Second Round Financing for Start-ups with Convertible Notes Under Asymmetric Information: A Real Options Approach

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Abstract

Convertible notes are commonly used in start-ups financing. Focusing on the relationship between two different types of investors, a second-round equity investor and a first-round convertible note holder, this paper simulates how the value of option to convert of the convertible note holder can be affected by the trustworthiness of the second-round equity investor to the entrepreneur's ability to procure funds and to increase firm's value in each financing round. The result of a real case simulation with a real options approach suggests, firstly, that the improvement of the investor's trustworthiness to the entrepreneur's ability has a great impact on the amount that the entrepreneur should offer and procure when the second-round equity investors are quite doubtful about the entrepreneur's ability and the convertible notes are chosen more preferably than equity. Secondly, the preference of convertible notes over equity becomes less strong when the investor's trustworthiness to the entrepreneur's ability is improved, especially in the lower range of trustworthiness.

Keywords: second round financing, convertible notes, adverse selection, real options

1. Introduction

Convertible notes are chosen as a common method for start-up financing. They are often used especially in the early stage investment. According to the survey by Marianne Hudson who is the ACA (Angel Capital Association) Executive Director in 2015, 78% of ACA members had done at least one convertible note in the recent 18 months until that year. When considering financing with convertible notes, there are at least three participants, that is, the entrepreneur/shareholder, the second-round equity investor (or new equity investor), and the first-round convertible note holder (or simply, convertible note holder). The interests of these three players are not always aligned and may be the cause of conflict of interest. If the business of start-up goes well and its firm's value or its share price increases, the second-round equity investor will be able to provide funds fully as the entrepreneur desires. Furthermore, the convertible note holder will convert its debt to equity in the next financing round. However, when the convertible note is converted to equity, the issue of dilution of the equity investor's share percentage emerges. On the other hand, if the business of start-up does not go well, or if the second-round equity investor does not believe it goes well, the second-round equity investors will be reluctant to invest and the entrepreneur/shareholder will decrease its value. Even in this situation, the value of the convertible note holder will not be changed because it is a debt contract and its conversion is an option especially in the case of voluntary conversion.

This paper focuses on this potential issue, especially between the convertible note holder and the second-round equity investors. The main objective of this paper is to analyse how the value of the convertible note holder's conversion option can be affected by the trustworthiness of the second-round equity investor to entrepreneur's ability to procure funds and to increase firm's value in each financing round. The issue of asymmetric information is incorporated in this analysis. Many researches argue that there exists an asymmetric information problem in the start-up financing. For example, with respect to venture capital's decision-making, Trester (1998) shows that preferred stock dominates debt and common stock in early venture capital's financing choice under asymmetric information. For entrepreneur's financing choice, Revest

and Sapio (2012) point out that European technology-based small firms finance new investments by relying primarily on internal funds, due to capital market failures induced by asymmetric information.

The existence of this type of asymmetric information problem can lead to select staged financing. Neher (1999) explains that the venture capitalists cannot observe whether the project has become a failure without bearing a monitoring cost. Therefore, staging the investment over time helps to mitigate this kind of problem. The model of Wanga and Zhoub (2004) shows that staged financing plays two roles of controlling risk and of mitigating moral hazard in venture capital. The method of alleviating this asymmetric information problem in venture capital's investment is not limited to the staged financing. In practice, convertible notes have become almost de facto standard when implementing the investment in start-ups (Feld and Mendelson, 2016). In academic research, Bascha (2001) argues that the ex-ante agreed optimal exit policy (=contract design) can be implemented with convertible securities in venture capital finance. Moreover, in their empirical study, Lewis et al. (2003) point out that convertible debt can be designed to mitigate different combinations of debt- and equity-related costs of external finance.

Recently, the asymmetric information problem in start-ups financing is often discussed as moral hazard issues. Bettignies et al. (2007) address the entrepreneur's financing choice problem between bank finance and venture capital from the viewpoint of moral hazard. Fairchild (2011) develops a game-theoretic model that analyses the effects of economic and behavioural characteristics on an entrepreneur's choice of financier (venture capitalist or angel) with the view of moral hazard. From the viewpoint of investors, however, the adverse selection problem is more critical. In the context of financing contract design, it is the investors that are usually the ones who have the bargaining power in the financing negotiation. Although the investors can make an investment decision, they cannot have enough information about the entrepreneur's ability to achieve its goal and the probability of increasing the firm's value. Krishnaswami and Yaman (2008) argue that moral hazard and adverse selection are important determinants of the likelihood of issuing convertible bonds over straight bonds. However, moral hazard costs do not influence bond structure, while adverse selection costs are somewhat important in determining the structure. Thus, this paper focuses on the concept of adverse selection problem.

Several papers discuss about the convertible notes under adverse selection. Stein (1992) insists that corporations may use convertible bonds as an indirect way to get equity into their capital structures when adverse selection problems make a conventional stock issue unattractive. Lewis (1998) suggests that some issuers design convertible debt to mitigate asset substitution problems, while others design it to reduce adverse selection problems. However, these discussions are based on the principal-agent model where the convertible bond investor is principal and the borrower of entrepreneur is agent. Although the discussion about the relationship between the equity investors and the convertible notes holder is also important, there are few researches discussing about this point. Dealing with this topic is one of the unique features of this paper. Moreover, incorporating an actual start-up financing practice is another feature. In practice, the conversion criterion of the convertible note depends on the amount of the investment from the second-round equity investor in the next financing round, not the share price. The convertible note is not converted to equity if the investment amount of the equity is small. Thus, the equity investment amount is a critical factor for the convertible note holder. On the contrary, such a conversion to equity causes the dilution of the equity investor's share. If this dilution is expected to be large, the equity investor would be reluctant to provide funds. Thus, the convertible note holder's option to convert is also a critical factor for the equity investor. Considering that the adverse selection problem is explained with the relationship between the equity investor and the entrepreneur, the convertible note holder should be the key player in the financing round.

The convertible notes are converted to equity when the amount of investment from second-round equity investors exceeds the threshold value set by the convertible note holder. There are two ways of voluntary and mandatory conversion in practice. The former characteristics can be described as a real option in particular because the investment amount that the entrepreneur can really obtain in the next financing round is uncertain and the value of the flexibility whether the convertible note holder convert can be thought. Here is another unique feature of this paper that this value of the conversion option to equity is introduced, thus, the real options approach analysis is conducted with the adverse selection model. The real options analysis is used for capturing the value of flexibility in decision-making by its nature, especially in the discrete type analysis (see for instance Corpland and Antikarov (2003)). In order to do so, however, the probability of each payoff must

be known. In this case, to resolve this issue, the probability in the concept of adverse selection can be used. For making this theoretical analysis be more understandable, the simulation is conducted by using data from a real case.

The structure of this paper is as follows. In section 2, the model is set up. In section 3, the application to convertible note investment is developed. In section 4, the real case simulation is conducted. In the section 5, conclusion is remarked.

2. The model

Applying the principal-agent contract model is suitable to analyse the relationship between investors such as venture capitalists and an entrepreneur in the situation of a financing contract. In particular, the effect of asymmetric information should be incorporated into this model. This kind of modelling process has already been proposed quite clearly by Macho-Stadler and Pérez-Castrillo (2001), thus, the first step to set-up the valuation model follows their way.

1. Base model with the concept of adverse selection

The idea of base model building derives from the concept proposed by Macho-Stadler and Pérez-Castrillo (2001). There are three assumptions (p.5):

- The principal designs the contract, or set of contract, that she will offer to the agent.
- The agent accepts the contract if she so desires, that is if the contract guarantees her greater expected utility than the other opportunities available to her.
- The agent carries out an action of effort on behalf of the principal.

Let x be the monetary value of the result to be obtained, w the wage or pay-off to the agent, and e the effort that the agent dedicates to the task. According to the second assumption above, when the utility function of agent is defined as $U(.)$, it can be written as $U(w,e) = u(w) - v(e)$ where $v(e)$ is called as disutility of effort function. Considering the Pareto efficient condition of the principal's utility function in the case of no adverse selection problem, the objective functions for agent, $u(w)$, can be described as follows.

$$u(w) = -Aexp(-r_a w) \qquad \cdots (1)$$

When there is an adverse selection problem between both participants of contract, preparing several possible states of nature of agent is quite effective to build up the model. According to Macho-Stadler and Pérez-Castrillo (2001), the main idea of this modelling is that principal cannot recognise that agent is either "Good type" (denoted by G) agent or "Bad type" (denoted by B) one, thus the optimal contract condition should be modified and the result would deviate from the one under symmetric information. In the context of start-ups, "Good type" can be interpreted as an entrepreneur having an excellent ability to procure more funds in each financing round and to achieve the firm's value increase. On the other hand, "Bad type" can be interpreted as an entrepreneur who has a lower ability than "Good type". Thus, this can be also defined as investor's "trustworthiness" toward entrepreneur. This modelling concept is also adapted by Koufopoulos (2009).

Before obtaining the final condition for optimal contract, the modelling process starts with describing the utility function of each type of agent, assuming that the principal made contracts with two different agents. In this situation, adverse selection problem is not yet to be considered.

Let $\Pi(e)$ be an expected payoff to principal with the assumption that the effort of agent is associated with this. Then, this can be written as $\Pi(e) = \sum_{i=1}^{n} p_i(e)\, x_i$ where $p_i(e)$ is the probability of possible results.

In the case of no adverse selection problem, the principal's optimal contract would be determined by solving the following problem:

$$\max_{e,w} \Pi(e) - w \qquad \text{s.t.} \quad u(w) - v(e) \geq \underline{U}$$

$\underline{U}$ refers to the agent's reservation utility. Assuming that the principal made two contracts with two different agents, the optimal contract condition for each type agent can be described as follows:

$$\Pi'(e^G) = \frac{v'(e^G)}{u'(w^G)}, \quad u(w^G) - v(e^G) = \underline{U}$$

$$\Pi'(e^B) = \frac{kv'(e^B)}{u'(w^B)}, \quad u(w^B) - kv(e^B) = \underline{U}$$

The coefficient of k in the disutility of effort function of Bad type can be assumed to be greater than 1 because Bad type agent would usually think that he needs more effort to accomplish his responsibility. The adverse selection problem has not yet been considered until this step.

For the last step of modelling process, the condition about adverse selection problems should be incorporated. Under this circumstance, it is not possible for the principal to know whether agent is Good type or Bad type. Now that introducing the variable q is effective, which represents the probability that the principal thinks of agent as being Good type. Then, the principal's problem to solve for obtaining an optimal contract is rewritten as follows:

$$\max_{\{(e^G,e^G),(e^B,e^B)\}} q[\,\Pi(e^G) - w^G\,] + (1-q)[\,\Pi(e^B) - w^B\,]$$

$$\text{s.t.} \quad u(w^G) - v(e^G) \geq \underline{U}\ , \quad u(w^B) - kv(e^B) \geq \underline{U}$$

$$u(w^G) - v(e^G) \geq u(w^B) - v(e^B)\ , \quad u(w^B) - kv(e^B) \geq u(w^G) - kv(e^G)$$

$\{(e^G, w^G), (e^B, w^B)\}$ is called as menu of contracts. The first two conditions are participation constraint. The last two conditions are called as self-selection or incentive compatibility, which is designed for being matched to each type, rather than other's.

Considering two Lagrange multipliers for the first two constraints above respectively, the following optimal condition can be obtained with Kuhn-Tucker theorem.

$$u(w^G) - v(e^G) = \underline{U} + (k-1)v(e^B) \qquad \cdots (2)$$

$$u(w^B) - kv(e^B) = \underline{U} \qquad \cdots (3)$$

$$\Pi'(e^G) = \frac{v'(e^G)}{u'(w^G)} \qquad \cdots (4)$$

$$\Pi'(e^B) = \frac{kv'(e^B)}{u'(w^B)} + \frac{q(k-1)}{1-q}\frac{v'(e^B)}{u'(w^G)} \qquad \cdots (5)$$

2. Application of base model to investment (in general condition)

Under the situation that adverse selection exists, the investor would invest their funds within the restrictions above. Thinking about the practical phase of investment, some variables in these theoretical conditions can be determined. The aim of this paper is to propose the practitioner-oriented model, simplifying the model to the degree not to deteriorate theoretical accuracy is desirable.

In the theoretical setting, x is defined as the monetary value of the result to be obtained. In the case of an investment for a start-up, this can be interpreted as the set of expectations by the investor. w represents the amount of investment from the investor. When assuming I to be defined as the desired amount of investment that the entrepreneur demands to the investor in a financing round, this variable should be $0 \leq w \leq I$. For simplifying the discussion, the measures of absolute risk-averse of both participants can be equal. It leads the value of r_a in the equation (1) to be ½. According to the model building under asymmetric information, setting of $w^G = I$ and $w^B = \beta I$ is possible. β $(0 \leq \beta < 1)$ is defined as the percentage of the amount invested by the investor. Then, the equation (3) can be rewritten as follows:

$$u(w^G) = -Aexp\left(-\frac{1}{2}I\right), \quad u(w^B) = -Aexp\left(-\frac{1}{2}\beta I\right) \qquad \cdots (6)$$

Considering the disutility function of $v(e)$ is the representation of something like cost in exchange of entrepreneur's effort, $v(e) = e$ is possible for simplicity.

The equation (4) expresses the optimal condition for the nature of "Good type". It is clear that the optimal condition is when $\beta = 1$, which means that the investor provides the full amount that the entrepreneur demands. It is also verifiable $\Pi'(e^G) = 0$ when $\beta = 1$.

Applying the equation (6) to the equation (5) of $\Pi'(e^B) = 0$, the following condition can be obtained;

$$q = \left[kexp\left(-\frac{1}{2}I\right)\right] / \left[kexp\left(-\frac{1}{2}I\right) - (1-k)exp\left(-\frac{1}{2}\beta I\right)\right] \qquad \cdots (7)$$

This equation represents the optimal contract condition for the principal under the existence of an adverse selection problem. This can also explain that if the real value of β (=percentage of amount obtained) is known, it is possible for the entrepreneur (agent) to estimate the probability that the investor (principal) thinks of the entrepreneur being "trustworthy" when the investor decides whether to invest. Thus, it allows the agent to take a further strategy when the entrepreneur offers the amount of investment she needs to investors.

3. Application to convertible notes investment

1. Basic scheme of convertible notes investment

The condition (7) is quite useful for applying to the valuation with convertible notes. Before applying it, confirming the value structure with and without convertible notes in each financing round is essential.

The valuation of share price is quite difficult for start-ups. Unlike the case of listed company's shares, there are few reliable price reference resources because almost all the start-ups' shares are not traded publicly. Thus, as mentioned above, both investor and entrepreneur actually want to avoid valuation if possible.

The practical common valuation scheme begins with the offer of the amount from the entrepreneur that he wants to procure as equity (=I_N), and the investment is implemented if investors agree to this amount. When the pre-money value of the start-up (or its venture project) has also been agreed among investors and entrepreneur, the post-money is determined (see. Fig.1). Similarly, the entrepreneur also offers the amount of investment he desires (=I_C) as convertible notes. If investors agree to this, investment is also implemented.

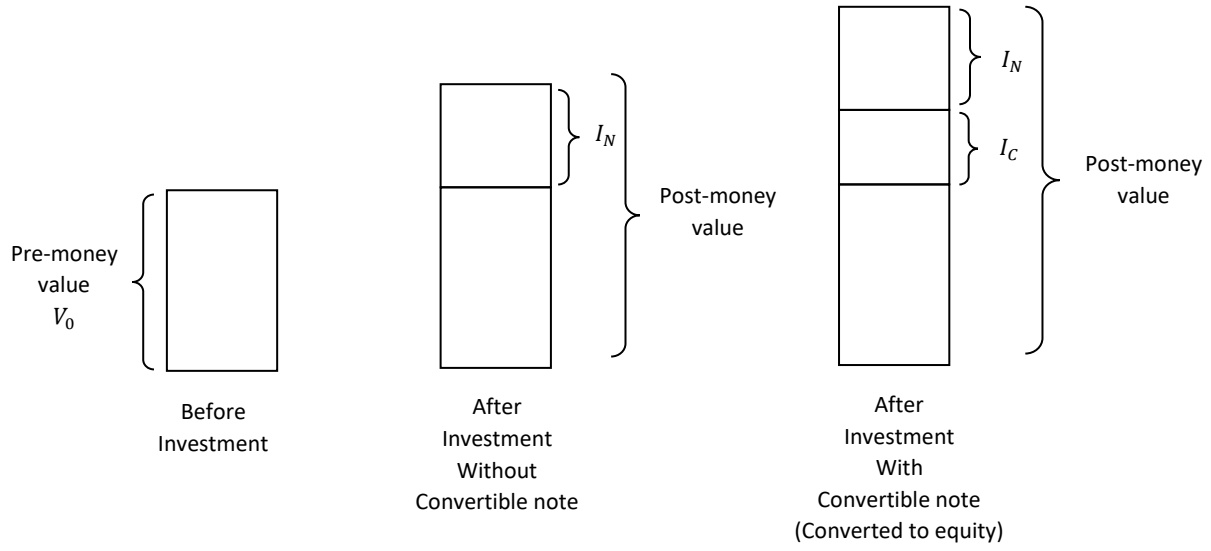


Fig.1 The values in the financing rounds

At least three methods for determining the share price are known, however, the one based on the pre-money is often chosen in practice, and convertible notes investment calculation is implemented with this price. This paper adopts this method. Now assume that the entrepreneur has N_0 shares before any investment in the financing round. Thus, the price of share is $\frac{V_0}{N_0}$. When equity investment is done by the new investors, the number and percentage of shares are changed as shown in Table 1.

Table 1: After investment without convertible note

	< Number of shares >	< Percentage of shares >
Entrepreneur	N_0	$\frac{V_0}{V_0 + I_N} \times 100$
Second-round equity investors	$\frac{I_N}{V_0} N_0$	$\frac{I_N}{V_0 + I_N} \times 100$
Convertible notes holder	-	-
< Total >	$\left(1 + \frac{I_N}{V_0}\right) N_0$	100

The convertible note holders can buy the share at a discount price when converting its debt to equity. Assume that the discount rate is α $(0 \leq \alpha < 1)$, the price of share for the convertible holders is $\frac{V_0}{N_0}(1 - \alpha)$. When equity investment is done by the new investors and convertible note is converted, the number and percentage of shares are changed as shown in Table 2.

Table 2: After investment with convertible notes converted to equity

	< Number of shares >	< Percentage of share s>
Entrepreneur	N_0	$\frac{V_0}{V_0 + I_N + I_C/(1-\alpha)} \times 100$
Second-round equity investors	$\frac{I_N}{V_0} N_0$	$\frac{I_N}{V_0 + I_N + I_C/(1-\alpha)} \times 100$
Convertible notes holder	$\frac{I_C}{V_0(1-\alpha)} N_0$	$\frac{I_C/(1-\alpha)}{V_0 + I_N + I_C/(1-\alpha)} \times 100$
< Total >	$\left(1 + \frac{I_N}{V_0} + \frac{I_C}{V_0(1-\alpha)}\right) N_0$	100

2. Application of the model with the effect of adverse selection: Real Options Approach

The investment into convertible notes has a unique characteristic that is suitable for introducing the concept of asymmetric problem. There is a question why convertible notes would be chosen favourably in both practice

and academic. It is often said in practice that the usage of convertible notes makes investment quick because of its less paper work and it does not require the valuation before investment. On the other hand, Wang et al. (2009) explain two academic approaches for this question. One is the incomplete-contract approach, and the other is the asymmetric-information approach. The former focuses on the renegotiation possibilities after investment. This approach is suitable for listed company's investment, and well explains the under-pricing problem of convertible bonds (see for instance Chan and Chen (2007)). However, as Hsu (2010) sets the assumption for his model, it is natural to think that the main goal of an entrepreneur is to maximize a probability of raising funds in the next financing round, while the one of an investor is to maximize the value of firm or project. Therefore, the latter asymmetric-information approach is appropriate for the convertible notes investment. This means that the convertible note investment is more favourable than equity investment when asymmetric information problem is suggested.

Along with the discussion above, applying the equation (7) to the convertible notes investment allows the following prediction. The percentage of the amount of investment β is agreed in the previous financing round, thus entrepreneur can estimate the degree to which investors trust him as being G type agent by calculating q. This estimation can be used for entrepreneur to predict how much amount he will actually be able to obtain. It is predicted as βI_N (see. Fig.2).

Shifting the focus back onto the behaviours of investors, the difference of characteristics between second-round equity investor and convertible notes holder must be taken into consideration. It is often the case in practice that convertible notes holders convert their debt obligation to equity when the amount of investment from second-round equity investors exceeds the threshold which convertible note holder set beforehand. This threshold is equivalent to strike price in the context of call option, and it can be said that convertible notes holders have an option to choose, that is a real option. In the discussion of why convertible notes are chosen favourably, Wang et al. (2009) also argue that the asymmetric-information approach explains that convertible notes give the firms a back door to equity and give investors an opportunity to wait and see if the project is worth investing in. Practically speaking, the legal term of mandatory conversion or voluntary conversion is

included in the term sheet of financing contract. The latter voluntary conversion demonstrates the characteristic of real options clearly.

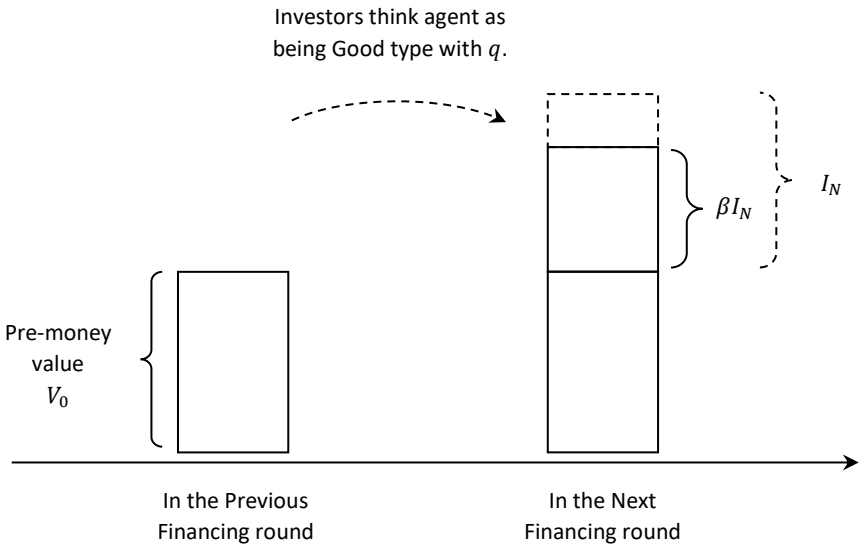


Fig.2 The expected investment amount in the next financing round

On the contrary, second-round equity investors have not such an option at this phase though it is possible to think of other kinds of options for them. However, a dilution problem is crucial for them under the existence of convertible note holder because they might give up investment if the degree of dilution is large. The second-round equity investors' decision-making would be affected significantly by the one of convertible holder especially when voluntary conversion is allowed. Therefore, the entrepreneur must take all these elements into consideration when he wants to raise funds.

The threshold amount of conversion to equity I^* can be determined after negotiating between entrepreneur and convertible note holder. Thinking about the difference of bargaining power, it should be natural that convertible note holder has the initiative to determine it.

As mentioned above, the entrepreneur offers the amount she desires first in each financing round. Let this amount be I_{offer}.

(1) $I_{offer} < I^*$

If the amount offered by the entrepreneur is smaller than the threshold amount for convertible note holder, no dilution of second-round equity investors' share will occur. However, this small amount offering would give a signal to investors that this venture project or start-up are neither lucrative nor attractive to invest. Thus, the entrepreneur should avoid this situation because the investment may not be implemented.

(2) $I_{offer} \geq I^*$

Although it looks attractive for the convertible note holder when this case is achieved, the effect of dilution for second-round equity investors should be considered as shown in fig.3. Even if second-round equity investors admit this investment is favourable, the possibility of dilution of their share may cause to be reluctant to provide funds.

The second-round equity investors would not decide to invest in the situation of being offered below I^{**} in Fig.3 unless their share level before dilution is guaranteed. On the contrary, if the offered amount is above I^{**}, and if this procurement is achieved, the second-round equity investors could be satisfied even in the existence of the effect of dilution. Thus, the entrepreneur should offer at least I^{**} strategically. Until this point, the effect of adverse selection is not considered. As discussed above, even if the entrepreneur offers a certain amount, he would not be able to obtain the full of it under asymmetric information. Instead, as keeping the probability q of investors (principal) unchanged, the estimation of the amount actually obtained, I^{***} should be $\frac{1}{\beta} I^{**}$. Therefore, the entrepreneur must offer the amount that is greater than $\frac{1}{\beta} I^{**}$ under asymmetric information.

The condition that I^* and I^{**} must satisfy can be described as follows:

$$\frac{I^*}{V_0 + I^*} = \frac{I^{**}}{V_0 + I^{**} + I_C/(1-\alpha)} \Leftrightarrow I^{**} = I_{offer} = \left(1 + \frac{I_C}{V_0(1-\alpha)}\right) I^*$$

This I^{**} represents the minimum amount that the entrepreneur should offer under no asymmetric information. Therefore, the amount under asymmetric information should be as follows:

$$I_{offer} = \frac{1}{\beta} I^{**} = \frac{1}{\beta}\left(1 + \frac{I_C}{V_0(1-\alpha)}\right) I^* \quad \cdots (8)$$

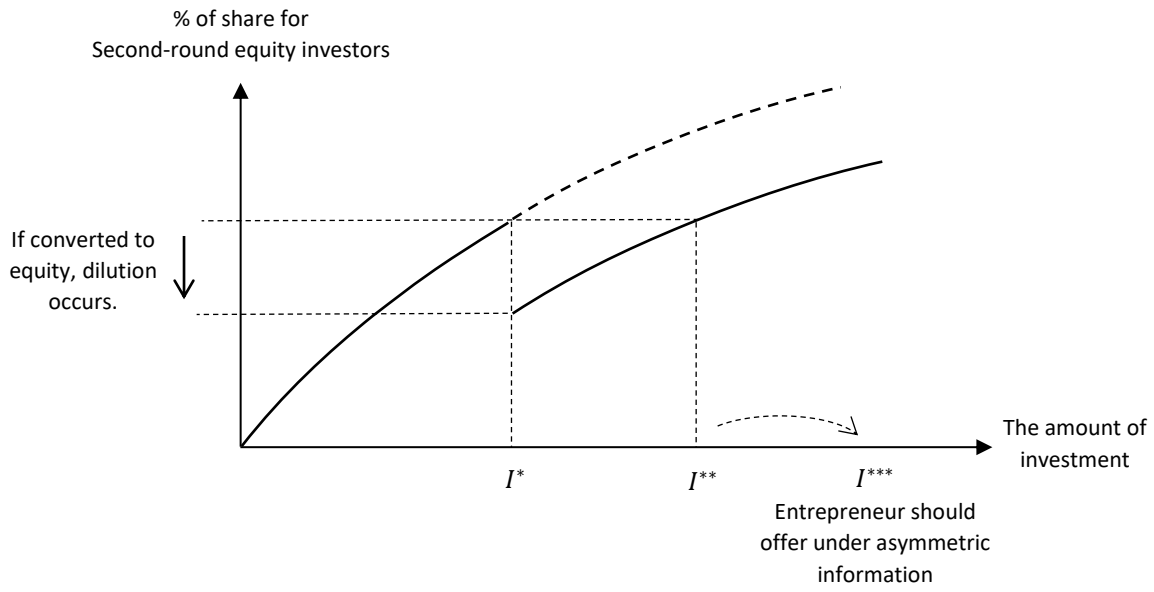


Fig.3 The dilution when the conversion to equity occurs

It is beneficial for the entrepreneur to be able to estimate the amount, which she should offer at the next financing round. Until the discussion above, the probability q that investors think agent as being "trustworthy" (=Good type) is unchanged. Then, β can be used as it is. However, what if this probability is changed in the next financing round? The simulation based on this probability change is also quite useful for the entrepreneur.

According to the equation (7), it is obvious that q is the function of β. Thus, it can be rewritten for simplicity as $q = f(\beta) \Leftrightarrow \beta = f^{-1}(q)$, where f^{-1} is defined as the inverse function. Let $\frac{I_C}{V_0}$ be replaced into γ, which represents the ratio of the amount of convertible note against the one of initial firm's value, then, the equation (8) can be rewritten as follows:

$$I_{offer} = \frac{1}{f^{-1}(q)}\left(1 + \frac{\gamma}{(1-\alpha)}\right) I^* \quad \cdots (9)$$

The equation (9) implies that the amount that the entrepreneur should offer can be changed if the investors' recognition whether she is "trustworthy" changes. At the same time, it depends on the amount of threshold, which the convertible note holder sets in advance. The model proposed in this paper is based on the assumption that the principal (investor) provides a contract (investment) and what the agent (entrepreneur) can do is to accept or reject it. However, the entrepreneur's attempt to change the investor's trustworthiness to her is meaningful even if she cannot do it directly because we assume that the entrepreneur is a probability maximizer of raising funds in the next financing round (Hsu, 2010). Therefore, knowing the relationship among I_{offer}, q and I^*gives the entrepreneur a quite useful tool when she goes into negotiation about the amount.

With the assumption that the convertible note holder has the option to convert as real option and determines its threshold amount I^*, the probability q is applicable to calculate the value of conversion option to equity. If second-round equity investors think the entrepreneur is trustworthy, they will pay the full amount of I_{offer}, and if not, they will pay $\beta \times I_{offer}$. Thus, this option value, c, can be obtained as shown in Fig.4 where r_f is risk free rate and t is the time to the next financing round;

< Payoff >

$$c = \frac{q\left(I_{offer} - I^*\right) + (1-q)\left(\beta \times I_{offer} - I^*\right)}{\left(1 + r_f\right)^t}$$

q → $max\left[I_{offer} - I^*, 0\right]$

$1 - q$ → $max\left[\beta \times I_{offer} - I^*, 0\right]$

Fig.4 The option value of conversion to equity

4. Case Simulation

Now, all these discussions are applied to a practical case simulation. A medical start-up (it is called "Start-up X" in this paper) procured the funds as shown in Table 3. This data is based on a real medical start-up company case as a reference. In general, the survival rate of the start-ups in the medical field is lower than the ones in other sectors and the uncertainty for their success is higher. Thus, the investors tend to prefer to choose the convertible notes rather than equity when they provide funds in the early financing round. For selecting the simulation case with the convertible notes, therefore, the medical start-up is suitable.

Table 3: The realised investment amounts

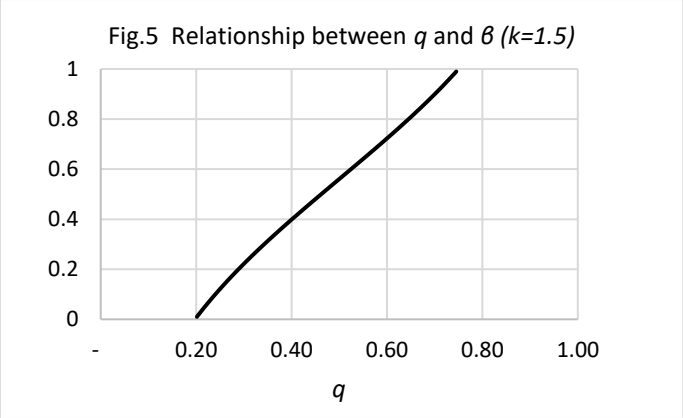
Date	**Type**	**Offered (million$)**	**Sold (million$)**
1/August/2007	equity	5.00	0.050
16/November/2007	Convertible Securities	2.00	0.805
7/January/2008	equity	5.00	0.025

Using the equation (7), the following results can be obtained.

V_0 (million$)	I_C (million$)	β (2007)	q (2007)	γ (2007)
0.05	0.805	0.01	0.20	16.09

The degree of impact of adverse selection can be captured by the probability of q. In this actual case, q, which represents the investor's "trustworthiness" toward the entrepreneur, can be obtained directly because β is known. In the discussions above, q is thought to be unchanged. However, it would be normal that it is changeable, thus its simulation is worth implementing.

The simulation results in Fig.5 shows the relationship between these two variables in the equation (7) when $k = 1.5$.



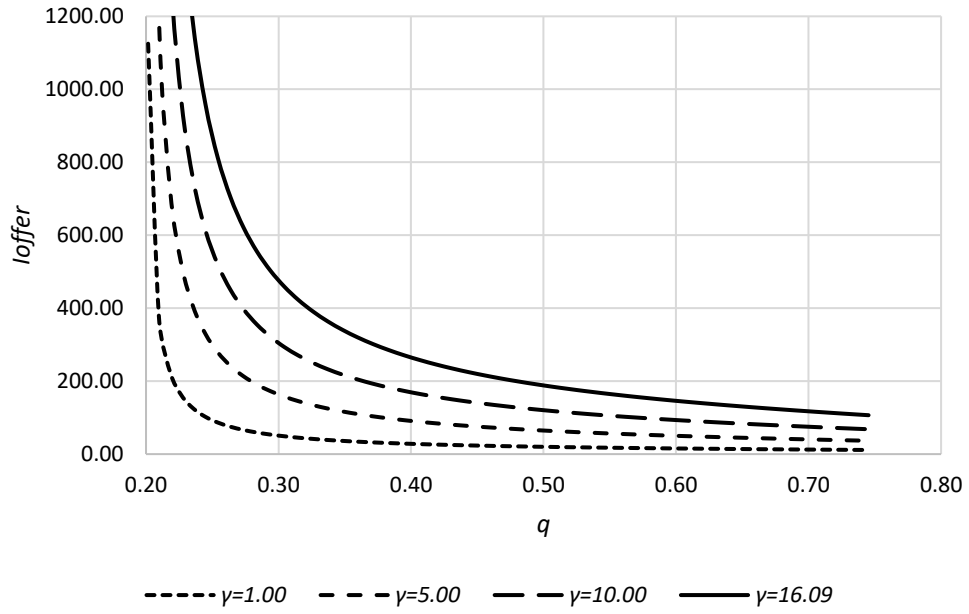
This graph explains that investors would increase their investment amount when the probability of investor's "trustworthiness" toward the entrepreneur rises even under adverse selection problem. This result is natural in practice. Thus, it can be said that this theoretical equation can capture the actual phenomenon well. According to this result, the entrepreneur of Start-up X was not evaluated as being trusty by investors because both β and q are quite low. One of the reasons might be that Start-up X is related to medical industry because the probability of failure is usually high in this sector in spite of the expectation of huge success with a handful of firms.

As the equation (9) implies, I_{offer} is the function of γ and I^*. For entrepreneur, how much of γ, which represents the proportion of convertible note value against the initial equity, is important for elaborating the negotiation strategies in the next financing round. $\gamma = 16.09$ of Start-up X in 2007 means that the proportion of convertible note is about 16 times than the equity existed. If this proportion is changed, what is the impact on the desired amount the entrepreneur offers under the situation of adverse selection? The result of this simulation is shown in Fig.6.

The result shows that I_{offer} becomes smaller drastically as the proportion γ of the amount of convertible note against the initial equity decreases, especially in the lower range of about 0.20 to 0.30 of q. In this range, even the slight change of level of q can have a great impact on the theoretical amount of I_{offer}. This tendency is reinforced as the proportion γ of convertible note against equity becomes smaller. This means that the improvement of the investor's trustworthiness to the entrepreneur's ability has a great impact on the amount that the entrepreneur should offer and procure when the second-round equity investors are quite doubtful

about the entrepreneur's ability and the convertible notes are chosen more preferably than equity. As shown by the fact of $q = 0.20$, the investor's trustworthiness to the entrepreneur of Start-up X is actually quite low. Therefore, it can be also implied that any efforts of the entrepreneur of Start-up X in order to acquire the trustworthiness from the investor toward the next financing round are quite effective when the entrepreneur has not yet convinced the investor about its ability to success.

Fig. 6 I_{offer}
(the amount that entrepreneur should offer)

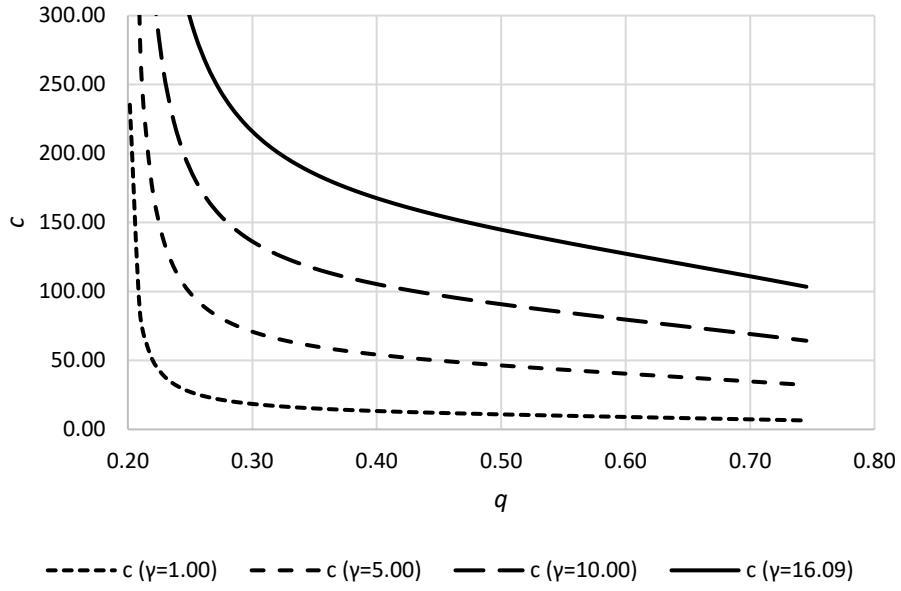


Here are the results of another simulation about the option value c of convertible note, shown in Fig.7 and 8. As convertible note holder has an initiative of determining I^*, the variation of I^* is also one of important factors to make a decision not only for investors but also for entrepreneur in the financing round. In this actual case, although the information about I^* of Start-up X is not revealed explicitly, as Start-up X offered \$5.00 million as equity, I^*can be estimated at least being equal to this amount.

The simulation result in Fig.6 is based on the assumption of I^* =5.00. As this result shows, the conversion option value c becomes smaller when the proportion γ of the amount of convertible note against initial equity decreases. Similarly, the degree of decrease of c becomes greater especially in the lower range of q as γ

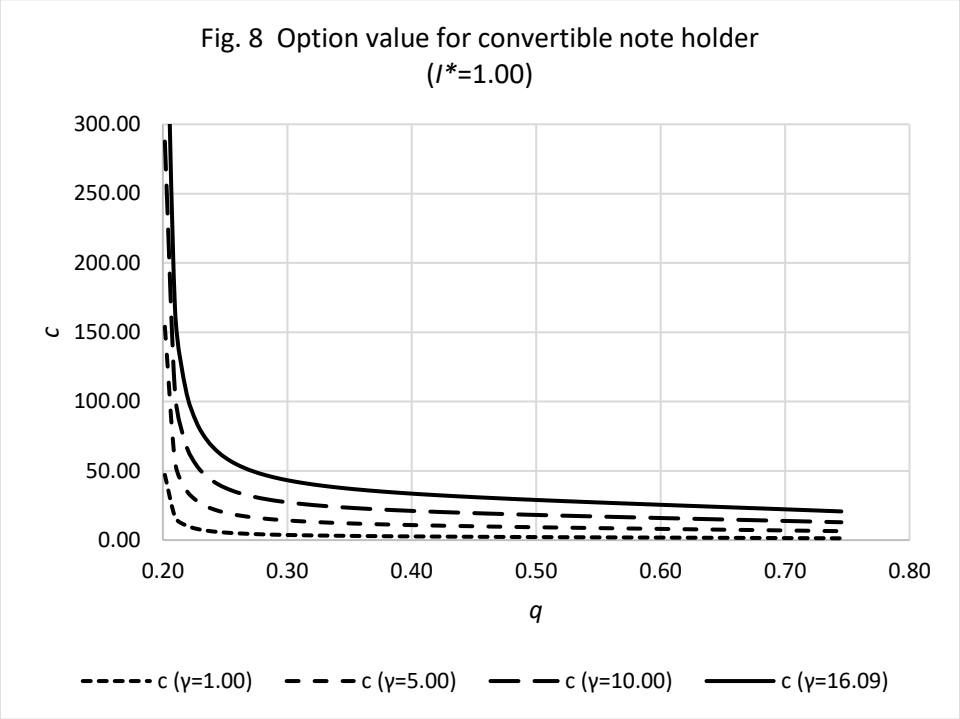
becomes low. This suggests that the preference or attractiveness of convertible note over equity becomes lower when the investor's trustworthiness to the entrepreneur's ability is improved, especially in the lower range of trustworthiness. Moreover, this tendency is reinforced as the proportion of convertible notes against equity becomes smaller. In other words, that investors prefer to choose equity rather than convertible note if they can trust entrepreneur to be able to increase their money. On the contrary, if they cannot trust it, they prefer to choose convertible notes. In effect, the proportion $\gamma = 16.09$ of convertible notes against equity of Start-up X is very high and $q = 0.20$ is very low. Considering the high uncertainty of success in the medical start-ups because of their feature of high costs and long periods, thus, this simulation result would be understandable.

Fig. 7 Option value for convertbile note holder
(I^*=5.00)



The threshold amount of conversion to equity is negotiable between the convertible note holder and the entrepreneur. Then, how does the convertible note holder set the threshold amount of conversion to equity lower? The similar trend can be read from the simulation result when I^* =1.00 shown in Fig.7. Comparing with the case of I^*=5.00, the value of c drops more drastically, especially in the lower range of q. Moreover, the value of conversion option is not so different in spite of γ in the range of $q > 0.30$. Thus, the reduction

tendency of the preference or attractiveness of convertible note over equity is magnified as the threshold amount of conversion to equity becomes smaller. When I^* as the threshold amount of conversion to equity is smaller, the firms value indicated as the post-money value will become smaller and the price of share will also become smaller. This is consistent with the reduction tendency of the preference or attractiveness of convertible note over equity. Therefore, it is implied that the effect of adverse selection in the lower rage of trustworthiness can be alleviated drastically especially when the proportion of convertible note is also low and the threshold amount of conversion to equity is small.



5. Conclusion

For discussing the start-up financing with the convertible notes, many researchers focus on the relationship between an investor and an entrepreneur. This paper contributes to the literature by investigating the relationship between two different types of investors, second-round equity investor and convertible note holder, and by introducing a real case simulation. We analyse how the value of option to convert of the

convertible note holder can be affected by the trustworthiness of the second-round equity investor to entrepreneur's ability to procure funds in each financing round and to increase firm's value.

The simulation results show that the improvement of the investor's trustworthiness to the entrepreneur's ability has a great impact on the amount that the entrepreneur should offer and procure when the second-round equity investors are quite doubtful about the entrepreneur's ability and the convertible notes are chosen more preferably than equity. Moreover, the preference or attractiveness of convertible note over equity becomes lower when the investor's trustworthiness to the entrepreneur's ability is improved, especially in the lower range of trustworthiness. In addition, this tendency is reinforced as the proportion of convertible notes against equity becomes smaller. Lastly, it is implied that the effect of adverse selection in the lower rage of trustworthiness can be alleviated drastically especially when the proportion of convertible note is also low and the threshold amount of conversion to equity is small.

In this paper, the calculation of the price of the share is based on the pre-money value. In practice, there are other methods, for example, which are based on the post-money value. Although this method seems to be not common, the simulation based on the post-money value is possible. The dynamics not only between entrepreneur and investor but also between investors should be emphasised in this case, thus, the interesting researches would be expected. Moreover, this paper assumes that both second-round equity investor and convertible note holder are a single entity. In practice, several investors make syndicate when investing. Further research could take this relationship into consideration.

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