ENTREPRENEURS, FINANCIERS AND GROWTH OPPORTUNITIES: MODELS FOR ENTREPRENEURIAL FINANCING DECISIONS BASED ON REAL OPTIONS

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This paper is a Working Paper submitted to the 2014 Real Options Conference: Please do not quote it

Abstract

This paper uses a real options approach to design a set of models to better understand entrepreneurial financing decisions, in a framework where a single shareholder and positive cash-flow generating firm are assessing its potential growth opportunities, for which it may require additional financial resources to be provided by a Venture Capitalist, provided that no debt financing will be available to fund such growth opportunities. In the base case, model outputs reveal that the envisaged profit growth should offset the ownership loss that the Entrepreneur will bear by allowing a Venture Capitalist to provide equity to fund the growth strategy. With the purpose of complementing this base case, a set of extensions was derived, including the case in which the Entrepreneur and the Venture Capitalist hold distinct profit growth prospects for the growth strategy, the case in which the Entrepreneur may hold a given minimum ownership requirement and the case in which, prior to the equity round, the Venture Capitalist is willing to acquire a ownership stake on the entrepreneurial firm. Provided that Entrepreneurs and Venture Capitalists find that the underlying profit flow of the Start-up Firm is modelled according to the same stochastic process, model prescriptions are valid for whichever stochastic process is chosen for the profit firm of the entrepreneurial firm.

Keywords: Venture Capital, Entrepreneurial Finance, Real Options, Growth Options, Entrepreneurship

JEL-Codes: G24, G31, G34, L26, M13

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1. INTRODUCTION

The relevance of Entrepreneurial Financing as a research topic is highlighted by the growing dynamics on the number of births of enterprises in Europe, which recorded a 0.79% compound annual growth rate between 2004 and 2011, totalling 2,265,182 firms in 2011 (Eurostat, 2014) or by the investment activity of venture capitalists («VCs»), which stood for 77.8% of the overall private equity and venture capital investment volumes between 2007 and 2013. In fact, this specific investment class has been gaining growing importance on the private equity and venture capital industry: while venture and growth stage investments stood for 72.1% of the total private equity and venture capital investment volumes and 14.8% of their investment amounts in 2007, these stood for 81.1% of deal volumes and 19.5% of investment amounts in 2013 (EVCA, 2014).

On this paper we intend to build on the existing literature, by designing a set of theoretical models to better understand which variables may have an impact on the Entrepreneurial Financing processes, and how these may influence their potential outcomes. Specifically, and taking the view that Entrepreneurial Financing decisions are a process where Entrepreneurs, VCs and Growth Opportunities interact simultaneously, we are interested in describing the conditions under which Entrepreneurs and VCs might reach an agreement towards the execution of a given growth strategy.. In addition, we take a demand-side view on the Entrepreneurial Financing process, highlighting the terms, which from the Entrepreneur's point-of-view, might be critical to undertake a decision on a given growth strategy jointly with an external equity provider.

The paper is structured as follows: Section 2 reviews major literature contributions on Entrepreneurial Financing processes, Section 3 outlines the basic model, Section 4 presents three extensions to the basic model: in Section 4.1 it is assumed that there are distinct perceptions on profit growth between the VC and the Entrepreneur, in Section 4.2 minimum shareholding requirements by the Entrepreneur are introduced, and in Section 4.3 an alternative deal structure in analysed, whereby the Entrepreneur sells part of the firm ownership prior to the equity round to be carried alongside the VC. Section 5 concludes by pointing out further model extensions and some of the research paths which are currently progressing.

2. LITERATURE REVIEW

Embedded on a given macroeconomic context, Entrepreneurial Financing decisions result from the interaction between three interdependent elements: an Entrepreneur, a Growth Opportunity and a Financier. As most of the literature tends to focus on these elements separately, we take the view that theoretical decision-making models for Entrepreneurial Financing processes shall essentially and holistically portray Entrepreneurial Financing as an alignment process between an Entrepreneur and a Financier, which is originated by a given Growth Opportunity.

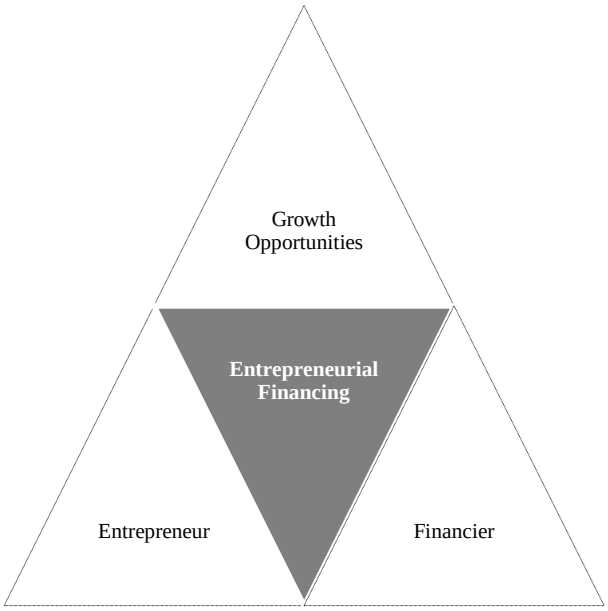

Figure 1. Building blocks of Entrepreneurial Financing decisions

As a result, we introduce the topic by presenting the most relevant literature contributions for each of building blocks of Entrepreneurial Financing decisions, followed by a summary of the main literature contributes to Entrepreneurial Financing as a process.

2.1. The Building Blocks of Entrepreneurial Financing Decisions

2.1.1. The Entrepreneur

Wealth maximization, wealth constraints, risk attitudes and individual goals form the main drivers of Entrepreneur's behavior. Rasmussen & Sørheim (2012) suggest that Entrepreneurs' perceptions, preferences, business case, relationship networks and the process of obtaining

financing are issues of key importance for understanding the outcomes of an Entrepreneurial Financing process, while Brush, Edelman & Manolova (2012) highlight the concept of «venture readiness». Sapienza, Korsgaard & Forbes (2003) propose that wealth maximization, self-determination and perceptions of the risks to self-determination are the primary motives driving Entrepreneurial Financing choices.

Muzyka, Birley, & Leleux (1996) acknowledge that the quality of the Entrepreneur is key to determine the funding decision. Wright, Robbie, & Ennew (1997) add that VCs do make extensive use of "serial entrepreneurs" who have exited from other VC's portfolios, primarily to lead management buy-ins. This is underlined by Gompers, Kovner, Lerner, & Scharfstein (2006) who have shown that (i) Entrepreneurs with a track record of success are more likely to succeed than first time Entrepreneurs and those who have previously failed, (ii) conversely, funding by more experienced VC firms enhances the chance of success, but only for Entrepreneurs without a successful track record, (iii) more experienced VCs are able to identify and invest in first time Entrepreneurs who are more likely to become serial entrepreneurs, and (iv) investments by VCs in successful "serial entrepreneurs" generate higher returns for their VC investors.

Similarly, Baron & Markman (2003) offered support for the hypothesis that the higher Entrepreneurs' social competence is, the greater their financial success. Even though pre-deal analysis rendered by VCs is pretty much focused on assessing the entrepreneurial team, Dimov & Shepherd (2005) have found that although general management team human capital had a positive association with the proportion of portfolio companies that went public, specific management team human capital (i.e., MBA, law education, or consulting experience) did not. On the other hand, specific management team human capital was negatively associated with the proportion of portfolio companies that went bankrupt.

Hsu (2007) investigated the sourcing and valuation of VC funding among Entrepreneurs with varied levels of prior start-up founding experience, academic training, and social capital, by examining venture valuation, as it reflects enterprise quality and entrepreneurs' cost of financial capital. Using data from a survey of 149 early stage technology-based start-up firms, Hsu (2007) found that (i) prior founding experience (especially financially successful experience) increases both the likelihood of VC funding via a direct tie and venture valuation,

(ii) Entrepreneur's ability to recruit executives via their own social network (as opposed to the VC's network) is positively associated with venture valuation and (iii) in the emerging (at the time) Internet industry, entrepreneurial teams with a doctoral degree holder are more likely to be funded via a direct VC tie and receive higher valuations, suggesting a signaling effect.

Chaganti, DeCarolis, & Deeds (1995) highlight that the prevailing paradigm on Entrepreneurial Financing decisions ignores factors such as owners' values or goals. Winborg (2000) showed that Entrepreneurs who sought financing to achieve higher growth sought more external funding, and that those who professed a need for external financing also held more positive attitudes towards it. Winborg (2000) talks about a "fear" (beyond economic loss) that Entrepreneurs have regarding external sources of funding.

Although the fear of loss of control or, alternatively, the drive for independence has been frequently mentioned has a key motivator for Entrepreneurs (Ang, 1992; Chaganti, DeCarolis, & Deeds, 1995), few attempt to sort out whether observed drive for self-determination is a means to achieve economic ends or a separate end in itself. Chaganti, DeCarolis, & Deeds (1995) posit that some Entrepreneurs are motivated by economic gain for themselves or their families and that others are motivated by their "desire" for control or their own affairs and to avoid dependence on others. Those driven by economic gain seek a different mix of external to internal financing mix than those driven by self-determination.

2.1.2. Growth Opportunities

Growth Opportunities carried by entrepreneurial firms feature significant uncertainty on future cash flow generation, involve considerable irreversible costs and benefit from flexible plans, forming investment opportunities that might be modelled as real options (Schwienbacher, 2007; Li Y., 2007). In addition, game options might be useful for modelling Entrepreneurial Financing as an interactive process, as shown by the pricing models for mergers and acquisitions by Lukas, Reuer & Welling (2012) and Yu & Xu (2011).

Debt financing is usually not an option for such Growth Opportunities, as entrepreneurial firms present low profits and lack tangible assets, driving Entrepreneurs to choose between distinct sources of equity (Fairchild, 2009; Andrieu & Groh, 2012; Schwienbacher, 2013),

rather than deciding between equity and debt financing (De Bettignies & Brander, 2007). Accordingly, we consider that, within an Entrepreneurial Financing context, Financiers are usually VCs in the broad sense that they are external equity providers and that these bring the relevant behavior and constraints to an Entrepreneurial Financing process.

Dixit & Pindyck (1994) present a comprehensive review of a wide range of growth options which may form distinct backgrounds for the setting in which the Entrepreneurial Financing process takes place. These include combined entry and exit strategies, growth options with lay-up reactivation and scrapping options, multistage projects, or growth options in competitive industry settings, covering distinct sources of uncertainty, from prices, to costs, to volumes.

2.1.3. Financiers

As Financiers of Growth Opportunities brought by Entrepreneurs, VCs might be regarded as profit maximizers with specific return on investment thresholds, which make use of a set of mechanisms to deal with information asymmetry and potential agency conflicts. Examples of these mechanisms include:

(i) The introduction of *contractual covenants* on VC contracts (such as cash-flow rights, voting rights, board rights, liquidation rights, as well as non-compete and vesting provisions) in order to mitigate information asymmetries and hold-up problems (Carter & Van Auken, 1994; Kaplan & Strömberg, 2001; Cumming, Schmidt, & Walz, 2006; Cumming & Binti Johan, 2007; Leisen, 2012);

(ii) The use of *preferred stock and convertible stock*, with the evidence that this may minimize expected agency problems associated with start-up and expansion stage investments and align the VCs and the Entrepreneur's interests with respect to different exit solutions (Sahlman, 1990; Gompers P. A., 1997; Kaplan & Strömberg, 2001; Bascha & Walz, 2001; Cumming, 2002), whereas debt and common stock are more appropriate at later stages of venture financing;

(iii) The use of *staged capital infusion schemes* and *rights of first refusal*, giving VCs the option to cut off badly performing ventures from new rounds of financing, thus minimizing the losses carried by early stage VCs, while controlling risk and mitigating moral hazard (Sahlman, 1990; Gompers P. A., 1995; Li, 2000; Gompers & Lerner, 2001; Wang & Zhou, 2004; Tian, 2011; Dahiya & Ray, 2012; Leisen, 2012);

(iv) The use of *compensation schemes* aligning the interests of VCs and Entrepreneurs, by providing a substantial fraction of compensation in the form of equity and options or by vesting stock options over a multiyear period, making it impossible to the Entrepreneur to leave the firm and take his or her shares (Gompers & Lerner, 2001). These are similar frameworks to those used to mitigate potential agency problems between VCs and their fund providers;

(v) The *duration of financing*, which is in turn related to the nature of the firm's assets, as higher industry ratios of tangible assets to total assets, lower market-to-book ratios, and lower research and development intensities are associated with longer funding duration (Gompers P. A., 1995).

In addition, VCs may provide non-financial contributions to Growth Opportunities and Entrepreneurs, such as value-adding monitoring (Croce, Marti, & Murtinu, 2013), professionalization (Kaplan & Strömberg, 2001) and firm certification (Hsu, 2004), which may be perceived as valuable by the Entrepreneur and therefore hold an impact on the outcomes of Entrepreneurial Financing processes.

Hsu (2004) has evaluated both the certification and value-added roles of reputable VCs, having tested and confirmed the proposition that Entrepreneurs are willing to accept a discount on the valuation of their start-up in order to access the capital of VCs with better reputations. Offers made by VCs with a high reputation are three times more likely to be accepted, and high-reputation VCs acquire start-up equity at a 10% to 14% discount. These results have shown that (i) VCs "extra-financial" – as also suggested by Fried & Hisrich

(1995), Steier & Greenwood (1995) or Hellmann & Puri (2002) – value may be more distinctive than their functionally equivalent financial capital and that (ii) affiliation is an ordinary economic good for which actors seeking association will face a price-reputation trade-off. In fact, Hsu (2004) follows a research stream that suggests that when the quality of a start-up cannot be directly observed, external actors rely on the quality of the start-up's affiliates as a signal of the start-up's own quality, as this certification-based approach may help legitimate start-ups and Entrepreneurs without a prior track record. This might be a possibile explanation for understanding how Entrepreneurs may choose between different sources of a same type of financing and equity, in particular.

While financial and non-financial contributions, alongside contractual mechanisms, form the grounds for the approach of VCs to Entrepreneurial Financing processes, the existing literature points out the impact that fund demography (i.e., fund size, fund location, fund age and fund experience) may also impact VCs decision-making (Franke, Gruber, Harhoff, & Henkel, 2006; Smart, 2000; Isaksson, Cornelius, Landström, & Junghagen, 2004; Dimov & Murray, 2008 and Tian, 2011).

2.2. Contributes to the Entrepreneurial Financing Process

The pecking-order theory (Myers & Majluf, 1984; Myers, 1994) is one of the most relevant contributes to understanding the Entrepreneurial Financing process. This hypothesis – stating that Entrepreneurs prefer internally generated funds first, debt next, and external equity last – incorporates an economically rational view of Entrepreneurs's financing preferences. Within a venture financing context, information asymmetry and uncertainty make the availability of external financing very limited and its cost prohibitively high. To compensate, Entrepreneurs must give up greater and greater control in order to "buy" funds needed to achieve the desired growth and profitability. However, according to this theory, Entrepreneurs are reluctant to accept external equity because of its accompanying threat of wealth dilution, unless they believe that financial opportunities made available through it exceed its financial costs. These greater costs for small and new firms arise out of the business and agency risks inherent in dealing with start-ups. The absence of a performance history for the venture and skill verification for the entrepreneurial teams lead to greater perceived risks of incompetence

and opportunism. Additionally, because executing due diligence (Harvey & Lusch, 1995; Harvey & Lusch, 1998) is as costly if done on a small firm as one on a large one, it is relatively more expensive for suppliers of capital to process funding for new firms. The risks would be lower for debt providers, to the extent that collateral exists.

Parhankangas (2007) underlines Myers (1994) by stating that, even though most Entrepreneurs prefer internal to external funding, few have sufficient funds to finance early stage projects themselves. It is also at this stage of development, when collateral-based funding from banks – the second-most preferred source of funding by Entrepreneurs – is often inappropriate or even potentially life-threatening to the new firm. Therefore, the alternative provision of VC becomes an attractive source of finance for potentially important companies operating on the frontier of emerging technologies and markets.

Adding to Trester (1998), Ueda (2004) has addressed the question of how start-up firms decide to raise funds from banks or from VCs. In order to do so, Ueda (2004) designed a model in which the VC can evaluate the Entrepreneur's project more accurately than the bank but can also threaten to steal it from the Entrepreneur. Consistent with evidence regarding VC finance, the model implies that the characteristics of a firm financing through VC are relatively little collateral, high growth, high risk, and high profitability. The model also suggests that tighter protection of intellectual property rights encourages Entrepreneurs to finance through VCs. The choice between the bank and the VC then depends then on two elements: (i) the severity of the asymmetric information problem between the Entrepreneur and the bank, and (ii) how strongly intellectual property rights are protected. Low collateral value, high growth, high return, and high risk of the project all raise the cost of the asymmetric information and thereby drive the Entrepreneur to finance through a VC fund.

Addressing a similar research question, De Bettignies & Brander (2007) examined the Entrepreneur's choice between bank finance and VC. With bank finance, the Entrepreneur keeps full control of the firm and has efficient incentives to exert effort. With VC finance, there is a two-sided moral hazard problem as both the Entrepreneur and VC provide unverifiable effort. The Entrepreneur benefits from the VC's managerial input but must surrender partial ownership of the venture, thus diluting the Entrepreneur's incentive to provide effort. Then, VC funding tends to be preferred to bank finance when VC productivity

is high and entrepreneurial productivity is low, as the choice between VC and bank finance is determined by the trade-off between VC productivity and the Entrepreneur's effort dilution.

Additionally, an equity share between an Entrepreneur and a VC holds a set of hidden costs, as this will dilute the Entrepreneur's incentive to provide appropriate effort and create potential problems or conflicts arising from Entrepreneur's loss of control, especially if the Entrepreneur is crucially important to the venture. The authors conclude that if debt commitments are available, VCs cannot survive as a pure financial intermediary: bank finance would always be preferred to a VC who could not provide managerial value-added to the venture.

Such prevailing economic perspectives provide a limited theoretical framework on venture financing, in the sense that although they posit a theoretical framework for how Entrepreneurs choose between different sources of financing, they do not address the question of how Entrepreneurs choose among financing sources of the same type. Literature points outs that this choice might grounded on (i) relationship features between Entrepreneurs and VCs (Sapienza & Korsgaard, 1996; Cable & Shane, 1997; De Clerq & Sapienza, 2001; De Clerq & Fried, 2005; and Zahra & Allen, 2007), (ii) Entrepreneur's personality traits (Landier, 2002; Sapienza, Korsgaard, & Forbes, 2003; and Hsu, 2004), (iii) VC industry features (Kanniainen & Keuschnigg, 2004; Mantell, 2005; and Shepherd, Armstrong, & Lévesque, 2005) and (iv) information asymmetry issues (Trester, 1998; Jungwirth & Moog, 2004; Gompers & Xuan, 2009; Elitzur & Gavious, 2011)

2.2.1. Final Remarks

Contributes to a better understanding of Entrepreneurial Financing processes are vast, coming from distinct theoretical perspectives, such as agency theory (Gompers P. A., 1995; and Sapienza & Villanueva, 2007), stewardship theory (Arthurs & Busenitz, 2003; Sapienza, Korsgaard, Goulet & Hoogendam, 2000) or procedural justice (Sapienza & Korsgaard, 1996; Busenitz, Moesel, Fiet, & Barney, 1997). As described on the previous sections, literature also reveals that there is a wide range of distinct decsion-making drivers within each of the Entrepreneurial Financial building blocks, covering financial, behavioral and demographic issues.

Acknowledging that such diversity would definitely provide a valuable contribution for modelling Entrepreneurial Financing processes and that such diversity may stand for a enriching source of potential extensions, we take a rational economic approach to the behavior of the two intervenient parties on Entrepreneurial Financing processes, by assuming that both Entrepreneurs and VCs are profit maximizers.

This choice is grounded (i) on the assertion that profit maximization still stands for one significant decision-making variable both for Entrepreneurs and VCs, (ii) on the purpose of making model outputs as tractable as possible, minimizing the number of variables and conditions required for obtaining results and (iii) on the perspective that Entrepreneurial Financing processes are essentially alignment processes between Entrepreneurs, Financiers and Growth Opportunities, in which Growth Opportunities may in turn become one of the alignment mechanisms between Entrepreneurs and Financiers. Therefore, as Growth Opportunities are basically changes on existing profit flows, we understand that a model that does not explictly address Entrepreneurs and VCs (at least) as profit maximizers through Growth Opportunities would not capture an essential trait of Entrepreneurial Financing decisions.

3. THE BASE CASE

On this section, we describe the model that was developed to analyse Entrepreneurial Financing decisions. Particular emphasis will be put on the equilibrium conditions which allow a given Entrepreneur and a Financier (taken as a VC) to jointly execute a given growth strategy. First, we describe the assumptions in which the basic model is grounded. Then, we derive the basic model and analyse its major outcomes. Then we will present three extensions on this base case.

(I)	**Assumptions on the Start-up Firm**	
(I.1)	Ownership	The Start-up Firm is owned by a single shareholder, which is assumed to be the E. It is assumed that E invested an initial capital k^i, $k^i > 0$ on the Start-up Firm.
(I.2)	Profitability	The Start-up Firm is currently delivering positive profits, which in turn generate a continuous continuous-time profit flow (π), which is assumed to follow a Geometric Brownian Motion (GBM) diffusion process.
(I.3)	Growth Strategy	The Start-up Firm holds an infinite set of alternative growth strategies comprising a given capital expenditure (k, $k > 0$) and a given multiplier on current profit flows (e, $e > 1$). This forms a growth option whose value adds to current profit flow generation. It is assumed that the firm does not generate enough financial resources internally to fund this given growth strategy.
(II)	**Assumptions on the Entrepreneur** (named as E)	
(II.1)	Objective Function	Taking a rational economic perspective, E is modelled as a profit maximiser.
(II.2)	Financial Resources to Support the Growth Strategy	E holds a limited set of financial resources to fund the start-up firm's growth strategy, given by k^a , $k^a > 0$. If capital expenditures required by the growth strategy are lower than k^a, then E would prefer to individually fund the growth strategy designed by Start-up Firm, as this would entitle to maximize her/his wealth without foregoing part of the Start-up Firm ownership. Therefore, $k > k^a$.
(III)	**Assumptions on the Financier** (taken as a VC)	
(III.1)	Type	Neither the Start-up Firm nor E have access to debt financing. As a result, it is assumed that the growth strategy will have to be funded through an external source of equity (i.e., a VC).
(III.2)	Objective Function	Taking a rational economic perspective, VC is modelled as a profit maximiser. Therefore, no specific features from business angels, corporate venture capitalists, captive venture capitalists or public venture capitalists are considered within the framework of this model.
(III.3)	Financial Resources to Support the Growth Strategy	VC has no funding restrictions to support the growth strategy designed by the Start-up Firm. As a result, and given (II.2), the execution of a given growth strategy would require E to obtain an external equity financing equal to (k - k^a > 0), which is equal to the amount of equity financing to be provided by the VC.
(IV)	**Assumptions on Deal Structuring**	
(IV.1)	Pre-Money Valuation	E and VC value the Start-up Firmat k^i before carrying the capital increase to execute the growth strategy, meaning that the equity value of the Start-up Firm is equal to its face value.
(IV.2)	Deal Type	The growth strategy will be exclusively funded by a capital increase, meaning that voting rights and profits will be proportionately shared between E and VC according to the amount of capital that each of these entities will provide to the Start-up Firm, meaning that this capital increase is made at no premium or discount. As a result, (i) parties will not provide capital to the Start-up Firm through a distinct source of capital, such as preferred shares or any quasi-equity instruments and that (ii) VC shall not acquire any ownership stake directly to E, before carrying the capital increase on the Start-up Firm.
(IV.3)	Contractual Covenants	No contractual covenants between E and VC with material impacts on their objective functions are agreed. These could include staged capital infusions, earn-outs, liquidation rights, or compensation schemes.

Table 1. Key assumptions of the basic model

3.1. Assumptions

The base case comprises a Start-up Firm, owned by a single Entrepreneur, which is generating positive profits and is designing its growth strategy, comprising an expansion of current profit flows at the expense of a given capital expenditure. In a world with no debt financing, such capital expenditure should be funded through an equity round baked by the Entrepreneur, who is assumed to own limited resources, and by an external financier, who is assumed to be a VC with no funding constraints.

A summary of the set of assumptions considered on the base case is presented on Table 1. Taking into account the assumptions (I.1), (II.2), (IV.1) and (IV.2) described on Table 1. Key assumptions of the basic model, and depending both on the growth strategy adopted (i.e., the set formed by k and e) which is chosen by the Entrepreneur and the VC, post equity round firm ownership held by the Entrepreneur E (given by Q^E) and by the VC fund (given by Q^{VC}) will come as follows:

Shareholder	As-Is	With Growth Strategy
Entrepreneur	$Q^E = 100\%$	$Q^E = \frac{k^i + k^a}{k^i + k} < 100\%$
Venture Capitalist	$Q^{VC} = 0\%$	$Q^{VC} = \frac{k - k^a}{k^i + k} > 0\%$

Table 2. Firm ownership split between E and VC with growth strategy

It is worth noting that, for a given k^a, Q^E is negatively related to k, implying that capital outlays for proceeding with a large expansion might be such that the Entrepreneur may lose firm ownership (i.e., $Q^E < 50\%$). This happens when $k > k^i + 2.k^a$. In such a context, the problem is not only about the dimension and profit expansion of the growth strategy, but also about the way that firm ownership shall be shared after the equity round and about potential hidden costs rising from diluting the Entrepreneur's ownership on the Start-up Firm.

Two relevant considerations should be highlighted. On the one hand, we assume that the Start-up Firm is unable to obtain any debt financing. We find this as a reasonable assumption,

considering that these firms typically (i) present a limited or inexistent historical firm performance, whereby banks can accurately assess credit risk for the entrepreneurial firm, (ii) do not own tangible assets which could serve as a collateral to debt financing, or would have to bear prohibitive interest costs otherwise, and (iii) debt financing could potentially lead to inadequate capital structures, with debt repayment schedules causing major cash-flow constraints to small and rapidly growing firms facing significant uncertainties. In addition, there could exist major credit restrictions due to macroeconomic and other exogenous factors that can exclude any debt financing alternatives.

On the other hand, we assume that that pre-money valuation is given equal to k^i and that an equity round backed by the Entrepreneur and the VC is made at no premium or discount. If no arbitrage is expected to occur during this process, this assumption demands that (i) both the Entrepreneur and the VC agree that the present value of the profit stream from the assets in place is given by k^i and that (ii) the value of the existing growth options held by the Start-up Firm is zero, or vice versa, meaning that the present value of the current profit stream is zero and the value of the existing growth options held by the Start-up Firm is equal to k^i. Although a comprehensive review of start-up valuation techniques is not the purpose of this paper, we understand that pre-money valuation based on invested capital is a practical and straightforward criteria used by VCs and Entrepreneurs when screening funding decisions as (i) even cash-flow positive start-up firms usually do not provide visible and appropriate returns on capital employed and (ii) their expansion options typically face high uncertainties, not only on business planning, but mostly on business plan execution, which may advise more conservative valuations.

3.2. Model derivation

Taking into account the assumptions described on the previous section, the Start-up Firm generates a continuous-time profit flow (π), which is assumed to follow a Geometric Brownian Motion diffusion process given by:

$$(1)\ d\pi = \alpha\pi dt + \sigma\pi dz,$$

where $\pi > 0$, α and σ stand for the trend parameter (i.e., the drift) and to the instantaneous volatility, respectively. Additionally, assuming that agents are risk neutral, $\alpha = r - \delta$, where $r > 0$ is the risk-free rate and $\delta > 0$ stands for the dividend yield. Finally, dz is the increment of a Wiener process. We assume that both the Entrepreneur and the VC understand that the continuous profit flow (π) follows the same stochastic process.

3.2.1. The decision to invest in the growth strategy for the Entrepreneur

Following the contingent-claim approach used by Dixit & Pindyck (1994), the value of the option held by E to invest in the expansion of the Start-up Firm, $E(\pi)$, must satisfy the following ordinary differential equation ("ODE"):

$$(2)\ \frac{1}{2}\sigma^2\,\pi^2\,E''(\pi) + (r-\delta)\pi\,E'(\pi) - r\,E(\pi) + \pi = 0,$$

where the last term on the left hand side of equation (2) refers to the current profit flow of the Start-up Firm and the remaining terms refer to the growth option held by the Start-up Firm. The general solution for (2) comes:

$$(3)\ E(\pi) = A\pi^{\beta_1} + B\pi^{\beta_2} + \frac{\pi}{\delta},$$

where A and B are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic, given by:

$$(4)\ \varrho(\beta) = \frac{1}{2}\sigma^2\,\beta\,(\beta-1) + (r-\delta)\beta - r = 0,$$

i.e.,

$$(5)\ \beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

$$(6)\ \beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

Assuming that π^*_E stands for the optimal trigger to carry the small expansion project, and considering that in order to execute the growth strategy $Q^E < 100\%$, according to assumptions

(I.1), (II.2), (IV.1) and (IV.2), the problem must be solved by considering the following boundary conditions:

$$(7)\ E(0) = 0$$

$$(8)\ E(\pi_E^*) = \frac{e.\pi_E^*}{\delta}.Q^E - k^a$$

$$(9)\ E'(\pi_E^*) = \frac{e}{\delta}.Q^E$$

Respecting condition (7) and noting that $\beta_2 < 0$, then B on the equation (2) must be equal to zero. Therefore, for the remaining of this paper, $\beta \equiv \beta_1$. The unknowns A and π^*_E are obtained by combining conditions (8) and (9). Solutions for the optimal profit trigger and for the option to invest then come:

$$(10)\pi_E^* = \frac{\beta}{\beta-1} \cdot \frac{\delta}{e.Q^E-1} k^a$$

$$(11)E(\pi) = \begin{cases} \frac{\pi}{\delta} + \frac{(e-1).\pi_E^*}{\delta\beta}\left(\frac{\pi}{\pi_E^*}\right)^\beta, for\ \pi < \pi_E^* \\ \frac{e.\pi.Q^E}{\delta} - k^a, for\ \pi \geq \pi_E^* \end{cases}$$

Our previous assumption that $e > 1$ ensures that $\pi^*_E > 0$, which is the only condition required for keeping economic meaning. Notice that as $e \rightarrow 1$, $\pi^*_E \rightarrow \infty$, meaning that for a given k and k^a, the lower the level of expansion, the higher will be the adequate profit flow that ensures an optimal investment decision.

Model outcomes reveal that the entrepreneurial profit trigger π^*_E is smaller, (i) the larger the profit expansion is $\left(\frac{\partial \pi_E^*}{\partial e} < 0\right)$ and (ii) the higher the post-project firm ownership retained is $\left(\frac{\partial \pi_E^*}{\partial Q^E} < 0\right)$, while this profit trigger π^*_E becomes higher, the higher the overall capital outlay for deploying the growth strategy k is $\left(\frac{\partial \pi_E^*}{\partial k} = \frac{\partial \pi_E^*}{\partial Q^E}\frac{\partial Q^E}{\partial k} > 0\right)$. In turn, the value of the option to proceed with a large expansion for the Entrepreneur increases both with the underlying profit expansion factor e and post-project firm ownership Q^E.

3.2.2. The decision to invest in the growth strategy for the VC

The value of the option held by the VC firm to support the growth strategy of the Start-up Firm, given by *VC(π)*, must satisfy the following ordinary differential equation, which does not hold the current profit flow π provided by the firm (as the underlying ODE presented on the Entrepreneur case), due to the fact that VCs can only profit by undertaking the expansion project, and not from current firm profitability, if they decide not to participate in this growth strategy:

$$(12)\frac{1}{2}\sigma^2\,\pi^2\,VC''(\pi) + (r-\delta)\pi\,VC'(\pi) - r\,VC(\pi) = 0.$$

The boundary conditions come as follows:

$$(13)VC(0) = 0$$

$$(14)VC(\pi_v^*) = \frac{e.\pi_{VC}^*}{\delta}.Q^{VC} - (k-k^a)$$

$$(15)VC'(\pi_{VC}^*) = \frac{e}{\delta}.Q^{VC},$$

where π^*_{VC} stands for the optimal profit trigger to support the growth strategy for the VC firm. Similarly to the Entrepreneur case, solutions to the unknowns underlying the option value are given by

$$(16)\pi_{VC}^* = \frac{\beta}{\beta-1}\cdot\frac{\delta}{e.Q^{VC}}(k-K_\alpha)$$

$$(17)VC(\pi) = \begin{cases} \frac{e.Q^{VC}.\pi_{VC}^*}{\delta\beta}\left(\frac{\pi}{\pi_{VC}^*}\right)^\beta, for\ \pi < \pi_{VC}^* \\ \frac{e.\pi}{\delta}.Q^{VC} - (k-k^a), for\ \pi \geq \pi_{VC}^* \end{cases}$$

It is implicitly assumed that the VC does not burden any additional opportunity costs from foregoing other potential investments in other companies or equivalently, it is assumed that the current investment opportunity is the best opportunity in which the VC fund may invest in.

3.2.3. Defining the equilibrium growth strategy

Now that we derived the value of the option to invest on the growth strategy held by the Entrepreneur and by the VC, we can now obtain the underlying conditions that ensure that a given growth strategy, formed by a given *k* and a given *e*, could be formed so that the Entrepreneur and the VC are jointly willing to support it. Formally, we shall now derive conditions for *k* and *e* that allow $\pi_E^* = \pi_{VC}^*$.

3.2.3.1. The capital outlay *k*

As for the capital outlay *k* that will allow the Entrepreneur and the VC to reach an agreement towards a joint investment on the growth strategy by the Start-up Firm, and considering that $Q^E = \frac{k^i + k^a}{k^i + k}$ and $Q^{VC} = \frac{k - k^a}{k^i + k}$, we obtain the following solution:

$$(18)\, k = k^i.(e-1)\text{, if } e > \frac{k^a + k^i}{k^i} \Leftrightarrow e > 1 + \frac{k^a}{k^i}$$

Such condition on *k* implies that, if current profit flow grows at least proportionately to the capital that the Entrepreneur provided to the Start-up Firm previously to and after executing the growth strategy, then the overall capital outlay *k* that will allow E and VC to carry this growth strategy shall be also proportional to the expected profit growth. In this case, $Q^E = \frac{k^i + k^a}{e.k^i} = \frac{1}{e}.\frac{k^a}{k^i}$

If such proportional effects do not take place, then the profit expansion of the Start-up Firm envisaged by the Entrepreneur would not offset the loss coming from a lower ownership on the Start-up Firm.

3.2.3.2. The profit growth multiplier *e*

Equivalently, as for the profit growth multiplier *e* that will allow the Entrepreneur and the VC to reach an agreement towards a joint investment on the growth strategy by the Start-up Firm, and considering that $Q^E = \frac{k^i + k^a}{k^i + k}$ and $Q^{VC} = \frac{k - k^a}{k^i + k}$, we obtain the following solution:

$$(19)\, e = \frac{k^i + k}{k^i}$$

This condition imposes that, without further restrictions on k^a or k, the equilibrium growth strategy should be such that profit flows expand proportionately to the capital employed in the Start-up Firm. Again, this is the condition that allows the Entrepreneur not to bear any losses arising from her/his ownership dilution.

One of the most interesting results on (18) and (19) lays on the fact that the growth strategy to be jointly set by the Entrepreneur and the VC does not depend on any of the parameters of the underlying stochastic process that governs the profit flow of the Start-up Firm.

4. MODEL EXTENSIONS

4.1. When Entrepreneurs and VCs hold different prospects on profit growth

On this extension, we will relax the assumption that Entrepreneurs and VCs share the same perspectives on profit growth, given by e. In fact, Entrepreneurs and VCs often hold distinct prospects over the same growth strategy, which could in turn lead to long discussions during negotiation stages. We argue that, even in these conditions, an agreement might be reached.

In this new setting, we take the profit growth envisaged by the Entrepreneur as e_E and the profit growth envisaged by VC as e_{VC} and re-write the boundary conditions and profit triggers for each of the options to invest on the growth strategy held by the Entrepreneur and the VC. As no further changes occur on the base case, we obtain the new boundary conditions and profit triggers simply by replacing e by e_E or e_{VC} respectively on equations (8), (9), (14) and (15), as presented on the table below.

For the Entrepreneur	For the VC firm
(20) $\boldsymbol{E(\pi_E^*) = \frac{e_E.\pi_E^*}{\delta}.Q^E - k^a}$	(23) $VC(\pi_{VC}^*) = \frac{e_{VC}.\pi_{VC}^*}{\delta}.Q^{VC} - (k - k^a)$
(21) $\boldsymbol{E'(\pi_E^*) = \frac{e_E}{\delta}.Q^E}$, leading to	(24) $VC'(\pi_{VC}^*) = \frac{e_{VC}}{\delta}.Q^{VC}$, leading to
(22) $\boldsymbol{\pi_E^* = \frac{\beta}{\beta-1}\cdot\frac{\delta}{e_E.Q^E-1}k^a}$	(25) $\pi_{VC}^* = \frac{\beta}{\beta-1}\cdot\frac{\delta}{e_{VC}.Q^{VC}}(k - k_\alpha)$

Table 3. Boundary conditions and profit triggers with different prospects on profit growth *e* for the Entrepreneur and the VC

Equating again $\pi_E^* = \pi_{VC}^*$ with the purpose of computing the capital outlay k under which the Entrepreneur and VC would be jointly willing to support the growth strategy of the Start-up Firm, we obtain the following solution:

$$(26)\, k = k^a.(e_E - e_{VC}) + k^i.(e_E - 1),$$
$$\text{if } e_E \geq 1 + e_{VC} \text{ or } e_E.\left(k^a + k^i\right) > e_{VC}.k^a + k^a + k^i$$

This extension on the base case reveals that the capital outlay k which underlies the Start-up Firm growth strategy should be such that:

(i) It is equal the proportional profit growth envisaged by the Entrepreneur, given by the term $k^i.(e_E - 1)$, which is similar to the one presented on the base case, added by

(ii) A term on $k^a.(e_E - e_{VC})$, meaning that the Entrepreneur should at least hold enough capital k^a to compensate any differences on the profit expansion prospects which may exist between the Entrepreneur and the VC (i.e., $e_E - e_{VC}$).

In this case, note that the solution on k only exists if there is a significant difference on profit growth expectations (i.e., $e_E \geq e_{VC} + 1$) or if $e_E \geq \frac{e_{VC}.x^a}{x^a + x^i} + 1$, i.e., if the profit growth envisaged by the Entrepreneur is at least equal to the proportional profit growth envisaged by VC on the total capital deployed by the Entrepreneur to execute the growth strategy, given by $(x^a + x^i)$.

Solving $\pi_E^* = \pi_{VC}^*$ for e_E in order to compute which profit growth envisaged by the Entrepreneur would allow her/him to reach an agreement with a VC to fund a given growth strategy, we obtain the following solution:

$$(27)\, e_E = \frac{k + k^i + e_{VC}.k^a}{k^i + k^a}$$

Results reveal that for an agreement to be reached between an Entrepreneur and a VC, it is enough that expectations on profit growth held by the Entrepreneur, on the one hand, offset the ownership loss that will be incurred with the execution of the growth strategy (i.e., generated by $\frac{k + k^i}{k^i + k^a}$) and, on the other hand, the capital to be deployed by the Entrepreneur to fund the growth strategy benefits from the profit growth which is expected to be generated by the VC (i.e., $\frac{e_{VC}.k^a}{k^i + k^a}$).

Again, results presented on (26) and (27) show that the growth strategy to be jointly set by E and VC does not depend on the underlying stochastic process that drives the profit flow of the Start-up Firm, provided that E and VC consider that the same stochastic process applies to the profit flow of the Start-up Firm.

4.2. When Entrepreneurs hold minimum ownership requirements

Fear of losing ownership control on the Start-up Firm is a key issue on Entrepreneurial Financing decisions considered by Entrepreneurs (Sapienza, Korsgaard, & Forbes, 2003). As a result, and considering that critical ownership thresholds to distinct levels of corporate control depend on exogenous variables, such as the prevailing legal framework or personal preferences, we shall build on the extended model of the previous section, by imposing one additional restriction, in which the Entrepreneur demands a given ownership (named *w*, $0 < w < 1$) to be retained so that the growth strategy is executed.

This additional restriction on equating $\pi_E^* = \pi_{VC}^*$ shall be formally written as $Q^E = \frac{k^i + k^a}{k^i + k} \geq w$. No changes on the boundary conditions and profit triggers presented on the previous section occur in this new setting. The new solution shall therefore comprise a new condition, dependent on *w*, i.e.,

$$(28)\ k = k^a.(e_E - e_{VC}) + k^i.(e_E - 1),$$
$$\text{if } e_E \geq \frac{e_{VC}.k^a}{x^a + k^i} + 1 \text{ and } 0 < w < \frac{k^a + k^i}{e_E.(k^a + k^i) - e_{VC}.k^a}$$

As a result, the Entrepreneur may aim to hold a ownership stake equal to the ratio between (i) the overall capital employed by the Entrepreneur (i.e., $k^a + k^i$) and (ii) the difference between the profit multiplicative effect the Entrepreneur expects to have on her/his total invested capital (i.e., $e_E.(k^a + k^i)$) and the multiplicative effect on profit growth that the Entrepreneur expects to benefit according to the VC on the additional capital that she/he will provide to the Start-up Firm (i.e., $e_{VC}.k^a$).

As before, the profit growth perceived by E to allow an agreement to be reach shall be at least equal to $\frac{e_{VC}.k^a}{x^a + k^i}$ as before.

4.3. When a combined equity sale and capital increase takes place

So far, we have assumed that the embedded deal structure on the Entrepreneurial Financing setting would comprise an equity round to be backed by the Entrepreneur and the VC. In this extension, we will consider an alternative deal structure, whereby the Entrepreneur may sell part of the Start-up Firm ownership prior to carrying the capital increase, in order to obtain an early additional cash-in. Such partial equity stake sale might be made at face value or at a given premium on face value (which will be named as p, $p > 0$).

We are interested in understanding (i) in which circumstances would the VC be willing to acquire an ownership stake in the Start-up Firm before participating on the equity round and (ii) which premium the VC would be willing to pay for a given equity stake on the Start-up Firm, so that an agreement may be reached with the Entrepreneur.

4.3.1. Computing the ownership stake to be sold prior to the equity round

On this case, profit triggers will be changed, provided that, (i) for the VC, the acquisition of a given ownership stake in the Start-up Firm will increase the required capital outlay for entering into this growth opportunity, but will also increase the share of the Start-up Firm held by the VC and, conversely, (ii) for the Entrepreneur, the sale of a given ownership stake in the Start-up Firm will decrease the required capital outlay for entering into this growth opportunity, at the expense of a lower share on the Start-up Firm and, consequently, of the growth opportunity.

Introducing this new setting into the previous model, naming z ($0 < z < 1$) as the ownership stake that the VC would be willing to acquire prior to participating on the equity round to fund the growth strategy, and assuming that such acquisition would be made at face value (i.e., would require a cash-out equal to $z.k^i$), will lead to the changes on the profit triggers held by the VC and the Entrepreneur described on the table below.

	For the Entrepreneur	For the VC firm
Required capital outlay	(29) $k^a - z.k^i$	(30) $k - (k^a - z.k^i)$
Post equity round ownership	(31) $\frac{k^a+(1-z).k^i}{(k+k^i)}$	(32) $\frac{k-k^a+z.k^i}{(k+k^i)}$
Profit trigger	(33) $\frac{\beta}{\beta-1} \cdot \frac{\delta}{e_E \frac{k^a+(1-z).k^i}{(k+k^i)}-1}(k^a - z.k^i)$	(34) $\frac{\beta}{\beta-1} \cdot \frac{\delta}{e_{VC} \cdot \frac{(k-k^a+z.k^i)}{k^i+k}}(k - k_\alpha + z.k^i)$

Table 4. Required capital outlays, Start-up Firm ownership and profit triggers when there is a ownership sale at face value prior to the equity round

In order to ensure that a net capital outlay occurs for the Entrepreneur, we will have to set $k^a - z.k^i > 0$. Otherwise, the Entrepreneur would make no capital outlay to benefit from the growth strategy, which would require an alternative valuation approach. In addition, with the purpose of simplifying modelling outcomes, we will relax the conditions described on the previous extension, whereby the Entrepreneur could hold minimum ownership requirements.

As a result, equating (33) and (34), and solving for *z*, we obtain the following solution

$$(35)\ z = \frac{k+k^a(e_{VC}-e_E)-k^i(e_E-1)}{k^i(e_{VC}-e_E)} = \frac{k}{k^i(e_{VC}-e_E)} + \frac{k^a}{k^i} - \frac{e_E-1}{e_{VC}-e_E}, \text{ subject to}$$

(i) If the Entrepreneur expects a greater profit growth than the VC, then:

a. The capital outlay to be made by the Entrepreneur (given by k^a) should be at least compensate the loss on the initial capital outlay which shall be shared with the VC (given by $k^i e_E$ or $k^i e_{VC}$), i.e.,

$$(36)\ \frac{k^i+k-k^i e_E}{e_E-e_{VC}} < k^a \leq \frac{k^i+k-k^i e_{VC}}{e_E-e_{VC}}$$

b. The total capital outlay required by the growth strategy (given by *k*) should be at least be proportional to the envisaged profit growth by the Entrepreneur – implying that $k > k^i(e_E - 1)$) – but no greater than the one adjusted by the excess profit

growth that the VC expects to be generated above the Entrepreneur – meaning that $k \leq \frac{k^i(e_E-1)}{(1+e_{VC}-e_E)}$ – and therefore

$$(37)\, k^i(e_E - 1) < k \leq \frac{k^i(e_E-1)}{(1+e_{VC}-e_E)}$$

(ii) If the VC expects a greater profit growth than the Entrepreneur, then:

a. Condition (36) should remain in place;

b. Conversely to (37), the total capital outlay required by the growth strategy (given by *k*) should be at least greater than the proportional to the initial capital on the Start-up Firm, adjusted by the excess profit growth that the VC expects to be generated above the Entrepreneur – meaning that $k > \frac{k^i(e_E-1)}{(1+e_{VC}-e_E)}$ – but no greater than the one which would be proportional to the envisaged profit growth by the Entrepreneur – given by $k < k^i(e_E - 1)$ – leading therefore to

$$(38)\, \frac{k^i(e_E-1)}{(1+e_{VC}-e_E)} < k \leq k^i(e_E - 1)$$

As a result, we can interpret the solution on (35), which stands for the ownership stake *z* that would allow an agreement to be set between the Entrepreneur and the VC over the sale of an ownership stake in the Start-up Firm at face value and over the terms of the equity round to be put in place to fund the growth strategy.

The target ownership stake *z* presented on (35) states that *z* is as influenced by three additive factors, which are always dependent on the profit growth that the VC expects the growth strategy to reach over the one estimated by the Entrepreneur, which is formally given as $e_{VC} - e_E$:

(i) *The size effect*, given by $\frac{k}{k^i(e_{VC}-e_E)}$, which stands for the proportion of the overall capital outlay *k* against the initially invested capital k^i: the greater the total capital outlay *k* is against the initially invested capital k^i, the greater would be the ownership stake to be sold to the VC so that an agreement is set with the Entrepreneur;

(ii) *The over profit growth effect*, given by $\frac{k^a}{k^i}$, which stands for the additional profit growth that the VC is expecting to have against the Entrepreneur, over the capital outlay k^a that the Entrepreneur shall provide to support the execution of the growth strategy: the greater the capital outlay k^i that the Entrepreneur shall provide to the Start-up Firm is and the greater the excess profit growth that the VC is expecting to have against the Entrepreneur, the greater will be the ownership stake to be sold to the VC so that an agreement is reached. This translates the gain to be generated on the VC by the equity apportions previously made (given by k^i) and to be made by the Entrepreneur (given by k^a) that will also contribute to the profit growth of the Start-up Firm, in the same way as the capital apportions to be made by the VC (given by $k - k^a$);

(iii) *The profit growth effect*, given by $\frac{e_E - 1}{e_{VC} - e_E}$, which stands for the profit growth that the Entrepreneur expects to be generated through the growth strategy: the greater this profit growth is – given by $(e_E - 1)$ – over the profit growth that the VC expects to achieve above the Entrepreneur – given by $e_{VC} - e_E$ – the greater is the ownership stake that has to be sold to the VC so that an agreement is reached.

4.3.2. Computing the premium on face value that the VC would be willing to pay

In this new setting, we will build on the previous case by introducing a variable *p* (p > 0) that will stand for the premium on face value that the VC would be willing to pay to the Entrepreneur for acquiring a given ownership *z* in the Start-up Firm, prior to participate in the equity round of the Start-up Firm alongside the Entrepreneur.

Therefore, while Start-up Firm ownership will remain equal to the previous case, the required capital outflows and profit triggers change by $(1 + p)$ as presented on Table 5. Required capital outlays, Start-up Firm ownership and profit triggers when there is a ownership sale with a premium over the face value prior to the equity round Similarly, in order to equate profit triggers, we will relax the existence of minimum ownership requirements posed by the Entrepreneur and we will assume that

$k^a - z.k^i.(1+p) > 0$, so that the Entrepreneur bears a net capital outflow to proceed with the envisaged growth strategy.

	For the Entrepreneur	For the VC firm
Required capital outlay	(39) $k^a - z.k^i.(1+p)$	(40) $k - k^a + z.k^i.(1+p)$
Post equity round ownership	(41) $\frac{k^a+(1-z).k^i}{(k+k^i)}$	(42) $\frac{k-k^a+z.k^i}{(k+k^i)}$
Profit trigger	(43) $\frac{\beta}{\beta-1}\cdot\frac{\delta}{e_E\frac{k^a+(1-z).k^i}{(k+k^i)}-1}(k^a - z.k^i.(1+p))$	(44) $\frac{\beta}{\beta-1}\cdot\frac{\delta}{e_{VC}.\frac{(k-k^a+z.k^i)}{k^i+k}}(k - k_\alpha + z.k^i.(1+p))$

Table 5. Required capital outlays, Start-up Firm ownership and profit triggers when there is a ownership sale with a premium over the face value prior to the equity round

Equating the resulting profit triggers for the Entrepreneur and the VC – i.e., (43) = (44) – and solving this for the premium *p*, we obtain the following result

$$(45)\ p = \frac{(k-k^a+iz)}{iz}.\frac{k^a(e_E-e_{VC})+k^i((e_E-1)-z(e_E-e_{VC}))-k}{k^a(e_{VC}-e_E)-k^i[(e_E-1)+z(e_{VC}-e_E)]-k(e_{VC}-1)}, \text{ subject to}$$

(i) A set of conditions on the profit growth multipliers envisaged by the Entrepreneur and the VC, in particular:

a. The profit growth envisaged by the Entrepreneur should be at least proportionately greater than the overall capital outlay to fund the growth strategy (*k)* relative to the overall invested capital in the business by the Entrepreneur after selling ownership *z* to the VC – i.e., $k^a + k^i(1-z)$ – and therefore

$$(46)\ e_E > \frac{k+k^a+k^i(1-z)}{k^a+k^i(1-z)}, \text{ and}$$

b. The profit growth envisaged by the VC should be greater than the ratio between (i) the total invested capital on the Start-up Firm – given by $k + k^i$ – net from the return that the Entrepreneur expects to generate from the capital she/he will deploy on the

business – given by $e_E\left(k^i(1-z)+k^a\right)$ – and (ii) the stake that the VC will acquire on the Start-up Firm ($z.k^i$), net from the capital outlay to be provided by the Entrepreneur (k^a), i.e.

$$(47)\ e_{VC} > \frac{k+k^i+e_E(k^i(1-z)+k^a)}{k^i(1-z)-k^a}$$

(ii) Or, a set of conditions on the ownership stake *z* that the Entrepreneur will sell to the VC and on the envisaged profit growth by the Entrepreneur:

a. The ownership *z* should be lower than the capital outlay to be made by Entrepreneur to execute the growth strategy (k^a) relative to the capital initially deployed on the Start-up Firm (k^i), i.e.,

$$(48)\ 0 < z < \frac{k^a}{k^i}, \text{ and}$$

b. The expected profit growth envisaged by the Entrepreneur should be more than proportional to the total amount of capital to be deployed on the Start-up Firm ($k+\ k^i$) against the total capital deployed on the Start-up Firm by the Entrepreneur – i.e., ($k^a + k^i(1-z)$ – and lower than the capital outlay required for the growth strategy (k) relative to the total capital deployed on the Start-up Firm by the Entrepreneur – i.e., $k^a + k^i(1-z)$ – and, therefore

$$(49)\ \frac{k+\ k^i}{k^a+k^i(1-z)} < e_E \leq \frac{k+k^a+k^i(1-z)}{k^a+k^i(1-z)}$$

(iii) Or, finally, a set of conditions on the capital outlay to be made by the Entrepreneur to fund the growth strategy (k^a), in which:

a. The capital available for the Entrepreneur to participate in the equity round (k^a) is at least proportional to the total capital outlay to execute the growth strategy (k) given the envisaged profit growth by the Entrepreneur ($e_E - 1$), net from the initial

invested capital (k^a) in the Start-up Firm that the Entrepreneur retains after selling $(1-z)$ to the VC, and

b. The capital available for the Entrepreneur to participate in the equity round (k^a) is lower than the total capital outlay to execute the growth strategy (k) less the return on the initially invested capital – given by $k^i(e_E-1)$ – relative to the excess profit growth that the Entrepreneur expects to have over the VC (e_E-e_{VC}), and the additional ownership that the VC will held on the Start-up Firm ($z.k^i$), i.e.

$$(50)\ \frac{k}{e_E-1}-k^i.(1-z)<k^a\leq\frac{k-k^i(e_E-1)}{e_E-e_{VC}}+z.k^i$$

Looking into the solution for (45) as a product of two factors which may be analyzed separately, we may interpret them as follows:

(i) The term $\frac{(k-k^a+iz)}{iz}$ stands for a Entrepreneur's *compensation for the ownership dilution*, as it indicated that p should be greater, the greater the capital outlay to be provided by the VC to execute the growth strategy (i.e., $k-k^a+iz$) relative to the part of the ownership which might be sold the VC, given by iz;

(ii) As for the numerator of the second term, we understand that:

a. $k^a(e_E-e_{VC})$ stands for the share on the return of the capital outlay to be performed by the Entrepreneur k^a that offsets VC's expectations of profit growth (e_E-e_{VC});

b. $k^i[(e_E-1)+z(e_{VC}-e_E)]$ stands for the share (i) on the return that the initial capital deploy shall generate according to the Entrepreneur's expectations $k^i(e_E-1)$ net from (ii) on the ownership loss the Entrepreneur will bear by selling the ownership z to the VC, given by $z(e_{VC}-e_E)$;

c. As a result, p should be greater, the greater (i) the expected profit growth by the Entrepreneur is above the one held by the VC (e_E-e_{VC}), (ii) the expected profit

growth by the Entrepreneur (e_E) is, (iii) the expected profit by the VC is above the one held by the Entrepreneur ($e_{VC} - e_E$) and (iv) the greater the difference between the return on the capital to be provided by the Entrepreneur to support the growth strategy (k^a) and the return on the capital initially employed on the Start-up Firm (k^i) against the overall capital to be deployed to be execute the growth strategy (*k*).

(iii) As for the denominator of the second term, we conversely understand that:

a. $k^a(e_{VC} - e_E)$ stands for the share on the return of the capital outlay to be performed by the Entrepreneur (k^a) that offsets Entrepreneur's expectations of profit growth, given by $(e_{VC} - e_E)$;

b. The term $k^i[(e_E - 1) + z(e_{VC} - e_E)]$ is symmetrical to the one described on (ii)b. above, and translates the gain that the VC will earn by acquiring the stake *z* in the Start-up Firm and obtaining higher profits than those envisaged by the Entrepreneur, i.e., $z(e_{VC} - e_E)$;

c. The term $k.(e_{VC} - 1)$ stands for the return on capital expected by the VC on the overall capital which shall be deployed on the Start-up Firm, given by *k*.

We can derive that *p* therefore depends on the relationship between (i) what the Entrepreneur may benefit from jointly executing the growth strategy with the VC and losing the ownership *z* in the Start-up Firm – where terms in "+" stand for a positive effect and terms in "-" for negative effects and (ii) what the VC may benefit by executing such growth strategy, and paying a premium for a given ownership stake *z* in the Start-up Firm.

5. CONCLUSION AND FUTURE RESEARCH PATHS

This paper highlights the role that initial capital outlays, the available capital for scale expansion by the Entrepreneur, and profit growth perceptions hold on Entrepreneurial Financing processes. It also shows how ownership requirements and alternative deal structures combining both capital increases with ownership sale may hold on the outcomes of

Entrepreneurial Financing decisions. In the base case, model outputs reveal that the envisaged profit growth should offset the ownership loss that the Entrepreneur will bear by allowing a VC to provide equity to fund the growth strategy. Provided that Entrepreneurs and VCs find that the underlying profit flow of the Start-up Firm is modelled according to the same stochastic process, model prescriptions are valid for whichever stochastic process is chosen for the profit firm of the entrepreneurial firm

Relevant model extensions might be generated by introducing some of the constraints or terms which are usually part of financing contracts, either brought by Entrepreneurs or VCs. In addition, model outcomes might be extended by introducing some of the personal preferences of the Entrepreneur within its decision-making process, including self-determination, decision control risk or perceived decision control risk, or by introducing potential agency conflicts or additional information asymmetry issues between Entrepreneurs and VCs.

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