The Value of Waiting to Invest in a Liquidity Trap

George Dotsis

[Preliminary and Incomplete, Please do not quote without permission from the author]

In this paper I examine optimal investment rules when interest rates are near the zero lower bound. Extant approaches produce an ambiguous relationship between investment and interest rates and are difficult to reconcile with prolonged periods of interest rates near the zero lower bound and low investment. I use the shadow-rate model of Black (1995) for modeling interest rate uncertainty and show that when interest rates are at the lower bound and the shadow rate is substantially below the bound it is always optimal to defer investment and wait for resolution of uncertainly about interest rates. So long as the interest rate volatility is positive, the shadow interest rate approach is consistent with low investment and interest rates near the zero lower bound.

[*]Department of Economics, University of Athens, Sofokleous & Aristidou str., 5th floor, office 519. Tel: 210-36894545, email: gdotsis@econ.uoa.gr.

1. Introduction

In response to the 2007-8 global financial crisis, central banks of advanced economies implemented a variety of policies to help stabilize the economy and the financial system. Amongst others, the sine qua non of monetary policy action put forward by central banks was the reduction of the level of short -term interest rates to near zero. However, despite record low interest rates since 2008, GDP growth remains anemic and unemployment is persistent. The phenomenon in which economic growth is weak and interest rates are close to zero is described by many economists as a "liquidity trap". According to the theory of liquidity traps, this happens when the natural rate of interest- the short-term real interest rate consistent with full employment-is negative but the nominal rate is stuck at the zero lower bound. This leads to ineffective monetary policy and to a spiral of excessive savings, a shortage of spending and investment and deflationary expectations.[1]

In this paper I examine optimal investment rules when interest rates are near the zero lower bound. Why do low interest rates fail to spur investment and do not induce investors to undertake projects with positive net present values? Long time ago studies in the finance literature have stressed the importance of option like characteristics in investment appraisal (see Dixit and Pindyck, 1994). When there is irreversibility and sunk costs an investor can either undertake a project today or defer and wait until more information accrues and decide whether to invest at a future time period. McDonald and Siegel (1986) show that when the cashflows of a project fluctuate stochastically over time, the option to wait is valuable and project benefits must be substantially above costs (e.g., twice the cost) to trigger investment decision. Besides cashflows,

[1] Krugman (1998), Eggertsson and Woodford (2003) and Bernanke and Reinhart (2004) examine optimal monetary policy rules when interest rates are at zero lower bound.

another important determinant of investment appraisal is the level of interest rates. If cashflows are deterministic but the level of interest rate is stochastic, the option to wait and delay investment is still valuable when there is anticipated resolution of uncertainly about interest rates and therefore project's cost of capital. Ingersoll and Ross (1992) is the first study that examines optimal investment rules with uncertain interest rates and deterministic cashflows. They show that interest rate uncertainty can delay investment and produces an ambiguous relationship between investment and the level of interest rates; a reduction in interest rates does not necessarily lead to an increase in investment.

In a single period model I show that the value of waiting versus investing today when the interest rates are uncertain is affected by three components. The first component is the expected change in the interest rates. If future interest rates are expected to rise, it is optimal to invest today and finance the investment at a lower cost. Conversely, if future interest rates are expected to fall it is optimal to wait and invest when interest rates are lower. The second component is uncertainly with respect to future interest rates. If interest rate volatility is high it is optimal to wait until uncertainty is resolved. The last component is the cost of waiting due to foregone interest. The cost of waiting is substantial when the level of interest rate is high and diminishes as the interest rate approaches zero. The relationship between investment and interest rates is ambiguous because it depends on the individual impact of each component. For example, a decrease in the interest rate will not necessarily rise investment if at the same time uncertainty about future interest rates increases.

Conventional one-factor models of interest rate dynamics are difficult to reconcile with prolonged periods of interest rates near the zero lower bound and low investment. For example, mean reverting one-factor model dynamics such as gaussian models (e.g., Vasicek, 1977), the square root process (e.g., Cox, Ingersoll & Ross (1985)) and the lognormal (Black and Karasinski, 1991) are not fully consistent with the empirical behavior of interest rates. Gaussian interest rate models allow for negative nominal interest rates while under lognormal models the zero boundary is unattainable.[2] The square root model is a borderline case since zero can be unattainable, an absorbing barrier or a reflecting barrier. A reflecting barrier that bounces the interest rate off zero is obviously unrealistic and the introduction of an absorbing barrier makes modeling quite complicated since it requires additional specifications to determine the probability of the interest rate becoming positive again. Moreover, the square-root model zero assumes that the volatility of the interest rate vanishes as the interest rate approaches zeros, which is inconsistent with the empirical behavior of the short term interest rates. [3] Even if we disregard for the moment the zero lower bound problem, conventional mean reverting interest rate models have also another important caveat. They predict that when the short-term interest rate is near zero, the expected change is positive to pull back the interest rate toward the long run mean (typically in the range of 2%-3%) and that decreases the value of waiting to invest. Therefore, prolonged periods of low interest rates and low investment can only occur through increased interest rate volatility.

[2] The zero is unattainable because under the lognormal model the volatility of the interest rate vanishes when the interest rate approaches zero and the mean reversion component is pulling the interest rate toward the long run mean
[3] Interest rates have substantial volatility.....

To circumvent the zero lower bound problem I use the shadow interest rate model proposed by Black (1995) for the dynamics of the short rate. In his very thoughtful paper, Black (1995) observes that nominal interest rates cannot become negative since investors will choose to hold physical currency instead of an asset that pays negative interest. He suggests that the short interest rate has option like characteristics since it can be viewed as a call option on the shadow interest rate with a zero strike price. The shadow interest rate can take both positive and negative values and it is the rate that would have prevailed in the market should the zero lower bound restriction did not exist. Recent studies find that shadow rate models provide a better fit to yield curve dynamics when interest rates are near the zero lower bound and are more informative about the monetary policy stance (see, Kim and Singleton (2012), Bauer and Rudebusch (2013), Krippner (2012), Wu and Xia (2014), Christensen and Rudebusch (2014).

I show that when the interest is well below the zero lower bound the expected change in interest rates and the cost of waiting is zero, and the decision to invest is driven solely by the uncertainty over future interest rates. So long as the interest rate volatility is positive, the shadow interest rate approach is consistent with low investment and interest rates near the zero lower bound. I also extend the model to a multi-period context and solve the problem numerically using the method Least-Square Monte Carlo method.

2. Single Period Model

I make the assumption that the term structure is flat and the interest rate follows a one-factor diffusion process:

$$dr_t = \kappa(\theta - r_t)dt + \sigma r_t^{\gamma} dZ_t \tag{1}$$

where r_t is the interest rate at time t, Z_t is a standard Wiener process, κ is the speed of mean reversion, θ is the long run mean σ is the volatility and γ is the elasticity. The one-factor diffusion in (1) nests several popular one-factor interest rate models. When γ=0 the diffusion is the Ornstein–Uhlenbeck process proposed by Vasicek (1972) for describing the evolution of interest rates. Under this model, the interest rate is normally distributed and can assume negative values with positive probability. For γ=1/2, model (1) becomes Feller's (1952) square root diffusion process introduced by Cox, Ingersoll and Ross (1985). In this model the variability of the interest rate depends on the square root of its level and declines as the interest rate approaches zero. If $2\kappa\theta \geq \sigma^2$, zero is an unattainable boundary. Otherwise, zero can be an absorbing barrier or a reflecting barrier. If γ is treated as a free parameter, the process is the mean reverting constant elasticity of variance proposed by Chan et al. (1992). The parameter γ captures the sensitivity of interest rate variability to the level of r and Chan et al. (1992) find that models with $\gamma \geq 1$ capture better the dynamics of interest rates.

For the purposes of the one-period setting I use an Euler discretization scheme of the continuous time model in (1), $r_{t+\Delta t} = r_t + \kappa(\theta - r_t)\Delta t + \sigma r_t^{\gamma} \varepsilon \sqrt{\Delta t}$, where ε is a random term from the standardised normal distribution. Suppose that the investment opportunity is a perpetuity with a riskless cash-flow per period equal to C. The cost of the investment is K. If the net present value at time t is positive, $I_t = \frac{C}{r_t} - K$, the investment is profitable and should be taken. Suppose that the investor can wait one period before deciding whether to invest. Using a second order Taylor expansion the expected value of the investment is given by:

$$\begin{aligned} &\mathrm{E}_t\left[I_{t+\Delta t}\right] = I_t + \mathrm{E}_t(r_{t+\Delta t} - r_t)\frac{\partial I_t}{\partial r_t} + \frac{1}{2}\mathrm{E}_t(r_{t+\Delta t} - r_t)^2 \frac{\partial^2 I_t}{\partial r_t^2} \\ &\Rightarrow \mathrm{E}_t\left[I_{t+\Delta t}\right] = I_t - \mathrm{E}_t(r_{t+\Delta t} - r_t)\frac{C}{r_t^2}\Delta t + \mathrm{E}_t(r_{t+\Delta t} - r_t)^2 \frac{C}{r_t^3} \end{aligned} \tag{2}$$

The difference between the one-period discounted expected investment value and the current investment value is equal to:

$$\frac{\mathrm{E}_t\left[I_{t+\Delta t}\right]}{(1+r_t\Delta t)} - I_t = \underbrace{\frac{I_t}{(1+r_t\Delta t)} - I_t}_{\text{cost of waiting}} + \underbrace{\frac{1}{(1+r_t\Delta t)}\left[-\mathrm{E}_t(r_{t+\Delta t} - r_t)\frac{C}{r_t^2} + \mathrm{E}_t(r_{t+\Delta t} - r_t)^2\frac{C}{r_t^3}\right]}_{\text{benefit of waiting}} \tag{3}$$

The first term is cost of waiting due to foregone interest. With nonstochastic interest rates the cost of waiting is the only term that determines the decision to invest and as long as interest rates are positive it is always optimal to invest immediately if the current NPV is positive. The second term captures the benefit of waiting and depends on the particular forms of the drift and diffusion components. For a linear interest rate drift and level dependent volatility the relationship in (3) is equal to:

$$\frac{\mathrm{E}_t\left[I_{t+\Delta t}\right]}{(1+r_t\Delta t)} - I_t = \underbrace{\frac{I_t}{(1+r_t\Delta t)} - I_t}_{\text{cost of waiting}} + \underbrace{\frac{\Delta t}{(1+r_t\Delta t)}\left[-\kappa(\theta - r_t)\frac{C}{r_t^2} + \sigma_t^2 r_t^\gamma \frac{C}{r_t^3}\right]}_{\text{benefit of waiting}} \tag{4}$$

The impact of the drift component is mixed and can have either a positive or a negative effect on the value of waiting. If θ-r>0, the interest rate is below the long run mean and the incentive to wait decreases since interest rates are expected to rise. Conversely, if θ-r<0, the interest rate is above the long run mean and the incentive to wait increases since interest rates are expected to fall. The last term captures the variability of the interest rate and has always a positive effect on the value of waiting. The option to wait is valuable when there is anticipated resolution of uncertainly about interest rates and therefore project's cost of capital. Given that the elasticity γ is found to be in the range of 1 to 1.5, the value of waiting is also inversely related to the level of interest rate. Ceteris paribus, when interest rates approach zero the value of waiting increases. The relationship between interest rates and investment is ambiguous since it depends on the individual impact of each component in relationship (3).

When interest rates are near the zero lower bound the relationship between interest rates and investment remains ambiguous since is depends on the effect of both the drift and the volatility component (the cost of waiting is almost zero). The value of waiting decreases due to the expected increase of the interest toward the long run mean (the long run mean is typically in the range of 2%-3%). On the other hand, the low level of interest rate increases the value of waiting.

Following Black (1995), suppose that the interest rate is the maximum of the shadow rate s_t and a lower bound r_L:

$$r_t = \max(s_t, r_L) \tag{5}$$

The interest rate is equal to the shadow rate whenever the shadow rate is above the lower bound or otherwise equal to the lower bound. If the interest rate follows (5), expression (4) becomes:

$$\frac{\mathrm{E}_t\left[I_{t+\Delta t}\right]}{(1+r_t\Delta t)} - I_t = \underbrace{\frac{I_t}{(1+r_t\Delta t)} - I_t}_{\cos t\ of\, waiting} + \underbrace{\frac{\Delta t}{(1+r_t\Delta t)}\left[-\mathrm{E}_t(\max(r_L, s_{t+1}) - r_t) + \mathrm{E}_t(\max(r_L, s_{t+1}) - r_t)^2 \frac{C}{r_t^3}\right]}_{benefit\ of\, waiting} \tag{6}$$

I assume that shadow rate follows the process, $ds_t = \kappa(\theta - s_t)dt + \sigma dZ_t$, and hence can take negative values. If the option is sufficiently in-the-money, eg., the shadow rate is well above the lower bound, the evolution of the nominal interest rate is driven by the Ornstein–Uhlenbeck process , $r_{t+1} = \max(r_L, s_{t+1}) = s_{t+1}$, with drift $\kappa(\theta - s_t)$ and volatility σ. The decision to invest is determined again by (4) with γ =0. How is the value of waiting affected when the shadow rate is well below the lower bound? To examine this case it is convenient to decompose the interest rate as follows:

$$r_{t+\Delta t} = \max(r_L, s_{t+\Delta t}) = \max(r_L, 0) + \max(0, s_{t+\Delta t}) , \tag{7}$$

which is the sum of a digital option that delivers the lower bound whenever the shadow interest rate is negative and zero otherwise and a call option on the shadow interest rate with a strike price of zero. Suppose that $s_{t+\Delta t} - N(\mu_t, \sigma\sqrt{\Delta t})$, where $\mu_t = s_t + \kappa(\theta - s_t)\Delta t$. The conditional expectations of the two components are given below:

$$\begin{aligned} E_t(\max(r_L, 0)) &= r_L \times N\left(\frac{r_L - \mu_t}{\sigma\sqrt{\Delta t}}\right) \\ E_t(\max(0, s_{t+1})) &= \mu_t\left[1 - N\left(-\frac{\mu_t}{\sigma\sqrt{\Delta t}}\right)\right] + \sigma\sqrt{\Delta t}\, f\left(-\frac{\mu_t}{\sigma\sqrt{\Delta t}}\right) \end{aligned} \tag{8}$$

where N and f are the cumulative distribution and the density function , respectively, of the standard normal variable. When $s_t \to -\infty$, $E_t(\max(r_L, 0)) = r_L$ and $E_t(\max(0, s_{t+1})) = 0$. If the shadow interest rate is negative and well below the lower bound, the expected interest rate change is zero since the nominal interest rates behaves as a deep-out-of the money option and $E[\max(r_L, r_{t+1})]$ converges to r_L.. The volatility term remains positive since there is still a probability, albeit small, that the interest rate will exceed the lower bound. A single path suffices to ensure that $\mathrm{E}_t(\max(r_L, s_{t+1}) - r_L)^2$ remains positive. Given that $r_L \Delta t \approx 0$, expression (6) is now equal to:

$$\mathrm{E}_t\left[I_{t+\Delta t}\right] - I_t = \Delta t \underbrace{\left[\mathrm{E}_t(\max(r_L, s_{t+1}) - r_L)^2 \frac{C}{r_L^3}\right]}_{benefit\ of\ waiting} \succ 0 \tag{9}$$

This is the main result of the paper. When interest rates are stuck at the lower bound and the shadow rate is well below the lower bound it is always optimal to wait and decide whether to invest at a future time period as more information accrues and uncertainty is resolved.

3. Multi-Period Model (Incomplete)

In this section I examine the optimal investment rule in continuous time and then solve the problem numerically using the Least-Square Monet Carlo method. To fix notation, the present value of a project that yields a stream of cash flows from t to t+T is given by:

$$V(r_t, t, T) = E_t\left[\int_t^{t+T} \exp\left\{-\int_t^s r_u du\right\} ds\right] = \int_t^T P(r_t, t, s) ds \tag{10}$$

where $P(r_t, t, s)$ is the price of a zero coupon bond that matures at time *s* given that *s*>*t*. The optimal time to invest in the project is given by the solution to the following optimal stopping problem:

$$F(r_t) = \sup E_t\left[\exp\left(-\int_t^{\tilde{\tau}} r_u du\right)\left(V(r_{\tilde{\tau}}, \tilde{\tau}, T) - I\right)\right] \tag{11}$$

where $\tilde{\tau}$ is a random stopping time and I is the sunk cost of the investment.

The short interest rate r is the maximum of the shadow rate s_t and a lower bound r_L, $r_t = \max(s_t, r_L)$ and the shadow rate follows the process, $ds_t = \kappa(\theta - s_t)dt + \sigma dZ_t$. The price of the zero coupon bond that pays one dollar at time *s* is given by:

$$P(X_t, t, s) = E_t^Q\left[\exp\left(-\int_t^s r_u du\right)\right] = E_t^Q\left[\exp\left(-\int_t^s \max(s_u, r_L) du\right)\right] \quad (12)$$

Under the shadow rate approach, the conditional expectation in (11) does not have a closed-form solution. Gorovoi and Linetsky (2004) provide and approximation solution of bond prices based on Weber-Hermite parabolic cylinder functions.

(**description of discrete time algorithm for LSMC**)

Appendix

The conditional expectation of the first component of the interest rate is given by**:**

$$E_t(\max(r_L, 0)) = r_L \times P(s_{t+1} \leq r_L) \tag{13}$$

The conditional expectation of the second component of the interest rate is given by**:**

$$E_t(\max(0, s_{t+1})) = E_t(s_{t+1} \big| s_{t+1} \geq 0) = \int_0^{+\infty} s_{t+1} \frac{1}{\sqrt{2\pi\sigma^2 \Delta t}} e^{-\frac{(s_{t=1} - \mu_{t+1})^2}{2\sigma^2 \Delta t}} ds_{t+1} \tag{14}$$

Define $h_{t+1} = \dfrac{s_{t+1} - \mu_{t+1}}{\sigma\sqrt{\Delta t}}$ so $dh_{t+1} = \dfrac{ds_{t+1}}{\sigma\sqrt{\Delta t}}$.

$$\int_{-\frac{\mu_{t+1}}{\sigma\sqrt{\Delta t}}}^{+\infty} s_{t+1} \frac{1}{\sqrt{2\pi\sigma^2\Delta t}} e^{-\frac{h_{t+1}^2}{2}} \sigma\sqrt{\Delta t} dh_{t+1} = \int_{-\frac{\mu_{t+1}}{\sigma\sqrt{\Delta t}}}^{+\infty} (h_{t+1}\sigma\sqrt{\Delta t} + \mu_{t+1}) + \frac{1}{\sqrt{2\pi}} e^{-\frac{h_{t+1}^2}{2}} dh_{t+1}$$

$$= \mu_{t+1} \int_{-\frac{\mu_{t+1}}{\sigma\sqrt{\Delta t}}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{h_{t+1}^2}{2}} dh_{t+1} + \sigma\sqrt{\Delta t} \int_{-\frac{\mu_{t+1}}{\sigma\sqrt{\Delta t}}}^{+\infty} h_{t+1} \frac{1}{\sqrt{2\pi}} e^{-\frac{h_{t+1}^2}{2}} dh_{t+1}$$

$$= \mu_{t+1} \left[1 - N\left(-\mu_{t+1} \big/ \sigma\sqrt{\Delta t}\right)\right] + \sigma\sqrt{\Delta t}\, f\left(-\mu_{t+1} \big/ \sigma\sqrt{\Delta t}\right)$$

References

Bauer, M., and G. Rudebusch, 2013. Monetary Policy Expectations at the Zero Lower Bound, FRBSF Working Paper, 2013-18.

Bernanke, B., and V. Reinhart, 2004. Conducting Monetary Policy at Very Low Short-Term Interest Rates, American Economic Review, 94, 85–90.

Black F. and P. Karasinski, 1991. Bond and Option Pricing When Short Rates Are Lognormal, Financial Analysts Journal, 47, 52.59.

Black, F., 1995. Interest Rates As Options, Journal of Finance, 50, 1371–1376.

Chan, K.C., Karolyi G.A., Longstaff F.A., and A.B. Sanders, 1992. An Empirical Comparison of Alternative Models of the Short-Term Interest Rate. Journal of Finance, 47, 1209-1227

Christensen, J. H. E., and G.D. Rudebusch, 2014. Estimating Shadow-Rate Term Structure Models with Near-Zero Yields, Journal of Financial Econometrics, forthcoming.

Cox, J.C., Ingersoll, J.E., and S.A. Ross, 1985. A Theory of the Term Structure of Interest Rates, Econometrica, 53, 385–408.

Dixit, A. and R. Pindyck, 1994. Investment Under Uncertainty, Princeton University Press, Princeton, NJ.

Eggertsson, G, and M. Woodford, 2003. The Zero Bound on Short-Term Interest Rates and Optimal Monetary Policy, Brookings Papers on Economic Activity,1:139–211.

Feller, W., 1952. The Parabolic Differential Equations and the Associated Semi-Groups of Transformations, Annals of Mathematics, 55, 468–519.

Gorovoi, V., and V. Linetsky, 2004. "Black's Model of Interest Rates as Options, Eigenfunction Expansions, and Japanese Interest Rates, Mathematical Finance, 14, 49–78.

Kim, D., and K. Singleton, 2012. Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields, Journal of Econometrics, 170, 32–49.

Krippner, L., 2012. Modifying Gaussian Term Structure Models When Interest Rates are Near the Zero Lower Bound" Reserve Bank of New Zealand Discussion Paper 2012/02.

Krugman, P. 1998. It's Baaack! Japan's Slump and the Return of the Liquidity Trap, Brookings Papers on Economic Activity, 2: 137–87.

McDonald, R. and D. Siegel, 1986. The Value of Waiting to Invest, Quarterly Journal of Economics, 101, 707–28.

Vasicek, O., 1977. An Equilibrium Characterization of the Term Structure, Journal of Financial Economics, 5, 177-188.

Wu, C., and F. Dora Xia, 2013. Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. Manuscript, University of California at San Diego.