VALUING REPLACEABLE ASSETS

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Abstract

We focus on the time-varying real option value (ROV) of replaceable assets whose operating cost and salvage value deteriorate stochastically. As the operating cost approaches the threshold justifying an immediate replacement, the ROV increases while the net present value of the operating asset declines. The ROV increases with the number of replacement opportunities, with large salvage values and volatile operating costs, possibly enhancing the equity value for those replaceable asset owners. Use of similar one or two factor models may undervalue replaceable assets, possibly misleading investors and corporate decision makers.

HIGHLIGHTS

- **Replacement ROV depends on operating costs, salvage value and tax depreciation.**
- **Greater ROV if salvage value is large and stable.**
- **Greater ROV if numerous replacement opportunities.**
- **Greater ROV if volatile operating costs.**
- **Cost increases reduce net present value, increase ROV.**
- **Practitioner focus on cost & salvage level and volatility, in funding, disposals and replacement actions.**

1. Introduction

What are the critical aspects of valuing replaceable assets that a chief real options manager (CROM) should consider? What is the current real value of her position, for purposes of selling (or acquiring) partial interests (equity) in that position; or disposing of (or buying) those assets in the second hand market? Naturally, a further concern is when to obtain the salvage value and buy new assets, the traditional concern of real option authors. As a part of that process, forecasting the input parameter values (cost deterioration and volatility, salvage value drift and volatility, correlation between cost and salvage) is critical, especially where there is a history of second hand equipment prices enabling quantification of expected "market indicated" deterioration rates.

We focus on the value of replaceable assets and the sensitivity of that value to (i) the number of replaceable opportunities, (ii) changes in expected critical parameter values, and (iii) whether the essential assumptions of the models herein are valid, or if not, what adjustments the CROM might make. Some empirical illustrations are based on a time series of dry bulk cargo ship new and second hand values by age, and salvage values.

For assets with a significant second-hand market value, or a notable scrap value such as ships, salvage value may be a crucial ingredient to the replacement decision because of the cash flow implications. Adkins and Paxson (2015) provide analytical solutions for the after-tax optimal timing boundary and real option value for a replaceable asset characterized by a deteriorating and stochastic operating cost and stochastic salvage value. Having determined the optimal replacement boundary, and therefore the real option coefficient and power parameter values, we focus on the value of owning a replaceable asset prior to reaching that boundary accompanied with the opportunity to replace that asset at a given investment cost. We assume this opportunity may be created by an actual arrangement with an equipment supplier (which may be exercised at any time in perpetuity) with no penalties for not exercising the option, due to customary arrangements and perhaps the owner's respected market position or operating competence[1].

[1] Obtaining a delivery option in new building ships and aircraft may require an initial deposit or pre-payment with additional deposits over time approaching physical delivery, which could be considered real option premiums.

There are several studies of the optimal boundaries for replaceable assets, but few focus on the interim real option value prior to the point of optimal replacement. Zambujal-Oliveira and Duque (2011) propose a two-factor model with stochastic operating cost and (autonomous) stochastic value, with depreciation following a negative exponential function, but did not quantify asset values. Adkins and Paxson (2011) derive real option values of renewable assets but only at the renewal boundaries. Adkins and Paxson (2013a) show the real option value for equipment with the possibility of technological progress, but ignore salvage value and taxation (and depreciation). Adkins and Paxson (2013b) consider the real option value of replaceable assets along with three different depreciation schemes, but do not quantify these values. Adkins and Paxson (2013c) derive the real option values for replaceable assets in a deterministic setting, ignoring salvage value and taxation (depreciation) but do not quantify these asset values.

There are two main contributions of this article. First, we supplement the existing real option literature on replacement by examining the differential impacts of operating cost, salvage value and depreciation on the policy and their implications for managers. In a recent literature review, Hartman and Tan (2014) note that there only a few asset replacement models which consider stochastic deterioration in continuous-time. The second contribution is an empirical evaluation of some model parameter values for dry bulk ships, an aspect some theoretical studies ignore.

The rest of this article is as follows. In Section 2, we summarize a quasi-analytical method for identifying the real option value of replaceable assets given some current operating cost level (and also salvage value and depreciation level). Section 3 shows a sensitivity analysis of the real option value at some assumed operating cost level to changes in many of the critical parameter values. Section 4 presents a short review of some parameter values from a database of second hand ship values. Section 5 suggests some apparent incentives for alert CROMs, and reviews which of the critical assumptions may not be entirely realistic, so presenting opportunities for further research.

2. Replacement Opportunity with Salvage and Tax Depreciation

There are examples of free options (without premiums) such as some hotel and other accomodation and even educational reservations over limited periods of time.

Three similar but distinct formulations of the replacement model involving stochastic operating cost and salvage vale and deterministic depreciation are presented based on a contingent-claims style of analysis and maximizing expected cash flows. Model 1 (identified by subscript 1) examines the replacement policy under multiple replacements while Model 1s restricts its scope to a single replacement. Model 2 (identified by subscript 2) investigates the effects of excluding depreciation on the replacement policy.

2.1 Valuation Function

Following Adkins and Paxson (2015) we determine the real-option replacement policy for a durable perpetual productive asset, without technological innovations, subject to operating cost decay and a deteriorating salvage value in a monopolistic situation whose output yields a constant revenue[2] P, assuming other flexibilities are inadmissible. The relevant cash flows crucial to the replacement decision are those associated with the operating costs, the depreciation charge and the salvage value. While annual operating cost and salvage value, denoted by C and S, respectively, are treated as stochastic factors following geometric Brownian motion processes, the annual depreciation charge, denoted by D, is a deterministic factor. The replacement policy, represented by an optimal timing boundary separating the decision regions of continuance and replacement, is defined over a three-dimensional cost-salvage-depreciation (C-S-D) space. The tax rate τ is applicable to all cash flows, both positive and negative, and regardless of whether they represent income or capital gains. At replacement, the operating cost, salvage value and depreciation level for the newly installed succeeding asset are set to their known initial levels of C_I, S_I and D_I, respectively. The replacement re-investment cost is a known constant K. To avoid round-tripping, $S_I < K$.

The asset value together with its embedded replacement option depends on the prevailing factor levels and is denoted by $F_1 = F_1(C, S, D)$. Following standard analysis, the replacement contingent claim is expressed by the following partial differential equation:

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[2] It is straightforward to recast the model in terms of net revenue instead of operating costs.

$$\begin{aligned}&\tfrac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 F_1}{\partial C^2} + \rho\sigma_C\sigma_S CS \frac{\partial^2 F_1}{\partial C \partial S} + \tfrac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_1}{\partial S^2} \\ &+ \theta_C C \frac{\partial F_1}{\partial C} + \theta_S S \frac{\partial F_1}{\partial S} - \theta_D D \frac{\partial F_1}{\partial D} - rF_1 + \left(P_I - C\right)\left(1-\tau\right) + D\tau = 0,\end{aligned} \tag{1}$$

where $r > 0$ is the constant risk-free rate of interest, P_I is the revenue assumed to be constant, σ_C and σ_S are the constant volatilities, $\rho_{C,S}$ is the correlation of C and S, and θ_C and θ_S are the respective risk-neutral drift rates, assumed to be equal to the expected C deterioration and S drift rates[3]. Following Adkins and Paxson (2011), the function satisfying (1) is:

$$F_1 = A_1 C^{\eta_1} S^{\gamma_1} D^{\lambda_1} + \frac{P_I\left(1-\tau\right)}{r} - \frac{C\left(1-\tau\right)}{r-\theta_C} + \frac{D\tau}{r+\theta_D}. \tag{2}$$

In (2), the expression $A_1 C^{\eta_1} S^{\gamma_1} D^{\lambda_1} \geq 0$ represents the replacement real option value (ROV) , so $A_1 \geq 0$. The last three terms represent the net present value (NPV) of operating the asset. Substituting (2) in (1) yields the characteristic root equation:

$$\begin{aligned}Q_1\left(\eta_1,\gamma_1,\lambda_1\right) = &\tfrac{1}{2}\sigma_C^2\eta_1\left(\eta_1-1\right) + \rho\sigma_C\sigma_S\eta_1\gamma_1 + \tfrac{1}{2}\sigma_S^2\gamma_1\left(\gamma_1-1\right) \\ &+ \theta_C\eta_1 + \theta_S\gamma_1 - \theta_D\lambda_1 - r = 0.\end{aligned} \tag{3}$$

Replacement is optimally triggered when the factor levels C, S, D attain their threshold levels $\hat{C}_1, \hat{S}_1, \hat{D}_1$, respectively, where $\hat{C}_1 \geq C_I, \hat{S}_1 \leq S_I, \hat{D}_1 \leq D_I$. This occurs when at exercise, the incumbent value and the successor value less the replacement cost net of salvage value and any depreciation recapture are in exact balance. After eliminating the constant P_I from both sides, the value matching relationship is:

$$\begin{aligned}&A_1\hat{C}_1^{\eta_1}\hat{S}_1^{\gamma_1}\hat{D}_1^{\lambda_1} - \frac{\hat{C}_1\left(1-\tau\right)}{r-\theta_C} + \frac{\hat{D}_1\tau}{r+\theta_D} \\ &= A_1 C_I^{\eta_1} S_I^{\gamma_1} D_I^{\lambda_1} - \frac{C_I\left(1-\tau\right)}{r-\theta_C} + \frac{D_I\tau}{r+\theta_D} + \left(1-\tau\right)\hat{S}_1 + \hat{D}_1\tau/\theta_D - K.\end{aligned} \tag{4}$$

[3] Adjustments for risks is an important research topic. We assume the discount rates are independent of cost volatility. Possibly the sensitivities for changes in interest rates indicate the significance of this research area.

Optimality is assured by the smooth-pasting conditions, one for each factor $C, S, D,$ which can be expressed in a reduced form by:

$$A_1 \hat{C}_1^{\eta_1} \hat{S}_1^{\gamma_1} \hat{D}_1^{\lambda_1} = \frac{\hat{C}_1 (1-\tau)}{\eta_1 (r-\theta_C)} > 0, \tag{5}$$

$$\frac{\hat{C}_1 (1-\tau)}{\eta_1 (r-\theta_C)} = \frac{\hat{S}_1 (1-\tau)}{\gamma_1} > 0, \tag{6}$$

$$\frac{\hat{C}_1 (1-\tau)}{\eta_1 (r-\theta_C)} = \frac{\hat{D}_1 \tau r}{\lambda_1 \theta_D (r+\theta_D)} > 0. \tag{7}$$

A reduced form value matching condition is:

$$\frac{\hat{C}_1 (1-\tau)}{\eta_1 (r-\theta_C)} \left[\eta_1 + \gamma_1 + \lambda_1 - 1 + \frac{C_I^{\eta_1} S_I^{\gamma_1} D_I^{\lambda_1}}{\hat{C}_1^{\eta_1} \hat{S}_1^{\gamma_1} \hat{D}_1^{\lambda_1}} \right] = K + \frac{C_I (1-\tau)}{r-\theta_C} - \frac{D_I \tau}{r+\theta_D}. \tag{8}$$

The C-S-D model 1 is composed of four simultaneous equations: the reduced form value matching relationship, two reduced form smooth pasting conditions, and the characteristic root equation. The optimal timing boundary and real option value can be determined by solving simultaneously four equations for $\hat{C}_1, \eta_1,$ γ_1 and λ_1, given assumptions about $\hat{S}_1$ and $\hat{D}_1$, and using these values in (2) along with some current C, S and D to determine $F_1 = ROV_1 + NPV_1$.

2.2 Single Replacement Opportunity

If there exists only one available remaining replacement opportunity, then the value-matching relationship for the multiple replacement model has to be amended to exclude replacement option value for the succeeding asset to become:

$$\begin{aligned} A_{1s} \hat{C}_{1s}^{\eta_{1s}} \hat{S}_{1s}^{\gamma_{1s}} \hat{D}_{1s}^{\lambda_{1s}} - \frac{\hat{C}_{1s} (1-\tau)}{r-\theta_C} + \frac{\hat{D}_{1s} \tau}{r+\theta_D} \\ = -\frac{C_I (1-\tau)}{r-\theta_C} + \frac{D_I \tau}{r+\theta_D} + (1-\tau) \hat{S}_{1s} + \hat{D}_{1s} \tau / \theta_D - K, \end{aligned} \tag{9}$$

where the subscript s refers to the single replacement opportunity. Since the smooth-pasting conditions are identical, except for the inclusion of the subscript s, the reduced form value-matching relationship is obtained by eliminating A_{1s} from (9):

$$\frac{\hat{C}_{1s}\left(1-\tau\right)}{\eta_{1s}\left(r-\theta_C\right)}\left[\eta_{1s}+\gamma_{1s}+\lambda_{1s}-1\right]=K+\frac{C_I\left(1-\tau\right)}{r-\theta_C}-\frac{D_I\tau}{r+\theta_D}. \tag{10}$$

This reveals that for a single replacement to be economically justified, the after-tax operating cost threshold has to exceed the re-investment cost plus the after-tax operating cost value for the replica less its depreciation tax shield value. By comparing (10) with (8), then $\hat{C}_{1s}\geq\hat{C}_1$ since our conjecture treats: $\frac{C_I^{\eta_1}S_I^{\gamma_1}D_I^{\lambda_1}}{\hat{C}_1^{\eta_1}\hat{S}_1^{\gamma_1}\hat{D}_1^{\lambda_1}}<1$. For any salvage value threshold, $\hat{C}_1$ is always less than $\hat{C}_{1s}$, since its re-investment cost can be recouped over multiple replacements instead of only one. The single replacement policy is determined by solving the four equations (i) the reduced form value-matching relationship, (ii) and (iii) two reduced form smooth-pasting conditions, modified, and (iv) the characteristic root equation $Q_1\left(\eta_{1s},\gamma_{1s},\lambda_{1s}\right)=0$ for $\hat{C}_{1s}$, η_{1s},γ_{1s} and λ_{1s}, given assumptions about $\hat{S}_{1s}$ and $\hat{D}_{1s}$.

2.3 C-S Model

The multiple C-S value-matching relationship is found in a similar way by excluding all terms involving depreciation:

$$A_2\hat{C}_2^{\eta_2}\hat{S}_2^{\gamma_2}-\frac{\hat{C}_2\left(1-\tau\right)}{r-\theta_C}=A_2C_I^{\eta_2}S_I^{\gamma_2}-\frac{C_I\left(1-\tau\right)}{r-\theta_C}+\left(1-\tau\right)\hat{S}_2-K. \tag{11}$$

Two smooth-pasting conditions, one for each of the two factors, C and S, are, respectively:

$$A_2\hat{C}_2^{\eta_2}\hat{S}_2^{\gamma_2}=\frac{\hat{C}_2\left(1-\tau\right)}{\eta_2\left(r-\theta_C\right)}=\frac{\hat{S}_2\left(1-\tau\right)}{\gamma_2} \tag{12}$$

The reduced form value-matching relationship can be expressed as:

$$\frac{\hat{C}_2(1-\tau)}{\eta_2(r-\theta_C)}\left[\eta_2+\gamma_2-1+\frac{C_I^{\eta_2}S_I^{\gamma_2}}{\hat{C}_2^{\eta_2}\hat{S}_2^{\gamma_2}}\right]=K+\frac{C_I(1-\tau)}{r-\theta_C}. \tag{13}$$

The C-S model 2 involves solving three simultaneous equations: (i) the reduced form value-matching relationship (13), (ii) the reduced form smooth-pasting condition (12), and (iii) the characteristic root equation $Q_2(\eta_2,\gamma_2)=0$.

3. Illustrative Real Option Value Results

A salvage value threshold is initially pre-specified, and then the optimal timing boundary is found for the two remaining factors, operating costs and depreciation, by varying the depreciation threshold level. If this procedure is repeated for alternative pre-specified salvage value thresholds, a representative set of optimal timing boundaries $\hat{C}_1$ can be constructed, along with the power parameter values.

3.1 Multiple Opportunity C-S-D Real Option Value

The real option values for the C-S-D 1 model given current C, S, D are illustrated in Table 1. The Excel formulae are shown below.

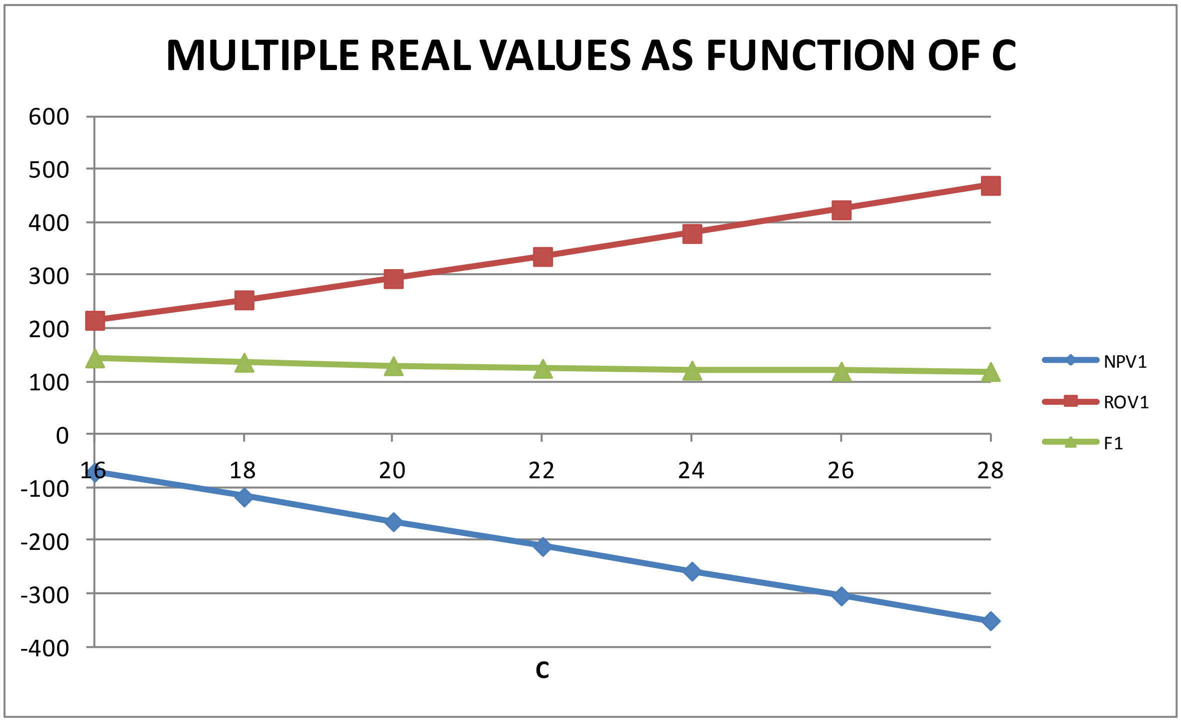
Q(η,γ,λ)	0.5*(B8^2)*B29*(B29-1)+0.5*(B9^2)*B30*(B30-1)+B10*B8*B9*B29*B30+B12*B29+B13*B30-B14*B31-B11
SP1	B32*(1-B18)/(B29*(B11-B12))-B15*(1-B18)/B30
SP2	B32*(1-B18)/(B29*(B11-B12))-(B16*B18*B11)/(B31*B14*(B11+B14))
VM	B25*(B29+B30+B31-1+B26)-B27
PART 1	B32*(1-B18)/(B29*(B11-B12))
PART 2	((B4^B29)*(B6^B30)*(B7^B31))/((B32^B29)*(B15^B30)*(B16^B31))
PART 3	B5+B4*(1-B18)/(B11-B12)-(B7*B18)/(B11+B14)
NPV_1	B3*(1-B18)/B11-B19*(1-B18)/(B11-B12)+B16*B18/(B11+B14)
ROV_1	**B35-B33**
F_1	IF(B19<B32,B38*((B19^B29)*(B15^B30)*(B16^B31))+B33,B36)
ROV EX	B38*((B4^B29)*(B6^B30)*(B7^B31))+B37+B3*(1-B18)/B11
NPV EX	-B4*(1-B18)/(B11-B12)+B7*B18/(B11+B14)+B15*(1-B18)+B16*B18/B14-B5
A_1	B25/((B32^B29)*(B15^B30)*(B16^B31))
PDE	0.5*(B8^2)*(B19^2)*B41+0.5*(B9^2)*(B15^2)*B43+B12*B19*B40+B13*B15*B42-B14*B16*B44-B11*B35+(B3-B19)*(1-B18)+B16*B18
Δ F1 C	B29*B38*((B19^(B29-1))*(B15^B30)*(B16^B31))-(1-B18)/(B11-B12)
Γ F1 C	B29*(B29-1)*B38*((B19^(B29-2))*(B15^B30)*(B16^B31))
Δ F1 S	B30*B38*((B19^B29)*(B15^(B30-1))*(B16^B31))
Γ F1 S	B30*(B30-1)*B38*((B19^B29)*(B15^(B30-2))*(B16^B31))
Δ F1 D	B31*B38*((B19^B29)*(B15^B30)*(B16^(B31-1)))+1*B18/(B11+B14)

The depreciation threshold level can be expressed as an asset age, $\hat{T}_1$, that is: $\hat{T}_1 = \frac{1}{\theta_D} \ln\left(\frac{D_I}{\hat{D}_1}\right)$.

	A	B	C	D	E	F	G	H	I
1	**Multi-factor Multiple Replacement Option with Salvage & Depreciation**								
2	**INPUT**	Stochastic	S & C & Deterministic D			Table 1			
3	PI	30.00	30.00	30.00	30.00	30.00	30.00	30.00	
4	CI	10.00	10.00	10.00	10.00	10.00	10.00	10.00	
5	K	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
6	SI	60.00	60.00	60.00	60.00	60.00	60.00	60.00	
7	DI	10.00	10.00	10.00	10.00	10.00	10.00	10.00	
8	σC	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
9	σS	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
10	ρ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
11	r	0.07	0.07	0.07	0.07	0.07	0.07	0.07	
12	θC	0.040	0.040	0.040	0.040	0.040	0.040	0.040	
13	θS	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	
14	θD	0.10	0.10	0.10	0.10	0.10	0.10	0.10	
15	S*	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
16	D*	1.35	1.35	1.35	1.35	1.35	1.35	1.35	
17	T*	20.0	20.0	20.0	20.0	20.0	20.0	20.0	
18	τ	0.30	0.30	0.30	0.30	0.30	0.30	0.30	
19	**C**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	
20	**OUTPUT**		SOLVER: SET I28=0, CHANGING B29:H32						
21	Q(η,γ,λ)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 3
22	SP1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 6
23	SP2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 7
24	VM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 8
25	PART 1	467.38	467.38	467.38	467.38	467.38	467.38	467.38	
26	PART 2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
27	PART 3	315.69	315.69	315.69	315.69	315.69	315.69	315.69	EQ 8 RHS
28	SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	η1	1.3923	1.3923	1.3923	1.3923	1.3923	1.3923	1.3923	
30	γ1	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	
31	λ1	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	
32	C*	27.889	27.889	27.889	27.889	27.889	27.889	27.889	
33	NPV_1	-70.95	-117.61	-164.28	-210.95	-257.61	-304.28	-350.95	Ex K
34	**ROV_1**	**215.62**	**254.04**	**294.18**	**335.93**	**379.19**	**423.89**	**469.97**	
35	F_1	144.67	136.43	129.90	124.98	121.58	119.62	119.02	EQ 2
36	ROV EX	B38*((B4^E	119.02	119.02	119.02	119.02	119.02	119.02	
37	NPV EX	-297.63	-297.63	-297.63	-297.63	-297.63	-297.63	-297.63	
38	A_1	4.147	4.147	4.147	4.147	4.147	4.147	4.147	
39	PDE	0.00	0.00	0.00	0.00	0.00	0.00	0.00	EQ 1
40	Δ F1 C	-4.57	-3.68	-2.85	-2.07	-1.34	-0.63	0.04	
41	Γ F1 C	0.46	0.43	0.40	0.38	0.36	0.34	0.33	
42	Δ F1 S	0.32	0.38	0.44	0.50	0.57	0.63	0.70	
43	Γ F1 S	-0.02	-0.02	-0.02	-0.02	-0.03	-0.03	-0.03	
44	Δ F1 D	2.33	2.44	2.54	2.65	2.77	2.89	3.01	

Note C is shown in a range of 16 to 28, which includes a level which is slightly above the C that justifies replacement, when then the intrinsic value upon replacement is shown. The NPV is also shown, the last three terms of equation 2, (assuming P_I=30> C), assuming the current revenue is P_I. The last few rows show that given the calculated deltas and gammas (first and second derivatives of equation 2) equation 1 is solved.

Figure 1



The operating cost threshold levels $\hat{C}_1$ are evaluated for the assumed salvage threshold level $\hat{S}_1$ =20 and time threshold level $\hat{T}_1$ =20 years, by solving (3), (6), (7), and (8). ROV_1, F_1 and NPV_1 are from (2).

The ROV_1 increases as the current operating costs increases, since the replacement option value nears exercise, but the NPV_1 declines, so the overall function F_1 declines, shown in Figure 1.

3.2 Single Opportunity C-S-D Model

The real option values for the single opportunity C-S-D (1s) model are illustrated in Table 2. The real option values increase as the current operating costs increase, but the overall function F_{1s} declines, as the NPV_{1s} declines, as shown in Figure 2.

	A	B	C	D	E	F	G	H	I
1	**Multi-factor Single Replacement Option with Salvage & Depreciation**								
2	**INPUT**	Stochastic	S & C & Deterministic D			Table 2			
3	PI	30.00	30.00	30.00	30.00	30.00	30.00	30.00	
4	CI	10.00	10.00	10.00	10.00	10.00	10.00	10.00	
5	K	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
6	SI	60.00	60.00	60.00	60.00	60.00	60.00	60.00	
7	DI	10.00	10.00	10.00	10.00	10.00	10.00	10.00	
8	σC	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
9	σS	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
10	ρ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
11	r	0.07	0.07	0.07	0.07	0.07	0.07	0.07	
12	θC	0.040	0.040	0.040	0.040	0.040	0.040	0.040	
13	θS	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	
14	θD	0.10	0.10	0.10	0.10	0.10	0.10	0.10	
15	S*	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
16	D*	1.35	1.35	1.35	1.35	1.35	1.35	1.35	
17	T*	20.0	20.0	20.0	20.0	20.0	20.0	20.0	
18	τ	0.30	0.30	0.30	0.30	0.30	0.30	0.30	
19	**C**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	
20	**OUTPUT**		SOLVER: SET I27=0, CHANGING B28:H31						
21	Q(η,γ,λ)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 3s
22	SP1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 6s
23	SP2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 7s
24	VM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 9
25	PART 1	788.25	788.25	788.25	788.25	788.25	788.25	788.25	
26	PART 3	315.69	315.69	315.69	315.69	315.69	315.69	315.69	EQ 9 RHS
27	SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	η1	1.3806	1.3806	1.3806	1.3806	1.3806	1.3806	1.3806	
29	γ1	0.0178	0.0178	0.0178	0.0178	0.0178	0.0178	0.0178	
30	λ1	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	
31	C*	46.640	46.640	46.640	46.640	46.640	46.640	46.640	
32	NPV$_1$	-70.95	-117.61	-164.28	-210.95	-257.61	-304.28	-350.95	Ex K
33	**ROV$_1$**	**179.96**	**211.74**	**244.89**	**279.33**	**314.99**	**351.79**	**389.69**	
34	F$_1$	109.02	94.13	80.61	68.39	57.38	47.51	38.75	EQ 2
35	ROV EX	98.69	98.69	98.69	98.69	98.69	98.69	98.69	
36	NPV EX	-297.63	-297.63	-297.63	-297.63	-297.63	-297.63	-297.63	
37	A$_1$	3.710	3.710	3.710	3.710	3.710	3.710	3.710	
38	PDE	0.00	0.00	0.00	0.00	0.00	0.00	0.00	EQ 1
39	Δ F1 C	-7.80	-7.09	-6.43	-5.80	-5.21	-4.65	-4.12	
40	Γ F1 C	0.37	0.34	0.32	0.30	0.29	0.27	0.26	
41	Δ F1 S	0.16	0.19	0.22	0.25	0.28	0.31	0.35	
42	Γ F1 S	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	
43	Δ F1 D	2.05	2.10	2.15	2.20	2.26	2.32	2.38	

Real option values are evaluated using the base parameter values, for the salvage threshold level $\hat{S}_{1s}$ =20 and time threshold level $\hat{T}_{1s}$ =20, by solving (3s), (6s), (7s), and (8s) for η_{1s}, γ_{1s} and λ_{1s}. ROV$_{1s}$, F$_{1s}$ and NPV$_{1s}$ are from (2), assuming the current revenue is P$_I$.

Figure 2

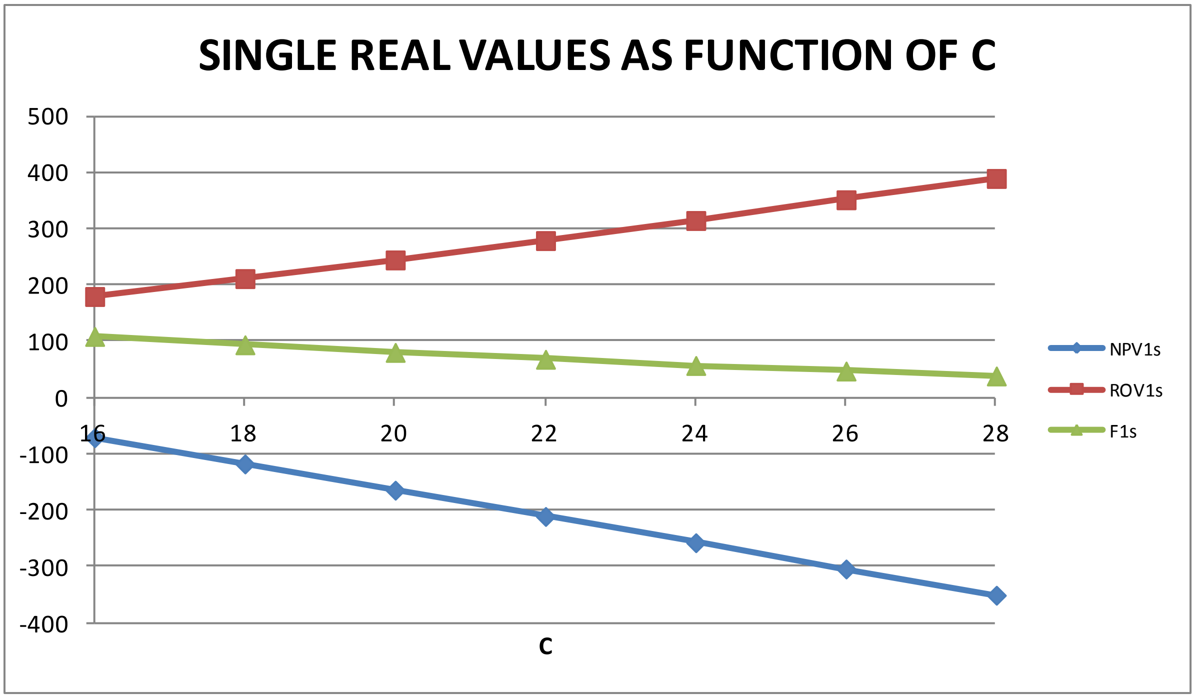
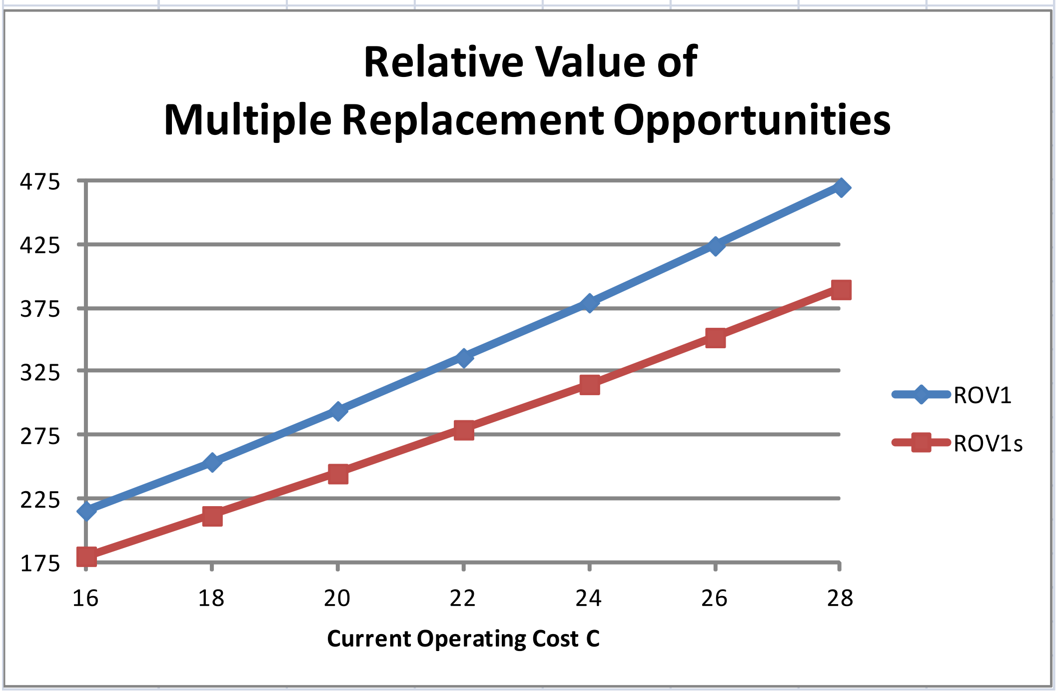


Figure 3

C	16	18	20	22	24	26	28
ROV_1	215.62	254.04	294.18	335.93	379.19	423.89	469.97
ROV_{1s}	179.96	211.74	244.89	279.33	314.99	351.79	389.69
ROV M-S	**35.65**	**42.30**	**49.29**	**56.59**	**64.20**	**72.10**	**80.27**



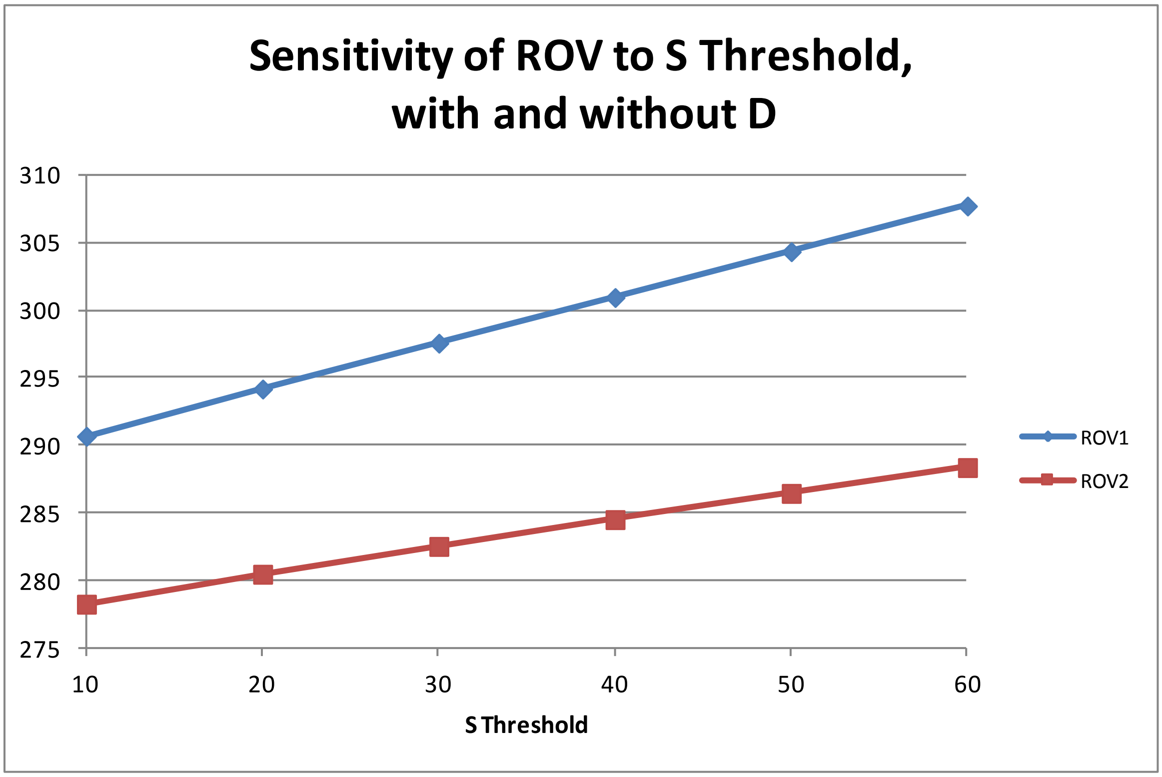
Real option values are evaluated using the base parameter values, for the salvage threshold level $\hat{S}_{1s}$ =20 and time threshold level $\hat{T}_{1s}$ =20, by solving (3s), (6s), (7s), and (8s) for η_{1s} , γ_{1s} and λ_{1s} . ROV_{1s} and ROV_1 are from (2).

NPV_1=NPV_{1s}, but ROV_1>ROV_{ls}, so F_1>F_{1s}. It is valuable to have multiple replacement opportunities, which also motivates more frequent replacements with lower C thresholds as shown in Figure 3.

3.3 The C-S Model

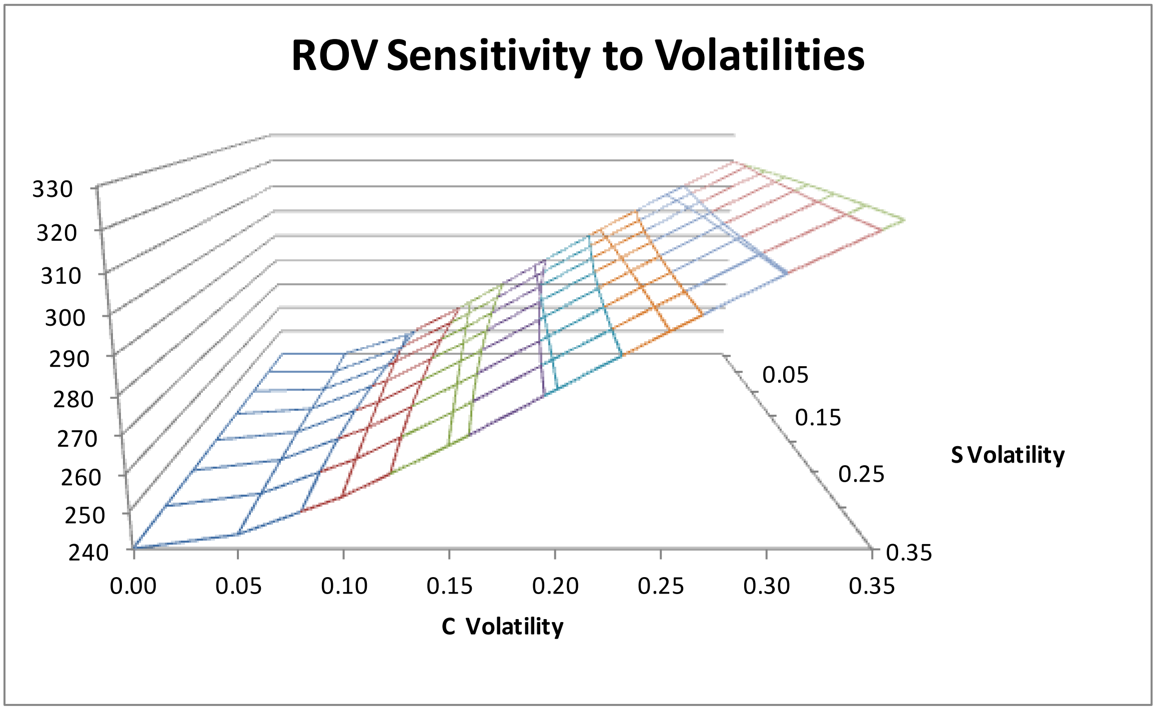
A model which does not consider depreciation (but considers taxation on operating profits and salvage value gains) is likely to have lower real option replacement values with fewer factors to consider in the replacement decision. This case is illustrated in Figure 4, which shows the ROV_1 for the C-S-D model compared to the ROV_2 for the C-S model as the S threshold increases.

Figure 4



3.4 Volatility

Figure 5

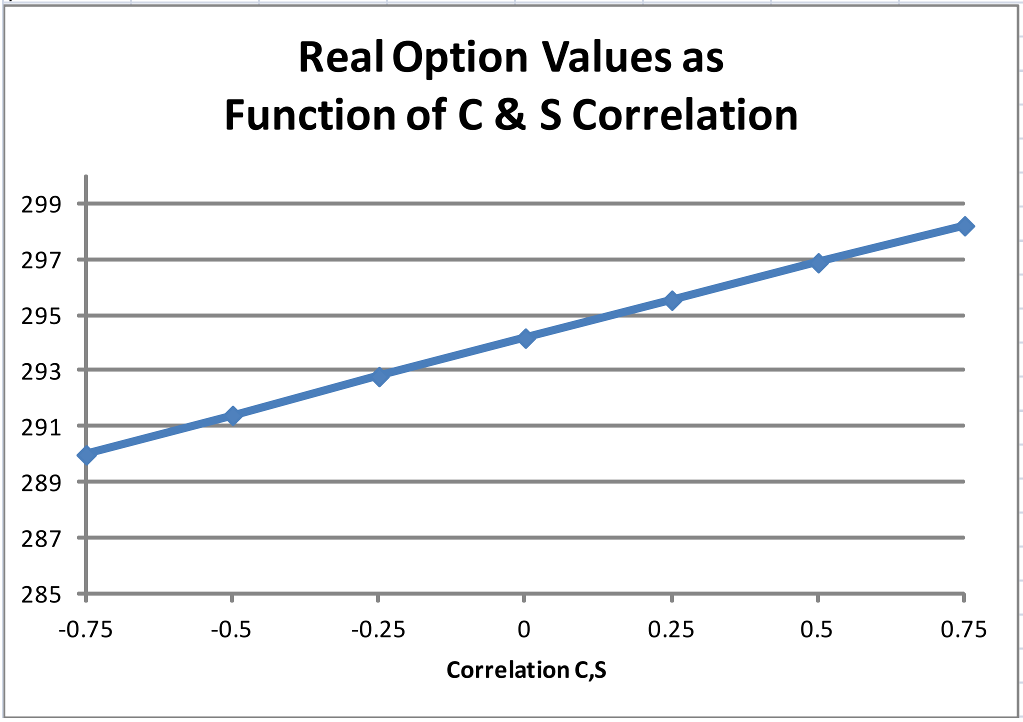


The real option values are highly sensitive to changes in expected operating cost volatilities, but not to changes in expected salvage value volatilities, if the correlation between C and S is not high as shown in Figure 5.

3.5 Correlation

Figure 6

ROV1	289.99	291.40	292.80	294.18	295.54	296.88	298.21
NPV1	-164.28	-164.28	-164.28	-164.28	-164.28	-164.28	-164.28
F1	125.71	127.12	128.52	129.90	131.26	132.61	133.93
ρ	-0.75	-0.5	-0.25	0	0.25	0.5	0.75

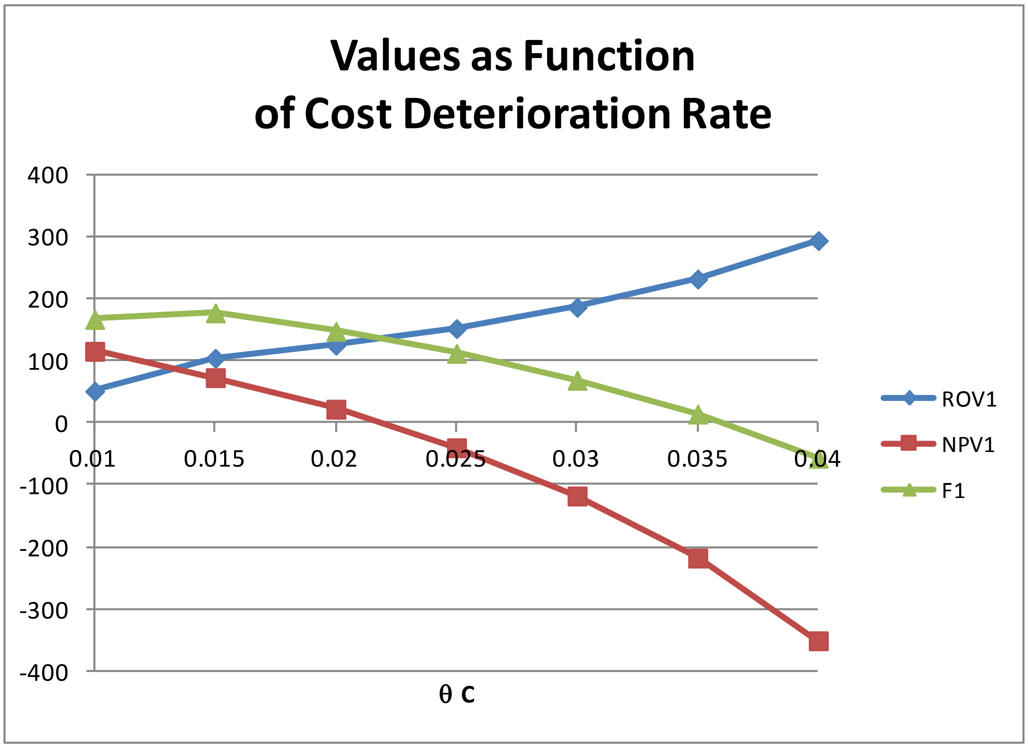


The changes in the real option value shown in Figure 6 are almost a linear function of increases of the correlation of C and S, but the effect is not very significant. NPV_1 does not change, so F_1 just increases slightly as the correlation moves from highly negative to highly positive. This is perhaps an example where the assumption of risk neutrality is not realistic, since there is a weak "natural hedge" if S changes are not related to C changes, as in some ship values over certain times.

3.6 Cost Deterioration

Figure 7 shows that the effect of changes in cost deterioration rates is quite dramatic, with the real option replacement value increasing substantially as the rate of C deterioration increases, but the NPV_1 declines. So the net effect is that F_1 declines, showing that cost deterioration is not favourable for the CROM, even if the ROV_1 increase offsets some of the NPV_1 decline[4].

Figure 7



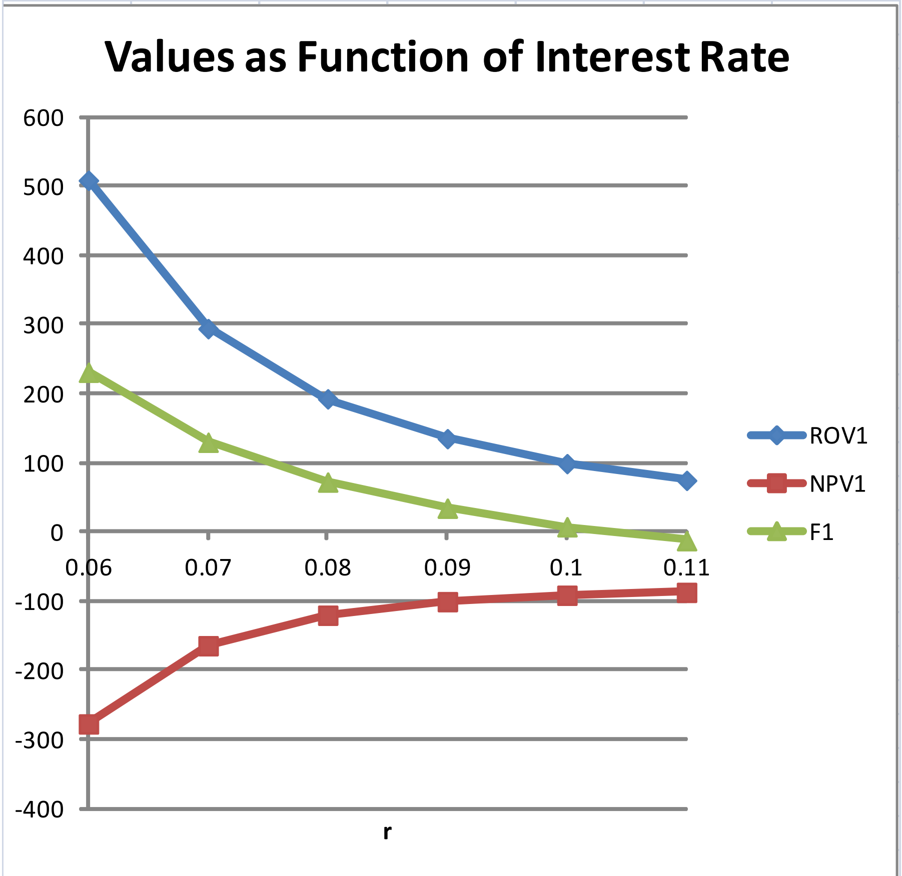
3.7 Discount Rate

Figure 8

[44] There is a constraint in the model that $\theta_C<r$, which also applies to Figure 8.

.

ROV1	508.61	294.18	192.62	135.10	98.98	74.75
NPV1	-277.46	-164.28	-120.24	-100.53	-91.30	-87.16
F1	231.15	129.90	72.38	34.57	7.68	-12.41
r	0.06	0.07	0.08	0.09	0.1	0.11

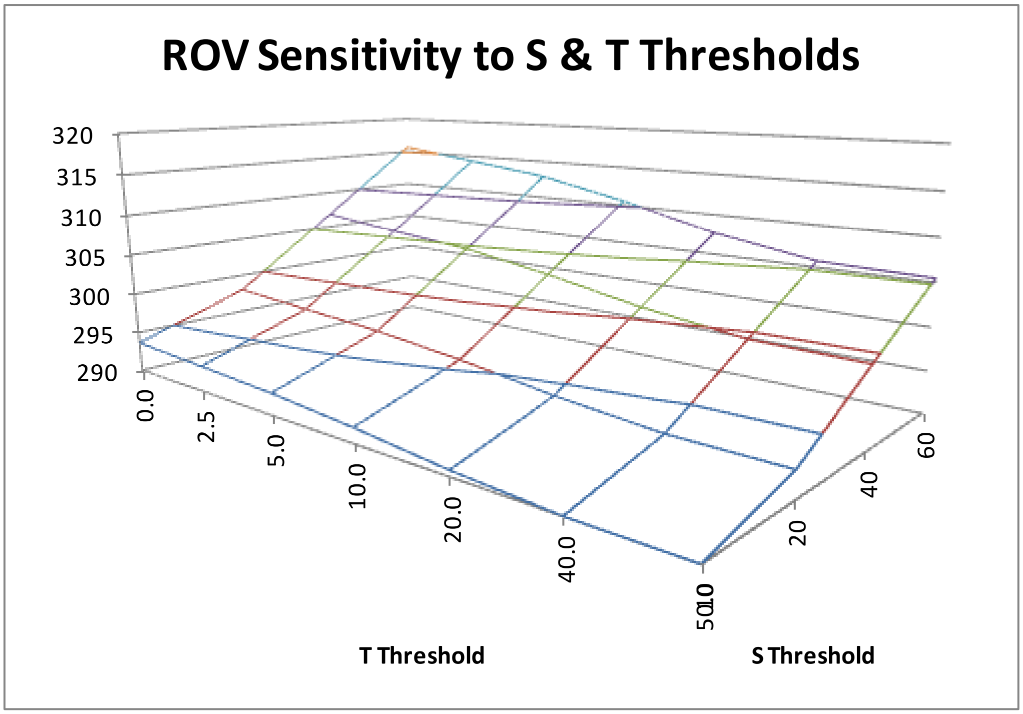


The effect of changes in interest rates is also quite dramatic, but in the opposite direction. The real option replacement value decreases substantially (as ROV_1 is required to "earn" a higher return), but the NPV_1 increases as the discount on C increases as in Figure 8. The net effect is that F_1 declines, showing that interest rate increases are not favourable for the CROM, even if the ROV_1 decrease is offset by some of the NPV_1 increase.

3.8 $\hat{S}$ and $\hat{T}$

One would expect that the real replacement option value would increase as the level of the salvage value increases, as shown in Figure 9, but the ROV_1 is not very sensitive to decreases in the level of depreciation, here proxied as the asset age.

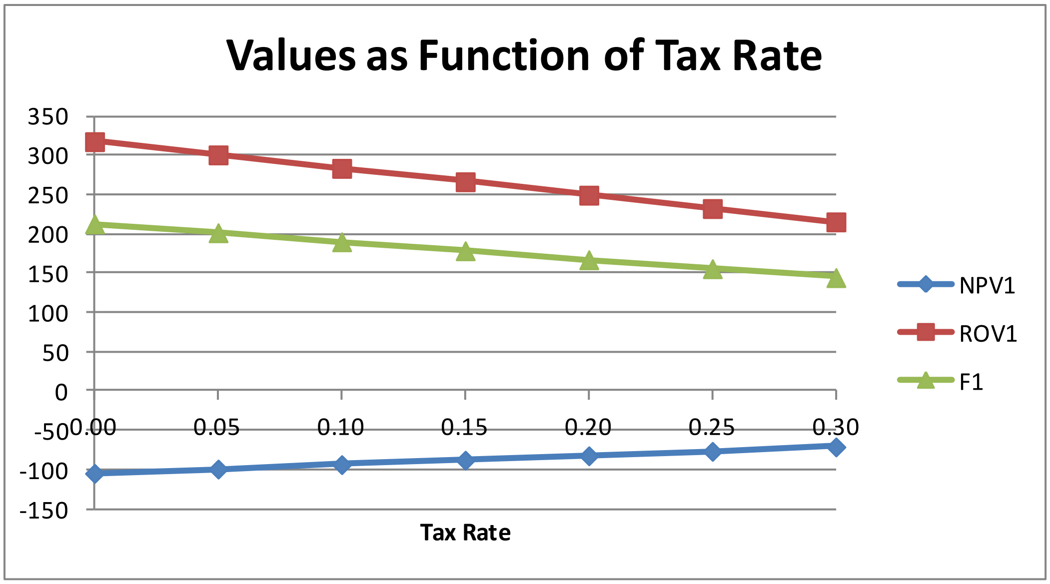
Figure 9



3.9 Tax Rate

It is assumed that the owner of the replaceable asset has taxable income, so the net revenue is subject to corporation tax, along with the salvage value gain/loss, relieved by the allowable depreciation charge. Figure 10 shows the effect of increases in the tax rate from 0 to 30%, assuming that C=16, so there is current taxable income if $P=30=P_I$. Note that increases in the tax rate reduce the ROV_1 but increase the NPV_1 at these parameter values, so the net effect on F_1 is tax increases do not benefit the asset owners even with depreciation allowances.

Figure 10

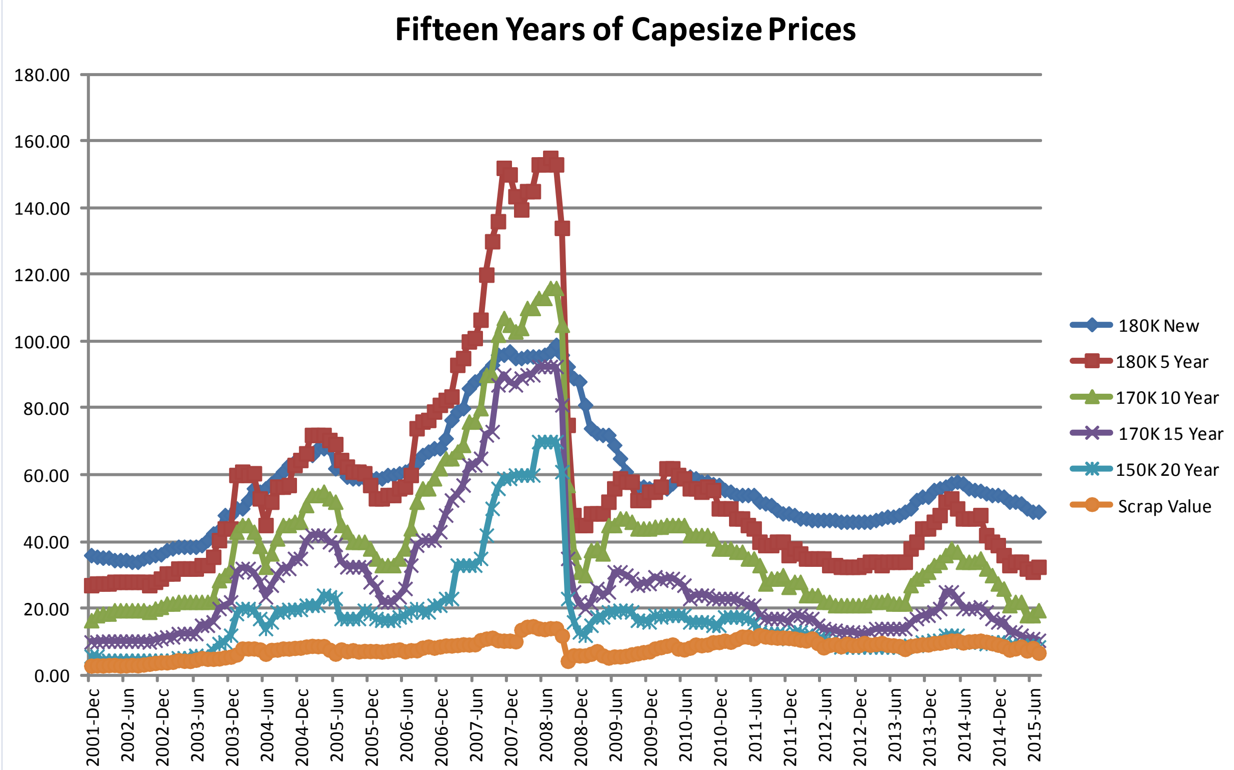


4. **Empirical Evaluation of Some Parameter Values for Dry Bulk Ships**

From a time series database of monthly dry bulk ship market values for the last fifteen years, it is possible to estimate a reasonable range for some Table 1 parameter values for a particular industry. Although new build and second-hand prices follow a relatively similar pattern, the new build price seems to be less exposed to shocks due to freight rate volatility compared to relatively young second-hand ships. New build prices are heavily dependent on new build costs, primarily steel. Of primary concern is how volatile are operating costs and scrap values, are these correlated, and what is a reasonable expectation for the deterioration rates of operating costs over time and usage/age. A first observation from Figure 11 is that ship values are highly volatile, but the distribution of volatility by ship age is partly dependent on whether the "abnormal boom years" 2006-2008 are considered. During that period, 5 and 10 year ships were more valuable than new buildings, since building typically takes a couple of years. But in general ship values decline with age, the mean shown in the MEAN row, and July 2015 prices by age in the previous row.

Figure 11

	CAPESIZE PRICES					
	180K New	180K 5 Year	170K 10 Year	170K 15 Year	150K 20 Year	Scrap Value
2015-Jul	**49.00**	**32.50**	**19.50**	**10.50**	**8.50**	**6.78**
MEAN	58.45	56.77	41.21	28.88	17.87	8.20
STDEV	16.40	31.15	23.78	21.01	14.59	2.62



In order to estimate cost deterioration over time and age, an imperfect proxy is "effective market deterioration" (EMD) defined as $\ln(\frac{5yearvalue}{K})/5$ =% per annum for 5 year old ships, where K is the new build price. Note that this operating cost deterioration sharply rises with ship age, but this proxy surely is contaminated with elements other than operating costs (especially for 20 year ships with the implicit abandonment option value). Note that the EMD for 5 year ships is positive during the 2006-2008 boom years, and is not "well behaved" for 20 year ships. The standard straight-line accounting depreciation over 25 years with some assumed scrap value (in this case the average scrap value over the 15 year period) in the last column does not reflect this eccentric price behaviour of the past shown in Figure 12.

Figure 12

	EFFECTIVE MARKET DETERIORATION					
	New-5 Year	5-10 Year	10-15 Year	15-20 Year	20 Year-Scrap	SL DEP
2015-Jul	-3.42%	-4.26%	-5.16%	-1.76%	-1.88%	-7.15%
MEAN	-2.09%	-6.75%	-8.44%	-10.36%	-12.13%	-7.71%
STDEV	4.63%	1.57%	2.69%	3.70%	10.06%	1.07%

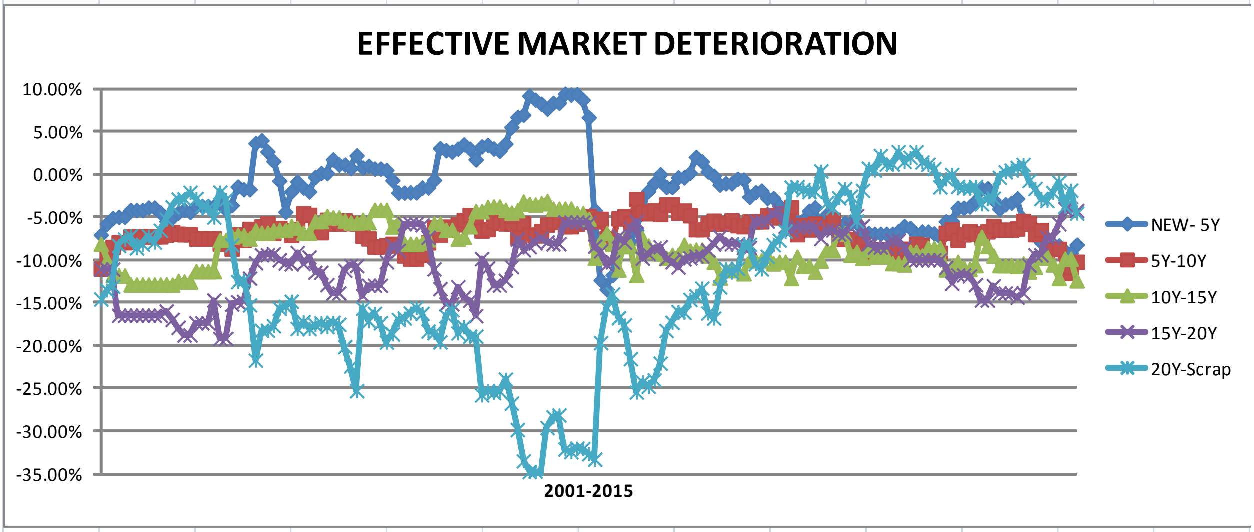


Table 3 shows the historical volatility of ship prices by age over the last fifteen years, and the correlation of different aged ships with scrap values.

Table 3

	Annualized Return					
	180K New	180K 5 Year	170K 10 Year	170K 15 Year	150K 20 Year	Scrap Value
MEAN	2.06%	1.63%	1.92%	0.36%	2.86%	**6.53%**
STDEV	9.08%	28.41%	32.58%	37.08%	**42.88%**	**38.06%**
MAX	118.13%	372.19%	354.56%	411.53%	519.44%	382.14%
MIN	-110.85%	-696.42%	-733.09%	-1006.92%	-1170.46%	-1209.57%
COUNT	164	164	164	164	164	164

Correlation Matrix

180K 5 Year	0.3433				
170K 10 Year	0.3010	0.8795			
170K 15 Year	0.3505	0.8456	0.8448		
150K 20 Year	0.2666	0.8084	0.7672	0.8025	
Scrap Value	0.1218	0.4289	0.4335	0.5629	**0.4947**

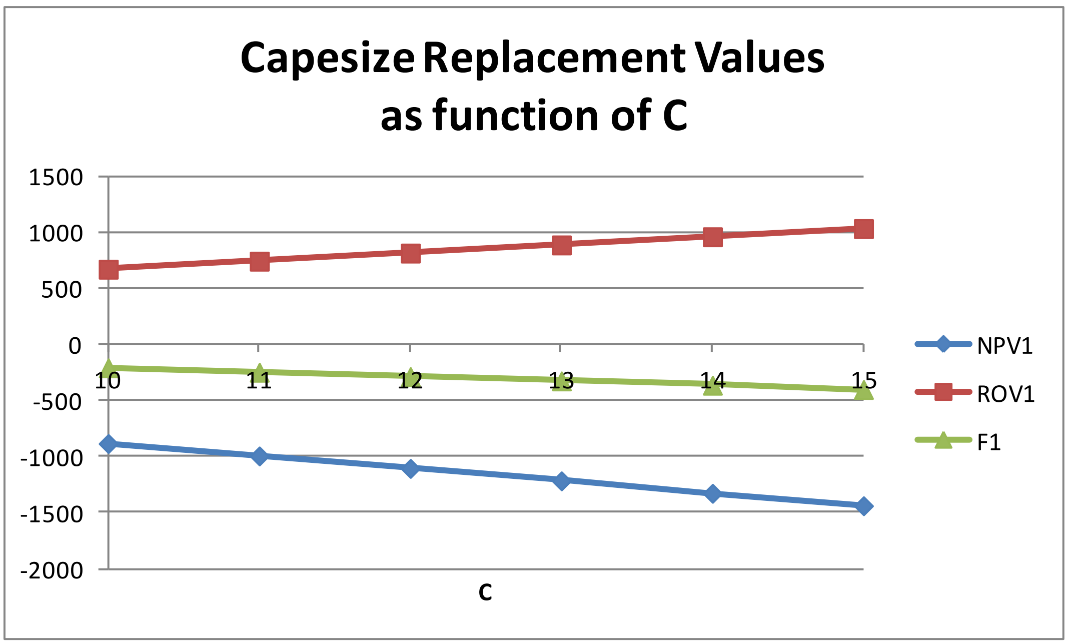
New building ships are not highly correlated with ships of any age, or with scrap value. The volatility of ship values increases with age, as does the correlation with scrap values.

Table 4

	A	B	C	D	E	F	G	H
1	**Multiple Replacement Option with Salvage & Depreciation**							
2	**INPUT**	Stochastic S & C & Deterministic D						
3								
4	PI	16.00	16.00	16.00	16.00	16.00	16.00	Estimate
5	CI	9.80	9.80	9.80	9.80	9.80	9.80	.2K
6	K	49.00	49.00	49.00	49.00	49.00	49.00	F9 July 15
7	SI	8.20	8.20	8.20	8.20	8.20	8.20	F9 Mean
8	DI	4.90	4.90	4.90	4.90	4.90	4.90	
9	σC	0.43	0.43	0.43	0.43	0.43	0.43	T3
10	σS	0.38	0.38	0.38	0.38	0.38	0.38	T3
11	ρ	0.49	0.49	0.49	0.49	0.49	0.49	T3
12	r	0.07	0.07	0.07	0.07	0.07	0.07	
13	θC	0.061	0.06	0.06	0.06	0.06	0.06	F9 M EMD/2
14	θS	-0.065	-0.07	-0.07	-0.07	-0.07	-0.07	T3 July 15
15	θD	0.10	0.10	0.10	0.10	0.10	0.10	
16	S*	6.78	6.78	6.78	6.78	6.78	6.78	F9 July 2015
17	D*	0.35	0.35	0.35	0.35	0.35	0.35	
18	**C**	**10**	**11**	**12**	**13**	**14**	**15**	
19	**OUTPUT**		SOLVER: SET H27=0, CHANGING B28:G31					
20	Q(η,γ,λ)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 3
21	SP1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 6
22	SP2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 7
23	VM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 8
24	PART 1	1675.95	1675.95	1675.95	1675.95	1675.95	1675.95	
25	PART 2	0.60	0.60	0.60	0.60	0.60	0.60	
26	PART 3	1109.07	1109.07	1109.07	1109.07	1109.07	1109.07	EQ 8 RHS
27	SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	η1	1.0585	1.0585	1.0585	1.0585	1.0585	1.0585	
29	γ1	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	
30	λ1	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	
31	C*	15.966	15.966	15.966	15.966	15.966	15.966	
32	NPV_1	-880.48	-991.59	-1102.70	-1213.81	-1324.93	-1436.04	Ex K
33	**ROV_1**	**675.86**	**747.61**	**819.74**	**892.22**	**965.03**	**1038.14**	
34	F1	-204.62	-243.98	-282.97	-321.60	-359.90	-397.90	EQ 2
35	A1	58.663	58.663	58.663	58.663	58.663	58.663	
36	NPV_1	B4/B12-B18/(B12-B13)+B17/(B12+B15)						
37	**ROV_1**	**B34-B32**						
38	F1	IF(B18<B31,B35*((B18^B28)*(B16^B29)*(B17^B30))+B32,B89)						
39	A1	B26/((B31^B28)*(B16^B29)*(B17^B30))						

The basis of some of the parameter values is described in column H in Table 4. The reversionary operating costs are a function of the new build price (K), the operating cost deterioration is at the July 2015 EMD rate (even though that price deterioration consists of more than increases in operating costs), the current operating costs in row 18 are above the assumed reversionary costs, and the revenue row 4 is arbitrarily determined. Of course, operating shipping companies would have better estimates of all of these inputs.

Figure 13



However, given these inputs, the current assumed operating costs are less than the cost which justifies current replacements. The operating NPV are negative, due to the assumed discount rate for operating costs even when there is a positive spread between current revenue and operating costs. The replacement option values for ships at all operating costs are significant if treated as perpetuities with multiple replacement opportunities and no competition, so the overall value function is positive in Figure 13. F_1 appears to decrease as operating costs increase, but this is partly dependent on the more or less arbitrary assumptions as to current revenue and operating costs. Any ship operator CROM can improve on these inputs, and probably on the interpretation of the outputs. Should the value function F_1 be considered the equity value of an alert ship operator with opportunities to replace repeatedly her ships at these newbuilding prices?

5. Conclusion

We apply a quasi-analytical method to find the after-tax timing boundary and real option value for replacing an incumbent asset when both its operating cost and salvage value deteriorate and are stochastic. It is intuitive that increases in operating costs enhance the real option value of the opportunity to replace assets. But increases in the ROV_1 at best offset some of the loss in NPV_1 from C increases, so the net effect is just to modify the overall "replaceable asset" value function F_1. Clearly the opportunities to replace assets multiple times are more valuable than either no replacement opportunity, or a single opportunity. Surprisingly, not considering depreciation results in lower real option values (although this may be a problem with a model which taxes operating profits and salvage values but does not allow for taxation relief on investment expenditures).

There are some assumptions which are logical for the CROM to encourage, such as reducing C (Figure 1), increasing the number of replacement opportunities (Figure 7), reducing the rate at which operating costs deteriorate (Figure 6), or the general level of interest rates (Figure 8), all of which increase the overall value function F_1, either through the ROV_1 and/or the NPV_1. Similarly the CROM should welcome circumstances where the salvage value is high (Figure 9), which results in increases in both the ROV_1 and the value function F_1. However, increases in cost volatility which result in higher real option replacement values is not altogether logical (Figure 5). The "vegas" (increases in call option value as expected future volatility of the underlying asset increases) are positive for traded financial options, where the holder does not necessarily own a stake in the underlying asset. Positive vegas for replaceable assets are perhaps another matter. There is no allowance in these models for risk aversion, but some might believe that at least the discount rate should be increased for risky operating costs. Finally, selling direct or indirect interests in these replaceable asset options has not been considered, perhaps because it is not obvious that the option value can be detached from the asset, perhaps awaiting innovations from real financial engineers.

There are further qualifications in the proposed replacement methodology, and analysis: investment costs are considered constant or deterministic; replacements are assumed to be identical so not allowing for technical innovation; no account has been given to alternative evolutionary processes; the possibility of sudden failure has been ignored; the replacement decision for the asset under consideration is examined in isolation from the other assets of the firm; alternatives from infinite multiple to single replacements have not been explicitly considered; competition among firms has been ignored; and no empirical comparisons have been made with actual replacement decisions, for specific firms or industries. Replacing the assumption that these replaceable assets are perpetuities with finite annuities with declining cash flows over time is not a problem, but simultaneously determining the annuity age and the optimal replacement times is problematical. Further research is required to investigate these matters, to examine the feasibility of a quasi-analytical method for overcoming these shortcomings, such as stochastic investment costs, technological innovation and/or failure, and strategic considerations. The possibility of revealing new insights in optimal replacement policy is at the risk of raising model complexity and lowering transparency.

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