Natural resource extraction with production target: the real option value of variable extraction rate

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Abstract

We consider the problem faced by mining companies to set their extraction rate over time in order to meet their production targets, from a real option perspective.

As part of their strategic planning, mining companies usually set a production target for a given, short-term to medium-term, time horizon. The extraction rate is constrained by this target, but remains flexible around a certain base rate. Because of the uncertainty in commodity prices, this flexibility has a value. Basically, by slowing down production when prices are low and speeding up production when prices are high, the miner can still meet its production target exactly, while at the same time significantly increase the value of the mine.

Mathematically, this resource extraction problem can be described as a multi-regime constrained stochastic control problem. The control is the extraction rate, to be set for every time step, and the constraint is the total extraction volume over the whole time period. We solve this problem numerically using an extension of the simulation-based Least-Squares Monte Carlo algorithm. To deal with the endogenous reserve variable, we use the so-called control randomization technique.

The estimated mine value increase would be nothing without the roadmap to achieve this improvement in practice. Fortunately, our numerical solution provides the optimal extraction regime to apply over time, for any commodity price level and any reserve level. We summarize the information about the optimal regime switching surfaces into intuitive graphics that can assist industry for their sequential production decisions under uncertainty in practice.

Key words: Mine valuation, multiple switching stochastic control, least squares Monte Carlo, control randomization, switching boundary

1 Introduction

The idea of using real option ideas to optimize mining operations in the face of metal price uncertainty is not new: as early as 1985, Brennan and Schwartz [1985] proposed a simple, stylized model to quantify the value of temporarily closing, or even abandoning, mines as a response to a drop in metal price. Though mathematically elegant, this kind of approach was never embraced by mine managers. On the contrary, it has been so rejected that "real option" now has a very bad name in the mining industry, on the erroneous belief that real option were limited to the simplistic abrupt shut-down option studied in Brennan and Schwartz [1985]. One major complaint from mining companies against the on-off type of strategy considered in Brennan and Schwartz [1985] is that social costs make the option of temporarily closing a mine very unappealing. Building a mine, hiring and training local workers, bringing skilled workers to remote mining areas, are all huge efforts that make a temporary closure hard to revert. The sheer size and scale of a mining project urges compellingly for steady and predictable operations, at odds with the real option mentality of great flexibility in the face of uncertainty.

For this reason, we decide in this paper to make two major modifications to the Brennan and Schwartz [1985] framework to make it more realistic and more appealing. Firstly, we assume that the mining companies set a fixed production target for a given time frame as part of their strategic planning. This kind of decision is common in practice; it can come from long-term contractual commitments to extract a specified amount of ore for major clients (see Armstrong and Galli [2013], Zhang and Dimitrakopoulos [2014] for examples). Secondly, instead of a binary all-or-nothing choice for the production rate, we assume that it can vary on a finer grid of production regimes, allowing for smaller adjustments over time. Remark that a small production rate adjustment can be achieved without affecting the volume extracted from the mine, by digging blocks known to have a higher or lower ore grade. In other words, small production rate adjustments can be achieve with little impact to day-to-day mining operations, and no need for dramatic and impractical workforce adjustment.

To sum up, we consider a fixed production target to be met at some specified time horizon, while allowing for a variable extraction rate over time. Even though the production target limits the scope of extraction rate variability, it is still possible to, say, compensate a low output period by a high output period later. Because of that, there is some optionality, and therefore value, to adjust extraction rates over time. Indeed, if the mine production is sold on the spot market, the uncertainty in metal price makes it valuable to decrease output when prices are low, and increase output when prices are high, all the while meeting the specified production target. This is the kind of flexibility we aim at quantifying and optimizing.

Mathematically, this dynamic extraction optimization under price uncertainty can be expressed as a multi-regime stochastic optimal control problem, where the control variables are the extraction rates, and where the state variables are the metal spot price (exogenous risk factor) and the reserve of ore in the mine (endogenous, as digging decreases the available reserve), with an additional total production constraint over the time horizon considered. To solve this problem numerically, we favor simulation-based methods over classical PDE methods, in order to avoid limitations on the possible stochastic dynamic model for the metal price. More specifically, we use an extension of the classical least-squares Monte Carlo algorithm from the mathematical finance literature (Longstaff and Schwartz [2001], Tsitsiklis and Van Roy [2001]). Here, the reserve level is an endogenous state variable, ie. its dynamics depends on the control. In order to deal with it within the least-squares Monte Carlo framework, and without resorting to reserve discretization, we use the control randomization technique (Kharroubi et al. [2014]): the reserve level is first replaced by a dummy random factor in the forward Monte Carlo simulation loop. This variable is then treated as an additional regression factor for the least-squares estimation of conditional continuation values in the backward induction loop, and optimized. In the meanwhile, the optimal policies computed during the backward induction are stored for later use (stress-testing and graphical display for example).

Although dynamic production strategies improve substantially the value of mining projects, the complexity of stochastic control and the associated algorithms hinders the adoption of modern real option theory by industry. To overcome this fear, it is key to produce intuitive, graphical display of the results obtained by real option analysis. An intuitive graphical summary of the best production regime to adopt at each time, for each price and each reserve level, would assist and educate the industry with optimal sequential decision-making under financial and geological uncertainties in practice. Therefore, we adapt the summary graphics from Chen et al. [2015] to the constrained extraction model developed in this paper.

The outline of the paper is as follows. Section 2 provides a mathematical description of the constrained extraction problem. Section 3 describes the simulation-based numerical method we use to solve the problem numerically. Section 4 illustrates our results, with tables highlighting the value of flexibility, and graphics describing the best policies to adopt to take advantage of this flexibility in practice. Section 5 concludes the paper.

2 Problem Formulation

We study how to optimize the operating strategy of a mining company which has a fixed production target for a given mine so that the maximum expected value can be obtained over the planning horizon T. The company has the option to adjust the extraction rate over time in response to commodity price fluctuation and remaining reserve level. This mine valuation problem can be formulated as a discrete time, finite horizon stochastic control problem under constraint.

2.1 Definitions

We assume that the manager has the option to change operating regimes at pre-specified discrete decision time $t_n = n\Delta t, \quad n = 0, 1, ..., N-1$, where $\Delta t = T/N$ using uniform time mesh. To make comparisons to the existing literature easier, we model the commodity price as a univariate geometric Brownian motion.

$$S_{n+1} = S_n e^{\left(\mu-\delta-\frac{\sigma^2}{2}\right)\Delta t+\sigma(W_{n+1}-W_n)}, \qquad W_{n+1} - W_n \sim \mathcal{N}(0, \Delta t) \quad \text{i.i.d.}$$

where μ is the drift rate rate, δ is the instantaneous convenience yield of the commodity, and σ is the volatility of the spot price S. However, we emphasize that the simulation-based numerical method we use (described in Section 3) is independent from the specific dynamics chosen for S, so that calibrating a more realistic dynamics later is perfectly possible (and highly recommended), with no change to the algorithm itself apart from simulating prices from the new dynamics.

Let Q be the total reserve to be extracted before T (ie. the production target), and let $q = Q/T$ be the average extraction rate . We define the set of possible operating regimes as $\mathbb{Z} = \{0,\ 0.5q,\ q,\ 1.5q,\ 2q\}$. One can of course use a finer grid if needed.$\alpha(S_t,\ Q_t,\ a,\ t) \in \mathbb{Z}$ denotes the control at time t, with previous regime a, commodity price S_t, remaining reserve Q_t. We write $\alpha_t = c_t \cdot q$ for short where $c_t \in \{0, 0.5, 1, 1.5, 2\}$.

The change in extraction rate at decision time t from regime α_t^- to α_t incurs a switching cost $k(\alpha_t^-, \alpha_t, t) = Ke^{\pi t}|c_t - c_{t-1}|$ which is proportional to the change in extraction rate, where K is the unit switching cost at time t_0 and π is the inflation rate.

Let $\Pi(t, S_t, \alpha_t)$ be the instantaneous cash-flow generated by the mine at time s when the metal price is equal to S_t and the operating regime is equal to $\alpha_t \in \mathbb{Z}$. When $c_t \neq 0$: $\Pi(t_n, S_n) = c_t q(S_n - A_{t_n}) - Tax(S_n)$, where $A_n = A_0 e^{\pi t_n}$ is the operating cost, $Tax(S_n) = p_1 c_t q S_n + p_2 c_t q(S_n(1-p_1) - A_n)$ is the total income tax and royalties, with p_1 and p_2 denoting the royalty rate and the income tax respectively. When $c_t = 0$: $\Pi(t_n, S_n) = -M_n$, where M_0 is the initial maintenance cost, and $M_n = M_0 e^{\pi t_n}$ is the after-tax maintenance cost.

The value function of the problem is given by

$$V(t,S,Q,i) = \sup_{\alpha\in\mathcal{A}_{t,i}} \mathbb{E}\left[\int_t^T e^{-\tilde{r}(s-t)}\Pi(s,S_s,\alpha_s)ds - \sum_{t\le\tau_n\le T} e^{-\tilde{r}(\tau_n-t)}k(\tau_n,\alpha_{\tau_n^-},\alpha_{\tau_n})\,|(S_t,Q_t,\alpha_t)=(S,Q,i)\right] \quad (2.1)$$

$$\text{s.t. } Q_T = 0 \quad (2.2)$$

where $\tilde{r} = \mu + \lambda$ is the discount rate, with λ being the property tax rate. Notice the constraint (2.2) that $Q_T = 0$, ie. that the whole target volume $Q = Q_0$ be dug out when reaching time T.

Our goal is now to solve the constrained stochastic control problem (2.1)-(2.2) numerically.

3 Numerical Method

Problem (2.1)-(2.2) is a stochastic control problem with a target constraint at the final time T. This kind of problem is known to be associated with a system of Hamilton-Jacobi-Bellman partial differential equations. This is why the most common approach in the literature to deal with this kind of problem is to use numerical methods for partial differential equations (see Evatt et al. [2010] or Haque et al. [2014] for recent examples in mining). The main limitations of this approach are that it is limited to basic stochastic dynamics for the risk factors, and that it can only deal with a few risk factors. To get rid of these limits, we favor simulation-based methods, which have no constraints for the dynamics (anything that can be simulated can be used), which paves the way for calibrations of realistic dynamics, and can easily deal with many risk factors (see Langrené et al. [2015] for a simple example with three risk factors: metal price, estimated remaining reserve and ore quality).

The least-squares Monte Carlo algorithm (more generally Regression Monte Carlo algorithm), initially developed to solve American option pricing problem (Longstaff and Schwartz [2001], Tsitsiklis and Van Roy [2001]) can actually be extended to any kind of Markovian stochastic control problem. An extension of this Monte Carlo algorithm has been used successfully in Chen et al. [2015] and Langrené et al. [2015] for mine valuation problem. One uncommon feature with mining problems is that the mine reserve (or remaining reserve to reach a production target) Q is a state variable that is dynamically affected by the control (extraction speed). One possibility to extend the Regression Monte Carlo algorithm to such endogenous state variables is to use the control randomization technique developed in Kharroubi et al. [2014]. A short description of it in a mining valuation context is provided in Langrené et al. [2015], in addition to implementation tricks.

The control randomization technique requires to first simulate all the state variables with a random control independent from the other sources of uncertainty. One difficulty specific to problem (2.1)-(2.2) is the constraint that $Q_T = 0$, as it puts a constraint on the possible random control to use. Therefore, it is worth describing more precisely how to properly implement control randomization for problem (2.1)-(2.2).

The state variable $Q = (Q_t)_{0 \leq t \leq T}$ corresponds to the excess reserve in the mine above the production target. Thus Q_0 is equal to the production target, and the goal is to reach $Q_T = 0$ at time T. The dynamics of Q is given by

$$Q_{t_{i+1}} = Q_{t_i} - q \cdot c_{t_i} \cdot \Delta t$$

where $c_{t_i} \in \mathbb{Z} = \{0, 0.5, 1, 1.5, 2\}$ in our example. In other words, our goal here, in order to use the control randomization technique, is to simulate an i.i.d. sample from a random vector $\mathbf{C} = [C_0, C_1, \ldots C_{N-1}] \in \mathbb{Z}^N$ such that

$$q\Delta t \sum_{i=0}^{N} C_i = Q_0 \;\; a.s. \tag{3.1}$$

One idea would be to enumerate all the possible combinations $(C_0, C_1, \ldots C_{N-1})$ that satisfy (3.1), and select one of them randomly. The problem with this enumeration approach is that the list can quickly become unmanageable when the mesh of the production grid $\mathbb{Z}$ becomes small, which is why we look for a more efficient alternative method, described below.

First, remark that if $\mathbf{C}$ can take continuous values instead of being constrained to $\mathbb{Z}^N$, then simulating $\mathbf{C}$ is easy: simulate first $\tilde{C}_i \leftarrow 2U_i$, where U_i are i.i.d. uniform random variables, and then set $C_i \leftarrow \tilde{C}_i \times Q_0 / (q\Delta t \sum_{i=0}^{N} \tilde{C}_i)$.

Then, one would need to project this continuous vector into the discrete grid $\mathbb{Z}^N$. Rounding each component to the closest point in the grid would not work, as the constraint (3.1) might not be satisfied anymore. This projection problem is fortunately identical to an old problem from a very different context:

how to allocate seats to each political party after a proportional parliamentary election? Similarly to our problem, rounding may not work, as the number of seats is fixed and needs to be matched exactly. Over the years, different algorithms have been proposed to solve this problem, cf. Benoit [2000] for a comparison of several of them. For our problem, we currently use the famous d'Hondt method (D'Hondt [1882]), though we plan to switch to the Sainte-Laguë method (Sainte-Laguë [1910]), as Benoit [2000] suggests it is less biased.

4 Numerical Results

In this section, we test numerically the algorithm described in section 3 to the optimal extraction problem introduced in section 2, on a specific example.

4.1 Numerical Example

In their seminal paper, Brennan and Schwartz [1985] studied a continous, infinite horizon, unique extraction rate, finite resource copper mine value problem. Our problem (2.1)-(2.2) can be seen as a modified Brennan&Schwartz problem where a predefined production target needs to be met by the end of a fixed time horizon T, with a larger set of possible extraction rates. To make our results easy to compare to this classical example, we choose our parameters as in Brennan and Schwartz [1985]: $q = 10^6$ tons/year for the standard operating rate, $Q_0 = 150 \times 10^6$ pounds, $A_0 = \$0.5$/ton, $k(0) = \$200\ 000$, $M_0 = \$500\ 000$/year, $\sigma = 0.08$, $\delta = 0.01$, $\mu = 0.1$, $p_1 = 0.02$, $p_2 = 0.5$, and $\pi = 0.08$. We set the time horizon to $T = 15$ years and the production target to Q_0 (ie. we force the mine to be empty at time $T = 15$ years). To investigate in detail the value of extraction rate flexibility, we tested different sets of extraction rates:

1) A set of five variable extraction rates (VR5) $\{0.0, 0.5, 1.0, 1.5, 2.0\} \times q$;

2) A set of three extraction rates (VR3) $\{0, 1, 2\} \times q$, and

3) A full open strategy (FO) with constant extraction rate q.

Our results are summarized in Table 4.1. A few observations can be made:

1. As expected, the more flexible the extraction rates the more valuable the mining project: $VR5 > VR3 > FO$. This flexibility is successfully captured by our Regression Monte Carlo algorithm approach with control randomization, which can help to take advantage of it in practice.

2. The dynamics strategies can significantly increase the value of a mine, especially when the price S_0 is low. When the initial price is high (eg. $S_0 = 1$), the accelerated extraction rates ($1.5q$, $2.0q$) options give more opportunities to the period when digging is more profitable.

S_0	VR5	VR3	FO	VR5-VR3	VR5-FO	VR3-FO
0.4	−2.35	−3.74	−4.26	1.39	1.91	0.52
0.5	3.33	2.30	1.87	1.05 (46.0%)	1.47 (78.8%)	0.42 (22.5%)
0.6	9.21	8.40	8.00	0.80 (9.5%)	1.21 (15.1%)	0.41 (5.1%)
0.7	15.20	14.60	14.12	0.60 (4.1%)	1.07 (7.6%)	0.48 (3.4%)
0.8	21.31	20.87	20.25	0.45 (2.1%)	1.06 (5.3%)	0.62 (3.0%)
0.9	27.60	27.30	26.38	0.30 (1.1%)	1.22 (4.6%)	0.92 (3.5%)
1.0	34.11	33.83	32.51	0.28 (0.8%)	1.60 (4.9%)	1.32 (4.1%)

Table 4.1: Comparison of mine value with different initial price and extraction flexibility

4.2 Constructing the Switching Surfaces

To construct and display the optimal regimes and the switching surfaces between them, we use the same methodology as in Chen et al. [2015]. The structure of the objective function and the Bellman induction indicate that at each decision time t, the optimal extraction rate q_t depends on the operating regime α_t^- at the previous decision time, the current commodity price S_t, and the remaining target Q_t. The regression Monte Carlo algorithm described in Section 3 provides, as a by-product, a mapping between the simulated state variables and the optimal operating regimes, and therefore provides an estimate of the switching sets, which can then be plotted.

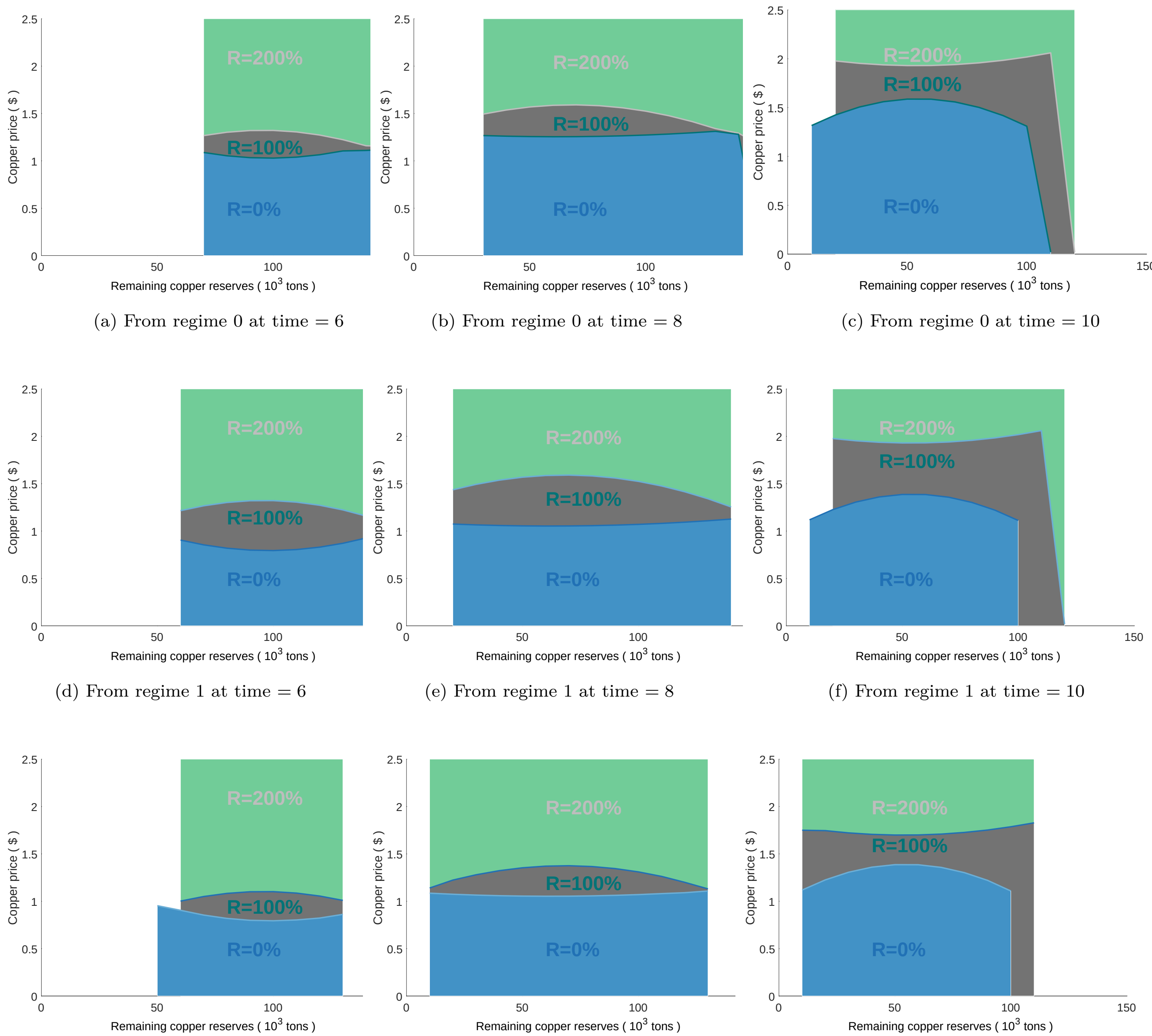
To illustrate this, using our numerical example, we choose to display the switching boundaries of the VR3 case, which has three operating regimes. Problems with five or more regimes can be deal with in the same way. As explained in Chen et al. [2015], each decision time t_n has switching sets $\Lambda^{i,j}(t_n)$ from each operating mode $i \in \mathbb{Z} = \{0,\, q,\, 2q\}$ to each other regime $j \in \mathbb{Z}$. Let the number $0, 1, 2$ be the index of the regime in set $\mathbb{Z}$, so that we denote the switching set $\Lambda^{0,1}(t_n)$ as the switching set from 0 to q at time t_n, and so on. There are 6 switching sets: $\Lambda^{0,1}(t_n)$,$\Lambda^{0,2}(t_n)$,$\Lambda^{1,0}(t_n)$,$\Lambda^{1,2}(t_n)$,$\Lambda^{2,0}(t_n)$ and $\Lambda^{2,1}(t_n)$. Our numerical results illustrate the fact that the switching sets form connected components in the (S, Q) plane. The frontiers between these sets form the switching boundaries. More precisely, at decision time t_n, the switching boundary between the operating modes i and j contains all the critical combination of commodity price S_n^* and reserve Q_n that trigger a regime switch from i to j.

In figure 4.1, we produce the switching boundaries from each regime to the other three at three different decision times: $t = 6, 8$ and 10 years. Each row corresponds to the current extraction rate ($q_t = 0$, q or $2q$), and each column corresponds to a different decision time ($t = 6$, 8 or 10). For example, the first row (subfigures 4.1a, 4.1b, 4.1c) shows the optimal switching regimes for a mine which has $q_t = 0$ extraction rate at time $t = 6, 8, 10$ respectively. The first column (subfigures 4.1a, 4.1d, 4.1g) shows the optimal switching regimes at time $t = 6$ from different regimes at the previous time. From these figures, we can observe the following features of the structures of switching regions:

1. The feasible reserve levels, which corresponds to the width of the colored regions, is a function of time: $Q^t_{\min} < Q_t < Q^t_{\max}$, where $Q^t_{\min} = \max\{0,\, Q_0 - 2q(T-t)\}$ and $Q^t_{\max} = \min\{Q_0,\, 2q(T-t)\}$. So a moving window for the feasible reserves can be observed as time passes.

2. As time goes by, the shape of the switching boundary between Regime 1 ($q_t = q$) and Regime 2 ($q_t = 2q$) changes from convex to concave, while the switching boundary between Regime 1 and Regime 0 ($q_t = 0$) changes from concave to convex. This is due to the target constraint, whereas in the uncontrained problem in Chen et al. [2015], the curvatures of both curves are almost the same.

3. A sharp drop occurs from time $t = 8$ in subfigure 4.1b and can be seen more clearly later at $t = 10$ from the subfigures 4.1c, 4.1f and 4.1i (third column). Indeed, when one gets closer to the time horizon T, a mine with large remaining reserve is forced to open or/and accelerate production so that it can meet the production target.



(a) From regime 0 at time = 6 (b) From regime 0 at time = 8 (c) From regime 0 at time = 10

(d) From regime 1 at time = 6 (e) From regime 1 at time = 8 (f) From regime 1 at time = 10

(g) From regime 2 at time = 6 (h) From regime 2 at time = 8 (i) From regime 2 at time = 10

Figure 4.1: Switching boundaries for 3 different regimes at time $t = 6, 8, 10$

5 Conclusion

In this paper, we propose to use a Regression Monte Carlo method combined with control randomization technique to solve a mine valuation problem with fixed production target but adjustable production rate, formulated as a finite horizon, constrained stochastic optimal multiple switching problem. This not only provides the best value for the mine, but also the practical roadmap for decision makers to make optimal decisions in response to the financial market uncertainties at different future decision times. We carry on numerical tests with different operating flexibilities to demonstrate the effectiveness of our method on taking advantage of uncertainties though the operating options. The switching boundaries generated from our algorithm also bring deep insight into the structure of the optimal regimes and can be used as intuitive decision support tool. In the future, we aim at performing the same numerical analysis on even more elaborate decision-making problems for mineral industry, taking into account more risk factors such as geological uncertainties, major incidents, and optimal scheduling.

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