# The Valuation of R&D Projects with on Option to Expand

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**Abstract**

We model a multistage finite life investment problem subject to several sources of cost and price uncertainty, as typical of start up firms, complex industrial and construction initiatives and research and development projects for software, drugs and technology ventures. Additionally, the firm also has an option to expand the original project to take advantage of derivative investment opportunities once all the development stages are successfully completed, which is modeled as an American style option. An important characteristic of these initiatives is that as the firm incurs cost and invests, it learns both about the difficulty of developing and implementing the project and also about market conditions, and updates its prospects of timely completion and of expansion accordingly. This information can then be used to optimally decide whether further investments is warranted or not, given the expected future revenues of the whole venture.

*Keywords:* Valuation, Real Option, R&D, Lattice Methods

## 1. Introduction

Most capital budgeting problems involve analysing the tradeoffs between a fixed and certain capital investment and an uncertain stream of future cash flows. On the other hand, a large class of problems such as the valuation of start up firms, complex industrial and construction ventures, and research and development projects such as software, drugs and technology initiatives, include complications not captured by this simple model. In addition to the uncertainty over the final payoffs of the venture, for these types of problems there is also considerable uncertainty over the cost and timing of the total investment required and over the quality and performance of the final product. The design of an advanced microprocessor chip, the development of a new aircraft, a start up firm or a new drug, for example, all involve investing an uncertain amount of capital and time in order to obtain and to bring to the market a product whose performance characteristics or final quality is uncertain.

Another characteristic of these projects is that by investing, the firm learns about the difficult of designing and building a new product or of performing research on a new drug,

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and updates its prospects of a successful development, timely completion and the quality of the final product as it progresses during each development phase. This new information also allows the firm to optimally determine whether to abandon or to continue investing in the project at any time. This gives these projects the characteristics of a contingent claim over the value of the completed project, and we analyse the problem from a real options approach.

Additionally, once successfully completed, through additional investment the project provides the firm an opportunity to optimally expand the scope of the project to new market segments that were not originally contemplated, which may involve new uses for the original product or making improvements or modifications that would allow it to be marketed for marginally different uses. As an example, a low power consumption version of a successful desktop micro processor design may expand the original market for this product to include the notebook computer market, or a drug that is targeted to the adult patients may be altered to also be used by children. Obviously, the prerequisite for a profitable expansion is that the development of the original product be successfully completed and with an adequate quality level. We assume that the project is subject to a finite economic life due to technical obsolescence, increased competition or patent expiration.

These projects are subject to several different types of stochastic cost and demand uncertainties. We model the investment cost as a diffusion process with a negative drift equivalent to the instantaneous rate of investment, in an approach that follows [1], where the firm starts out with an exogenously defined expected cost to completion and updates this expected cost as new information becomes available.

Prior to the beginning of the project, the firm specifies a set of performance characteristics the final product is expected to have, which we hereby refer to as the "quality" of the product in a broad sense. This can be the clock speed of a new microprocessor chip design, the operating range of a new aircraft, the maximum sustainable output of a power plant or the effectiveness of a new drug. As the firm invests in the project it also learns about any deviations from the expected quality and updates this information as the progresses through each stage. We assume that the final product quality is correlated with the deviations from the expected cost to completion in each stage.

There is also uncertainty concerning the changes in the competitive and market environment, both during and after product introduction, that may render the project worthless, such as a preemptive move by a competitor or a technological breakthrough that makes the product obsolete. This is modeled as an exogenous Poisson death process.

All these uncertainties are assumed to be firm or project specific private risks, uncorrelated with the market, and accordingly, command no risk premium. A different type of uncertainty relates to the market risks associated with the future cash flows generated by the completed project, which are a function of the market demand for the product, and thus, a component of systematic risk.

We adopt a discrete lattice approach to obtain a dynamic programming solution for the value of this more complex multistage investment problem and show that this method has some computational advantages over other approaches such as simulation models or numerical solutions. We first model the more general case of an $n$-stage investment project

subject to several sources of uncertainties and then analyze the case of a three stage drug research and development project in the pharmaceutical industry. We obtain a discrete solution to this application and show the comparative statics.

This paper is organized as follows. In the next section we discuss previous work in this field that is related to our work. In Section 3 we present our basic model, the notation and the valuation equations. In Section 4 we apply the model to value a R&D project in the pharmaceutical industry and show analytical results. In Section 5 we draw our conclusions. The more technical proofs and details of the discretization processes are shown in the Appendix.

## 2. Related Work

The valuation of R&D projects as a contingent claim on the value of the completed project has been subject to much interest in recent years. [2] analyze a multistage investment problem where the value of the project upon completion is uncertain. The problem of cost uncertainty in an irreversible investment was first modeled as a real options problem by [1], where he addressed the issue of an uncertain investment cost subject to both market and private risks and where the value of the completed project is known with certainty. By undertaking R&D activities and incurring costs, the firm not only produces an R&D output but also learns about the difficulty of the research project and gets better information about the expected remaining time and costs to completion. Based on this information, the firm can optimally choose to continue its R&D efforts or abandon the project altogether. The project is valued as a single period contingent claim on a fixed asset value and a closed form solution is obtained.

[3] extended this valuation model to include a market uncertainty for the project revenues and the possibility of catastrophic events in a multi period setting to allow for different investment rates for each stage, where the firms has the option to abandon its R&D efforts at any time. Since no closed solution exists, they solve by numerical methods. [4] further extend the model to include the effects of market competition in a duopolistic market setting. Neither of these papers discusses the option to expand the project after completion of the R&D stages.

[5] analyze a multi stage investment project and study the impact of the interaction between simultaneous private and market risks in the dynamics of the risk premia. They conclude that the required risk premium for the R&D stages is significantly higher than it would be were the R&D complete and the venture a traditional cash producing project. This occurs even if the private risks are purely idiosyncratic in nature and are a result of the fact that these projects have compound options on the systematic (market) uncertainty. Even though the private risk merits no risk premium itself, due to the existence of the options, the resolution of the private uncertainty affects the risk premium earned on the whole project.

Market practice by major firms in the industry seems to support this idea. While a pharmaceutical firm may agree to a relatively high upfront payment and royalty fee for late term development from a biotech firm, they are reluctant to pay as much for early stage developments from independent research labs or academic institutions, or even refuse to do

so at all [6]. [7] extends this model to a two firm competitive setting and concludes that competition tends to accelerate the pace of innovation and lower the cost to consumers at the expense of the firm's profitability. Both these papers use numerical methods to solve the value equations.

As more uncertainties are added to the problem, the complexity of the model increases. [8] examine the response of a pharmaceutical firm involved in a two stage R&D process pf a new drug to different types of government incentives. They model this as a discrete two period project where the firm has the option to abandon development only at the end of each stage. The novel aspect of this work is that it incorporates uncertainty in quality of the R&D output that is endogenously specified and which in turn affects the efficiency and the market size for the product, and then use simulation to value both the option to abandon and the project itself.

This paper is closed to ours in the sense that it adopts a real options discrete time approach to value the project. Their simulation model though, only allows the option to abandon to be exercised at the end each stage, and thus requires the firm to complete any stage already initiated regardless of the cost that will be incurred by doing so. Even if it becomes clear to the firm that further investment is not warranted, the firm is assumed to be incapable of interrupting the investment and abandon its research efforts until this stage is completed. Since there is no possibility of exercise while a stage is in progress, any learning that occurs before a particular stage is completed has no effect on that stage, as any costs already incurred are sunk costs, and thus, irrelevant for the investment decision of the firm. The only benefit of the learning that does occur is through the correlation between the actual cost of the current stage and the expected initial cost of the subsequent one. This way, if a stage suffers delays and a corresponding increase in costs, the initial expected cost for the next stage also increases. This is an essential feature of this model without which there would be no optionality involved in the problem, and allows the project, once initiated, to have a negative value.

Our model differs from [8] in that it has no such limitations on the timing of the exercise of the American option. We allow the firm to continuously decide on the optimal operating strategy and, if necessary, abandon the project before a particular stage is completed, and thus avoid committing resources to a project that will offer a negative expected NPV. Due to this, there will usually be a positive probability that any particular stage will not be completed due to the early exercise of the option to abandon. We also model the project cash flows and the endogenous quality variable differently. Besides being affected by market uncertainties, in our model the cash flows have an additional dimension as they are also a function of the uncertain time required to complete the project. Quality, on the other hand, is defined as a function of a stochastic variable that is negatively correlated with the deviations from the expected cost to completion of each stage, whereas [8] resort to random draws from a Beta distribution.

Our work also differs from these other previous efforts in two other aspects. First, we analyze and incorporate the effects of an option to expand the original project during the production and marketing phase once development of the new product is successfully completed. The existence of such an opportunity, which is analyzed as a compound option,

has important consequences on the prior decision of whether to abandon the project during any of the development stages and may alter the optimal investment strategy of the firm, and is, to the best of our knowledge, a novel addition to the literature on the valuation of R&D projects.

Secondly, we approximate the continuous time problem with the corresponding discrete time problem and solve using a lattice approach to model the value function and the project options, and to obtain the solution. The discrete time problem does not have an exact solution since it can be broken down into an infinite number of time steps, but we can define the desired level of precision and let the model determine the granularity that is required to achieve this precision. The solution is very flexible in its handling of complex models, and, although it is computationally intensive, reasonable processing times can be obtained for coarse approximations.

## 3. The Model

Consider a multistage investment project where the first research and development stage (1) ends at time $t = \tau_1$ the second stage (2) ends at time $t = \tau_2$ and the last R&D stage ($n$) ends at time $t = \tau_n$. If successfully completed, the project provides the firm with a continuous stream of stochastic cash flows $\tilde{C}(t)$ during the subsequent market phase, where $t \in [\tau_n, \tau_m]$. The project is subject to uncertainties concerning the total cost of the investment required to complete each stage, the final quality of the finished product, the level of the cash flow streams and the risk of catastrophic failure that would instantaneously render the project worthless. All these uncertainties are assumed to be project and firm specific idiosyncratic risks, uncorrelated to the market, and the firm and its shareholder are assumed to be adequately diversified. Accordingly, we assign no risk premium to these risks and discount them at the risk free rate $r$. The stochastic cash flows the firm receives once all stages are completed and the product is marketed is the only source of systematic risk of the project, and thus commands a risk adjusted discount rate $\mu > r$. The figure 1 shows the stages of the general model.

The initial expected costs to complete each stage are $E_0[\tilde{K}_i], i = 1, 2, .., n$, and it is assumed that each stage has a fixed rate of investment $I_i$. Prior to the beginning of the project, the firm also specifies the expected quality of the final product $E_0[\tilde{Q}(\tau_n)]$ where $0 \leq E_0[\tilde{Q}(\tau_n)] \leq 1$.

Without uncertainty, the actual investment costs in each stage are equal to their expected costs and there will also be no deviations from the expected quality level of the final product. The solution is straightforward since the project presents no managerial flexibility other than the decision on whether to commit to the project or not at the outset, and the value $V(\tau_n)$ of the expected cash flows the firm will receive upon completion of the project at $t = \tau_n$ is

$$V(\tau_n) = E\left[\int_{\tau_n}^{\tau_m} \tilde{C}(t) e^{-\mu(t-\tau_n)} dt\right] \tag{1}$$

Considering that $K_i = E_0[\tilde{K}_i], i = 1, 2, .., n$, and $Q = E_0[\tilde{Q}(\tau_n)]$, the value $F(V, K_i, Q)$ of the project is

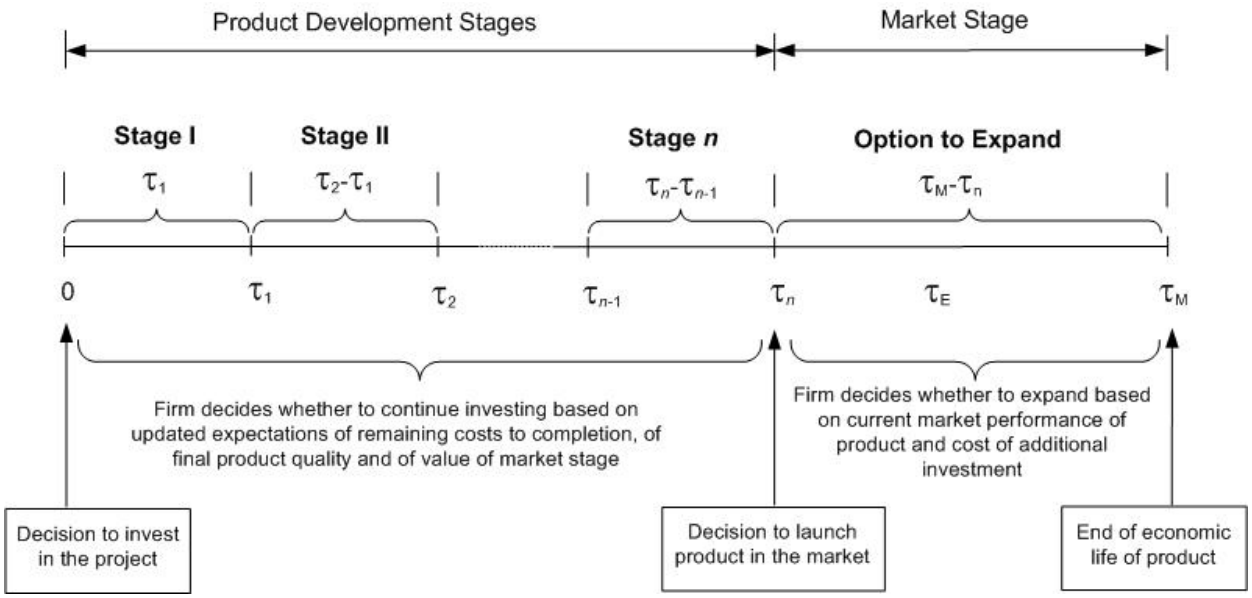


Figure 1: Stages and decisions of the general model

$$F(V, K_i, Q) = max\left[\left(V(\tau_n)e^{-\mu\tau_n} - \sum_{i=0}^{n-1} e^{-r\tau_i}\int_{\tau_i}^{\tau_{i+1}} I_{i+1}e^{-r(t-\tau_i)}dt|K_i, Q\right), 0\right] \qquad (2)$$

where $\tau_0 = 0$, $\tau_1 = \frac{K_1}{I_1}$ and $\tau_n = \sum_{i=1}^{n} \frac{K_i}{I_i}$

The first term is the value of the completed project cash flows discounted to the current time $t = \tau_0$, while the remaining terms represent the present value of the total investment costs of each stage of an $n$-stage project. Integrating the costs over all the development stages we arrive at

$$F(V, K_i, Q) = max\left[\left(V(\tau_n)e^{-\mu\tau_n} + \sum_{j=0}^{n-1} \frac{I_{j+1}}{r}\left(e^{-r\tau_{j+1}} - e^{-r\tau_j}\right)\right)|K_i, Q, 0\right] \qquad (3)$$

The optimal rule when no uncertainty exists is to invest whenever $F(V, K_i, Q) > 0$, which is simply the traditional discounted cash flow investment criteria. Since there are no uncertainties to be resolved in time, no learning occurs and whatever decision the firm makes at the onset (to invest or not to invest) remains optimal throughout the full life of the project. Without any new information forthcoming, the firm has no reason to change its decision at any future time $t > \tau_0$.

Under uncertainty, the firm can decide not only whether to invest in the project or not, but, once the project is underway, whether it should continue its investment efforts until all stages are successfully completed. At any time the firm observes the expected revenues it will receive once all stages are completed and the product is taken to market, the expected costs remaining to complete the project, and optimally decides whether to continue or abandon.

As the firm invests in its research and development efforts, it also learns about the difficulties, rate of progress and probable quality of the final product, and updates its expectations of total cost, time to completion and future revenues of the project. With this update, the firm again verifies if the progress made so far warrants continued investment in the project and decides optimally whether to continue or to abandon. The learning that occurs by investing in each of the stages gives the research effort the characteristic of a contingent claim on the value of the completed project, where the exercise price is the stochastic investment cost.

We now present the model in more detail.

*3.1. The Investment Cost Model*

The total cost of the investment and the time required to complete each stage are uncertain. We follow closely the model described by [1], where the expected cost of each stage $i$ is $E_0[\tilde{K}_i]$ and there is a constant investment rate $I_i$ in each stage. We assume that the expected costs in each stage follow a random walk with a negative drift term that reflects the instantaneous (negative) investment rate of this cost stage, and that this diffusion process is given by:

$$dK_i = -I_i dt + \sigma_i dz_i \qquad \tau_{i-1} < t < \tau_i \tag{4}$$

where $dz_i$ s the standard Wiener process that governs the investment cost process for stage $i$.

As the firm invests, the expected cost to completion tends to decrease, but is also subject to random shocks which are assumed to be the consequence of purely idiosyncratic (private) risks which can be totally diversified away. Depending on direction and intensity of these shocks, the firm may take a longer or a shorter time to complete a particular stage than expected, which will cause the actual investment cost to be respectively greater or less than expected.

We assume that the initial values of the expected costs to completion for the stages are correlated across consecutive stages. This implies that the expected cost to completion of the next stage is affected by the learning that occurs in the immediately preceding stage, and undue delays that may occur in a development stage negatively affect the initial expected costs to completion of the following stage. Likewise, if a stage is developing faster and at a lower cost than expected, then the expected cost to completion of the following stage will also decrease, even though this next stage has not yet begun. The information about the actual cost and timeliness of a particular stage i is conveyed by the random component $\sigma_i dz_i$ of the cost diffusion process. In addition to the process described by Equation 4, for any two consecutive stages $i$ and $i+1$ we will also have in stage $i$ the process described by Equation 5:

$$dK_{i+1} = \sigma_{i+1} dz_{i+1} \qquad \tau_{i-1} < t < \tau_i \tag{5}$$

where $\rho dt = E[dz_i dz_{i+1}]$ is the instantaneous correlation between $dz_i$ and $dz_{i+1}$. The expected cost to completion of stage $i+1$ will be updated during the time period $\tau_{i-1} < t < \tau_i$

prior to the beginning of this stage through the correlation between the two processes, even though stage $i+1$ has not yet begun.

*3.2. Learning*

The firm may abandon the project at any point in time, including during any particular stage up to the beginning of the market phase. At each instant it assesses the expected value of the project given the actual costs incurred up to then, the expected final quality level conditional on the deviation from the expected times to completion and the value of the future expected cash flow that will be generated by the project. The probability that the project may suffer a sudden and catastrophic ending is also taken into account and factored into the final value of the project. A negative value indicates that the expected future revenues will be insufficient to cover the actual and expected costs and the firm will be better off if it abandons the project at that point. If this value is positive, then the firm continues on until the next instantaneous exercise time when it again repeats this analysis with updated information.

Once all stages are successfully completed and the product is ready to go to market at $t = \tau_n$, the firm has one final opportunity to abandon the venture prior to the beginning of the market phase. This option to abandon will only be exercised if the expected value of the future project revenues net of any capital expenditure investment required for the production of the final goods is negative, since all prior costs incurred to develop the project are now sunk costs. The level of the capital investment at $t = \tau_n$ will depend on the nature of the R&D venture, ranging from low for intellectual property goods such as software and pharmaceutical drugs, to high for technologically complex products such as aircrafts and microprocessors.

As the product is brought to market, the firm immediately begins to analyze possible enhancements into derivative products and product extensions in order to capture additional market segments that may not have been targeted initially. The value of this expansion opportunity is a function of the actual performance of the current product in the market, which must be sufficiently large to warrant the additional investment required for the expansion. We assume that the investment cost required to implement the expansion is a fraction of the total costs originally incurred to develop and manufacture the product and involves little or no uncertainty.

*3.3. Learning*

During the design phase and prior to the beginning of the project, the firm defines a set of technical specifications that the completed product is expected to achieve. This may include, for example, the sustainable generating output of a power plant, the cruising speed, consumption and maintenance cost of an aircraft, the reliability of a complex system, the clock speed of a processor chip or the effectiveness of a drug against a particular ailment. Once the project is initiated and investment begins, the firm obtains information that allows it to revise these expectations to adjust them to the facts that are being uncovered by the research, development or construction that is taking place. For example, the actual speed and maneuverability of a new oil tanker or submarine are only known with certainty in the

final stage of the project, when the vessel undergoes extensive testing under actual working conditions in what is known as "sea trials". Shipbuilders are typically rewarded for exceeding specifications and penalized for not meeting them, so the actual performance or "quality" of the finished product can have a significant impact on the expected cash flows of the project. For the purposes of this paper, we use the term "quality" as a single measure to refer to the overall performance characteristics that impact on the cash flows that the project is expected to generate in the future. Extending the model to include additional measures of performance is straightforward.

We now define our quality model in more detail. The firm exogenously determines the time zero expected quality of the final product, conditional on the evolution of the investment stages of the project, which may be actual construction phases or R&D stages. As more information becomes available and the firm learns about the ease or difficulty of completing each stage, the firm updates this expectation. We assume that an increase in the time and cost to completion of a stage from the original estimate indicates that unforeseen problems and difficulties are negatively affecting the project and the quality of the final product will suffer. Conversely, a stage that is completed quicker and at a lower cost than expected indicates that difficulties are being resolved efficiently and that the final quality of the product will be higher than initially expected. In all, the quality is affected by the learning that occurs in each stage, and if there is no deviation from the expected costs, then no learning occurs and the initial quality level assessed remains unchanged. This implies that this stochastic variable has no drift, although random events may affect the quality of the final product.

The expected final product quality at time zero is $E_0[\tilde{Q}(\tau_n)]$ where $\tilde{Q}(\tau_n)$ is the (uncertain) quality of the final product at the end of all the R&D stages. During each stage i the firm assesses its expectation of the quality of the final product based on the learning that occurs in that stage. Although this assessment is done continuously, we assume that the firm only updates its expectation of the quality $E_{\tau_i}[\tilde{Q}(\tau_n)]$ at the end of each stage, after all the information for that particular stage becomes available. This updated expectation is then the starting point for the quality assessment during the next stage. This process ends at $t = \tau_n$ when the actual value $Q(\tau_n) = E_{\tau_n}[\tilde{Q}(\tau_n)]$ is finally determined.

We model the quality process $\tilde{Q}(\tau_n)$ as a function of a diffusion process $\tilde{Q}(t)$, $0 \leq t \leq \tau_i - \tau_{i-1}$, fluctuating stochastically within the interval $[0, 1]$, which acts as an absorbing barrier. We define the stochastic variable $S(t)$ as a driftless random walk over $(-\infty, +\infty)$ as shown in Equation 6, which is negatively correlated with the deviations from the expected investment costs associated with each stage $i$ as defined by Equation 4. This is in keeping with the assumption that the expected quality must remain unchanged if no learning occurs during the development stages.

$$dS(t) = \sigma_S(t) dz_S \tag{6}$$

where $\rho_S dt = E[dz_S dz_i]$, $i = 1, 2, ...n$, $\rho_S \leq 0$ is the instantaneous correlation between $dz_S$ and the Wiener process $dz_i$ from Equation 4, and $S(0)$. We assume that the expected quality at the end of stage $i$ $E_{\tau_i}[\tilde{Q}(t)]$ is a function of $\tilde{S}(t)$ as specified in Equation 7, where

$t = \tau_i - \tau_{i-1}$ and which assures that $E_{\tau_i}[\tilde{Q}(\tau_n)]$ remains within the range of $[0, 1]$.

$$E_{\tau_i}[\tilde{Q}(\tau_n)] = 1 - \exp\left(e^{S(t)} \log\left(1 - E_{\tau_{i-1}}[\tilde{Q}(\tau_n)]\right)\right) \tag{7}$$

We take the deviations between the actual and expected costs in each stage as a proxy for the learning that occurs during each R&D stage. At the end of stage $i$, the expected quality of the final product $E_{\tau_i}[\tilde{Q}(\tau_n)]$ will then be a function of the initial expected quality level $E_{\tau_{i-1}}[\tilde{Q}(\tau_n)]$ of the previous stage and the correlation of $\tilde{S}(t)$ with the deviations from the expected investment costs $E_0[\tilde{K}_i]$, and the only source of drift from the expected value of $\tilde{S}(t)$ will come from the correlation with the deviations in cost. As information flows and learning occurs, the quality expectation is updated at the end of each stage and $E_{\tau_i}[\tilde{Q}(\tau_n)]$ becomes the revised expectation for the quality of the final product. The process is repeated for each stage until all development stages are completed and the final product quality is determined.

If actual R&D costs are the same as the expected costs for a particular stage, then no learning has occurred, $S(t)$ remains at the zero level and the expected quality remains constant since $e^{S(t)} = 1$ and $E_{\tau_i}[\tilde{Q}(\tau_n)] = 1 - \left(1 - E_{\tau_{i-1}}[\tilde{Q}(\tau_n)]\right) = E_{\tau_{i-1}}[\tilde{Q}(\tau_n)]$.

If product development is progressing at a fast pace with no major hurdles in a particular stage, the actual time and costs of this stage will be less than expected and the cost deviation will be negative. $S(t)$ will then increase and become greater than zero, and the expected final quality will be revised upwards. For very high values of $S(t)$ we have $e^{S(t)} \to \infty$

On the other hand, if the R&D stage takes longer than expected and costs increase beyond the expected costs for this stage, the deviation will be positive and $S(t)$ will decrease in value and become negative. This will cause a similar decrease in the expected quality of the product, indicating that unforeseen problems are hampering the R&D efforts and that product quality will suffer. For large cost overruns the corresponding decrease in $S(t)$ may result in large negative values of $S(t)$ and we will have $e^{S(t)} \to 0$ and $E_{\tau_i}[\tilde{Q}(\tau_n)] = 1 - \left(1 - E_{\tau_{i-1}}[\tilde{Q}(\tau_n)]\right)^0 = 0$.

With this model, the stochastic quality of the final product evolves endogenously, spanning all stages of the project. The value function of the project must then incorporate this uncertainty, in addition to the uncertainty in the investment cost, risk of catastrophic failure and market demand for the product. This is done by multiplying the initial cash flows of the market phase by the quality factor $\Omega$, which we define in Equation 8.

$$\Omega = \left(\frac{Q(\tau_n)}{E_0[Q(\tau_n)]}\right)^{\theta} \tag{8}$$

where $\theta \geq 1$ is the sensitivity of the initial market cash flows to deviations in the quality of the final product. The sensitivity parameter $\theta$ can be used to calibrate the model to reflect the expected impact of the product quality on the value of the project.

*3.4. Risk of Catastrophic Failure*

At any time, including in the market phase, the project may be rendered worthless due to a preemptive move by a competitor, loss of the technical capability required to undertake the necessary research such as the loss of strategic personnel, or changes in regulations that may require the firm to withdraw the product from the market. The firm may also be forced to abandon its research efforts due to new safety concerns. Changes in the political and social climate may prohibit certain types of research or force the firm to forego their patent protection, such as the ban on stem cell research in the US or the loss of patent protection due to a life threatening epidemic outbreak. If such an event occurs, for whatever reason, the result is that the value of the project for the firm goes instantly to zero.

We model this uncertainty with a Poisson process, where at any time $t$ the probability of a catastrophic event occurring in the next small interval of time $dt$ is $\lambda dt$. Suppose a project producing a cash flow of $P$ in perpetuity is subject to this uncertainty. Then, the probability that it will continue for the short time period $dt$ is $1 - \lambda dt$. The value of the project is $V(P) = \int_0^\infty E[P]e^{-rt}dt$, where $E[P] = (1 - \lambda dt)^n P$, and $(1 - \lambda dt)^n = e^{-\lambda t}$. Substituting in the value function, we arrive at $V = \int_0^\infty e^{-\lambda t} P e^{-rt} dt = \frac{P}{r+\lambda}$. Therefore the impact of the probability of a catastrophic event that suddenly ends the project is equivalent to an increase in the discount rate by an amount $\lambda$. If this value is different for each of the development stages, we then represent this uncertainty as $\lambda_i$, where $i$ represents the $i^{th}$ stage.

*3.5. Market Phase*

Once all the R&D stages are successfully completed at $t = \tau_n$ and the product is approved for release, the firm has the final option to bring the product to market or to abandon the project altogether. Bringing the product to market entitles the firm to an uncertain cash flow stream for the duration of the market phase whose value at $t = \tau_n$ is $\tilde{C}(\tau_n, Q(\tau_n))$. At this point, except for cases where significant capital expenditures are required to begin the manufacture of the final product, all investment costs already been irreversibly expended, so only the expectation of a negative net cash flow stream would lead the firm to abandon the project.

The initial level of these cash flows $C(\tau_n, E_0[Q(\tau_n)])$ at the start of the project ($t = \tau_0$) is exogenously defined and reflects the firm's expectation of the commercial success of the developed product and final product quality. These cash flows are subject to systematic risk over market uncertainties concerning pricing and demand for the final product, and are assumed to evolve according to a Geometric Brownian Motion diffusion process with growth rate of $\alpha$ and volatility of $\sigma_C$ as the project progresses through the development and market phases, as shown in Equation 9. At any point in time they represent the net cash flows that would accrue to the firm were the project instantaneously completed at that time.

$$dC(t) = \alpha C(t)dt + \sigma_C C(t)dz \qquad \tau_0 < t < \tau_m \tag{9}$$

While the firm does not receive any cash flows until the project is completed, it updates this value during the investment and development stages as it observes the market and obtains more information about the true potential of the product. The information

imparted by this variable also impacts the project value and the firm's optimal investment and operational strategies.

Given that it takes time to develop and complete the project, the actual level of the cash flows that will occur during the market phase is uncertain and will depend on how soon the project will be completed, among other factors. While the firm can observe the level of these cash flows at any time, it only starts to receive them when, and if, the project is completed at $t = \tau_n$.

Under market equilibrium, the risk adjusted rate of return of the firm's shares according to the CAPM is $\mu = r + \varphi\sigma\rho_{m,S}$ where $r$ is the risk free rate, $\varphi$ is the market price of risk, $\sigma$ is the volatility of the stock returns, and $\rho_{m,S}$ is the correlation between the returns of the market and that of the stock of the firm. We assume that the project is typical of all the projects of the firm and thus the stochastic changes in the value of the project are spanned by the firm's stock so that the volatility of the project is the same as the volatility $\sigma$ of the stock, and the risk adjusted discount rate for the project market cash flows is also $\mu$.

If we consider only the net revenues associated with the original product assuming that the project offers no managerial flexibility during the market phase, the value of the project at any time $t$ during the market phase can be determined by simply taking expectations and discounting at the risk adjusted discount rate $\mu$ as shown in Equation 10, where $\tau_n < t < \tau_m$. This is also the standard value that is obtained from the traditional discounted cash flow method.

$$V(C,t) = E_t\left[\int_t^{\tau_m} C(\tau)e^{-\mu(\tau-t)}dt|C(\tau_n), Q(\tau_n)\right] \tag{10}$$

*3.6. Option to Expand*

Once a project is successfully completed, the firm engages in the market phase where it begins to manufacture the product and receive the net revenues associated with its sales. The firm may also choose to continue research on the product beyond the R&D stages in order to develop derivative products and/or additional markets segments. This presents the firm with an opportunity to expand the original market with tailored versions of the product and to develop extensions of the basic product. This is standard practice in the software industry where, for example, lower priced "academic" versions with fewer features are developed for the student market, after the full feature "professional" versions targeted towards the corporate market are launched. In the aviation industry, successful aircraft models may be modified for "long range" performance or "stretched" to accommodate increased passenger payloads. In the movie industry a successful film will be a likely candidate for a sequel, and a pharmaceutical drug can be modified to target additional markets that were not initially contemplated.

Usually the firm knows well in advance of the actual development of the product both the expected benefits and the capital costs involved and also whether such an opportunity will be available or not for any particular product, and thus we assume that if exercised, the project value is increased by a factor of $\kappa$ at a cost of $\Psi$.

The decision of whether to exercise the option to expand during the market phase is modeled as an American type option and the value of this opportunity will be a function of the cash flow stream generated by the original product, the benefits of the expansion, the volatility of the project value, the risk free rate, the time to expiration and the exercise cost, which is the investment required to implement and market the modified product. At each instant $t$, $\tau_n < t < \tau_m$, the firm assesses the expected value of the remaining project cash flow stream $V(t)$ and compares this to the expected value of the project were the expansion to take place at that time, $V'(t) = \kappa V(t) - \Psi$, net of the required capital investment $\Psi$ and chooses the greatest of the two. The instantaneous value of the project with the option to expand during the market phase is then $F(t) = maxV(t), V'(t)$, $\tau_n < t < \tau_m$. We assume that the firm has a single opportunity to exercise the option during the market phase and does so at an uncertain time $t = \tau_E$, when $V'(\tau_E) > V(\tau_E)$.

*3.7. Valuation and Solution*

As is usual in dynamic programming, we begin by assessing the value function from the terminal value of the project and work our way backwards. We rely on the equivalent risk neutral valuation and modify the cash flow diffusion process accordingly while discounting future cash flows under the risk neutral measure at the risk free rate $r$, as is standard in the option pricing literature. The modified cash flow process will be

$$dC'(t) = (r - \delta)C'(t)dt + \sigma_C C'(t)dz \qquad \tau_0 < t < \tau_m \tag{11}$$

The Bellman equation for the value of this cash flow stream is shown in Equation 12.

$$rV(C', t) = C(t) + \frac{1}{dt}E[dV(C', t)] \tag{12}$$

Applying Ito's Lemma to $dV(C', t)$ we obtain

$$dV(C', t) = \left(\frac{\partial V(C', t)}{\partial t} + (r - \delta)\frac{\partial V(C', t)}{\partial C'(t)}C'(t) + \frac{1}{2}\frac{\partial^2 V(C', t)}{\partial C'(t)^2}\sigma_C^2 C'(t)^2\right)dt + \frac{\partial V(C', t)}{\partial C'(t)}\sigma_C C'(t)dz$$

Substituting this in Equation 12, we arrive at the value equation 13 that the project value must satisfy.

$$\frac{1}{2}\frac{\partial^2 V(C', t)}{\partial C'(t)^2}\sigma_C^2 C'(t)^2 + (r - \delta)\frac{\partial V(C', t)}{\partial C'(t)}C'(t) + \frac{\partial V(C', t)}{\partial t} - rV(C', t) + C'(t) = 0 \tag{13}$$

The project value is monotonically decreasing due to the cash flows that accrue to the firm and their shareholder during the market phase, and thus the boundary condition at $t = \tau_m$ is $V(C', \tau_m) = 0$, which states that the terminal value is zero. The solution to this partial differential equation is shown in Equation 14, which is the risk neutral version of Equation 10.

$$V(C', t) = E_t\left[\int_t^{\tau_m} C'(\tau)e^{-r(\tau - t)}d\tau | C(\tau_n), Q(\tau_n)\right] \tag{14}$$

We now include both the option to expand the project and the probability of sudden catastrophic failure of the project, which is represented by the Poisson death parameter $\lambda_m$. For simplicity of notation we drop the subscript $C'$ of $V(C', t)$. If it is optimal to expand during the market phase we assume this will occur at time $t = \tau_E$, and the remaining expected future cash flows would increase by a factor of $\kappa$ while the project would suffer an immediate and instantaneous investment cost of $\Psi$. The value $V'(\tau_E)$ of the project with this expansion and also considering the possibility of catastrophic failure would then be

$$V'(\tau_E) = E\left[\kappa \int_{\tau_E}^{\tau_m} C'(t) e^{-(r+\lambda_m)(t-\tau_E)} dt | C(\tau_n), Q(\tau_n)\right] - \Psi \tag{15}$$

With this in mind, we can now determine the value of the project at time $t = \tau_n$, prior to the beginning of the market commercialization of the product. All the development stages have been successfully completed and all investment expenses are now sunk costs. If we ignore any additional capital expenditures, the firm will proceed to market if the expected value of the market phase is positive. The value of the future cash flows at this point is then:

$$V(\tau_n) = E\left[\int_{\tau_n}^{\tau_E} C'(t) e^{-(r+\lambda_m)(t-\tau_n)} dt + F(\tau_E) e^{-(r+\lambda_m)(\tau_E-\tau_n)} | C(\tau_n), Q(\tau_n)\right] \tag{16}$$

At time $t = \tau_n$ the firm gains $V(\tau_n)$ if it continues and nothing if it abandons. Accordingly, the project will be abandoned only if $V(\tau_n) < 0$. The value of the project will then be $F(\tau_n) = max\{V(\tau_n), 0\}$.

At any time prior to $\tau_n$, the firm must assess and deduct the expected cost of the investment still to be incurred from the discounted value of the future project cash flows, while continuously determining whether is optimal to abandon or to continue investing conditional on the expected quality of the final product. While this process is continuous, the expected investment costs, investment rates and other parameters such as cost correlation across contiguous development stages may be different for each development stage.

At the beginning of the last development stage $n$ at time $t = \tau_{n-1}$, the firm must invest an uncertain amount $K_n$ at an investment rate $I_n$ in order to complete this stage. The value of the future project cash flows at $t = \tau_{n-1}$ is

$$V(\tau_{n-1}) = E\left[\int_{\tau_{n-1}}^{\tau_n} I_n e^{-(r+\lambda_n)(t-\tau_{n-1})} dt + F(\tau_n) e^{-(\mu+\lambda_n)(\tau_n-\tau_{n-1})} | C(\tau_0), E_{\tau_{n-1}}[K_n], E_{\tau_{n-1}}[Q_{\tau_n}]\right] \tag{17}$$

and given that the firm will abandon the project if $V_{\tau_{n-1}}$ is negative, the value of the project at $t = \tau_{n-1}$ is $F(\tau_{n-1}) = max\{V(\tau_{n-1}), 0\}$. As the firm progresses in this development stage it updates its expectation of the cost to completion while amounts already invested become sunk costs, and this new information is then used to optimally exercise the option to abandon at any time during this stage. As previously mentioned, we assume the firm only updates its expectation of the quality of the final product at the end of each stage.

We progress steadily back in time continuously through all the development stages in the same fashion, adopting the different values for the variables and parameters for each of these stages. For example, the value of the project during the first development stage will be

$$V(t) = E\left[\int_t^{\tau_1} I_1 e^{-(r+\lambda_1)(\tau-t)} d\tau + F(\tau_1) e^{-(\mu+\lambda_1)(\tau_1-t)} | C(\tau_0), E_0[K_1], ..., E_0[K_n], E_0[Q_{\tau_n}]\right] \tag{18}$$

where $\tau_0 < t < \tau_1$. The value of the project at time $t$ is then $F(t) = max\{V(t), 0\}$, and we proceed in this manner until time $t = \tau_0$ when

$$F(0) = max\{V(0), 0\} \tag{19}$$

Figure 2 presents the project valuation model where the value of the project is shown at discrete times at the beginning of each development and market phases of the project.

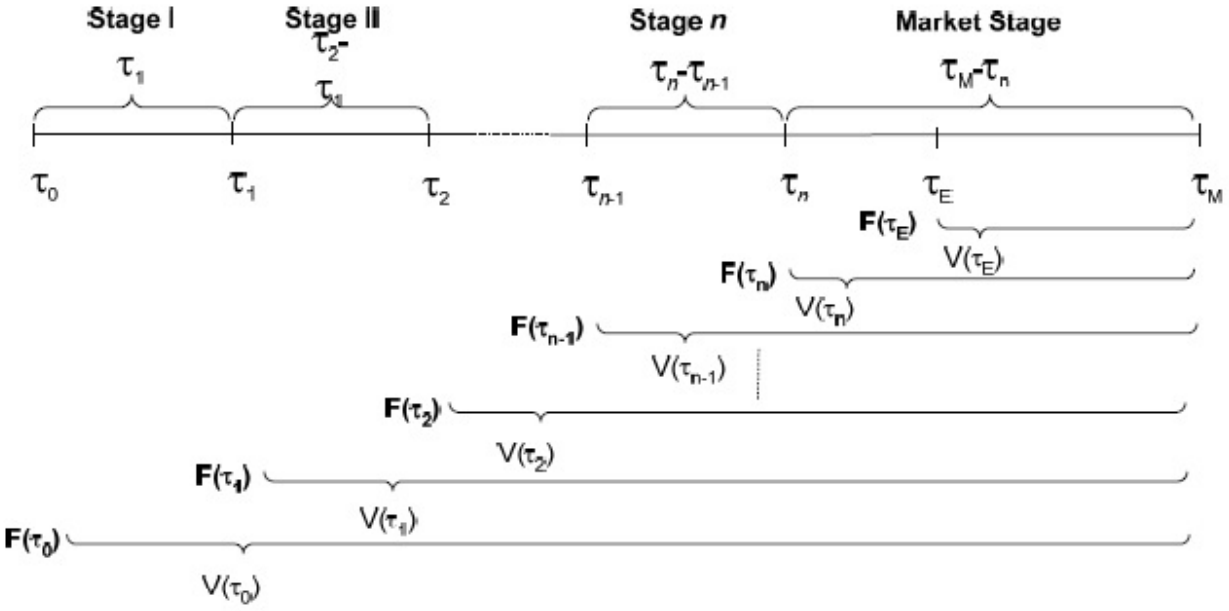


Figure 2: Dynamic Programming Valuation Model

## 4. Application: R&D in the Pharmaceutical Industry

Drugs are molecules that present biological activity against a targeted disease. Unfortunately, not all molecules are drugs, so the task of the drug maker is to discover which ones are. The drug discovery method is an iterative process involving trial and error where a targeted disease is bombarded with a series of compounds, which are traditionally obtained from dirt samples, until one is found to be active against it. Then the focus of the search is

to find what the active molecule in the compound is so that testing it for effectiveness and toxicity can begin.

Prior to World War II, pharmaceutical innovation was largely the result of luck and observation, such as the discovery of penicillin from a mold in a bacteria culture by Sir Alexander Fleming in 1928, or a matter of a chance discovery that a naturally derived chemical compound happened to be effective for a particular ailment. Aspirin, for example, which is a purified form of salicylic acid derived from the bark of the willow tree, was first synthesized in 1853. However, its properties as a painkiller, fever reducing and antiallergic drug were only discovered much later, and the drug was only introduced by Bayer in 1899.

In the post war years great scientific and medical advances brought about the development of antibiotics, hormones and tranquilizers, which by 1965, along with analgesics, were the four most commonly prescribed drug classes. Since then, the development of pharmaceutical drugs has evolved significantly and has since become an increasingly capital intensive and risky investment. The advent of biotechnology in the late 1970s created a better understanding of key diseases at the molecular level and fundamentally altered the drug discovery process, as biotech companies rushed to reproduce natural proteins and hormones, such as human growth hormone and insulin to compensate deficiencies of the body. Genetically engineered proteins and antibodies for treatment of cancer, heart disease, arthritis and other diseases followed and biotech companies such as Genentech, Amgen and Biogen became familiar names to investors. Nowadays, the development of a new drug may involve an independent research lab or an academic institution in the early stages until it is picked up by a biotechnology firm for further development. But currently only the large pharmaceutical firms have the expertise and the capital to turn a raw scientific discovery into a useful and profitable drug.

R&D efforts in the pharmaceutical industry involve significant amount of capital. The PhRMA association, which represents the leading research-based pharmaceutical and biotechnology companies in the United States, estimates that its member companies invested $33.2 billion dollars in 2003 on research to develop new treatment for diseases, on sales of $196 billion dollars [9] .

Before a firm can apply for approval by the Federal Drug Administration (FDA) to take a drug to market, the firm must successfully complete several well defined research stages and clinical trials in order to establish its efficacy, safety, dosage and other parameters. Figure 3 shows the required stages for full development of a drug in the United States.

Each of these stages provides the firm with additional information about the medical and economic feasibility of the compound as an effective and profitable drug, and based on this information the firm can decide whether to continue or abandon the project. Each stage carries a significant chance of failure and ultimately only a small fraction of the drugs that enter into pre clinical testing get to the FDA approval phase. The speed and success rate at which a drug is developed depends on the difficulties associated with the particular type of drug and disease and also on the technical competence of the firm, and are assumed to be uncorrelated with the market.

Typical R&D stages involve the discovery stage, where a new molecule is identified, synthesized and screened as a potential drug. Most of the potential molecules are abandoned

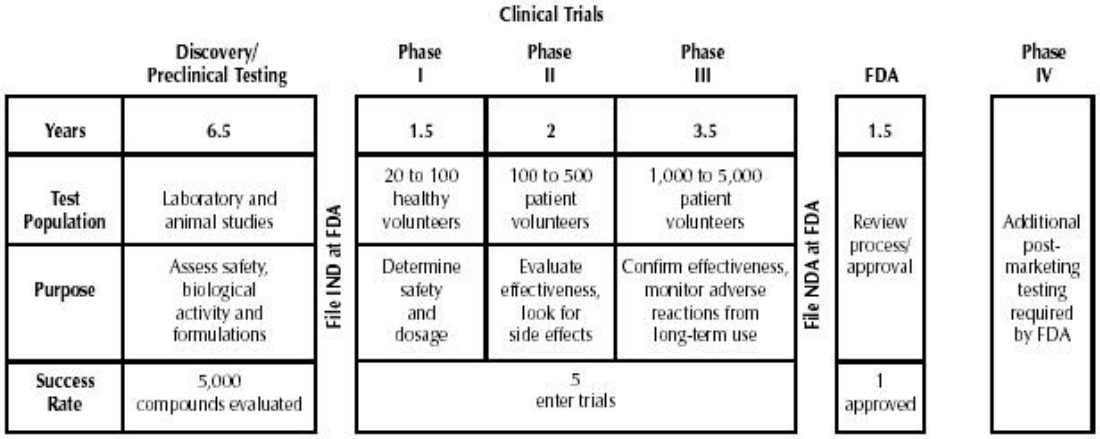


Figure 3: R&D Stages for Drug Development, [10]

during this stage, and only about 250 out of 5000 candidate molecules progress to preclinical trials, where they are then analyzed for pharmacological activity against the targeted disease and toxicity using animals. If results of this stage indicate that the drug is safe enough to be tested on humans, which occurs with approximately 5 of the 250 potential drugs that entered this stage, the firm then files a IND (Investigational New Drug Application) request to inform the FDA that it intends to enter clinical trials. The Discovery/Preclinical testing stage is by far the most lengthy and costly of all development stages, consuming roughly a third of all R&D expenditures involved in the development of a new drug and taking an average of seven years to complete.

Each of the three stages that comprise the clinical trials involve a growing number of human subjects and has the objectives of testing the potential drug for safety, appropriate dosage, negative side effects and of assessing its effectiveness for the intended use in controlled groups. If all three stages of clinical trials are successfully completed, the firm then files a NDA (New Drug Application) where it requests FDA authorization to market the drug to the general population. The data obtained during the clinical trials are the primary source of the information submitted to the FDA and may run up to 100,000 pages long. Only one in five drugs that enter clinical trials are eventually approved by the FDA.

Once the project is successfully completed and FDA approval is secured, the drug is brought to market. As is typical of risky ventures, this step presents the firm with the opportunity to expand and extend the original market for the product to include additional patient groups or markets by making improvements and/or modifications to the existing drug. The feasibility of this expansion will center on the cost of these improvements and the revenue volume of the existing drug, which in turn is a function of the quality and the market demand for the drug. Once the market phase is initiated, the firm measures the level of revenues and decides whether an expansion is warranted. For simplicity, we assume that only one such opportunity may exist during the patent lifetime of the drug, but this can easily be extended to incorporate additional expansion opportunities into different market segments

or applications. Given that the marginal cost of production of intellectual property products such as drugs is usually small compared to development efforts, it is extremely unlikely that the firm will abandon a project once development is complete and the product is brought to market, since the cash flows generated at this stage will almost certainly be positive. Due to this, during the market phase we only model the option to expand.

An opportunity to expand the project may take many different forms and can offer significant gains. For example, a drug designed to be delivered in solid form as a pill may be modified to be administered as an injectable dosage in order to target the hospital market, which may generate the opportunity to increase project revenues. Likewise, the firm may extend the target market of a drug to address the requirements of related diseases if the initial product proves to be successful for the disease for which it was originally developed, or it can modify the dosage and concentration in order to create a line extension to target the infant population. Eli Lilly & Co analyzed the feasibility of launching extensions to an anti-thrombotic drug it was developing against blood clotting that occurs during angioplasty. The extension would also allow the drug to be marketed for angina pectoris and acute myocardial infraction depending on how successful it was in the angioplasty market, and the firm estimated the incremental value from these extensions to be 2 to 2.5 times the value of the original project. Similarly, having developed a new compound (Tribactan) that increases the effectiveness of antibiotics, GlaxoSmithKline estimated that market extensions for this product could increase project value by up to 60% (Real Options Group, 2001). The costs associated with the decision to expand include the costs of future development, of completing clinical trials, filing requests with the FDA and implementing additional production facilities.

Although the United States provides a 20 year patent life for a new drug, the average commercial life during which the firm has exclusive marketing rights is estimated to be 12 years. This is due to the fact that the patent application is usually filed and granted while the drug is still under development, which reduces the effective patent duration. Increased competition between research based pharmaceutical companies can reduce the market value of a drug even further if a similar or more advanced drug for the same ailment is launched before the expiration of the patent protection. We assume that this possibility is already considered for in the reduction of project value modeled in the Poisson death process for catastrophic events; thus we make no further allowance for loses due to competitive strikes. We assume that once the patent expires, competitive forces will drive prices down to an equilibrium level where continuing operation will yield no further value creation for the firm and the project expires at the end of its effective patent life with no terminal value.

For the purpose of applying the ideas developed in our model, we analyze a hypothetical drug development project involving three development stages, followed by a market phase equal to the duration of its effective patent protection, where the firm earns a stream of uncertain cash flows and has the opportunity to expand its operations.

### *4.1. Expected Results*

The industry data and parameters adopted are assumed to be typical of the industry and are shown in Table 1 (Development Stage) and Table 2 (Market Phase).

Table 1: Parameters for Development Stage.

| Development Stage | | Stage I | Stage II | Stage III |
|---|---|---|---|---|
| Expected Investment Cost | $K_i$ | 30 | 50 | 90 |
| Investment Rate | $I_i$ | 15 | 25 | 30 |
| Cost Volatility | $\sigma_i$ | 10 | 10 | 15 |
| Cost Correlation | $\rho$ | - | 0.30 | 0.30 |

Table 2: Parameters for Market Phase.

| Market Phase | | | Other Parameters | | |
|---|---|---|---|---|---|
| Initial Cash Flow rate | $C_0$ | 35 | Expected Quality Level | $Q$ | 0.80 |
| Cash Flow growth rate | $\alpha$ | 0.08 | Quality Correlation | $\rho_Q$ | 0.30 |
| Cash Flow volatility | $\sigma_C$ | 0.30 | Quality Volatility | $\sigma_Q$ | 0.40 |
| Duration of market Phase | $D$ | 12 years | Sensitivity to Quality | $\theta$ | 2.0 |
| Risk adjusted discount rate | $\mu$ | 0.15 | Risk free rate | $r$ | 0.05 |
| Expansion Cost | $\psi$ | 100 | Poisson Parameter | $\lambda$ | 0.08 |
| Expansion Factor | $\kappa$ | 0.50 | Number of time steps per year | $n$ | 9 |

For computational purposes we divide each calendar year into nine equal time periods (a finer lattice mesh can be obtained by simply increasing the time step parameter $n$) and have the program sequentially scan each possible discrete development path that may occur, beginning with the shortest and least costly ones. Longer cost paths have decreasing probability of occurrence, and a cutoff point is required in order to limit the time required for the computations. While the time step $n$ and the precision parameters chosen provide a level of discretization that is adequate for the valuation purpose of this application, it may be coarse for graphic displays and the approximation errors appear as jagged lines in some of the graphs. Greater detail can be obtained by adopting a finer mesh and higher precision levels at the cost of increased computer processing times.

We begin our analysis by initially considering only the opportunity to abandon the project if the prospects of profitable completion are unlikely, and compare this to the results from the traditional DCF valuation model. Later we incorporate and analyze the impact of each of the additional features of the model such as the option to expand during the market phase, investment cost correlations and the uncertainty over the quality of the final product.

Figure 4 shows that the threshold level for the DCF analysis is \$23.45 million, and since the initial value of the cash flow stream is \$35 million, the project will no be undertaken. We can also see that increases in the cost correlation across consecutive development periods increases the option value of abandonment due to an increase in the volatility of the cost process. The option to abandon, considering a cost correlation of 0.3, increases the project NPV to \$1.86 million and decreases the threshold level to \$31.45 million.

Figure 5 shows the effect of the option to expand will depend on the parameters used. This shows how changes in the expansion factor ($\kappa$) impact the project value for different

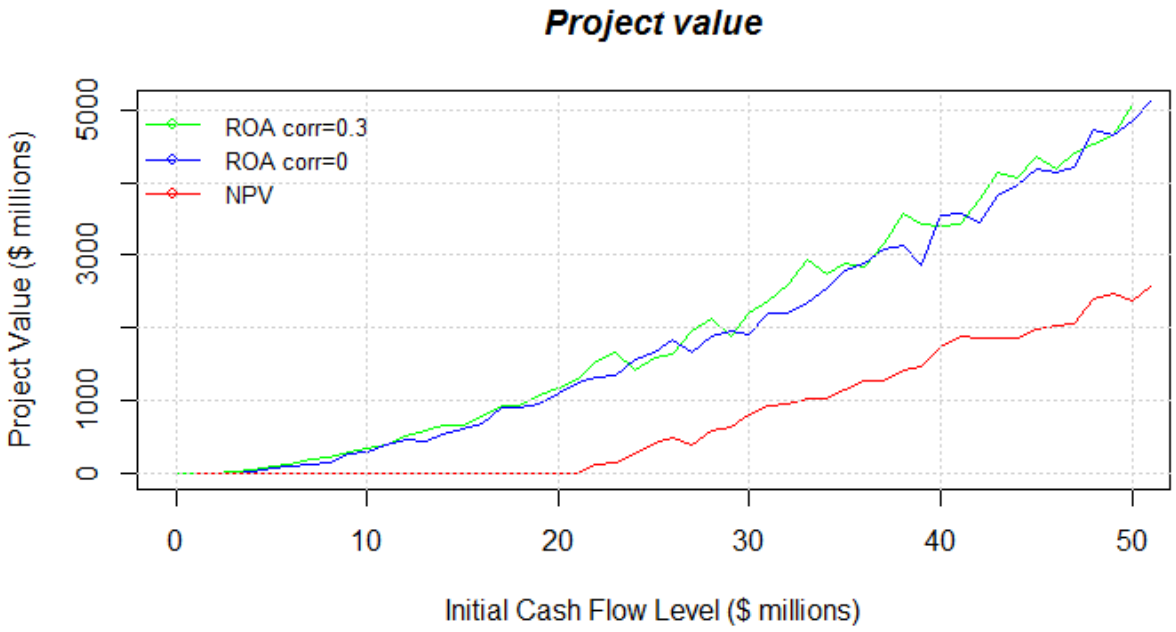


Figure 4: Project Value with Option to Abandon

initial cash flow levels, and we can see that significant increases in value are obtained for small increments in the expansion factor.

Finally, we note that the initial market cash flow stream, were the project completed today, is assumed to be $35 million. Under traditional discounted cash flow analysis (DCF), where no consideration is given to project options, investment cost correlations or product quality, the value of the project will grow linearly as a function of the initial level of the cash flow stream once the minimum cash flow investment threshold level is overcome. This threshold represents the minimum cash flow level at which the firm is indifferent about whether to invest or not in the project. With the option to abandon, the firm can opt out of the project if it learns that development is lagging, and thus is able to avoid a potential loss. This has the net effect of increasing the project value whenever the option is exercised, which is more likely to occur when the initial cash flow level is close to the threshold level.

## 5. Conclusions

The analysis of multistage investment problems is a typical application of real options analysis given the characteristics of a compound option problem. We modeled a R&D investment problem with multiple sources of uncertainty and compound abandon and expansion options as a discrete approximation to a continuous time problems using a lattice approach.

We show that the existence of this opportunity to further expand the market revenues of the finished product once the development stages have been successfully completed can significantly affect the value of the project and, as a result, the optimal investment decision.

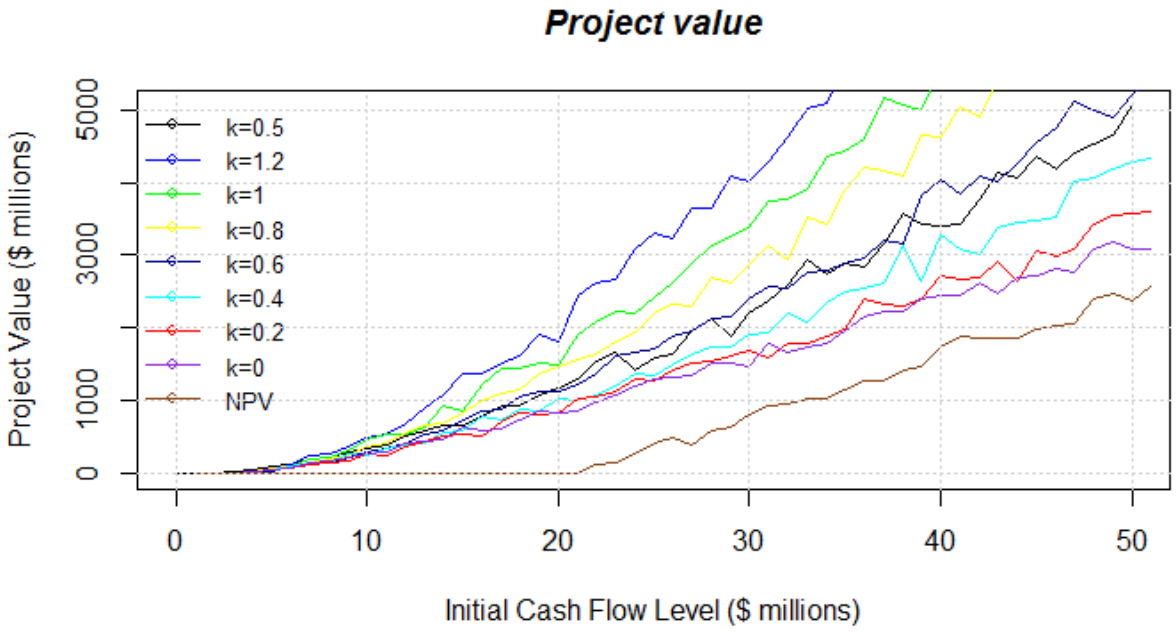


Figure 5: Initial Cash Flow Level ($millions)

In particular, we show that the option to expand after product development is completed can increase the project value by an order of magnitude.

Additionally, the uncertainty over the quality of the finished product also impacts the valuation problem, and is in turn affected by the difficulties that arise from the product development.

Traditional real options models usually require simplifying assumptions that limit the complexity of the model in order to maintain tractability of the solution. In the drug development case presented, the opportunity to expand into new markets adds an additional level of complexity which is more difficult to model with traditional continuous time methods.

The valuation problem has no closed form solution and was solved using a discrete lattice approach under dynamic programming. The lattice approach provides an efficient computation tool to numerically solve the valuation function of the model. Greater precision can be obtained by adopting a finer mesh for the lattice, which will approach the continuous time solution at the limit. While the degree of precision must be weighted against the time necessary for the computations, reasonable approximations can be obtained even with coarse discretizations. The model presented here is very flexible and can be used as a general framework to develop more detailed and sophisticated models. It allows for the inclusion of additional development stage uncertainties, increased numbers of options and the modeling of government induced industry incentives.

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