Real options for investment decisions in mobile TV infrastructure

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Abstract

This work presents a strategic investment framework for mobile TV infrastructure. We address the question of whether an operator should enter the mobile TV market and, if yes, when to do so. We consider a realistic setting where the mobile TV network is mainly relying on a DVB infrastructure whose coverage can be complemented by the cellular network. As several actors may be involved in this service setting, we consider a dynamic game theoretical framework combining real option theory with coalition games. We consider two main sources of uncertainty: user demand and network operation cost. We then propose a novel a bi-level dynamic programming algorithm that solves the underlying maximization problem. Our numerical results illustrate the decisions of both actors and the impact of the system parameters and the degree of uncertainty on the investment dates.

I. INTRODUCTION

Mobile TV service has been considered for a long time as an important service that will allow a significant growth in the telecommunications market. This promise has lead to an important standardization activity both in broadcast and cellular standardization bodies, leading to DVB-H standard (pushed by broadcasters) and eMBMS one (defined by cellular network actors). However, mobile TV service has been until now a commercial failure, with almost no large scale deployments. This failure led to the abandoning of the DVB-H technology by broadcasters, while mobile TV service offered by mobile network operators is still limited to the inefficient unicast technology with no current plans for deploying eMBMS technology.

In order to remedy to this situation, new standardization activities have been launched in order to cope with the technological limitations of the past standards. In particular, a mobile extension of the fixed broadcast standard DVB-T2, called DVB-T2 Lite, has been defined with a larger flexibility allowing to serve handheld devices with better quality and coverage [1]. On the other hand, the convergence between DVB and cellular technologies is being discussed in the context of the new DVB-Next Generation Handheld (DVB-NGH) standard [2][3], where DVB is intended to operate in conjunction with LTE eMBMS technology. Such a convergence, even if not mandatory for a proper operation of the mobile TV service, reduces drastically the network cost and increases the perceived Quality of Service (QoS) (see for instance [4], [5] and [6]).

This technological convergence is however not sufficient for allowing the development of mobile TV service; a clear business model that clarifies the relationships between actors and constructs the service from an economic point of view is equally needed, and this is the focus of the present work.

We specifically propose a new framework using real option theory coupled with coalition games. We consider the mobile TV network deployment as a strategic investment whose value depends on the different market uncertainties and on the behavior of the main actors. The main contributions of this work are the following:

- We develop a real option framework for investment decision in mobile TV networks and show how broadcasters can incorporate the uncertainties related to demand and network operation costs, in this case electricity price, in their decisions.
- We develop a novel decision making framework combining the real options method with coalition game theory. We show how a decision maker can incorporate in its decision the future possibility of being joined cooperatively by another actor which may increase its profits and reduce its costs. We make use of the Shapley value to derive the profits and costs of the different actors in case of a cooperative DVB/LTE network and show how to incorporate this result in the investment decision.
- We propose a bi-level dynamic programming algorithm to solve numerically the developed real option game. The dynamic programming technique is introduced to solve the real option problem, while the strategic aspects related to game theory are tackled using the bi-level algorithm. To the best of our knowledge, this is the first time this kind of algorithms is proposed in the literature.

The remainder of this paper is organized as follows. In Section II, we present a literature review on real option games and bi-level dynamic programming techniques. In section III, we describe the hybrid LTE/DVB system and derive the induced costs in a stand-alone DVB network and a cooperative DVB-LTE one. Section III presents the strategic investment framework considering the DVB-only network where the decision is whether to deploy the network or not, and if yes, when to do so, taking into account the uncertainties related to the demand and network operation cost. Section IV extends this framework in order to include, in the investment decision, the reaction of the mobile network operator. This latter may decide to join cooperatively the broadcaster in the investment, bringing thus more customers and reducing the network cost. Concluding remarks and some future work perspectives are given in the last section.

II. Related works

A. Economic aspects of cooperative DVB/cellular networks

Very few works addressed economical issues in convergent DVB/cellular networks. Authors in [9] considered cooperative broadcast and mobile telecommunication networks. They assessed the efficiency of the management of the convergent network and derived policies that increase the network profitability. They however did so in a centralized way, as if both operators acted as one. Authors in [6] proposed a method to dimension the converged network and to share profits between the main actors in the mobile TV value chain. This work was however done in a static case, considering one snapshot of the market state. Work in [10] tried to identify the emerging cooperation models between the various stakeholders piloting mobile broadcasting in Europe, based on the analysis of different technological trials. This analysis was however solely qualitative; no quantitative measures were given in the results.

B. Real option games

The success of mathematical models of financial markets, starting from the Black and Scholes model in 1973 [11], has lead to a large development of the usage of financial options. This tendency had an impact on capital budgeting (investment decisions) and led to the emergence of the real options theory, a term that has been first introduced by Myers in 1977 to evaluate the future opportunity to invest in uncertain environments [12]. Since then, the real options method has spread across different disciplines, ranging from natural resource investments [13], R&D projects [14], to information technology infrastructure and telecommunication equipment investments (see [15][16] for instance). For a deeper comprehension of real options theory and its applications, please refer to [17].

Unlike financial options, real investment opportunities are however rarely held by a single firm in isolation and most investment projects are open to several firms in the same industry or line of business, subject of course to the core competencies of each firm. This is not reflected in the large majority of works dealing with real options as they ”*mainly consider single decision maker problems of firms operating in monopoly or perfect competition markets*” [18]. A new research field, incorporating strategic considerations in investment decisions by combining game theory with real options, is gaining in importance (see for instance [19], [20] and [21]). In the recent paper [22], the authors review two decades of real option game models and concluded that there are very few models considering cooperation between firms. This is precisely the main objective of our paper.

C. Bi-level dynamic programming

Dynamic programming is a well-known technique used to solve complex optimization problems under uncertainty. This technique is particularly efficient for solving some optimization problems in stochastic environment with specific features like Markov Decision Process (MDP) problems, see reference [23]. The basic idea of dynamic programing is simple. It is based on dividing a complex optimization problem into sub-problems where each sub-problem is linked to another one through so-called Bellman equations. Considering an MDP problem in finite horizon, the algorithm starts by solving the problem in the latest time (the time horizon). This technique is called backward induction. Backward induction is the process of reasoning backwards in time, from the end of a problem or situation, to determine a sequence of optimal actions. It is traditionally used in the context of real options in order to numerically evaluate investment opportunities when explicit closed-form solutions are difficult to obtain [17].

In our paper, we deal with a particular real option problem that takes into account two decision makers that interact sequentially like a bi-level optimization problem. In bi-level programing problems [24], the variables are divided into two classes, namely the upper-level and lower-level variables. There are also upper-level and lower-level objective functions and constraints which define the mathematical problem. Upper-level constraints invoke variables from both levels. For the best of our knowledge, only one paper deals with a dynamic programming algorithm to solve a bi-level optimization problem [25] where the leader controls the size of the knapsack and the follower solves the original knapsack optimization problem. The authors do not consider however a finite horizon MDP problem. In our work, we consider a bi-level problem in which each leader/follower problem is a finite horizon MDP problem. To the best of our knowledge, using backward induction technique to solve a stochastic leader/follower game is new and has never been proposed.

III. Mobile TV network cost

In this section, we focus on technological aspects of mobile TV networks and derive the network deployment costs in two cases: stand-alone DVB network and cooperative DVB/LTE one.

A. Stand-alone DVB network

DVB-T2 Lite network is conceived as a mobile extension of DVB-T2 which was originally designed for rooftop TV receivers. The existing DVB infrastructure is thus reused with a larger transmit power in order to be able to cover handheld receivers. Let the *coverage area* of the DVB transmitter be defined as a circular area of radius R_S and the area covered by the mobile TV service be of radius $R_D \leq R_S$ (note that R_D has to be equal to R_S if the broadcaster wants to ensure a universal coverage in a stand-alone DVB network).

As the same DVB-T infrastructure is reused for mobile TV, the cost of the DVB network depends mainly on its operation, mostly in terms of the consumed energy. It is thus very important to determine the minimal transmit power so that coverage is ensured. This transmit power P_D depends on the number of TV channels to be served, denoted by K_{TV}, and the target coverage radius R_D. The calculation details of $P_D(R_D, K_{TV})$ are given in the appendix. To ease the reading, we drop in the following the dependence on the number of TV channels, as we will consider a pre-determined number of TV channels.

For the sake of illustration of our calculations, let us consider a DVB cell with radius (for fixed TV services) $R_S = 25$ Km. If 8 TV channels are to be served, the DVB-T2 power needed for covering this area is equal to 21 dBW while a power of 56 dBW is needed for a complete mobile TV coverage (using DVB-T2 Lite with $16-QAM$ $3/5$ modulation). If the broadcaster wants to keep the initial planned power of 21 dBW, it can only cover mobile TV users in a small area of radius $R_D = 3$ Km (see Figure 7 in the appendix).

We now compute the cost of the DVB system. The consumed power $P_c(R_D)$ is higher than the Electromagnetic Radiated Power (ERP). It includes a component that is proportional to the ERP and another one that is consumed independently of the average transmit power [28]:

$$P_c(R_D) = \alpha_n P_D(R_D) + \beta_n \tag{1}$$

where α_n is a scaling coefficient due to amplifier and feeder losses as well as cooling of sites and β_n is an offset due to signal processing, battery backup, etc.

Eventually, the total monthly cost of one DVB transmitter covering a region of radius R_D is composed of power consumption costs and equipment costs E_D [29] and is given by:

$$C_D(R_D) = \alpha_e P_c(R_D).H + E_D \tag{2}$$

where α_e is the electricity cost of one kWh and H is the number of operating hours per month.

B. A cooperative DVB/LTE network

As observed above, due to many transmission penalties between fixed and mobile services, the DVB coverage for mobile devices is lower than its coverage for fixed receivers unless the transmitted power is increased drastically. However, in the case of a cooperative DVB/LTE network, the DVB network can reduce its transmission power if the LTE network complements the mobile TV coverage using the eMBMS feature. The DVB coverage region is modeled as a circular area around the DVB transmitter, smaller than the fixed TV service area, which corresponds to the inner region in Fig. 1. On the other hand, LTE cells, primarily dimensioned for handheld services are able to ensure the mobile TV service in the outer area in Fig. 1. LTE has thus to upgrade its network by adding the eMBMS feature to cells in the outer area and, eventually, to add more sites in order to ensure the mobile TV service without degrading the Quality of Service (QoS) of its unicast users (voice and data). This is why the resulting LTE deployment is not uniform, as depicted in Fig. 1. Note that we assume that user terminals are assumed to have both 3GPP and DVB receivers.

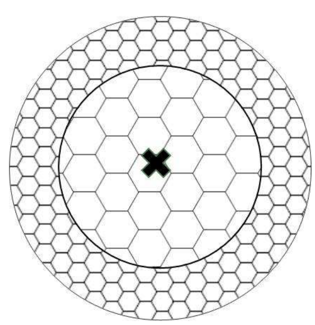

Fig. 1. Service Area

Let the original LTE cell radius (designed for unicast services only) be denoted by R_L^u, and the cell radius when LTE has to serve, in addition to the unicast services, K_{TV} TV channels be denoted by $R_L^b(K_{TV})$ (a simple methodology for calculating these radii is given in the appendix). For a given service area of radius R_S, when the inner area (served by the DVB transmitter) has a radius of R_D, the additional number of LTE sites to be deployed in the outer region can be easily computed by:

$$\Delta N_{cell}(R_D) = ((R_S)^2 - (R_D)^2)(\frac{1}{(R_L^b)^2} - \frac{1}{(R_L^u)^2}) \tag{3}$$

The cost for upgrading the LTE network is thus computed by

$$C_L(R_D) = \Delta N_{cell}(R_D) E_L \tag{4}$$

where E_L is the monthly deployment and operation cost of an LTE cell (assuming that the deployment cost can be credited on a monthly basis).

IV. Investment decision in mobile TV networks: Single decision-maker case

The previous section derived a model for mobile TV network costs. This section builds on this technical and economical analysis and considers the mobile TV network deployment as a strategic investment decision. We focus on the case of a stand-alone DVB network where the broadcaster decides the investment date and the Mobile Network Operator (MNO) does not play any role. The case of a joint DVB/LTE network is studied in the next section.

A. Uncertainties

When taking the investment decision, the broadcaster has to take into account the different uncertainties impacting the project. In addition to the classical uncertainty related to the evolution of the demand (i.e., the number of customers that will pay for the service), there is an important uncertainty in DVB networks related to the electricity prices as DVB networks are highly energy consuming. The evolution of the two random processes representing the demand level d_t and energy costs x_t are described as follows. We start with the latter.

1) Electricity price modeling : We consider the Geometric Ornstein-Uhlenbeck (Geometric O-U) model as it is widely used in the literature for modeling electricity prices [35]. This model assumes that the logarithm of the price follows the O-U process suitable for modeling the mean reverting behavior (the price will always return to the mean). Let $Y_t = \log(x_t)$ denote the log of the energy price x_t at time t. It follows the mean reversion process:

$$dY_t = \alpha \cdot (\xi - Y_t) \cdot dt + \sigma \cdot dw_t \tag{5}$$

where α is the mean reversion rate, ξ is the mean value of the logarithms of spot prices (half-hour prices of wholesale market electricity), σ is the volatility of the logarithms of spot prices, dw_t is the increment of a standard Wiener process.

The expected value and the variance of Y_t given the beginning state Y_i at time T_i are, respectively, given by [35]:

$$\mathbf{E}_{Y_i}[Y_t] = e^{-\alpha \cdot (t - T_i)} \cdot Y_i + \xi \cdot (1 - e^{-\alpha \cdot (t - T_i)}) \tag{6}$$

$$\mathbf{V}_{Y_i}[Y_t] = (1 - e^{-2\alpha \cdot (t - T_i)}) \frac{\sigma^2}{2\alpha} \tag{7}$$

And so, the electricity price x_t is log-normally distributed with mean

$$\begin{aligned} \mathbf{E}_{x_i}[x_t] &= e^{\mathbf{E}_{Y_i}[Y_t] + \frac{1}{2}\mathbf{V}_{Y_i}[Y_t]} \\ &= e^{e^{-\alpha \cdot (t - T_i)} \cdot \log(x_i) + \xi \cdot (1 - e^{-\alpha \cdot (t - T_i)}) + \frac{\sigma^2}{4}(1 - e^{-2\alpha \cdot (t - T_i)})} \end{aligned} \tag{8}$$

2) Demand modeling: The Geometric Brownian Motion (GBM) is a widely used model for demand in financial market. This model is however not suitable for modeling demand in almost mature markets as is the case for the telecommunication market in developed countries. We consider instead a more realistic model for the demand, developed in [34] for population evolution in large cities, whose expected value remains bounded. In this model, the demand evolves following the differential equation:

$$dd_t = \mu(t) d_t + \sigma d_t dw_t \tag{9}$$

where

$$\mu(t) = \frac{-\mu(1 - K)e^{-\mu t}}{K + (1 - K)e^{-\mu t}} \tag{10}$$

It is easy to prove that the mean of this latter process at time t given the beginning state d_i at time T_i is given by:

$$\mathbf{E}_{d_i}[d_t] = d_i \frac{K + (1 - K)e^{-\mu_d \cdot t}}{K + (1 - K)e^{-\mu_d \cdot T_i}} \tag{11}$$

Note that the framework developed in this paper is general and can be applied to other demand models.

B. Real options framework

We assume that the broadcaster can invest in the mobile TV network until a certain time denoted by T_D. This flexibility in investment opportunity over time is not considered in the classical decision method based on cost-benefit analysis (Discount Cash Flow (DCF)) [36]. The analogy between the opportunity to invest in the mobile TV project and the holding of a financial call option argues for a real options approach. In fact, the firm has the right but not the obligation to buy an asset (the project) at a future time, at an exercise price (the total cost, uncertain in our case).

We then apply this approach to answer the following question: until when is it preferable to delay the investment and how much is the value of this opportunity (option to defer)? We formulate the problem as a classical discrete-time real option problem where the aim of the decision-maker is to maximize its utility over the periods before and after the investment.

The time period is divided into epochs of length δ (in months) and, at each time epoch $t \in \{0, \delta, 2\delta, ..., T_D\}$, the broadcaster decides to invest or not based on the expected net benefit obtained from t until the obsolescence of the technology (say at $T_{end} > T_D$). The profit of the operator at a given time epoch is the difference between the revenues that it gains from subscription fees and the network costs. This utility is thus equal to 0 before the investment. After the investment, it is computed using equation (2):

$$u(t, x_t, d_t) = (A \cdot d_t - x_t \cdot P_c(R_S)H - E_D)\delta \tag{12}$$

where

- d_t is the demand level (number of subscribers) at time t,
- A is the per-user subscription fee,
- $P_c(R_S)$ is the power consumption necessary for covering the whole area (of radius R_S) around the transmitter (equation (1)),
- x_t is the energy cost at time epoch t,
- E_D is the equipment cost,
- cost parameters A, H and E_D are given per month.

The expected aggregated discounted net profit if the operator decides to invest at time t is thus given by:

$$\begin{aligned} &U(t, x_i, d_i) = \\ &= \sum_{\tau=t}^{T_{end}} \frac{(A \cdot \mathbf{E}[d_\tau] - \mathbf{E}[x_\tau] \cdot P_c(R_S)H - E_D)\delta}{(1+r)^{(\tau-t)}} \end{aligned} \tag{13}$$

where r is the discount rate. Note that we discount the future cash flows using the risk free interest rate r since we assume that the projects risk can be diversified.

The value of the option at time t is thus the maximum between the expected net profit if the investment occurs at t and the value of waiting until the next epoch:

$$O(t) = \max\left[U(t, x_t, d_t), W(t, x_t, d_t)\right] \tag{14}$$

where the value of waiting is equal to the expected discounted value of the option at time $t + \delta$:

$$W(t, x_t, d_t) = \frac{E[O_{x_t,d_t}(t+\delta)]}{1+r} \tag{15}$$

C. Dynamic programming algorithm

In order to solve the above defined real option problem, we adopt a backward dynamic programming approach:

1) Discretize the demand and electricity price into discrete values $d_k, k \in [1, N_d]$ and $x_j, j \in [1, N_x]$, where N_d and N_x are the number of possible values for the demand and the electricity price, respectively. Details of how to perform this discretization are given in [34] and [35]. Let $\mathbf{s}_{k,j} = (d_k, x_j)$ be the different possible states.
2) Compute $p(t, \mathbf{s}_{k,j}, \mathbf{s}_{k',j'})$, the probabilities that the system moves from state $(\mathbf{s}_{k,j})$ at time t to state $(\mathbf{s}_{k',j'})$ at time $t+\delta$. As the two processes (demand and electricity price) are independent, this joint probability is simply the product of the individual probabilities of passing from d_k to $d_{k'}$ and from x_j to $x_{j'}$. These latter can be found in the literature (e.g., [34], [35]). Compute also $p(t, \mathbf{s}_{k,j})$, the probabilities of being at state $\mathbf{s}_{k,j}$ at time t, as the product of the probabilities of the electricity price being equal to x_j and the demand being equal to d_k at time t.
3) Start at the maturity date T_D at which a now or never decision should be taken. At $t = T_D$, the option for state $\mathbf{s}_{k,j}$ is calculated as:

$$O(T_D, \mathbf{s}_{k,j}) = \max[U(T_D, \mathbf{s}_{k,j}), 0] \tag{16}$$

for all states $\mathbf{s}_{k,j}$.

4) Move back one period to $t = T_D - \delta$ and calculate the value of waiting as:

$$W(T_D - \delta, \mathbf{s}_{k,j}) = \frac{\sum_{k',j'} p(T_D - \delta, \mathbf{s}_{k,j}, \mathbf{s}_{k',j'}) O(T_D, \mathbf{s}_{k',j'})}{(1+r)} \tag{17}$$

The value of the option is thus:

$$O(T_D - \delta, \mathbf{s}_{k,j}) = \max[U(T_D - \delta, \mathbf{s}_{k,j}), W(T_D - \delta, \mathbf{s}_{k,j})] \tag{18}$$

5) Continue moving back until computing the value of the option at time 0.

For each system state $\mathbf{s}_{k,j}$, the first time t at which the value of investment is larger than the value of waiting is the optimal time to invest. We show in Appendix C how to compute $p_{inv}(t)$, the probability of investing at time $t \in [0, T_D]$.

D. Numerical illustration

In order to illustrate the dynamic programming approach, we consider a system with the parameters of Table I (we use only parameters related to the broadcaster in this section).

	DVB	LTE
Initial cell coverage (Km)	R_S =25	R_L^u =0.5
Equipment cost (Euros/month)	E_D =0	E_L =833
Other cost parameters	α_n =10;β_n =80	
Maturity date (months)	T_D =24	T_L =36

TABLE I
SYSTEM PARAMETERS

We apply the dynamic programming approach to the broadcaster decision. Table II illustrates the decisions at different time epochs and for different possible demand values, for a given electricity price (the complete decision table is 3-Dimensional). We observe that, for higher electricity prices, the operator has to wait for higher demand level to be reached before investing.

Electricity	Demand level					
price	10264	10791	11318	11845	12373	12900
0.1312	1	1	1	1	1	1
0.2005	0	0	1	1	1	1
0.3064	0	0	0	0	0	0

TABLE II
AN EXTRACTION OF THE DECISION TABLE OF THE BROADCASTER AT TWO INSTANTS. ZEROES CORRESPOND TO DELAY ACTIONS WHILE ONES CORRESPOND TO IMMEDIATE INVESTMENT DECISIONS.

Figure 2 shows the impact of the initial energy price on the average investment date. It is clear that this later increases with the initial price. In fact, low prices will not got any better in the future so operator invest directly. However, high prices will decreases in the future since they follow a mean reversion process so it is better to postpone a little bit the investment.

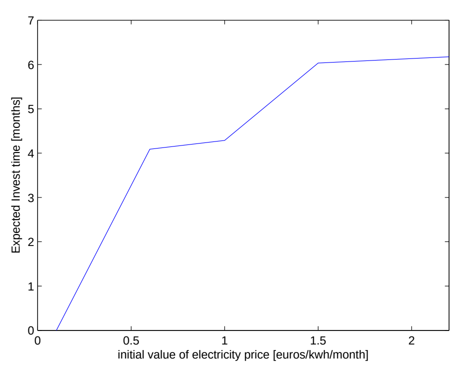

Fig. 2. Impact of the initial energy price on the expected investment date

V. Investment decision in mobile TV networks: multiple decision-maker case

In this section, we extend the real options framework developed in the previous section to account for the possibility of constructing a coalition comprising the broadcaster and the MNO. This coalition would increase the overall project profit (by bringing more customers from the MNO customer database) and reduce its costs by allowing a reduction of the DVB transmission power. We begin by considering a static case (no timing) and show how the profits and costs are shared in a cooperative LTE/DVB network using the so-called Shapley value [32]. We then move to the dynamic case where the first mover (the broadcaster) takes the first decision of investment. The MNO acts as a second mover and can decide (immediately or after some time) to join the broadcaster and forms a coalition with him if this corresponds to a win-win situation.

A. *Profit sharing in the static case*

The idea with Shapley value is that each player will have a profit share proportional to its contribution in the network setting and the added value it brings to the overall value chain.

We denote by N the set of players and S a given coalition formed by a subset of these players. The worth function $V(S)$ denotes the weight or payoff of coalition S. The Shapley value $\phi_i(S,V)$ defined by L. Shapley [26] is the share gained by player i when she is in coalition S. This value is given by:

$$\phi_i(S,V) = \frac{1}{N!}\sum_{\pi\in\Pi} \Delta_i(V,S(\pi,i)), \forall i \in N \tag{19}$$

where Π is the set of all $N!$ players permutation, $S(\pi,i)$ is the coalition formed by players from rank 1 till i in a given permutation $\pi\in\Pi$ and $\Delta_i(V,S(\pi,i)) = V(S) - V(S\backslash\{i\})$ is the marginal contribution of player i in coalition S, defined as the difference between the worth functions of (S) and $(S\backslash\{i\})$, and representing the benefits or losses that player i could bring if she entered coalition $(S\backslash\{i\})$.

The Shapley distribution is stable if it is in the core of the game. The latter is defined in [33] as "*the set of feasible payoff vectors for the grand coalition that no coalition can upset*". So the Shapley value profit sharing is stable if we cannot find any coalition whose players may earn more than if they stick to the largest coalition (the grand coalition). Formally, it should verify the following condition:

$$\sum_{(i\in S)} \phi_i(P',V') \geq V'(S) \quad \forall S \subseteq P' \tag{20}$$

In our present case, the set of players includes the LTE operator with d^L subscribers and the DVB operator with d^D subscribers. The revenue worth function of a certain subset of players defined by coalition S is equal to:

$$V_r(S) = A(d^L + d^D)\cdot I_{\{L\in(S),Din(S)\}} + Ad^D\cdot I_{\{D\in(S),L\notin(S)\}} \tag{21}$$

where $I_B = 1$ if condition B is true and 0 otherwise, A is the subscription fee per user.

Given (S,V_r), we consider each player and eliminate the other one and apply Shapley distribution (Eq. (19)) to it. We aggregate then the elementary shares to simply obtain the revenue shares for the two actors:

$$\phi_L(d^D,d^L) = A\frac{d^L}{2} \tag{22}$$

$$\phi_D(d^D,d^L) = A(d^D + \frac{d^L}{2}) \tag{23}$$

As of costs, they are also shared between the two actors using the Shapley value, i.e., the MNO pays parts of the DVB costs and vice-versa. The same analysis as above gives the MNO and the broadcaster cost shares when the cooperative network is constructed (the DVB network covers an area of radius R_D and the LTE network complements the remaining coverage):

$$\Psi_L(x,R_D) = \frac{E_L\Delta N_{cell}(R_D) - x(P_c(R_S) - P_c(R_D))H}{2} \tag{24}$$

and

$$\Psi_D(x,R_D) = \frac{E_L\Delta N_{cell}(R_D) + x(P_c(R_S) + P_c(R_D))H}{2} + E_D \tag{25}$$

Based on this cost and revenue share analysis, we have the following result:

Theorem 1. *Under the Shapley value, LTE and DVB operators have always the incentive to cooperate in offering mobile TV service and both have the same optimal configuration in terms of inner area radius (R_D^*).*

Proof. It is easy to show, using equations (24) and (25), that the DVB radius that minimizes the costs of both operators is given by:

$$R_D^* = \arg\min_{R_D}[E_L\Delta N_{cell}(R_D) + xP_c(R_D)H] \tag{26}$$

Using equations (1), (2) and (4), the optimal DVB radius becomes:

$$R_D^* = \underset{R_D}{\arg\min}[xH\alpha_n P_D(R_D) - E_L(\frac{1}{(R_L^b)^2} - \frac{1}{(R_L^u)^2})(R_D)^2] \tag{27}$$

The revenue shares being independent of R_D (see equations (22) and (23)), the inner radius that minimizes the cost shares (the solution of equation (27)) maximizes the revenues of both operators.

□

Figure 3 illustrates the optimal DVB coverage radius R_D^* as a function of the electricity price. It can be seen that a larger electricity price implies that the area covered by DVB shrinks and the role of the LTE operator in the mobile TV coverage increases. Knowing these optimal radii, Figure 4 illustrates the profit shares of both operators. We observe that a higher electricity price implies a higher profit for the LTE operator, since this latter will be paid by DVB operator for using the cellular infrastructure to serve the DVB clients in the outer region (Figure 1).

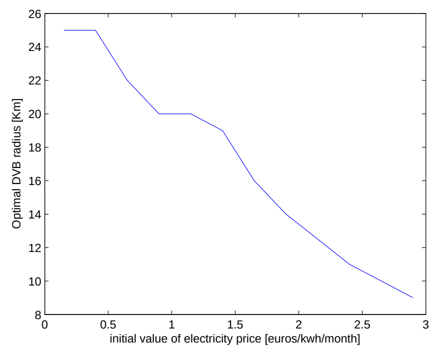

Fig. 3. Optimal radius R_D^* for the DVB coverage function of the electricity price.

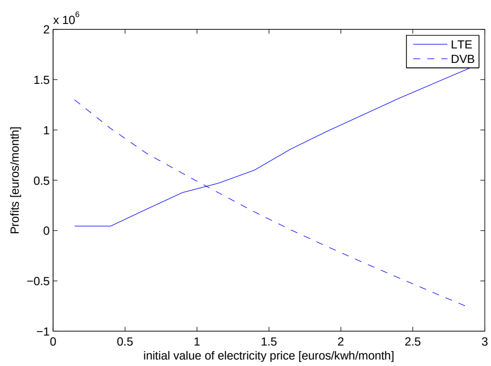

Fig. 4. Profit shares for DVB and LTE operators function of the electricity price. Demand level is fixed to 153 customers per Km^2.

B. A game theoretical real options framework

We now move to the dynamic case where the broadcaster takes the first move and decides to invest in the network, but has to incorporate in its decision the possibility to be joined, immediately or later on, by the MNO. In this latter case, the MNO may increase the revenues by bringing a new set of subscribers and reduce the cost by complementing the coverage of the DVB network which will be able to reduce its transmission power. The Shapley value framework that we presented in the previous section can thus be used to derive the revenue and cost share of each of the actors.

We consider three sources of uncertainty: the electricity price x_t, the number of subscribers brought by the broadcaster d_t^D and the number of subscribers brought by the MNO d_t^L. The utility of the broadcaster at time t will be equal to:

$$u_D(t, x_t, d_t^D, d_t^L) = \begin{cases} (A.d_t^D - x_t.P_c(R_S)H - E_D)\delta & \text{DVB only} \\ (\phi_D(d_t^D, d_t^L) - \psi_D(x_t, R_D^*))\delta & \text{cooperation} \end{cases} \tag{28}$$

and the utility of the MNO is computed by:

$$u_L(t, x_t, d_t^D, d_t^L) = \begin{cases} (\phi_L(d_t^D, d_t^L) - \psi_L(x_t, R_D^*))\delta & \text{cooperation} \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

where R_D^* is the optimal coverage of the DVB transmitter (Theorem 1).

Knowing that the broadcaster takes the first move and LTE follows it or not, the problem can be decoupled as two inter-related decision problems:

1) Decision of the MNO: Supposing that the broadcaster decides, at time $\tau \leq T_D$ and when the system is in state $(d_\tau^D, d_\tau^L, x_\tau)$, to invest in the project, the MNO has the choice to join the network immediately, to delay its decision, or to abandon the investment. This is a classical real options problem, similar to that described in section IV-B, but where the time origin is at t and the initial state of the market is $(d_\tau^D, d_\tau^L, x_\tau)$. The MNO can decide to invest until his proper maturity date $T_L \in [\tau, T_{end}]$ and, after investment, he can exploit the project until T_{end}. The utility of the MNO at time $t \geq \tau$ is given by equation (29).

For each starting state given by τ (the broadcaster's investment date) and $(d_\tau^D, d_\tau^L, x_\tau)$ (the market state when the broadcaster invests), the value of the option for the MNO is thus defined as in section IV-B.

2) Decision of the broadcaster: Even if the broadcaster takes the first move, he must take into account the possibility of being joined later by the MNO in a coalition that may reduce its costs and increase its profits. This new source of uncertainty has to be integrated within the option's value. At each date τ and for each state of the market $(d_\tau^D, d_\tau^L, x_\tau)$, the expected net profit, given by equation (13) in the case of a stand-alone DVB network, has thus to incorporate the future decision of the MNO:

$$U(\tau, d_\tau^D, d_\tau^L, x_\tau) = \sum_{t=\tau}^{T_{end}} \frac{\hat{u}_D(t, x_t, d_t^D, d_t^L)}{(1+r)^{\tau - t}} \tag{30}$$

where the expected utility at time t incorporates the MNO's decision:

$$\begin{aligned} \hat{u}_D(t, x_t, d_t^D, d_t^L) = \\ (A.E[d_t^D] - E[x_t].P_c(R_S)H - E_D)\delta p^L(t|\tau, d_\tau^D, d_\tau^L, x_\tau) \\ +E[(\phi_D(d_t^D, d_t^L) - \psi_D(x_t, R_D^*))]\delta(1 - p^L(t|\tau, d_\tau^D, d_\tau^L, x_\tau)) \end{aligned} \tag{31}$$

C. The bi-level dynamic programming approach

In order to solve this compound real option problem, we introduce a bi-level dynamic programming algorithm as follows:

- For each time $\tau \in [0, T_D]$ and each system state $s_{i,j,k} = (d_i^D, d_j^L, x_k)$, perform a dynamic programming algorithm, like the one described in section IV-C, to evaluate the option of the MNO, knowing that the broadcaster decides to invest at time τ and that the initial market state is $s_{i,j,k}$. Compute the corresponding probability that the MNO did not invest at any $t \in [\tau, T_L]$, as in Appendix C.
- Perform a dynamic programming algorithm to evaluate the option of the broadcaster. This algorithm has to incorporate, in the net utility of the broadcaster, the future decision of the MNO as in equation (30). This calculation takes as input the output of the dynamic programming algorithm relative to the decision of the MNO, introduced in the previous step.

As an output of this bi-level dynamic programming approach, the global value of the project can be computed and the expected investment times for both operators can be derived.

Note that the stochastic processes describing the evolution of the demands of DVB and LTE operators can be regarded as independent or correlated processes, but the latter assumption is more realistic as the interest of customers depends more on the offered service than on the network technology. We choose in this paper a completely correlated model where the global demand is modeled as a stochastic process d_t (as the one described in section IV-A2), and each operator has a proper demand which is proportional to its market penetration. This reduces the dimension of the problem to 2. Any other model of correlated processes can, however, be used.

D. Numerical illustration

The system parameters are those reported in Table I. Figure 5 shows the expected investment dates for both operators as a function of the initial electricity price. We observe that for low electricity price, the DVB operator will invest directly, whereas the LTE one will wait for higher electricity prices until the DVB will need him to compensate the high operation costs.

At higher initial electricity prices, the DVB operator will wait some time before investing hoping that this price decreases (due to the mean reversion nature of the process). The LTE operator will however invest at the same investment date as DVB because the electricity price is already high and there is an immediate need for him to enter the coalition and any delay in the investment will decrease the subscription fees accumulated during the fixed project lifetime.

Figure 6 shows the impact of the mean reversion rate on the LTE investment date. We observe that this time decreases when the rate increases. In fact, a high mean reversion rate means that the price will return faster to the mean value, and so, if the electricity price was not favorable at this instant, it is not worthy to postpone the investment date, because the gain in the accumulated subscription revenues will compensate the high cost in this small period of time.

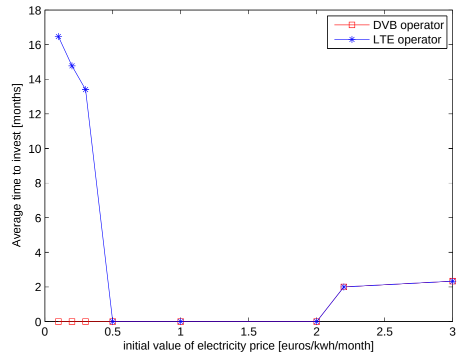

Fig. 5. Expected investment dates for both operators function of the initial electricity price. The initial demand is fixed to 137.5 (subscriber/km^2)

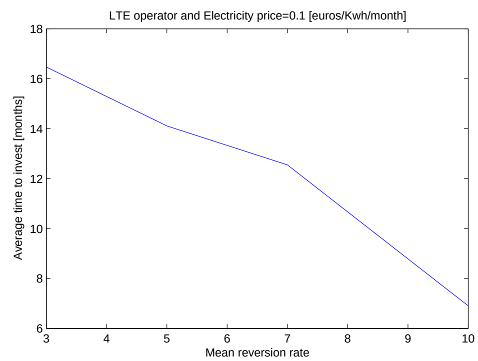

Fig. 6. Expected investment date for the LTE operator function of the mean reversion rate

VI. Conclusion

In this paper, we developed a framework for investment decisions in mobile TV networks, based on real options theory, and which takes into account two main sources of uncertainty related to demand and network operation cost. We considered the presence of two main actors in the offering of the mobile TV service: DVB operators who rely on their DVB towers and the DVB-T2 lite technology and mobile network operators who would use their classical cellular networks along with LTE eMBMS technology. The LTE operators would complement the DVB coverage if the two operators decide to make a coalition for delivering the mobile TV service. This is achieved through the definition of a novel game theoretical real options methodology that incorporates the possible formation of a future coalition in the investment decision of the broadcaster.

In order to solve this investment decision making problem in practice, we proposed a novel bi-level dynamic programming algorithm based on backward induction and applied it to both players. The proposed framework is flexible and applicable to different types of uncertainties and other investment problems.

As of future works, we aim to extend our framework to the case with more than two actors/coalitions. An interesting case to analyze is when several LTE operators compete for forming the coalition with the broadcaster or when several coalitions with different actors are possible.

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Appendix

A. DVB network dimensioning

In DVB-T2 Lite technology [1], each TV channel TV$_i$, $i \in [1, .., K_{TV}]$ where K_{TV} is the number of TV channels, is carried in an independent Physical Layer Pipe (PLP), with the possibility of having different modulation and coding rate schemes (MODCOD), to meet different reception conditions.

The net data rate of a PLP as well as the total capacity of the system vary with different transmission parameters (MODCOD of the PLPs) and super frame configuration [3][27]. For example, we can serve a maximum of 4 TV channels, requiring 512 Kbps each, if the chosen modulation is QPSK 3/5. We can, however, serve up to 8 TV channels with 16-QAM 3/5 [27].

However, the choice of the modulation and coding scheme is not sufficient to design the system; the transmission power, which we denote by P_D, has also to be planned accordingly in order to cover the desired service area. In order to have a good coverage, a required Carrier to Noise Ratio (C/N) of TV services has to be ensured, knowing the propagation environment and the possible locations of the receivers (on rooftops for terrestrial services and handheld for mobile TV). A simple link budget analysis allows deriving the maximal permitted signal degradation due to the pathloss, when the transmission powers are known (see for instance the link budget analyses of papers [4], [5] and [6]). We present in Figure 7 this DVB transmission powers as a function of the required coverage radius, for both fixed (rooftop) and mobile devices.

It is obvious that a much higher power than that of DVB-T2 (targeting fixed reception) is needed for DVB-T2 Lite in order to have the same system coverage. Knowing the target radius area R_D, we can thus obtain the required DVB-T2 Lite emission power using Fig. 7.

B. LTE network dimensioning

Due to lack of space and for clarity purposes, we do not present here a detailed dimensioning framework for LTE networks carrying both unicast and broadcast services. The basic idea is that the LTE resources (namely the Physical Resource Blocks (PRBs)) have to be shared between all services. Let C_L^u be the overall throughput of the LTE cell when it is dedicated to unicast services and C_L^b its overall throughput when it is entirely dedicated to broadcast services ($C_L^b > C_L^b$ as the Single Frequency Network (SFN) feature of eMBMS allows increasing the rates for broadcast services by reducing inter-cell interference [30][31]).

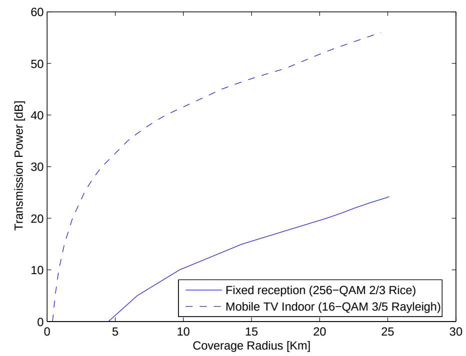

Fig. 7. The required transmitter power P_D function of the target coverage radius

When a number K_{TV} of TV channels are to be served, each requiring a throughput of T_{TV}, a proportion $p_b(K_{TV}) = \frac{K_{TV}T_{TV}}{C_l^b}$ of the cell resources is to be dedicated to eMBMS, and the remaining capacity (equal to $(1-p_b)C_L^u$ can be used for unicast services. For a unicast traffic density during the busy hour of V_u (in bit/s/Km^2), the following procedure can be used for dimensioning the LTE network in the outer region of Figure 1:

1) For a pure unicast network (with no TV service), the LTE cell radius has to be determined for being able to serve the target data volume. Assuming, for simplicity, that the LTE cell has a circular shape of radius R_L^u, the optimal LTE cell radius is calculated by:

$$R_L = \sqrt{\frac{C_L^u}{\pi V_u}} \tag{32}$$

2) When broadcast services are also to be offered, only a proportion of $(1-P_b)$ of LTE resources are available for unicast services, so the cell radius becomes:

$$R_L^b(K_{TV}) = \sqrt{\frac{(1-p_b(K_{TV}))C_L^u}{\pi V_u}} \tag{33}$$

C. *Probability of investing at time t*

For computing this probability of investment, we first define two types of events, $\mathbf{I}_t$ the event of investing at time t and $\bar{\mathbf{I}}_t$ the event of not investing. The optimal time to invest is the first time at which we get the value of the discounted net profit higher than the waiting value. $p_{inv}(t)$ is the probability of investing at time t and do not at any given time $t_i \in [0, t]$.

$$p_{inv}(t) = P(\mathbf{I}_t, \bar{\mathbf{I}}_{t-1}, ..., \bar{\mathbf{I}}_0) \tag{34}$$

The system is a markov chain, since that each state (t, x_t, d_t) depends only on the previous one. So,

$$p_{inv}(t) = \prod_{i=0}^{t-1} P(\mathbf{I}_{t-i}/\bar{\mathbf{I}}_{t-(i-1)})P(\bar{\mathbf{I}}_0) \tag{35}$$

$P(\mathbf{I}_{t-i}/\bar{\mathbf{I}}_{t-(i-1)})$ is given by $\frac{P(\mathbf{I}_{t-i},\bar{\mathbf{I}}_{t-(i-1)})}{P(\bar{\mathbf{I}}_{t-(i-1)})}$ with

$$\begin{aligned} P(\mathbf{I}_{t-i}, \bar{\mathbf{I}}_{t-(i-1)}) = \\ \sum_{k,j,k',j'} p(t-1, \mathbf{s}_{k,j}) I_{U(t-1,\mathbf{s}_{k,j}) \le W(t-1,\mathbf{s}_{k,j})} \\ \times p(t-1, \mathbf{s}_{k,j}, \mathbf{s}_{k',j'}) I_{U(t,\mathbf{s}_{k',j'}) > W(t,\mathbf{s}_{k',j'})} \end{aligned} \tag{36}$$

$$\begin{aligned} P(\bar{\mathbf{I}}_{t-i}, \bar{\mathbf{I}}_{t-(i-1)}) = \\ \sum_{k,j,k',j'} p(t-1, \mathbf{s}_{k,j}) I_{U(t-1,\mathbf{s}_{k,j}) > W(t-1,\mathbf{s}_{k,j})} \\ \times p(t-1, \mathbf{s}_{k,j}, \mathbf{s}_{k',j'}) I_{U(t,\mathbf{s}_{k',j'}) > W(t,\mathbf{s}_{k',j'})} \end{aligned} \tag{37}$$

On the other hand, we can obtain the probability of not investing at any $t_i \in [\tau, t]$ by calculating $P(\bar{\mathbf{A}}_t, \bar{\mathbf{A}}_{t-1}, ..., \bar{\mathbf{A}}_\tau)$in the same way as above.