The optimal regrade of conventional to appropriate technology versus abandonment

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Abstract

A mature physical asset regrade, in contrast to a like-for-like replacement, describes a switch to a technological alternative more appropriate for the depleted state of an underlying resource. Off-shore oil rigs are an illustration, since their technological scale designed for very large output flows becomes inappropriate as their operational efficiency declines later in life and facing a dwindling output flow, so a more appropriate extraction technology becomes economic. A real option representation is formulated on a stochastic oil price and deteriorating output volume. The resulting two-factor model yields analytical results that switching is increasingly deferred as the cost structure for the regrade becomes adverse and the volatility decreases, but is advanced by increasing depletion rate and convenience yield. Under certain conditions, the oil rig is divested and not regraded, an occurrence common for some North-Sea fields.

1 Introduction

While a replacement constitutes a like-for-like exchange of a deteriorated productive asset for a brand new version, a regrade is defined here as a switch to a more appropriate technology used specifically in extracting an exhaustible resource. The erosion in the economic prospects for a conventional technology due to continuous deterioration in productivity and efficiency, usually associated with cumulative output, is often a prompt for appraising its qualities relative to an appropriate technology operating at a lower output level but having a more favourable cost structure. Off-shore platforms (installations, rigs) are an interesting illustration. Typically, these installations with their large-scale extraction facilities, suitable for the largest discoveries, carry commensurately large capital and operating costs. Even if the high installation capital expenditures have been amortized, their viability becomes increasingly questionable during their end-of-life stage due to the reduction in the extraction rate and decline in the reserve volume, a business state that becomes increasingly acute as oil prices decline. The inevitable outcome is abandonment unless a regrade to a more suitable, small-scale, appropriate technology having significantly lower operating expenditures can be economically justified. We investigate the viability of implementing a regrade policy under declining periodic output volumes and volatile oil prices. This is formulated as a two-factor real option model, which provides the revenue threshold justifying a switching from a conventional to a more appropriate technology and compares the economics of this policy with abandonment, all in an analytical form.

A technology regrade with its more favourable properties can be conceived as a discrete sequential investment style model for an active productive asset that can assume more than one state. The earliest formulations of this style of model are founded on the continual switching between an active and suspended state as presented by Brennan and Schwartz (1985) and Dixit (1989). In an extension to multiple states, Paxson (2005) develops a circular contraction-expansion model having contraction as one of its states, characterized by a more favourable cost structure in the presence of an output decline. In contrast, Malchow-Møller and Thorsen (2005) propose a repeated investment model of potentially ever-improving technological advances, in which an advance is installed whenever a sufficient productivity deterioration is suffered. Siddiqui and Maribu (2009) formulate a one-factor investment model for the electricity generation industry to examine the economic justification for a distribution upgrade and show the significance of volatility on the policy decision. Siddiqui and Fleten (2010) develop a process representation to model a real option formulation of a switch to an alternative energy technology having a more favourable cost structure. Kort et al. (2010) show that despite the intrinsic flexibility of a stepwise versus a lumpy investment strategy, greater uncertainty makes the latter more attractive. However, the former is always superior if a choice exists on the installed capacity level, Chronopoulos et al. (2016). Chronopoulos and Siddiqui (2015) assess the merits of alternative strategies through an optimal timing model for new technology replacement in the presence of price and technological arrival uncertainty. In an extension, Chronopoulos and Lumbreras (2017) assess the effect of risk aversion on the reluctance to switch between regimes under market and technological uncertainty to show that changes in volatility, risk aversion and innovation significantly affect the optimal policy decision. Expressing the motivation for technological advancement in terms of a duopolistic framework, Huisman and Kort (2003), Huisman and Kort (2004) demonstrate that the effect of competition is to encourage earlier adoption.

Decisions associated with oil extraction are effectively investigated through a real option formulation due to the inherent uncertainty and managerial flexibility, Dias (2004). One of the earliest expositions is presented by Tourinho (1979), who advocates the inclusion of a holding cost to ensure exercise, see also Adkins and Paxson (2011c). Paddock et al. (1988) show analytically that lease values increase with greater volatility, Ekern (1988) applies a binomial lattice approach while the model of Bjerksund and Ekern (1990) is based on the analytical familiar American perpetuity model. Laughton (1998) shows that both oil price and reserve volume uncertainties enhance the prospect value but distinctively influence the exercise of the various decisions. McCormack and Sick (2001) discuss the use of real options in valuing undeveloped reserves. Chorn and Shokhor (2006) apply a jump diffusion model for evaluating the emergence of new information. Guedes and Santos (2016) assess the value of an offshore oil development installation involving a sequence of interdependent decisions modelled as options to show the high value associated with abandonment. The role of CO_2 in enhancing oil recovery rates as well as mitigating its potentially harmful effects on the environment is assessed by Compernolle et al. (2017). In works possibly closest to our own formulation, Adkins and Paxson (2011a) and Støre et al. (2016) examine two-factor models on making a viable irreversible switch between two different inputs (outputs) to show that both sources of uncertainty are crucial in the optimal decision.

Our formulation is an abstracted illustration of the current state of the off-shore United Kingdom continental shelf (UKCS) oil business, representing one of the most mature off-shore basins with operating asset ages exceeding 30 years. From its peak in 1999, production has steadily declined until 2014 at an annual rate of about 7%[1], Wood (2014). His report focuses on maximizing economic recovery, since 70% of the UKCS decline is considered to be due to production inefficiencies, compounded by a lack of investment in new technology. Although the report advocates achieving savings from greater collaboration amongst the players facilitated through a less light-touch regulatory framework, it also recommends increased asset stewardship through technological advancements having the potential to increase efficiencies and enhance recovery rates and volumes, while maximizing the economic extension of the field life. In the absence of any effective policies on asset stewardship and field-life extension, the installations involved in oil extraction and transportation become vulnerable to suspension and divestment. The fields most susceptible to abandonment are classified as marginal. These are characterized as having unattractive prospects due to low recoverable reserves, low volume production, unfavourable economics, which is the focus of this paper, but isolation and challenging reservoir oil properties are also unfavourable attributes.

Off-shore oil field abandonment policy is treated in different ways. Kemp (1992) examines the economic and fiscal aspects of making an abandonment decision by considering the nature of the costs incurred, the adequacy of the selected timing criterion, the role of fiscal relief and security concerns. In identifying the relevant factors, not only should the ongoing costs and abandonment expenditures be addressed, but also the effects of a fluctuating oil price on the timing decision should be considered in conjunction with fiscal relief, field interdependency and technological progress. A net present value analysis shows a greater incentive to postpone abandonment in the presence of a less steep production decline rate, higher abandonment costs and a higher discount rate. Given the presence of uncertainty and managerial flexibility, Kemp (1992) omits any mention of employing a real option formulation. The externalities associated with decommissioning off-shore installations for all maritime users, including environmental groups and governments as well as the oil companies are considered by Osmundsen and Tveterås (2003). Disposal costs can vary by

[1] In contrast, 2015 experienced a recovery due to new field openings. Data is available from https://www.gov.uk/government/organisations/department-for-business-energy-and-industrial-strategy

field even for the same geographic region, while differences in disposal strategies can lead to varying reputational and cost consequences apportioned amongst the players. Parente et al. (2006) extend this scope by considering the role of a periodic ex-ante deductibility of the decommissioning cost despite constituting an ex-post expense and the question of assigning decommissioning responsibility for an installation following a transfer of rights during its project life. They argue for a dedicated trust fund to be established that functions as a secure source to fund the decommissioning cost at a time when the net revenue from the oil installation is probably nil and possibly when revenues from other installations in the portfolio are eroded by low oil prices. The alienation of rights especially to inexperienced and unscrupulous agents creates a moral hazard issue that needs to be government managed through assiduous data collection, intense scrutiny and effective policing.

Our aim is to develop and analyse a discrete sequential investment problem characterizing the regrade trade-off between continuing to use the conventional or incumbent technology for oil extraction versus installing an appropriate technology having more favourable properties, under price uncertainty and a dynamic declining periodic output volume. Our real option formulation employs a perpetuity to value the residual reserve volume. However, a divestment option is introduced to ensure a finite time termination as advocated by Preinreich (1940) for analytical appraisal models based on an infinite lifetime assumption. The divestment option incurs a significant decommissioning cost on exercise and provides the means for terminating the implied infinitely lived asset.

There are three principal contributions made by this paper. First, we develop a two-factor representation in which the output price follows an assumed stochastic process and the output volume declines deterministically with cumulative production, but despite this complexity it yields an analytical solution amenable to detailed examination. Second, the analysis reveals the thresholds signalling a conventional technology divestment without regrade, and an appropriate technology regrade but with the conventional technology divested, thereby producing the condition that discriminates between the two policies. A regrade is economically justified when it is accompanied by a fall in the value of the periodic extracted oil output, which is compensated by a commensurate fall in the total value of the net costs incurred in installing the regrade. Numerical sensitivity analysis is used to show that an increase in the oil price volatility produces a decline in the regrade threshold, but since the periodic extracted oil output value is naturally deteriorating, this decline is interpreted as a policy deferment. Finally, the results can be applied by oil companies in assessing the requisite properties of an appropriate technology to be viable as well as the conditions prompting abandonment.

Several simplifying assumptions are introduced to make the real-option switching model analytically tractable. Switching between the two technologies or between the incumbent technology and divestment is treated as irreversible except for possible subsequent reinvestments in an idle oil reserve. It is assumed that switching occurs instantaneously with any associated costs or payoffs, which are treated as known and constant, incurring immediately. Any periodic operating cost is assumed to be fixed and known. If an oil reserve becomes idle, any holding cost required for maintaining it for subsequent exploitation is treated as zero. Although royalties and tax are excluded from the analysis, it is straightforward to include them as well as government subsidies, but tax depreciation allowances are ignored because of the additional level of complexity, Adkins and Paxson (2017).

The paper is organized as follows. The real option model derivations are developed in section 3, following a description in the next section of some fundamental findings. Section 4 explores numerical sensitivity analysis to gain further insights into the model solution. The paper ends with a conclusion.

2 Fundamentals

In formulating the periodic value of extracted oil from a reserve, the output price for oil and the periodic extracted volume are assumed to be the two key dynamic variables influencing decision making on expansion-contraction style policies. For convenience, the oil price p, treated as stochastic, is described by a geometric Brownian motion process having the form:

$$\mathrm{d}p = \alpha_p p\mathrm{d}t + \sigma p\mathrm{d}W , \quad (1)$$

where α_p denotes the known drift rate, σ the price volatility, and $\mathrm{d}W$ an increment of the standard Wiener process. In contrast, the periodic extraction volume q is deterministic and follows the declining balance form:

$$\mathrm{dq} = -\theta q\mathrm{d}t , \quad (2)$$

where $\theta > 0$ denotes a known depletion rate. For any prevailing q, the residual reserve volume is q/θ, so for two reserves with identical periodic extraction volumes, that having the lower depletion rate has the greater reserve volume, or with identical depletion rates, that having the higher periodic extraction volume has the greater reserve volume.

The policy change assessment is based on the two-factor risk-neutral valuation relationship derived from (1) and (2) using Ito's Lemma, Brennan and Schwartz (1985):

$$\tfrac{1}{2}\sigma^2 p^2 \frac{\partial^2 F}{\partial p^2} + (r-\delta) p \frac{\partial F}{\partial p} - \theta q \frac{\partial F}{\partial q} - rF = 0, \tag{3}$$

where F denotes the relevant option value, $\delta > 0$ the oil convenience yield and $r > \alpha_p$ the risk-free rate. A valuation function satisfying (3), Adkins and Paxson (2011b), Adkins and Paxson (2017), takes the form:

$$F = A_1 p^{\beta_1} q^{\gamma_1} + A_2 p^{\beta_2} q^{\gamma_2}, \tag{4}$$

where A_1 and A_2 are two non-negative coefficients, and the generic parameters β and γ are related through the characteristic equation:

$$Q(\beta,\gamma) = \tfrac{1}{2}\sigma^2 \beta(\beta-1) + (r-\delta)\beta - \theta\gamma - r = 0. \tag{5}$$

Subsequently, it is shown $\gamma_1 = \beta_1$, $\gamma_2 = \beta_2$, Paxson and Pinto (2005), so from (5) β_1 and β_2 are the respective positive and negative roots of:

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta-\theta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta-\theta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{6}$$

with $\beta_1 > 1$ and $\beta_2 < 0$. From (6), β_1, β_2 vary with the depletion rate θ. Since $\gamma_1 = \beta_1$, $\gamma_2 = \beta_2$, (3) and (4) can be framed without loss in terms of the single variable $v = pq$, which defines the periodic output value. The elements of F in (4), $A_1 p^{\beta_1} q^{\gamma_1}$ and $A_2 p^{\beta_2} q^{\gamma_2}$, represent the expansion (investment) and contraction (divestment) option, respectively.

2.1 Divestment

An owner, operating an oil rig and incurring a periodic fixed cost denoted by f, faces divestment whenever its known divestment value denoted by D adequately compensates the value from continuously operating the rig, which is composed of its net operating value and the divestment option. If $D<0$ then divesting entails a decommissioning cost. The value matching relationship capturing this condition is:

$$\frac{\hat{p}_D \hat{q}_D}{\delta+\theta}-\frac{f}{r}+A_2 \hat{p}_D^{\beta_2} \hat{q}_D^{\gamma_2}=D, \tag{7}$$

where $\hat{p}_D$ and $\hat{q}_D$ represent the divestment thresholds for p and q, respectively. The two respective smooth pasting conditions justify the equality between β_2 and γ_2, and enable (7) to be expressed as:

$$\hat{v}_D=\hat{p}_D \hat{q}_D=\frac{-\beta_2}{1-\beta_2}\left(D+\frac{f}{r}\right)(\delta+\theta), \tag{8}$$

with:

$$A_2=\frac{\hat{v}_D^{1-\beta_2}}{-\beta_2(\delta+\theta)}=\left(\frac{D+f/r}{1-\beta_2}\right)^{1-\beta_2}\left(-\beta_2(\delta+\theta)\right)^{-\beta_2} \tag{9}$$

The divestment option value characterized by A_2 is an increasing function of the divestment value D, the periodic fixed cost f, the oil convenience yield δ and the depletion rate θ. Given the non-negativity of the divestment option and threshold, $D+f/r>0$, so $D<0$ is permissible provided $f/r>-D$.

2.2 Investment

The investment cost denoted by K is committed to operationalizing an oil rig provided that the net value generated by the investment including any embedded options adequately compensates the value of the investment opportunity. If divestment is the only embedded option, then this condition is captured by the following value matching relationship:

$$A_1 \hat{p}_K^{\beta_1} \hat{q}_K^{\gamma_1}=\frac{\hat{p}_K \hat{q}_K}{\delta+\theta}-\frac{f}{r}-K+A_2 \hat{p}_K^{\beta_2} \hat{q}_K^{\gamma_2} \tag{10}$$

where $\hat{p}_K$ and $\hat{q}_K$ represent the investment thresholds for p and q, respectively. The two respective smooth pasting conditions justify the equality between β_1 and γ_1, and enable (10) to be expressed as:

$$\hat{v}_K=\hat{p}_K \hat{q}_K=\frac{\beta_1(\delta+\theta)}{\beta_1-1}\left(K+\frac{f}{r}\right)-\frac{(\beta_1-\beta_2)(\delta+\theta)}{\beta_1-1}A_2 \hat{v}_K^{\beta_2}, \tag{11}$$

where $\hat{v}_K$ is the periodic output value threshold, with:

$$A_1 = \frac{\hat{v}_K^{1-\beta_1}}{\beta_1(\delta+\theta)} + \frac{\beta_2}{\beta_1} A_2 \hat{v}_K^{\beta_2-\beta_1}$$
$$= \frac{\hat{v}_K^{-\beta_1}}{(\beta_1-\beta_2)} \left[(1-\beta_2)\frac{\hat{v}_K}{(\delta+\theta)} + \beta_2\left(K+\frac{f}{r}\right) \right]. \tag{12}$$

From (11), since $(\beta_1-\beta_2)/(\beta_1-1)>0$, $\hat{v}_K$ is less than that when the divestment option is absent, so the divestment value depresses the investment threshold and produces a hastening of the investment decision. Further, since A_1 is an increasing function of D, it makes the investment opportunity with a divestment option more desirable than one without.

3 Regrading Models

A regrade[2] to the appropriate technology is expected to transform the cost structure. Although its adoption should produce a beneficial reduction in the fixed operating cost, being appropriate entails that its extraction properties more closely match those of the marginal field. In particular, the appropriate technology is designed for a lower reserve volume and a lower rate of extraction than that for large-scale conventional platforms, which are typically deployed for substantial oil discoveries with commensurate rates of extraction. But, the reduced extraction rate associated with the appropriate technology implies a periodic revenue lower than that for the incumbent technology. The periodic revenue loss due to the regrade has to be compensated by a reduction in the fixed operating cost. Moreover, for a regrade from the conventional to the appropriate technology to be viable, the fixed operating cost reduction has to compensate not only the fall in periodic revenue but also the net investment cost incurred by the regrade.

The optimal condition signalling a regrade from the conventional to the appropriate technology depends on the state of the oil reserve. A reserve defined to be active entails the oil being actively extracted using the prevailing technology, whereas when in an idle state, the process of oil extraction has already been terminated and the associated assets divested. For an active reserve deploying the conventional technology, a regrade is only feasible provided that the owner has access to and can acquire the appropriate technology and the

[2] The binary representation of technology as being conventional and appropriate is an abstraction, since in reality, offshore oil platforms vary in size with fixed platforms and compliant towers being the largest and floating production storage offloading (FPSO) facilities, possibly unattended and remotely controlled, being the smallest. The economics are commensurate with their size. Specialist firms, such as MFDevCo, offer switching advice on effective facilities conducive to marginal field economics.

trigger signalling a regrade occurs before that for a divestment. If the divestment trigger is activated before that for a regrade, then the associated assets are divested and the reserve becomes idle. When the reserve is idle, any regrade to the appropriate technology corresponds to a straightforward investment opportunity. In representing these possibilities, we introduce a subscript notation. In addition to the subscripts D and K denoting respectively divestment and investment, we use R to denote a regrade. The conventional and appropriate technologies are denoted by X and Y, respectively. Since the power parameters depend on the depletion rate, β_1, β_2 for technology X are denoted by β_{X1}, β_{X2} where $\theta = \theta_X$ and for Y by β_{Y1}, β_{Y2} where $\theta = \theta_Y$. A list of the key variables in our analysis is presented in Table 1.

**** Table 1 about here ****

3.1 Active Reserve

The reserve owner is assumed to be actively extracting oil from the reserve incurring a fixed periodic operating expenditure f_X, having previously installed an oil extraction facility based on the conventional technology bearing a capital expenditure K_X. Over the passage of time, the owner may wish to re-appraise the viability of the incumbent technology whenever the residual reserve approaches depletion, extraction becomes economically marginal, or during times of significantly low oil prices. In deciding its prospects, the owner should consider the comparative merits of continuing with the conventional technology to extract the oil. The two alternatives to continuation are divestment without regrade and regrade with subsequent divestment. If divestment without regrade is adopted, then the owner foregoes the net revenue value from oil extraction under the conventional technology and instead receives the divestment value D_X from the disposal of the productive assets, but pays a decommissioning cost if $D_X < 0$. If regrade with divestment is adopted, the owner receives the change in net revenue value due to the regrade, which, owing to periodic fixed operating expenditure under the appropriate technology f_Y being less than that under the conventional technology f_X, $f_Y < f_X$, is expected to be positive, but incurs a regrade capital expenditure R_Y net of any divestment value D_X. For convenience, we assume an identical divestment value for either divesting without regrade or for regrading with divestment[3]. We consider first divestment without regrade, since its threshold discriminates between an active and an idle reserve, and because of this it plays a pivotal role in determining the optimal policy.

The owner continues to actively extract reserve oil deploying the conventional technology until the periodic output value v falls to the divestment threshold $\hat{v}_{DX}$ or below, at which point the oil extraction process is terminated and the accompanying facilities are divested. From (8):

$$\hat{v}_{DX} = \hat{p}_{DX}\,\hat{q}_{DX} = \frac{-\beta_{X2}}{1-\beta_{X2}}\left(D_X + \frac{f_X}{r}\right)(\delta+\theta_X). \tag{13}$$

Also, the divestment option coefficient from (9) is:

$$A_{2DX} = \left(\frac{D_X + f_X/r}{1-\beta_{X2}}\right)^{1-\beta_{X2}} \left(-\beta_{X2}(\delta+\theta_X)\right)^{-\beta_{X2}}. \tag{14}$$

While oil extraction under the conventional technology remains active, with a periodic output value exceeding the divestment threshold, $v \geq \hat{v}_{DX}$, the owner may deliberate on the merits of deploying the incumbent compared to the appropriate technology. In the absence of optionality, the net revenue value for an active reserve under the conventional technology is $pq_X/(\delta+\theta_X) - f_X/r$, where q_X denotes its periodic output. The owner holds an embedded option to regrade the conventional large-scale incumbent extraction technology with its high capital and operating expenditures to a more appropriate smaller-scale technology having a relatively lower capital and operating expenditures. At regrade, the net revenue value for the residual oil reserve under the appropriate technology is $pq_Y/(\delta+\theta_Y) - f_Y/r$, where q_Y denotes its periodic output. Since the residual oil reserve volume at regrade are equal under each technology, then $q_X/\theta_X = q_Y/\theta_Y$. Further, the periodic extraction volume for the appropriate technology is expected to be lower, so $q_Y < q_X$ with $\theta_Y < \theta_X$. Ignoring any optionality, and since any viable regrade requires the net capital expenditure to be at least covered, the incremental gain derived from operating at a lower fixed cost $(f_X - f_Y)/r > 0$ has to more than outweigh the loss in revenue incurred from operating at a lower output level, which is:

[3] It is a minor adjustment to amend for different divestment values associated with the policies of divestment without regrade and regrade with divestment.

$$\frac{f_X - f_Y}{r} + pq_Y\left(\frac{1}{\delta+\theta_Y} - \frac{\theta_X}{\theta_Y(\delta+\theta_X)}\right) > 0,$$

where:

$$\frac{1}{\delta+\theta_Y} - \frac{\theta_X}{\theta_Y(\delta+\theta_X)} = -\frac{\delta(\theta_X-\theta_Y)}{\theta_Y(\delta+\theta_X)(\delta+\theta_Y)} < 0. \tag{15}$$

For a regrade to be viable, the value attained from the regrade by adopting a technology having a lower fixed operating cost has to compensate the accompanying loss in value of the ensuing revenue decline.

The optimal regrade threshold is defined by $\hat{v}_{RY} = \hat{p}_R \hat{q}_{RY}$, which is expressed in terms of the appropriate technology. Since a regrade to the appropriate technology is feasible provided extraction under the conventional technology remains active, the periodic output value v_X has to at least exceed the divestment threshold, $v_X \geq \hat{v}_{DX}$, which is expressed in terms of the conventional technology. To show that a regrade is feasible because it happens during the active state, the divestment and regrade thresholds have to be expressed in identical units. At regrade, the residual volumes under either technology are identical, $q_X/\theta_X = q_Y/\theta_Y$, so the adjusted divestment threshold expressed in the units of the appropriate technology $\hat{v}_{DX}^{adj}$ can be specified as:

$$\hat{v}_{DX}^{adj} = \frac{\theta_Y}{\theta_X}\hat{v}_{DX}. \tag{16}$$

Provided $\hat{v}_{RY} \geq \hat{v}_{DX}^{adj}$, a regrade is feasible and implementable.

Since both regrade and divestment are motivated by inadequately low values for the periodic output value, their option properties are similar. So the regrade option is inversely related to both the oil price and the periodic output volume and specified by $A_{2RY}\hat{p}^{\beta_{Y2}}\hat{q}_Y^{\gamma_{Y2}}$, where the power parameters reflect depletion rate for the appropriate technology. A regrade is economically justified provided the generated incremental net gains at least compensate the associated incremental net opportunity costs, where both gains and costs are interpreted to include any embedded options. The incremental net gains are determined from three distinct sources: (i) the net periodic output value from using the appropriate technology, less (ii) the capital expenditure incurred in its installation net of the divestment value yielded by abandoning part or all of the conventional technology, and (iii) the divestment option of the appropriate technology. The associated net incremental costs are represented by (i) the net periodic output value foregone due to the conventional technology being regraded, and (ii) the regrade option value. The periodic output threshold levels signalling an optimal regrade are denoted by $\hat{q}_{RX}$ and $\hat{q}_{RY}$ for the conventional and appropriate technologies, respectively. Because the residual oil reserve volume is identical under either technology, $\hat{q}_{RX}/\theta_X = \hat{q}_{RY}/\theta_Y$. We denote the corresponding optimal oil price threshold by $\hat{p}_R$. The value matching relationship for regrade expressed in terms of $\hat{q}_{RY}$ is:

$$\frac{\hat{p}_R\hat{q}_{RY}\theta_X}{\theta_Y(\delta+\theta_X)} - \frac{f_X}{r} + A_{2RY}\hat{p}_R^{\beta_{Y2}}\hat{q}_{RY}^{\gamma_{Y2}} = \frac{\hat{p}_R\hat{q}_{RY}}{\delta+\theta_Y} - \frac{f_Y}{r} - (R_Y - D_X) + A_{2DY}\hat{p}_R^{\beta_{Y2}}\hat{q}_{RY}^{\gamma_{Y2}}, \tag{17}$$

where A_{2DY} is the divestment option coefficient for the appropriate technology, which is found in the exactly the same way as for (14):

$$A_{2DY} = \left(\frac{D_Y + f_Y/r}{1-\beta_{Y2}}\right)^{1-\beta_{Y2}} \left(-\beta_{Y2}(\delta+\theta_Y)\right)^{-\beta_{Y2}}. \tag{18}$$

The smooth-pasting condition with respect to p associated with (17) can be expressed[4] as:

$$\frac{\hat{p}_R\hat{q}_{RY}\theta_X}{\theta_Y(\delta+\theta_X)} + \beta_{Y2}A_{2RY}\hat{p}_R^{\beta_{Y2}}\hat{q}_{RY}^{\gamma_{Y2}} = \frac{\hat{p}_R\hat{q}_{RY}}{\delta+\theta_Y} + \beta_{Y2}A_{2DY}\hat{p}_R^{\beta_{Y2}}\hat{q}_{RY}^{\gamma_{Y2}}, \tag{19}$$

which, in the absence of any divestment optionality, demonstrates $\beta_{Y2} < 0$. By comparing the two smooth-pasting conditions with respect to price and output, $\gamma_{Y2} = \beta_{Y2}$. Combining (17) and (19) yields the optimal threshold $\hat{v}_{RY} = \hat{p}_R\hat{q}_{RY}$ for the periodic output value under the appropriate technology, where:

$$\hat{p}_R\hat{q}_{RY}\left[\frac{1}{\delta+\theta_Y} - \frac{\theta_X}{\theta_Y(\delta+\theta_X)}\right] = \frac{\beta_{Y2}}{\beta_{Y2}-1}\left[(R_Y - D_X) + \frac{f_Y - f_X}{r}\right]. \tag{20}$$

By (15), for a viable solution to exist, $f_X > r(R_{YM} - D_{XM}) + f_Y$, so the gain from foregoing the conventional technology fixed cost has to more than compensate the return on the net capital expenditure and the appropriate technology fixed cost. Since $0 < \beta_{Y2}/(\beta_{Y2}-1) < 1$, a regrade is only economically justified if the output value difference between the appropriate and the conventional technology is less than the cost value difference including the net

[4] This is derived by multiplying the smooth pasting condition by $\hat{p}_R$.

regrade cost. Alternatively, the output value foregone has to exceed the overall cost value saving. Also, the regrade threshold $\hat{v}_{RY}$ is linearly dependent on R_Y, f_Y, D_X, f_X, (20), the association being negative for the former two but positive for the latter two. A more unfavourable cost structure for the appropriate technology (increases in R_Y and f_Y) entails a decrease in $\hat{v}_{RY}$ leading to a deferral of the regrade decision. In contrast, a more unfavourable cost structure for the conventional technology (increases in f_X and D_X) produces a threshold increase and an advancement of the regrade decision. For a positive decommissioning cost $(D_X < 0)$, a rise is associated with a decision deferment. Finally, we also have:

$$A_{2RY} = \frac{\hat{v}_{RY}}{\beta_{Y2}}\left[\frac{1}{\delta+\theta_Y} - \frac{\theta_X}{\theta_Y(\delta+\theta_X)}\right]\hat{v}_{RY}^{-\beta_{Y2}} + A_{2DY} > A_{2DY}. \tag{21}$$

Since $A_{2DY} > 0$, the unexercised regrade option is always greater in the presence of the divestment option, making it more valuable. Further, in the absence of divestment optionality with $A_{2DY} = 0$, then for $A_{2RY} > 0$ again we require $\theta_X > \theta_Y$.

3.2 Reactivating an Idle Reserve

When an oil reserve enters an idle state, a regrade is classified to have been infeasible. However, an investment opportunity in the appropriate technology continues to exist provided the cost of keeping this option open including maintenance are minimal. In the absence of a feasible regrade with $\hat{v}_{RY} < \hat{v}_{DX}^{adj}$, oil extraction is terminated when the periodic output value v_X falls to the divestment threshold, $\hat{v}_{DX}$, the facility is shut down, any divestment value is recovered (returned) and the reserve becomes idle. At shut-down, the oil price is p_{DX} and the residual reserve volume q_{DX}/θ_X where $p_{DX}q_{DX} = \hat{v}_{DX}$. Subsequently, while the residual reserve volume remains unchanged, the price fluctuates both positively and negatively according to its stochastic process. If reactivated, the residual reserve volume is valued at zero because of being a sunk cost[5] despite carrying a value exceeding zero. The idle reserve may be reactivated under the appropriate technology during a favourable oil price period provided the ensuing revenue value is sufficient to bear the installation and operating costs as well as the investment option value net of any divestment option value. Given that

[5] It is possible for the idle reserve to have a non-zero price in the presence of an actively traded market; the current analysis is straightforward to amend to reflect this.

the opportunity value of investing in the appropriate technology is $A_{1KY}p^{\beta_{Y1}}q^{\gamma_{Y1}}$, the value-matching relationship from (10) is:

$$A_{1KY}\hat{p}_{KY}^{\beta_{Y1}}\hat{q}_{KY}^{\gamma_{Y1}} = \frac{\hat{p}_{KY}\hat{q}_{KY}}{\delta+\theta_Y} - \frac{f_Y}{r} - K_Y + A_{2DY}\hat{p}_{KY}^{\beta_{Y2}}\hat{q}_{KY}^{\gamma_{Y2}}, \tag{22}$$

where $\hat{p}_{KY}$ and $\hat{q}_{KY}$ denote the respective thresholds for the price and periodic volume. In (22), the periodic volume is treated as a variable because it is inherently dynamic, and not as a known constant. The solution is expressed in terms of the periodic revenue threshold level, $\hat{v}_{KY} = \hat{p}_{KY}\hat{q}_{KY}$, where by setting $\hat{q}_{KY} = q_{DX}\theta_Y/\theta_X$ enables the price threshold $\hat{p}_{KYM}$ to be obtained as a single value. Following §2.2, then from (11), (12), (8) and (9), we have, respectively:

$$\hat{v}_{KY} = \frac{\beta_{1Y}(\delta+\theta_Y)}{\beta_{1Y}-1}\left(K_Y + \frac{f_Y}{r}\right) - \frac{(\beta_{Y1}-\beta_{Y2})(\delta+\theta_Y)}{\beta_{Y1}-1}A_{2DY}\hat{v}_{KY}^{\beta_{Y2}} \tag{23}$$

$$A_{1KY} = \frac{\hat{v}_{KY}^{1-\beta_{Y1}}}{\beta_{Y1}(\delta+\theta_Y)} + \frac{\beta_{Y2}}{\beta_{Y1}}A_{2DY}\hat{v}_{KY}^{\beta_{Y2}-\beta_{Y1}} \tag{24}$$

$$\hat{v}_{DY} = \hat{p}_{DY}\,\hat{q}_{DY} = \frac{-\beta_{Y2}}{1-\beta_{Y2}}\left(D_Y + \frac{f_Y}{r}\right)(\delta+\theta_Y) \tag{25}$$

$$A_{2DY} = \frac{\hat{v}_{DY}^{1-\beta_2}}{-\beta_2(\delta+\theta_Y)} = \left(\frac{D_Y + f_Y/r}{1-\beta_{Y2}}\right)^{1-\beta_{Y2}}\left(-\beta_{Y2}(\delta+\theta_Y)\right)^{-\beta_{Y2}} \tag{26}$$

We surmise that investment threshold $\hat{v}_{KY}$ for the appropriate technology has to exceed the adjusted divestment threshold $\hat{v}_{DX}^{adj}$. If a regrade is not feasible, then any difference between the two thresholds, $\hat{v}_{KY}$ and $\hat{v}_{DX}^{adj}$, reflects an oil price variation because the residual reserve volume remains unchanged. A ratio $\hat{v}_{KY}/\hat{v}_{DX}^{adj} > 1$ implying a favourable price change is treated as essential for overcoming the relative unattractiveness of implementing the appropriate technology investment compared to the divest-regrade policy. Further, from (20) we observe a negative linear association between the regrade threshold $\hat{v}_{RY}$ and the investment cost R_Y (and fixed cost f_Y). A feasible regrade with $\hat{v}_{RY} \geq \hat{v}_{DX}^{adj}$ is a more likely occurrence for relatively smaller R_Y (and f_Y) values, whilst relatively higher R_Y (and f_Y) values create an infeasibility. Since reactivating entails any regrade to be infeasible, we can expect the R_Y (and f_Y) values to be relatively high, which feeds into a relatively high investment threshold $\hat{v}_{KY}$. Consequently, it is reasonable to expect $\hat{v}_{KY} / \hat{v}_{DX}^{adj} > 1$.

3.3 Divestment Deferral

A regrade may not entail a full divestment of the conventional technology, since a part of the existing installation and infrastructure may be essential to implementing and operationalizing the appropriate technology. If we denote by φ the proportional value of the conventional technology divested at the switch, then φD_X is the redeemed (expended) amount from divesting the inessential part of the conventional technology at regrade, while the remainder $(1-\varphi) D_X$ is redeemed (expended) at the time of divesting the appropriate technology, yielding a then total divested amount $(1-\varphi) D_X + D_Y$. Although our numerical evaluations constrains $0 \le \varphi \le 1$, we recognize the possibility of $\varphi < 0$ implying at a regrade a decommission charge if $D_X > 0$ or a divestment gain if $D_X < 0$. The regrade threshold for a deferred divestment is denoted by $\hat{v}_{RY(\varphi)}$. From (17), the value matching relationship becomes:

$$\frac{\hat{v}_{RY(\varphi)} \theta_X}{\theta_Y (\delta + \theta_X)} - \frac{f_X}{r} + A_{2RY(\varphi)} \hat{v}_{RY(\varphi)}^{\beta_{Y2}} = \frac{\hat{v}_{RY(\varphi)}}{\delta + \theta_Y} - \frac{f_Y}{r} - (R_Y - \varphi D_X) + A_{2DY(\varphi)} \hat{v}_{RY(\varphi)}^{\beta_{Y2}}, \qquad (27)$$

where $A_{2RY(\varphi)}$ is the switch option coefficient and $A_{2DY(\varphi)}$ the divestment option coefficient for the appropriate technology, which is given from (18) by:

$$A_{2DY(\varphi)} = \left(\frac{(1-\varphi) D_X + D_Y + f_Y / r}{1 - \beta_{Y2}} \right)^{1-\beta_{Y2}} \left(-\beta_{Y2} (\delta + \theta_Y) \right)^{-\beta_{Y2}}. \qquad (28)$$

The appropriate technology divestment threshold $\hat{v}_{DY(\varphi)}$ from (8) is given by:

$$\hat{v}_{DY(\varphi)} = \frac{-\beta_{Y2}}{1 - \beta_{Y2}} \left((1-\varphi) D_X + D_Y + \frac{f_Y}{r} \right) (\delta + \theta_Y). \qquad (29)$$

From the smooth pasting condition associated with (27), the switch threshold, similar to (20), is expressed as:

$$\hat{v}_{RY(\varphi)} \left[\frac{1}{\delta + \theta_Y} - \frac{\theta_X}{\theta_Y (\delta + \theta_X)} \right] = \frac{\beta_{Y2}}{\beta_{Y2} - 1} \left[(R_Y - \varphi D_X) + \frac{f_Y - f_X}{r} \right] \qquad (30)$$

For a feasible solution to exist, φ is not allowed to violate $(1-\varphi)D_X + D_Y + f_Y/r > 0$ from (28) and $r(R_Y - \varphi D_X) + (f_Y - f_X) > 0$ from (30). The regrade option coefficient $A_{2RY(\varphi)}$ is evaluated as A_{2RY} in (21) after setting $\hat{v}_R = \hat{v}_{R(\varphi)}$ and $A_{2DY} = A_{2DY(\varphi)}$.

In the presence of decommissioning charges, postponing a part of the conventional technology divestment makes a regrade more attractive due to the deferral of the implied negative cash flow. If $D_X < 0$, then increases in φ are associated with decreases in the switch threshold (30) but increases in the divestment threshold for the appropriate technology (29). A divestment deferral advances the timing of a switch to the appropriate technology owing to the decreased foregone value of the fixed and divestment cost for the conventional technology, but also defers the timing of the divestment for the appropriate technology owing to the increased decommissioning cost. By deferring part of the conventional technology decommissioning charge, the appropriate technology with its more favourable cost structure is implemented earlier and operated for longer before being inevitably divested.

4 Numerical Illustrations

Further insights into the behaviour of the solution are obtained through numerical sensitivity analysis. The simulations are in the main generated from the base case, presented in Table 2. The conventional technology is seen to be more expensive, in terms of both investment and operating cost, but it has the merit of having a greater depletion rate due to the requirement $\theta_X > \theta_Y$. At divestment, both technologies incur decommissioning costs (or negative divestment values), with respective absolute divestment value-investment cost ratios of 5.0% and 3.0%. The values in Table 2 are selected so that the two conditions on divestment $f > -rD$ and upgrade $f_X > r(R_Y - D_X) + f_Y$ are satisfied. The values for the option power parameters are also presented, because of their difference due to the different depletion rates.

**** Table 2 about here ****

4.1 Active Reserve

The divestment and investment-divestment results for the two stand-alone technologies evaluated using the Table 2 values are presented in Table 3. This clearly shows that the investment and divestment decisions are exercised at a lower periodic output value for the appropriate technology because of its more favourable cost structure. Although our interest lies in the regrade decision, the conventional technology divestment threshold $\hat{v}_{DX} = 797.48$ is critical because it represents the lower threshold bound for a regrade to be feasible. This bound has to be expressed in units applicable to the appropriate technology. At a regrade, the residual oil reserve remains intact, $q_X/\theta_X = q_Y/\theta_Y$, and since any regrade takes place instantaneously with no change in the oil price, an optimal regrade is feasible provided $\hat{v}_{RY}$ at least equals $\hat{v}_{DX}^{adj} = \hat{v}_{DX}\theta_Y/\theta_X = 318.99$.

Installing an upgrade and switching to the appropriate technology should generate a fixed and capital cost saving of $(f_X - f_Y) - r(R_Y - D_X) = 985$, which decreases for a constant f_X with increases in either the fixed or the investment cost for the appropriate technology or in the decommissioning cost for the conventional technology. This saving compensates for the loss incurred in the periodic output value caused by the regrade due to the fall in depletion rate. Based on Table 2 values, the regrade threshold $\hat{v}_{RY} = 700.44$ and associated option coefficient $A_{2RY} = 953846.1$ are evaluated from (20) and (21), respectively. The regrade is feasible since $\hat{v}_{RY} \geq \hat{v}_{DX}\theta_Y/\theta_X$. When the equality holds, we are indifferent between conventional technology divestment and regrading with divestment, so $\hat{v}_{RY}^{b/e} = \hat{v}_{DX}\theta_Y/\theta_X$ is the break-even threshold discriminating between divest and regrade-divest. The break-even regrade investment cost $R_Y^{b/e}$ is obtained from (20):

$$R_Y^{b/e} = \hat{v}_{RY}^{b/e}\frac{\beta_{Y2}-1}{\beta_{Y2}}\left[\frac{1}{\delta+\theta_Y} - \frac{\theta_X}{\theta_Y(\delta+\theta_X)}\right] - \left(\frac{f_Y - f_X}{r} - D_X\right), \tag{31}$$

with a similar expression for the break-even fixed cost for the appropriate technology $f_Y^{b/e}$. The respective break-even values are $R_Y^{b/e} = 17728.4$ and $f_Y^{b/e} = 676.42$. Finally, the regrade threshold declines as the cost structure for the appropriate technology becomes increasingly more unfavourable. This finding agrees with Dixit and Pindyck (1994), since for both cases an unfavourable change in cost is associated with an unfavourable change in threshold.

While $f_Y^{b/e}$ and $R_Y^{b/e}$ are linearly dependent on $\hat{v}_{RY}^{b/e}$, the break-even fixed cost $f_X^{b/e}$ and divestment value $D_X^{b/e}$ for the conventional technology are not. Figure 1 illustrates the impact of f_X on the divestment threshold $\hat{v}_{DX}$ and regrade threshold $\hat{v}_{RY}$. Although $\hat{v}_{DX}$ and $\hat{v}_{RY}$ are both positively related to f_X, the latter's greater slope enables a break-even to occur at $f_X^{b/e} = 829.15$ with $\hat{v}_{RY}^{b/e} = 152.28$. For $f_X \geq f_X^{b/e}$, a regrade is both justified and feasible, but infeasible if otherwise. Conventional technologies with higher periodic fixed costs are more likely to be regraded. This suggests that a process of improving efficiencies that leads to incremental reductions in the conventional technology fixed cost may enhance the cash flow, but may also cause a regrade to become less likely and even debar its enactment. A similar pattern is observed for the conventional technology divestment value. Figure 2 illustrates impact of D_X on $\hat{v}_{DX}$ and $\hat{v}_{RY}$. The break-even value for D_X occurs at $D_{XM}^{b/e} = -17917.1$ with $\hat{v}_{RYM}^{b/e} = 152.28$. For $D_X \geq D_X^{b/e}$, a regrade is feasible. Conventional technology rigs with high decommissioning costs are more likely to lead to infeasible regrades and the creation of a seascape of idle oil fields. In a world of heightened environmental concerns and exacting health & safety regulations, the consequences of a cost rise in decommissioning a conventional technology rig are seen not only in a deferred divestment but also in a regrade becoming less likely.

**** Figure 1 and Figure 2 about here ****

Infeasibility can also arise due to variations in the parameters that determine the option power parameters. For a typical investment opportunity, increases in volatility and the resulting threshold are positively related, Dixit and Pindyck (1994). However, in the current context, both a regrade investment and a conventional technology divestment are economically stimulated not by increases but decreases in the periodic revenue, so we can expect a negative relationship. This feature is revealed in Figure 3, which illustrates the volatility-threshold relationship. Both the periodic revenue threshold signalling regrade and the adjusted divestment threshold are presented, since each is dependent on the volatility. The figure shows that a volatility increase produces a decline in the thresholds for both regrade and divestment, implying increasing oil price turbulence leads to a deferral in regrade and divestment decisions. For our data set and coverage, the regrade threshold is always greater, but clearly this result cannot be generalized.

**** Figure 3 about here ****

Variations in the two depletion rates influence the solution through their presence and their impact on the power parameters, but differentially. Since any increase in θ_X enhances the attractiveness of the conventional relative to the appropriate technology, this is expected to be reflected in a decreased regrade threshold and a deferred regrade decision, while an increase in θ_Y creates the opposite effect since the appropriate technology is made to be relatively more attractive, the regrade threshold increases and the regrade decision is advanced. Figures 4 and 5, which illustrate the effects of θ_X and θ_Y, respectively, on the solution, endorse this expectation. For $\theta_X > \theta_Y$, Figure 4 reveals that falls in the regrade threshold are associated with increases in θ_X, with a steep rise in the regrade threshold as θ_X approaches θ_Y because a fixed cost saving is being gained at the expense of a decreasing revenue value loss. For $\theta_Y < \theta_X$, Figure 5 reveals that an increase in θ_Y produces an accelerating increase in the regrade threshold as θ_Y approaches θ_X because the appropriate technology becomes increasingly more attractive, but as θ_Y tends to 0 and the attractiveness of the appropriate technology fades, regrade no longer remains feasible and conventional technology divestment is preferred. A θ_X increase makes continuing with the incumbent technology relatively more attractive and divestment or regrade less attractive, while in contrast, a θ_Y increase leads to the incumbent becoming less attractive and divestment or regrade more attractive.

**** Figure 4 and Figure 5 about here ****

Also, the convenience yield and risk-free rate impact on the thresholds through the solution and the power parameters, but in quite different ways. Figures 6 and 7 illustrate their respective effects on the regrade and divestment threshold. Figure 6 reveals that as δ increases the regrade threshold falls, deferring the adoption of the appropriate technology and making it less attractive to the incumbent, while the divestment threshold rises, implying in the absence of a regrade that the divestment decision is advanced and becomes relatively more attractive. In contrast, in Figure 7 r increases enhance the incumbent's attractiveness relative to both regrade and divestment. It shows both thresholds declining for an increase in r, with the eventuality of a regrade becoming infeasible. Clearly, risk-free rate increases signal holding on to the incumbent.

**** Figure 6 and Figure 7 about here ****

4.2 Idle Reserve

When a reserve is in the idle state, the opportunity to deploy the appropriate technology is characterized as a standard investment option, except for the existence of the ensuing divestment option. Based on the Table 2 values, the results are presented in Table 3. The presence of the divestment option impacts significantly on the magnitude of the results. Despite a negative divestment value, the investment threshold falls from 1960.0 to 1191.4, without and with the divestment option, respectively, while the investment option coefficient increases from 0.07662 to 0.10620. Having a subsequent divestment opportunity makes the investment opportunity not only more attractive due to the greater option coefficient, but also to be exercised earlier due to having a lower threshold. Although incurring a negative terminal charge, decommissioning is conceived as adding value to the pure investment opportunity, since the effect of the divestment option is to create a floor threshold for the periodic output value and to prohibit even more adverse cash flow values from occurring.

The reserve becomes idle from being active when $v \leq \hat{v}_{DX}$ and remains idle until the periodic revenue under the appropriate technology rises from $\hat{v}_{DX}^{adj} = 318.99$ to attain the investment threshold $\hat{v}_{KY} = 1191.4$ when the option is exercised. Over that passage of time, the reserve volume remains unchanged so any change in periodic revenue is solely due to a change in the oil price. This implies that the oil price change from the time of the conventional technology divestment to the appropriate technology investment is $\hat{v}_{KY} / \hat{v}_{DX}^{adj} = 3.75$, which is not exceptional given the oil price history over the past 20 years[6]. However, the price multiple is derived assuming a regrade is infeasible so $\hat{v}_{RY} \leq \hat{v}_{DX}^{adj}$. To create the underlying conditions conducive both to a regrade infeasibility and a viable investment in the idle state requires increasing $\hat{v}_{DX}^{adj}$ and lowering $\hat{v}_{KY}$ for the ratio $\hat{v}_{KY} / \hat{v}_{DX}^{adj}$ to yield a plausible multiple, but these two requirements act in opposition. Therefore, committing an appropriate technology investment for the idle state while possible is not very probable.

4.3 Divestment Deferral

The effect of partly deferring the divestment of the conventional technology on the attractiveness of a regrade is illustrated in Figure 8 based on Table 2 values for $0 \leq \varphi \leq 1$. It

[6] Between January 2nd 1997 and January 23rd 2017, the minimum and maximum Brent crude oil prices are \$9.10 and \$143.9, respectively.

reveals that both the regrade threshold and option coefficient are negatively sloped for φ increases. Deferring an increasing proportion of the decommissioning charge is desirable due to the increase in the regrade option value because of the time value of money. Further, deferring raises the regrade threshold, which results in the earlier exercise and implementation of the appropriate technology with its more favoured net revenue flow. Finally, although deferral results in a fall in the coefficient for the divestment option of the appropriate technology (including the deferred divestment of the conventional technology) thereby making divestment less valuable, there is an accompanying decrease in the divestment threshold. This suggests that the duration of actively extracting oil using the appropriate technology is prolonged not only by the earlier exercise of regrade but also by the delayed divestment.

**** Figure 8 about here ****

5 Conclusion

We formulate a real option model for determining the optimal regrade decision to switch from conventional to appropriate technology for a marginal off-shore oil installation. As an active field becomes increasingly marginalised, conventional technologies lose their viability and relevance and without a regrade the residual oil in the reserve becomes economically trapped and inaccessible. Our model is formulated on oil price uncertainty described by a geometric Brownian motion process and a dynamic declining output volume, assumptions which enable a tractable analytical solution to be derived for the two-factor representation based on the periodic revenue level. The formulation incorporates divestment options not only because of the high cost in decommissioning an expended rig and its resulting impact on the divestment decision, but also because of its role in terminating an infinitely lived asset implied by the American perpetuity representation. However, the presence of abandonment affects whether a regrade can exist, since any regrade to be feasible has to be implemented while the oil reserve is active and not subsequent to divestment when the reserve is idle. Other assumptions made are typical for most other analytical real option models.

The periodic revenue threshold signalling an optimal regrade from conventional to appropriate technology is positively related to cost structure improvements in the appropriate relative to the conventional technology. Individually, a decrease in the cost for a regrade or the fixed cost for the appropriate technology or an increase in the fixed cost or divestment value for the conventional technology produce a higher periodic revenue threshold, which implies an earlier regrade decision because both the oil price and initial output volume are assumed to be favourable at the time when the conventional technology rig was installed. Despite its immediate benefits, a continual programme of driving down conventional technology fixed costs defers the regrade decision making it possibly infeasible, as does higher decommissioning costs due to more stringent health and safety regulations. The consequences are potential losses from closed oil reserves and a legacy of derelict oil rigs.

In common with other studies, increases in oil price volatility lead to a postponement of both the regrade and conventional technology divestment decisions. Greater oil price turbulence delays regrade and committing to the appropriate technology. If extractors when selling oil are using risk sharing agreements or other mechanisms for lowering price volatility, then regrades should be made earlier. A lower depletion rate for the conventional technology advances the regrade decision because its relative attractiveness lessens. In contrast, a lower depletion rate for the appropriate technology lowers its relative attractiveness and defers regrade, and for a sufficiently low depletion rate it makes a regrade actually infeasible. Appropriate technology suppliers should have an interest in designing their offerings with the highest compatible depletion rate to motivate earlier adoption. An increase in either the convenience yield or risk-free rate leads to a regrade deferral. A risk-free rate rise makes a switching investment less attractive with the possibility of a regrade becoming infeasible.

Deferring the conventional technology decommissioning charge is interpreted as favourable because of the time value of money. In our investigation of divestment deferral, the analytical solution demonstrates that increasing the proportion of the delayed conventional technology divestment value produces a rise in both the regrade threshold and its option coefficient. This facility is attractive not only because of the increase in value it renders, but the advance in committing to the regrade as well as the elongation of time the appropriate technology is actually deployed before abandonment. Since some regrades may be accompanied with a transfer of ownership, governments should encourage divestment deferral despite any adverse issues caused by rights alienation.

There are consequences for governments intent on pursuing a policy of maximizing economic recovery for oil fields in a near depleted state. If this policy involves motivating extractors to adopt alternative technologies such as regrade, which are more appropriate for the given prevailing reservoir volume, governments may wish to stimulate their uptake through offering research subsidies to suppliers to increase the depletion rate for their technology or reduce its fixed operating cost, or investment subsidies to extractors. The downside of a subsidy programme is the current cost to the government treasury. In contrast, divestment deferral is seen to advance the regrade decision as well as creating value for the extractor, without the government suffering any downside cost. Measures that defer part or all of the cost in divesting the conventional technology result in benefits both to the extractor because of the deferred high decommissioning costs and to the government because of the implied tax relief (assuming that sufficient profits are being generated). However, divestment deferral may be accompanied with rights alienation, which necessitates effective government action to ensure these risks are completely avoided.

The analysis can be extended in several ways. The model in its presented form investigates the comparative merits of a regrade investment assuming known regrade properties and the absence of contending alternatives. The increasing global prevalence of marginal fields may result in innovation with the possibility of the arrival of a more sophisticated appropriate technology with properties outperforming those studied here. This raises the question of whether the extractor should enact the regrade decision promptly or wait until the new innovation emerges. Second, the analysis is performed in isolation of economic alternatives such as gaining improvements through production and cost efficiency gains. These developments should lead to a richer and more insightful representation of the economics of marginal fields.

Table 1
Notation

p	Oil price
q	Periodic output volume
$v = pq$	Periodic output value (revenue)
f	Periodic fixed cost
D	Divestment value (decommissioning cost)
K	Investment cost
R	Regrade cost
$\hat{v}_{DX}$	Optimal divestment threshold for technology X
$\hat{v}_{DX}^{adj}$	Optimal divestment threshold for technology X expressed in terms of Y
$\hat{v}_{DY}$	Optimal divestment threshold for technology Y
$\hat{v}_{RY}$	Optimal regrade threshold for technology Y
$\hat{v}_{KY}$	Optimal investment threshold for technology Y
A_{2DX}	Option coefficient for divesting technology X
A_{2DY}	Option coefficient for divesting technology Y
A_{2RY}	Option coefficient for regrading to technology Y
A_{1KY}	Option coefficient for investing in technology Y

In our notation, lower-case variables represent continuous quantities, such as the periodic output volume and fixed cost, while upper-case variables are one-off quantities, such as the regrade cost and divestment value. Optimal thresholds are denoted by $\hat{}$. All quantities are expressed in terms of the technology specified in the subscript, except for $\hat{v}_{DX}^{adj}$ which is expressed in terms of technology Y. Option coefficients have the subscript 1 or 2, representing investment-style or divestment-style (regrade) opportunities, respectively.

Table 2

Base Case Values

Parameter	Symbol	Value
Risk-free rate	r	5.0%
Convenience yield	δ	3.0%
Volatility	σ	30.0%
Incumbent Technology		
Investment cost	K_X	50000
Divestment value	D_X	-2500
Periodic fixed cost	f_X	1600
Depletion rate	θ_X	5.0%
Option power parameter	β_{X1}	2.1770
	β_{X2}	-0.5104
Appropriate Technology		
Investment cost	$K_Y = R_Y$	7000
Divestment value	D_Y	-210
Periodic fixed cost	f_Y	140
Depletion rate	θ_Y	2.0%
Option power parameter	β_{Y1}	1.6667
	β_{Y2}	-0.6667

Table 3

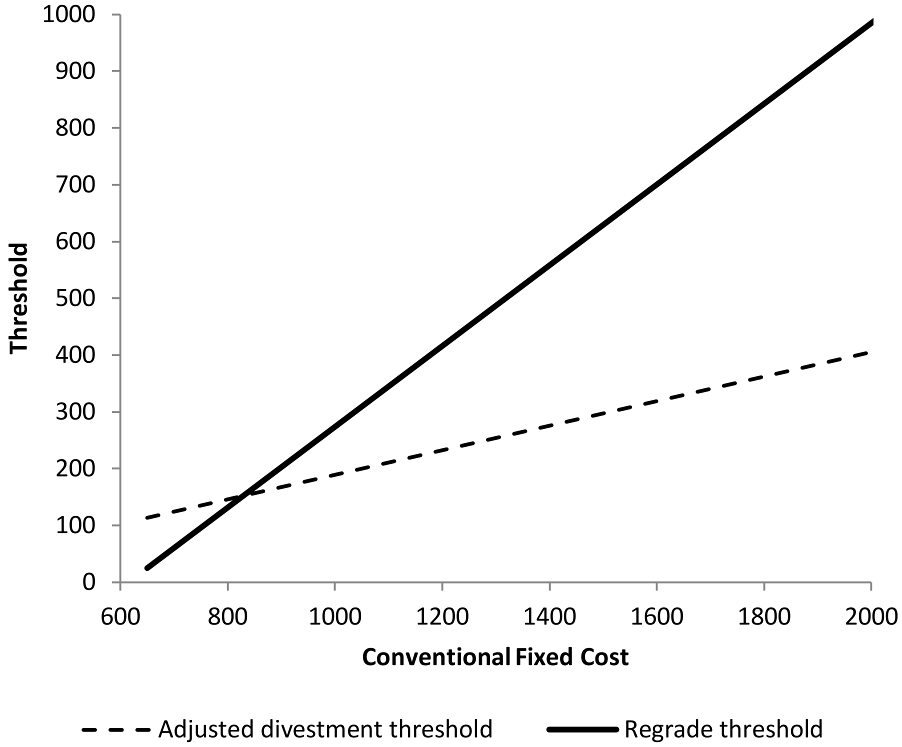
Thresholds and Option Coefficients for Divestment and Investment with Divestment

For the Conventional X and Appropriate Y Technologies

	Equation	Symbol	X	Y
Divestment				
Threshold	(8)	$\hat{v}_D$	797.47	51.80
Option coefficient	(9)	A_2	591159.4	21594.2
Investment with divestment				
Threshold	(11)	$\hat{v}_K$	11207.4	1191.4
Option coefficient	(12)	A_1	9.65E-05	0.106204

Figure 1

The Impact of Variations in the Conventional Fixed Cost $\left(f_X\right)$ on the Regrade Periodic Revenue $\left(\hat{v}_{RY}\right)$ and Adjusted Divestment Thresholds $\left(\hat{v}_{DX}^{adj}\right)$



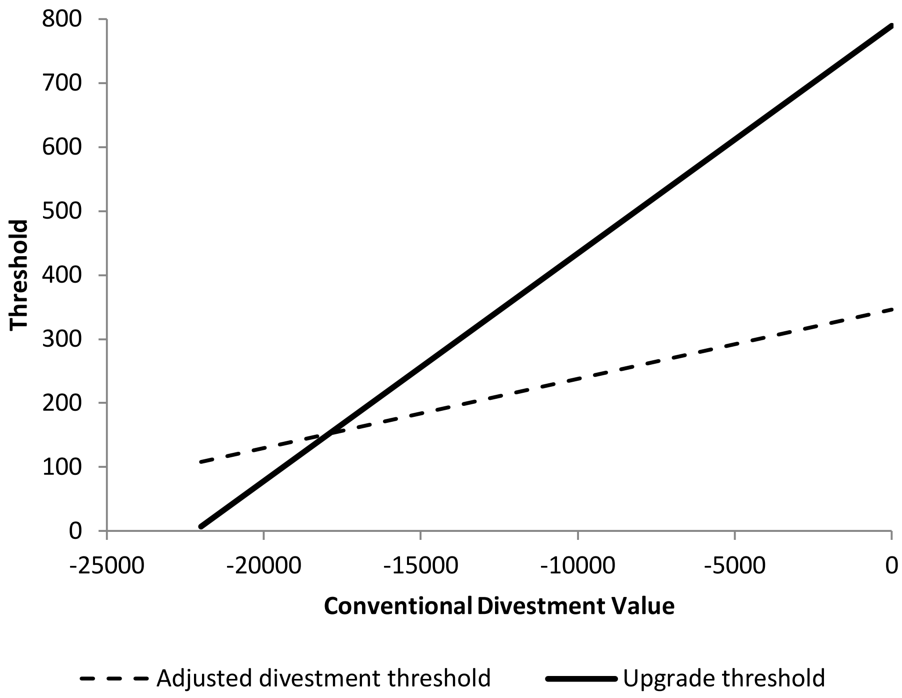
This figure is evaluated from (20) based on Table 2 values except for f_X, while respecting the constraint $f_X > r\left(R_{YM} - D_{XM}\right) + f_Y$. Typical values are given in the following table:

f_Y	A_{2DX}	$\hat{v}_{DX}$	$\hat{v}_{DX}^{adj}$	A_{2DY}	$\hat{v}_{RY}$
800	181530.5	364.95	145.98	79018.1	131.56
1000	268643.0	473.08	189.23	216387.6	273.78
1200	366609.5	581.21	232.48	412794.7	416.00
1400	474378.7	689.34	275.74	660233.5	558.22
1600	591159.4	797.48	318.99	953846.1	700.44
1800	716327.8	905.61	362.24	1290229.6	842.67

with $A_{2DY} = 21594.2$ and $\hat{v}_{DY} = 51.8$.

Figure 2

The Impact of Variations in the Conventional Divestment Value $\left(D_X\right)$ on the Regrade Periodic Revenue $\left(\hat{v}_{RY}\right)$ and Adjusted Divestment Thresholds $\left(\hat{v}_{DX}^{adj}\right)$

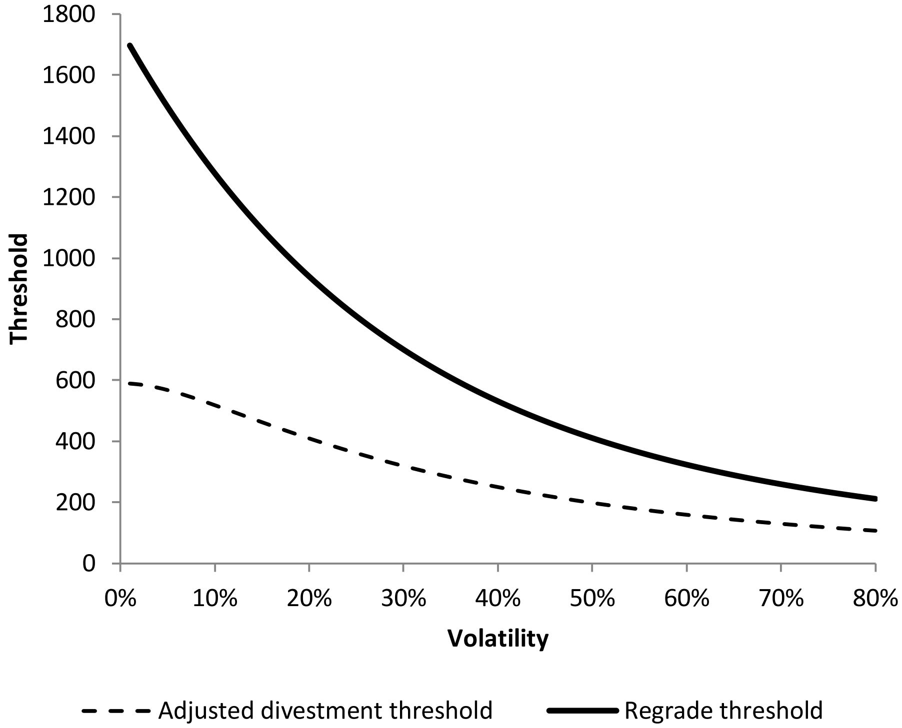


This figure is evaluated from (20) based on Table 2 values except for f_X, while respecting the constraint $D_{XM} > -\left(f_X - f_Y\right)/r + R_{YM}$.

Figure 3

The Impact of Variations in Price Volatility (σ) on the

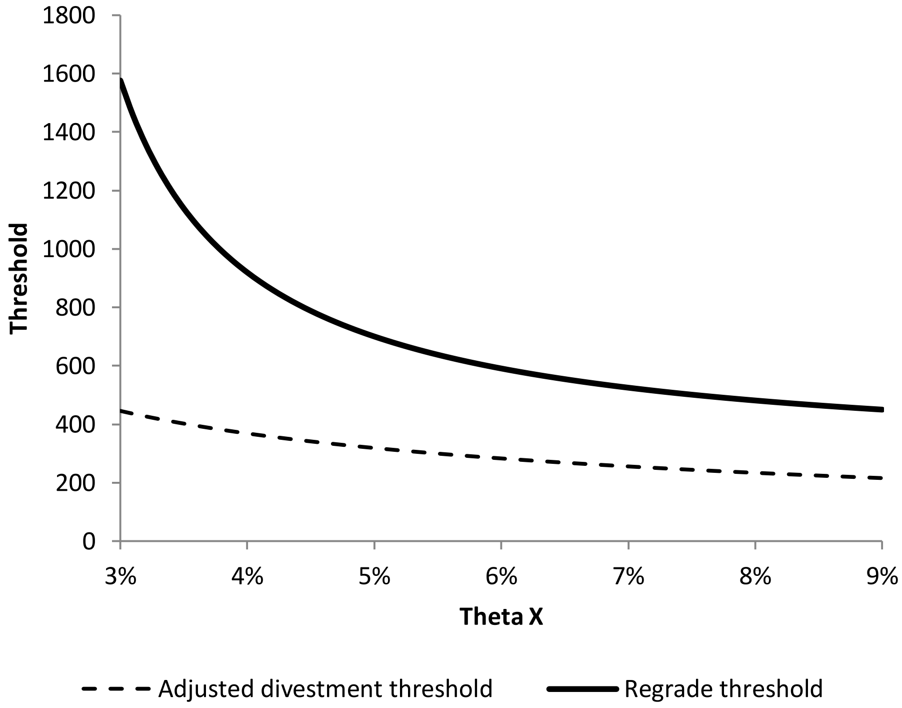
Regrade Periodic Revenue $(\hat{v}_{RY})$ and Adjusted Divestment Thresholds $(\hat{v}_{DX}^{adj})$



This figure is evaluated from (20) based on Table 2 values except for σ.

Figure 4

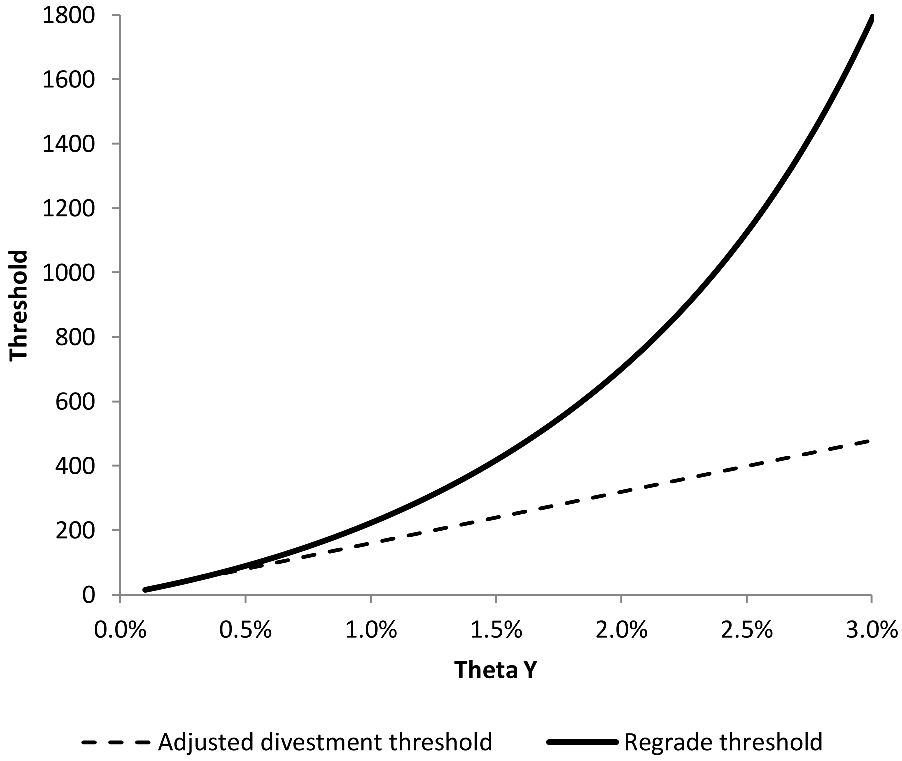
The Impact of Variations in Conventional Technology Depletion Rate (θ_X) on the Regrade Periodic Revenue $(\hat{v}_{RY})$ and Adjusted Divestment Thresholds $(\hat{v}_{DX}^{adj})$



This figure is evaluated from (20) based on Table 2 values except for θ_X .

Figure 5

The Impact of Variations in Appropriate Technology Depletion Rate (θ_Y) on the Regrade Periodic Revenue $(\hat{v}_{RY})$ and Adjusted Divestment Thresholds $(\hat{v}_{DX}^{adj})$

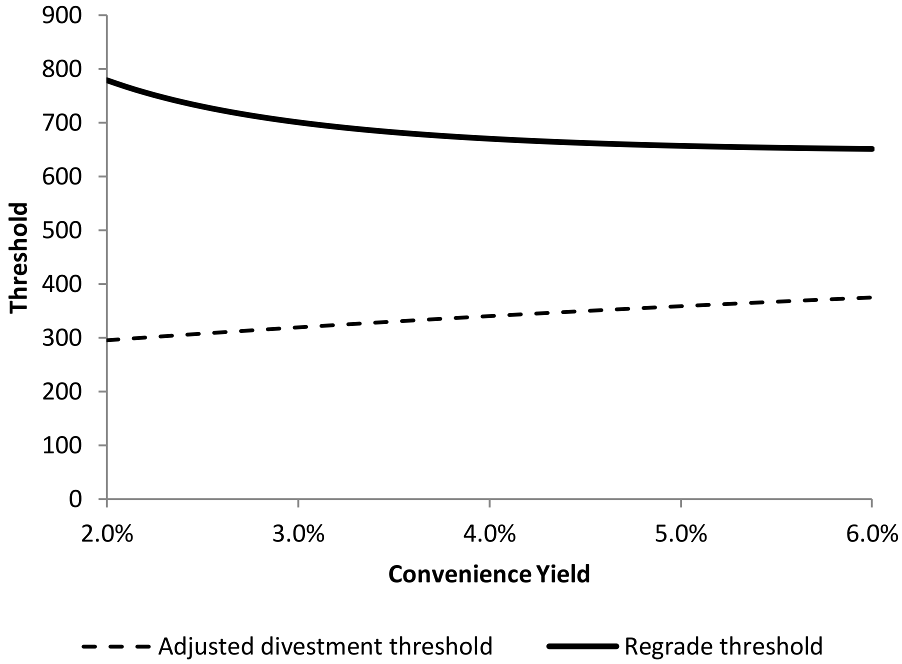


This figure is evaluated from (20) based on Table 2 values except for θ_X .

Figure 6

The Impact of Variations in Convenience Yield (δ) on the

Regrade Periodic Revenue $(\hat{v}_{RY})$ and Adjusted Divestment Thresholds $(\hat{v}_{DX}^{adj})$

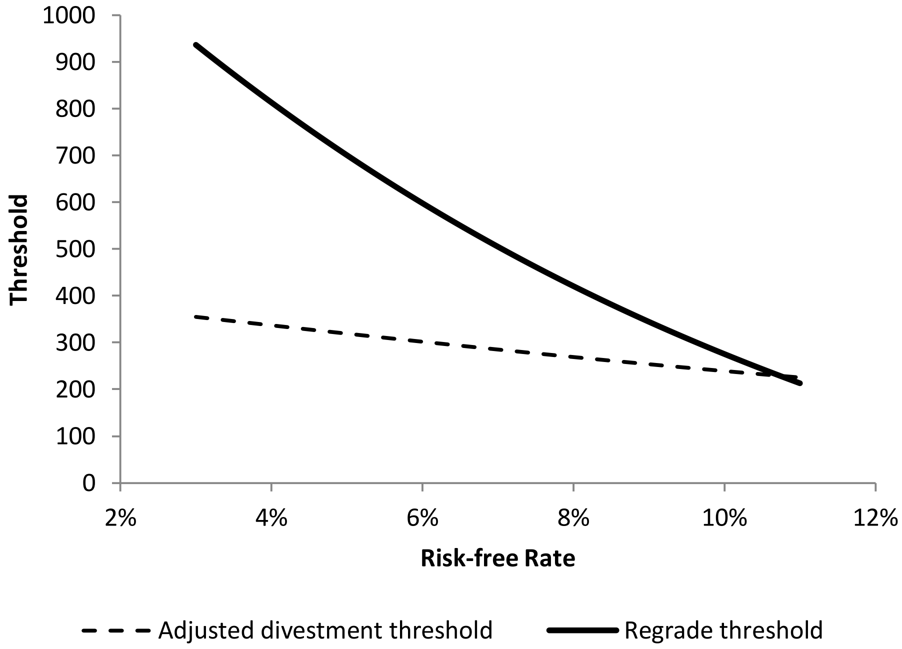


This figure is evaluated from (20) based on Table 2 values except for δ.

Figure 7

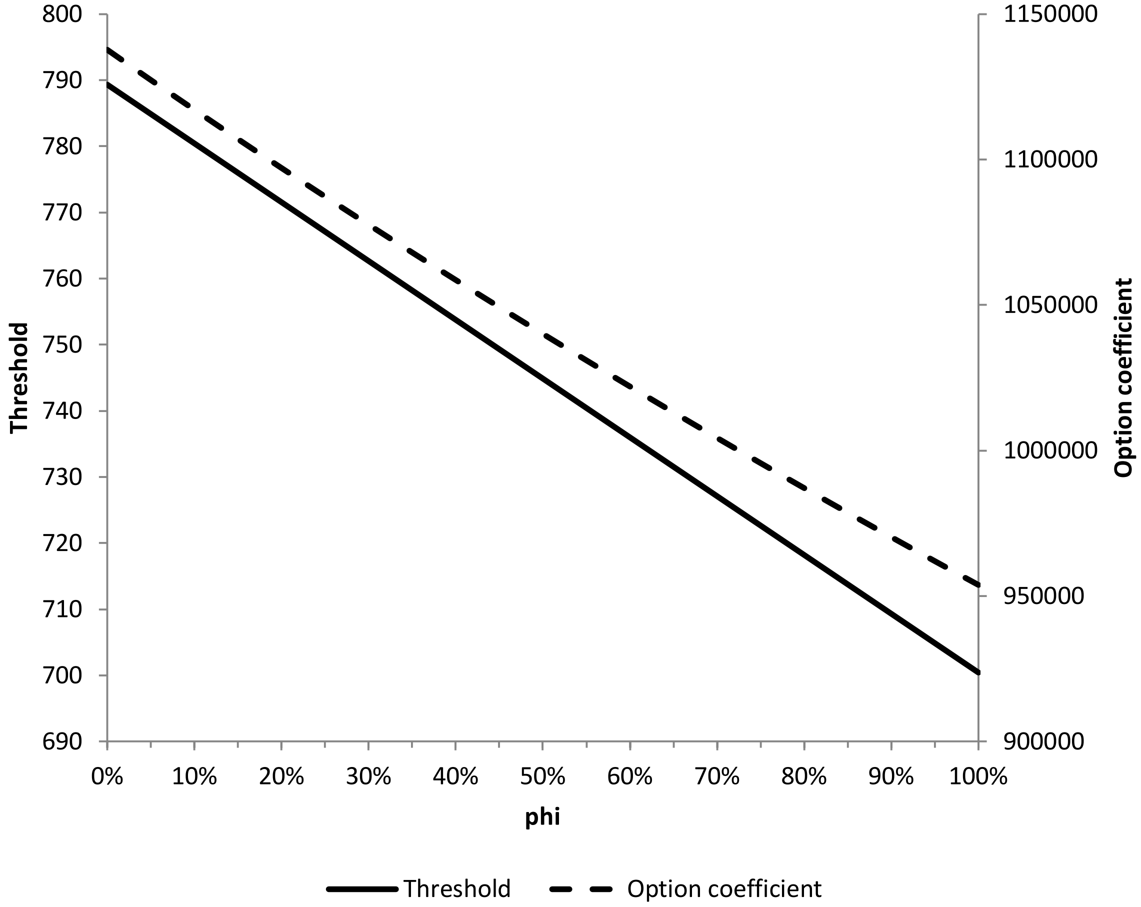
The Impact of Variations in Risk-free Rate (r) on the

Regrade Periodic Revenue $(\hat{v}_{RY})$ and Adjusted Divestment Thresholds $(\hat{v}_{DX}^{adj})$



This figure is evaluated from (20) based on Table 2 values except for r.

Figure 8
The Impact of Variations in the
Proportion of the Non-Deferred Conventional Technology Divestment (ϕ) on the
Regrade Periodic Revenue $\left(\hat{v}_{RY(\phi)}\right)$ and Option Coefficient $\left(A_{2RY(\phi)}\right)$



This figure is evaluated from (30) based on Table 2 values.

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