The Innovator’s Dilemma revisited: Investment Timing, Capacity Choice, and Product Life Cycle under Uncertainty

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Abstract:

Innovative industries like the semiconductor or flat-panel industry have been a driving momentum of global growth for decades. However, global competition, constant decline of output prices, rocketing costs, and shorter life-cycles put great stress on making the right investment policy. By means of a Markov-regime switching model we model the simultaneous choice of optimal investment timing and capacity under uncertainty in continuous time. Our results indicate that, the threat of disruptive technological change lead to install less capacity later. If uncertainty levels in each demand regime are different, we find that both optimal capacity and timing threshold become ambiguous. For low degrees of uncertainty in the decline regime, an increase of uncertainty leads to an increase of the investment threshold and a decrease of the optimal capacity in the growth regime.

Keywords: product life cycle; investment timing; investment size; real options
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1. Introduction

Moore's Law is dead - Long live Moore's Law! It is little bit longer than half a century ago when IBM's co-founder Gordon Moore predicted that every 18 months the number of transistors on a die will double (Moore, 1965). Ever since, the semiconductor industry has become a global key industry worth $335.2 billion in 2015 and making-off roughly 10% of the world's gross domestic product (GDP).[1] Concurrent to this development Moore's projected technological progress has spawned three distinct economic mechanisms that characterize this industry today. Firstly, since the first commercial microprocessors were introduced in the early 1970s, each chip generation has shown a unique sales pattern over time. As Figure 1a exemplifies, there is a certain bell-shape feature immanent indicating that after a period of remarkable growth the sales of a particular chip generation soon start to decline -after a neglectable mature phase- due to *technological obsolescence.* Second, the constant drive for innovation has triggered a *permanent decline of chip prices*, which encompass not only intra- but inter-chip-generation periods of time (see Figure 1b). Thirdly, cracking the physical gap size between the millions of transistors on a microprocessor down to 22 nanometers has caused an enormous shift in capital spending over the years. This is mainly driven by the fact that the more density packed transistors on a chip the more complex the manufacturing process. Exemplary, building a state-of-the-art manufacturing facility –fab- for the next generation of microprocessors cost approximately US-$ 7 bill. while this could rise up to US-$ 16 bill. by 2020 which represents almost one-third of Intel's current annual revenue (The Economist, 2016). Obviously,

[1] See Semiconductor Industry Association (SIA) at www.semiconductor.org

most of this capital is sunk and thus *irreversible* due to the asset specifity of these fabs.

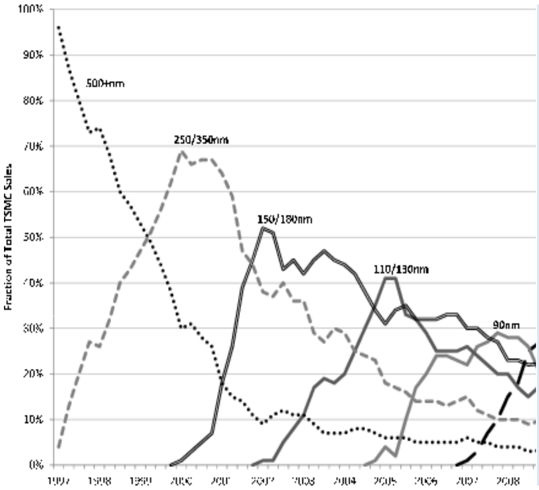

Fig. 1a: Annual Sales of Chip Generations between 1970-2010 of Taiwan Semiconductor Manufacturing Company (TSMC)
Source: TSMC Quarterly Reports

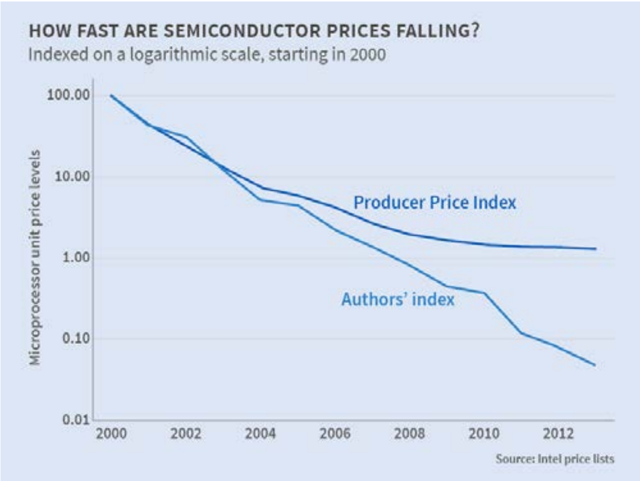

Fig. 1b: Output Prices of Semiconductor Chips between 2000-2012,
Source: Byrne et al. (2015)

The stylized facts presented demonstrate that decision makers in such and other innovative industries are confronted more that others with what is coined the innovator's dilemma, i.e. the permanent choice of incumbent firms between holding onto an existing market and risk suffering from future disruptive technologies or undertake risky investments to promote new innovations (Christensen, 2000). While this dilemma put great stress on one dimension of strategic financial decision making, i.e. *invest* or *not invest,* and timing the *switch* to another technology, respectively it neglects another important dimension, i.e. investment scale. Going back to the semiconductor industry example, the installed capacity increased steadily despite the increased sunk cost and accelerated technological obsolescence over years. Given these figures, it remains unclear why to sunk so much irreversible capital, build up significant fab capacity if the possibilities to recoup the cost shrinks constantly?

Apparently, the irreversibility of the capital expenditures, the significant degree of economic and technological uncertainty, as well as the permanent necessity to choose the right capacity at the right time to remain part in the global pool of innovators make these decision problems an ideal object for investigation by means of real option analysis. However, analyzing the effect of product-life-cycles (PLC) as well as joint decision making with respect to timing-and-scale has only recently gained momentum.

In the following paper, we thus want to analyze the effect life-cycle uncertainty, deteriorating product prices and irreversibility have on the simultaneous choice of timing and scaling an investment. Obviously, our paper is not the first to investigate these features in the domain of real option modeling. A significant number of papers has emerged investigated the timing-scaling relationship of an individual firm (see e.g. Dixit, 1993; Bengtsson & Olhager, 2002; Bøckman, et al., 2008; Fontes, 2008 Wong, 2010; Sarkar, 2011; Della Seta et al. 2012, among others). A key finding of this research is that larger project uncertainty delays investment thereby increasing the scale of the project. Notably, Della Seta et al. (2012) show that in industries which exhibit learning economies, learning can have a moderating effect on the timing-scale trade-off, i.e. faster knowledge accumulation motivate early entry on a smaller scale even if uncertainty is high. A second strain of literature deals with alternative ways to model the cyclical nature of cash flow and output quantities, respectively (see e.g. Farzin, Huisman, & Kort, 1998; Bollen, 1999; Driffill et al., 2003; Gutiérrez 2005;Funke & Chen, 2010; Gutiérrez & Ruiz-Aliseda 2011, Koussis et al. 2013, among others)

A closer look reveals an important shortcoming that our paper tries to overcome. So far, the timing-scale trade-off and the optimal investment policy, respectively have always been analyzed under the premise of exponential growth that is rarely observed in reality. However, there are three recent exceptions found in the literature that our work tries to bridge and to advance. First, the paper by Oshikoji (2016) is the only one we know of that discusses the simultaneous choice of timing and scaling by means of a regime-switching model. Here, output prices follow a Markov-switching process and the technological uncertainty controls for the switch between two possible exponentially growing output prices. His findings reveal that once the capacity and timing decision is made under both price and technological uncertainty the firm invests both earlier and in limited capacity. Second, Lukas, Spengler, Kupfer, & Kieckhäfer (2017) apply a stochastic version of the Bass-model to investigate the timing-scale trade-off in the context of dimensioning a facility. In such a setting, the optimal investment threshold becomes S-shaped, is monotonically increasing over the PLC, and segmented according to the optimal capacity to be installed. Moreover, the way how technology is expected to diffuse over time significantly shapes the optimal investment policy and the results indicate that the optimal investment threshold and capacity choice may become ambiguous over time.

Finally, the paper by Hagspiel, Huisman, Kort, & Nunes (2016) models a situation where the firm operates in a declining market and has the choice between either exit this industry or invest in a new technology. The optimal switching policy is characterized not only by an optimal investment timing but optimal capacity decision, too. Although the firm is only allowed to switch from a declining market to a growing or another declining market thereby neglecting technological risk and the characteristics of the new product's life-cycle the results foreshadow that if quantities have a strong negative effect on prices the firm might invest later in less capacity.

Our attempt is to combine the different aspects of this literature. In particular, we want to enrich the literature on the timing-scaling trade-off in the domain of real option modeling by explicitly taking deteriorating product prices and a product generation life-cycle into account. Hence, we borrow from Oshikoji (2016) the modeling idea of disruptive technological change by means of a Markov-regime switching approach. But instead of modeling a regime switch between exponentially growing output prices we focus on a switch between positive and negative demand over time in order to stronger emphasize the PLC pattern in innovative industries.[2] Moreover, we borrow from Hagspiel, Huisman, Kort, & Nunes (2016) the idea of declining markets by taking a downward sloping output price into account.

The rest of the paper is organized as follows. First, we outline the model assumption and present the optimal investment policy. Second, we perform a comparative-static analysis to discuss the impact of major inter- and intra-new product generations on the firm's optimal investment policy. Finally, we conclude and provide an outlook for further studies.

[2] It should be noted, that Bollen (1999) was among the first to model a regime-switching model in the context of the semiconductor industry thereby taking account of both a growing and a declining regime. While the model allowed for constanly updating capacity choices during the life of a technology it neglects, however, the ability to delay the investment.

2. The Model

Assume a company which discounts with the riskless rate r and observes the quantity $q(t)$ of a product sold at any time t which is governed by a geometric Brownian motion Markov switching process

$$dq(t) = \alpha_i q(t)dt + \sigma_i q(t)dW(t) \qquad for\ i = 1, 2 \tag{1}$$

where $dW(t)$ is the increment of a Wiener process and α_i is the drift parameter and σ_i is a measure for uncertainty for the current state i.

We denote $i = 1$ as the growth state during which the cash flows from product sales are expected to increase and $i = 2$ as the decline state in which cash flows start to decline. The two states are connected by the transition matrix

$$\begin{bmatrix} 1 - \lambda dt & \lambda dt \\ \theta dt & 1 - \theta dt \end{bmatrix}, \tag{2}$$

where λdt denotes the probability to switch from growth to decline and θdt denotes the probability to switch from decline to growth. We will assume that once the product has entered the decline phase it will not return to growth, so that $\theta = 0$.

Furthermore, the company observes declining prices $p(t)$ of this product which follow the process

$$dp(t) = -\omega p(t)dt. \tag{3}$$

where ω denotes the rate at which prices deteriorate over time. Thus, the cash flows $x(t)$ of the products sold at time t are the multiple of the price and quantity at this time, i.e. $x(t) = p(t)q(t)$. Hence, infinitesimal changes in earnings flow $x(t)$ can be calculated by following Dixit & Pindyck (1994, p. 81 f.) :

$$dx(t) = \frac{\partial x(t)}{\partial p}dp + \frac{\partial x(t)}{\partial q}dq + \frac{1}{2}\frac{\partial^2 x(t)}{\partial p \partial q}dpdq. \tag{4}$$

Using $\frac{\partial x(t)}{\partial p} = q$, $\frac{\partial x(t)}{\partial q} = x$ and $\frac{\partial^2 x(t)}{\partial p \partial q} = 1$ we get

$$dx(t) = p(t)dq + q(t)dp + dpdq. \tag{5}$$

Substituting with Eq. (1), Eq. (3) and ignoring terms that go to zero faster than dt $dx(t)$ can be expressed by the following stochastic differential equation:

$$dx(t) = (\alpha_i - \omega)x(t)dt + \sigma_i x(t)dW(t) \qquad for\ i = 1, 2 \tag{6}$$

which is again a geometric Brownian motion.

In the following, we will first derive the current expected value of the future cash flows $V(t)$, i.e. the value of an actively sold product. As the product sales cannot switch from decline to growth, we will begin to value the project in the decline phase. The value of the project is the value of the current cash flows $x(t)$ over the next small time intervall dt and its continuation value which can be stated as

$$V_2(x(t)) = x(t)dt + \mathbb{E}[V_2(x(t) + dx)e^{-rdt}]. \tag{7}$$

This can be expanded using Ito's Lemma and expressed as

$$\begin{aligned} V_2(x(t)) = x(t)dt \\ + \left[(\alpha_2 - \omega)x(t)\frac{\partial V_2(x(t))}{\partial x} + \frac{1}{2}\sigma_2^2 x^2(t)\frac{\partial^2 V_2(x(t))}{\partial x^2}\right]dt \\ + (1 - rdt)V_2(x(t)) \end{aligned} \tag{8}$$

and after dividing by dt and rearranging as

$$\frac{1}{2}\sigma_2^2 x^2(t)\frac{\partial^2 V_2(x(t))}{\partial x^2} + (\alpha_2 - \omega)x(t)\frac{\partial V_2(x(t))}{\partial x} - rV_2(x(t)) + x(t) = 0. \tag{9}$$

The homogeneous part of the equation has the solution $Ax^{\beta}(t)$, when β is the root of the fundamental quadratic equation

$$\frac{1}{2}\sigma_2^2\beta(\beta - 1) + (\alpha_2 - \omega)\beta - r = 0. \tag{10}$$

Thus, the general solution of the homogenous part is a combination of the solutions $A_1 x^{\beta_1}(t)$ and $A_2 x^{\beta_2}(t)$ to which we need to a particular solution of the equation. It can be shown that $\frac{x(t)}{r-\alpha_2+\omega}$ is a possible solution so that complete solution is

$$V_2(x(t)) = A_1 x^{\beta_1}(t) + A_2 x^{\beta_2}(t) + \frac{x(t)}{r-\alpha_2+\omega}. \tag{11}$$

The last term on the right hand side is simply the expected current value of the future cash flows $x(t)$ when they are assumed to accrue infinitely. We can rule out the first term of the right hand side as the value of the project will turn to zero as the geometric Brownian motion turns to zero. The second term is attributed to speculative bubbles as $x(t) \rightarrow \infty$ and will also be ruled out (Dixit & Pindyck, 1994, p. 181 f.). This leaves the value of the active project in the decline phase to be

$$V_2\big(x(t)\big) = \frac{x(t)}{r-\alpha_2+\omega} = V_2\big(p(t)q(t)\big) = \frac{p(t)q(t)}{r-\alpha_2+\omega}. \tag{12}$$

Next, we turn to the growth phase. Equivalently to the decline phase, over the next small time interval dt the project is expected to yield the cash flow $x(t)$. However, over this time interval there is also the probability λdt that the growth phase will switch to the decline phase. We can express this as

$$V_1(x(t)) = x(t)dt + (1-\lambda dt)\mathbb{E}[V_1(x(t)+dx)e^{-rdt}] + \lambda dt\mathbb{E}[V_2(x(t)+dx)e^{-rdt}]. \tag{13}$$

Using Ito’s Lemma and after rearranging we get

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 V_1\big(x(t)\big)}{\partial x^2} + (\alpha_1-\omega)x(t)\frac{\partial\, V_1\big(x(t)\big)}{\partial x} - rV_1\big(x(t)\big) + x(t) + \lambda\Big(V_2\big(x(t)\big) - V_1\big(x(t)\big)\Big) = 0 \tag{14}$$

or

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 V_1\big(x(t)\big)}{\partial x^2} + (\alpha_1-\omega)x(t)\frac{\partial\, V_1\big(x(t)\big)}{\partial x} - (r+\lambda)V_1\big(x(t)\big) + x(t) + \lambda V_2\big(x(t)\big) = 0. \tag{15}$$

We can now easily substitute the value of $\mathrm{V}_2\big(\mathrm{x(t)}\big)$ into the equation and get

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 V_1\big(x(t)\big)}{\partial x^2} + (\alpha_1-\omega)x(t)\frac{\partial\, V_1\big(x(t)\big)}{\partial x} - (r+\lambda)V_1\big(x(t)\big) + x(t) + \lambda\frac{x(t)}{r-\alpha_2+\omega} = 0. \tag{16}$$

Again, we can try the solution $Hx^{\gamma}(t)$ for the homogeneous part of the equation, where γ is the root of the fundamental quadratic equation

$$\frac{1}{2}\sigma_0^2\gamma(\gamma-1) + (\alpha_1-\omega)\gamma - (r+\lambda) = 0. \tag{17}$$

Hence,

$$V_1\big(x(t)\big) = H_1 x^{\gamma_1}(t) + H_2 x^{\gamma_2}(t) + \frac{\frac{x(t)}{r-\alpha_2+\omega}}{r-\alpha_1+\omega+\lambda} + \frac{x(t)}{r-\alpha_1+\omega+\lambda}. \tag{18}$$

represents a possible particular solution. As before, we assume that $V_1(0) = 0$ to rule out the first term on the right hand side and exclude speculative bubbles to eliminate the second term. Collecting the last to terms we can thus write the value of the project in the growth state as

$$V_1\big(x(t)\big) = \frac{x(t)(r-\alpha_2+\omega+\lambda)}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} = V_1\big(p(t)q(t)\big) = \frac{p(t)q(t)(r-\alpha_2+\omega+\lambda)}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)}. \tag{19}$$

In both regime, we assume that the investment can be scaled. The investment costs and the cash flows of the product will both depend on the choice of the endogenous capacity parameter $c_i \in (0,1)$ with $i = (1,2)$, i.e. we assume that investment costs are

$$I_i(c_i) = \frac{c_i}{1-c_i} \tag{20}$$

and in return the investor will not get the full value of the future cash flows V_i but only the proportion $c_i V_i$. Thus, the expected value upon investment π_i in the growth or decline phase respectively will be

$$\pi_i(x, q_i) = c_i V_i(x) - I(c_i). \tag{21}$$

At the time the investment takes place it is optimal to maximize this expected value by the choice of c_i. Thus, we will require that

$$\frac{\partial \pi_i(x, c_i)}{\partial c_i} = 0. \tag{22}$$

Using Eq. (12) and Eq. (19) respectively we get the optimal capacity for the decline regime

$$c_2(x) = \frac{x - \sqrt{x(r-\alpha_2+\omega)}}{x} \tag{23}$$

and for the growth regime

$$c_1(x) = \frac{x(r-\alpha_2+\omega+\lambda) - \sqrt{x(r-\alpha_2+\omega)(r-\alpha_2+\omega+\lambda)(r-\alpha_0+\omega+\lambda}}{x(r-\alpha_2+\omega+\lambda)} \tag{24}$$

in dependence of the current cash flows.

The corresponding project values of Eq. (21) are

$$\pi_2(x) = \frac{x}{r-\alpha_2+\omega} + 1 - \frac{2\sqrt{x(r-\alpha_2+\omega)}}{r-\alpha_2+\omega}. \tag{25}$$

and

$$\pi_1(x) = \frac{x(r-\alpha_2+\omega+\lambda)}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} + 1 - \frac{2\sqrt{x(r-\alpha_2+\omega+\lambda)(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)}}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)}. \tag{26}$$

In the second step, we can derive the value of the option to invest in this new product. The value of the option to invest $\mathrm{F_i}\big(\mathrm{x(t)}\big)$ for each state i in the continuation region is given by the Bellman equation

$$rF_i\big(x(t)\big)dt = \mathbb{E}\left(dF_i\big(x(t)\big)\right). \tag{27}$$

Similar to the approach above, we expand the right hand side with Ito's Lemma so that each of the both states as a differential equation with

$$\frac{1}{2}\sigma_2^2 x^2(t)\frac{\partial^2 F_2\big(x(t)\big)}{\partial x^2} + (\alpha_2-\omega)x(t)\frac{\partial F_2\big(x(t)\big)}{\partial x} - rF_2\big(x(t)\big) = 0. \tag{28}$$

as the value in the decline phase and

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 F_1\big(x(t)\big)}{\partial x^2} + (\alpha_1-\omega)x(t)\frac{\partial\, F_1\big(x(t)\big)}{\partial x} - (r+\lambda)F_1\big(x(t)\big) + \lambda F_2\big(x(t)\big) = 0. \tag{29}$$

as the value in the growth phase. Note that these equations are quite similar to the ones above except for the cash flow term $x(t)$ as the option will not generate cash flow streams. Once more, it is useful to evaluate the decline phase first. From the discussion above it is obvious that full solution of the equation only consists of the homogeneous solution Bx^{β} with β as the root of the quadratic equation

$$\frac{1}{2}\sigma_2^2\beta(\beta-1) + (\alpha_2-\omega)\beta - r = 0. \tag{30}$$

This yields the solution

$$F_2\big(x(t)\big) = B_1 x^{\beta_1} + B_2 x^{\beta_2}, \tag{31}$$

where B_1 is a constant to be determined. As before we can rule out the second term as the value of the option will go to zero as the geometric Brownian motion goes to zero so that $F^1(0) = 0$ must hold. The rest of the solution must follow boundary conditions at the investment trigger x_2^* where it is optimal for the company to invest the amount $I_2(c_2)$ to receive the expected value of the future cash flows $V_2\big(x(t)\big)$.

These conditions are the value-matching condition

$$F_2(x_2^*) = c_2(x_2^*)V_2(x_2^*) - I_2\big(c_2(x_2^*)\big) \tag{32}$$

and smooth pasting condition

$$\frac{\partial F_2(x_2^*)}{\partial x} = \frac{\partial c_2(x_2^*)V_2(x_2^*)}{\partial x} - \frac{\partial I_2\big(c_2(x_2^*)\big)}{\partial x} \tag{33}$$

Together with (22) we now have three equations to find the three unknowns B_1, x_2^*, and c_2^*. We find that the threshold is

$$x_2^*, = \left(\frac{\beta_1}{\beta_1 - 1}\right)^2 (r - \alpha_2 + \omega), \tag{34}$$

the optimal capacity is

$$c_2^* = \frac{1}{\beta_1}, \tag{35}$$

and

$$B_1 = \frac{\left(\left(\frac{\beta_1}{\beta_1 - 1}\right)^2 (r - \alpha_2 + \omega)\right)^{-\beta_1}}{(\beta_1 - 1)^2}. \tag{36}$$

We now turn to the investment option in the growth stage. The differential equation (29) for the value of the option $F_1\big(x(t)\big)$ contains solution for the option in the decline stage. It is important to note that this option value depends on the value of $x(t)$, so that

$$F_2(x(t)) = \begin{cases} B_1 x^{\beta_1} & for\ x < x^{*1} \\ c_2\big(x(t)\big)V_2(x(t)) - I_2\left(c_2\big(x(t)\big)\right) & for\ x \geq x^{*1} \end{cases}. \tag{37}$$

We will first consider the region $x \geq x_2^*$. In this region, there is the probability λdt that a switch to the decline state occurs. In this case, the company will immediately exercise the option to receive the expected payoff of the project. Thus, we can state equation (29) as

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 F_1(x(t))}{\partial x^2} + (\alpha_1 - \omega)x(t)\frac{\partial F_1(x(t))}{\partial x} - (r+\lambda)F_1(x(t)) + \lambda\left(\frac{x}{r-\alpha_2+\omega} + 1 - \frac{2\sqrt{x(r-\alpha_2+\omega)}}{r-\alpha_2+\omega}\right) = 0. \tag{38}$$

This has again the well-known general solution Cx^{γ} where γ is again solution to the quadratic equation (17). We can write the full solution as

$$F_1(x) = C_1 x^{\gamma_1} + C_2 x^{\gamma_2} + \frac{\lambda x}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} + \frac{\lambda}{r+\lambda} - \frac{\lambda 16\sqrt{x}}{(8r - 4\alpha_1 + 8\omega + 8\lambda + \sigma_1^2)\sqrt{r-\alpha_2+\omega}}. \tag{39}$$

Here, we cannot rule out the second term of the solution as the option will not become worthless as it reaches the lower boundary x_2^*.

Now consider the region $x(t) < x_2^*$. The state may again switch to decline at any time with probability λdt so that the company will go over to hold the option value $B_1 x^{\gamma_1}$. Thus, we can write equation (29) as

$$\frac{1}{2}\sigma_1^2 x^2(t)\frac{\partial^2 F_1(x(t))}{\partial x^2} + (\alpha_1 - \omega)x(t)\frac{\partial F_1(x(t))}{\partial x} - (r+\lambda)F_1(x(t)) + \lambda B_1 x^{\beta_1} = 0. \tag{40}$$

The solution is now Gx^{γ} where γ has the known two solutions given by equation (17). Hence, following Dixit & Pindyck (1994, p. 203) we can show that the full solution is

$$F_1(x(t)) = G_1 x(t)^{\gamma_1} + B_1 x(t)^{\beta_1}. \tag{41}$$

To find the optimal investment threshold x_1^* in the growth phase, we need two additional boundary conditions. Again, these conditions are the value-matching condition

$$F_1(x_1^*) = c_1(x_1^*)V_1(x_1^*) - I_1(c_1(x_1^*)) \tag{42}$$

and smooth pasting condition

$$\frac{\partial F_1(x_1^*)}{\partial x} = \frac{\partial c_1(x_1^*)V_1(x_1^*)}{\partial x} - \frac{\partial I_1\big(c_1(x_1^*)\big)}{\partial x} \tag{43}$$

For the further analysis we need to know whether $x_1^* < x_2^*$ or $x_1^* > x_2^*$ and use the corresponding $F_1(x_1^*)$.

In the case $x_1^* > x_2^*$, we would have

$$\begin{aligned} C_1 x_1^{*\gamma_1} + C_2 x_1^{*\gamma_2} + \frac{\lambda x_1^*}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} + \frac{\lambda}{r+\lambda} \\ - \frac{\lambda 16\sqrt{x_1^*}}{(8r-4\alpha_1+8\omega+8\lambda+\sigma^2)\sqrt{r-\alpha_2+\omega}} \\ = c_1(x_1^*)V_1(x_1^*) - I_1\big(c_1(x_1^*)\big) \end{aligned} \tag{44}$$

and

$$\begin{aligned} \gamma_1 C_1 x_1^{*\gamma_1-1} + \gamma_2 C_2 x_1^{*\gamma_2-1} + \frac{\lambda}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} \\ - \frac{8\lambda}{\sqrt{x_1^*}(8r-4\alpha_1+8\omega+8\lambda+\sigma^2)\sqrt{r-\alpha_2+\omega}} \\ = \frac{\partial c_1(x_1^*)V_1(x_1^*)}{\partial x} - \frac{\partial I_1\big(c_1(x_1^*)\big)}{\partial x} \end{aligned} \tag{45}$$

However, as for the option value $F_1\big(x(t)\big)$ the investment threshold x_2^* is not a decision node in state *i*=1, the option value $F_1\big(x(t)\big)$ must be continuously differentiable at x_2^*, i.e. equations (39) and (41) as well as their derivatives must be equal at x_2^*.

$$\begin{aligned} C_1 x_2^{*\gamma_1} + C_2 x_2^{*\gamma_2} + \frac{\lambda x_2^*}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} + \frac{\lambda}{r+\lambda} \\ - \frac{\lambda 16\sqrt{x_2^*}}{(8r-4\alpha_1+8\omega+8\lambda+\sigma^2)\sqrt{r-\alpha_2+\omega}} \\ = G_1 x_2^{*\gamma_1} + B_1 x_2^{*\beta_1} \end{aligned} \tag{46}$$

and

$$\begin{aligned} \gamma_1 C_1 x_2^{*\gamma_1-1} + \gamma_2 C_2 x_2^{*\gamma_2-1} + \frac{\lambda}{(r-\alpha_2+\omega)(r-\alpha_1+\omega+\lambda)} \\ - \frac{8\lambda}{\sqrt{x_2^*}(8r-4\alpha_1+8\omega+8\lambda+\sigma^2)\sqrt{r-\alpha_2+\omega}} \\ = \gamma_1 G_1 x_2^{*\gamma_1-1} + \beta_1 B_1 x_2^{*\beta_1-1}. \end{aligned} \tag{47}$$

We now have four equations to find the four unknowns x_1^*, G_1, C_1, and C_2 so that we can also derive $c_1^*(x_1^*)$.

In the case $x_1^* < x_2^*$ We require that

$$G_1 {x_1^*}^{\gamma_1} + B_1 {x_1^*}^{\beta_1} = c_1(x_1^*)V_1(x_1^*) - I_1\big(c_1(x_1^*)\big), \tag{48}$$

and

$$\gamma_1 G_1 {x_1^*}^{\gamma_1 - 1} + \beta_1 B_1 {x_1^*}^{\beta_1 - 1} = \frac{\partial c_1(x_1^*)V_1(x_1^*)}{\partial x} - \frac{\partial I_1\big(c_1(x_1^*)\big)}{\partial x}, \tag{49}$$

This yields two equations with two unknowns G_1 and x_1^*, from which we can also derive $c_1^*(x_1^*)$.

3. Results

In the following, we will assume that the firm can invest in a new product which demand follows a product life cycle, i.e in the standard case the quantities sold grow with $\alpha_1 = 0.08$ in the expansion phase while demand declines at a rate of $\alpha_2 = -0.08$ once a new technology enters the market. It is expected that a new technology will arrive at a rate of $\lambda = 0.2$, i.e. in expectancy every five years. For the sake of simplicity, the uncertainty is assumed to be equal in the expansion and decline phase with $s_1 = s_2 = 0.2$. Moreover, the product price is expected to decline with the factor $\omega = 0.02$ p.a. and the firm discounts at the riskless interest rate $r = 0.1$.

In this case the optimal investment threshold in the growth phase is $x_1^* = 0.25$ and $x_2^* = 0.28$ in the decline phase. Put different, we observe a lower investment threshold in the growth phase and a higher investment threshold in the decline phase.[3] With respect to the optimal capacity, the company should choose a higher capacity of $c_1^* = 0.3$ in the growth phase and a lower capacity of $c_2^* = 0.15$ in the decline phase. To what extend do these findings change if uncertainty increases? For the moment, let's assume that prices are constant, i.e. $\omega = 0$. To what extend do these findings change if uncertainty increases? For the moment, let's assume that prices are constant, i.e. $\omega = 0$ and the firm is operating in a single regime, i.e. the demand growth exponentially over time.

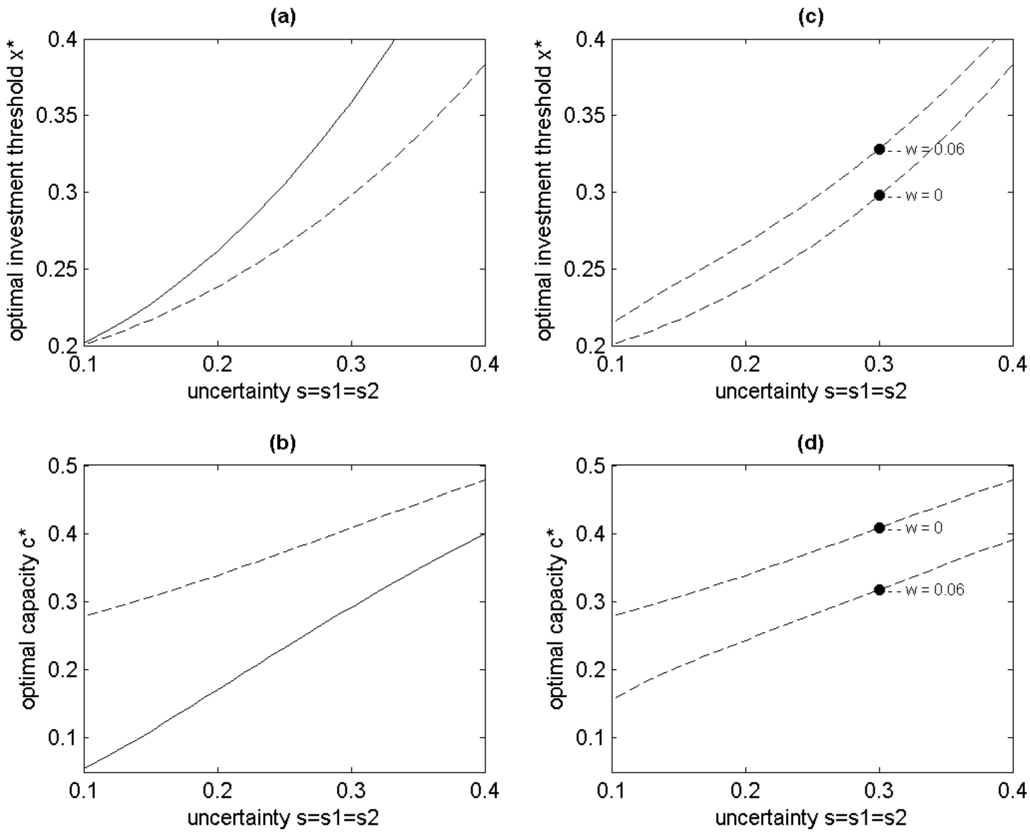

Figure 1: The optimal investment threshold and capacity in the growth phase (dotted if $x_1^* > x_2^*$ and dashed if $x_1^* > x_2^*$) and decline phase (solid) in dependence of the decline regime's uncertainty σ_2 for $\omega = 0$ in (a) and (b) as well as for for $\omega = 0.06$ in (c) and (d).

As Figure 1a and 1b indicate, the corresponding investment threshold, i.e. x_1^*, increases as uncertainty increase. Moreover, the associated optimal capacity c_1^*

[3] This is in line with the findings of Lukas et al. (2017) who also observe a growing investment threshold over time.

also increase as uncertainty increases indicating that higher uncertainty leads to more capital invested later which replicates the classical findings. However, should the regime switch occur, and the firm has not yet invested, it is no longer optimal to follow that particular investment policy, i.e. $\Omega = (x_1^*, c_1^*)$; Rather, the firm will prefer to invest less later, i.e. $x_1^* < x_2^*$ and $c_2^* < c_1^*$. Should the industry be subject to deteriorating output prices, then the firm will prefer to invest an even lower amount later irrespectively what uncertainty level prevails (see Figure 1c and 1d). Hence, although modeled differently, these results seem to foster recent findings that declining prices lead to less capacity later (see e.g. Hagspiel, Huisman, Kort, & Nunes (2016))

However, it seems unrealistic to assume the same uncertainty level in both regimes. Looking closer at innovative industries and the PLC of innovation, respectively it makes more sense to differentiate between those situations, where the growth phase is subject to more uncertainty than the decline phase and vice versa. While the first scenario is plausible for most new product launches the latter scenario might reflect situations, where the innovation is no longer new to the market but to the firm and hence the competing forces predominantly trigger cash flow uncertainty.

An innovator would typically assume high uncertainty in the growth regime when the product is new and market sales can hardly be estimated, while he would predict lower uncertainty in a mature and declining market. We consider this case in Figure 2 for $\sigma_1 = 0.5$. If uncertainty in the decline regime is much lower, we observe in Figure 2a that the investment threshold in the growth regime x_1^* becomes larger than in the decline regime. During the very uncertain market growth the innovator prefers to delay investment until demand for the new product is significantly high enough, and considering decreasing product prices even higher. With respect to capacity, Figure 2b indicates that the innovator would invest in a large capacity if demand does rise high enough. If the product would switch from the growth to the decline regime, the new investment threshold x_2^* would be lower. The innovator would invest sooner due to lower uncertainty, even though the cash flows are expected to decrease. Nevertheless, after the switch the market would only justify a lower capacity. That is, the investor would invest earlier but lower.

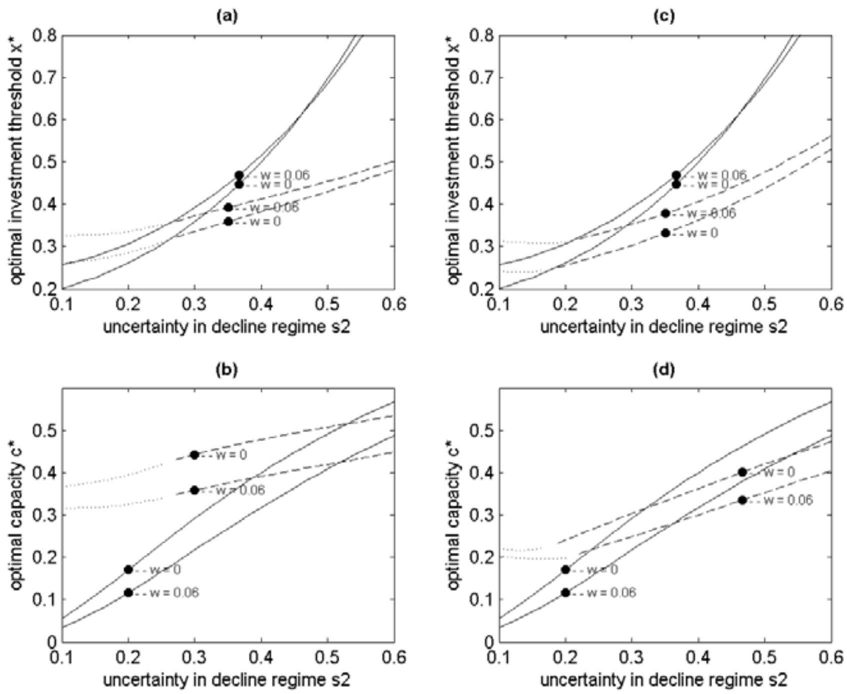

Figure 2: The optimal investment threshold and capacity in the growth phase (dotted if $x_1^* > x_2^*$ and dashed if $x_1^* > x_2^*$) and decline phase (solid) in dependence of the decline regime uncertainty σ_2 with $\lambda = 0.2$ in (a) and (b) as well as for for $\lambda = 0.7$ in (c) and (d) for $\omega = [0, 0.06]$.

In contrast, when uncertainty in the decline regime is close to the uncertainty in the growth regime, we observe again that $x_1^* < x_2^*$. The innovator has no advantage to wait for an almost equally uncertain decline regime. Furthermore, we find that the effect of an increase of the price reduction rate ω on the investment threshold in the decline regime x_2^* depends on the degree of uncertainty in this regime. For low degrees of uncertainty a stronger price reduction, i.e. a higher ω, leads to a higher investment threshold. In contrast, in case of high uncertainty the investment threshold decreases with stronger price reduction. Thus, if uncertainty in the decline regime is high as well and prices are expected to decline the company has an incentive to invest earlier as both quantity and price of the product are expected to decline rapidly. Nevertheless, in case of high uncertainty and falling prices, the innovator would invest in a larger capacity in the decline than in the growth regime.

The innovators investment decision does not only crucially depend on uncertainty within the respective regime but also on the duration for which the innovator expects the new product to stay in the growth market. If the probability λdt to switch from the growth to the decline regime increases, the product would be expected to be less favorable for the investor. Figure 2c illustrates the effect of a shorter expected growth period on the optimal investment threshold. While the probability to switch has no effect on the investment threshold in the declining market, the optimal threshold in the growth regime is lower for every degree uncertainty. While x_1^* is still larger than x_2^*, we observe an ambiguous effect of uncertainty on x_1^* for low degrees of uncertainty in the decline regime. First, if uncertainty is very low, an increase of uncertainty leads to a decrease of the optimal investment threshold. If uncertainty increases further the optimal investment threshold starts to increase. Additionally, we observe the same ambiguous result in Figure 2d with respect to the optimal capacity. In the case of very low uncertainty in the decline regime, an increase of uncertainty first leads to a lower optimal capacity to be installed while an additional increase eventually leads to increasing optimal capacity. Overall, an in expectancy short and highly uncertain growth regime is strongly influenced by the prospect of a more secure future market, even though this market is expected to decline.

4. Conclusion

Even in the face of deteriorating output prices and *technological obsolescence* we find that firms quite often invest, rather than discard, thereby installing a considerable amount of capacity. The paper's main objective is to analyze the impact of a product's life cycle and deteriorating output prices on the simultaneous choice of investment scale and timing under both technological and economic uncertainty. By means of a Markov-regime switching model we model the simultaneous choice of optimal investment timing and capacity under uncertainty in continuous time. Our results indicate that, the threat of disruptive technological change leads to install less capacity later and even in declining markets stimulate investment. If uncertainty levels in each demand regime are different, we find that both optimal capacity and timing threshold become ambiguous. In case of low uncertainty, an increase in uncertainty in the decline regime leads to an earlier investment as well as a smaller optimal capacity to be installed in the growth regime.

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