Corporate liquidity, and dividend policy under uncertainty

Nicos Koussis[1], Spiros H. Martzoukos[2], and Lenos Trigeorgis[3]

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1. Lecturer, Department of Economics, Finance and Accounting, Frederick University Cyprus, 7 Frederickou Street, Nicosia, 1036, CYPRUS. Email: bus.kn@ac.cy

2. Associate Professor, Department of Accounting and Finance, University of Cyprus, P.O. Box 20537, CY 1678 Nicosia, CYPRUS. : Email: baspiros@ucy.ac.cy

3. Professor of Finance, King's College London and Bank of Cyprus Chair Professor of Finance, Department of Accounting and Finance, University of Cyprus, P.O. Box 20537, CY 1678 Nicosia, CYPRUS. Email: lenos@ucy.ac.cy

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Abstract

We examine firm valuation with optimal liquidity (retained earnings) and dividend choice under revenue uncertainty that incorporates debt financing and bankruptcy costs. We revisit the conditions for dividend policy irrelevancy and the role of retained earnings and dividends. Retained earnings have a net positive impact on firm value in the presence of growth options, high external financing costs and low default risk. High levels of retained earnings enhance debt capacity but have a negative effect on equity value. Opposite directional effects of retained earnings on equity and debt values lead to a U-shaped relation with firm value. Agency conflicts over dividend policy among equity and debt holders are more prevalent for firms with higher profitability, low volatility and high level of growth options.

1. Introduction

Miller and Modigliani (1961) have shown that dividend policy is irrelevant in a frictionless market, however, dividend irrelevancy does not hold in the presence of costly external financing, default risk, bankruptcy costs or costly growth options. Recent developments using a contingent claim approach provide a promising framework for the analysis of investment, optimal capital structure and dividend policy incorporating default risk and growth options. This strand of literature, building on Leland (1994) (see also Mauer and Sarkar, 2005, Sundaresan and Wang, 2006) is however yet largely agnostic about corporate liquidity and dividend policy. This is because these models assume that any excess cash is distributed in the form of dividends to the equity holders of the firm, while, in periods of negative cash flows, the firm resorts to external financing to finance the shortfall. One of the difficulties of incorporating liquidity choice in a contingent claims framework is in dealing with path-dependency arising from the fact that one needs to keep track of the history of cash balances retained. In this paper we build the theoretical framework in order to investigate these issues in a contingent claim setting. We provide a numerical model that incorporates revenue uncertainty, path-dependent liquidity choice (retained earnings), growth options, debt financing with risk of default, and costly external financing. In our model retained earnings are held in the form of a liquid asset that earns a fixed per period interest and can be used to reduce future external financing costs and reduce the risk of costly default and the incurrence of bankruptcy costs. A number of new implications are derived and a simple benchmark analytic model with growth options is used to verify the numerical solutions developed.

First, we focus on an unlevered firm and we show that the irrelevancy of retained earnings and dividends holds only in the absence of default risk, under the condition that the return earned on retained earnings is equal to the risk-free rate (used as discount rate) and in the absence of external financing costs. We show that the presence of default risk can create a negative impact of retained earnings on firm value since accumulated cash from earlier periods may be foregone if the firm goes bankrupt.

Secondly, our results show that the higher the expected benefits of the growth option and the higher the external financing costs the more important the role of retained earnings. On the other hand, we show that external financing costs have minor impact on firm values when their only role is to finance liquidity shortages necessary to avoid default. Lins et al. (2010) survey of CFOs from 29 countries and have shown that the main driver of holding liquidity is indeed the financing of future investment opportunities. Brown and Petersen (2011) show that cash balances may enable a firm to smooth R&D spending which is also in line with evidence of the role of cash balances in financing growth. Riddick and Whited (2009) develop a theoretical model allowing a precautionary motive of holding cash and Palazzo (2009) shows that cash rich firms may earn superior returns due to precautionary motives. Our model highlights the risks of default may result in the opposite effect, i.e., the risk of foregone cash flows reducing or even eliminating the precautionary motive of accumulated cash.

Thirdly, we show that the incentive to keep high retained earnings (low dividends) is affected by firm profitability and the initial (accumulated) cash balances available. For firms with low profitability or low initial cash balances, retained earnings may not be sufficient to avoid default, thus it is optimal for the firm shareholders to reduce cash balances by paying higher dividends (and resort to external financing in the future if needed). For firms with high profitability or high initial cash balances it is optimal for the firm shareholders to increase the accumulation of cash balances further to avoid costly external financing in the future. The role of initial level of cash balances highlights important differences for the behavior of young firms (with low initial cash balances) and more mature firms (with higher accumulated cash).

Fourthly, we show that higher revenue volatility increase default risk and thus reduce the importance of retained earnings since equity holders fear that they may be foregone.

Fifthly, we study the role of retained earnings in the presence of debt financing. We consider the cases of first-best (firm value maximization) which also assumes that retained earnings are obtained by debt holders in the event of default versus the second-best solution (equity-value maximization) which also assumes that retained earnings are distributed to equity holder prior to default. In the presence of debt the aforementioned effects of profitability, volatility, growth options and external financing costs continue to hold but apply for equity value. On the other hand, higher retained earnings have an enhancing role in debt capacity both under a first-best and under a second-best solution since the risk of default is reduced. The opposite directional effects of retained earnings on equity and debt value may have a U-shape effect on firm value. We find that under first-best firm value is more likely to be maximized at high plowback when the risk of default is low (high profitability, low volatility) and in the presence of high value of growth options and high external financing costs. A low plowback is preferred at high levels of default risk and when the value of growth options and external financing costs are low. Under a second-best solution we always obtain a solution of low plowback. Agency costs over dividend policy exist when there are deviations between the optimal dividend policy between the first-best and second-best solution. These deviations are more prevalent for firms with higher profitability, lower volatility, higher levels of growth options and external financing costs. Agency costs estimates do not exceed 2% of firm value under realistic parameter values.

Finally, the role of investment timing within this context is also considered. We show that with low initial profitability (and low accumulated earnings from earlier periods) the firm has an advantage to delay the exercise of the option in order to avoid immediately incurring high external financing. When the firm has a high initial profitability (and high accumulated earnings from earlier periods) then early exercise becomes more attractive since it can enhance revenues early-on without the firm having to incur high external financing costs.

Our paper is organized as follows. Section 2 provides a literature review, Section 3 develops the theoretical framework, Section 4 provides sensitivity analysis and model predictions and section 5 concludes. An appendix provides a benchmark analytic model and provides accuracy tests of the numerical model.

2. Literature review

Early theories of dividends include Miller and Modigliani (1961) dividend irrelevancy in a frictionless market, theories incorporating the effect of taxes (e.g., Brennan, 1970 and Miller and Scholes, 1978) and the use of dividends as a signaling for the future growth prospects of the firm (e.g., Miller and Rock, 1985). [1] The pecking order theory (see Myers, 1984) which is based on asymmetric information suggests that retained earnings should be the first source of financing followed by debt and then equity. Although our model does not capture asymmetric information it provides new implications about a complementary relationship between retained earnings and debt which is not highlighted in pecking order theory. Asymmetric information can be implicitly captured in our context by increasing the costs of external financing.

Agency theories also provide prominent explanations of firms' dividend decisions. Easterbrook (1984) points out that higher dividend may provide a disciplinary mechanism that can reduce the manager-shareholder conflicts since the have to resort to the markets for financing of investment opportunities. This is similar to the "free-cash flow" argument of Jensen (1986) which also predicts that larger dividends may be optimal so as to reduce the incentives of managers to expropriate value from large accumulated cash balances.[2] In our model agency conflicts of this sort can be implicitly captured by reducing the return earned on accumulated cash balances (see also Asvanunt et al. 2010) reflecting the increasing agency costs of maintaining high cash balances. Agency conflicts over dividend policy between equity and debt holders

[1] Our paper does not incorporate the effects of personal taxes and signalling.

[2] A recent article in the Economist with a title "The Rise of Distorporation" on Oct.26, 2013 points the creation of many firms in the US adopting structures such as Master Limited Liability (MLP) which keep no retained earnings in order to reduce the payment of corporate taxes and enforce a market disciplinary role for managers. Allen et al. (2000) point out that another positive side effects of higher dividends is the increased monitoring role of institutional investors which are clienteles in firms paying higher dividends.

are under-researched. We solve for first-best (firm maximization) and second-best (equity-value maximization) optimal retention policies and also discuss the impact of the risk of equity holders distributing accumulated earnings prior to default on claim holders value and the value of the firm. Hirth and Uhrig-Homburg (2010) show the positive role of liquidity in mitigating the agency costs between debt and equity holders.

More recent theoretical developments include the general framework of Gamba and Triantis (2008) who analyze a firm's dynamic financing, investment and cash retention policies. With respect to the positive effects of increased liquidity, they show (similarly to us) that accumulated cash can be used to finance growth and to avoid issuance costs. On the negative side, they show that accumulated cash has a tax disadvantage relative to debt financing. The presence of default risk with fire-sales or liquidation at a discount increases the role of cash balances, a result we verify in our setting. Our model shares several similarities with their context, however, we show that the presence of debt financing and bankruptcy costs results in a U-shape relationship with retained earnings. Our simpler setting also allows us to carefully examine the effects leading to the irrelevancy of retained earnings and dividends, the role of profitability and revenue volatility.

Recently, Copeland and Lyasoff (2013) analyze an unlevered firm retention policy in the presence of a growth option and costly external financing, however, their model does not allow for default risk and does not accommodate the effects of bankruptcy costs. Kisser (2013) shows that cash have a real option value since they can be used to avoid or reduce issuance costs; however, he does not discuss the role of default risk, debt and bankruptcy costs. His model allows quadratic agency costs of free-cash flows which implies an amplification of agency issues when cash balances are high. With respect to volatility, Kisser

(2013) shows similarly to our analysis that higher volatility reduces the value of holding cash. However, in his model agency costs drive this result whereas in our model the main driver of this result is default risk.[3]

Boyle and Guthrie (2003) study the effect of cash balances on the optimal timing of investment in the presence of constraints. They show a V-shape investment trigger as a function of cash balances: for small level of cash balances the investment trigger is reduced (firm accelerates investment) and for high cash balances an increase in cash balances results in an increase in the investment trigger (firm delays investment). Hirth and Viswanatha (2011) explain that the U-shape of the investment trigger with respect to cash balances may be caused when the firm balances the trade-offs between present and future financing costs. We show that in the absence of constraints the opposite result may hold: firms with low cash balances may delay investment to avoid external financing costs, whereas, firms with high level of initial profitability and initial cash balances may invest to accelerate the enhanced benefits of the growth option since external financing costs can be reduced with the use of available cash. Asvanunt et al. (2010) model is also closely related to our model. However, in Asvanunt et al. (2010) the cash balances can only be retained for one period while our model allows cash balances to be accumulated over longer horizons.

Empirical evidence on dividend policy generally shows that dividend payouts are positively related to profitability (measured by ROA) and size (measured by assets), and negatively related to growth opportunities (measured by R&D/Assets) (see Fama and French, 2001). Fama and French (2002) connect these findings with the predictions of trade-off and pecking order theories. They point out that the empirical result that firms with higher profitability and lower growth opportunities pay less dividends is consistent with trade-off theory. However, to our knowledge no formal model makes these predictions explicitly. Our

[3] In his empirical analysis, Kisser (2013) finds that cash balances do not have a statistical significant impact on firm value, a result that demonstrates dividend irrelevancy. Our analysis shows under which conditions such irrelevancy may hold.

model provides the explicit link of dividend payout with growth opportunities showing that dividend payout is indeed negatively associated with growth opportunities; however, we show that the effect of profitability on dividend payouts is much more subtle. We show that equity holders in firms that are less profitable firms may actually have an incentive to increase dividends so as to avoid foregoing accumulated cash in the future in the event of default. On the other hand when firms operate with sufficiently high profits the firm may more safely plowback earnings in order to avoid external financing csots (since the risk of default is low). Thus, we point out that the observed positive relationship between dividend payouts and profitability may be due to other factors not explained by trade-off theory.[4] Bates et al. (2009) show an increase in cash balances over time and that cash holdings are positively associated with volatility, market to book, R&D and negatively associated with leverage. The effect of market to book and R&D is in alignment with our model's predictions of using accumulated cash to finance growth. Furthermore, the negative effect of cash holdings with leverage is consistent with second-best solutions where the higher the risk of default the more likely that firm value is optimized at lower levels of plowback. The positive effect of volatility on cash holdings in our setting can only be justified if the increased volatility is driven by a higher value of growth options in which case firms accumulate cash in order to finance the growth options.

3. The theoretical framework

3.1. The model

We assume that the firm operates in 3 periods, $t = 0$, $t = T_1$ and $t = T_2$. The last year of operations occurs at T_2 where we assume that the firm stops its operations and distributes all extra cash as dividends or defaults.[5] Let P denote the firm's present value of revenues over a horizon T_1 which follow a Geometric Brownian Motion (GBM) of the following form:

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[4] Clientele effects may provide a plausible explanation since clientele effects may be more prevalent in large firms attracting large institutional investors. Thus, the positive relationship between dividend payout and profitability (and the size of the firm) may be due to efforts of the large firms to attract large institutional investors.

[5] For simplicity we will set the incremental period ($T_2 - T_1$) to be equal to the first stage period T_1.

$$\frac{dP}{P} = \alpha dt + \sigma dZ \tag{1}$$

where α denotes a constant drift, σ is a constant volatility and dZ is the standard Weiner process. Under risk-neutrality or spanning assets assumption the constant drift is replaced with the risk-free rate *r*. Further, it is assumed that the firm's revenues actually grow at a rate g = *r*-*δ*. Thus, *δ* defines a form of dividend-yield adjustment that captures competitive erosion reducing the growth of revenues. We allow per period operating costs to be different, thus, the operating costs are C_0, C_1, C_2 for period 0, 1 and 2 respectively. A corporate tax rate τ applies, thus, the after tax profits of the firm per period are $(P\text{-}C_i)\,(1\text{-}\tau)$, i=0,1,2.

We also assume that the firm has an existing cash balance at $t = 0$ equal to X_0 and that the firm has the possibility to raise debt at $t = 0$. Debt will subsequently require a coupon payment C_R per period in periods 1 and 2. In the event of default debt holders obtain the value of unlevered assets by incurring proportional bankruptcy costs *b*.

Furthermore, assume x_t is the proportion of revenues retained in period $t = 0, T_1$. In the last period T_2 any retained earnings accumulated are assumed to be paid out to the shareholders. Then, $(1 - x_t)$ defines the dividend payout of the firm in the corresponding periods. It is assumed that liquid assets are invested in an asset that earns r_x annually continuously compounded. We also assume that if earnings net of taxes in a period and the available cash balances retained from earlier periods are not sufficient the firm needs to resort to external financing which requires a cost I_E. The external financing cost may include a fixed cost component F_E and variable cost v_E which is proportional to the level of financing required. In practice, the cost of external financing includes a fixed component for the services of the underwriter (e.g., for the road show, legal advice and preparation of prospectus) and also a variable component depending on the amount raised in the new issue which may also reflect spread costs. However, one may also consider that the costs of external financing include agency costs arising from asymmetric information. In this case the firm may be required to issue securities at a discount to their fair value thus implicitly incurring an additional cost at the time of the issuance.

The firm has the option to expand revenues (growth option) by $e_1 > 1$ at T_1 by incurring an additional cost X_E which may also result in an increase in revenues in the last period by $e_2 > 1$. When the expansion of revenues starts after investment we set $e_1 = 1, e_2 > 1$. In this section we assume no investment timing exists. We relax this assumption in subsequent sections.

In order to obtain a solution to the above problem we build a forward-backward algorithm of exhaustive search on a binomial tree by optimizing among a discrete set of choices for retained earnings $x_0 \in [0,1]$ and $x_1 \in [0,1]$. At t = 0 the firm chooses $x_0 \in [0,1]$ which defines the level of retained earnings and dividends paid to shareholders. The profits after tax (Π), retained earnings (R) and the dividends (D) for period t = 0 conditional on the choice of x_0 are thus:

$$\Pi_0 = (P - C_0)(1-\tau)$$

$$R_0 = x_0\left[(P - C_0)(1-\tau) + X_0\right] \tag{2}$$

$$D_0 = (1 - x_0)\left[(P - C_0)(1-\tau) + X_0\right]$$

Let $\tilde{B}_t$ and $\tilde{V}_t$ denote the expected present value of debt and levered firm value at t respectively. The value of the levered firm obtained by shareholders at t = 0 is equal to the sum of dividends payments obtained D_0, the expected present value of the levered firm from future activities $\tilde{V}_0$ and the value of debt financing raised $\tilde{B}_0$:

$$V_0 = \max_{x_0}(D_0 + \tilde{V}_0 + \tilde{B}_0) \tag{3}$$

The expected present value of the levered firm and the value of debt will be determined by taking expectations using the binomial tree probabilities (see below).

The levered firm value can also be equivalently found as the sum of dividends and the expected value of unlevered assets $\tilde{V}_{0U}$ and the expected present value of the net tax benefits of using debt $T\tilde{B}_0$:

$$V_0 = \max_{x_0}(\Pi_0 + \tilde{V}_{0U} + T\tilde{B}_0) \qquad (3')$$

The expected present value of levered firm, debt, unlevered assets and tax benefits are obtained under risk-neutrality using a binomial tree approach. We use a standard formulation of the lattice parameters for the up and down jumps and the up and down probabilities (see Cox, Ross and Rubinstein, 1979) which requires that $u = e^{\sigma\sqrt{dt}}$, $d = e^{-\sigma\sqrt{dt}} = \frac{1}{u}$, $p_u = \frac{e^{(r-\delta)dt} - d}{u-d}$, $p_d = 1 - p_u$, where $dt = \frac{T_1}{N_1}$. We use N_1 steps for the first T_1 years. Then for *each* end state value of *P* of the lattice at T_1 a new a lattice for the remaining $T_2 - T_1$ years with size $N_2 = (T_2 - T_1)/T_1 N_1$ is built. Thus, several lattices emanate, one from each ending node at T_1.

Retained earnings of period *t* = 0 earn interest r_X and thus become $R_1 = x_0(P - C_0)(1-\tau)e^{(r_x T_1)}$ in period T_1.

3.1.1. No financing needs at T_1

At T_1 the mode of operations can be either to remain with the option not exercised (mode *S*) or exercise the growth option (mode *E*). When the profits net of any investments costs are positive, i.e, when:

$$\Pi_1^S = (P_1 - C_1 - C_R)(1-\tau) \geq 0 \text{ or } \Pi_1^E = (e_1 P_1 - C_1 - C_R)(1-\tau) - X_E \geq 0 \qquad (4)$$

then there are no financing needs.

The firm optimizes retained earnings at T_1 by choosing $x_1 \in [0,1]$ so as to maximize the sum of the dividends at T_1 and expected value of dividends of next period. First, define the unlevered profits at T_1 as:

$$\Pi_{1U}^{S} = (P_1 - C_1)(1-\tau) \text{ or } \Pi_{1U}^{E} = (e_1 P_1 - C_1)(1-\tau) \tag{5}$$

Conditional on the new choice of retained earnings x_1 at $t = T_1$ and depending on the mode of operations being either S or E we have:

$$R_1^i = x_1(\Pi_1^i + x_0\left[(P - C_0)(1-\tau) + X_0\right]e^{(r_X T_1)})$$

$$D_1^i = (1 - x_1)(\Pi_1^i + x_0\left[(P - C_0)(1-\tau) + X_0\right]e^{(r_X T_1)}) \tag{6}$$

$$B_1^i = C_R + \tilde{B}_1^i$$

$$V_{1U}^i = \Pi_{1U}^i + \tilde{V}_{1U}^i$$

$$TB_1^i = \tau C_R + \widetilde{TB}_1^i$$

$$V_1^i = D_1^i + \tilde{V}_1^i,\ i = S, E.$$

In equation (6) the new retained earnings at T_1 are determined as a proportion x_1 of the sum of the profits of the current period and the accumulated earnings of the prior period. The remaining of this sum $(1 - x_1)$ is distributed as dividends. Since there is no default under these states debt value is obtained as the sum of current coupon plus the expected present value of debt arising from next period. The expected values of all variables are evaluated by building a tree forward conditional on the current state of revenues at T_1, the retained earnings kept at T_1 and the state (normal mode S or growth E). Finally note that $V_1^i = D_1^i + \tilde{V}_1^i > 0$, $i = S, E$ since $D_1^i \geq 0$ and $\tilde{V}_1^i > 0$.

3.1.2. Positive financing needs at T_1

At T_1 and depending on the mode being S or E if:

$$\Pi_1^S = (P_1 - C_1 - C_R)(1-\tau) < 0 \text{ or } \Pi_1^E = (e_1 P_1 - C_1 - C_R)(1-\tau) - X_E < 0 \tag{7}$$

then the firm can only choose retained earnings from the available liquid assets accumulated from earlier period thus:

$$R_1^i = x_1(x_0\left[(P - C_0)(1-\tau) + X_0\right]e^{(r_X T_1)}), i = S, E \tag{8}$$

Since in this state $\Pi_1^i < 0, i = S, E$ only the amount of accumulated earnings from previous period that was not distributed as dividend will be used to finance the shortfall. We call this the amount of financing from retained earnings (F) and it is given by:

$$F_1^i = (1 - x_1)(x_0\left[(P - C_0)(1-\tau) + X_0\right]e^{(r_X T_1)}), i = S, E \tag{9}$$

If $\Pi_1^i + F_1^i < 0, i = S, E$ then the firm needs to resort to external financing and incur issuance costs I_E (if it chooses to stay in operations). The issuance costs include a fixed and a variable component and are thus equal:

$$I_E = F_E + v_E(F_1^i + \Pi_1^i), i = S, E \tag{10}$$

where the amount $F_1^i + \Pi_1^i, i = S, E$ defines the deficit that requires financing.

This means that the value at T_1 under this scenario is:

$$V_1^i = \max(\Pi_1^i + F_1^i - I_E + \tilde{V}_1^i, 0), i = S, E \tag{11}$$

If $\Pi_1^i + F_1^i > 0$ then costly external financing can be avoided and thus:

$$V_1^i = \max(\Pi_1^i + F_1^i + \tilde{V}_1^i, 0), i = S, E. \tag{12}$$

If $V_1^i > 0$ then default is avoided and so debt value is:

$$B_1^i = C_R + \tilde{B}_1^i$$

$$TB_1^i = \tau C_R + T\tilde{B}_1^i,\ i = S, E. \tag{13}$$

If $V_1^i = 0$ then default is triggered and so:

$$B_1^i = (1-b)(\Pi_{1U}^i + \tilde{V}_{1U}^i + R_1)$$

$$TB_1^i = 0,\ i = S, E. \tag{14}$$

3.1.3. Optimization at T_1 and values at T_2

The optimal value at T_1 will be obtained as the maximum between the value under the normal (*S*) or growth option (*E*) state:

$$V_1^* = \max_{x_1}(V_1^S, V_1^E) \tag{13}$$

In the last period we assume the firm distributes all profits and retained earnings as dividends. Thus:

$$V_1^i = \max(\Pi_2^i + R_2^i, 0) \tag{14}$$

where $R_2^i = e^{(r_X(T_2 - T_1))} R_1^i$, $\Pi_2^S = (P_2 - C)(1-\tau), \Pi_2^E = (e_2 P_2 - C)(1-\tau)$, $i = S, E$

If $V_1^i > 0$ then

$$B_2^i = C_R,\ TB_2^i = \tau C_R,\ i = S, E\ . \qquad (15)$$

One the other hand, if $V_1^i = 0$ then default is triggered so:

$$B_2^i = (1-b)(\Pi_{2U}^i + R_2^i) \text{ and } TB_2 = 0,\ i{=}S{,}E. \qquad (16)$$

The appendix provides an analytic benchmark model for the special case of no debt and liquidity choice and in the absence of external financing costs. This model is used to test the accuracy of the numerical model. The results indicate that the numerical model provides an accurate solution both in the absence and in the presence of growth options which is re-assuring of using the numerical method to draw conclusions on issues of liquidity choice in the presence of debt, costly bankruptcy and costs of external financing which have been incorporated in the numerical model.

3.2. Model implications

In this section we discuss the main assumptions and forces driving retained earnings/dividend choice in our model and form our predictions which are then verified in the following section via extensive sensitivity results.

First, our assumption of revenues following a Geometric Brownian Motion (GBM) follows standard practice in contingent claims models. This can be considered to be driven by the value of a commodity that the firm sells which drives the uncertainty in the model (e.g., Mauer and Sarkar, 2005). Other papers use the GBM assumption as the driving process for the earnings before interest and taxes (EBIT) (e.g., see Goldstein et al., 2001, Hackbarth, et al. 2007). Our assumption implies that the variance of revenues increases with time and that longer the intervals between decisions increase both the upside and downside of firm's profitability. This may have substantial effects on the choice of retained earnings (as opposed to for example choice of using a mean-reverting process for revenues). A higher upside potential for revenues may imply that retained earnings may not eventually be needed in the future in order to finance shortfalls or even the growth option. In the case of a high upside potential, the firm may choose to distribute the earnings retained without further jeopardizing that they are foregone in the future so the risk of foregone cash flows is mitigated. On the other hand, however, an increase in the downside values of revenues can result in an increase in the risk that the firm's equity holders will forego the retained earnings of the previous periods (if the firm defaults). The GBM assumption also has implications for multi-stage decisions since it well-known that in compound option settings the correlation between the Brownian motions at different points has an impact on firm value (see Koussis, et al., 2013). The larger the distance between decision points (in our setting between T_2 and T_1) the lower the correlation between the Brownian motions which implies less predictability of revenues of T_2 as of T_1. This obviously has an effect on planning and will make commitment of resources either through new investments or higher retained earnings less likely the longer the distance between the decision points.

Secondly, our analysis adds two basic trade-offs for the firm's shareholders regarding their choice of the level of retained earnings. On the positive side, higher retained earnings reduce external financing costs needed to finance a future shortfall or a growth option. In the presence of debt, retained earnings can also be used to avoid costly default and bankruptcy costs and thus may enhance the debt capacity of the firm and the tax benefits of debt. These roles of retained earnings can be clearly seen in equations (9), (11) and (12) where the retained earnings can be used to finance shortfalls or the growth option. Also in equation (14) the retained earnings accumulated can used to reduce the risk of default in the last period. On the negative side, retained earnings may be foregone in the presence of default risk (see equation 11, 12 and 14 where firm value can become zero and accumulated retained earnings will be lost). Despite the risk faced by equity holders that retained earnings are lost in the event of default, the debt holders are benefited by the higher level of retained earnings since in the event of default the form part of the assets that can be recovered net of bankruptcy costs (see equations 14 and 16). This implies that higher retained earnings may enhance debt capacity. In a subsequent section we also consider the second-base case where accumulated cash balances are distributed to equity holders prior to default, however, as we show debt values are still enhanced at higher levels of retained earnings since the risk of default is reduced.

Thirdly, our setting provides insights with respect to the role of retained earnings in the presence of growth options. First, in the case where the growth option expansion of revenues starts immediately at T_1, then $e_1 > 1$ and the external financing needs may be reduced since available cash from the growth option itself can be used to finance shortages (see equations 4 and 7). In this case the role of retaining earnings from earlier periods will be reduced. When however the enhancement of revenues of the growth option accrues in a future period $(e_1 = 1, e_2 > 1)$ while the investment cost X_E needs to paid immediately then the retaining earnings from earlier periods becomes important in order to reduce external financing costs. The role of retained earnings is expected to be higher the higher the accrued benefits (e_2) and the higher the external financing costs (see equations 9,10 and 11).

Fourthly, our model assumes that retained earnings are held in the form of a liquid asset that earns a return r_x per period until they are eventually used to finance growth investments or liquidity shortages, distributed as dividends or end up in the hands of debt holders in the event of default (or in the hands of equity holders in the case of second-best solution). In contrast, Boyle and Guthrie (2003) use a separate process to describe the evolution of cash balances which allows a role for the volatility of cash balances.[6] However, their approach does not allow for the analysis of retention policies. Our assumption of fixed return of retained earnings implies that retained earnings can be used to reduce volatility since in low states the firm can use retained earnings to alleviate default. As pointed out earlier this may have a beneficial role in the presence of debt since it may enhance debt capacity. On the other hand retained earnings may be foregone if unfavorable states materialize.

[6] They show that higher cash flow volatility results in the firm investing earlier to avoid foregoing cash flows in the future.

Finally, we note that the simple three period framework does not have an impact on the implications drawn from our study. Our model allows for retained earnings to have a role in two consecutive periods capturing the essential dynamics and trade-offs involved in the decision process. Furthermore, although our model does not distinguish between new debt or equity issues at T_1 it still allows for external financing, thus capturing the basic dynamic financing choices of the firm so that the role of retained earnings is not overstated.

4. Numerical Results

In subsection 4.1 and 4.2 we analyse the conditions under which our model predicts dividend irrelevancy and the factors that affect retained earnings/dividend choice for an unlevered firm. In section 4.3 we incorporate the effect of debt and bankruptcy costs and in subsection 4.4. we investigate the effect of investment timing.

4.1.Dividend/retained earnings irrelevancy for an unlevered firm

In this subsection we present the numerical results and the cases where our model predicts dividend policy/retained earnings to be irrelevant for an unlevered firm. In this and subsequent results we set the operating costs of the initial period $C_0 = 0$ so as to allow for a sufficiently high retained level if the firm chooses a high plowback.[7]

Figure 1 shows numerical results both in the presence and in the absence of a growth option. In both these cases we assume that operating costs and coupons are zero in all periods (no default risk). We use P =100, $C_0=C_1=C_2=C_R=X_0=0$, $\tau = 0.3$, $r = \delta = 0.05$, $\sigma = 0.2$, $T_1=5$, $T_2=10$. Our parameters for the risk-free rate, the volatility of revenues and the corporate tax rate are generally consisent with prior studies (e.g., see

[7] Equivalent results can be obtained when the initial operating cost is positive and we allow for an initial cash balance X_0 (see equation 2).

Leland, 1994, Mauer and Sarkar, 2005, Goldstein et al.2001 and Henessy et al. 2007). The 5-year horizon reflects typical horizons for firm product development (e.g., see Pennings and Lint, 1997, for a particular case of Philips in the multimedia business).[8] We discuss the implications of varying other parameters in subsequent sections. We also assume that the return earned on retained earnings equals the discount rate, i.e, $r_x = r = 0.05$. In the "without growth" case of panel A we use $e_1=e_2=1$ and in the "growth" case of panel B we use an option cost $X_E = 100$ and $e_1=1$, $e_2=3$ which assumes that the enhancement of revenues of the growth option accrue in the future and thus the growth option cannot be self-financed. We analyze the cases without ($v_E = 0$) and with external financing costs ($v_E = 0.1$). Henessy and Whited (2007) estimate external financing costs to be between 5%-10.7% depending on firm size. All fixed external financing costs are for simplicity set to zero.

[Insert Figure 1 & 2]

Figure 1, panel A shows that in the absence of default risk and growth options, firm value is invariant to the proportion of profits retained earnings (equivalently the amount paid as dividends). As expected the level of external financing costs does not affect this result since there are no expected financing needs in the absence of operating costs and risk of default. Our investigation of the decisions at T_1 also reveals that a plowback equal to zero is optimal irrespective of the level of revenues. We should note that a small reduction in the return earned on retained earnings from $r_x = 0.05$ to $r_x = 0.04$ would result in a break-down of dividend/retained earnings irrelevancy. In the case with $r_x = 0.04$ ($<r=0.05$) we find that firm value decreases with retained earnings from 166.97 (plowback=0) to 163.56 (plowback =1). Similarly, if $r_x = 0.06$

[8] A 5 year horizon may be considered long since it creates a high range of potential revenue outcomes at T_1 and T_2 and reduces the potential of planning, in particular with respect to retained earnings and future financing needs (see the discussion of previous section). We explore shorter horizons in additional sensitivity results and report our results next. Our general implications do not change although we verify that shorter horizons allow for better planning and an improved role for retained earnings.

(>r=0.05) then firm value is increasing in retained earnings with a firm value of 169.77 (plowback =0) to 177.13 (plowback =1).

In Figure 1, in panel B we investigate the case with a growth option. Now, in the presence of external financing costs retained earnings add some value since retained earnings can be used to finance the growth option. In the absence of external financing costs, retained earnings/dividends remain irrelevant even in the presence of a growth option.

Figure 2 analyzes the case with positive operating costs thus allowing for the possibility of default. In Figure 2 we keep C_0=C_R=X_0=0 and set C_1 = C_2 = 80 (thus adding risk of default). In the "growth" case of panel B we set an option cost X_E = 100 and e_1=1, e_2 =3. Figure 2, panel A shows that even in the absence of external financing costs, dividend/retained earnings becomes relevant when there is a risk of default. In this case an increase in plowback (retained earnings) actually reduces firm value. This result holds both in the presence and in the absence of a costly growth option (see Figure 2, panel B) and even when we add variable external financing costs of 10%. [9]

The reason behind the negative relationship between retained earnings and firm value is that in the presence of default risk accumulated cash balances may be foregone in the future. In the presence of a growth option (see panel B) the role of retained earnings may be enhanced in the presence of external financing costs. However, for this particular set of parameters any positive level of retained earnings is found to be suboptimal. We will discuss in the subsequent section cases were the role of retained earnings will be further enhanced. At this stage it is important to first summarize the following result that relates to the irrelevancy of dividend policy.

[9] The case with external financig costs shows only minor differences with the case without external financing costs and the lines essentially overlap.

Result 1 (Dividend/Retained earnings irrelevancy for an unlevered firm). The amount of retained earnings/dividends are irrelevant when:

a) the discount rate (r) equals the return earned on retained earnings (r_x)

b) there is no default risk (zero operating costs) or costly growth options

or

c) there are no external financing costs.

Condition a) is necessary to obtain the retained earnings/dividend irrelevancy result. When for example conditions b) or c) hold and condition a) is violated then the role of retained earnings is either enhanced (when $r_x>r$) or reduced (when $r_x<r$). On the other hand either condition b) or c) should also hold. If there are no operating costs or costly growth options obviously external financing is not needed so condition c) is not necessary. On the other hand if there financing needs (i.e., condition b is violated) then condition c) should hold to retain dividend/retained earnings irrelevancy. Dating back to the seminal work of Miller and Modigliani (1961) dividend irrelevancy was linked with a frictionless market which may be related to condition c) above. Result 1 however highlights the important impact of default risk and costly growth options in dividend/policy, results which are new and has not been highlighted in earlier work. In the following section we investigate the various factors affecting the optimal level of retained earnings and dividends.

Before we move on to examine other factors affecting the retained earnings/dividend choice we explain some of the observed results in more detail. To do that we use the case of Figure 2 with positive operating costs $C_1=C_2=80$ in the presence of a growth option using a smaller number of lattice steps N=4 so that we can better illustrate the decisions at each step. We provide the values and the decision for three alternative decisions at t=0, zero retained earnings, 50% retained earnings and 100% retained earnings. For each of this case we obtain firm values of 114.16, 111.13 and 112.32 for each of these cases respectively. The values and optimal decisions at T_1 are provided in Table 1. Table 1 also provides the differences between the value at T_1 between the three different cases pertaining to show the advantages and disadvantages of each policy. We note that under all policies and all states at T_1 the optimal decision is to keep zero retained earnings.

The results demonstrate that the benefit of a higher level of retained earnings at $t = 0$ is to reduce external financing costs. For example, when the retained earnings are 50% for any revenue level higher or equal to 100 the firm saves some external financing costs. This also helps the firm to exercise the growth option in states where the firm would otherwise remain in normal operations (not exercising the growth option). A higher level of retained earnings is also shown to help the firm avoid default by decreasing the default threshold (in fact when the retained earnings is 100% at t=0 default in this case is completely avoided). However, from the perspective of the shareholders this amounts to undertaking losses by foregoing the profits that would have otherwise been paid as dividends to finance the deficits. In some cases default cannot be avoided and all retained earnings from earlier period are foregone as is the case for example when the retained earnings are 50% at t=0 where we observe that at a revenue level of 40.88 default is triggered and the accumulated earnings from the previous period (amounting to 44.94) are foregone. In other cases were the firm avoids default the firm uses retained earnings to finance the deficits and remain active so that it can at least have a claim on some part of the retaining earnings (whereas with zero retained earnings would have default).

[Insert Table 1]

A second observation is that the difference between firm values at different level of retained earnings once a certain level of retained earnings is reached remains small. This could also be seen in Figure 2 where we observe that the line showing the relationship between firm value and retained earnings remains rather flat at higher level of retained earnings. The intuition of this result (which can also be verified at the simpler case of Table 1) is a certain level of retained may be adequate to reduce external financing costs in particular for the intermediary levels of revenues where the firm is still valuable but the firm may have to cover temporary shortfalls. For very high values of revenues the extra retained earnings do not add extra value while for very low values their added value is small while the firm faces the risk that the extra accumulated cash will be lost.

4.2. The effect of profitability, volatility and growth options for an unlevered firm

In this subsection we investigate factors that may enhance the role of retained earnings and reduce the importance of dividends (and vice versa). We focus on the effect of firm profitability, the volatility of revenues and the impact of growth options for an unlevered firm.

First, we investigate the role of profitability (defined as the level of revenues relative to operating costs) in Figures 3 and 4. Intuitively, in the presence of positive external financing costs a low or even negative initial profitability may increase financing needs and thus enhance the importance of keeping retained earnings, however, as our results shortly show we obtain the opposite result. We explain below the economics driving this result. In Figure 3 we consider the no growth option case and we use like before P =100, $C_0=X_0=0$, $\tau = 0.3$, $r=\delta=r_x= 0.05$, $\sigma = 0.2$, $T_1=5$, $T_2=10$ and variable issuance costs $v_E = 0.1$. In Figure 4 we add a growth option with cost $X_E = 100$, $e_1=1$, $e_2=3$. Both in Figure 3 and in Figure 4 we vary the operating cost C_1 and C_2 to be 110, 80 or 50 thus varying the profitability from low to high.

In Figure 3 in the absence of a growth option we find in all cases that higher level of retained earnings reduces firm value. However, the results of Figure 3 show, contrary to intuition, that at higher operating costs retained earnings have a substantially higher negative impact on firm value. In fact, when operating costs are low (firm profitability is high) the negative impact of retained earnings becomes negligible.

[Insert Figure 3]

The reason behind the higher negative impact of retained earnings at higher operating costs (low profitability) is that the risk of default becomes higher and thus the risk that retained cash flows are lost in future periods increases. Our analysis reveals that with zero retained earnings the firm defaults when revenues reach 91.44 when the operating costs are equal to 110 per period whereas when operating costs are 50 per period default is triggered at 40.89. At sufficiently high profitability the risk of default is limited and this tends to satisfy condition b) of Result 1 approximating dividend irrelevancy. Note that condition a) is also satisfied since we use $r=r_x$.

At low profitability levels retaining cash balances is value reducing since despite the fact that they reduce the risk of default and external financing costs the firm faces a risk that when default is triggered that accumulated cash balances will be lost. For example, consider the case with operating costs equal 110 and the firm retains all profits at t =0. Indeed, in this case we find that the revenue level triggering default is reduced to 37.39. However, for any level of revenues lower than that the firm defaults and loses a significant amount of retained earnings amounting to 89.88 ($=100 \cdot (1-0.3)\exp(0.05 \cdot 5)$). The risk of foregone cash flows appears to dominate the resulting savings in external financing costs.

Figure 4 shows that a similar result holds in the presence of a growth option, i.e., a higher level of profitability enhances the role of retained earnings. In this case we observe that for sufficiently high profitability the firm's optimal decision is to keep the maximum level of retained earnings. At high level of profitability and with a high level of growth option benefits accruing in the future the firm's exercise of the investment option is triggered earlier (at lower revenue levels). Anticipating this, the firm's optimal decision is to keep a maximum level of retained earnings so that they are used to finance the growth option and reduce financing costs. Indeed, when operating costs are equal to 50 the revenue level triggering investment in the growth option 91.44 with zero retained earnings while with operating costs equal 110 it is 109.35. When the firm retains all earnings (plowback =1) then the optimal investment trigger when operating costs is equal to 50 is brought at even lower levels reaching 83.62 (while for operating costs of 110 remains at a revenue level equal to 100). At the level of revenues of 83.62 where the firm optimally triggers investment in the growth option the financing needs are (83.62-50)(1-0.3)-100 = 76.47. This would require financing costs 7.647 (10% on the financing deficit). Instead, the firm alleviates these costs by retaining the after tax profits of period 0 (which amount to 70) which grow at $r_x = 0.05$ (continuously compounded) to become 89.89. This more than covers the financing needs of the firm at the investment trigger.

[Insert Figure 4]

We now summarize the result about the effect of profitability on the importance of retained earnings/dividends.

Result 2 (The effect of profitability for an unlevered firm). At lower level of firm profitability, indicating a higher level of default risk, a higher level of retained earnings (dividends) reduces (increases) unlevered firm value.

In Figures 5 and 6 we analyze the effect of revenue volatility. First note that higher volatility increases firm value due to the existence of options effects (see for example, Trigeorgis, 1996). In the case with no growth option the presence of positive operating costs creates an operational flexibility option value due to the option to abandon (default) which increases in value at higher volatility. In the case with the growth option the volatility can further enhance value. Our results show however that at higher revenue volatility (σ) retained earnings have a larger negative impact on firm value. This can be seen by the steeper negative effect of increasing retained earnings at higher volatility levels. This result is in line with Kisser (2013), however, in our case we show that this result is driven by default risk and holds both in the presence and in the absence of a growth option. Effectively, at higher σ the risk that accumulated cash balances will be foregone increases and this reduces the value of retained earnings. We have verified that this result holds for various parameters and varying the level of growth option expansion factors (see also the results that follow).

[Insert Figure 5&6]

We thus summarize the following result with respect to the effect of volatility.

Result 3 (The effect of volatility for an unlevered firm). Higher revenue volatility reduces (enhances) the importance of retained earnings (dividends) since the risk of default and foregone cash flows increases.

Result 3 holds because of the presence of default risk which increases the risk of foregoing cash flows. At higher volatility the chance of very favorable and very unfavorable states increases. In both these cases accumulated cash balances would not be value-enhancing. On the highly favorable states the firm does not need the accumulated earnings anyway since it can use existing profits to finance the growth option. In the unfavorable states the firm faces the risk that these cash balances will be lost.

Retained earnings may be value enhancing when there are significant benefits of the growth option accruing in the future. Figure 7 demonstrates this result by varying the expansion factor e_2 that captures the accrued benefits of the growth option following investment. The Figure shows that a higher level of e_2 enhances the role of retained earnings. This occurs since with this high level of future benefits accruing for the growth option the firm will most likely invest in the expansion option. Given the high level of the investment cost the firm will then have to incur a high level of external financing costs if it does not accumulate retained earnings from earlier periods. Thus, in this case a higher level of retained earnings may be value-enhancing.

[Insert Figure 7&8]

In Figure 8 we investigate the same case but now we assume higher external financing costs. We now see that the role of retained earnings is further enhanced. Compared with Figure 7 (where we used lower financing costs of 10%) we see that in Figure 8 (using variable financing costs of 30%) firm value may now be increasing in the plowback even for lower expansion factors. We note that a high level of external financing costs may implicitly capture issues of asymmetric information since in those cases the firm has to issue securities at a discount. We can now summarize the following result.

Result 4 (The effect of growth options and external financing costs for an unlevered firm). When the level of expansion factor of growth options is higher the importance of retained earnings (dividends) is enhanced (reduced). The importance of retained earnings (dividends) is improved (reduced) at higher external financing costs.

4.3. The role of debt financing

In the presence of debt it is important to consider the relative impact of retained earnings/dividends on equity and debt values due to the presence of default risk. One should consider two possibilities: a first-best solution with firm value maximization implying the absence of agency conflicts between debt and equity holders or a second-best solution amounting to equity instead of firm value maximization. A related issue is to consider whether equity holders have the ability to distribute any accumulated retained earnings as dividends prior to default or whether retained earnings are obtained by debt holders at default (see also Morrelec, 2001). We consider this latter effect to be part of a solution involving the second-best. In section 4.3.1. we analyze the first-best case where retained earnings are obtained by debt holders in the event of default and where firm value maximization is used. A first-best solution may be achieved in the presence of covenant restrictions, corporate governance mechanisms or audit controls which alleviate the risk for the debt holders that equity holders will distribute accumulated cash prior to default. In section 4.3.2. we investigate the second-best case where equity holders may use the proceeds from retained earnings to increase dividend payouts prior to default thus diverting wealth from debt holders.

4.3.1. Retained earnings obtained by debt holders at default

In the presence of debt the risk of default is inevitably introduced. This means that in the presence of debt, dividend irrelevancy does not hold even in the absence of operating costs, costly growth options and zero bankruptcy costs. Assuming that retained earnings may be captured by debt holders at default implies that debt capacity may be enhanced at higher retained earnings. Similarly, Morrelec (2001) has also shown that increasing liquid assets may enhance debt capacity when there are debt covenants protecting them from shareholders' disposition of liquid assets. We show next that even under a first-best solution the optimal plowback is determined by the relative impact of retained earnings on on equity and debt value. As is also shown next the risk of default is low (low volatility, low operating costs/high profitability) a higher level of retained earnings enhances debt more than the reduction in equity and thus firm value is optimized at higher level of retained earnings. In fact we obtain a corner solution of 100% plowback. When the risk of default is substantial then the negative impact of retained earnings on equity value is more significant than the positive impact on debt values. In general due to the opposite direction of equity and debt values as a function of retained earnings we may observe a U-shape in firm value with respect to retained earnings. A positive role of retained earnings exists in the presence of growth options when retained earnings and high external financing costs (just like we have found for the case of an unlevered firm).

Figure 9 shows the impact of retained earnings on firm, equity and debt values for two different levels of profitability. Importantly, the results show that a higher level of retained earnings reduces equity and increases debt values. The overall effect on firm values depends on the relative impact on equity and debt values and a U-shape relationship between firm value and retained earnings may appear. In general we obtain corner solutions of either 0 or 100% plowback depending on the relative impact of retained earnings on equity and debt values.

As the figures show, equity values are decreasing in the level of retained earnings due to the risk that retained earnings may be foregone while debt values are increasing in the level of retained earnings. The results show that with high profitability, firm value may be enhanced at a higher level of retained earnings. In this case of higher profitability the drop in equity value is more than balanced by the increase in debt value thus leading to firm value maximized at a high level of retained earnings (100% plowback). On the other hand, for lower the level of firm profitability the decrease in equity value is more substantial compared to the increase in debt value (and we thus obtain a solution with 0 plowback). We summarize the following results:

Result 5 (Retained earnings irrelevancy and U-shape of retained earnings with debt). In the presence of debt financing, retained earnings/dividend irrelevancy *does not* hold because of the existence of default risk. The effect of retained earnings on firm values depends on the relative impact of retained earnings on equity and debt values. Retained earnings have a negative effect on equity and a positive on debt values and may result in a U-shape in firm value.

Result 6a (The effect of profitability on equity). In the presence of debt financing a high level of profitability substantially reduces the negative impact of retained earnings on equity values.

Result 6b (The effect of profitability on debt). At low level of profitability the risk of default increases and thus retained earnings can substantially increase debt values by reducing default risk and the recoverable amount in the event of default.

Result 6c (The effect of profitability on firm value). Firm value is optimized at higher level of retained earnings when the profitability is high since the negative impact on equity value is less important than the positive impact on debt value. The opposite result obtains when firm profitability is low.

Figure 10 shows the effect of volatility and growth options on firm value. First, notice that a higher volatility results in higher firm values due to the options involved (option to default and growth option). However, the figures show that both in the absence and in the presence of a growth option, an increase in volatility substantiates the negative impact of retained earnings on firm value. Evidently, an increase in volatility increases the risk of default and increases the negative impact of higher retained earnings on equity value. Thus, despite the positive impact on debt values of higher retained earnings, the overall effect on firm value is that firm values are reduced at higher retained earnings (the separate impact on equity and debt values is not shown for brevity). Furthermore, the role of retained earnings is shown to be enhanced in the presence of the growth option since the firm can reduce external financing costs. This is more evident in the case of the growth option with relatively low volatility (σ = 0.2). Extensive sensitivity results at higher expansion factors confirm this finding.

We thus summarize the following results:

Result 7 (The impact of revenue volatility in the presence of debt). An increase in revenue volatility creates a more significant negative impact of retained earnings on firm value. This arises because at higher volatility there is a more significant negative impact of higher retained earnings on equity value compared to the positive impact on debt value.

Result 8 (The impact of growth options in the presence of debt). The higher the level of the expansion factor of the growth options the more important the role of retained earnings since they can be used to reduce external financing costs.

4.3.1. Retained earnings obtained by equity holders just prior to default

The analysis of the previous section assumed that equity holders do not divert any accumulated cash balances into higher payouts prior to default. As pointed out by Morrelec (2001) who investigated a related issue of liquidations of assets this assumption may have an effect on debt capacity.

We investigate this issue by running the model under a second-best solution of equity optimization and under the assumption that debt holders do not obtain any accumulated cash balances at default. In the event of default we assume that equity holders distribute the accumulated cash as dividends.[10]

[10] In practice the distribution of dividends takes place before default. To keep the model simple we maintain the simplifying assumption that accumulated cash flows are distributed to equity holders when default is reached. Debt holders then only receive the value of unlevered assets net of bankruptcy costs.

Despite the fact that retained earnings may be obtained by equity holders at default, our results show that equity values remain decreasing in the level of retained earnings except for high levels of plowback where a slight increase is achieved. This implies that we always obtain a second-best solution where the optimal policy is zero retained earnings. In some cases this may match the solution which would have been reached under a first-best solution where a zero retained level would be optimal. As we have shown in the previous subsection a low level of plowback would be optimal for high levels of default risk (high volatility, high operating costs/low profitability) or when growth options are negligible. In this case the agency costs will be non-existent. In other cases we find that agency costs do not exceed 2%.

Our results (also not shown for brevity) show that debt values are enhanced at higher level of retained earnings even when retained earnings are eventually diverted to equity holders in the event of default. This enhancement in the value of debt at higher retained earnings exists since the risk of default is reduced with the equity holders defaulting at lower revenue levels. Interestingly, the U-shape of firm value remains also under a second-best solution.

Table 2 shows four cases that illustrate the severity of agency conflicts for different levels of profitability and volatility and in the presence or absence of a growth option.

The results show that with low levels of default risk (high profitability, low volatility) and in the presence of growth options the optimal solution between first-best and second-best may deviate leading to agency costs. In these cases the optimal solution under first-best is 100% plowback of profits whereas a second-best solution that caters only for the interests of the shareholders would result in zero plowback. The agency costs are shown not to exceed the 2% level. Agency costs appear more significant in the presence of growth options, low volatility and high firm profitability.

In contrast, at low levels of profitability, high levels of volatility and when growth options value is small we do not obtain any differences in the optimal decision between first-best and second-best solution (which is a zero plowback). Thus under these cases the agency costs over dividend policy are zero. We summarize the following result.

Result 9 (Agency costs over dividend policy). Agency costs over dividend policy exists when first-best and second-best policies diverge. This is more likely to occur when: firm profitability is high, volatility is low and growth options are significant. Agency costs over dividend policy (based on our simulations) do not exceed 2% of firm value.

4.4. The role of investment timing

In this section we investigate a variation of our model which allows for investment timing either at $t = 0$ or at $t = T_1$. With investment timing the firm can start investment earlier which allows for early enhancement of revenues.[11] In the particular setting analyzed in this section we assume that with early exercise revenues are not only enhanced at T_2 by $e_2 = 2$ but the enhancement starts earlier at T_1 (also by $e_1=2$). Despite the benefit of earlier enhancement of revenues arising from early exercising the option the firm may have to possibly incur higher financing costs early-on needed to finance the option since cash available at that stage may not be sufficient to cover the investment cost needed to finance the option.

Figure 10 shows firm values of delaying (W) and of early exercising (EE) the growth option for various levels of plowback and at two levels of initial profitability. We choose to analyze a low and a high level of initial profitability since in the former case the financing costs of early-exercising the option will be high while in the latter will be reduced. We note that the EE strategy is invariant to plowback since the firm under both scenarios considered does not have cash left following the financing of the growth option to keep as retained earnings. The results show that when the profitability is low and thus there are not sufficient cash balances to finance the investment cost the firm will find it optimal to delay exercise of the investment

[11] For brevity we do not provide details of the numerical implementation of investment timing which is available uppon request.

option.[12] Firm value under EE is substantially lower since the firm lacks enough initial cash balances and thus has to incur high external financing costs early-on to finance the growth option. On the other hand, by waiting the firm has an option value to reduce external financing costs since it may enter a more favorable state of revenues next period which will reduce the costs of financing. In the second figure, the situation is reversed. Since now the initial profitability is higher, the external financing costs of early-exercising the growth option are small and the firm can benefit from early enhancement of revenues. We summarize the following result:

Result 7 (The effect of optimal timing) When the initial level of profitability or the accumulated earnings from earlier periods is low the advantage of early exercise is reduced since the firm has to incur high external financing costs in order to finance the growth option. At higher initial profitability or high accumulated earnings from earlier periods early exercise becomes more beneficial since it can enhance revenues early-on without the firm incurring high external financing costs.

5. Conclusions

In this paper we developed a contingent claims model that accommodates revenue uncertainty, retained earnings (dividend) choice and debt financing with risk of default and bankruptcy costs. Our model provides predictions for dividend/retained earnings irrelevancy and the factors that affect retained earnings/dividend choice. The predictions of our model can be contrasted with the predictions of other theories and thus form an invaluable tool for empirical researchers working in the area.

We find that dividend policy irrelevancy critically depends on the absence of default risk. In the presence of growth options, dividend irrelevancy remains if there are no costs of external financing. We find that lower firm profitability and higher volatility encourages higher distribution of dividends since there is a higher risk of default and loss of accumulated cash balances in future periods. We emphasize the possible

[12] Even though firm value is decreasing in plowback when the firm waits we observe that firm values for the delay strategy (W) are higher than the value when the firm early exercises (EE).

complementary role of retained earnings and debt financing: a higher level of retained earnings may enhance debt capacity in particular with higher risk of default (low profitability and high volatility). Agency costs over dividend policy are prevalent for firms with higher profitability, lower volatility and a high value of growth options and external financing costs. The timing of investment depends critically on current profitability and currently available cash balances. When current profitability and cash balances are high then the firm can avoid high external costs of financing the growth option by using available cash balances and this encourages early exercise of the growth option, else, the firm prefers to delay in anticipation that future external financing costs of the growth option will be lower.

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Appendix

A benchmark three stage model

In this appendix we develop an analytic solution for the unlevered firm in the absence of debt financing and liquidity choice which is used as benchmark for the more complex numerical model of the subsequent section. Revenues are assumed to follow a Geometric Brownian Motion (see equation 1 of the main text)

We split the operational phase of the firm in 3 periods with operations and decisions occurring at t = 0, t $=T_1$ and t = T_2. We also assume that the firm may choose to abandon operations at any time (with zero bankruptcy costs). At T_1, the firm will decide whether to abandon operation obtaining zero (mode "A"), whether to continue operations (mode "S") which will result in profits $(P-C)(1-\tau)$ or invest X_E and expand revenues by e_1(mode "E") thus driving revenues to $e_1 P$. In the last period, the firm decides whether to operate or not, and, depending on the decision to expand or not will either receive P or $e_2 P$ (where e_2 may be different than e_1) . Note that in general there are three regions in the intermediary time T_1 which have the following sequence (starting from lower values of P): A, S and E. The value of the firm at t = 0 can thus be obtained by:

$$\begin{aligned}
V(P) &= (P-C)(1-\tau) + P(1-\tau)e^{-\delta T_1}(N(a_{S,1}) - N(a_{E,1})) - C(1-\tau)e^{-rT_1}(N(a_{S,2}) - N(a_{E,2})) \\
&+ e_1 P e^{-\delta T_1} N(a_{E,1}) - (C(1-\tau) + X_E)e^{-rT_1} N(a_{E,2}) \\
&+ e^{-\delta T_2} P(1-\tau)(N(a_{S,1}, b_{S,1}, \rho) - N(a_{E,1}, b_{S,1}, \rho)) - C(1-\tau)e^{-rT_2}(N(a_{S,2}, b_{S,2}, \rho) - N(a_{E,2}, b_{S,2}, \rho)) \\
&+ e^{-\delta T_2} e_2 P(1-\tau) N(a_{E,1}, b_{E,1}, \rho) - C(1-\tau)e^{-rT_2} N(a_{E,2}, b_{E,2}, \rho)
\end{aligned} \qquad \text{(A1)}$$

The value of the firm consists of the value of operating in first period (first term), the value of the option to operate under normal mode in second period receiving revenues and paying costs (second and third term), the value of the option to expand revenues in the second period by incurring an additional investment cost (fourth and fifth term), the value of cash flows under normal mode at T_2 assuming the firm remains in normal mode at T_1 (sixth and seventh term) and the value of cash flows under expanded mode at T_2 assuming the firm invests in the expansion option at T1 (eighth and ninth term). Note that the term (

$N(a_{S,2}) - N(a_{E,2}))$ captures the probability of entering the region of operate, i.e., probability that ($P_S^* \le P_{T_1} \le P_E^*$) and $(N(a_{S,1}, b_{S,1}, \rho) - N(a_{E,1}, b_{S,1}, \rho))$ captures the probability that the firm operates under normal state at T1 and stays in operation at T_2, i.e, the probability that $(P_S^* \le P_{T_1} \le P_E^*) \cap (P_{T_2} > C)$.

The following apply for regions r, r =S, E:

$$a_{r,1} = \frac{\ln(P / P_r^*) + (r - \delta + 0.5\sigma^2)T_1 + 0.5\sigma^2}{(\sigma\sqrt{T_1})}$$

$$a_{r,2} = a_{r,1} - \sigma\sqrt{T_1} \quad \text{(A2)}$$

$$b_{r,1} = \frac{\ln(P / C) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}}, \qquad b_{r,2} = b_{r,1} - \sigma\sqrt{T} \qquad \rho = \sqrt{\frac{T_1}{T_2}} \quad .$$

Since in general there are three regions in the intermediary time T_1 having the sequence (starting from the lower values of P) A, S and E, this requires obtaining two thresholds. The first one is between A and S which is obtained from solving the following (trigger) equation:

$$(P_S^* - C)(1-\tau) + P_S^*(1-\tau)e^{-\delta(T_2 - T_1)}N(b_{S,1}) \\ - C(1-\tau)e^{-r(T_2 - T_1)}N(b_{S,2}) = 0$$

$$b_{S,1} = \frac{\ln\left(P_S^* / C\right) + (r - \delta + 0.5\sigma^2)(T_2 - T_1)}{\sigma\sqrt{(T_2 - T_1)}} \quad \text{(A3)}$$

$$b_{S,2} = b_{S,1} - \sigma\sqrt{(T_2 - T_1)}$$

and the second one is the threshold between S and E which is obtained from the following (trigger) equation:

$$(e_1P_E^* - C)(1-\tau) + e_2P_E^*(1-\tau)e^{-\delta(T_2-T_1)}N(b_{S,1}) - C(1-\tau)e^{-r(T_2-T_1)}N(b_{S,2}) - X_E$$
$$= (P_E^* - C)(1-\tau) + P_E^*(1-\tau)e^{-\delta(T_2-T_1)}N(b_{E,1}) - C(1-\tau)e^{-r(T_2-T_1)}N(b_{E,2})$$

$$b_{E,1} = \frac{\ln\left(e_2P_E^*/C\right) + (r-\delta+0.5\sigma^2)(T_2-T_1)}{\sigma\sqrt{(T_2-T_1)}} \quad \text{(A4)}$$

$$b_{E,2} = b_{E,1} - \sigma\sqrt{(T_2-T_1)}$$

The terms $b_{s,1}$ and $b_{s,2}$ are obtained from equation (A3).

It is possible that the growth option region completely dominates the normal region at T_1. This would be the case if $P_E^* < P_S^*$ which means the growth option is triggered earlier than the normal operating mode. In that case there are only two regions at T1, A or E and equation (2) simplifies to:

$$V(P) = (P-C)(1-\tau) + e_1Pe^{-\delta T_1}N(a_{E,1}) - (C(1-\tau)+X_E)e^{-rT_1}N(a_{E,2})$$
$$+ e^{-\delta T_2}e_2P(1-\tau)N(a_{E,1}, b_{E,1}, \rho) - C(1-\tau)e^{-rT_2}N(a_{E,2}, b_{E,2}, \rho) \quad \text{(A5)}$$

In this case we need to obtain the threshold of switching from the abandonment mode to the growth option which is obtained by solving:

$$(e_1P_E^* - C)(1-\tau) + e_2P_E^*(1-\tau)e^{-\delta(T_2-T_1)}N(b_{E,1})$$
$$- C(1-\tau)e^{-r(T_2-T_1)}N(b_{E,2}) - X_E = 0$$

$$b_{E,1} = \frac{\ln\left(e_2P_E^*/C\right) + (r-\delta+0.5\sigma^2)(T_2-T_1)}{\sigma\sqrt{(T_2-T_1)}} \quad \text{(A6)}$$

$$b_{E,2} = b_{E,1} - \sigma\sqrt{(T_2-T_1)}$$

Accuracy of the numerical lattice model

The following table tests the accuracy of the numerical model provided in section 3 by providing a comparison of the analytic model developed in the previous section of the appendix against the numerical lattice model. The results indicate that the numerical lattice model is fairly accurate and can thus be used to accommodate more realistic features including liquidity choice, external financing costs, debt and bankruptcy costs.

[Insert Table A1]

Table A1. Accuracy of the numerical model

σ	No growth Analytic	No growth Numerical	e_1=e_2	With growth Analytic	With growth Numerical
0.05	33.639	33.639	1.0	42.723	42.725
0.10	35.781	35.781	1.1	45.050	45.091
0.15	39.058	39.064	1.2	52.609	52.599
0.20	42.723	42.725	1.3	60.757	60.760
0.25	46.518	46.530	1.4	69.246	69.250
0.30	50.331	50.323	1.5	77.999	77.999
0.35	54.106	54.116	1.6	86.955	86.953
0.40	57.806	57.823	1.7	96.067	96.066
0.45	61.408	61.407	1.8	105.301	105.300
0.50	64.893	64.876	1.9	114.629	114.627
0.55	68.248	68.254	2.0	124.029	124.025

Note: We use P =100, C= 80, τ = 0.3, r = 0.05, δ = 0.05, T_1 =5, T_2=10. In panel A (No growth) we use e_1=e_2=0 (no growth) and volatility σ varies from 0.05-0.55. In panel B (Growth) we use σ = 0.2, cost of exercising the growth option X_E = 10 and vary e_1, e2 from 1-2 (e_1=e_2). For the analytic solution we use equation (A1). For the numerical model we use the lattice based solution of section 3. with N_1 = 200 steps (assuming no debt, no liquidity choice and zero issuance costs).

Table 1. A simple example illustrating the differences between retained earnings/dividend policies

	Plowback =0 at t=0		Plowback = 50% at t=0		Plowback = 100% at t=0		Differences		
Revenue	Firm (1)	Decision	Firm (2)	Decision	Firm (3)	Decision	(2)-(1) $+R_2$	(3)-(1) $+R_3$	(3)-(2) $+R_{(3-2)}$
244.59	371.63	Growth	416.57	Growth	461.51	Growth	0.00	0.00	0.00
156.39	160.99	Growth	210.43	Growth	255.53	Growth	4.49	4.65	0.16
100.00	29.61	Normal	74.77	Growth	123.82	Growth	0.22	4.33	4.11
63.94	0.00	Default	37.45	Normal	82.39	Normal	-7.49	-7.49	0.00
40.88	0.00	Default	0.00	Default	62.93	Normal	-44.94	-26.96	17.98

Note: We use P =100, C_0=C_R =0, X_0 =0, C_1 = C_2 = 80, τ = 0.3, r =r_x= δ = 0.05, σ = 0.2, T_1 =5, T_2=10 and a growth option with a cost X_E = 100 and e_1=1, e_2 =3. We use variable issuance v_E = 0.1 (fixed external financing costs are set to zero). The decision "Growth" is to exercise the growth option, the decision "Normal" is not to exercise the growth option and remain at a normal state of operations and the decision "Default" is to abandon operations. R_2 are the retained earnings kept from period 0 which amount to 44.94 when the plowback =50% and 89.88 when the plowback is 100%. Firm (2) and Firm (3) includes these retained earnings since the firm optimally keeps zero retained earnings at T_1 for both policies.

Table 2. A comparison of first-best and second-best solutions and agency costs

	First-best			**Second-best**			
	Firm	**Equity**	**Debt**	**Firm**	**Equity**	**Debt**	**AC**
No growth							
Low cost	126.31	102.99	23.31	124.76	104.90	19.86	0.012
High cost	86.68	79.87	6.81	86.68	79.87	6.81	0.000
Growth							
Low volatility	118.64	96.89	21.75	116.93	103.37	13.56	0.015
High volatility	145.82	135.11	10.71	145.82	135.11	10.71	0.000

Note: We use P =100, C_0=0, X_0 =0, , C_R =20, τ = 0.3, b =0.2, r =r_x = δ = 0.05, T1 =5, T2=10 and e_1= e_2=1 (No growth option) and e_1=1, e_2=3, X_E = 100 (Growth). For high profitability we use C_1 = C_2 = 50 and for low profitability we use C_1 = C_2 = 110. For high volatility we use σ =0.4 and for low volatility σ =0.2. Variable issuance costs v_E = 0.1 and fixed cost of external financing zero.

Figure 1. Optimal level of retained earnings without default risk

Panel A. No growth

Panel B. With a growth option

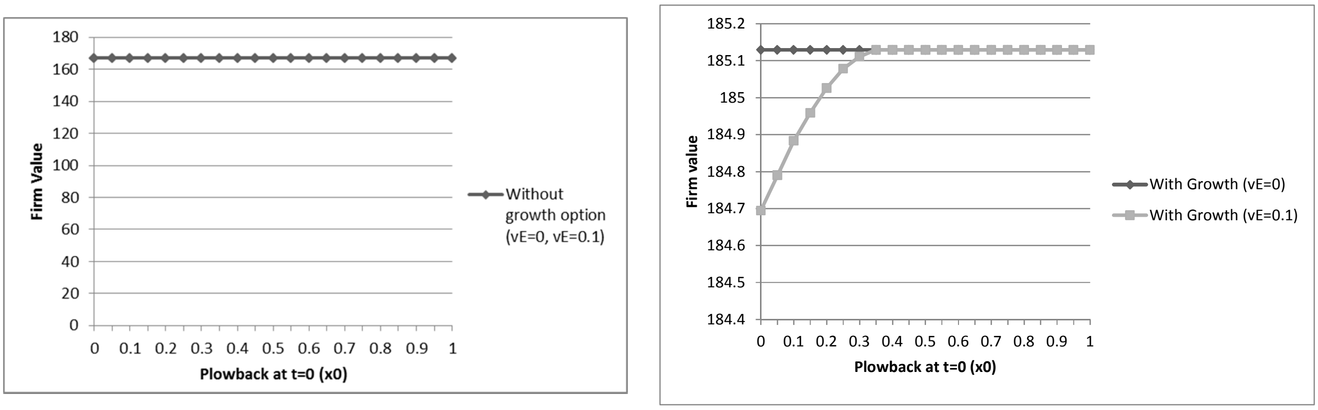
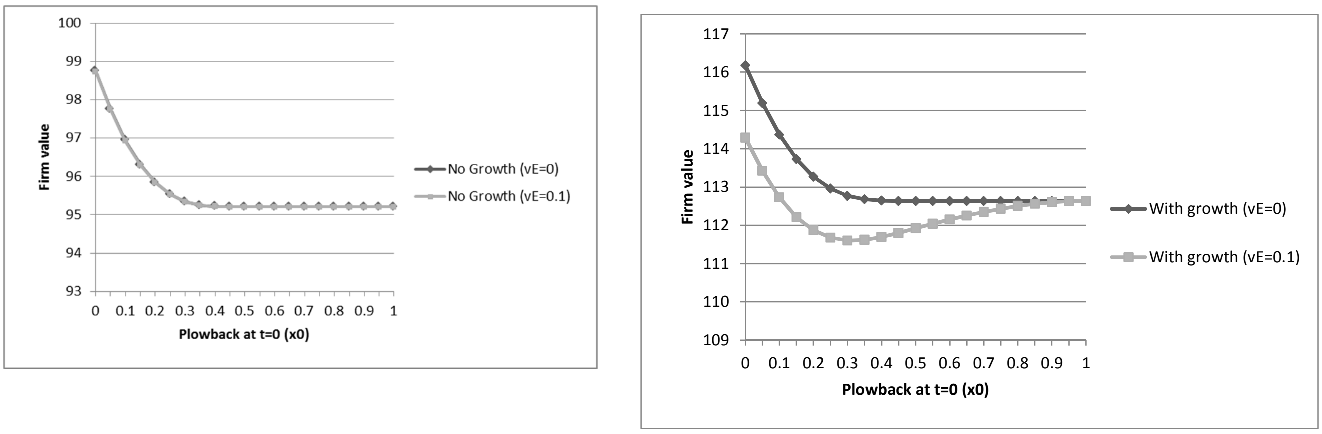


Figure 2. Optimal level of retained earnings with default risk

Panel A. No growth

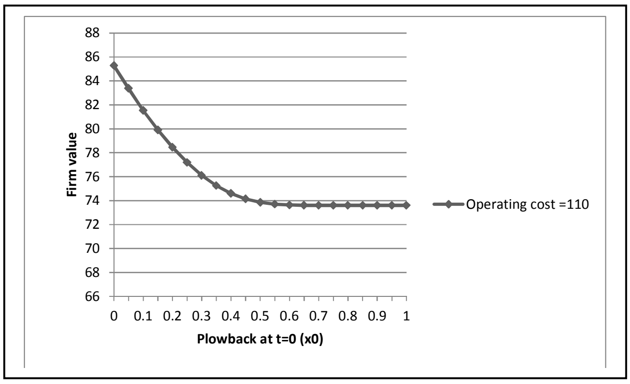
Panel B. With a growth option



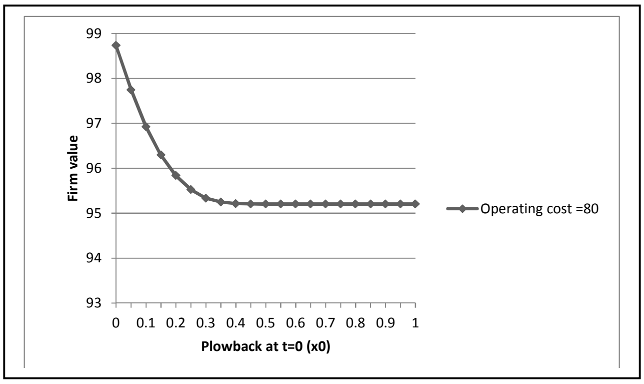
Note: In figure 1 we use P =100, C_0=C_1 C_2=C_R =0, X_0 =0, τ = 0.3, r =r_x= δ = 0.05, σ = 0.2, T_1 =5, T_2=10. In "without growth" case we use e_1=e_2=1 and in "growth" case growth option cost X_E = 100 and e_1=1, e_2 =3. We use variable issuance costs v_E = 0 or v_E = 0.1 (fixed external financing costs are set to zero). In Figure 2 we set C_0=C_R=0 and C_1 = C_2 = 80 (adding risk of default) and in "growth" case growth option cost X_E = 100 and e_1=1, e_2 =3.

Figure 3. The impact of firm profitability (No growth option)

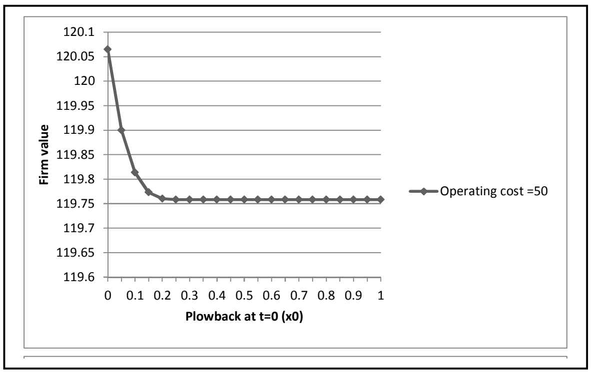
Panel A. Low profitability



Panel B. Medium profitability



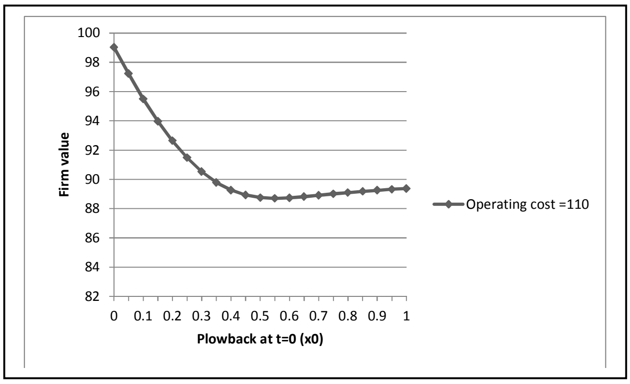
Panel C. High profitability



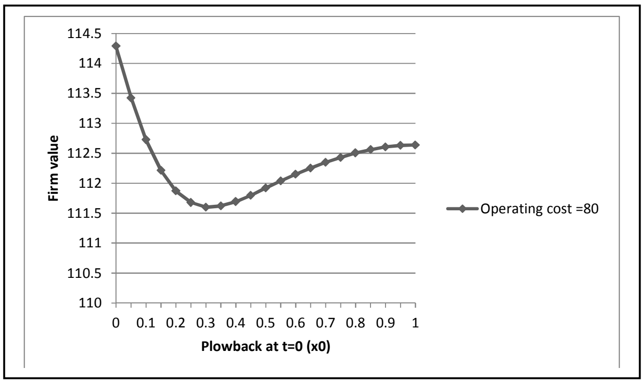
Note: We use P =100, $C_0=C_R=X_0=0$, $\tau = 0.3$, $r=r_x=\delta = 0.05$, $\sigma = 0.2$, $T_1=5$, $T_2=10$ and variable issuance costs $v_E = 0.1$. We vary firm profitability by varying the operating cost C_1 and C_2 to be 110, 80 or 50.

Figure 4. The impact of firm profitability (With growth option)

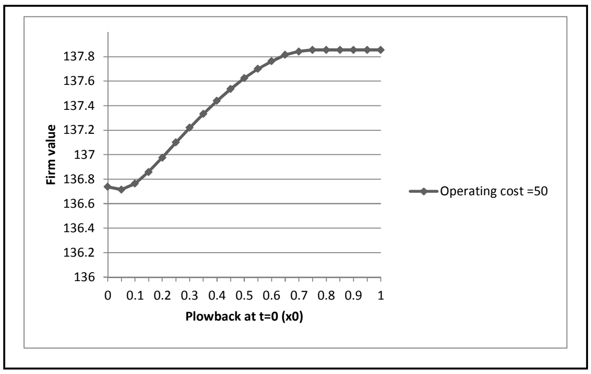
Panel A. Low profitability



Panel B. Medium profitability



Panel C. High profitability



Note: We use P =100, C_0=C_R =X_0 =0, τ = 0.3, r =r_x=δ = 0.05, σ = 0.2, T_1 =5, T_2=10 and variable issuance costs v_E = 0.1, and a growth option with cost X_E = 100, e_1=1, e_2 =3. We vary firm profitability by varying the operating cost C_1 and C_2 to be 110, 80 or 50.

Figure 5. The impact of revenue volatility (No growth option)

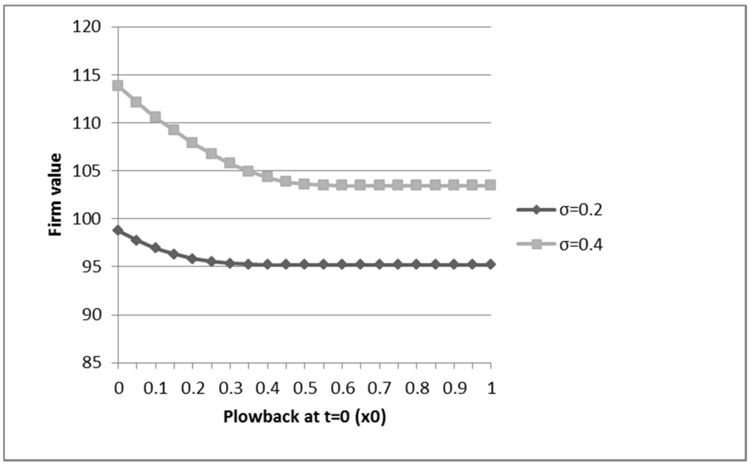
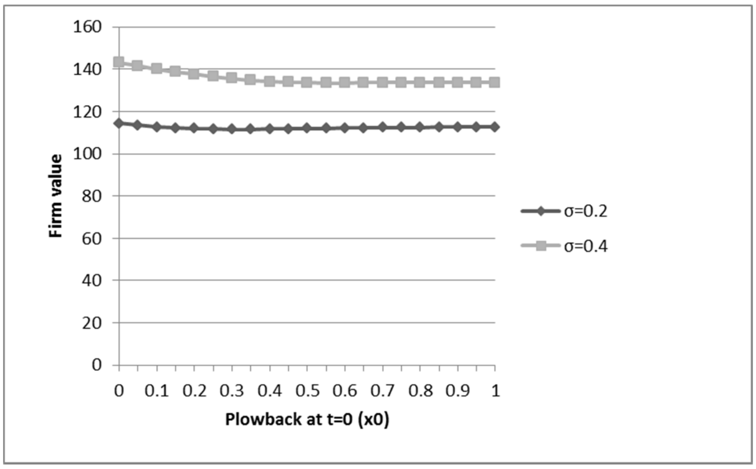


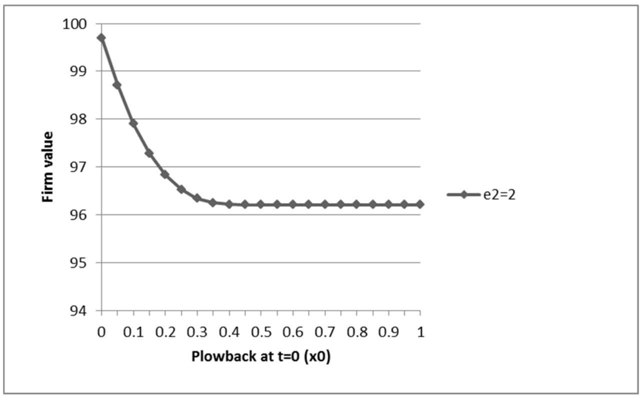
Figure 6. The impact of revenue volatility (With growth option)

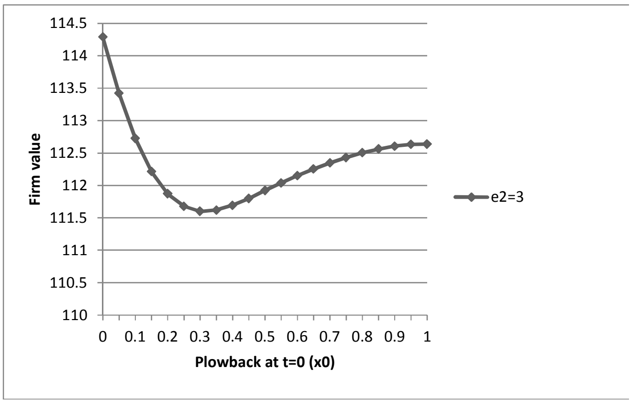


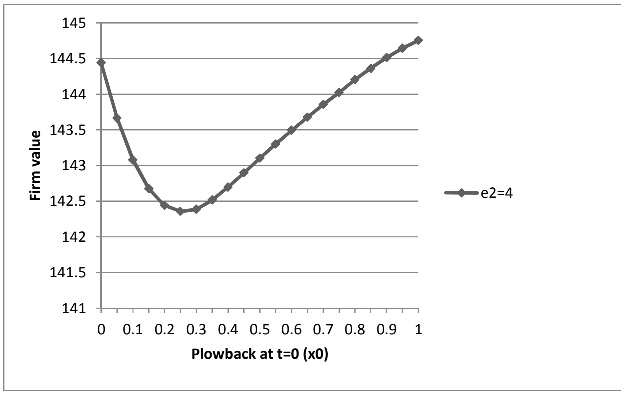
Note: In figure 5 we use P =100, C_0=0, X_0 =0, C_1 = C_2 = 80, τ = 0.3, r =r_x =δ= 0.05, σ = 0.2, T_1 =5, T_2=10 and variable issuance costs v_E = 0.1, and e_1=e_2=1 (no growth). In figure 6 we add a growth option with cost X_E = 100, e_1=1, e_2 =3. In both figure 5 and 6 we vary revenue volatility between 0.2 and 0.4.

Figure 7. The impact of expansion option

Variable financing costs = 10%







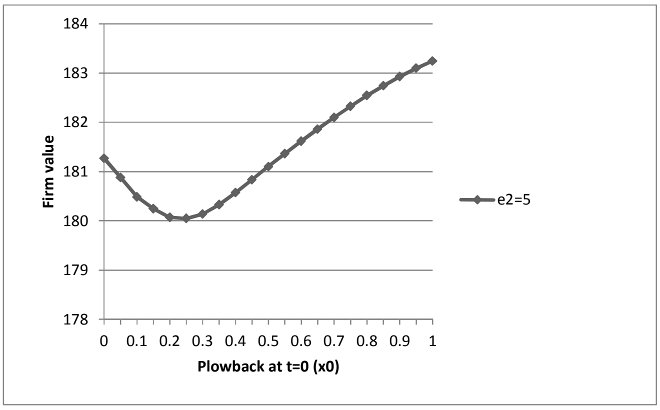
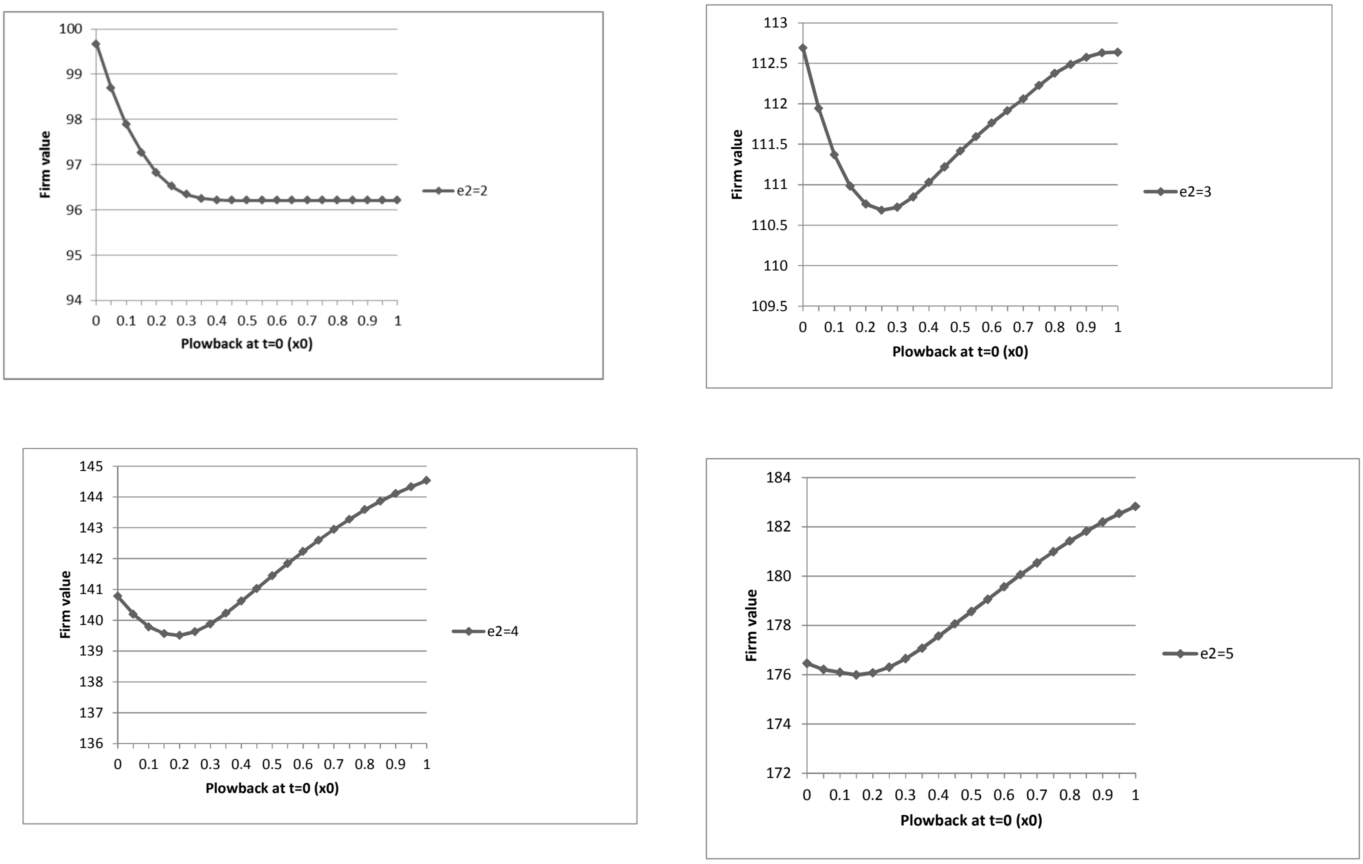


Figure 8. The impact of expansion option

Variable financing costs = 20%

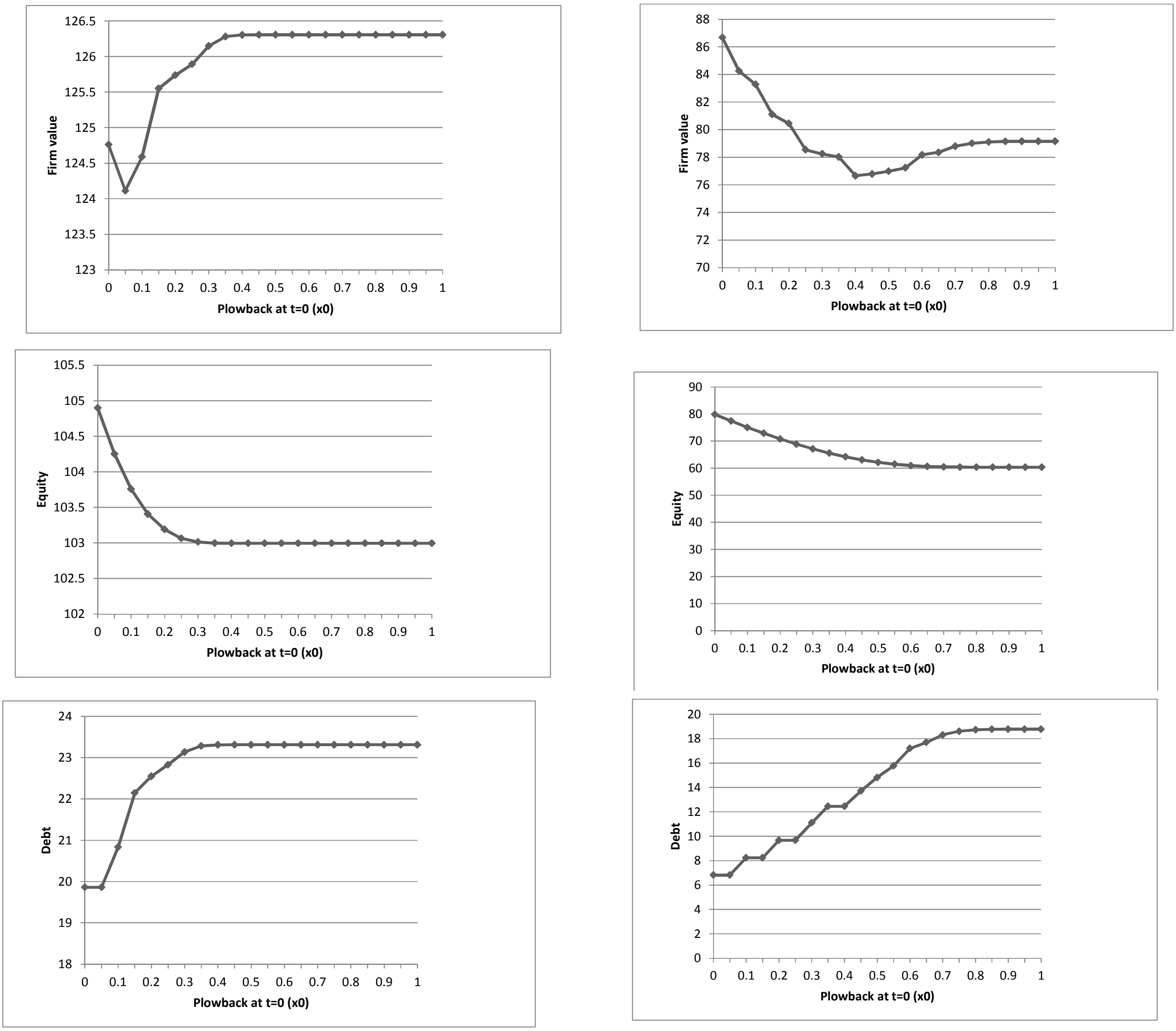


Note: In figure 7 and 8 we use P =100, C_0=0, X_0 =0, $C_1 = C_2 = 80$, $\tau = 0.3$, $r = r_x = \delta = 0.05$, $\sigma = 0.2$, T_1 =5, T_2=10 with a growth option with cost X_E = 100, e_1=1, and e_2 =2, 3,4 and 5. In Figure 7 we use variable external financing costs v_E =0.1 and in figure 8 v_E = 0.2. We assume fixed cost of external financing are zero in both figures.

Figure 9. The impact of debt financing and profitability

High profitability

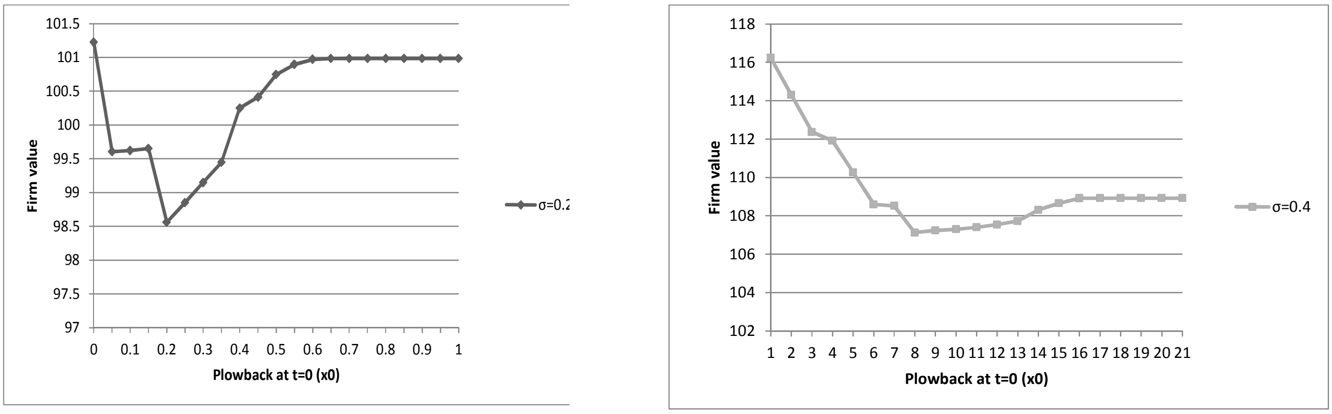
Low profitability



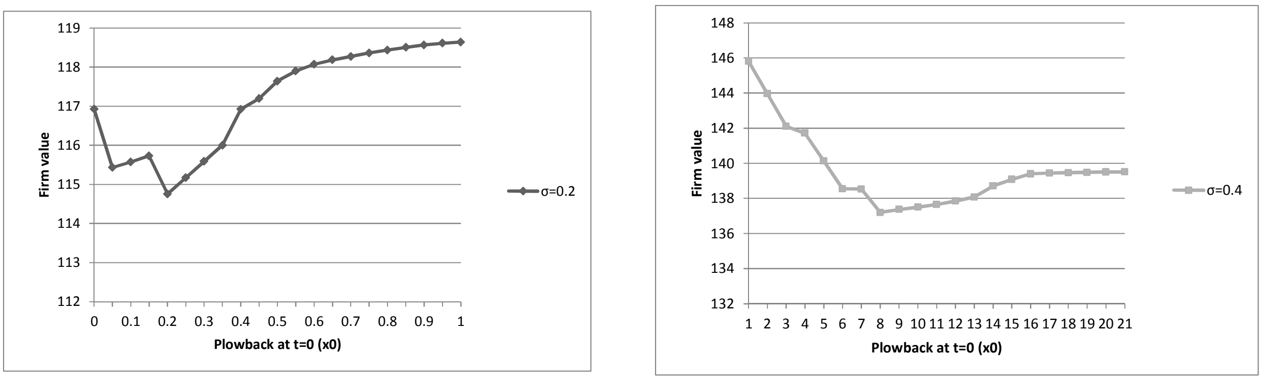
Note: We use P =100, C_0=0, X_0 =0, , C_R =20, τ = 0.3, b =0.2, r =r_x = δ = 0.05, σ = 0.2, T_1 =5, T_2=10 and e_1= e_2=1 (No growth option). For high profitability we use C_1 = C_2 = 50 and for low profitability we use C_1 = C_2 = 80. Variable issuance costs v_E = 0.1 and fixed cost of external financing zero.

Figure 10. The impact of revenue volatility

No Growth



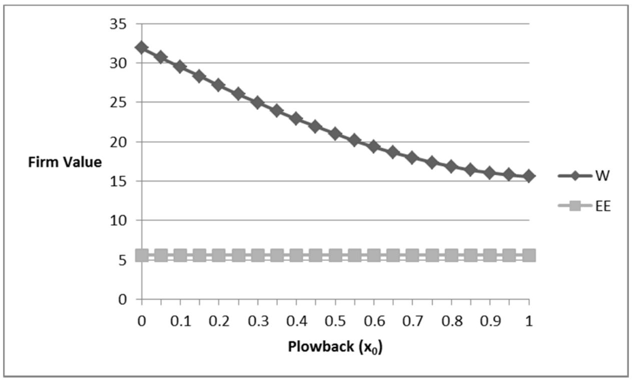
Growth



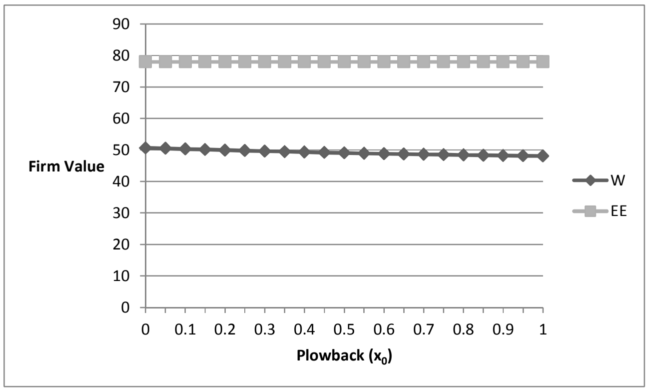
Note: We use P =100, C_0=0, X_0 =0, , C_1=C_2=80, C_R =20, τ = 0.3, b =0.2, r =r_x = δ = 0.05, T_1 =5, T_2=10 and e_1= e_2=1 (No growth option) and e_1=1, e_2=3, X_E = 100 (Growth). For high volatility we use σ =0.4 and for low volatility σ =0.2.Variable issuance costs v_E = 0.1 and fixed cost of external financing zero.

Figure 11. The impact of investment timing

Low profitability (P=100)



High profitability (P=150)



Note: We use P =50 or 100, C_0= C_1 = C_2 = 80, X_0 =0, , C_R =0, τ = 0.3, b =0, r =r_x =δ = 0.05, σ = 0.2, T_1 =5, T_2=10 and e_1=1, e_2=2, X_E = 50. Variable issuance costs v_E = 0.1 and fixed cost of external financing zero. "W" defines the value of the firm when it does not early exercise the option to invest (waits) and "EE" defines the value of the firm if it decides to early exercise the option. In the latter case the firm needs to finance the shortage with external financing implying that the firm cannot keep any level of retained earnings.