - P - P_{Strat}) \\ &+ G(y; 0, M) \cdot (P - P_{Salv}) - P_{Strat} \cdot G(y; M, \infty)\big] f_{X_\tau}(y)dy.\end{aligned} \tag{7}$$

The equation for $\gamma^{-1}(M)$ is based on the underlying distribution that is used to approximate the unknown demand. Appendix C shows an example of the derivation of $\gamma^{-1}(M)$ for the normal distribution. Taking Equation 7; differentiating it with respect to M and and setting it equal to zero while applying the integration initiated previously provides us with a final psuedo-analytical expression for the optimal offshore order quantity M^* which can be solved using simple numerical methods or computer modeling

$$0 = C_U - C_M + (P + P_{Strat} - C_U) \cdot F_{X_\tau}(\gamma^{-1}(M)) - (P - P_{Salv} - P_{Strat}) \int_{-\infty}^{\gamma^{-1}(M)} F_{X_T|X_\tau}(M|y) f_{X_\tau}(y) dy. \tag{8}$$

4 Results

Preliminary results are presented in this section. Figure 1 below shows the importance of choosing a distribution which best represents the expected demand of the underlying asset. Running the model, all parameters were kept constant, modifying only the stochastic process between Brownian motion and geometric Brownian motion with varying initial expected demand X_0.

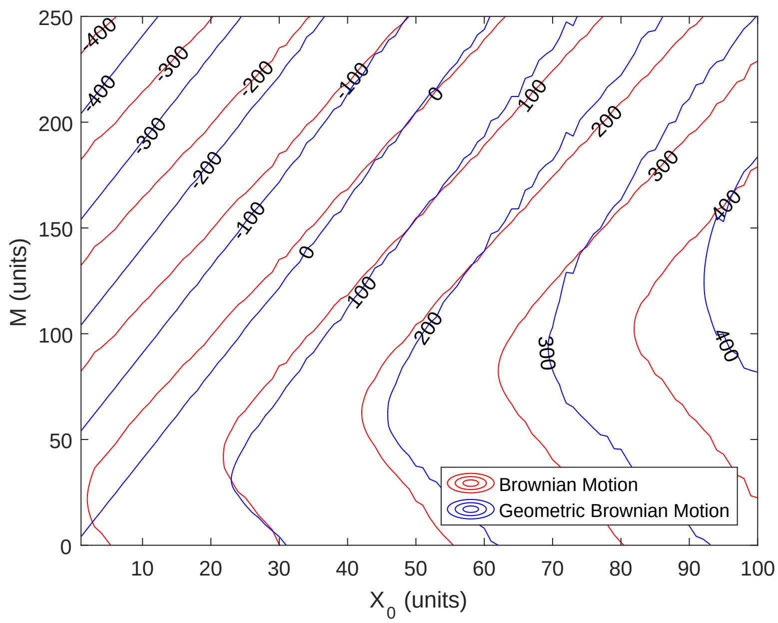


Figure 1: Expected profit based on stochastic process;
Brownian Motion: Drift = 20; Volatility = 30
Geometric Brownian Motion: Drift = 0.2; Volatility = 0.3
$P = 10, P_{Salv} = 3, P_{Strat} = 4, C_U = 6, C_M = 5, T = 5/12, \tau = 2/12$

The expected profit (contours) based on the initial expected demand and varying offshore production quantities (M) fluctuates greatly depending on the distribution used. This is especially true with increasing initial expected demand, as the contours between chosen processes begin to cross boundaries and can provide highly inaccurate results. It is imperative that the users of this model understand the expected behavior of their underlying asset either through historical experience or other stochastic processes.

Alongside, the implied drift and variance also have an effect on the expected profit. Using a Brownian motion process while modifying drift and variance, Figure 2 shows a contour of the expected profit with respect to the initial expected demand X_0.

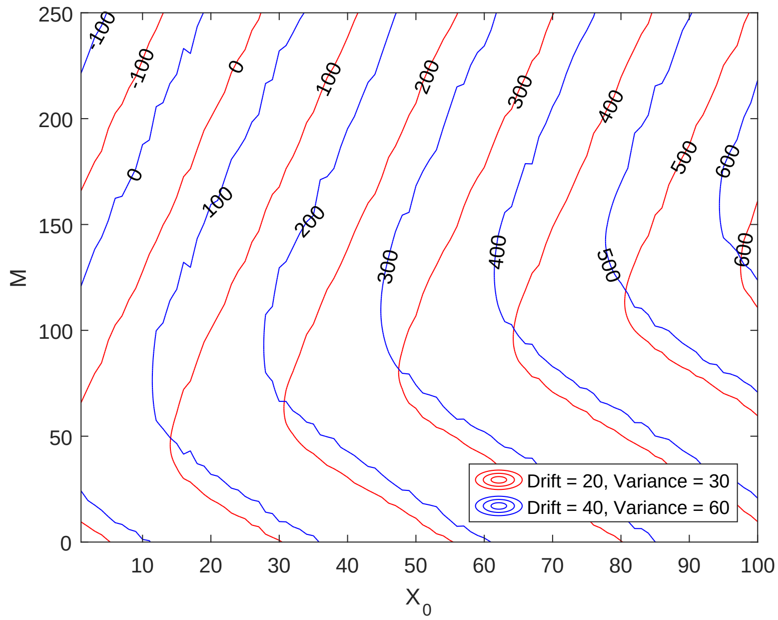
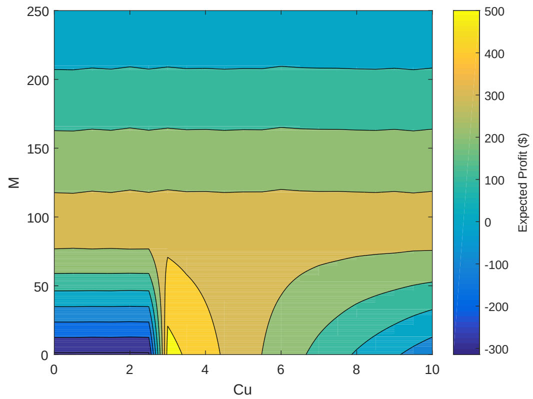


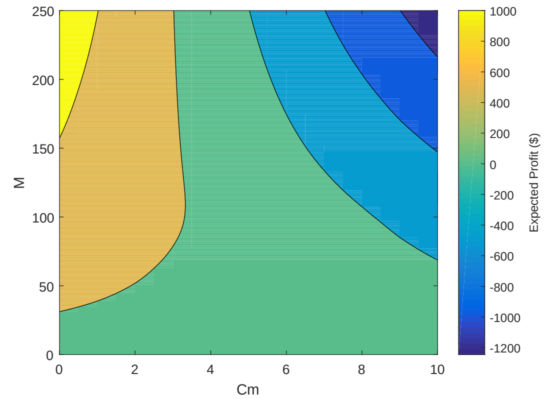
Figure 2: Effect of drift and variance on expected profit

As expected, an increase in drift will typically lead to an increase in the expected profit under constant X_0 and M. This can be explained by the fact that the LC is expecting a sharper increase in demand as time progresses, as is noticed by seasonal items and much less so for year-round goods. Modification of variance affects the expected profit as well, but nowhere near to the the extent that the underlying distribution or drift do. A large variance causes disturbances and uncertainties in pinpointing the optimal offshore and onshore order quantities. However this is usually negligible compared to determining an accurate drift and stochastic process assumption.

The psuedo-analytical model does have its limitations. For example, when $C_U = P_{Salv}$, the expected profit is undefined and approaches infinity as visible in Figure 3a below. This behavior is expected and consistent with Equation 4 as the inverse cumulative distribution function simplifies to $F^{-1}_{X|X_\tau}(1; y) = \infty$.



(a) $P_{Salv} = 2.75,\ C_M = 5,\ P = 10$



(b) $P_{Salv} = 2.75,\ C_U = 5,\ P = 10$

Figure 3: Expected profit contour

For low values of C_U ($C_U < P_{Salv}$), the offshore order quantity M has a much higher impact on the expected profit. When the local production cost is less than the salvage value, ordering a small quantity of units from the offshore source causes a large loss in potential profit. And as expected, it can also be seen that the offshore order quantity has little to no effect on the expected profit when the offshore and local production costs are equal ($C_U = C_M$).

In contrast, the offshore production price C_M is not limited by the salvage value of the asset as shown in Figure 3b. The expected profit based on the offshore production price behaves conventionally, providing maximum profit when C_M is minimized.

The local order time τ and end time T have little effect on the maximum expected profit as shown in Figure 4 below under most circumstances. Only in the case where the local order time is equal to the the end time ($\tau = T$), does the maximum expected profit drop significantly as shown by the large downward spikes in profit. This is because the time the local order is placed is typically based on the lead time set by the local manufacturer. Therefore, orders placed at a time where $\tau = T$ cause a large decrease in maximum expected profit because the goods cannot be manufactured instantaneously and the LC is effectively not taking advantage of the hedging option.

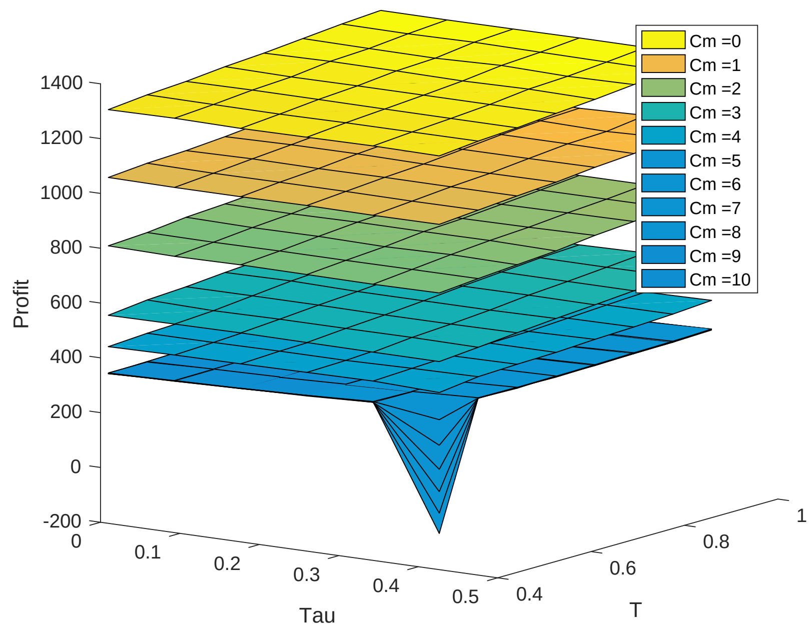


Figure 4: Maximum expected profit;
$C_U = 5$, $\tau = 0 : \frac{5}{12}$, $T = \frac{5}{12} : 1$

Conversely, the maximum expected profit is stagnating once $C_M = C_U$. As noted before, this behavior is predicted because the LC would instead then choose the option to attain all of their goods from the local manufacturer. Once it does become profitable for the LC to acquire a portion of their assets from the offshore manufacturer ($C_M > C_U$), we see the maximum expected profit increase; creating a larger profit margin at each index of C_M.

5

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Appendices

A Supplementary Derivation of U*

Equation 3 can be separated into sections $A1 - A4$ as follows:

$$A1 = \int_{-\infty}^{\infty} (1_{x<M+U} \cdot x + 1_{x>M+U} \cdot (M+U)) \cdot P \cdot f_{X_T|X_\tau}(x|y)dx, \tag{9}$$

$$\begin{aligned}\frac{dA1}{dU} &= \int_{\infty}^{\infty} 1_{x>M+U} \cdot P \cdot f_{X_T|X_\tau}(x|y)dx \\ &= \int_{M+U}^{\infty} P \cdot f_{X_T|X_\tau}(x|y)dx \\ &= P \cdot (1 - F_{X_T|X_\tau}(M+U|y)),\end{aligned} \tag{10}$$

$$A2 = \int_{-\infty}^{\infty} P_{Salv} \cdot 1_{x<M+U} \cdot (M+U-x) \cdot f_{X_T|X_\tau}(x|y)dx, \tag{11}$$

$$\begin{aligned}\frac{dA2}{dU} &= \int_{\infty}^{\infty} P_{Salv} \cdot 1_{x<M+U} \cdot f_{X_T|X_\tau}(x|y)dx \\ &= \int_{0}^{M+U} P_{Salv} \cdot f_{X_T|X_\tau}(x|y)dx \\ &= P_{Salv} \cdot F_{X_T|X_\tau}(M+U|y),\end{aligned} \tag{12}$$

$$A3 = \int_{-\infty}^{\infty} P_{Strat} \cdot 1_{x>M+U} \cdot (x-M-U)) \cdot f_{X_T|X_\tau}(x|y)dx, \tag{13}$$

$$\begin{aligned}\frac{dA3}{dU} &= -\int_{\infty}^{\infty} P_{Strat} \cdot 1_{x>M+U} \cdot f_{X_T|X_\tau}(x|y)dx \\ &= -\int_{M+U}^{\infty} P_{Strat} \cdot f_{X_T|X_\tau}(x|y)dx \\ &= -P_{Strat} \cdot (1 - F_{X_T|X_\tau}(M+U|y)),\end{aligned} \tag{14}$$

$$A4 = UC_U, \tag{15}$$

$$\frac{dA4}{dU} = C_U, \tag{16}$$

where $F_{X_T|X_\tau}(M+U|y)$ is the cumulative distribution function of X at time $t = \tau$ and $y = X_\tau$.

B Reduction of Equation 6

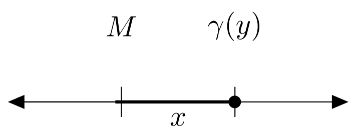
Similar to the derivation of U^*, Equation 6 can be separated into

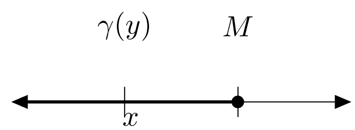
$$
\begin{aligned}
B_1 &= \int_0^\infty (1_{x<M+U} \cdot x + 1_{x>M+U} \cdot (M+U)) \cdot P \cdot f_{X_T|X_\tau}(x|y)dx, \\
B_2 &= \int_0^\infty (1_{x<M+U} \cdot (M+U-x)) \cdot P_{Salv} \cdot f_{X_T|X_\tau}(x|y)dx, \\
B_3 &= \int_0^\infty (1_{x>M+U} \cdot (x-(M+U))) \cdot P_{Strat} \cdot f_{X_T|X_\tau}(x|y)dx, \\
B_4 &= \begin{cases} -MC_M - (\gamma(y)-M)C_U & \text{if } \gamma(y) \geq M \\ -MC_M & \text{if } \gamma(y) < M. \end{cases}
\end{aligned} \tag{17}
$$

Appropriate boundary conditions can now be applied to the functions in Equation 17, as can be seen for $B1$:

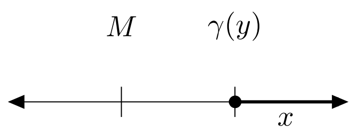
$$B_1 = \int_0^\infty (1_{x<M+U} \cdot x + 1_{x>M+U} \cdot (M+U)) \cdot P \cdot f_{X_T|X_\tau}(x|y)dx$$

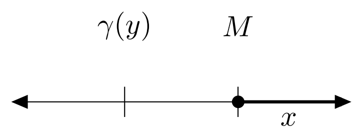
$$1_{x<M+(\gamma(y)-M)^+} \rightarrow \begin{cases} 1_{x<\gamma(y)} & \text{if } \gamma(y) \geq M \\ 1_{x<M} & \text{if } \gamma(y) < M \end{cases} = 1_{\gamma(y)>M} \cdot 1_{x<\gamma(y)} + 1_{\gamma(y)<M} \cdot 1_{x<M}$$





$$1_{x>M+(\gamma(y)-M)^+} = 1_{\gamma(y)>M} * 1_{x>\gamma(y)} + 1_{\gamma(y)<M} * 1_{x>M}$$





$$\therefore B_1 = P \begin{cases} \displaystyle\int_0^{\gamma(y)} x * f_{X_T|X_\tau}(x|y)dx + \int_{\gamma(y)}^\infty \gamma(y) * f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ \displaystyle\int_0^M x \cdot f_{X_T|X_\tau}(x|y)dx + \int_M^\infty M \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) < M. \end{cases} \tag{18}$$

Applying a similar process, functions $B2$ and $B3$ simplify to

$$B_2 = P_{Salv} \begin{cases} \displaystyle\int_0^{\gamma(y)} (\gamma(y)-x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ \displaystyle\int_0^M (M-x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) < M \end{cases} \tag{19}$$

$$B_3 = P_{Strat} \begin{cases} \displaystyle\int_{\gamma(y)}^\infty (x-\gamma(y)) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ \displaystyle\int_M^\infty (x-M) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) < M. \end{cases} \tag{20}$$

By defining $G(y;a,b) = \int_a^b x f_{X_T|X_\tau}(x|y)dx$ we are able simplify

$$\int_0^{\gamma(y)} (\gamma(y) - x) \cdot f_{X_T|X_\tau}(x|y)dx = \gamma(y) \cdot F_{X_T|X_\tau}(\gamma(y)|y) - G(y;0,\gamma(y)); \tag{21}$$

and apply this to Equations 18, 19, and 20 to reduce them to

$$\therefore B_1 = P \begin{cases} G(y;0,\gamma(y)) + \gamma(y) \cdot (1 - F_{X_T|X_\tau}(\gamma(y)|y)) & \text{if } \gamma(y) \geq M \\ G(y;0,M) + M \cdot (1 - F_{X_T|X_\tau}(M|y)) & \text{if } \gamma(y) < M \end{cases} \tag{22}$$

$$\begin{aligned} \therefore B_2 =& P_{Salv} \begin{cases} \int_0^{\gamma(y)} (\gamma(y) - x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ \int_0^{M} (M - x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) < M \end{cases} \\ =& P_{Salv} \begin{cases} \gamma(y) \cdot F_{X_T|X_\tau}(\gamma(y)|y) - G(y;0,\gamma(y)) & \text{if } \gamma(y) \geq M \\ M \cdot F_{X_T|X_\tau}(M|y) - G(y;0,M) & \text{if } \gamma(y) < M \end{cases} \end{aligned} \tag{23}$$

$$\begin{aligned} \therefore B_3 =& P_{Strat} \begin{cases} \int_{\gamma(y)}^{\infty} (x - \gamma(y)) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ \int_{M}^{\infty} (x - M) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) < M \end{cases} \\ =& - P_{Strat} \begin{cases} \gamma(y)[1 - F_{X_T|X_\tau}(\gamma(y)|y)] - \int_{\gamma(y)}^{\infty} (x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \\ M[1 - F(M);y)] - \int_{M}^{\infty} (x) \cdot f_{X_T|X_\tau}(x|y)dx & \text{if } \gamma(y) \geq M \end{cases} \end{aligned} \tag{24}$$

$$= - P_{Strat} \begin{cases} \gamma(y) - \gamma(y) \cdot F_{X_T|X_\tau}(\gamma(y)|y) - G(y;\gamma(y),\infty) & \text{if } \gamma(y) \geq M \\ M - M \cdot F_{X_T|X_\tau}(M|y) - G(y;M,\infty) & \text{if } \gamma(y) < M \end{cases} \tag{25}$$

which can be substituted back into the original equation and reduced to the form presented in Equation 7.

C Derivation of $\gamma^{-1}(M)$

$$\begin{aligned} F(M + U^*(y), y) &= \frac{P + P_{Strat} - C_U}{P + P_{Strat} - P_{Salv}} \\ \frac{1}{2} + \frac{1}{2} erf\left(\frac{M + U^*(y) - y}{\sigma\sqrt{2}}\right) &= \frac{P + P_{Strat} - C_U}{P + P_{Strat} - P_{Salv}} \\ U^*(y) &= \underbrace{y + \sigma\sqrt{2} * erf^{-1}\left[2\left(\frac{P + P_{Strat} - C_U}{P + P_{Strat} - P_{Salv}}\right) - 1\right]}_{\gamma(y)} - M \end{aligned}$$

Noting that $\gamma(y)$ has to be greater than M allows us to rearrange the equation to:

$$\gamma^{-1}(M) = M - \sigma\sqrt{2} * erf^{-1}\left[2\left(\frac{P + P_{Strat} - C_U}{P + P_{Strat} - P_{Salv}}\right) - 1\right]. \tag{26}$$