Discrete Time Approximation of a Mean Reverting Jump Diffusion; with Application to Natural Gas Storage

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Commodity prices exhibit characteristic features that are different from stock or other financial asset prices. They are inherently seasonal, mean-reverting and often contain jumps, which is particularly the case for energy commodities such as electricity and natural gas. Valuation of commodity financial and real options requires realistic price models that reflect these characteristic features. In this paper, we develop a novel discrete time approximation approach for mean-reverting jump-diffusion (MRJD) processes, which allows for valuing American-style financial and real commodity options.

We combine the approximation scheme for jump diffusions proposed by Amin (1993) with the general method from Nelson and Ramaswamy (1990) for approximating diffusion processes using recombining binomial trees. Compared to alternative approaches, our method avoids the mathematical complexity of solving partial differential equations with finite difference methods. Moreover, it also sidesteps the potential issues with estimation errors and the choice of appropriate basis functions for the least square Monte Carlo approach of Longstaff and Schwartz (2001). Our approach is relatively easy to implement and it is flexible for valuing a wide range of derivatives with a variety of exercise features and payouts.

We apply our approach to determine value and the optimal injection/extraction strategy for a natural gas storage facility, where we use NYMEX Natural Gas

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historical prices to calibrate the price model parameters. The gas storage valuation is based on the assumption that ownership has an expiration date, so it can be viewed as a contract which grants temporary access to a storage facility. To provide a realistic setup, we allow for extraction and injection rates that vary with the inventory level. The parameters such as the maximum storage capacity, extraction and injection rates are based on a salt cavern storage facility in the U.S., described in detail in Thompson et al. (2009).

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