Optimal resource rent taxation when nations are credit constrained

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First, preliminary version; comments are welcome

Abstract

The Resource Rent Tax suggested by Garnaut and Clunies Ross (1975) has been influential in resource rich countries and in academic literature. Several authors show that it distorts investments through asymmetric treatment of profits and losses. Neutrality can only be achieved if authorities commit to treating these symmetrically, guaranteeing loss offset through payouts if necessary. Risks are substantial, both in output (technology, geology) and (output and factor) prices. Many nations will be unable or unwilling to take risks involved in such guarantees. We analyze optimal rent taxation in this situation, maximizing tax revenue subject to a constraint of no loss offset.

KEYWORDS: Resource rent tax, neutrality, credit constraints

JEL CLASSIFICATION NUMBERS: G31, H25, Q32

1 Introduction

In most countries, nonrenewable natural resources are owned by the nation.[1] The net value of a deposit is called the (resource) rent, and at the outset, it then belongs to the nation. Governments invite private-sector companies to participate in extraction activities because these companies have the necessary technology and personnel, perhaps also capital. After some process, some companies get licenses to explore and extract. In some countries there are auctions (cash-bonus bidding), in which the nation hopes to receive from the winning company a payment close to the total rent of a particular area. In other countries, there is little or no up-front payment for licenses, but companies are subject to rent taxes, sometimes in addition to corporate income taxes. Some countries combine auctions and taxation. Boadway and Keen (2010, 2014) and IMF (2012) discuss the pros and cons of these alternatives, but draw no definite conclusion. Since a large number of countries rely on rent taxation, we concentrate on this alternative.

The simplest model of optimal rent taxation is a pure cash flow tax at a rate close to one hundred percent.[2] The tax is neutral under the assumption of value additivity, i.e., that a company attaches a value to x percent of a (risky) cash flow stream which is equal to x percent of the value it attaches to the whole cash flow stream. The company will then make the same decisions (exploration, development, operation, shut-down) under the pure cash flow tax as if there were no taxes, in order to maximize the total value of the prospective deposits.

[1] In federal nations, the ownership may be by the states/regions/provinces. We neglect this distinction here.

[2] By a pure cash flow tax, we mean a proportional tax on the company's non-financial cash flows, as suggested by Brown (1948). In years with a negative cash flow, this system gives payouts of the negative taxes. In terms of cash flows, this is similar to equity participation by the government. The government may prefer the tax over equity participation in order not to have to participate in decision making.

Such a tax would be optimal if the government attaches the same value to risky future cash flows as do the companies, and the government is able and willing to commit to the payouts that it may have to make if output quantities or prices turn out to be low. The guarantees have to be credible to such a degree that the companies regard them as certain. Another condition for the system to be optimal, is that the government is able to prevent transfer problems, i.e., companies using transfer pricing or real transfers to inflate the costs or deflate the gross revenues in its activities.[3]

In practice, most governments choose tax systems that do not involve payouts. The typically large initial costs are not expensed but deducted later through depreciation allowances and similar investment-related deductions. As shown by Fane (1987) and Bond and Devereux (1995), the tax system can still be neutral if the present value of deductions are maintained. This can be achieved by carrying forward negative cash flows with accumulation of interest at a risk free interest rate, provided that eventual deduction is guaranteed. Norway has tried to implement elements of such a system for its petroleum sector, cf. Lund (2014b).[4] The question is again whether the guarantee is credible. Historically in Norway, the guarantees only occurred after a sizable sovereign wealth fund had been accumulated, and the system would perhaps not work for less wealthy nations.

Many countries will be unable or unwilling (or both) to make these payouts or guarantees. Some nations have resource sectors that are large compared to the national economy, and, in particular, to the government budget. If rent tax rates well above fifty percent should be applied, the nations would have problems financing these payouts immediately (under the pure cash flow tax system) or guaranteeing the future payouts credibly (under systems with deductions carried forward). One such case may be Greenland, as described

[3]See Lund (2002) for discussion of the transfer problem.

[4]Something similar was proposed in Australia under the name Resource Super Profits Tax, cf. Lund (2011).

in Nielsen (2013). Both kinds of problems (payout now versus guaranteed future payouts) are clearly related to the country's ability to borrow at reasonable interest rates. In this version of the paper, we concentrate on credit constraints as the motivation for not relying on neutral rent taxes.

There can be many alternatives to neutral rent taxation. One simple alternative would be to tax positive outcomes, but give (and promise) no payouts in case of negative outcomes. This is the alternative we explore below. In the next section we give a brief account of the discussion of such tax systems in the economics literature.

Another possible motivation for not relying on neutral rent taxes could be risk aversion on part of the government. This could be imagined as the government representing the median citizen, who is risk averse. It is difficult for poor nations to diversify well in international capital markets. Thus it may be reasonable to model the nation as risk averse when it comes to investment in its resource sector. This possibility is not treated in the current version of the paper.

In order to discuss what is the best achievable rent tax system for a government, we need to formalize the valuation, by the government, of its future risky rent tax revenues. In the current version of the paper, the government attaches values to its tax claims in accordance with theories from financial economics, and these values are the same (but with opposite sign) as those that are attached to the same claims by the companies. This simplifying assumption is not completely satisfactory, since the valuation model assumes that there are no credit constraints. It also assumes spanning and complete diversification, so it is not compatible with the risk averse behavior by a government that would have liked to diversify away some of the risk from its resource tax revenue.

Nevertheless, this is the approach of this first version of the paper, since we would like to analyze the simplest version of our problem first. If we had introduced different valuation functions for governments and companies, a neutral tax system would be sub-optimal, and an important function of the tax system would have been to correct for this deviation. Instead, in the present version, a neutral tax would have been optimal if the government could afford it.

Clearly, the word "optimal" in the title should be interpreted with caution. Our results will only be optimal under the set(s) of assumptions we use, and some of these are overly simplified or controversial. But this paper differs from most of the literature in trying to find some criteria for optimality, instead of presenting neutrality as the aim, or criticizing neutrality.

2 Neutral taxation versus the Resource Rent Tax

We use the Resource Rent Tax (capitalized, as a proper name) (RRT) to denote the tax system proposed by Garnaut and Clunies Ross (1975, 1979). The tax base is defined on the basis of a company's yearly cash flow (from a license area or from the whole resource sector in a country). When the cash flow is negative, it is carried forward with interest accumulation and deducted in next year's cash flow. When the cash flow is positive after deduction of such accumulated losses, it is taxed at a constant, proportional rate.[5] If the company never gets sufficient revenues to deduct the accumulated negative cash flows, nothing is paid out from the government. This is an important asymmetry, and differs from the neutral systems mentioned above (Fane, 1987, Bond and Devereux, 1995). Garnaut and Clunies Ross (1975) suggest to use the company's cost of capital as the interest rate for accumulation.

[5]The multi-tier version of the RRT is not considered here.

Several authors (Mayo, 1979, Ball and Bowers, 1983, Smith, 1999) notice that the system is a disincentive for investment because the deductions are not guaranteed. One could imagine that a higher interest rate would be appropriate to achieve neutrality when deductions are risky. Smith (1999) shows in some detail why different circumstances (geology, prices, costs, time lags) will lead to different appropriate rates, so that neutrality cannot be achieved in practice.

This is an important criticism of the RRT, which was not taken well care of in the original articles by Garnaut and Clunies Ross (1975, 1979). But the theoretical satisfactory remedy, to guarantee the deductions and use the risk free interest rate, is only available to countries that can afford it. When they cannot, the RRT may be the best achievable system. The design of such a system should be made under the assumption that the companies realize that deductions are risky and act accordingly. In the next section, a very stylized model is used, with only one production period. In such a model, the firm's optimal investment (under decreasing returns to scale) is found analytically. The government will choose the tax parameters in order to maximize the value of the tax claim, given the assumed behavior of the companies. When parameters vary across many companies and licenses, it may be informationally, politically and/or legally impossible to differentiate the tax system accordingly. The best solution will then be some kind of average, but this is not explicitly solved for in the current version.

Even though there is only one year with extraction, we refer to the system as the RRT, since it has the important asymmetry mentioned above. But the present model exaggerates the effect of asymmetry, since the possibility of loss carry-forward to later years is ignored. The system will have two parameters, the tax rate and the deduction rate, i.e., the "rate" at which an investment can be deducted from future income (in one year). A nice feature of the model is that it gives an explicit solution for the optimal tax rate, which is not often found in models of resource rent taxation, cf. Lund (2009, section 6.5).

3 The model

The government will maximize the value of taxes using the Resource Rent Tax (RRT) system, under the condition that the taxpaying firm maximizes the value of its net after-tax profits. The firm chooses investment I in a production process with decreasing returns to scale, i.e., more is invested up to a point where the value of the marginal product of an additional unit of money invested is equal to one unit of money.

The output from investment $Q = \kappa I^{\nu}$ (where κ, ν are positive constants, $\nu < 1$) will appear t years later and will be sold then at the uncertain price P_t.[6] The valuation will rely on standard assumptions in the literature on real options and option-analogous taxes, cf. references below. We assume that the price process is a geometric Brownian motion with drift. From the time of investment, both the firm and the tax authorities attach a value $V_0(P_t) = P_0 e^{-\delta t}$ to claims to one unit of output to be received at t, where P_0 can be interpreted as today's price (at time 0) and δ is a positive constant (the "rate-of-return shortfall" of the price process).[7]

The tax will only work in year t. It will allow a deduction $c \cdot I$ then (with c being a positive constant, known as the "depreciation rate" or (rate of) "allowance for corporate capital" or the (rate of) accumulated negative cash flow). The tax base will be the difference

[6] To make the model somewhat realistic, this t could be set to some average time difference between investment and extraction from an average deposit.

[7] If extracted units of output had been an investment asset in itself, like gold, one could have imagined $\delta = 0$ (when all agents are at interior optima in their choices of how much gold to hold, and storage and insurance costs are negligible). But for most exhaustible resources, there will be a positive rate-of-return shortfall, and only those agents who have some gain, in addition to a possible price increase, from holding extracted units of the asset, will do so. That additional gain is called the convenience yield, and it may be modeled as identical to a rate-of-return shortfall.

$P_tQ - cI$ if the difference is positive. This is taxed proportionally at a rate $\tau \in (0,1)$, but the tax gives no refund if $P_tQ < cI$. The tax is thus $\tau \max(P_tQ - cI, 0)$. The tax claim is similar to a claim to a European call option. This analogy has been used to find the value of the tax claim by, e.g., Ball and Bowers (1983), Majd and Myers (1985), Green and Talmor (1985), MacKie-Mason (1990), Jacoby and Laughton (1992) and Lund (1992). At time zero, both the firm and the authorities attach a value to the tax payment which is determined by the McDonald and Siegel (1984) modification of the Black and Scholes (1973) formula,

$$\tau \left[P_0 e^{-\delta t} Q N(z_1) - cI e^{-rt} N(z_1 - \sigma\sqrt{t}) \right], \tag{1}$$

where N is the standard normal cumulative distribution function, and

$$z_1 = \frac{\ln(P_0 Q) - \ln(cI) + (r - \delta)t}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}. \tag{2}$$

The expression in (1) is, of course, the absolute value, i.e., for the firm, this counts negatively. The expression can easily be interpreted using the concept of risk-neutral probabilities which is well-known from financial economics (see, e.g., McDonald (2006, p. 321)). The second $N()$ expression in (1) gives the risk-neutral probability of the tax base being strictly positive, so that the deduction is earned. The first $N()$ is a conditional probability, so that the first term is the expected present value of P_tQ under the risk-neutral price process, conditional on the tax base being positive (which, in itself, depends on the outcome of P_t).

In standard applications of the model for pricing European call options, and the analogous tax claims, the probabilities are assumed to be exogenous. Lund (2014a) shows how to endogenize them. As shown in the appendix, when the production function is as described above, and the firm maximizes after-tax value, z_1 will be determined implicitly by the equation

$$z_1 = \frac{1}{2}\sigma\sqrt{t} + \frac{1}{\sigma\sqrt{t}}\left\{rt + \ln\left[1 - \tau c e^{-rt} N(z_1 - \sigma\sqrt{t})\right] - \ln(c\nu) - \ln\left[1 - \tau N(z_1)\right]\right\}. \quad (3)$$

This follows from the first-order condition for an optimum, which will give the interior maximum under one additional condition, that $1 > \tau c e^{-rt} N(z_1 - \sigma\sqrt{t})$. This condition can be interpreted as "no gold plating incentives," i.e., that the valuation of the additional tax deduction that follows from an additional investment should not exceed that additional investment. If it did, there would be incentives for completely unproductive investments. This will put upper limits on τ (for a given c) and c (for a given τ).[8]

For the interior solution, the optimal I is then given by

$$I = \left\{\frac{P_0 e^{-\delta t}\kappa\nu\left[1 - \tau N(z_1)\right]}{1 - \tau c e^{-rt} N(z_1 - \sigma\sqrt{t})}\right\}^{1/(1-\nu)}. \quad (4)$$

The formula shows the trade-off at the margin between the after-tax value of the marginal product of investment, which appears in the numerator, and the after-tax cost of investing, in which the tax value deduction is subtracted, appearing in the denominator.

The model will be solved numerically. A reasonable vector of exogenous parameters $(P_0, \delta, t, \kappa, \nu, r, \sigma)$ will be specified as a base case. The computer makes a two-dimensional grid search over the two tax parameters. For each pair (τ, c), the company's optimal investment is found analytically, and the value of the tax claim can be calculated (see above). The government's optimal (τ, c) is that pair which maximizes the value of the tax

[8] Whether and when the limits will be binding, come out as results of the simulations below.

claim. An easy extension would be to consider a pair (τ, c) that maximizes a weighted average of the tax claim and the after-tax value to the company.

4 Results

The base case will be $(P_0, \delta, t, \kappa, \nu, r, \sigma) = (10, 0.03, 10, 1, 0.5, 0.05, 0.3)$. Neither P_0 nor κ will influence the results, apart from the magnitudes of rents and taxes.[9] The nominal interest rate is set at 5 percent, which is not unreasonable by historical standards. The rate-of-return shortfall, δ, should be less than r, and is set at 3 percent. The elasticity of the production function, ν, is set at 0.5, which is an intermediate value in the assumed $(0, 1)$ range, although this does not imply that 0.5 is more reasonable than, say, 0.34.[10] When ν is closer to unity, one is closer to a situation with constant returns to scale. There will be less rent, and the probability of being out of tax position will be higher.[11] With one year as the unit of time, the volatility, σ, is set at 0.3, not unreasonable for, e.g., oil, while t is set at 10 years. This t could be seen as an average time lag between investment and oil extraction, perhaps representative for offshore activities. For onshore and smaller deposits, a shorter time lag would be more representative.

When looking at the results, one should keep in mind that as long as $\sigma > 0$, the tax system can at most be neutral for some particular configuration of parameters. With uncertainty, there will be some probability that the tax base is negative, in which case

[9] Neither P_0 nor κ appear in equation (3), which determines z_1. However, if the government wants to maximize its tax revenue by using the same tax parameters for a number of different deposits, the average which was mentioned in section 2 will clearly put more weight on larger deposits, i.e., those with higher values of κ.

[10] The value $\nu = 0.55$ is used by Lund (1992) and Blake and Roberts (2006). For a lower number, an anecdote tells that increasing the number of extraction platforms on the Norwegian Statfjord offshore oil field from two to three, increased the predicted extraction by 15 percent. If platforms are equal and the time profile of extraction is unaffected, this suggests an elasticity of about 0.34.

[11] Consequences of this elasticity for the risk of the tax claim is studied in Lund (2014a).

there is incomplete loss offset. If we compare this tax system with one that guarantees full loss offset, the incomplete-loss-offset system will discourage investment, given that the parameters (τ, c) are the same. If $c = e^{rt}$ and $\tau \in (0, 1)$, the system with full loss offset will be neutral (Fane, 1987, Bond and Devereux, 1995). For some given τ, it may be possible to compensate for the lack of loss offset by increasing c to some number greater than $c = e^{rt}$. If such a c value exists, for which the firm chooses the same I as in the no-tax or neutral-tax cases, it will clearly depend on the other parameters of the model (Smith, 1999).

4.1 Base case and variation in elasticity and rent

For the base case, the optimal investment in a no-tax situation would be $I = 13.7$, which would lead to the maximal before-tax (net) value of 13.7, the same number (which follows when $\nu = 0.5$). With $c = e^{rt} = 1.649$ and the tax rate set as high as $\tau = 0.9$, the after-tax optimal investment is $I = 3.47$, i.e., substantially reduced, but the before-tax (net) value would only be reduced to 10.33, i.e., to 75 percent of what it would have been in the absence of taxation. Clearly, decreasing returns means that a very substantial reduction in investment leads to only a moderate reduction in value. The risk-neutral probability of a positive tax base, $N(z_2)$, is as high as 0.837, even when ten years will pass with $\sigma = 0.3$.

If we consider the higher $\nu = 0.7$, but maintain the other parameters from the base case, as well as the two tax parameters, the optimal investment in a no-tax situation would be $I = 241$, which would lead to the maximal before-tax (net) value of 103. There is less rent relative to investment. The after-tax optimal investment would be $I = 17.9$, i.e., an even stronger relative reduction. The before-tax (net) value would be reduced to 37.9, i.e., to 37 percent of what it would have been in the absence of taxation. Clearly, not-so-decreasing returns means that a very substantial reduction in investment now leads to a substantial reduction in value. The risk-neutral probability of a positive tax base is now 0.766, but this is endogenous. The firm counter-acts the higher risk from lower rent by choosing a lower investment, resulting in not-too-low rent in relative terms, and not-so-substantial reduction in risk.

A change of the elasticity of the production function, ν, in the opposite direction, to $\nu = 0.3$, gives more rent (in relative terms). The optimal investment in a no-tax situation would be $I = 3.13$, which would lead to the maximal before-tax (net) value of 7.30. The after-tax optimal investment would be $I = 1.52$, i.e., a more moderate reduction. At this investment, the before-tax (net) value would be 6.88, i.e., 94.2 percent of what it would have been in the absence of taxation. The risk-neutral probability of a positive tax base is now 0.908.

The conclusion so far is that the impact of incomplete loss offset depends very much on the amount of inframarginal rent. Only one parameter has been varied so far, the elasticity, ν. The tax rate is set high (by international standards), and the effects of changing ν may look different at lower tax rates. What we have found in the case considered is that a lower elasticity of the production function means more rent, lower probability of being out of tax position, and thus lower relative distortion from the tax system. Moreover, the absolute numbers are difficult to interpret, but relative numbers and probabilities lend themselves to good, intuitive interpretations.

4.2 Optimal choice of tax parameters

The next experiment is a first attempt at finding optimal tax parameters, starting from the base case described above. The resulting values of the tax claim are shown in table 1. The first column shows the tax rate, varying from 0.54 to 0.99. The first three rows show

c	1.2365	1.4426	1.6487	1.8548	2.0609	2.2670	2.4731	2.6792	2.8853	3.0914
ce^{-rt}	0.7500	0.8750	1.0000	1.1250	1.2500	1.3750	1.5000	1.6250	1.7500	1.8750
$\ln(c)/t$	0.0212	0.0366	0.0500	0.0618	0.0723	0.0818	0.0905	0.0986	0.1060	0.1129
τ										
0.5400	7.9266	7.8483	7.6856	7.4583	7.1872	6.8902	6.5811	6.2701	5.9639	5.6670
0.5900	8.3267	8.3343	8.2304	8.0369	7.7792	7.4811	7.1610	6.8326	6.5056	6.1859
0.6400	8.6199	8.7424	8.7224	8.5812	8.3498	8.0588	7.7330	7.3907	7.0447	6.7034
0.6900	8.7787	9.0523	9.1484	9.0836	8.8947	8.6212	8.2960	7.9436	7.5809	7.2195
0.7400	8.7632	9.2333	9.4890	9.5332	9.4083	9.1654	8.8486	8.4906	8.1141	7.7339
0.7900	8.5116	9.2369	9.7141	9.9140	9.8824	9.6875	9.3889	9.0310	8.6436	8.2464
0.8400	7.9210	8.9776	9.7724	10.2003	10.3054	10.1822	9.9147	9.5636	9.1691	8.7570
0.8900	6.8024	8.2801	9.5624	10.3470	10.6588	10.6420	10.4227	10.0872	9.6900	9.2653
0.9400	4.7688	6.6728	8.8242	10.2597	10.9104	11.0554	10.9086	10.5999	10.2057	9.7711
0.9900	1.0396	1.9583	6.0868	9.6599	10.9944	11.4034	11.3662	11.0997	10.7153	10.2740

Table 1

Tax values in base case as function of tax parameters (τ, c)

the deduction rate values, varying from 1.24 to 3.09, and two alternative presentations of these values, the present value using the risk free interest rate, ce^{-rt}, and the risk-adjusted interest rate, $\ln(c)/t$ that would result in this c. The latter is known from the RRT system.[12]

Under the assumptions of the model, it may be reasonable for the government to try to maximize revenue. This will be particularly relevant if the company is owned by foreigners, in which case the government may find it reasonable to attach no welfare weight to their after-tax profits. It was assumed above that the government attaches a value to a risky tax claim which follows from the models of financial economics, in this case an option value. This is the ex ante meaning of "tax revenue" in the model, and the government looks for the highest possible number in the table.

[12]Let $r_x = \ln(c)/t$. Then $e^{r_x t} = c$, which means that an interest accumulation at the rate r_x is equivalent to the deduction rate c, assuming that the time lag is known in advance. Notice, in particular, the third lowest value of c, $c = 1.6487$. This is chosen to that the present value is unity (for given $r = 0.05, t = 10$), and would be a relevant rate if uncertainty was very low (or loss offset was guaranteed). It corresponds to $r_x = r = 0.05$. The other c values add or subtract eighths of this, but r_x does not grow linearly.

In the current version of the model, the transfer problem is explicitly ignored. This opens for the possibility of very high tax rates without the problems of base erosion and profits shifting (BEPS). In future versions we hope to incorporate a BEPS mechanism based on convex concealment costs along the lines of Lund (2002).

The numerical results in table 1 can be summarized as follows.

For each of the lower values of c, up to $c = 1.85$, the tax value is first increasing, then decreasing as function of τ. For each of the higher values, however, the results suggest that the tax value is increasing all the way to $\tau = 0.99$.

Looking instead along each row in the table: For each value of τ apart from the lowest, the tax value is first an increasing, then decreasing function of c. This means that for a given tax rate, the deduction rate is indeed able to compensate to some extent for the imperfect loss offset. In the base case, it turns out that when τ is 0.64 or lower, the optimal deduction rate is even lower than 1.6487, the value for c that would be used in a neutral system with full, guaranteed loss offset.

The overall maximum in table 1 occurs for $(\tau, c) = (0.99, 2.47)$. Of course, this is only an approximate result, dependent on the chosen grid.[13] The fact that the optimum is a corner solution is an additional motivation for including a BEPS mechanism in future versions.

Two other tables show additional characteristics of the results in table 1. In table 2, the risk-adjusted probabilities of positive tax bases are shown for the same 100 configurations of tax parameters for the base case. This shows that the probability $N(z_2)$ is as low as 0.54 at the government's optimum, $(\tau, c) = (0.99, 2.47)$. The combination of an extremely high

[13] In particular, there is no indication that $\tau = 0.999$ should not give a higher tax value. But see the comment above on BEPS, which makes the results gradually less realistic as τ approaches unity from below. Even higher values of τ will lead the model to collapse, even under its own assumptions, as the company will first be indifferent (at $\tau = 1$) and then choose not to produce (at $\tau > 1$).

c	1.2365	1.4426	1.6487	1.8548	2.0609	2.2670	2.4731	2.6792	2.8853	3.0914
ce^{-rt}	0.7500	0.8750	1.0000	1.1250	1.2500	1.3750	1.5000	1.6250	1.7500	1.8750
$\ln(c)/t$	0.0212	0.0366	0.0500	0.0618	0.0723	0.0818	0.0905	0.0986	0.1060	0.1129
τ										
0.5400	0.8199	0.7548	0.6898	0.6278	0.5704	0.5182	0.4713	0.4294	0.3920	0.3587
0.5900	0.8347	0.7691	0.7025	0.6384	0.5789	0.5248	0.4763	0.4331	0.3947	0.3607
0.6400	0.8509	0.7850	0.7166	0.6501	0.5882	0.5320	0.4817	0.4371	0.3976	0.3627
0.6900	0.8687	0.8028	0.7324	0.6631	0.5984	0.5398	0.4876	0.4414	0.4007	0.3648
0.7400	0.8885	0.8231	0.7505	0.6779	0.6099	0.5484	0.4939	0.4459	0.4039	0.3670
0.7900	0.9106	0.8466	0.7716	0.6950	0.6229	0.5580	0.5008	0.4508	0.4073	0.3694
0.8400	0.9353	0.8746	0.7971	0.7152	0.6379	0.5687	0.5084	0.4562	0.4110	0.3718
0.8900	0.9621	0.9091	0.8292	0.7399	0.6555	0.5810	0.5169	0.4620	0.4149	0.3744
0.9400	0.9880	0.9533	0.8735	0.7723	0.6772	0.5954	0.5264	0.4684	0.4192	0.3772
0.9900	1.0000	0.9991	0.9545	0.8210	0.7055	0.6127	0.5374	0.4754	0.4238	0.3802

Table 2
Risk-adjusted probabilities for positive tax bases in base case as function of tax parameters (τ, c)

tax rate and a fairly high deduction rate induces the company to choose an investment which makes the government's tax revenue risky. This is nevertheless the best solution for the government, because of the high tax rate.

In table 3, the relative distortions in before-tax values are shown. These are the ratios of the before-tax value when the company maximizes after-tax value, to the maximal before-tax value. A number just below unity means that the tax system does not distort much, so that almost the maximal before-tax value is obtained. The table shows that at the government's optimum, $(\tau, c) = (0.99, 2.47)$, there is just a modest distortion of 5 percent. The deduction rate is sufficiently high to induce investment not too much below the before-tax optimum. For the same tax rate, a deduction rate of $c = 1.65$ would lead to less than half in before-tax value. The general picture in the table is that distortions are modest, except for the south-west corner.

c	1.2365	1.4426	1.6487	1.8548	2.0609	2.2670	2.4731	2.6792	2.8853	3.0914
ce^{-rt}	0.7500	0.8750	1.0000	1.1250	1.2500	1.3750	1.5000	1.6250	1.7500	1.8750
$\ln(c)/t$	0.0212	0.0366	0.0500	0.0618	0.0723	0.0818	0.0905	0.0986	0.1060	0.1129
τ										
0.5400	0.9181	0.9412	0.9588	0.9718	0.9810	0.9875	0.9920	0.9950	0.9970	0.9983
0.5900	0.8948	0.9244	0.9472	0.9639	0.9759	0.9842	0.9899	0.9937	0.9963	0.9979
0.6400	0.8659	0.9035	0.9328	0.9543	0.9697	0.9803	0.9874	0.9922	0.9954	0.9974
0.6900	0.8297	0.8773	0.9148	0.9425	0.9621	0.9755	0.9846	0.9905	0.9944	0.9969
0.7400	0.7838	0.8438	0.8921	0.9278	0.9529	0.9699	0.9811	0.9885	0.9933	0.9963
0.7900	0.7242	0.7999	0.8627	0.9092	0.9415	0.9630	0.9771	0.9862	0.9919	0.9956
0.8400	0.6442	0.7402	0.8233	0.8851	0.9271	0.9546	0.9722	0.9834	0.9904	0.9947
0.8900	0.5314	0.6528	0.7671	0.8524	0.9087	0.9442	0.9664	0.9802	0.9887	0.9938
0.9400	0.3597	0.5053	0.6764	0.8049	0.8840	0.9311	0.9593	0.9763	0.9866	0.9928
0.9900	0.0762	0.1435	0.4473	0.7233	0.8487	0.9141	0.9506	0.9718	0.9842	0.9916

Table 3

Relative distortions in base case as function of tax parameters (τ, c)

A second experiment shows the effect of a higher volatility, $\sigma = 0.4$. Apart from this change, all other parameters of the base case are unaltered, as well as the grid for (τ, c) values. The results are shown in table 4.

With the higher value of σ, the variance of P_t is higher, and one striking feature of table 4 follows from this: The values in the cells towards the north east of the table, and in particular also the maximum value in the south east corner, exceed the values in the corresponding cells in table 1. This is a well known phenomenon from financial economics: When all else is equal, the value of a European call option is increasing in the volatility. The intuition here is that the government benefits from the higher outcomes of P_t, but does not suffer from the lower outcomes. In the present model, there is a complication that the company chooses I and thus z_1 endogenously, so that its unfavorable position can be partly counteracted by this choice.

c	1.2365	1.4426	1.6487	1.8548	2.0609	2.2670	2.4731	2.6792	2.8853	3.0914
ce^{-rt}	0.7500	0.8750	1.0000	1.1250	1.2500	1.3750	1.5000	1.6250	1.7500	1.8750
$\ln(c)/t$	0.0212	0.0366	0.0500	0.0618	0.0723	0.0818	0.0905	0.0986	0.1060	0.1129
τ										
0.5400	7.7789	7.7603	7.7007	7.6084	7.4909	7.3547	7.2054	7.0473	6.8839	6.7178
0.5900	8.0840	8.1314	8.1252	8.0743	7.9877	7.8734	7.7386	7.5891	7.4295	7.2636
0.6400	8.2728	8.4039	8.4679	8.4732	8.4299	8.3478	8.2359	8.1018	7.9516	7.7904
0.6900	8.3227	8.5580	8.7122	8.7918	8.8073	8.7702	8.6915	8.5810	8.4468	8.2953
0.7400	8.2022	8.5655	8.8349	9.0119	9.1060	9.1301	9.0977	9.0210	8.9107	8.7754
0.7900	7.8644	8.3840	8.8011	9.1066	9.3059	9.4129	9.4437	9.4140	9.3378	9.2265
0.8400	7.2344	7.9440	8.5539	9.0328	9.3762	9.5968	9.7144	9.7495	9.7207	9.6432
0.8900	6.1778	7.1144	7.9868	8.7134	9.2640	9.6467	9.8865	10.0119	10.0488	10.0185
0.9400	4.4016	5.5788	6.8445	7.9772	8.8636	9.4985	9.9209	10.1768	10.3066	10.3422
0.9900	1.0334	1.8379	3.8553	6.1906	7.8779	9.0058	9.7406	10.2013	10.4689	10.5988

Table 4

Tax values as function of tax parameters (τ, c) with $\sigma = 0.4$

Another striking feature is that at this high value of σ, we have not found an interior solution for c at the row for $\tau = 0.99$. Based on the model, the optimal solution is a very high tax rate and a very high deduction rate.

5 Discussion and directions for further research

There are a number of possible additional analyses that can be made on the basis of the current model. Of course, the sensitivity of the results to changes in other parameters can be analyzed, in particular, r, δ, and t. Moreover, there is an interesting question whether and how the optimal c depends on r and t. Since the deduction in the RRT proposal (Garnaut and Clunies Ross, 1975) was an accumulation of a risk-adjusted interest rate, reflecting the company's cost of capital, one could analyze whether this holds here when r and t change. This can be linked to the average and marginal systematic risk of the company's cash flow, as analyzed by Lund (2014a).

Extending the model, one may consider the transfer possibilities that are particularly relevant when the tax rate is high. This can be done, again in a very stylized way, based on the convex concealment cost in Lund (2002). The solution to the company's maximization problem is then perhaps not analytically tractable, but it may be solved numerically. To keep the model manageable, it will be helpful to restrict the number of tax parameters to two, which means that the element of "gross taxation" (or royalty) in Lund (2002) could be represented through the insufficient deduction for costs via c. A richer model might give even more results, but may be difficult to keep track of.

Another extension, which adds realism, would be to consider many deposits and/or many time periods. Based on limited information the government may have to apply the same tax parameters to all deposits, and it may have to rely on a simple rule for interest accumulation to implement a time-varying c. This may lead to trade-offs between to harsh taxation in some situations and too lenient taxation in others. An overall sector maximization may show how to arrive at a useful average solution.

6 Conclusion

Based on valuation functions from financial economics, we have shown how to solve the problem of optimal taxation of a company operating under decreasing returns to scale when exogenous output price uncertainty is the only source of risk. The model is highly stylized with only one period of extraction. The optimization of two parameters of the tax system was made under the assumption that there would be no loss offset. This is quite common in practice, and can be motivated for some nations by their inability to give a credible guarantee of payouts in the future in case the output price turns out to be low. Another motivation could be risk aversion on behalf of citizens, due to lack of international diversification. Outside the model, there is also the possibility of quantity uncertainty for technical or geological reasons.

The numerical results of the current version lead to tax rates approaching one hundred percent from below. This is an unrealistic feature, which will motivate an extension where the company tries to transfer costs into the sector, either by transfer pricing or real transfers.

The results also show how the optimal deduction rate for costs will depend on the tax rate and on the other parameters of the model. The model may be used to investigate whether the deduction rate can be interpreted as an interest accumulation at a risk-adjusted rate, which was the original formulation of the Resource Rent Tax.

7 Appendix

This appendix shows how to derive the first-order conditions for the firm's maximization. There is also a discussion of the condition to avoid gold-plating incentives.[14]

The original Black-Scholes formula had five arguments. The value of a European call option is typically written as

$$C(S, K, \sigma, r, t) = SN(d_1) - Ke^{-rt}N(d_2),$$

[14]The derivation is similar to what is found in Lund (2014a, p. 593), but with two changes: In the present model, there is no immediate deduction when investment is done, unlike the "investment tax credit" at rate a which is included in the previous model. This could easily have been included here, but is seldom part of an RRT, and would make equations a bit more complicated. Moreover, in the present model, the time between investment and output, t, can be varied, while in the previous model, this was fixed at unity, one year. (A multi-period extension is found in Lu (2012).) Furthermore, there are changes in notation, τ here was t there. Also, the discount factor was $1/(1 + r)$ there, but is e^{-r} here.

where

$$d_1 = \frac{1}{\sigma\sqrt{t}}\left[\ln(S/K) + rt\right] + \frac{1}{2}\sigma\sqrt{t},$$

$$d_2 = d_1 - \sigma\sqrt{t}.$$

The derivatives of the Black-Scholes formula are well known, but not quite trivial to derive (see, e.g., McDonald (2006, app. 12B)). Below, we need

$$\frac{\partial C}{\partial S} = N(d_1),$$

and

$$\frac{\partial C}{\partial K} = -e^{-rt}N(d_2).$$

With a rate-of-return shortfall of δ, McDonald and Siegel (1984) show that the option has a value of $C(Se^{-\delta t}, K, \sigma, r, t)$.

When the tax claim is of the form given in the main text, its value as seen from period 0 is thus $\tau C(P_0 e^{-\delta t}Q, cI, \sigma, r, t)$, which can be rewritten as equations (1) and (2).

The firm's maximand is the market value after tax,

$$P_0 e^{-\delta t}\kappa I^{\nu} - \tau\left[P_0 e^{-\delta t}\kappa I^{\nu}N(z_1) - ce^{-rt}N(z_1 - \sigma\sqrt{t})\right] - I$$

$$= P_0 e^{-\delta t}\kappa I^{\nu} - I - \tau C(P_0 e^{-\delta t}\kappa I^{\nu}, cI, r, \sigma, t).$$

The first-order derivative of the maximand w.r.t. I is

$$P_0 e^{-\delta t}\kappa\nu I^{\nu-1} - 1 - \tau\left[P_0 e^{-\delta t}\kappa\nu I^{\nu-1}N(z_1) - ce^{-rt}N(z_2)\right].$$

The first-order condition sets this to zero, which yields

$$P_0 e^{-\delta t}\kappa\nu\left[1-\tau N(z_1)\right]I^{\nu-1} = 1 - ce^{-rt}\tau N(z_2).$$

If the right-hand side is positive, this yields an interior solution, which is a maximum (assuming that the second-order condition is satisfied). As long as the right-hand side is negative, however, an interior solution is not yet found, and the firm will gain from increasing I, since the first-order derivative is positive. If this holds for all values of I, no interior solutions exist, and the firm will increase I indefinitely.

When the interior solution exists, we can solve for the optimal I, equation (4). This solution (or, equivalently, an expression for $P_0 e^{-\delta t}\kappa I^{\nu-1}$ from the first-order condition) can be plugged into (2) to obtain (3). Two of the intermediate steps are shown: Equation (2) can be rewritten as

$$z_1 = \frac{\ln(P_0 e^{-\delta t}\kappa I^{\nu-1}) - \ln(c) + rt}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}.$$

The argument of the first logarithm is recognized from the first-order condition above, which is plugged in, under the assumption that the expression is positive, so that the logarithm is well-defined, and an interior solution exists:

$$z_1 = \frac{\ln(\frac{1-ce^{-rt}\tau N(z_2)}{\nu[1-\tau N(z_1)]}) - \ln(c) + rt}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t},$$

which can be rewritten as (3).

References

Ball, Ray, and John Bowers (1983) 'Distortions created by taxes which are options on value creation: The Australian Resources Rent Tax proposal.' *Australian Journal of Management* 8, 1–14

Black, Fischer, and Myron Scholes (1973) 'The pricing of options and corporate liabilities.' *Journal of Political Economy* 81, 637–654

Blake, Andon J., and Mark C. Roberts (2006) 'Comparing petroleum fiscal regimes under oil price uncertainty.' *Resources Policy* 31, 95–105

Boadway, Robin, and Michael Keen (2010) 'Theoretical perspectives on resource tax design.' In *The Taxation of Petroleum and Minerals: Principles, Problems and Practice,* ed. Philip Daniel, Michael Keen, and Charles McPherson (New York: Routledge) pp. 13–74

_ (2014) 'Rent taxes and royalties in designing fiscal regimes for non-renewable resources.' Working Paper 4568, CESifo, Munich. Forthcoming in David Layton and Robert Halvorsen (Eds.) *Handbook on the Economics of Natural Resources,* Edward Elgar

Bond, Stephen R., and Michael P. Devereux (1995) 'On the design of a neutral business tax under uncertainty.' *Journal of Public Economics* 58, 57–71

Brown, E. Cary (1948) 'Business income, taxation, and investment incentives.' In *Income, Employment, and Public Policy: Essays in Honor of Alvin H. Hansen,* ed. Lloyd Metzler, Harvey S. Perloff, and Evsey D. Domar (New York: Norton) pp. 300–316

Fane, George (1987) 'Neutral taxation under uncertainty.' *Journal of Public Economics* 33, 95–105

Garnaut, Ross, and Anthony Clunies Ross (1975) 'Uncertainty, risk aversion and the taxing of natural resource projects.' *Economic Journal* 85, 272–287

_ (1979) 'The neutrality of the Resource Rent Tax.' *Economic Record* 55, 193–201

Green, Richard C., and Eli Talmor (1985) 'The structure and incentive effects of corporate tax liabilities.' *Journal of Finance* 40, 1095–1114

IMF (2012) 'Fiscal regimes for extractive industries: Design and implementation.' Report 15 August, International Monetary Fund, Washington, D.C.

Jacoby, Henry D., and David G. Laughton (1992) 'Project evaluation: A practical asset pricing method.' *The Energy Journal* 13(2), 19–47

Lu, Zongwei (2012) 'Effects of taxes on the after-tax cost of capital: A simulation approach for multi-period models.' Master's thesis, University of Oslo. Available at http://urn.nb.no/URN:NBN:no-31553

Lund, Diderik (1992) 'Petroleum taxation under uncertainty — contingent claims analysis with an application to Norway.' *Energy Economics* 14, 23–31

_ (2002) 'Rent taxation when cost monitoring is imperfect.' *Resource and Energy Economics* 24, 211–228

_ (2009) 'Rent taxation for nonrenewable resources.' *Annual Review of Resource Economics* 1, 287–308

_ (2011) 'Neutrality of the resource super profits tax.' *Australian Economic Review* 44, 233–238

_ (2014a) 'How taxes on firms reduce the risk of after-tax cash flows.' *FinanzArchiv/Public Finance Analysis* 70, 567–598

_ (2014b) 'State participation and taxation in Norwegian petroleum: Lessons for others.' *Energy Strategy Reviews* 3, 49–54

MacKie-Mason, Jeffrey K. (1990) 'Some nonlinear tax effects on asset values and investment decisions under uncertainty.' *Journal of Public Economics* 42, 301–327

Majd, Saman, and Stewart C. Myers (1985) 'Valuing the government's tax claim on risky corporate assets.' Working Paper 1553, National Bureau of Economic Research, Cambridge, Mass.

Mayo, Wayne (1979) 'Rent royalties.' *Economic Record* 55, 202–213

McDonald, Robert L. (2006) *Derivatives Markets,* 2nd ed. (Boston: Addison Wesley)

McDonald, Robert L., and Daniel Siegel (1984) 'Option pricing when the underlying asset earns a below-equilibrium rate of return: A note.' *Journal of Finance* 34, 261–265

Nielsen, Søren B. (2013) 'Exploitation of natural resources and the public sector in Greenland.' Technical Report, Committee for Greenlandic Mineral Resources to the Benefit of Society. Available at http://hdl.handle.net/10398/9050

Smith, Ben (1999) 'The impossibility of a neutral Resource Rent Tax.' Working paper in economics and econometrics 380, Australian National University, Canberra