Investment under Uncertainty and the Recipient of the Entry Cost

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Abstract

The typical model of investment under uncertainty where firms pay an irreversible cost in order to produce is revisited, this time with a novel focus on the recipient of this payment. This recipient is modeled as a firm that sells a resource (or a right) necessary for the production of the final good. We find the optimal price that the resource owner sets for its resource, and study how it depends on the characteristics of the market for the final good. The analysis reveals that one of the main results of the literature on investment under uncertainty – that firms delay their investment even when its NPV is positive – may not survive the endogenization of the investment cost, or at least loose much of its plausibility.

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1. Introduction

Usually, the models on investment under uncertainty deal with the decisions of a firm that has to pay a certain irreversible exogenous cost in order to start producing and make profits.[1] In this article we remain within the traditional framework used in those models but shift the focus of attention elsewhere – to the recipient of this cost. We model this recipient as a firm that sells a certain resource which is necessary for the production of a final durable good. The producers of the final good face the typical investment under uncertainty problem studied in the literature, as the cost of each unit of the resource is an exogenous irreversible cost from their point of view. The firm that sells the resource faces a problem not yet studied – it must decide at what level to set the price of its resource in order to maximize the value of its sales.

The common result in the literature on investment under uncertainty is that the optimal policy for the firm is to delay investment until profits from the investment are sufficiently large. In particular it has been found that a positive Net Present Value is not enough to trigger investment, as the firm seeks to cover not only its direct investment cost but also the opportunity cost of the forgone option to delay investment. Clearly, these results cannot survive a full endogenization of the investment cost in which the resource owner can change the price of its resource at any time and with no cost. In that case the resource owner changes the price of the resource in response to any swing in the demand for the final good and thus strips the producers of the final good from any profit and keeps them at constant indifference as to whether to invest immediately or delay investment.

Abstracting therefore from full endogenization and assuming that there are some exogenous components to the investment costs revives the delay results,

[1] For detailed surveys of this literature see Pindyck (1991) or Dixit and Pindyck (1994).

although its relevancy is nonetheless weakened as the set of parameter values and market conditions it requires is narrowed.

Specifically, we assume that the resource owner cannot continuously change the price of its resource, possibly due to technical limitations or to costs on doing so. Due to that, the price the resource owner sets for its resource is fixed for at least a certain amount of time. In setting this price the resource owner faces the following dilemma: Pushing the price up yields, on the one hand, more upon selling, but on the other hand it may delay the timing of these sales because it may induce the producers of the final good to delay their purchases until the demand they face sufficiently rises.

For simplicity, we take the assumption of non-continuous changes in the price of the resource to the extremity in which once the resource owner sets the price of the resource it cannot change it anymore. Although extreme, this assumption is in fact the standard one in the relevant literature as all models in that literature assume that the potential investor faces an investment cost that is fixed over time.

We find that if buying this resource is the only cost for the producers of the final good then wishing to receive payments early is the overriding consideration and the resource owner sets its price low enough to induce immediate investment. This occurs for all levels of the demand for the final good. In particular, when this demand is very low the resource owner sets an accordingly low price for its resource rather than set a higher price which it could enjoy later when demand would eventually rise. This happens because when the demand is low the probability of a large surge in it is accordingly small under the standard assumption of a geometric process taken here for the demand dynamics.

A situation where the producers of the final good delay their investments is possible therefore only later on in time if the demand for the final good falls sufficiently below its initial level. Note that this possibility hinges on the assumption that the resource owner is limited in its ability to change its price over time. Relaxing this assumption and allowing the resource owner to change the price of its resource over time will make the occurrence of such delay periods even less plausible.

The finding that the resource owner sets its price low enough to induce immediate investment is avoided if the investment costs of the producers of the final good contain not just the endogenously set price of this resource but also another component, exogenous in its nature. We find that this exogenous component alters the relative force of the two factors in the dilemma described above, and thus enables the result that the resource owner sets a price which sends the market to a period of delayed investments. Specifically, we find that the resource owner does so if the demand for the final good is sufficiently low, and then investments take place only later on when the demand for the final good sufficiently rises. Only if initially the demand for the final good is sufficiently high does the resource owner set a price that induces immediate investment.

As stated above, the subject at the focus of this article has never to our knowledge been studied. The study closest to ours is Yu et al. (2007) who examine a case where the irreversible investment cost of the firms is subject to the endogenous decisions of a government. Yet, it does not share the key element of our study, namely that the irreversible cost of the producers of the final good is the cost of a necessary resource or right determined endogenously by its owner. Specifically, they compare two policy alternatives for a host country wishing to draw in FDI: entry cost subsidy and tax rate reduction. Another study that is somewhat close to the current article is de Villemeur, Ruble and Versaevel (2014) who analyze a model in which an upstream producer is choosing the price it requires from a downstream producer for an input required to the production of a final good. Yet, in that article there is also an exogenous investment cost that the upstream producer has to pay in order to produce the input it then sells to the downstream producer. The focus of that article is on questions of efficiency of this market structure, and by that it strongly differs from our study which focuses on the way that endogenizing the investment cost affects the main results of the relevant literature. In particular, in their model the upstream producer has no knowledge of the current demand and the current price in the market for the final good. Therefore, by construction, they cannot study if and how the producer of the necessary input adjusts the price it sets for that input according to the demand for the final good.

Several articles studying the possibility of speeding-up investment by subsidizing the investment cost are also close to the current study due to the partly endogenous net investment cost they model. The current study differs from them mainly in our focus on maximizing profits from receiving the investment costs, rather on public welfare from speeding-up investment. See Di Corato (2016) for a typical model of that literature, as well as for a survey of its main articles.

The article is organized as follows. In section 2 the model is presented and the value of the resource to its owner is analyzed. In section 3 the resource owner's choice of the price of the resource and the resulting immediate market situation – sales or inaction – is analyzed for the case where the resource owner maximizes its profits. Section 4 offers some concluding remarks.

2. The Model

Consider the market for the durable good X. Production of X requires the resource N. The seller of N is a monopoly that sets the price k per each unit of N. Once k is set – it cannot be changed anymore. We assume that the X producers can buy N any time they choose and that each time they do so they must transform it to X immediately. All X producers are risk-neutral and have the same production process: a unit of N is transformed to a unit of X at a cost w. The demand for X is given by:

(1) $$P = \frac{A}{Q^{\alpha}},$$

where Q is the aggregate amount of good X and P is the price of X. The parameter α is positive and $-1/\alpha$ is the demand elasticity. A is a geometric Brownian motion and its dynamics are described by the following rule:

(2) $$dA = \mu A dt + \sigma A dZ,$$

where Z is the standard Wiener process satisfying at each point in time:

(3) $$E(dZ) = 0, \qquad E[(dZ)^2] = 1.$$

μ and σ are constants and $\sigma > 0$. By Itô's lemma and (1), when Q is unchanged the evolution of P is governed by:

(4) $$dP = \mu P dt + \sigma P dZ,$$

which means that P is a geometric Brownian motion too.

We denote the discount rate relevant to the X producers and to the resource owner by *r*. Following Dixit (1989) we assume that $r > \mu$, an assumption that makes the expected rate of growth of *P* smaller than the discount rate, preventing thus the value of the firms that produce *X* from going to infinity.

Under this modeling, the *X* market is the same market studied by Leahy (1993). As Leahy (1993) shows, under this setup there is a threshold price, P_H, that characterizes the optimal policy of each single *X* producer: when $P < P_H$ the *X* producer does nothing, when P hits P_H the *X* producer buys some *N* and produces *X* from it. This optimal policy is the same for all *X* producers since they are identical. The firms' purchases of *N* increase the supply of *X* and prevent *P* from rising above P_H. As Leahy (1993) shows, the value of P_H is:[2]

(5) $$P_H = \frac{\beta}{\beta - 1}(r - \mu)(k + w),$$

where β is the positive root of the quadratic:

(6) $$\tfrac{1}{2} \cdot \sigma^2 \cdot Y^2 + \left(\mu - \tfrac{1}{2} \cdot \sigma^2\right) \cdot Y - r = 0 .$$

Applying $Y = 0$ and then $Y = 1$ and using the assumption that $r > \mu$ shows that one root of this quadratic (denoted γ) is negative and the other one, denoted β, exceeds unity.

Given the initial values of *A* and *Q* the resource owner sets a value of *k*

[2]Throughout most of his paper, Leahy (1993) studies a more general case than the one presented here. In page 1119, though, the analysis takes several assumptions that make it entirely equivalent to the current model. The second equation in p.1199 is equation (5) of the current paper. Some notational differences should be mentioned: the investment threshold is denoted P_H here and $\overline{P}$ there; the irreversible cost of producing a unit is denoted there by *k* while here it is $k + w$; the positive root of equation (6) is denoted β here while denoted by Leahy as α. All the other notations are identical.

optimally. In the following sub-section 2.1 we start with the case where this value of k is sufficiently high to make the producers of X delay their purchases of N, i.e., the case where $A/Q^{\alpha} \leq P_H(k)$. By (5), in this case k is in the range:

$$k \geq \frac{(\beta-1)A}{\beta(r-\mu)Q^{\alpha}} - w \equiv k^{*} \tag{7}$$

Next, in sub-section 2.2, we analyze the case where the resource owner sets a value of k in the range $0 < k < k^{*}$. This leads to $A/Q^{\alpha} > P_H(k)$ and induces immediate purchase of N by the X producers, purchases that increase Q until $A/Q^{\alpha} = P_H(k)$.

2.1 Delaying purchases of *N*

In this case, the X producers delay their purchases of N because the resource owner sets a value of k that leads to an investment threshold, P_H, which is above the market price. P.

Throughout the article we use the term "the value of the resource" for the present value of the stream of revenues that the resource owner extracts from selling N to the X producers. Let $V(A, Q, k)$ denote this value in the range defined by (7) given the current levels of A and Q and given a value of k. By Itô’s lemma,

$$dV(A, Q, k) = \left[V_A(A,Q,k)\mu A + \tfrac{1}{2}V_{AA}(A,Q,k)\sigma^2 A^2\right] dt + V_A(A,Q,k)\sigma A dZ \tag{8}$$

and due to (3):

(9) $$\frac{E[dV(A,Q,k]}{dt} = V_A(A,Q,k)\mu A + \tfrac{1}{2}V_{AA}(A,Q,k)\sigma^2 A^2$$

Equation (9) captures the resource owner's expected capital gain due to the change in A over time. The no-arbitrage condition implies that this expected capital gain should equal the normal return to the resource. This implies:

(10) $$\frac{E[dV(A,Q,k]}{dt} = rV(A, Q, k)$$

Applying (9) in (10) and rearranging yields:

(11) $$V_A(A, Q, k)\mu A + \tfrac{1}{2}V_{AA}(A, Q, k)\sigma^2 A^2 - rV(A, Q, k) = 0$$

(11) is a second-order homogenous differential equation. Trying a solution of the form $V(A, Q, k) = C(Q, k)A^Y$ yields the quadratic captured by (6). Recall that the two roots of this quadratic satisfy $\gamma < 0$ and $\beta > 1$. Thus:

(12) $$V(A, Q, k) = H(Q, k)A^{\gamma} + B(Q, k)A^{\beta}$$

where $H(Q, k)$ and $B(Q, k)$ are to be determined using two benchmark requirements. To do so recall that $V(A, Q, k)$ captures the value of future sales of Q that occur each time A is sufficiently high so that P hits the investment threshold P_H. However, if A is close to 0 then the probability of A ever rising so high is zero as well. In that case, therefore, the value of the resource is 0. Formally:

(13) $\underset{A \to 0}{Lim} V\left(A, Q, k\right) = 0$

Since γ is negative, (13) implies that $H(Q, k) \equiv 0$.

We now turn to finding $B(Q, k)$. As appendix A shows, the condition for a no-arbitrage evaluation of the value of the resource in the time instants when there are changes in Q, i.e., when $A/Q^{\alpha} = P_H$, is the following Value Matching Condition:

(14) $V_Q(A, Q, k) = - k.$

Thus, by (12), (14) and $H(Q, k) \equiv 0$, when $A/Q^{\alpha} = P_H$:

(15) $B_Q(Q, k)A^{\beta} = - k.$

Applying $A/Q^{\alpha} = P_H$ in (15) and rearranging it, yields that when $A/Q^{\alpha} = P_H$:

(15') $B_Q(Q, k) = -\frac{k}{Q^{\alpha\beta} {P_H}^{\beta}}$.

Straightforward integration of $B_Q(Q, k)$ leads to:

(16) $B(Q, k) = \frac{k}{(\alpha\beta - 1) Q^{\alpha\beta - 1} {P_H}^{\beta}} + C$

As Q goes to infinity P goes to 0 and the probability of P ever reaching P_H goes to zero as well. This implies that the resource owner is not going to sell any N in the future and its value is therefore 0, i.e.:

(17) $$Lim_{Q\to\infty}B(Q, k) = 0.$$

This benchmark dictates a distinction between two cases based on the value of α. We start with a case in which $\alpha < 1/\beta$. In this case, Q in the denominator of the first term at the RHS of (16) is raised by a negative power and as it goes to ∞ the entire term goes to -∞. This, taken together with (17), implies that C goes to ∞, and therefore that so are $B(Q, k)$ and $V(A, Q, k)$ for each finite level of Q.

The economic logic underlying the infinite value of the resource in this case is based on the relation between α and the elasticity of demand which is $-1/\alpha$. The smaller α, the larger the demand elasticity and therefore the larger the increase in Q each time that P hits P_H. Thus, the smaller α, the faster the process of sales of the resource N and the less heavily discounted are its revenues. This drives the value of future sales of N to infinity when α is sufficiently small, namely – below $1/\beta$. This case is not in the focus of this study and from here on we assume $\alpha > 1/\beta$.

Returning to (16) and (17), now with $\alpha>1/\beta$, the first term at the RHS of (16) goes to zero as Q goes to infinity, implying that C=0. Applying (5), C=0, (7), (16) and $H(Q, k)\equiv0$ in (12) yields:

(18) $$V(A, Q, k) = \frac{(k^* + w)^\beta Q}{\alpha\beta - 1} \cdot \frac{k}{(k + w)^\beta},$$

From the first order condition $V_k(A, Q, k) = 0$ it follows that the value of k that maximizes $V(A, Q, k)$ is:

(19) $$k_1 \equiv \frac{w}{\beta - 1}.$$

Applying (19) in (7) shows that k_1 is in the range $k>k^*$, in which $V(A, Q, k)$ represents the value of the resource, iff the following condition holds:

(20) $$\frac{A}{Q^\alpha} < \frac{\beta^2 (r-\mu) w}{(\beta - 1)^2} \equiv P^*.$$

(20) implies that the resource owner will set a value of k that is sufficiently large to make the X producers delay their purchases N if current demand in the X market is sufficiently low so that the market price is below P^*.

Note that if w=0, i.e., if the X producers do not face costs except for the purchase of N, then condition (20) cannot hold. In that case the function $V(A, Q, k)$ is strictly decreasing and maximized at the lower boundary of its definition range, namely at $k=k^*$, implying that the resource owner does not set k high enough to send the market to an inaction period.

From continuity it follows, by applying (7) in (18), that at instants in which $A/Q^\alpha = P_H$ and the x producers invest, the value of the resource is:

(21) $$V(A, Q, k) = \frac{k}{\alpha\beta - 1} Q.$$

2.2 Inducing immediate purchases of *N*

The X producers immediately purchase N when the market price, $P = A/Q^\alpha$, exceeds the threshold P_H. This investment immediately raises Q to Q_1 so that the price becomes $P = A/Q_1^{\alpha} = P_H$. This rise from Q to Q_1 rewards the resource owner with $k(Q_1 - Q)$.

Let $G(A, Q, k)$ denote the value of the resource in the range where immediate investment takes place, $k < k^*$, given the current levels of A and Q and also for a given value of k. Equation (22) below shows $G(A, Q, k)$ as the sum of two factors: First, the immediate proceeds $k(Q_1 - Q)$; Second, the value of the resource after the quantity immediately becomes Q_1, as described by (21).

(22) $$G(A, Q, k) = k(Q_1 - Q) + \frac{k}{\alpha\beta - 1} Q_1,$$

Note that $Q_1 > Q$ and therefore $G(A, Q, k) > 0$ throughout the range $k < k^*$ in which $G(A, Q, k)$ is defined. Simplifying (22) and applying $Q_1 = (A/P_H)^{1/\alpha}$ and (5) in it yields:

(23) $$G(A, Q, k) = JA^{\frac{1}{\alpha}} \frac{k}{(k + w)^{\frac{1}{\alpha}}} - Qk,$$

where:

(24) $$J \equiv \frac{\alpha(\beta - 1)^{\frac{1}{\alpha}}}{(\alpha\beta - 1)\beta^{\frac{1}{\alpha} - 1}(r - \mu)^{\frac{1}{\alpha}}} > 0.$$

The following *Proposition 1* shows some important properties of $G(A, Q, k)$.

Proposition 1:

(a) There exists a single value of k that brings $G(A, Q, k)$ to a maximum;

(b) This value of k, denoted by k_2, is an increasing concave function of A/Q^{α};

(c) k_2 is in the range $k<k^*$, the range in which $G(A, Q, k)$ represents the value of the resource iff $A/Q^{\alpha}>P^*$.

Proof: In the appendix. □

3. The optimal *k* when the resource owner maximizes its profits

In this section we analyze how the optimal k is chosen in the case where the resource owner is a profit maximizing firm. Based on the analysis in the previous sections, the value of the resource as a function of A, Q and k can be defined and denoted by:

(25) $$VG(A, Q, k) \equiv \begin{cases} G(A, Q, k) & if \;\; 0<k<k^* \\ V(A, Q, k) & otherwise \end{cases}$$

Note that $V(A, Q, k^*) = G(A, Q, k^*)$ as follows from applying (7) in (18) and then in (23). Two cases should be analyzed now: The case where $A/Q^{\alpha} < P^*$ and the case where $A/Q^{\alpha} > P^*$.

3.1 When $A/Q^{\alpha} < P^*$

In this case $k_2 > k^*$ as follows from part (c) of *Proposition 1*. Thus, in the range $k < k^*$, the value of the resource, represented by $G(A, Q, k)$, is increasing in k. From the analysis in sub-section 2.1 it follows that in the range $k > k^*$ the value of the resource, now represented by $V(A, Q, k)$, reaches a maximum at $k = k_1$. Thus, since $V(A, Q, k^*) = G(A, Q, k^*)$, the value of the resource, $VG(A, Q, k)$, reaches its maximum in $k = k_1$.

The line marked with circles in Figure 1 below presents $VG(A, Q, k)$ in this case. The thin line shows $V(A, Q, k)$ and the thick line shows $G(A, Q, k)$.

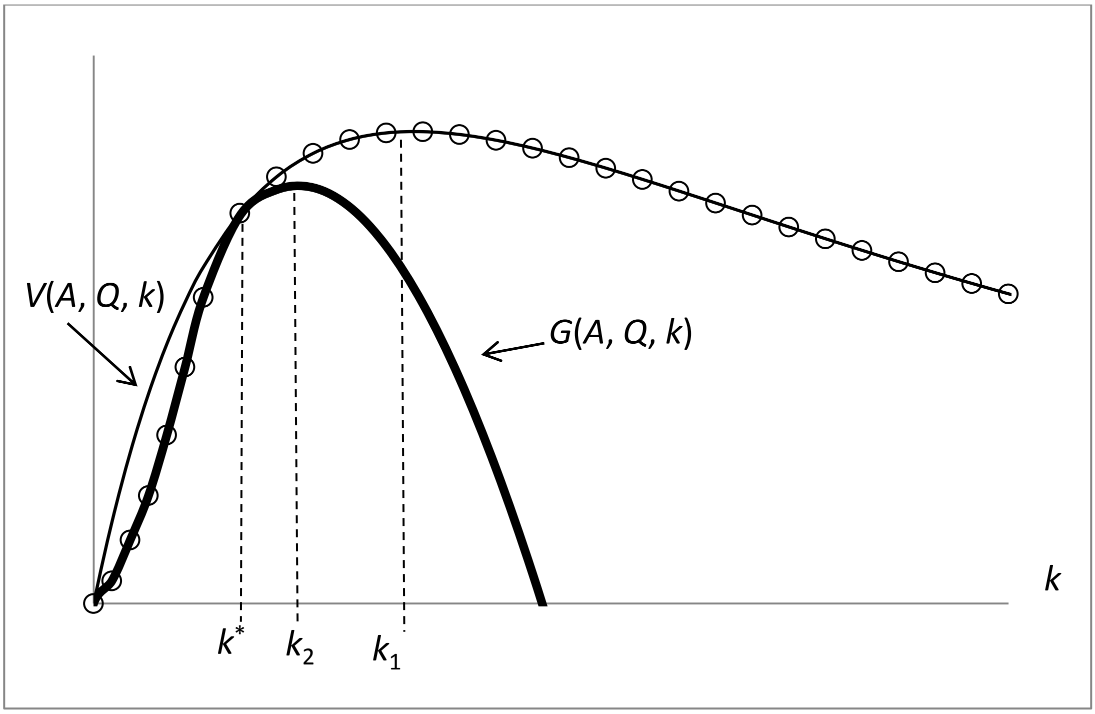

Figure 1: The resource firm's value, $VG(A, Q, k)$, when $A/Q^{\alpha} < P^*$. The thick line shows $G(A, Q, k)$, the thin line shows $V(A, Q, k)$ and the circles indicate $VG(A, Q, k)$. In this case $VG(A, Q, k)$ is maximized at $k = k_1 > k^*$ implying that the resource firm sets a value of k sufficiently high to delay purchases of N by the X producers.

3.2 When and $A/Q^{\alpha} > P^*$

In this case, in the range $k < k^*$, the value of the resource, which is represented by $G(A, Q, k)$, reaches a maximum at $k = k_2$ as follows from parts (a) and (c) of *Proposition 1*. Also, $k_1 < k^*$, as follows from sub-section 2.1. Thus, in the range $k > k^*$ the value of the resource, represented now by $V(A, Q, k)$, decreases in k. Therefore, since $V(A, Q, k^*) = G(A, Q, k^*)$, the value of the resource is maximized at $k = k_2$.

The line marked with circles in Figure 2 below presents $VG(A, Q, k)$ in this case. The thin line show $V(A, Q, k)$ and the thick line shows $G(A, Q, k)$.

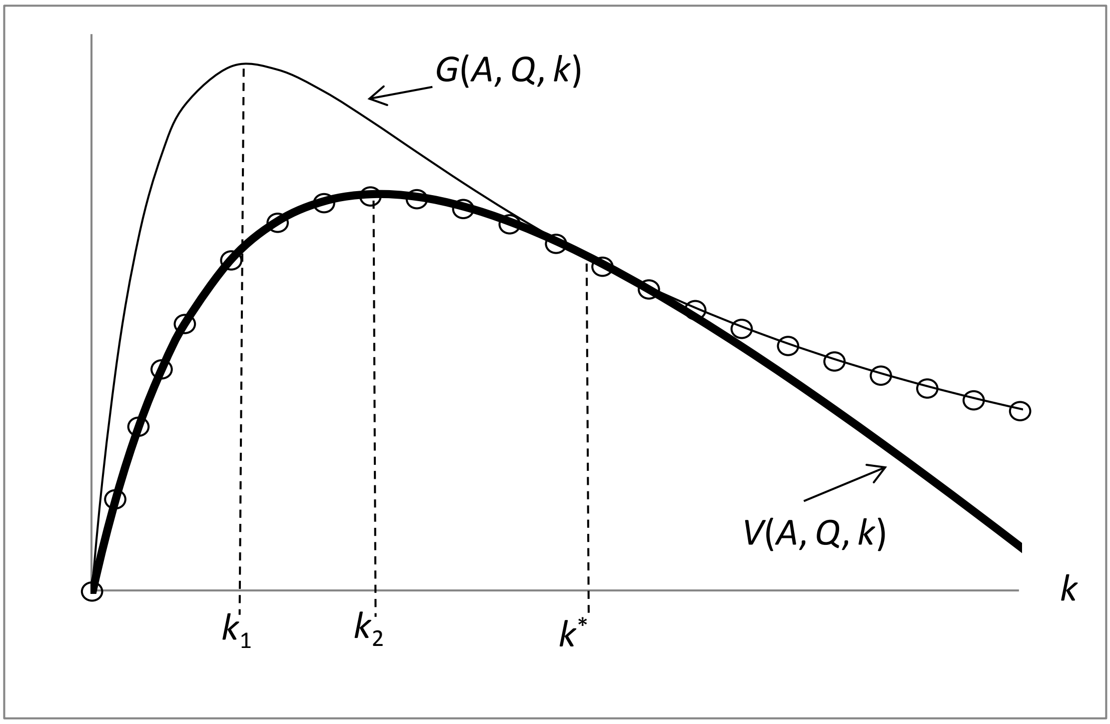

Figure 2: The resource firm's value, $VG(A, Q, k)$, when $A/Q^{\alpha} > P^{*}$. The thick line shows $V(A, Q, k)$, the thin line shows $G(A, Q, k)$ and the circles indicate $VG(A, Q, k)$. In this case $VG(A, Q, k)$ is maximized at $k = k_2 < k^{*}$ implying that the resource owner sets a value of k sufficiently low to induce immediate purchases of N by the X producers.

Based on the analysis of the two previous sub-sections, Figure 3 below shows the optimal k as a function of A/Q^{α}.

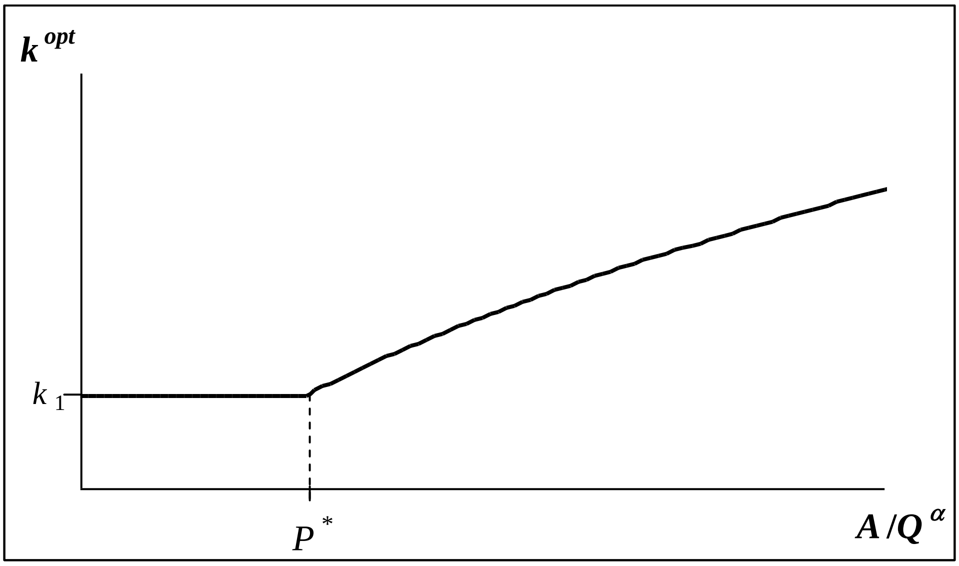

Figure 3: The optimal k as a function of A/Q^{α}.

4. Concluding Remarks

In this study, we returned to the typical model of investment under uncertainty and examined it from a new angle – that of the recipient of the investment cost. We modeled the recipient of this cost as a firm that sells a resource or a right that is necessary for the production of the final good. The focus of our study was on how the resource owner sets the price of its resource. Our main results was that the main result of the relevant literature, that firms should delay their investments even when their NPVs are positive, loses much of its plausibility. The first reason for that, too trivial to be analyzed in this study, is that continuous endogenous changes of the investment cost shall eliminate the option value of delaying investment. The second reason, analyzed here in detail, is that even if the investment cost cannot be changed after it is optimally set – the recipient of the cost will set it low enough to induce immediate investment and enjoy its receipts sooner.

A key assumption in the model was that the price of the resource cannot be changed on a continuous basis. For simplicity, we took this assumption to its extremity – i.e., once the price is set, the resource owner may not change it at all under any circumstances. While extreme, this is actually the assumption implicitly taken in all models of the relevant literature. Relaxing this assumption should not change the qualitative results of the analysis, so long as the more general assumption – that there are indeed some technical barriers or costs to changing the price of the resource continuously – is maintained.

Appendix

A. Establishing condition (14)

In this appendix we derive the benchmark condition (14) for the value of the resource at the time instants in which P hits P_H. Typically, articles in the relevant literature use value matching conditions of this type without providing a detailed derivation of them. Instead they merely refer to other articles using this condition in similar models. Here, we do offer a full derivation of this condition because under the unique viewpoint of the current paper, this condition characterizes the value function of the recipient of the investment cost rather then the value function of the investor.

To derive this condition we use the discrete approximation of a Brownian Motion presented in Dixit (1991). Since it is more convenient to perform this approximation for a Brownian Motion rather than for a Geometric Brownian Motion, the analysis is based on the function:

(A.1) $$F(a, Q, k) \equiv V(A, Q, k)$$

where $a \equiv \ln A$. Due to this definition, by Itô's lemma, a is a Brownian Motion since A is a Geometric Brownian Motion. The drift and variance parameters of a are denoted here by μ_a and σ_a^2. To approximate the motion of a we divide time to small intervals of length τ and the variable a space into steps of size ξ. The variable a now ranges over a discrete set of values a_i such that:

(A.2) $$a_{i+1} - a_i = \xi \quad \text{for all } i.$$

Starting at state a_i, time τ later the variable a takes with probability p a step down to the value of a_{i-1}, or takes with probability q=1-p a step up to the value of a_{i+1}. Two conditions relating τ, ξ, p and q to μ_a and σ_a should be used in order to make this process an approximation of the original Brownian Motion. First:

(A.3) $\mu\tau = q\xi + p(-\xi)$,

which leads to:

(A.4) $$q = \frac{1}{2}\left(1+\frac{\mu\tau}{\xi}\right), \qquad p = \frac{1}{2}\left(1-\frac{\mu\tau}{\xi}\right)$$

The condition regarding the variance of the process is:

(A.5) $$\sigma^2\tau = q(\xi-\mu\tau)^2 + p(-\xi-\mu\tau)^2 = \xi^2 + 2\mu\tau\xi(p-q) + (\tau\mu)^2$$
$$= \xi^2 - \mu^2\tau^2$$

Eliminating the term with τ^2 leaves:

(A.6) $$\sigma^2\tau = \xi^2$$

When a_i is such that $P = \frac{A}{Q^\alpha}$ is at the investment threshold P_H then, by (1):

(A.7) $$Q = \left(\frac{A}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}}$$

If time τ later a takes a step up, the endogenous investment by the X producers raises Q such that P remains at P_H. This implies that Q is raised to the level:

(A.8) $$\left(\frac{e^{a_i+\xi}}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} + \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}-1} \cdot \frac{1}{\alpha} \cdot \xi + \frac{\left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}-2} \cdot \frac{1}{\alpha} \cdot \frac{1-\alpha}{\alpha} \cdot \xi^2}{2} + \dots$$

The change in Q during that time is therefore:

(A.9) $$\Delta Q = \left(\frac{e^{a_i+\xi}}{P_H}\right)^{\frac{1}{\alpha}} - \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}-1} \cdot \frac{\xi}{\alpha} + \mathrm{o}(\xi),$$

where $\mathrm{o}(\xi)$ collects all the terms that go to zero faster than ξ, such that $\mathrm{o}(\xi)/\xi \to 0$ as $\xi \to 0$. Note from (A.6) that τ too falls under the category of $\mathrm{o}(\xi)$.

The Bellman equation for the value of the resource when a_i and Q are such that $P = P_H$ is:

(A.10) $$F(a_i, Q, k) = e^{-r\tau} [pF(a_{i-1}, Q, k) + qF(a_{i+1}, Q + \Delta Q, k) + qk\Delta Q]$$

(A.10) shows the value of the resource in this situation as the time τ later value of the resource discounted by $e^{-r\tau}$. With probability p the variable a takes a step down and the value of the resource becomes $F(a_{i-1}, Q, k)$. With probability q the variable a takes a step up. In this case, endogenous investment by the producers of X raises Q by ΔQ and the value of the resource becomes $F(a_{i+1}, Q + \Delta Q, k)$. In addition, in this case the resource owner also gains $k\Delta Q$ from sales to the X producers.

Expanding $e^{-r\tau}$ to a Taylor series, bearing in mind that that τ is $\mathrm{o}(\xi)$, yields:

(A.11) $$e^{-r\tau} = 1 + (-r\tau) + \frac{(-r\tau)^2}{2} + \frac{(-r\tau)^3}{6} + \ldots = 1 + o(\xi)$$

Applying this in (A.10) and expanding terms of (A.10) to Taylor series yields:

(A.12) $$F(a_i, Q, k) = p[F(a_i, Q, k) + F_a(a_i, Q, k)(-\xi) + o(\xi)]$$

$$+q[F(a_i, Q, k) + F_a(a_i, Q, k)(\xi) + F_Q(a_i, Q, k)\Delta Q + o(\xi) + k\Delta Q]$$

Using $p + q = 1$ and the result that τ is $o(\xi)$ by itself helps simplify (A.12) to:

(A.13) $$0 = (q - p)F_a(a_i, Q, k)\xi + q F_Q(a_i, Q, k)\Delta Q + qk\Delta Q + o(\xi)$$

By (A.4), $(q - p)\xi = \mu\tau = o(\xi)$ which simplifies (A.13) into:

(A.14) $$0 = F_Q(a_i, Q, k)\Delta Q + k\Delta Q + o(\xi)$$

Dividing by ΔQ and applying (A.9) yields:

(A.15) $$F_Q(a_i, Q, k) = -k - \frac{o(\xi)/\xi}{\frac{1}{\alpha}\left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} + o(\xi)/\xi}$$

By the definition of $o(\xi)$, as $\xi \to 0$ the numerator and the second addendum on the RHS of (A.15) approach 0 as well. This, together with $F_Q(a, Q, k) \equiv V_Q(A, Q, k)$, which follows from the definition of $F(a_i, Q, k)$ in (A.1), concludes establishing (14).

B. Proof of ***Proposition 1***

By (23) the first order condition for a maximum is

(B.1) $$G_k(A, Q, k) = -Q + JA^{\frac{1}{\alpha}} f(k) = 0,$$

where,

(B.2) $$f(k) \equiv \frac{k(\alpha - 1) + \alpha w}{\alpha (k + w)^{\frac{1+\alpha}{\alpha}}}.$$

Manipulating (B.1) and applying (1) in it, the first order condition (B.1) becomes:

(B.3) $$f(k) = \frac{1}{JP^{\frac{1}{\alpha}}}.$$

To establish existence of a root to (B.3) note from (B.2) that $f(k)$ approaches infinity when k approaches $-w$ and approaches 0 when k goes to infinity. Thus, by continuity, there exists a level of k in the relevant range (namely k>-w) for which $f(k)$ equals the positive term at the RHS of (B.3). To see that there is only one such level of k, note from (B.2) that:

(B.4) $$f'(k) = -\frac{k(\alpha - 1) + 2\alpha w}{\alpha (k + w)^{\frac{1+2\alpha}{\alpha}}}$$

If $\alpha \geq 1$ then clearly $f'(k) < 0$ for each level of k, implying that there can only be a single value of k for which $f(k)$ equals the positive term on the RHS of (B.2). If, on the other hand, $\alpha < 1$, then (B.4) reveals that $f'(k)$ can switch from being negative to being positive as we look at larger values of k. Yet, the linear numerator of (B.4) implies that this switch can occur just once, and therefore $f(k)$, which approaches 0 as k goes to infinity, can hit the positive term on the RHS of (B.2) only once. Thus, in both cases regarding α there is a single root for (B.3). Denoting this single root by k_2, the result that $f'(k_2) < 0$ also asserts that k_2 brings $G(A, Q, k)$ to a maximum since:

(B.5) $$G_{kk}(A, Q, k_2) = JA^{\frac{1}{\alpha}} f'(k_2) < 0.$$

(B.3) presents k_2 as an implicit function of P. Differentiating it leads to:

(B.6) $$\frac{dk_2}{dP} = -\frac{f(k_2)}{\alpha P f'(k_2)} > 0$$

where the inequality follows from $f(k_2)>0$ and $f'(k_2)<0$.

Applying (7), (24) and (B.2) in (B.3) and simplifying yields that $k_2=k^*$ if and only if P equals either 0 or P^*. Applying (7), (24), (B.2), (B.4) and (20) in (B.6) yields that when $P=P^*$ and $k=k^*$:

(B.7) $$\frac{dk_2}{dP} = \frac{\beta-1}{\beta(r-\mu)} \cdot \frac{\alpha\beta-1}{2\alpha\beta-1-\alpha} < \frac{dk^*}{dP}$$

The inequality follows from the assumptions that $\alpha\beta>1$.

k_2 and k^* are both increasing functions of P. They meet one another only when P=0 and when P=P^* and in that second meeting point $\frac{dk_2}{dP} < \frac{dk^*}{dP}$. From these properties it follows that k_2>k^* as long as P<P^* and vice verse.

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