Appraising a Portfolio of Interdependent Physical and Digital Urban Infrastructure Investments: A Real Options Approach

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Abstract

Massive capital investment is required into both existing and new urban infrastructures in order to address the historically unprecedented challenges faced by many cities around the world. However, traditional methods of appraisal and evaluation are widely regarded as inadequate since they do not correctly take into account the various sources of uncertainty nor the multiple interdependencies among investment projects. This paper presents two new real options-based appraisal frameworks for selecting a portfolio of physical and digital urban infrastructure investment projects: the first approach considers (strategic) interdependencies between real options within single investment projects but not between projects in the portfolio, whereas the second approach additionally takes into account interdependencies between urban infrastructure investment projects in the portfolio; the interdependencies considered are physical, cyber, geographical, and logical (resource and market, strategic, and budget). Representing the decision makers flexibilities through influence diagrams, we have used these two frameworks to formulate dynamic programming-based valuation problems that can be efficiently solved numerically by applying the least squares Monte Carlo approach. We expect our new frameworks to have substantial potential to enhance investment decisions, particularly with regard to timing, scale, and project selection, thus potentially creating significant value for investors. Future work will comprehensively evaluate the comparative performance of traditional and our two new approaches under a wide range of real-world case studies.

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1. Introduction

Today's cities face historically unprecedented challenges in managing their transition towards a sustainable, low-carbon future. In order to address present and future urban challenges, massive capital investment is required in both existing and new infrastructure systems in the so-called "urban silos" comprising energy, transport, water, waste, ICT, real estate, and others (Della Croce, 2012). It has been estimated that cumulative investments of at least USD 40 trillion will be required globally during the period 2005-2030 (Doshi et al., 2007; Ottesen, 2011) to modernise existing and build new urban infrastructures. These include investments in electricity systems (generation, transmission, and distribution), sewage treatment plants, public transportation systems, district heating networks, telecommunication systems, and others.

Besides investing in such traditional technologies, significant investments are expected towards new technologies and services collectively known-as "smart city technologies", such as wireless sensor networks, smart meters (e.g. for electricity, water, gas, etc.), and intelligent transport systems. Pike Research (2011) forecast global investment into smart city technologies – comprising the utilities, transport, building, and government sector – to total USD 108 billion between 2010 and 2020, with annual spending forecast to reach almost USD 16 billion by 2020. In another report, BIS (2013a,b) present a market assessment of smart city solutions in water, waste, energy, transport, and assisted living, and estimate the global market for such solutions including the services required for their deployment at USD 408 billion by 2020. Yet another study (MarketsandMarkets, 2014), considering an even wider area of application additionally including building automation, healthcare, education, and security, expects the global smart cities market to grow to some USD 1,266 billion by 2019.

Regardless of the actual amount to be invested, further investments in both smart and traditional urban infrastructures will have to be made in the context of enormous uncertainties. This includes the rather "typical" investment risks related to construction, operation, as well as costs and benefits, but also a number increased risks for investors since: technologies are often new, complex and unproven; the technologies' potential market success is generally difficult to predict; and the business case is often difficult to assess. In addition to these uncertainties and given the increasing vertical and horizontal integration of urban systems, the correct appraisal of investments into both physical and digital infrastructure will need to take into account multiple interdependencies and inter-linkages that potentially exist among systemic urban infrastructures.

However, traditional methods of appraisal and evaluation such as the ones based on simple temporal discounting or on standard option valuation models are widely regarded as inadequate since they do not correctly take into account the various sources of uncertainty nor the multiple interdependencies among investment projects. This paper introduces a real options based portfolio approach to these challenges that systematically incorporates potentially multiple uncertainties influencing the performances of the investments and multiple interdependencies both between real options within an investment project and between projects in a portfolio. We expect our new framework to have substantial potential to enhance investment decisions, particularly with regard to timing, scale, and project selection, thus potentially creating considerable economic value for investors.

Section 2 discusses the existing literature on real options in the appraisal of infrastructure investments and other works related to the research presented in this paper. Section 3 presents the approach taken in this paper to the appraisal of a portfolio of independent and of a portfolio interdependent urban infrastructure investment projects. The valuation method used to numerically solve these two valuation problems is presented in Section 4. We discuss our approach and provide some concluding remarks in Section 5.

2. Literature review

In this section, we review literature on real options in the appraisal of infrastructure investments, modelling approaches for infrastructure interdependencies and valuation methods for portfolios of real options.

2.1. Infrastructure investment appraisal

When compared with many other areas of applications, Real Options Analysis (ROA) has not been widely applied yet in the area of infrastructure investment appraisal (Garvin and Ford, 2012) and, as Gil and Beckman (2009) pointed out, applying ROA to infrastructure design "is still in its infancy". With regard to infrastructure design, Zhao and Tseng (2003) appraised flexible design alternatives for the construction of public parking garages. Arguing with the inappropriateness of complex option valuation techniques, De Neufville et al. (2006) proposed a simple spreadsheet approach for the valuation of the flexibility incorporated in the design of a parking garage. Another early study (Gil, 2007) on infrastructure design investigated the effects of modularization – i.e. product design modularity – when assessing safeguarding investments as part of airport expansions programmes. Garvin and Cheah (2004) applied options pricing on a case study of a toll road project to comparatively evaluate the project's economic viability under the NPV and options approach, thereby demonstrating the superiority of the latter by being able to capture strategic considerations (deferment option). A few years earlier but still considering a toll road infrastructure project, Rose (1998) valued complex interacting real options that represent contractual agreements by using Monte Carlo simulation. With regard to urban systems, investments into urban transportation infrastructure have been considered by Saphores and Boarnet (2004), whose modelling approach took into account the impact of the variation of a city's population on land rents and prices as well as on transportation costs.

In addition to the above, a number of papers have dealt with issues related to the provision and ownership of infrastructure systems. In the light of different forms of private sector participation arrangements such as PPPs, PFIs, and BOTs, Cheah and Garvin (2009) discussed the potential application of ROA in infrastructure projects, noting that such projects are (naturally or intentionally) "ripe with flexibility" with typical options being call, put, switching, timing, compound, and learning options. Ho and Liu (2002) proposed a quantitative model based on real options theory – considering both construction cost and cash flow risks – to evaluate the economic viability of privatised (BOT) infrastructure projects from the perspective of both the government and the project promoter. Having stressed the dominance of private over market risks in most infrastructure projects, Cheah and Liu (2006) investigated the case of the Malaysia-Singapore Second Crossing and developed a methodology to value governmental support in BOT infrastructure projects by modelling the government guarantee as a put option and the potential repayment (i.e. a cap on the return of the private sponsor) from the private sector participant to the government as a call option. More general, Chiara et al. (2007) argued that a revenue guarantee in a BOT infrastructure project can be modelled as a discrete-exercise real option (e.g. European, Bermudan, or Australian), while noting that currently applied valuation approaches, such as the one used by Cheah and Liu (2006), represent the government guarantee as a European styled option, thus modelling a rather "static contract". In order to provide a more flexible way to deal with the associated revenue risk, the authors developed a novel methodology that allows the valuation of "dynamic contracts" based on discrete-time American-type options and solved it numerically by the least squares Monte Carlo (LSM) approach. Alonso-Conde et al. (2007) applied ROA to analyse the contractual terms associated with the case of the PPP of the Melbourne CityLink Project. Krüger (2012) analysed the implications of PPP agreements on the execution of expansion options in road infrastructure.

Besides appraising investments in physical infrastructures, ROA has also been applied in the context of digital infrastructures like information technology (IT) infrastructures. One of the first attempts to link ROA and more broadly options thinking with information systems investments has been presented by Kambil et al. (1991), who recognised the growth options often embedded in such investments. Panayi and Trigeorgis (1998) applied a multi-stage (compound) real options on the case of an IT infrastructure investment faced by CYTA, the state telecommunications authority of Cyprus. Another ROA application on IT investments has been presented by Benaroch and Kauffman (1999), who argued that investments in IT infrastructures generally do not result in immediate expected paybacks, but rather can provide the basis for profitable future investment opportunities. Miller et al. (2004) applied ROA to evaluate the "Korean Information superhighway infrastructure" investment project. Benaroch (2002) stated that real options generally must be intentionally planned in an IT investment project, instead of being "inherently" embedded, and mainly focused on how ROA may be applied to manage the risks involved – particularly functionality and organisational ones – in an IT investment project. Furthermore, the author claimed that there currently exists a number of gaps between real options theory and what is required to adequately model and appraise real-world IT investments. One of these, the need to formulate and model a "custom-tailored analytical valuation model" in situations with more than two sources of risk involved concurrently, has been tackled by Kumar (2004). The author developed a novel general framework based on the "asset valuation" literature to evaluate IT infrastructure investments in the light of multiple sources of uncertainty.

2.2. Infrastructure interdependencies

Even though much research in recent years has focused on the modelling and simulation of interdependent infrastructure systems on a variety of scales and across different infrastructure sectors, the multidisciplinary science of interdependent infrastructures is still relatively immature (Rinaldi, 2004). Since our research is focusing on physical and digital infrastructure systems[1] on an urban scale, and in particular on the incorporation of interdependencies between those systems into the appraisal of investments in such systems, we start with briefly reviewing current modelling approaches and the types of interdependencies considered in these approaches.

[1]An infrastructure system is considered to be a grouping of subsystems and a subset of an "infrastructure" (Rinaldi et al., 2001).

Ouyang (2014) has recently presented a comprehensive review of studies in the field of infrastructure interdependencies and classified modelling approaches (and sub-approaches) into the following types: empirical, agent-based, system dynamic based, economic theory based (input-output or computable general equilibrium), network based (topology-based or flow-based), and "others" comprising hierarchical holographic modelling, petri-net, as well as bayesian network. These types of modelling approaches have been compared by the author with regard to a number of criteria comprising: quantity of input data; accessibility of input data; types of interdependencies; computation costs; maturity; and resilience.

With regard to the "types of interdependencies" criteria, there exist a myriad of types of classification. For example, Zhang and Peeta (2011) proposed a method that considers functional, physical, budgetary, as well as market and economic interdependencies; Eusgeld et al. (2011) developed an interdependency modelling approach and extended the classification of Rinaldi et al. (2001); Mendonça and Wallace (2006) to better describe subsystems in a system-of-systems context by considering input, mutual, co-located, shared, exclusive-or, physical, cyber, geographical, and logical interdependencies; Dudenhoeffer et al. (2006) presented an approach that takes into account physical, geospatial, policy, and informational interdependencies, and Pederson et al. (2006) extended the latter by additionally considering societal interdependencies. However, as previously noted by Ouyang (2014), when compared with other classifications it appears that the "self-contained" classification introduced by Rinaldi et al. (2001) is most suitable to describe the wide-reaching and multidimensional interdependencies among physical and digital urban infrastructure systems. Their definitions following Rinaldi et al. (2001) are:

1. Physical Interdependency: If the state of one infrastructure system is dependent on the material output(s) of another infrastructure system.

2. Cyber Interdependency: If the state of one infrastructure system is dependent on the information transmitted through the information infrastructure.

3. Geographical Interdependency: If a local environment event can create state changes in other infrastructure systems.

4. Logical Interdependency: If the state of one infrastructure system is dependent on the state of another one through a mechanism that is not a physical, cyber, or geographic link.

In addition to the above listed interdependencies, there exist a variety of other types of interdependencies between projects in a portfolio such as the ones often mentioned in the project portfolio management and portfolio selection literature. These include resource, technology, and market interdependencies (Verma and Sinha, 2002) as well as benefit (and/or cost) and outcome interdependencies (Schmidt, 1993). Above all, interdependencies between projects may vary over time and can be available at the same time (time-horizontal) or at different times (time-vertical), see Götze et al. (2015).

2.3. Portfolios of interdependent real options

Several attempts have been made in the last two decades to introduce the notion of "interdependency" into real options models, with approaches focusing on either single projects or portfolios of projects. With regard to single projects, Trigeorgis (1995) reviewed the literature and noted that the recent recognition of real options interdependencies (i.e when values of multiple real options interact), has the potential to widen the applicability of ROA to many practical situations. Wang and De Neufville (2005) stated that real options "on" projects do usually not feature interdependencies, whereas real option "in" projects are complex and interdependent, often even highly interdependent/path-dependent, which rapidly increases the associated computational costs; see the earlier paper (Wang and De Neufville, 2004) by the same authors. One of the first attempts to overcome the restriction to single investments was presented by Childs et al. (1998), who considered a firm that has the opportunity to invest in two (mutually exclusive) projects, more precisely in their development stage, but then only select a single project for implementation.

In the light of these challenges, ROA applied on a portfolio of possibly interdependent projects has recently been considered in a number of fields of application including energy (e.g. Wang and Min (2006) for electric power generation planning) and the pharmaceutical industry (e.g. Zapata and Reklaitis (2010) for R&D portfolio). However, it appears that particularly applications to portfolios of IT investment projects have received considerable attention by academics. For example, Bardhan et al. (2004) modelled a portfolio of IT investment projects, each of which embedding a single option, and proposed a real options portfolio optimisation algorithm that can be used to both prioritise projects and make optimal funding decisions for these projects given limited resources. As an extension to Bardhan et al. (2004), Bardhan et al. (2006) took into account time-wise project interdependencies and formulated the portfolio optimisation problem as a mixed integer programming model. Based on the MAD assumption[2] and a binomial lattice approach, Pendharkar (2010)

[2]Existence of a traded replicated portfolio is unnecessary (Borison, 2005), since the NPV of the investment project without flexibility "is the best unbiased estimate of the project were it a traded

developed a real options model that includes cash flow interdependencies amongst multi-stage IT investment projects. Generalising his own model to enable an application to more than the earlier considered two projects, Pendharkar (2014) proposed a decision-making framework to value an IT project portfolio containing project interdependencies and subsequently solved, as the author claimed, "easily" a project selection problem of 60 dependent projects.

However, there are several limitations inherent to ROA when used in a portfolio context, particularly path-dependency of options, curse of dimensionality, and combinatorial burden (Zapata and Reklaitis, 2010). In order to overcome these limitations, a number of further developments have been proposed in the academic literature. Some of these combine Monte Carlo simulation, introduced to the pricing of European call options by Boyle (1977), with dynamic programming in order to value American (e.g Barraquand and Martineau (1995); Broadie and Glasserman (1997)) and Asian (e.g. Broadie and Glasserman (1996)) styled options; Boyle et al. (1997) provided an overview of recent developments. Despite adding computational complexity, Monte Carlo techniques have significant advantages over traditional option pricing techniques such as analytical and lattice-based methods since they allow the consideration of multiple sources of risk and stochastic variables, multiple underlying assets, real options with complex features, etc. (Pringles et al., 2015).

The practical valuation approach for American options called "Least Squares Monte Carlo" (LSM) method, proposed by Longstaff and Schwartz (2001), has gained considerably attention from researchers in the last few years. Combining least-square regression used to approximate the conditional expectation function of the dynamic programming problem with Monte Carlo simulation of random variables' evolution over time, the LSM method is a simple and efficient numerical technique that can be applied to value complex and compound options, such as multidimensional American real options (Cortazar et al., 2008; Pringles et al., 2015). Besides being used to efficiently value American options, the LSM method can also be applied to value complex real capital investments with many, possibly interacting, embedded real options and in situations with multiple uncertain state variables (Abdel Sabour and Poulin, 2006). The method has been recently assessed and analysed in detail by Stentoft (2004a,b), confirming its computational advantages over other existing numerical methods.

An interesting approach that extends the valuation of individual options to a multi-option setting has been presented by Gamba (2003), who considered indepen-

asset" (Copeland and Antikarov, 2001).

dent, mutually exclusive, compound, and switching options[3]. The aim of the author's approach is to decompose a complex real options valuation problem with potentially multiple interacting options into a set of simple options, which can be combined to describe a wide range of more complex real world situations. Despite providing some interesting insights and, from a methodological perspective, a step forward towards integrating (strategic) interdependencies into real options models, it appears that the framework is rather suited to appraise individual investment projects, than a portfolio of interdependent and interlinked projects, where each of which may consist of a portfolio of interdependent real options.

Nevertheless, all the above cited publications consider only one type of project interdependency (almost always cash flow interdependencies), one type of real option available, and a very limited number of independent uncertainties (e.g. market and technological ones), whilst addressing capital budgeting situations in one specific field (e.g. R&D or IT investments). Yet the correct appraisal of a portfolio of interdependent urban infrastructure investments – potentially located in different urban silos, e.g. energy, transport, water, waste, ICT, etc. – necessitates an alternative and more general approach. The approach we have used in this study aims to overcome the limitations of earlier approaches by taking into account four types of interdependencies (physical, cyber, geographical, and logical[4]), many embedded real options, and various sources of uncertainty, thus introducing an appraisal framework that provides a lot of flexibility.

3. The urban infrastructure investment problem

In this section, we present the real options approach taken in this paper to appraise a portfolio of physical and digital urban infrastructure investments. These investments can be either in existing urban infrastructure assets (e.g. brownfield projects) or in new urban infrastructure assets (e.g. greenfield projects) and may have both physical and digital characteristics.

3.1. Appraisal of a portfolio of independent projects

We consider the appraisal of a portfolio of I independent investment projects, each of which consisting of a portfolio of $H_i, i \in \mathcal{I}$, interdependent real options, where $\mathcal{I} = \{1, 2, \ldots, I\}$ is the set of investment projects and $\mathcal{H}_i = \{1, 2, \ldots, H_i\}$ the

[3]It is important to note that the author considered interdependencies from a strategic perspective.

[4]See Ouyang (2014) for a recent review of infrastructure interdependencies.

set of real options available to project $i \in \mathcal{I}$. In order to model the flexibility the decision maker has and also the interdependencies between the real options of an individual investment project, we use an influence diagram. As noted by Sick and Gamba (2010), modelling real options with an influence diagrams allows to focus on decisions whilst leaving the risk analysis in the background.

3.1.1. Modelling real options in projects with influence diagrams

We model the graphical part of the influence diagram for project $i \in \mathcal{I}$ to be composed of two elements: A set of decision nodes $\mathcal{D}_i = \{1, 2, \dots, D_i\}$, which represents, for example, stages of development, operating modes, or states of the decision making process, and a set of directed edges $\mathcal{H}_i$ (i.e. real options), which describe the transitions one can make, or in other words the real option(s) one can exercise. Both elements together represent the directed graph $(\mathcal{D}_i, \mathcal{H}_i)$. In order to simplify this modelling approach, we assume the set of decision nodes $\mathcal{D}_i$, for all $i \in \mathcal{I}$, contains exactly one beginning and one ending node, which are characterised through not having incoming and outgoing transitions, respectively. Let the state of the decision making process of project $i \in \mathcal{I}$ at time $t \in \mathcal{T}_i^D$ be denoted by $S_{i,t} \in \mathcal{S}_i$, where $\mathcal{T}_i^D$ is the set of decision dates, and $\mathcal{S}_i$ is the discrete state-space of project i. In addition to the representation of the graph, we require functions that enable mapping from nodes to transitions and vice versa. Therefore, the functions used are:

- a^D: $\mathcal{S}_i \to \mathcal{H} \cup \emptyset$ $\quad a^D(S_{i,t})$ is the set of outgoing transitions when in state $S_{i,t}$.
- b^D: $\mathcal{S}_i \to \mathcal{H} \cup \emptyset$ $\quad b^D(S_{i,t})$ is the set of incoming transitions when in state $S_{i,t}$.
- a^H: $\mathcal{I} \times \mathcal{H}_i \to \mathcal{D}_i$ $\quad a^H(i,h)$ is the source node of transition h of project i.
- b^H: $\mathcal{I} \times \mathcal{H}_i \to \mathcal{D}_i$ $\quad b^H(i,h)$ is the terminal node of transition h of project i.

With regard to the flexibilities modelled with the influence diagram, important information is associated with a transition or real option. There are four elements to any transition (i.e. real option) $h \in \mathcal{H}_i$ of project $i \in \mathcal{I}$:

1. Option h of project i can be exercised in the interval $[T'_{i,h}, T''_{i,h}]$, where $T'_{i,h}$ can be zero or depend on the exercise policy of the preceding option, and $T''_{i,h}$ may coincide with $T'_{i,h}$, thus representing a situation in which the decision to exercise option h cannot be delayed. The duration of option h of project i (or transition time) is denoted by $\Delta_{i,h}$.
2. Decisions to exercise any of the available options in $a^D(S_{i,t})$ of project i in state $S_{i,t}$ are represented by a vector of binary indicator variables, $\delta = \big(\delta_1, \delta_2, \dots, \delta_{|a^D(S_{i,t})|}\big)$,

and have to satisfy a set of linear constraints denoted by the feasible region $\mathcal{A}(S_{i,t})$, which depends on the state $S_{i,t}$ at time $t \in \mathcal{T}_i^D$.

3. We use the transition function $S^M(S_{i,t}, \delta_h)$ to describe the evolution of the state $S_{i,t}$ of project i from time t to $t + \Delta_{i,h}$ after having chosen to exercise option h (i.e. $\delta_{i,h} = 1$).

4. Exercising option h of project i at time $t \in \mathcal{T}_{i,h}$ generates the stochastic cash flow $\{\tilde{V}_{i,h,t'}\}_{t' \in \mathcal{T}^c_{i,h}(t)}$, where $\mathcal{T}_{i,h}$ is the set of stopping times of option h and $\mathcal{T}^c_{i,h}(t)$ is the set of time periods when cash flows occur through exercising option h of project i at time t.

However, the form of the discrete stream of stochastic cash flows $\tilde{V}_{i,h,t'}$ generated by exercising option $h \in \mathcal{H}_i$ of project $i \in \mathcal{I}$ at time $t \in \mathcal{T}_{i,h}$ strongly depends on the type of real option and more specifically on the urban infrastructure investment project under consideration.

3.1.2. Cash flow model

We assume that cash flows occurring during the payoff horizon given by $\mathcal{T}^c_{i,h}(t)$ can be subdivided into two main components: a stochastic component that solely represents a cost cash flow and a stochastic component modelling rewards. The first component is represented by the cash flow $\tilde{C}_{i,h,t'}, t' \in \mathcal{T}^c_{i,h}(t)$, determined by the cost function $C_{i,h}(\cdot)$ in (3.1) and – depending on the type of option (e.g. option to develop, option to operate, etc.) – may characterise costs due to capital expenditures in terms of investment costs (which might represent costs for external financing) and costs for O&M, as well as costs due to procurement of commodities and/or services related to the exercise of the real option. Let the amount of commodity/service $m \in \mathcal{M}^a_{i,h}$ required during time interval $t' \in \mathcal{T}^c_{i,h}(t)$ by exercising option $h \in \mathcal{H}_i$ of project $i \in \mathcal{I}$ at time $t \in \mathcal{T}_{i,h}$ be denoted by $x_{i,h,t',m}$, where $\mathcal{M}^a_{i,h}$ is the set of inputs (commodities/services) required by the underlying of option h of project i. For all $i \in \mathcal{I}$, $h \in \mathcal{H}_i$, $t' \in \mathcal{T}^c_{i,h}(t)$:

$$\tilde{C}_{i,h,t'} = C_{i,h}\big(x_{i,h,t'}, c^{\text{om}}_{i,h,t'}, c^{\text{inv}}_{i,h,t'}, p^S_{t'}, q^S_{t'}\big), \tag{3.1}$$

where $x_{i,h,t'} = (x_{i,h,t',m})_{m \in \mathcal{M}^a_{i,h}}$ is the vector of all commodity/service inputs required during time interval t', $c^{\text{om}}_{i,h,t'}$ and $c^{\text{inv}}_{i,h,t'}$ are the $O\&M$ and investment costs associated with option h of project i at time t', respectively, and $p_{t'} = (p_{t',m})_{m \in \mathcal{M}^a_{i,h}}$ and $q^S_{t'} = (q^S_{t',m})_{m \in \mathcal{M}^a_{i,h}}$ are the vectors of prices and supply of commodities/services at time t', respectively. Depending on the type of the option, different inputs for the cost function $C_{i,h}(\cdot)$ might be needed. For example, an option to develop might require more input related to external financing such as interest rate on debt, equity ratio, credit period, whereas an option to operate might need the specification of additional information about procurement strategies, supply patterns, etc.

The second component is represented by $\tilde{R}_{i,h,t'}, t' \in \mathcal{T}^c_{i,h}(t)$, and models the cash flows related to the commercialisation and provision of commodities and services. These cash flows are determined by the reward function $R_{i,h}(\cdot)$ in (3.2) and, similar to the cost function above, the inputs of this function strongly depend on the type of option being modelled and its underlying physical asset. Let the amount of commodity/service $m \in \mathcal{M}^b_{i,h}$ provided during time interval $t' \in \mathcal{T}^c_{i,h}(t)$ by exercising option $h \in \mathcal{H}_i$ of project $i \in \mathcal{I}$ at time $t \in \mathcal{T}_{i,h}$ be denoted by $y_{i,h,t',m}$, where $\mathcal{M}^b_{i,h}$ is the set of outputs (commodities/services) provided by the underlying of option h of project i. For all $i \in \mathcal{I}$, $h \in \mathcal{H}_i$, $t' \in \mathcal{T}^c_{i,h}(t)$:

$$\tilde{R}_{i,h,t'} = R_{i,h}\big(y_{i,h,t'}, p^D_{t'}, q^D_{t'}\big), \tag{3.2}$$

where $y_{i,h,t'} = (y_{i,h,t',m})_{m \in \mathcal{M}^b_{i,h}}$ is the vector of all commodities/services provided during time interval t', and $p^D_{t'} = (p^D_{t',m})_{m \in \mathcal{M}^b_{i,h}}$ and $q^D_{t'} = (q^D_{t',m})_{m \in \mathcal{M}^b_{i,h}}$ are the vectors of prices and demand for commodities/services at time t', respectively. Additional input variables might be needed in situations where, for example, contracts for service provision need to be modelled or where the commercialisation of services requires a more adequate description of the infrastructure's business model.

In order to link the cost function with the reward function, we assume that there exists a commodity/service production function $Y(\cdot)$ which mathematically describes how commodities and services $y_{i,h,t'}$ are produced and provided, respectively, given input(s) $x_{i,h,t'}$. For all $i \in \mathcal{I}$, $h \in \mathcal{H}_i$, $t' \in \mathcal{T}^c_{i,h}(t)$:

$$y_{i,h,t'} = Y\big(x_{i,h,t'}, \vartheta_{i,h}, \Theta_{i,h,t'}\big), \tag{3.3}$$

where $\vartheta_{i,h}$ is a vector of technology parameters and $\Theta_{i,h,t'}$ is the state of the technology at time t'. The specification and properties of $Y(\cdot)$ largely depend on the type of option and complexity of the option's underlying technology, i.e. urban infrastructure system. As mentioned above, many types of options (e.g. option to operate, develop, scale-up, etc.) do not necessarily represent the production of goods and/or services, so characterising the output obtained by exercising them by means of a production function may not be appropriate. For such options, however, it may still be appropriate to use simplified cost and reward functions to represent financial transactions.

Consequently, the (positive or negative) net cash flow at time t' of option h of project i is $\tilde{V}_{i,h,t'} = -\tilde{C}_{i,h,t'} + \tilde{R}_{i,h,t'}$. When exercised at time $t \in \mathcal{T}_{i,h}$, the expected payoff of option h of investment project i and state $S_{i,t}$ can then be obtained taking the expected value of the net present value (NPV) of these net cash flows giving:

$$\Pi_{i,h,t}(S_{i,t}) = \mathbb{E}_t\left[\sum_{t' \in \mathcal{T}^c_{i,h}(t)} e^{-r(t'-t)} V_{i,h,t'}\right], \tag{3.4}$$

where r is the risk free rate.

3.1.3. Option valuation problem

Unlike the approach taken by Gamba (2003), who decomposed a portfolio of interacting real options into a set of independent, compound, mutually exclusive and switching options, in this paper we propose a single framework to value such interacting options where simple constraints and binary indicator variables are used to model strategic interdependencies of the considered real options. Even though we do not consider technical uncertainty as in (Gamba, 2003), our framework is flexible enough to incorporate the presence of technical uncertainty, for example through including chance nodes in the influence diagram, as shown by Charnes and Shenoy (2004). Let the value of option $h \in \mathcal{H}_i$ of project $i \in \mathcal{I}$ at time $t \in \mathcal{T}_{i,h}$ in state $S_{i,t} \in \mathcal{S}_i$ be denoted by $F_{i,h,t}(S_{i,t})$, as well as let the optimal value of the portfolio of options available to project i at time $t \in \mathcal{T}^D_i$ in state $S_{i,t}$ be denoted by $G_{i,t}(S_{i,t})$. Then, for all $S_{i,t} \in \left\{S' \in \mathcal{S}_i : b^D(S') \neq \emptyset\right\}$:

$$G_{i,t}(S_{i,t}) = \max_{\delta} \sum_{h \in b^D(S_{i,t})} F_{i,h,t}(S_{i,t}) \cdot \delta_h \tag{3.5}$$

$$\text{s.t. } \delta_h \in \{0,1\}, \qquad \forall h \in b^D(S_{i,t}), \tag{3.6}$$

$$\delta_h \in \mathcal{A}(S_{i,t}), \qquad \forall h \in b^D(S_{i,t}), \tag{3.7}$$

$$S_{i,t+\Delta_{i,h}} = S^M(S_{i,t}, \delta_h), \qquad \forall h \in b^D(S_{i,t}), \tag{3.8}$$

$$F_{i,h,t}(S_{i,t}) = \Pi_{i,h,t}(S_{i,t}) + \mathbb{E}_t\Big[e^{-r\Delta_{i,h}} G_{i,t+\Delta_{i,h}}(S_{i,t+\Delta_{i,h}})\Big], \forall h \in b^D(S_{i,t}), \tag{3.9}$$

where the value of an ending node, i.e. a node without outgoing transitions, is given by its terminal value $G^T_{i,t}(S_{i,t})$, for all $S_{i,t} \in \left\{S' \in \mathcal{S}_i : b^D(S') = \emptyset\right\}$. It is important to note that the value of making a transition, that is exercising a real option, is given by the sum of the payoff from exercising that real option and the expected value of being at the next decision node; the latter equals the optimal value of the opportunity to choose from a portfolio of real options available at the next decision node. This opportunity value provided by the compound feature is modelled in (3.9). Since the formulation in (3.5)-(3.9) considers only interdependencies within an investment project but not between projects, the value of the portfolio of projects at time 0 is the sum of the values of the projects optimised separately:

$$G_0^P\big(\{S_{i,0}\}_{i\in\mathcal{I}}\big) = \sum_{i\in\mathcal{I}} G_{i,0}(S_{i,0}). \tag{3.10}$$

In the following subsection we drop the assumption of independence between urban infrastructure investment projects in a portfolio and explicitly consider four types of infrastructure interdependencies.

3.2. Appraisal of a portfolio of interdependent projects

This subsection contains both the mathematical modelling of the interdependencies considered in this paper and the formulation of the valuation problem for a portfolio of interdependent urban infrastructure investment projects.

3.2.1. Modelling of infrastructure interdependencies

With regard to the interdependencies among different urban infrastructure investment projects, we consider the four interdependency types first defined by Rinaldi et al. (2001). These four types are physical, cyber, geographical, and logical interdependency. These interdependencies were modelled as follows:

- Physical and cyber: Urban infrastructure investment projects $i \in \mathcal{I}$ is physically dependent on and/or has a cyber interdependency with project $i' \in \mathcal{I}\backslash\{i\}$ if there exists an option $h \in \mathcal{H}_i$ of project i that, when exercised at time $t \in \mathcal{T}_{i,h}$, requires one or several commodities that are being provided through the exercise of option $h' \in \mathcal{H}_{i'}$ of project i'. In other words, $x_{i,h,t'} = f(y_{i',h',t'}, q_{t'}^S)$, where $f(\cdot)$ denotes the general dependence and $q_{t'}^S$ represents the vector of supply of commodities at time $t' \in \mathcal{T}_{i,h}^c(t)$ from one or several external sources, e.g. spot market. We will denote relationship between two "project-option" pairs by σ, which, using the above two pairs, is given by:

$$\sigma_{i,h}^{i',h'} = \begin{cases} 1 & \text{if } (i,h) \text{ is physically and/or digitally dependent on } (i',h') \\ 0 & \text{otherwise.} \end{cases} \tag{3.11}$$

  However, the dependence between two options of different projects may be significantly affected by the stopping times of these options. For example, if one of these two is being exercised early and the other one late, there may not be an overlap in terms of commodity flows. The set of ordered triples $(i', h', \tau_{i',h'})$, each of which representing a project-option pair and corresponding stopping time, which contains all the actual dependencies of option h of project i exercised at time $t \in \mathcal{T}_{i,h}$ is given by:

$$\kappa_{i,h}(t) = \Big\{(i', h', \tau_{i',h'}) : i' \in \mathcal{I} \setminus \{i\}, h \in \mathcal{H}_{i'}, \tau_{i',h'} \in \mathcal{T}_{i',h'}, \sigma_{i,h}^{i',h'} = 1, \\ \tau_{i',h'} \geq t, \min \mathcal{T}^c_{i',h'}(\tau_{i',h'}) \leq \max \mathcal{T}^c_{i,h}(t)\Big\} \tag{3.12}$$

- Geographical: The geographical interdependency caused by physical proximity of urban infrastructure systems is modelled by introducing the transition function $\Theta^M(B_{\cdot,t}, \cdot)$, which describes the evolution of the state of the technology $\Theta_{\cdot,\cdot,t}$ of each geographically interdependent investment project from time t to $t+1$. Given the spatial distributions of urban infrastructures, which often differ significantly, the geographical interdependence between urban infrastructures may follow particular patterns. Let $A_j \subseteq \mathcal{I}, j \in \mathcal{J}$, denote the j-th set of geographically interdependent urban infrastructures, where $\mathcal{J} = \{1, 2, \ldots, J\}$ is the index set describing the number of geographical interdependencies in $\mathcal{I}$. Hence, $\{A_j : j \in \mathcal{J}\}$ is the indexed family of sets describing all possible geographical interdependencies amongst investment projects in $\mathcal{I}$. For example, let us consider an investment portfolio of five urban infrastructure systems, giving $\mathcal{I} = \{1, 2, 3, 4, 5\}$, where geographical interdependencies exist between projects one and two and between projects three, four and five. Hence, $A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5\}$ and $\mathcal{J} = \{1, 2\}$.

- Logical: The logical interdependencies considered are resource and market, strategic, as well as budget interdependencies. These are modelled as follows:

  - Resource and market: Unlike the resource balance constraint presented in (Samsatli and Jennings, 2013), which also considers import, export and storage of resources, we use a simplified approach by considering that resource and market interdependencies are given by the competition of investment projects for limited supply of and demand for commodities, which are represented by the vectors $q^S_{t'}$ and $q^D_{t'}$, respectively, as well as by their corresponding purchasing and selling prices $p^S_{t'}$ and $p^D_{t'}$, respectively.
  - Strategic: Similar to the strategic interdependencies between options within an urban infrastructure investment project, real options of different projects

can be strategically interdependent such as independent, mutually exclusive, or may require a particular order of being exercised. These interdependencies can be modelled similar to the strategic interdependencies of real options within a project (e.g. see constraints in (3.7)) by adding linear constraints to the feasible region $\mathcal{A}(\cdot)$.

– Budget: We assume that all I projects compete for the same financial resources represented through a global budget and are thus, possibly, dependent on decisions made with respect to other investment projects in $\mathcal{I}$ at potentially different points in time. The budget balance equation used in this paper is partially based on (Brosch, 2008). We model the budget constraint by a balance that dynamically links the budget available at time t to be used during time period t' (where $t' \geq t$), $B_{t,t'}$, with the budget available at the preceding point in time $t-1$, the new funds available at time t', $\beta_{t'}$, and the sum of all expected investment cash flows of all options exercised at time t, $\sum_{i,h} \mathbb{E}_t\big[c^{\text{inv}}_{i,h,t'}\big], t' \in \mathcal{T}^c_{i,h}(t)$. Then, for all $i \in \mathcal{I}$, $S_{i,t} \in \mathcal{S}_i$, $h \in b^D(S_{i,t})$, at time $t = 0$:

$$B_{0,t'} = B_{0,t'-1} + \beta_{t'} - \sum_{i \in \mathcal{I}} \sum_{h \in b^D(S_{i,0})} \mathbb{E}_0\big[c^{\text{inv}}_{i,h,t'}\big]\delta_{i,h}, \quad t' = 0, \dots, \max_{i,h}\big\{\mathcal{T}^c_{i,h}(0)\delta_{i,h}\big\}, \tag{3.13}$$

where $B_{0,-1}$ is 0 and $\delta_{i,h}$ is the binary indicator variable that denotes if option h of project i is being exercised at time t.
If $t \geq 1$, $t' = t, \dots, \max\Big\{\text{length}\big(\hat{B}_{t-1}\big), \max_{i,h}\big\{\mathcal{T}^c_{i,h}(t)\delta_{i,h}\big\}\Big\}$:

$$B_{t,t'} = \begin{cases} \hat{B}_{t-1,t'} + \beta_{t'} - \sum_{i \in \mathcal{I}} \sum_{h \in b^D(S_{i,t})} \mathbb{E}_t\big[c^{\text{inv}}_{i,h,t'}\big]\delta_{i,h}, & t' \leq \text{length}\big(\hat{B}_{t-1}\big), \\ B_{t,t'-1} + \beta_{t'} - \sum_{i \in \mathcal{I}} \sum_{h \in b^D(S_{i,t})} \mathbb{E}_t\big[c^{\text{inv}}_{i,h,t'}\big]\delta_{i,h}, & t' > \text{length}\big(\hat{B}_{t-1}\big), \end{cases} \tag{3.14}$$

where the vector $\hat{B}_{t-1} = (B_{t-1,t'} - \beta_{t'})_{t'>t-1}$ has been determined and added to the state at time $t-1$.

3.2.2. Option valuation problem

Unlike the case of appraising independent urban infrastructure investment projects described in Subsection 3.1, where projects can be valued separately by solving the valuation problem given in (3.5)-(3.9) and the overall portfolio value then be determined by simply summing up the contributions of each project as in (3.10), taking interdependencies between investment projects into account requires decision making on a portfolio level since decisions made with respect to real options of one project might affect or be dependent upon decisions made with respect to another urban infrastructure investment projects. In other words, investment project value additivity (Trigeorgis, 1993) does not hold any more and the valuation problem for a portfolio of interdependent real options needs to be extended such that it also takes into account the effects and interdependency of decisions made with respect to other projects in the portfolio.

Let $G_t^P(S_t)$ represent the value of the portfolio of interdependent urban infrastructure investment projects when in state S_t at time $t \in \mathcal{T} = \bigcup_{i \in \mathcal{I}} \mathcal{T}_i^D$, where $\mathcal{T}$ is the set of all decision dates of all projects I in the portfolio and $\mathcal{T}_t$ the set of all decision dates available after time t, i.e. $\mathcal{T}_t = \{t' \in \mathcal{T} : t' > t\}$.

$$G_t^P(S_t) = \max_{(\delta_{t'})_{t' \geq t}} \Bigg\{ \sum_{i \in \mathcal{I}} \sum_{h \in b^D(S_{i,t})} \Pi_{i,h,t}\big(S_t, \overline{\delta}_{i,h,t}\big) \cdot \delta_{i,h,t} + \mathbb{E}_t\Bigg[\sum_{t' \in \mathcal{T}_t} e^{-r(t'-t)} \sum_{i \in \mathcal{I}} \sum_{h \in b^D(S_{i,t'})} \Pi_{i,h,t'}(S_{t'}, \overline{\delta}_{i,h,t'}) \cdot \delta_{i,h,t'} \Bigg] \Bigg\} \tag{3.15}$$

$$\text{s.t.} \quad \delta_{i,h,t}, \delta_{i',h',\tau_{i',h'}} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, h \in b^D(S_{i,t}), (i', h', \tau_{i',h'}) \in \kappa_{i,h}(t), \tag{3.16}$$

$$\delta_{i,h,t}, \delta_{i',h',\tau_{i',h'}} \in \mathcal{A}(S_t), \qquad \forall i \in \mathcal{I}, h \in b^D(S_{i,t}), (i', h', \tau_{i',h'}) \in \kappa_{i,h}(t), \tag{3.17}$$

$$S_{t+\Delta} = S^M\big(S_t, \delta_{i,h,t}, \overline{\delta}_{i,h,t}\big), \quad \forall i \in \mathcal{I}, h \in b^D(S_{i,t}), \tag{3.18}$$

where $\overline{\delta}_{i,h,t} = \big(\delta_{i',h',\tau_{i',h'}}, \forall (i', h', \tau_{i',h'}) \in \kappa_{i,h}(t), i \in \mathcal{I}, h \in b^D(S_{i,t})\big)$, is the vector of decision variables that effect the payoff of option h of project i in state $S_{i,t}$ if exercised at time $t \in \mathcal{T}$. Compared with the case of independent projects in (3.10), the value of the portfolio of interdependent urban infrastructure investment projects at time 0 is given by $G_0^P\big(S_0\big)$. It is important to note that the state variable of the above valuation problem, S_t, has to contain all the information we need to determine the option payoffs in (3.15), the feasible region in (3.17), and the transition function in (3.18) (Powell, 2014). Unlike the valuation problem given in equations (3.5)-(3.9), which aimed at making exercise decision for every project individually and thus considered a single state $S_{i,t}$ for every investment project $i \in \mathcal{I}$, we now consider one state S_t that represents the entire portfolio of investment projects in order to be able to make exercise decisions for investment projects within the portfolio concurrently. For example, the state variable for the above problem could be $S_t = (S_{1,t}, \ldots, S_{I,t}, \ldots)$, where the variables $S_{i,t}$ may itself represent vectors of information or history, thus making S_t become multidimensional and potentially extremely large.

Interdependencies between urban infrastructure investment projects are included in the above valuation problem through the payoff function in (3.15), the feasible region in (3.17), and the transition function in (3.18). Both physical and cyber interdependencies are represented through the payoff of option h of project i exercised at time t, $\Pi_{i,h,t}\big(S_t, \overline{\delta}_{i,h,t}\big)$, being dependent on past (stored in S_t), as well as present and future exercise decision $\big(\overline{\delta}_{i,h,t}\big)$ made with respect to other options. Consequently, if the payoff of option h of project i is independent then the vector $\overline{\delta}_{i,h,t}$ is empty and no information about other options' exercise decisions needs to be stored in the state variable regarding option h. Geographical interdependencies are taken into account by adding the state of the technology $\Theta_{\cdot,\cdot,t}$ at time t of every graphically interdependent investment project to the state variable S_t and by expanding the transition function $S^M(\cdot)$ correspondingly, thus allowing a local environment event to create state changes in these graphically interdependent urban infrastructure systems. With regard to logical interdependencies: While market interdependencies between projects are given by parameters underlying the payoff function of options given in (3.4), resource interdependencies are integrated into the valuation problem through a simplified resource balance constraint, which is modelled similar to the budget balance constraint but considers commodities instead of cash flows; we integrate the linear constraints that represent strategic interdependencies between options of different urban infrastructure investment projects into the feasible region $\mathcal{A}(S_t)$; lastly, interdependencies due to investment projects competing for limited budgets are taken into account by adding the vector $\hat{B}_{t-1}$ to the state S_{t-1} at time $t-1, t \geq 1$, and the vector $B_{t,t'}$, which denotes the available future budget at time t, to the feasible region $\mathcal{A}(S_t)$ (e.g. by demanding $B_{t,t'} \geq 0$, for all $0 \leq t \leq t'$).

4. Methods

In order to numerically solve the valuation problems for portfolios of independent and interdependent urban infrastructure investment projects, which are given by (3.5)-(3.9) and (3.15)-(3.18), respectively, we apply the least squares Monte Carlo (LSM) approach, which has been proposed by Longstaff and Schwartz (2001) and in a slightly different way by Carriere (1996); Tsitsiklis and Van Roy (2001). In particular, the LSM approach allows to estimate the options' expected payoff from continuation[5] in equations (3.5) and (3.15). This is done by regressing the discounted payoff from optimally exercising an option on functions of the state variables, more specifically on linear combinations of so-called basis functions. Using the optimal coefficients obtained by the least squares regression, the fitted value of the regression can then be used to determine the optimal exercise strategy of an option. It is important to note that the algorithm proposed by Longstaff and Schwartz (2001) uses only "in-the-money paths", i.e. paths where the option is in the money, not only when estimating the coefficients by the least squares regression, but also when determining the optimal exercise decision at each exercise time and path.

Even though originally proposed for valuing single option problems, the LSM approach can be applied to the valuation of multi-options problems using a few simple extensions. In addition to presenting a framework for the decomposition of a portfolio of strategically interacting real options, Gamba (2003) has also presented the corresponding extensions to value such portfolios of real options by the LSM algorithm. Building upon Gamba's framework, Areal et al. (2008) presented, amongst other things, an alternative algorithm to value mutually exclusive options which is faster and more accurate. In general, adapting the above described LSM algorithm to multi-option problems requires a few plain extensions. For the sake of brevity and since these have been described by Gamba (2003) Areal et al. (2008), we will not summarise the authors extensions to the original LSM algorithm and refer to their publications.

When it comes to the practical implementation of the LSM algorithm, a number of choices have to be made which potentially affect not only the computational efficiency of the algorithm but also the accuracy of the results obtained. These choices include the number of options exercise times (K), the total number of paths generated (Ω) through Monte Carlo simulation, and the polynomial family and the number of basis functions (M) chosen, as well as the regression algorithm applied. With regard to the number of discrete time steps, the larger K (i.e. the Bermudan option's possible exercise times), the more accurate the value of an American option is approximated. Early convergence results were provided by Longstaff and Schwartz (2001) and by Clément et al. (2002), who proved that for a given K and if both $\Omega \to \infty$ and $M \to \infty$ then the LSM algorithm almost surely converges and its normalised error is asymptotically Gaussian. More recently, Areal et al. (2008) tested the impact of the choice of the polynomial family, M, and of the number of paths generated, Ω, on the accuracy of the LSM approach when valuing American options and concluded that

[5]Since the LSM approach is approximating the value function, it can be classified as an approximate dynamic programming (ADP) strategy (Powell, 2011).

weighed Laguerre polynomials have slight advantages over other polynomial families in terms of accuracy, while in terms of computational speed the power functions get first. Interestingly, the authors also found that with a certain value for the number of basis functions (M), further improvement in accuracy can only be achieved by increasing the number of paths (Ω). Kohler (2010) recently provided a review on regression-based Monte Carlo methods used to value American options.

5. Discussion

This paper presents two new real options-based appraisal frameworks for selecting a portfolio of urban infrastructure investment projects. Firstly, we have developed a framework to appraise a portfolio of independent investment projects, where each project consists of a portfolio of strategically interdependent real options. Unlike the decomposition approach presented by Gamba (2003), we propose a single appraisal framework to value independent, mutually exclusive, compound and switching option, and formulate the corresponding valuation problem with binary indicator variables and simple linear constraints defining the interactions between real options. We then expand this framework by additionally considering interdependencies between investment projects in the portfolio while considering the following four types of infrastructure interdependency: physical, cyber, geographical, and logical, with the latter representing resource and market, strategic, and budget interdependencies.

However, there is a significant difference among the two frameworks in the way in which the considered interdependencies are implemented within them. While strategic interdependencies can be modelled effectively for every project separately via the feasible region and simple linear constraints, the modelling of infrastructure interdependencies requires several different approaches on a portfolio level. Geographical and logical (resource and market, strategic, and budget) interdependencies are included via the transition function and additional constraints, respectively, and thus have the ability to alter both the state of the system and feasible region. On the other hand, physical and cyber interdependencies between urban infrastructure investment projects, and more specifically between real options thereof, mean that the payoff of an option of one project may depend on whether an interdependent real option of another project has been exercised. This includes not only the case of "input" dependency, i.e. when the payoff of an options is dependent on the "commodities" provided by another option, but also the case of "output" dependency, i.e. when an option's payoff depends on another option consuming part or all of its commodity output.

Modelling the decision maker's flexibilities (i.e. real options) through influence diagrams, we apply the LSM approach to provide numerically efficient solutions to the two presented valuation problems. These new appraisal frameworks are relevant to risk-aware investors wishing to build an optimal urban infrastructure investment portfolio that potentially consists of a number of interdependent investment projects and has considerable flexibilities in terms of real options available to individual investment projects. Furthermore, we expect our new frameworks to have substantial potential to enhance investment decisions, particularly with regard to timing, scale, and project selection, thus potentially creating significant value for investors. Future work will comprehensively evaluate the comparative performance of traditional approaches like NPV and our two new approaches under a wide range of real-world case studies.

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