Investment in Technological Innovations under Rivalry and Uncertainty

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30 January 2015

Abstract

Investments in the area of technological innovations are particularly risky, since, apart from price uncertainty, firms must take into account not only uncertainty in the arrival of innovations but also the presence of potential rivals. Therefore, we develop an analytical framework for sequential investment in order to determine how duopolistic competition impacts a firm's technology adoption strategy. We assume that firms either adopt a compulsive strategy and invest sequentially in each technology that becomes available or a leapfrog/laggard strategy, whereby they first wait for a new technology to arrive and then decide whether to invest in a newer or an older version, respectively. Thus, we determine both the optimal technology adoption strategy, and, within each strategy, the optimal investment rule. Results indicate that the relative loss in the value of a leader due to the presence of a rival decreases as the first–mover advantage and the rate of innovation increase, yet increases with greater price uncertainty. Intriguingly, while technological uncertainty has a non–monotonic impact on the optimal investment threshold of a follower, it does not impact a leader's investment decision. Finally, we compare the investment strategies and illustrate how, unlike in the case of monopoly, the compulsive strategy always dominates the leapfrog/laggard strategy.

Keywords: Investment analysis, Real options, Competition

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1 Introduction

As a product of research and development (R&D), technological innovations occur at random points in time and the firm that adopts them first is able to capture a greater market share. Furthermore, since technological innovations are subject to frequent upgrades, a firm typically does not hold a single but rather a sequence of investment opportunities, and, as a result, it must determine both the optimal adoption strategy, and, within each strategy, the optimal investment rule taking also into account the presence of potential rivals. Although various models have been developed in order to analyse sequential investment under price and technological uncertainty, most of these assume a monopoly or perfect competition (Grenadier and Weiss, 1997; Farzin *et al.*, 1998; Doraszelski, 2001; Chronopoulos and Siddiqui, 2014), and, as a result, how strategic interactions impact sequential investment decisions under price and technological uncertainty remains an open question.

Incorporating such features in an analytical framework for sequential investment is crucial as these are pertinent to various industries, e.g., computer software, telecommunications, pharmaceutical, etc. For example, firms producing brand–name drugs enjoy high revenues so long as their patents are protected. Once their patent protection expires, a rival firm may launch a generic drug, thereby lowering prices (Wall Street Journal, 2013a). In the area of telecommunications, Apple's iPhone sales declined prior to the introduction of iPhone 4s in 2012, while, as the same time, Samsung's Galaxy S_3, the closest rival to Apple's market leading iPhone, took close to 18% of the market (Financial Times, 2012). Apple claimed that this was a result of Samsung's illegal adoption of technologies that are found in iPhones and demanded total damages of up to $2.75 billion. Even though no direct link was established between Samsung's alleged infringement and the subsequent increase in its market share, the legal debate between Apple and Samsung reflects a highly competitive environment, in which firms can potentially profit from adopting other firms' patented technologies. Of course, there are various other competitive advantages that a firm may have, however, their analysis is beyond of the scope of this paper. For example, while Apple relies on many suppliers to make parts for its devices, Samsung is more vertically integrated, and, thus, is able bring products to the market more quickly (Wall Street Journal, 2013b).

Real options theory finds particular application in such sectors as it facilitates the analysis of capital budgeting, yet models for analysing strategic interactions under price and technological uncertainty remain somewhat underdeveloped. Therefore, we consider the case of duopolistic competition, where two firms invest sequentially in technological innovations facing price and technological uncertainty. We analyse the case of proprietary duopoly, which occurs when a firm controls the innovation process, and, therefore, does not run the risk of pre–emption. The same setup is amenable to a non–proprietary duopoly which would occur if both firms fight for the leader's position, e.g., when the innovation process is exogenous. Hence, we contribute to the existing literature by first developing a theoretical framework for sequential investment in order to analyse the impact of price and technological uncertainty on investment under proprietary duopoly. Second, for each firm we determine the optimal technology adoption strategy, and, within each strategy, the value of the option to invest and the optimal investment rule. Finally, we provide managerial insights for investment decisions based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and formulate the problem in Section 3. In Sections 4 and 5, we consider a single investment option and analyse the impact of price and technological uncertainty on the optimal investment rule under monopoly and duopoly settings. In Section 6, we assume that firms adopt each technology that becomes available (compulsive strategy), and, in Section 7, we assume that a firm may either skip and old technology in order to adopt the next one (leapfrog strategy) or wait for a more efficient technology to arrive before adopting the previous one (laggard strategy). We provide numerical results for each case in Section 8 and illustrate how strategic interactions impact technology adoption strategies under price and technological uncertainty in order to enable more informed investment decisions, and conclude the paper in Section 9.

2 Related Work

The majority of real options models address the problem of optimal investment timing without considering competition (McDonald and Siegel, 1985 and 1986), while the ones that do, ignore either technological uncertainty or the sequential nature of such investments. Examples in the area of investment under technological uncertainty include Balcer and Lippman (1984), who analyse the optimal timing of technology adoption and find that technological uncertainty tends to delay adoption. Different technology adoption strategies, e.g., compulsive, leapfrog and laggard, have been analysed in Grenadier and Weiss (1997) within the context of sequential investment under technological uncertainty. They find that a firm may adopt an available technology even though more valuable innovations may occur in the future, while future decisions on technology adoption are path dependent. Farzin *et al.* (1998) investigate the optimal timing of technology adoption assuming technological uncertainty and irreversibility, yet ignore price uncertainty. Although they find that the real options theory yields the same investment rule as the NPV approach when a firm has a finite number of investment options, Doraszelski (2001) identifies an error in Farzin *et al.* (1998) and shows that, compared to the NPV approach, a firm will defer technology adoption when it takes the option value of waiting into account. Tsekrekos (2001) extends Weeds (1999) by allowing uncertainty over the implementation phase to affect not only the investment timing but also the profitability of a project. Results indicate that uncertainty over the implementation phase of a project may facilitate or delay investment compared to the corresponding certainty case. Although the aforementioned papers offer a comprehensive analysis of investment under technological uncertainty, they ignore the implications of strategic interactions.

In the area of competition, Smets (1993) first combined real options with game theory, thus developing a continuous–time model of strategic real options under product market competition, assuming stochastic demand and irreversibility. Spatt and Sterbenz (1985) analyse how the degree of rivalry impacts the learning process and the decision to invest and find that an increase in the number of players hastens investment and that the investment decision resembles the standard NPV rule. Via a deterministic model, Fudenberg and Tirole (1985) show that a high first–mover advantage results in a pre–emption equilibrium with dispersed adoption timings, as it increases a firm's incentive to pre–empt investment by its rivals. Extending the framework of Fudenberg and Tirole (1985), Huisman and Kort (1999) find that uncertainty creates a positive option value of waiting that increases with uncertainty, thereby raising the required investment threshold. More specifically, in deterministic models high first–mover advantage leads to a pre–emptive equilibrium, whereas in the stochastic case, increasing price uncertainty raises the required entry threshold for both firms. Finally, if first–mover advantages are sufficiently large for the pre–emptive equilibrium to result in the deterministic model, then high uncertainty results in simultaneous investment equilibrium. An analytical model that deals with the coordination problem that arises in pre–emptive competition is presented by Thijssen *et al.* (2012).

In the same line of work, Tsekrekos (2003) analyses the impact of temporary and permanent first mover advantage under price uncertainty, while Lambrecht and Perraudin (2003) incorporate incomplete information into an equilibrium model in which firms invest strategically. Paxson and Pinto (2005) develop a rivalry model under price and quantity uncertainty, and, among other results, they find that an increase in the correlation between the profits per unit and the quantity of units raises their aggregate volatility, and, in turn, the triggers of both the leader and the follower. Takashima *et al.* (2008) assess the effect of competition on the investment decision of firms with asymmetric technologies under price uncertainty. They show how mothballing options facilitate investment, thereby offering a competitive advantage to a liquified natural gas thermal power plant over a nuclear power plant. By contrast, lower variable and construction costs increase the competitive advantage that a nuclear power plant has over a coal or an oil–thermal power plant. Bouis *et al.* (2009) analyse investments in markets with more than two identical competitors, and, in the setting including three firms, they find that, if the entry of the third firm is delayed, then the second firm has an incentive to invest earlier so that it can enjoy the duopoly market structure for a longer time. This increases the incentive for the first firm to delay investment, as it faces a shorter period in which it can enjoy monopoly profits. Graham (2011) finds that an equilibrium may not exist when allowing for asymmetric information over revenues. Siddiqui and Takashima (2014), develop an analytical framework for sequential investment under price uncertainty and competition in order to explore the extent to which sequential decision making offsets the impact of competition.

In the area of competition under technological uncertainty, Weeds (2002) analyses irreversible investment in competing research projects under price uncertain assuming a winner–takes–all patent system. Results indicate that competition reduces the value of the option to invest and that, in a pre–emptive competition, firms invest sequentially. However, a symmetric outcome may also occur in which investment is more delayed than in the case of monopoly. Compared to the optimal cooperative investment pattern, investment is found to be more delayed when firms act non–cooperatively as each refrains from investing in the fear of starting a patent race. Huisman and Kort (2004) analyse a duopolistic competition under price and technological uncertainty and show that high arrival probability can turn a pre–emption game into a war of attrition and that price uncertainty induces the adoption of a new technology. Miltersen and Schwartz (2004) analyse how competition in the development and marketing of a product impacts investment in R&D. They find that competition in R&D not only increases production and reduces prices but also shortens the development stage and raises the probability of a successful outcome.

More pertinent to our analysis, is the working paper by Chronopoulos and Siddiqui (2014) who develop an analytical framework for sequential investment under price and technological uncertainty, yet ignore the impact of competition. They analyse a compulsive as well as leapfrog/laggard strategy and show how uncertainty regarding the availability of future versions may hasten investment in the current one. Additionally, they show how a firm adopts only a compulsive strategy when the output price is low, yet under a high ourput price, a firm may consider a leapfrog/laggard strategy as the rate of innovation increases. We extend this framework by considering two identical firms that invest sequentially in technological innovations and determine how competition interacts with price and technological uncertainty to affect the technology adoption strategy of each firm. As in Chronopoulos and Siddiqui (2014), price uncertainty is modelled via a geometric Brownian motion (GBM) whereas technological uncertainty via a Poisson process and reflects the random arrival of innovations. We analyse three strategies, i.e., compulsive, leapfrog, and laggard, and determine the feasibility of each strategy under different levels of price and technological uncertainty. Results indicate that relative loss in the value of the leader due to the presence of a rival decreases as the first–mover advantage and the rate of innovation increase. By contrast, the relative loss in the value of the leader increases with greater price uncertainty. Interestingly, while technological uncertainty has a non–monotonic impact on the optimal investment threshold of the follower, it does not impact the leader's decision to invest. Finally, unlike in the case of monopoly, the compulsive strategy always dominates the leapfrog/laggard strategy.

3 Problem formulation and assumptions

Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we assume that the revenue at time t, P_t, where $t \geq 0$ is continuous and denotes time, follows a GBM that is described in (1), where μ is the annual growth rate, σ is the annual volatility, and dZ_t is the increment of the standard Brownian motion. Also, $\rho > \mu$ denotes the subjective discount rate.

$$dP_t = \mu P_t dt + \sigma P_t dZ_t, \quad P_0 \equiv P > 0 \tag{1}$$

We let $b = m, \ell, f$ (denoting monopolist, leader, and follower, respectively). Moreover, we assume that each firm has $N < \infty$ investment options and that i is the technology that is currently in operation, j is the one that a firm wants to invest in, and k is the latest one available ($i < j \leq k$). There is no operating cost associated with technology i, while the investment cost is I_i $\left(I_i \leq I_j\right)$ and the corresponding output is $D_i(X)$, where $X = 0, 1, 2$ denotes the number of firms in the industry. Hence, $D_i(X)$ is decreasing in X and increasing in i. The state in which technology $i(j)$ is in operation and $X = 1, 2$ is denoted by $\underline{i}\left(\underline{j}\right)$ and $\bar{i}\left(\bar{j}\right)$, respectively. Depending on the number of firms in the industry, a firm's option to invest in technology j when it currently operates version i and k is the latest one available is $F^{b(N)}_{i,j,k}(\cdot)$, while $p^{b(N)}_{i,j,k}$, is the corresponding optimal investment threshold. For example, a leader's option to invest in technology j when both firms currently operate technology i is denoted by $F^{\ell(N)}_{\bar{i},\underline{j},k}(\cdot)$, while $p^{\ell(N)}_{\bar{i},\underline{j},k}$ is the corresponding optimal investment threshold. Similarly, the expected project value from operating technology i when k is the latest one available inclusive of embedded options is $\Phi^{b(N)}_{\bar{i},k}(\cdot)$ for $X = 2$. In order to have a trade off between an old and a more efficient technology, the point of indifference, $\tilde{p}$, where $\tilde{p} : \Phi^{b(N)}_{i,k}(\tilde{p}) = \Phi^{b(N)}_{k,k}(\tilde{p})$, must satisfy $\Phi^{b(N)}_{i,k}(\tilde{p}) > 0$. Finally, we assume that all investment options are perpetual and that technologies have an infinite lifetime.

We introduce technological uncertainty by assuming that technological innovations follow a Poisson process $\{M_t, t \geq 0\}$, which is independent of the price process, P_t, and is defined in (2).

$$M_t = \sum_{d \geq 1} \mathbb{1}_{\{t \geq T_d\}} \tag{2}$$

Notice that $T_d = \sum_{n=1}^{d} y_d$ and $\{y_d, d \geq 1\}$ is a sequence of independent and identically distributed random variables, with $y_d \sim \exp(\lambda)$. Intuitively, M_t counts the number of innovations that occur between 0 and t, and $h_n = T_n - T_{n-1}$ is the time interval between subsequent innovations. Hence, if no innovation has occurred for t years, then, with probability λdt, it will occur within then next short time interval dt, i.e:

$$dM_t = \begin{cases} 1 & \text{, with probability } \lambda dt \\ 0 & \text{, with probability } 1 - \lambda dt \end{cases}$$

4 Monopoly with $N = 1$

Although this case is analysed in Chronopoulos and Siddiqui (2014), we present the analysis here for ease of reference and to allow for comparisons. Once an innovation takes place, the firm moves from state (0,0) to $(0, \underline{1}, 1)$, where it can exercise the option to invest, and, thus, move to state $(\underline{1}, 1)$, where it receives a perpetually operating project. The expected NPV from immediate investment in the first technology is described in (3).

$$\Phi_{\underline{1},1}^{m(1)}(P) = \frac{PD_{\underline{1}}}{\rho - \mu} - I_1 \tag{3}$$

Next, the value of the investment opportunity in state $(0, \underline{1}, 1)$ is described in (4), where $\beta_1 > 1, \beta_2 < 0$ are the roots of $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0$, while the endogenous constant, $A_{0,\underline{1},1}^{m(1)}$, and investment threshold, $p_{0,\underline{1},1}^{m(1)}$, are determined via value–matching and smooth–pasting conditions between the two branches of (4) and are indicated in (A–3) (all proofs can be found in the appendix).

$$F_{0,\underline{1},1}^{m(1)}(P) = \begin{cases} A_{0,\underline{1},1}^{m(1)} P^{\beta_1} & , P < p_{0,\underline{1},1}^{m(1)} \\ \Phi_{\underline{1},1}^{m(1)}(P) & , P \geq p_{0,\underline{1},1}^{m(1)} \end{cases} \tag{4}$$

The dynamics of the firm's value function in state $(0, 0)$ are described in (5). Notice that over an infinitesimal time interval dt, either an innovation will take place with probability λdt and the firm will receive the option to invest, $F_{0,\underline{1},1}^{m(1)}(P)$, or, with probability $1 - \lambda dt$, no innovation will take place and the firm will continue to hold the value function $\Phi_{0,0}^{m(1)}(P)$.

$$\Phi_{0,0}^{m(1)}(P) = e^{-\rho dt}\left[\lambda dt \mathbb{E}_P\left[F_{0,\underline{1},1}^{m(1)}(P + dP)\right] + (1 - \lambda dt)\mathbb{E}_P\left[\Phi_{0,0}^{m(1)}(P + dP)\right]\right] \tag{5}$$

By expanding the right–hand side of (5) using Itô's lemma and solving the resulting ordinary differential equation, we obtain the expression of the value function in state (0,0) as indicated in (6). The endogenous constants $A_{0,0}^{m(1)}$ and $B_{0,0}^{m(1)}$ are determined via value–matching and smooth–pasting conditions between the two branches of (6) and are indicated in (A–5) and (A–6), respectively. Also, δ_1 and δ_2 are the roots of the quadratic $\frac{1}{2}\sigma^2\delta(\delta - 1) + \mu\delta - (\rho + \lambda) = 0$, with $\delta_1 = \beta_1$ and $\delta_2 = \beta_2$ for $\lambda = 0$.

$$\Phi_{0,0}^{m(1)}(P) = \begin{cases} A_{0,\underline{1},1}^{m(1)} P^{\beta_1} + A_{0,0}^{m(1)} P^{\delta_1} & , P < p_{0,\underline{1},1}^{m(1)} \\ \frac{\lambda P D_{\underline{1}}}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_{\underline{1}}}{\rho+\lambda} + B_{0,0}^{m(1)} P^{\delta_2} & , P \geq p_{0,\underline{1},1}^{m(1)} \end{cases} \tag{6}$$

The top part of (6) is the option to invest (first term) adjusted (via the second term) to the fact that an innovation has not taken place yet. The first two terms in the bottom part of (6) are the expected profit from the new technology, while the third term is the probability that the price will drop into the waiting region before an innovation occurs.

5 Proprietary duopoly with $N = 1$

We extend the previous framework by assuming that there are two firms in the industry. Since the proprietary leader does not run the risk of preemption, once an innovation occurs she will receive the option to invest and move to state $(0, \underline{1}, 1)$. At $p^{\ell^{(1)}}_{0,\underline{1},1}$, the leader will invest, thereby moving to state $(\underline{1}, 1)$ where she receives monopoly profits until the follower enters the market. Once the leader invests, the follower moves to state $(0,\overline{1},1)$ and receives the option to invest in the same technology as the leader. At $p^{f^{(1)}}_{0,\overline{1},1}$, the follower invests and both firms share the market, as in Figure 1.

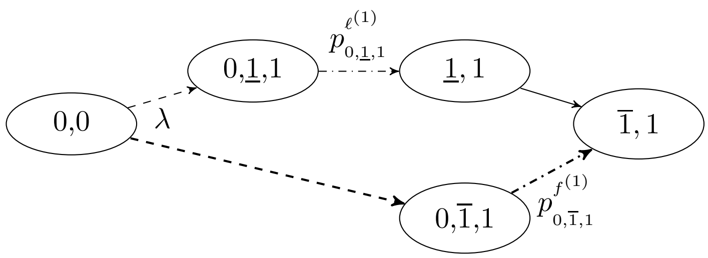

Figure 1: Proprietary duopoly with $N = 1$ under a compulsive strategy

We determine the value functions of the leader and the follower in each state via backward induction. The expected NPV from investing immediately in the first technology, i.e., state $(\overline{1}, 1)$, is described in (7), where $b = \ell, f$ and $D_{\overline{1}} < D_{\underline{1}}$

$$\Phi^{b^{(1)}}_{\overline{1},1}(P) \quad = \quad \frac{PD_{\overline{1}}}{\rho - \mu} - I_1 \tag{7}$$

while the value function of the follower in state $(0, \overline{1}, 1)$ is described in (8), where $A^{f^{(1)}}_{0,\overline{1},1}$ and $p^{f^{(1)}}_{0,\overline{1},1}$ are indicated in (A–7).

$$F^{f^{(1)}}_{0,\overline{1},1}(P) \quad = \quad \begin{cases} A^{f^{(1)}}_{0,\overline{1},1} P^{\beta_1} & , P < p^{f^{(1)}}_{0,\overline{1},1} \\ \Phi^{f^{(1)}}_{\overline{1},1}(P) & , P \geq p^{f^{(1)}}_{0,\overline{1},1} \end{cases} \tag{8}$$

The value function of the follower in state $(0, 0)$ is indicated in (10), where $A^{f^{(1)}}_{0,0}$ and $B^{f^{(1)}}_{0,0}$ are described in (A–8) and (A–9), respectively. Over an infinitesimal time interval dt, either an innovation will occur with probability λdt and the follower will receive the option to invest, $F^{f^{(1)}}_{0,\overline{1},1}(P)$, or no innovation will take place with probability $1 - \lambda dt$ and the follower will continue to hold the value function $\Phi^{f^{(1)}}_{0,0}(P)$.

$$\Phi^{f^{(1)}}_{0,0}(P) \quad = \quad e^{-\rho dt}\left[\lambda dt \mathbb{E}_P\left[F^{f^{(1)}}_{0,\overline{1},1}(P + dP)\right] + (1 - \lambda dt)\mathbb{E}_P\left[\Phi^{f^{(1)}}_{0,0}(P + dP)\right]\right] \tag{9}$$

The expression for $\Phi^{f^{(1)}}_{0,0}(P)$ is indicated in (10), where the first term in the top part of (10) is the option to invest, however, since the technology is not available yet, it must be adjusted via the second term. The first two terms on the bottom part of (10) reflect the expected profit and the third term the probability that the price will drop into the waiting region before an innovation takes place.

$$\Phi_{0,0}^{f^{(1)}}(P) \;=\; \begin{cases} A_{0,\overline{1},1}^{f^{(1)}} P^{\beta_1} + A_{0,0}^{f^{(1)}} P^{\delta_1} & , P < p_{0,\overline{1},1}^{f^{(1)}} \\ \frac{\lambda P D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + B_{0,0}^{f^{(1)}} P^{\delta_2} & , P \geq p_{0,\overline{1},1}^{f^{(1)}} \end{cases} \tag{10}$$

Assuming that the follower chooses the optimal investment policy, the value function of the leader in state $(\underline{1},1)$ for $P < p_{0,\overline{1},1}^{f^{(1)}}$ is described in (11). The first term is the monopoly profits from operating the first technology before the follower's entry and the second term reflects the expected reduction in the profits of the leader due to the entry of the follower.

$$\Phi_{\underline{1},1}^{\ell^{(1)}}(P) \;=\; \Phi_{\underline{1},1}^{m^{(1)}}(P) + A_{\underline{1},1}^{\ell^{(1)}} P^{\beta_1}, \quad A_{\underline{1},1}^{\ell^{(1)}} = \frac{{p_{0,\overline{1},1}^{f^{(1)}}}^{1-\beta_1} \left(D_{\overline{1}} - D_{\underline{1}}\right)}{\rho-\mu} < 0 \tag{11}$$

Prior to state $(\underline{1},1)$, the leader is in state $(0,\underline{1},1)$ and holds the option to invest. The value function of the leader in state $(0,\underline{1},1)$ is indicated in (12), where $A_{0,\underline{1},1}^{\ell^{(1)}}$ and $p_{0,\underline{1},1}^{\ell^{(1)}}$ are obtained via value–matching and smooth–pasting conditions between the two branches of (12) and are indicated in (A–10).

$$F_{0,\underline{1},1}^{\ell^{(1)}}(P) \;=\; \begin{cases} A_{0,\underline{1},1}^{\ell^{(1)}} P^{\beta_1} & , P < p_{0,\underline{1},1}^{\ell^{(1)}} \\ \Phi_{\underline{1},1}^{\ell^{(1)}}(P) & , P \geq p_{0,\underline{1},1}^{\ell^{(1)}} \end{cases} \tag{12}$$

As indicated in (13), in state (0,0) the leader will either receive the option to invest if, over an infinitesimal time interval dt, an innovation occurs with probability λdt, or she will continue to hold the value function $\Phi_{0,0}^{\ell^{(1)}}(P)$ if, with probability $(1-\lambda dt)$, no innovation takes place.

$$\Phi_{0,0}^{\ell^{(1)}}(P) \;=\; e^{-\rho dt}\left[\lambda dt \mathbb{E}_P\left[F_{0,\underline{1},1}^{\ell^{(1)}}(P+dP)\right] + (1-\lambda dt)\mathbb{E}_P\left[\Phi_{0,0}^{\ell^{(1)}}(P+dP)\right]\right] \tag{13}$$

The expression of the leader's value function in state $(0,0)$ is described in (14). The first term on the top part of (14) is the option to invest, while the second term reflects the correction to the first term since the technology is not available yet. The first two terms in the bottom part of (14) reflect the expected monopoly profits from investment, while the third term is the reduction in profit due to the presence of a rival. Notice that since the technology is not available yet, the third term must be adjusted via the fourth one. Finally, the last term reflects the likelihood that the price will drop before the arrival of an innovation. The endogenous constants $A_{0,0}^{\ell^{(1)}}$ and $C_{0,0}^{\ell^{(1)}}$ are obtain via value–matching and smooth–pasting conditions between the two branched of (14) and are indicated in (A–12) and (A–13) respectively, while $B_{0,0}^{\ell^{(1)}}$ is obtained by value matching the bottom part of (14) with the bottom part of (10) at $p_{0,\overline{1},1}^{f^{(1)}}$.

$$\Phi_{0,0}^{\ell^{(1)}}(P) \;=\; \begin{cases} A_{0,\underline{1},1}^{\ell^{(1)}} P^{\beta_1} + A_{0,0}^{\ell^{(1)}} P^{\delta_1} & , P < p_{0,\underline{1},1}^{\ell^{(1)}} \\ \frac{\lambda P D_{\underline{1}}}{(\lambda+\rho-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A_{\underline{1},1}^{\ell^{(1)}} P^{\beta_1} + B_{0,0}^{\ell^{(1)}} P^{\delta_1} + C_{0,0}^{\ell^{(1)}} P^{\delta_2} & , P \geq p_{0,\underline{1},1}^{\ell^{(1)}} \end{cases} \tag{14}$$

6 Compulsive strategy under proprietary duopoly

Unlike Section 5, now both firms hold two investment options and adopt a compulsive strategy, i.e., they invest in each technology that becomes available. Once an innovation takes place, the leader receives the option to invest and moves from $(0,0)$ to $(0,\underline{1},1)$. By contrast, the follower receives the option to invest when the leader adopts the first technology. The follower enters the market at $p^{f^{(2)}}_{0,\overline{1},1}$, and the same process is repeated once the second innovation takes place, as illustrated in Figure 2. Unlike in the case $N = 1$, in state $(\overline{1},1)$ the firms share the market, yet they do not hold the same value function. In fact, the leader (follower) holds the value of the active project with an embedded option to invest in the second technology first (second).

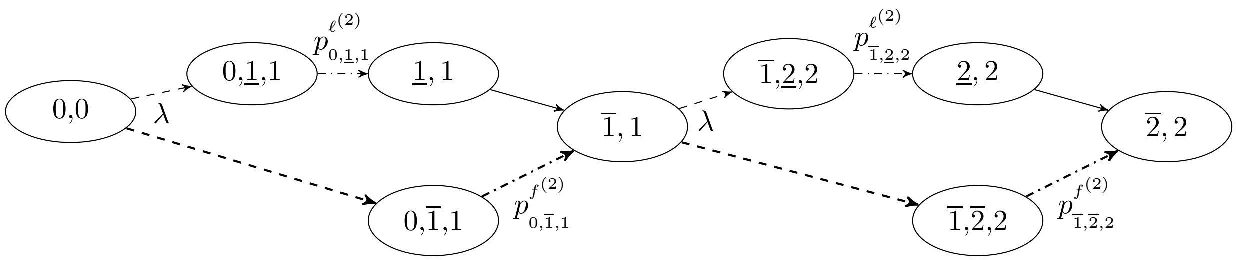

Figure 2: Proprietary duopoly with $N = 2$ under a compulsive strategy

We begin with state $(\overline{2},2)$, in which both firms operate the second technology. The expected project value is described in (15), where $b = \ell, f$ and $D_{\overline{2}} > D_{\overline{1}}$.

$$\Phi^{b^{(2)}}_{\overline{2},2}(P) = \frac{PD_{\overline{2}}}{\rho-\mu} - (I_1 + I_2) \tag{15}$$

In state $(\overline{1},\overline{2},2)$, the follower's option to invest in the second technology is described in (16), where $A^{f^{(2)}}_{\overline{1},\overline{2},2}$ and $p^{f^{(2)}}_{\overline{1},\overline{2},2}$ are obtaine via value–matching and smooth–pasting conditions between the two branches of (16) and are described in (A–14).

$$F^{f^{(2)}}_{\overline{1},\overline{2},2}(P) = \begin{cases} \Phi^{f^{(1)}}_{\overline{1},1}(P) + A^{f^{(2)}}_{\overline{1},\overline{2},2} P^{\beta_1} & , P < p^{f^{(2)}}_{\overline{1},\overline{2},2} \\ \Phi^{f^{(2)}}_{\overline{2},2}(P) & , P \geq p^{f^{(2)}}_{\overline{1},\overline{2},2} \end{cases} \tag{16}$$

Next, we step back and consider state $(\overline{1},1)$ in which both firms operate the first technology. Following the same reasoning as in (9), we can obtain the differential equation for the value function of the follower under technological uncertainty, and then solve it in order to obtain the expression for $\Phi^{f^{(2)}}_{\overline{1},1}(P)$, that is indicated in (17). The endogenous constants $A^{f^{(2)}}_{\overline{1},1}$ and $B^{f^{(2)}}_{\overline{1},1}$ are determined via value–matching and smooth–pasting conditions between the two branches of (17) and are indicated in (A–15) and (A–16), respectively.

$$\Phi^{f^{(2)}}_{\overline{1},1}(P) = \begin{cases} \Phi^{f^{(1)}}_{\overline{1},1}(P) + A^{f^{(2)}}_{\overline{1},\overline{2},2} E^{\beta_1} + A^{f^{(2)}}_{\overline{1},1} P^{\delta_1} & , P < p^{f^{(2)}}_{\overline{1},\overline{2},2} \\ \frac{E\left[\lambda D_{\overline{2}} + (\rho-\mu) D_{\overline{1}}\right]}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} - I_1 + B^{f^{(2)}}_{\overline{1},1} P^{\delta_2} & , P \geq p^{f^{(2)}}_{\overline{1},\overline{2},2} \end{cases} \tag{17}$$

In state $(0,\overline{1},1)$, the follower's option to invest in the first technology with an embedded option to invest in the second, which has yet to become available, is described in (18), where $A^{f^{(2)}}_{0,\overline{1},1}$ and $p^{f^{(2)}}_{0,\overline{1},1}$ are obtained numerically via (A–17) and (A–18).

$$F^{f^{(2)}}_{0,\overline{1},1}(P) = \begin{cases} A^{f^{(2)}}_{0,\overline{1},1} P^{\beta_1} & , P < p^{f^{(2)}}_{0,\overline{1},1} \\ \Phi^{f^{(2)}}_{\overline{1},1}(P) & , P \geq p^{f^{(2)}}_{0,\overline{1},1} \end{cases} \tag{18}$$

Finally, in state (0,0) no innovation has occurred yet and the value function of the follower is indicated in (19), where $A^{f^{(2)}}_{0,0}$ and $B^{f^{(2)}}_{0,0}$ are obtained via value–matching and smooth–pasting conditions between the two branches of (19) and are indicated in (A–19) and (A–20), respectively.

$$\Phi^{f^{(2)}}_{0,0}(P) = \begin{cases} A^{f^{(2)}}_{0,\overline{1},1} P^{\beta_1} + A^{f^{(2)}}_{0,0} P^{\delta_1} & , P < p^{f^{(2)}}_{0,\overline{1},1} \\ \frac{\lambda P D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{f^{(2)}}_{\overline{1},\overline{2},2} P^{\beta_1} + A^{f^{(2)}}_{\overline{1},1} P^{\delta_1} + B^{f^{(2)}}_{0,0} P^{\delta_2} & , P \geq p^{f^{(2)}}_{0,\overline{1},1} \end{cases} \tag{19}$$

Assuming that the follower chooses the optimal investment policy, the value function of the leader in state $(\underline{2},2)$ for $P < p^{f^{(2)}}_{\overline{1},\overline{2},2}$ is described in (20), where the first term reflects the monopoly profits from operating the second technology before the entry of the follower, and the second term is the expected reduction in profits due to the follower's entry.

$$\Phi^{\ell^{(2)}}_{\underline{2},2}(P) = \frac{PD_{\underline{2}}}{\rho-\mu} - (I_1 + I_2) + A^{\ell^{(2)}}_{\underline{2},2} P^{\beta_1}, \quad A^{\ell^{(2)}}_{\underline{2},2} = \frac{{p^{f^{(2)}}_{\overline{1},\overline{2},2}}^{1-\beta_1}\left(D_{\overline{2}} - D_{\underline{2}}\right)}{\rho-\mu} < 0 \tag{20}$$

Next, in state $(\overline{1},\underline{2},2)$ the leader's option to invest in the second technology is described in (21) where $A^{\ell^{(2)}}_{\overline{1},\underline{2},2}$ and $p^{\ell^{(2)}}_{\overline{1},\underline{2},2}$ are indicated in (A–21).

$$F^{\ell^{(2)}}_{\overline{1},\underline{2},2}(P) = \begin{cases} \Phi^{\ell^{(1)}}_{\overline{1},1}(P) + A^{\ell^{(2)}}_{\overline{1},\underline{2},2} P^{\beta_1} & , P < p^{\ell^{(2)}}_{\overline{1},\underline{2},2} \\ \Phi^{\ell^{(2)}}_{\underline{2},2}(P) & , P \geq p^{\ell^{(2)}}_{\overline{1},\underline{2},2} \end{cases} \tag{21}$$

The expression for the leader's value function in state $(\overline{1},1)$ is indicated in (22), where $A^{\ell^{(2)}}_{\overline{1},1}$ and $C^{\ell^{(2)}}_{\overline{1},1}$ are determined by value matching and smooth pasting the two branches of (22), while $B^{\ell^{(2)}}_{\overline{1},1}$ is obtained by value matching (22) with (17) at $p^{f^{(2)}}_{\overline{1},\overline{2},2}$.

$$\Phi^{\ell^{(2)}}_{\overline{1},1}(P) = \begin{cases} \Phi^{\ell^{(1)}}_{\overline{1},1}(P) + A^{\ell^{(2)}}_{\overline{1},\underline{2},2} P^{\beta_1} + A^{\ell^{(2)}}_{\overline{1},1} P^{\delta_1} & , P < p^{\ell^{(2)}}_{\overline{1},\underline{2},2} \\ \frac{P\left[\lambda D_{\underline{2}} + (\rho-\mu) D_{\overline{1}}\right]}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} - I_1 + A^{\ell^{(2)}}_{\underline{2},2} P^{\beta_1} + B^{\ell^{(2)}}_{\overline{1},1} P^{\delta_1} + C^{\ell^{(2)}}_{\overline{1},1} P^{\delta_2} & , P \geq p^{\ell^{(2)}}_{\overline{1},\underline{2},2} \end{cases} \tag{22}$$

Next, we step back and consider the leader's value function in state $(\underline{1},1)$. According to Proposition 6.1, the leader can not invest in the second technology before the follower enters the market, since $p^{\ell^{(2)}}_{\underline{1},\underline{2},2} > p^{f^{(2)}}_{0,\overline{1},1}$. Consequently, the leader will have to go through state $(\overline{1},1)$. Hence, the value function of the leader in state $(\underline{1},1)$ is indicated in (23), where $A^{\ell^{(2)}}_{\underline{1},1} < 0$ is obtained by value matching (23) with the leader's value function in state $(\overline{1},1)$, i.e., with the top part of (22), at $p^{f^{(2)}}_{0,\overline{1},1}$ and is indicated in (A–23). The first term on the right–hand side of (23) reflects the monopoly profits from the first technology, and, the second term, is the expected reduction in the leader's profits due to the follower's entry inclusive of the embedded option to invest in the second technology.

$$\Phi^{\ell^{(2)}}_{\underline{1},1}(P) \quad = \quad \Phi^{m^{(1)}}_{\underline{1},1}(P) + A^{\ell^{(2)}}_{\underline{1},1} P^{\beta_1} \tag{23}$$

Proposition 6.1 $p^{\ell^{(2)}}_{\underline{1},\underline{2},2} > p^{f^{(2)}}_{0,\overline{1},1}$.

In state $(0,\underline{1},1)$, the value of the leader's option to invest in the first technology with an embedded option to invest in the second, which has yet to become available, is described in (24).

$$F^{\ell^{(2)}}_{0,\underline{1},1}(P) \quad = \quad \begin{cases} A^{\ell^{(2)}}_{0,\underline{1},1} P^{\beta_1} & , P < p^{\ell^{(2)}}_{0,\underline{1},1} \\ \Phi^{\ell^{(2)}}_{\underline{1},1}(P) & , P \geq p^{\ell^{(2)}}_{0,\underline{1},1} \end{cases} \tag{24}$$

Interestingly, $p^{\ell^{(2)}}_{0,\underline{1},1}$ is not affected by technological uncertainty, as shown Proposition 6.2. Intuitively, this result is a consequence of the optimality of myopic behavior based on which a firm disregards subsequent investment decisions when evaluating the current one. The optimality of myopic behavior is not generally true but holds under certain assumptions. For example, it holds in the case of monopoly (Bertola, 1989; Pindyck 1988, 1993) and perfect competition, as well as within a context of strategic interactions provided that the profit is additively separable if more that one technologies are considered (Baldurson and Karatzas, 1997).

Proposition 6.2 $p^{\ell^{(2)}}_{0,\underline{1},1}$ *is independent of* λ.

Finally, in state (0,0), the value function of the leader is indicated in (25), where $A^{\ell^{(2)}}_{0,0}$ and $C^{\ell^{(2)}}_{0,0}$ are determined via value–matching and smooth–pasting conditions between the two branches of (25), while $B^{\ell^{(2)}}_{0,0}$ is determined by value matching the bottom part of (25) with the top part of (22) at the optimal investment threshold of the follower in the first technology, $p^{f^{(2)}}_{0,\overline{1},1}$.

$$\Phi^{\ell^{(2)}}_{0,0}(P) \quad = \quad \begin{cases} A^{\ell^{(2)}}_{0,\underline{1},1} P^{\beta_1} + A^{\ell^{(2)}}_{0,0} P^{\delta_1} & , P < p^{\ell^{(2)}}_{0,\underline{1},1} \\ \frac{\lambda P D_{\underline{1}}}{(\lambda+\rho-\mu)(\rho-\mu)} - \frac{\lambda I_{\underline{1}}}{\rho+\lambda} + A^{\ell^{(2)}}_{\underline{1},1} P^{\beta_1} + B^{\ell^{(2)}}_{0,0} P^{\delta_1} + C^{\ell^{(2)}}_{0,0} P^{\delta_2} & , P \geq p^{\ell^{(2)}}_{0,\underline{1},1} \end{cases} \tag{25}$$

As it will be illustrated numerically, the relative loss in the leader's value of investment opportunity, which is described in (26), is decreasing with λ. This happens because greater λ reduces the loss in option value due to technological uncertainty, thereby decreasing the discrepancy between the option value of the leader and the monopolist.

$$\frac{F^{m^{(2)}}_{0,\underline{1},1}(P) - F^{\ell^{(2)}}_{0,\underline{1},1}(P)}{F^{m^{(2)}}_{0,\underline{1},1}(P)} \tag{26}$$

7 Leapfrog/laggard strategies under proprietary duopoly

Instead of adopting each technology that becomes available, the leader may decide to ignore a technology temporarily and wait for another one to arrive before deciding which one to invest in, as illustrated in Figure 3. Thus, instead of moving from $(0, \underline{1}, 1)$ to $(\underline{1}, 1)$, the leader moves to $(0, \underline{1} \vee \underline{2}, 2)$, and, then, either invests in the first technology while holding the option to switch to the second, and, thus, moves to state $(\underline{1}, 2)$, or $(\vee)$ invests directly in the second technology, thereby moving to state $(\underline{2}, 2)$. In the former case, the follower receives the option to invest in the first technology and moves to state $(0, \overline{1}, 1)$, which implies that she will adopt both technologies. By contrast, in the latter case only the second technology will be commercialised, and, therefore, the follower will never invest in the first one.

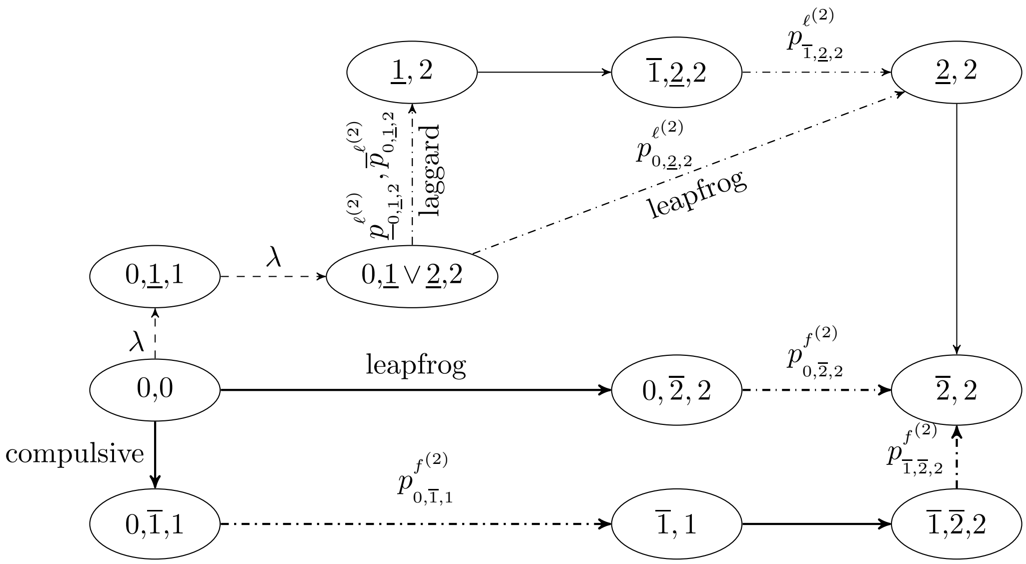

Figure 3: Proprietary duopoly with $N = 2$ under a leapfrog/laggard strategy

Since the value function of the leader in states $(\overline{2}, 2)$, $(\underline{2}, 2)$, $(\overline{1}, \underline{2}, 2)$, and $(\underline{1}, 2)$ is the same as in Section 5.2, we proceed directly to state $(0, \underline{1} \vee \underline{2}, 2)$, where the leader holds an option to choose between two alternative technologies. The expected payoff from immediate investment in the second technology is indicated in (27), where $\widetilde{A}^{\ell^{(2)}}_{\underline{2},2}$ is obtained by value matching (27) with the follower's value function at $p^{f^{(2)}}_{0,\overline{2},2}$.

$$\widetilde{\Phi}^{\ell^{(2)}}_{\underline{2},2}(P) = \frac{D_{\underline{2}}P}{\rho - \mu} - I_2 + \widetilde{A}^{\ell^{(2)}}_{\underline{2},2} P^{\beta_1}, \quad \widetilde{A}^{\ell^{(2)}}_{\underline{2},2} = \frac{\left(D_{\overline{2}} - D_{\underline{2}}\right) p^{f^{(2)^{1-\beta_1}}}_{0,\overline{2},2}}{\rho - \mu} \tag{27}$$

The value function of the leader in state $(\underline{1}, 2)$ is described in (28), where $A^{\ell^{(2)}}_{\underline{1},2}$ is obtained by value matching (28) with (21) at $p^{f^{(2)}}_{0,\overline{1},1}$.

$$\Phi^{\ell^{(2)}}_{\underline{1},2}(P) = \frac{D_{\underline{1}}P}{\rho - \mu} - I_1 + A^{\ell^{(2)}}_{\underline{1},2} P^{\beta_1}, \quad A^{\ell^{(2)}}_{\underline{1},2} = \frac{1}{p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1}} \left[\frac{\left(D_{\overline{1}} - D_{\underline{1}}\right) p^{f^{(2)}}_{0,\overline{1},1}}{\rho - \mu} + A_{\overline{1},\underline{2},2} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} \right] \tag{28}$$

Notice that in order to have a trade–off between the two technologies the condition indicated in (29) must be satisfied. Intuitively, if we denote by $\tilde{p}$ the point of indifference between the two projects, then the value functions (27) and (28) evaluated at $\tilde{p}$ should be positive. If this condition does not hold, then a firm would have no incentive to invest in the first technology.

$$\frac{D_n}{\sum_{i=1}^{n} I_i} < \frac{D_{n-1}}{\sum_{i=1}^{n-1} I_i},\ \forall n \tag{29}$$

From Décamps *et al.* (2006), we know that the value function in state $(0,\underline{1}\vee\underline{2},2)$ is described in (30). Notice that, due to the presence of the second technology, there exist two waiting regions, i.e., $P < \underline{p}^{\ell(2)}_{0,\underline{1},2}$ and $\overline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2}$. If $P < \underline{p}^{\ell(2)}_{0,\underline{1},2}$, then the firm will adopt a laggard strategy, i.e., wait until $\underline{p}^{\ell(2)}_{0,\underline{1},2}$ and then invest in the first technology, whereas if $\overline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2}$, then the firm can either adopt a laggard, if $P \downarrow \overline{p}^{\ell(2)}_{0,\underline{1},2}$, or a leapfrog strategy, if $P \uparrow \overline{p}^{\ell(2)}_{0,\underline{2},2}$.

$$F^{\ell(2)}_{0,\underline{1}\vee\underline{2}}(P) = \begin{cases} A^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_1} & , P < \underline{p}^{\ell(2)}_{0,\underline{1},2} \\ \Phi^{\ell(2)}_{\underline{1},2}(P) & , \underline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < \overline{p}^{\ell(2)}_{0,\underline{1},2} \\ G^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_2} + H^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_1} & , \overline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2} \\ \widetilde{\Phi}^{\ell(2)}_{\underline{2},2}(P) & , p^{\ell(2)}_{0,\underline{2},2} \leq P \end{cases} \tag{30}$$

Finally, in state $(0,\underline{1},1)$, either the second technology will become available with probability λdt and the leader will receive the option $F^{\ell(2)}_{0,\underline{1}\vee\underline{2}}(P)$, or no innovation will take place with probability $(1-\lambda dt)$ and the firm will continue to hold $F^{\ell(2)}_{0,\underline{1},1}(P)$. The solution is indicated in (31), where $A^{\ell(2)}_{0,1,1}$, $L^{\ell(2)}_{0,\underline{1},1}$, $U^{\ell(2)}_{0,\underline{1},1}$, $Q^{\ell(2)}_{0,\underline{1},1}$, $R^{\ell(2)}_{0,\underline{1},1}$, and $J^{\ell(2)}_{0,\underline{1},1}$ are determined analytically via the value–matching and smooth–pasting conditions between the branches of (31).

$$F^{\ell(2)}_{0,\underline{1},1}(P) = \begin{cases} A^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_1} + U^{\ell(2)}_{0,\underline{1},1} P^{\delta_1} & , P < \underline{p}^{\ell(2)}_{0,\underline{1},2} \\ \frac{\lambda D_{\underline{1}} P}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{\ell(2)}_{\underline{1},2} P^{\beta_1} + L^{\ell(2)}_{0,\underline{1},1} P^{\delta_1} + B^{\ell(2)}_{0,\underline{1},1} P^{\delta_2} & , \underline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < \overline{p}^{\ell(2)}_{0,\underline{1},2} \\ G^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_2} + H^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_1} + Q^{\ell(2)}_{0,\underline{1},1} P^{\delta_1} + R^{\ell(2)}_{0,\underline{1},1} P^{\delta_2} & , \overline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2} \\ \frac{\lambda D_{\underline{2}} P}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + \widetilde{A}^{\ell(2)}_{\underline{2},2} P^{\beta_1} + J^{\ell(2)}_{0,\underline{1},1} P^{\delta_2} & , p^{\ell(2)}_{0,\underline{2},2} \leq P \end{cases} \tag{31}$$

Having determined the value function of the leader in state $(0,\underline{1},1)$ under a compulsive as well as leapfrog and laggard strategies, we can compare these strategies, and, thus, determine the optimal strategy endogenously. Also, since the comparison between the strategies can be done in $(0,\underline{1},1)$, without loss of generality, the analysis of state (0,0) is omitted.

8 Numerical Examples

For the numerical results, the parameter values are $\mu = 0.01$, $\sigma \in [0, 0.3]$, $I_1 = 700$, $I_2 = 1800$, $D_{\underline{1}} = 16$, $D_{\overline{1}} = 8$, $D_{\underline{2}} = 24$, $D_{\overline{2}} = 16$, $\lambda \in \mathbb{R}^+$. For $N = 1$, the value functions of the monopolist as well as the leader and the follower are illustrated in Figure 4, where the thin (thick) lines reflect the value functions under (without) technological uncertainty. Notice that the proprietary leader can delay investment, and, therefore, the required investment threshold is 6.56, which is the same as that of the monopolist. Nevertheless, the leader's value of investment opportunity is lower compared to that of the monopolist due to the follower's entry. Once the follower enters at 13.13, both firms share the market. Notice also that greater λ increases the likelihood of an innovation and raises the expected value of both the leader and the follower, however, it does not affect the required entry thresholds.

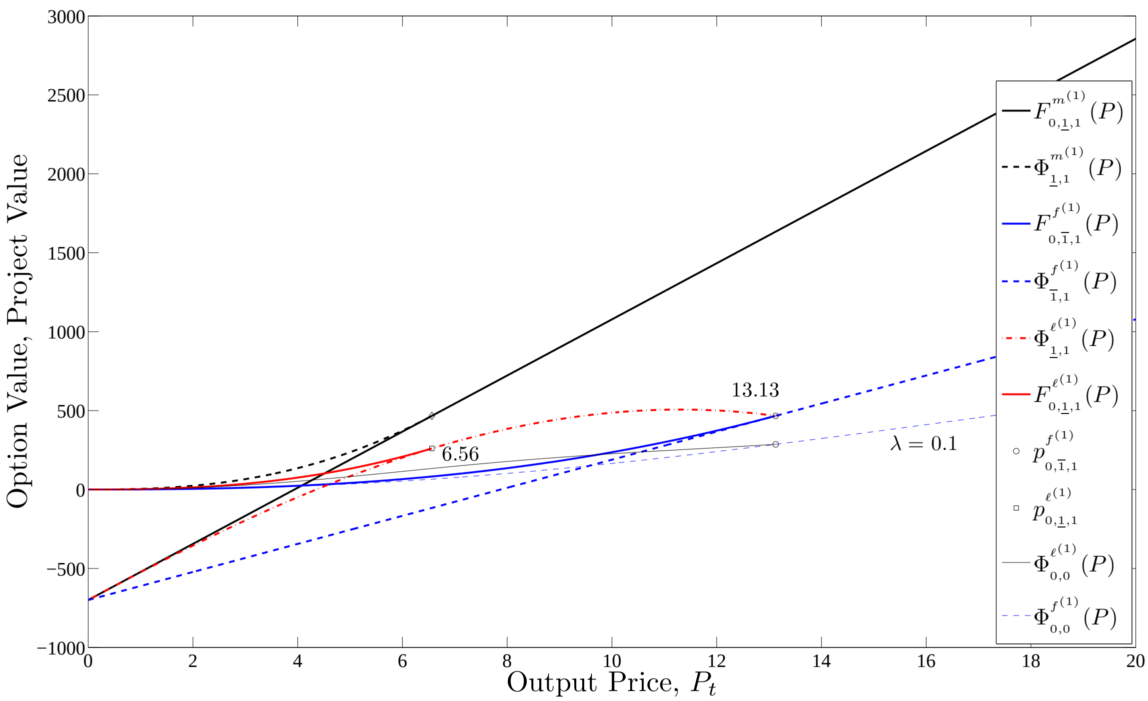

Figure 4: Value functions in a proprietary duopoly with $N = 1$ and $\sigma = 0.2$

Figure 5 illustrates the value function of the monopolist, the leader, and the follower for $N = 2$ assuming that the first technology is available while the second one has yet to arrive. Once the follower invests in the second technology at 33.75, then both firms share the market. By contrast, in state $(\overline{1}, 1)$ both firms have adopted the first technology, yet the value function of the leader is not the same as that of the follower, since the former will receive the value of the option to invest in the second technology first. Therefore, unlike state $(\overline{2}, 2)$, the value function of the leader value matches with her value function in state $(\overline{1}, 1)$ at $p^{f(2)}_{0,\overline{1},1} = 13.04$ and not with the follower's option to invest.

Notice that in state $(\underline{1}, 1)$ the leader can not invest directly in the second technology as shown in Proposition 6.1, and, as a result, she will have to go through state $(\overline{1}, 1)$. Interestingly, while the required investment threshold of the follower decreases with greater λ, the leader's decision to invest is not affected by technological uncertainty, as shown in Proposition 6.2.

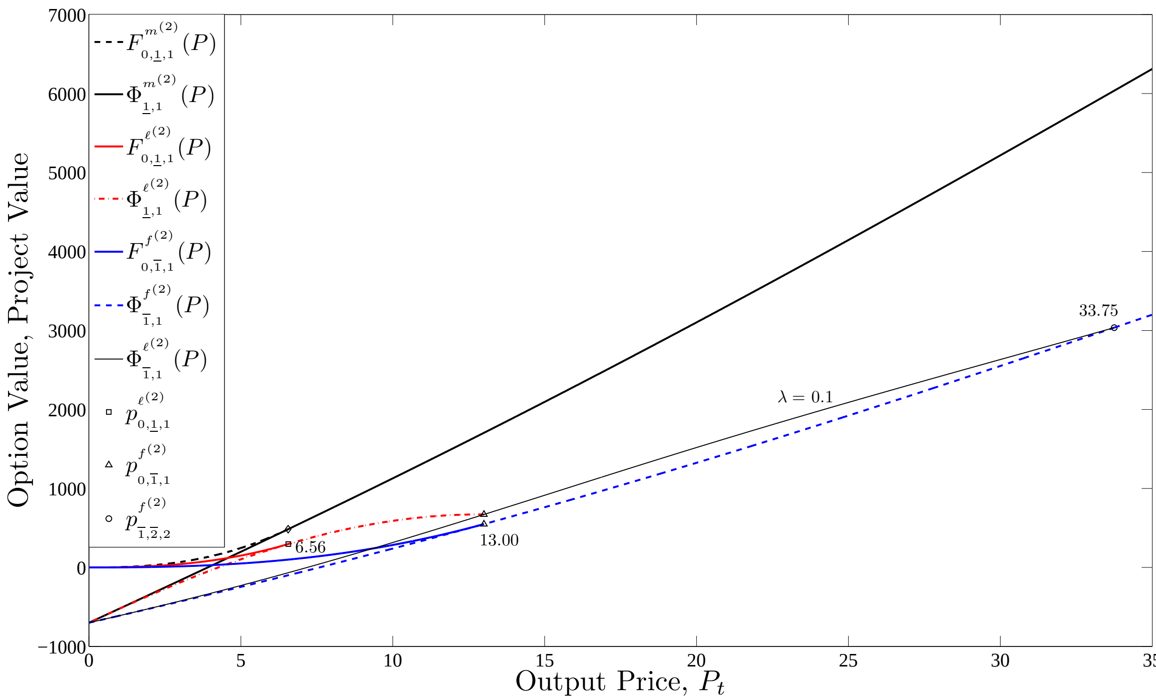

Figure 5: Value functions in a proprietary duopoly with $N = 2$ and $\sigma = 0.2$

The impact of price and technological uncertainty on the optimal investment threshold of the monopolist, the leader, and the follower is illustrated in Figure 6. Intriguingly, although the impact of λ on the required investment threshold of the monopolist and the follower is non–monotonic, the required investment threshold for the leader is not affected by technological uncertainty, as shown in Proposition 6.1. As the right panel illustrates, higher λ lowers the feasibility of a compulsive strategy, thereby increasing the follower's incentive to adopt the currently available technology in order to have a shot at the yet unreleased version. Nevertheless, this incentive is not applicable to the proprietary leader, whose decision to invest in the first technology is not affected by the likelihood of subsequent innovations. This happens because the leader can not invest in the second technology from state $(\underline{1}, 1)$. Indeed, before the leader invests in the second technology the follower will have already invested in the first one, i.e., $p^{f^{(2)}}_{0,\overline{1},1} < p^{\ell^{(2)}}_{\underline{1},\underline{2},2}$, thereby reducing the leader's profits in state $(\underline{1}, 1)$. This reduction, removes the incentive to invest sooner and induces the leader to adopt a myopic behaviour. Additionally, both for the leader and the follower, greater uncertainty raises the value of waiting by increasing the opportunity cost of investing, thereby decreasing the incentive to invest.

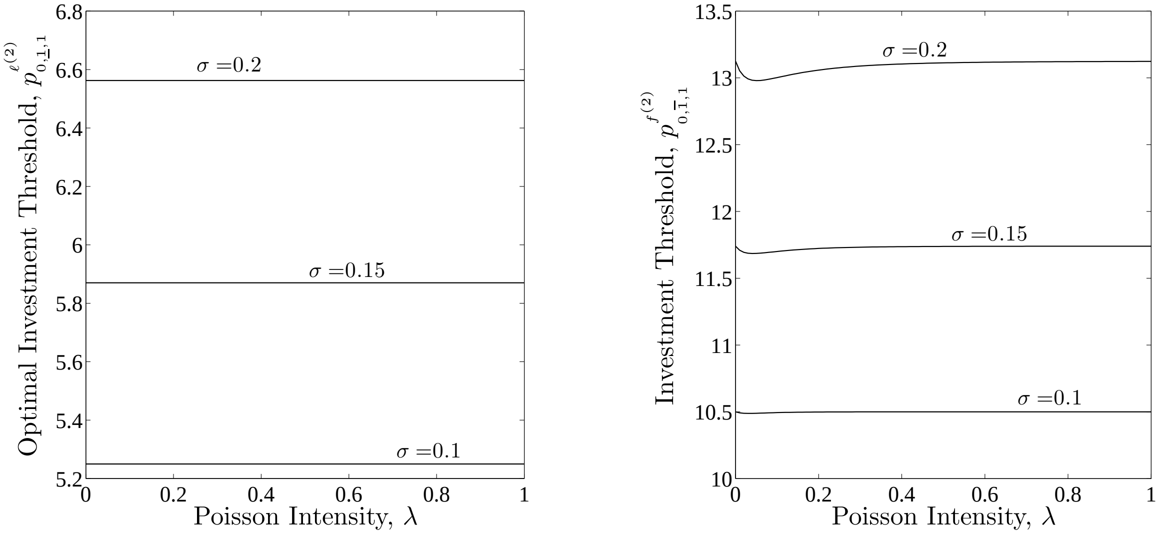

Figure 6: Optimal investment threshold $p^{\ell(2)}_{0,\underline{1},1}$ (left) and $p^{f(2)}_{0,\overline{1},1}$ (right) versus λ

The impact of price and technological uncertainty on the relative loss in the leader's value due to the entry of the follower under a proprietary duopoly is illustrated in Figure 7. As the left panel illustrates, the relative loss in the value of the leader decreases with higher λ. This happens because greater λ reduces the loss in value due to technological uncertainty and raises the expected option value in states $(\overline{1}, 1)$ and $(0, \underline{1}, 1)$. By contrast, greater uncertainty raises the relative loss in the value of the leader because it postpones the entry of the follower, thereby increasing the reduction in market share. Finally, a higher first–mover advantage lowers the relative loss in the value of the leader since it raises the incentive to invest, thereby allowing the leader to enjoy monopoly profits for longer time.

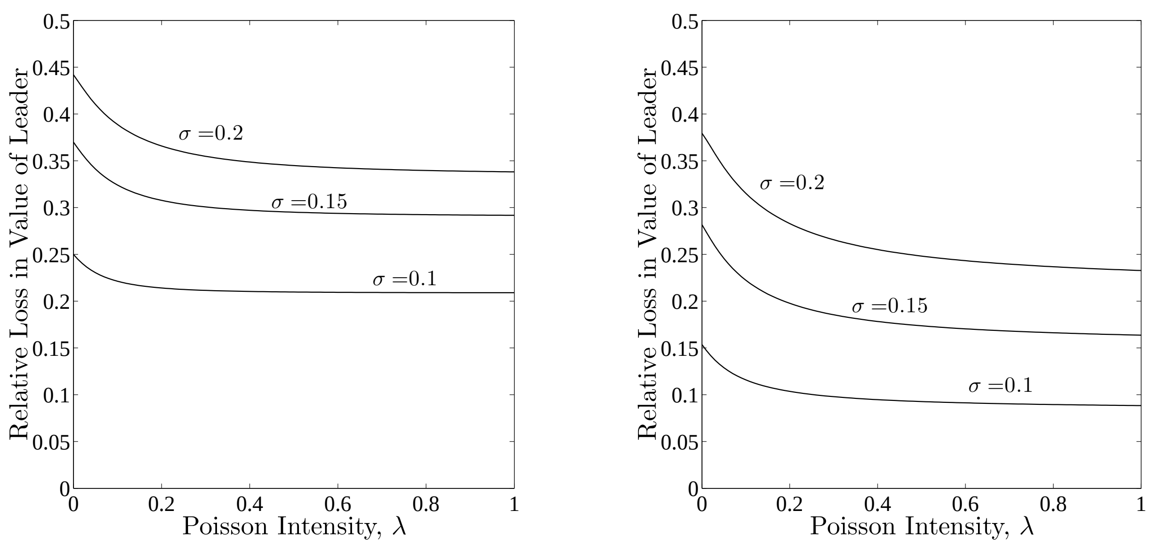

Figure 7: Relative loss in option value versus λ under low, i.e., $D_{\underline{1}} = 16$, $D_{\underline{2}} = 24$ (left) and high , i.e., $D_{\underline{1}} = 20$, $D_{\underline{2}} = 30$ (right) first mover advantage

Figure 8 illustrates the value functions of the proprietary leader and follower under leapfrog and laggard strategies. Notice that, if the output price is low, i.e., $P \in [0, 6.56]$, then it is optimal to wait until $P = 6.56$ and then invest in the first technology, while holding the option to switch to the second. The presence of the second technology creates an additional waiting region. Indeed, if $P \in [11.2, 13.41]$, then it is optimal to wait and if P drops to 11.2 and then invest in the first technology otherwise, if P increases to 13.41, then invest directly in the second one. More specifically, if the output price is low, then once the leader invests in the first technology, the follower will also receive the option to invest, and, at $p^{f^{(2)}}_{0,\bar{1},2}$, the two firms will share the market. Consequently, once the follower invests, the value function of the leader in state $(\underline{1}, 1)$ decreases and value matches with her value function in state $(\bar{1}, \underline{2}, 2)$, i.e., the value of operating the first technology together with the follower with an embedded option to invest in the second first.

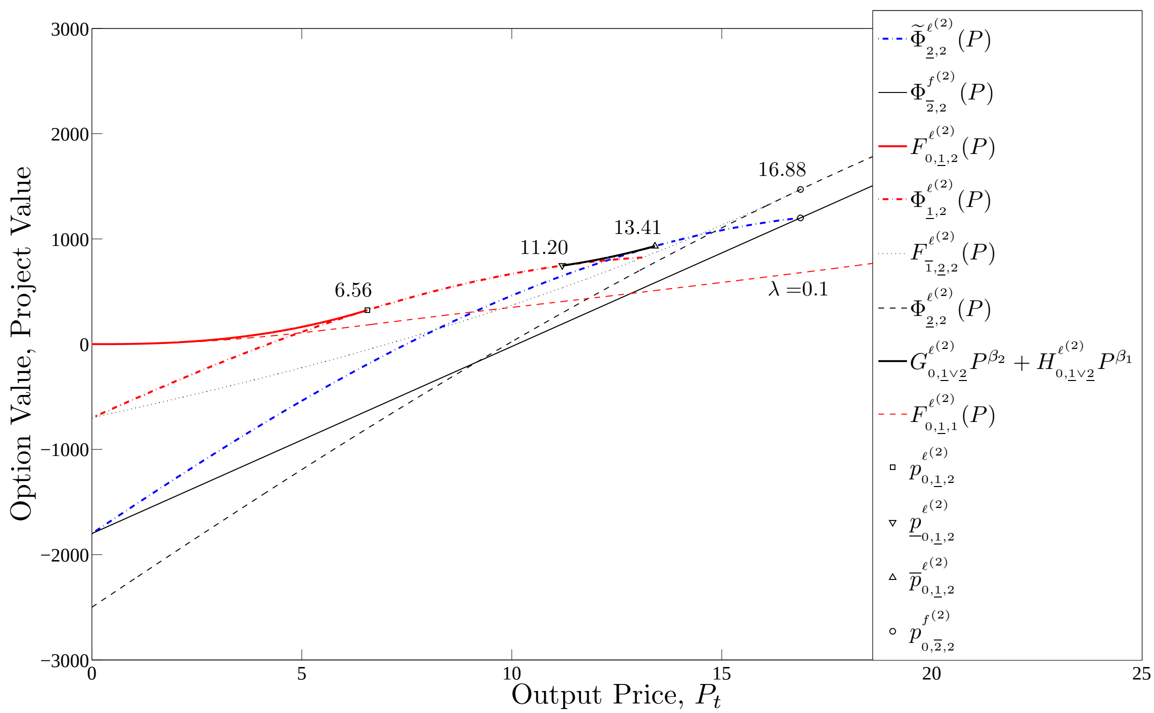

Figure 8: Value functions in a proprietary duopoly under leapfrog and laggard strategies with $N = 2$ and $\sigma = 0.2$

The impact of price uncertainty on the investment thresholds $p^{\ell^{(2)}}_{0,\underline{1},2}$, $\overline{p}^{\ell^{(2)}}_{0,\underline{1},2}$, and $\underline{p}^{\ell^{(2)}}_{0,\underline{1},2}$ is illustrated in Figure 9. Notice that, since both technologies are available, the investment thresholds are not affected by λ, however, greater price uncertainty does not decreases the likelihood of investing directly in the first technology. This is contrary to Chronopoulos and Siddiqui (2014) and Siddiqui and Fleten (2010) who find that $\overline{p}^{\ell^{(2)}}_{0,\underline{1},2}$ decreases with σ, and that $\overline{p}^{\ell^{(2)}}_{0,\underline{1},2} < p^{\ell^{(2)}}_{0,\underline{1},2}$ at high values of σ. In the current setting it is always possible to invest in the first technology.

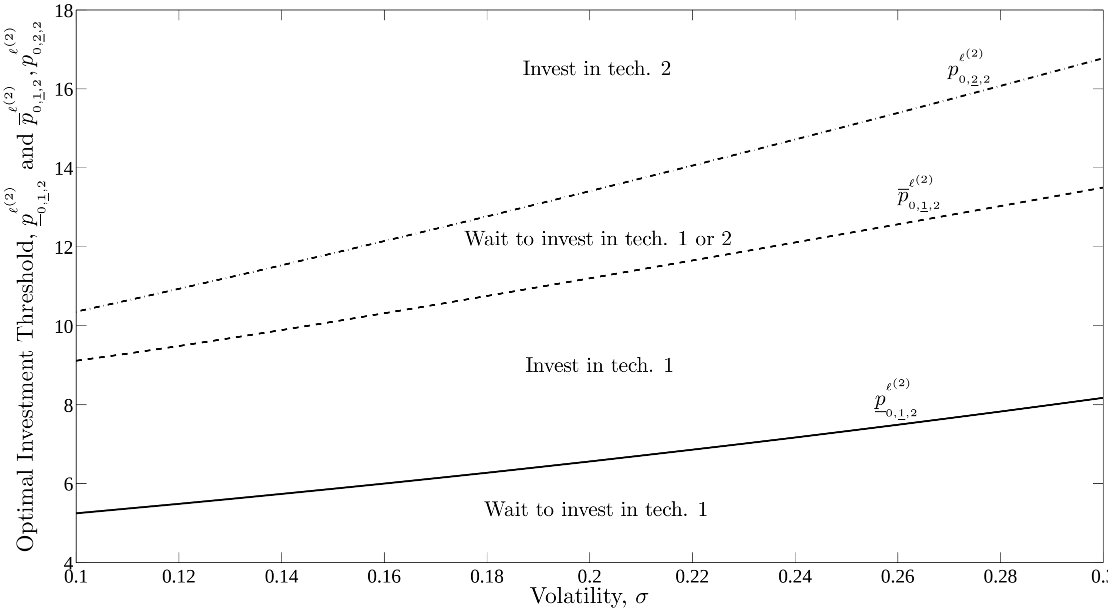

Figure 9: Investment thresholds under a leapfrog/laggard strategy versus σ

The relative value of the two strategies in state $(0, \underline{1}, 1)$ is indicated in (32) for low and high output price and is illustrated in Figure 10. For $P < \underline{p}^{\ell(2)}_{0,\underline{1},2}$ and $\underline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2}$ the relative value is obtained by dividing the top (bottom) branch of (24) with the top (bottom) branch of (31) as indicated in 32. In line with Figure 11, the relative value of the two strategies is strictly greater than one.

$$\frac{A^{\ell(2)}_{0,\underline{1},1} P^{\beta_1}}{A^{\ell(2)}_{0,\underline{1}\vee\underline{2}} P^{\beta_1} + U^{\ell(2)}_{0,\underline{1},1} P^{\delta_1}} \quad \text{and} \quad \frac{\Phi^{m(1)}_{\underline{1},1}(P) + A^{\ell(2)}_{\underline{1},1} P^{\beta_1}}{\frac{\lambda D_{\underline{1}} P}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{\ell(2)}_{\underline{1},2} P^{\beta_1} + L^{\ell(2)}_{0,\underline{1},1} P^{\delta_1} + B^{\ell(2)}_{0,\underline{1},1} P^{\delta_2}} \tag{32}$$

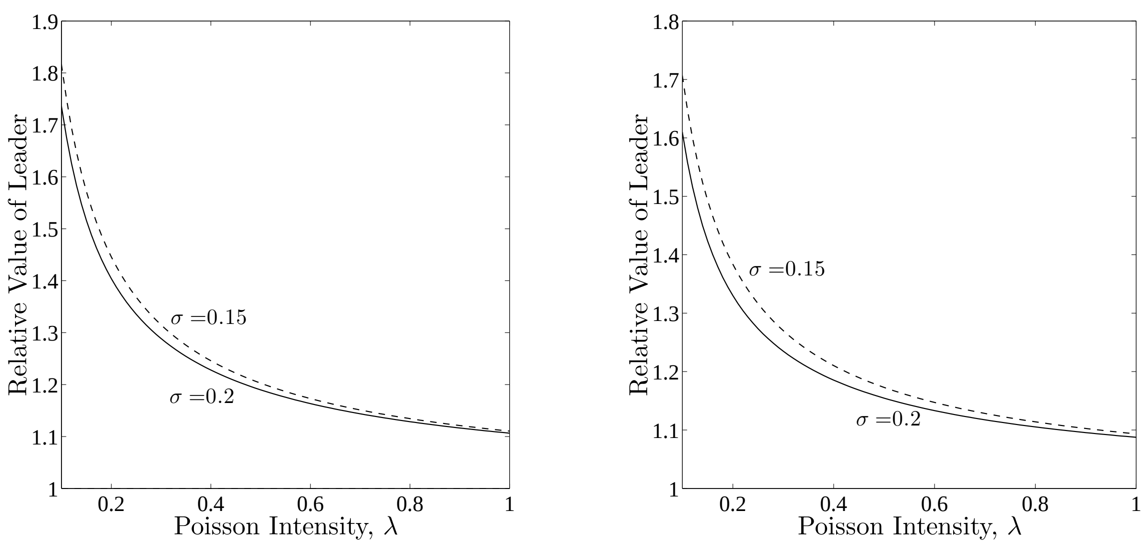

Figure 10: Relative value of the strategies in state $(0, \underline{1}, 1)$ versus λ under low, i.e., $P < \underline{p}^{\ell(2)}_{0,\underline{1},2}$, (left) and high, i.e., $\underline{p}^{\ell(2)}_{0,\underline{1},2} \leq P < p^{\ell(2)}_{0,\underline{2},2}$, (right) output price

Figure 11 illustrates the impact of λ on the value functions of the leader in state $(0,\underline{1},1)$ under compulsive and leapfrog/laggard strategies. The direction of the arrows indicates the increase in λ. Notice that the compulsive strategy dominates for all values of λ, which is in contrast to Chronopoulos and Siddiqui (2014) who show that, in the absence of competition, a firm may have an incentive to adopt a leapfrog or a laggard strategy under a high output price and λ. In this case, the reduction in the leader's profits due to the follower's entry makes the value function of the former less responsive to changes in λ. Indeed, the reduction in the leader's profits due to the presence of a rival reduces the convexity of the leader's value function.

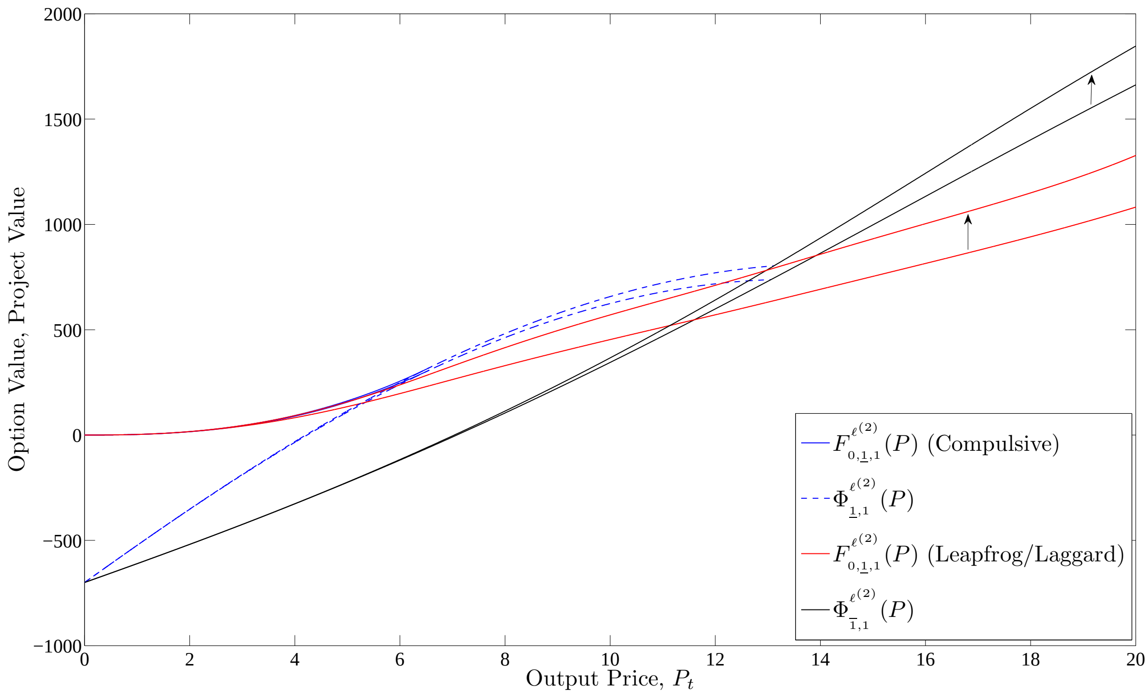

Figure 11: Compulsive versus leapfrog/laggard strategies for $\lambda = 0.2, 0.4$ and $\sigma = 0.2$

9 Conclusions

Investment in technological innovations are considerably risky as they must typically take into account both price and technological uncertainty. Further complicating the decision to invest is the highly competitive environment in which firms must consider the presence of potential rivals when developing a technology adoption strategy. While the problem of sequential investment in technolical innovations is ammenable to the real option theory, the vast majority of real options papers analyse the impact of price and technological uncertainty on the decision to invest under the assumption of monopoly and perfect competition, thus ignoring the implications of strategic interactions (Grenadier and Weiss, 1999).

In this paper, we develop an analytical framework for sequential investment in order to examine the impact of price and technological uncertainty on the optimal investment decision of a firm that faces competition. Thus, we extend the current literature by analysing how different technology adoption strategies, e.g., compulsive, leapfrog, and laggard, are impacted by price and technological uncertainty as well as strategic interactions due to the presence of potential rivals. We find that under a proprietary duopoly, the rate of innovation has a non–monotonic impact on the optimal investment threshold of the follower, yet does not affect the leader's optimal investment threshold. Also, the relative loss in the value of the leader relative to a monopolist decreases as the rate of innovation increases and increases with greater price uncertainty. Finally, we find that, under duopolistic competition, the compulsive strategy always dominates the leapfrog/laggard strategy.

APPENDIX

Monopoly with N = 1

The value of the investment option in state $(0,\underline{1},1)$ is indicated in (A–1).

$$F_{0,\underline{1},1}^{m^{(1)}}(P) = \begin{cases} (1-\rho dt)\mathbb{E}_P\left[F_{0,\underline{1},1}^{m^{(1)}}(P+dP)\right] & ,P < p_{0,\underline{1},1}^{m^{(1)}} \\ \Phi_{\underline{1},1}^{m^{(1)}}(P) & ,P \geq p_{0,\underline{1},1}^{m^{(1)}} \end{cases} \tag{A–1}$$

By expanding the first branch on the right–hand side of (A–1) using Itô's lemma, we obtain the the general solution that is indicated in (A–2).

$$F_{0,\underline{1},1}^{m^{(1)}}(P) = A_{0,\underline{1},1}^{m^{(1)}} P^{\beta_1} + C_{0,\underline{1},1}^{m^{(1)}} P^{\beta_2} \tag{A–2}$$

Notice that $\beta_2 < 0 \Rightarrow C_{0,\underline{1},1}^{m^{(1)}} P^{\beta_2} \to \infty$ as $P \to 0$. Hence, we must have $C_{0,\underline{1},1}^{m^{(1)}} = 0$. The endogenous constant, $A_{0,\underline{1},1}^{m^{(1)}}$, and the required investment threshold, $p_{0,\underline{1},1}^{m^{(1)}}$, are indicated in (A–3).

$$p_{0,\underline{1},1}^{m^{(1)}} = \frac{\beta_1}{\beta_1 - 1}\frac{I_1(\rho-\mu)}{D_{\underline{1}}} \quad \text{and} \quad A_{0,\underline{1},1}^{m^{(1)}} = \frac{{p_{0,\underline{1},1}^{m^{(1)}}}^{1-\beta_1}}{\beta_1}\frac{D_{\underline{1}}}{\rho-\mu} \tag{A–3}$$

Next, we consider the value function in state $(0,0)$, where the first technology has yet to become available. The expression for $\Phi_{0,0}^{m^{(1)}}(P)$ is indicated in (A–4), where, following the same reasoning as in (A–2), we can rule out the terms containing the negative exponents, β_2 and δ_2, in the top part of (A–4) and the term containing the positive exponent δ_1 in the bottom part of (A–4).

$$\Phi_{0,0}^{m^{(1)}}(P) = \begin{cases} A_{0,\underline{1},1}^{m^{(1)}} P^{\beta_1} + A_{0,0}^{m^{(1)}} P^{\delta_1} & ,P < p_{0,\underline{1},1}^{m^{(1)}} \\ \frac{\lambda D_{\underline{1}} P}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + B_{0,0}^{m^{(1)}} P^{\delta_2} & ,p_{0,\underline{1},1}^{m^{(1)}} \leq P \end{cases} \tag{A–4}$$

The endogenous constants $A_{0,0}^{m^{(1)}} < 0$ and $B_{0,0}^{m^{(1)}} > 0$ are determined via the value–matching and smooth–pasting conditions between the two branches of (A–4) at $p_{0,\underline{1},1}^{m^{(1)}}$ and are indicated in (A–5) and (A–6) respectively.

$$A_{0,0}^{m^{(1)}} = \frac{p_{0,\underline{1},1}^{m^{(1)^{-\delta_1}}}}{\delta_2 - \delta_1}\left[\frac{\lambda(\delta_2 - 1)D_1 p_{0,\underline{1},1}^{m^{(1)}}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\beta_1 - \delta_2)A_{0,\underline{1},1}^{m^{(1)}} p_{0,\underline{1},1}^{m^{(1)^{\beta_1}}} - \frac{\delta_2 \lambda I_1}{\rho+\lambda}\right] \tag{A–5}$$

$$B_{0,0}^{m^{(1)}} = \frac{p_{0,\underline{1},1}^{m^{(1)^{-\delta_2}}}}{\delta_1 - \delta_2}\left[\frac{\lambda(1 - \delta_1)D_1 p_{0,\underline{1},1}^{m^{(1)}}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\delta_1 - \beta_1)A_{0,\underline{1},1}^{m^{(1)}} p_{0,\underline{1},1}^{m^{(1)^{\beta_1}}} + \frac{\delta_1 \lambda I_1}{\rho+\lambda}\right] \tag{A–6}$$

■

Proprietary duopoly with N = 1

Follower

In state $(0, \overline{1}, 1)$, $A_{0,\overline{1},1}^{f^{(1)}}$ and $p_{0,\overline{1},1}^{f^{(1)}}$ are obtained via value–matching and smooth–pasting conditions between the two branches of (8) and are indicated in (A–7)

$$A_{0,\overline{1},1}^{f^{(1)}} = \frac{p_{0,\overline{1},1}^{f^{(1)^{1-\beta_1}}}}{\beta_1}\frac{D_{\overline{1}}}{\rho-\mu} \quad \text{and} \quad p_{0,\overline{1},1}^{f^{(1)}} = \frac{\beta_1}{\beta_1 - 1}\frac{I_1(\rho-\mu)}{D_{\overline{1}}} \tag{A–7}$$

By applying the same conditions to (10) we obtain $A_{0,0}^{f^{(1)}}$ and $B_{0,0}^{f^{(1)}}$, that are indicated in (A–8) and (A–9) respectively.

$$A_{0,0}^{f^{(1)}} = \frac{p_{0,\overline{1},1}^{f^{(1)^{-\delta_1}}}}{\delta_2 - \delta_1}\left[\frac{\lambda(\delta_2 - 1)p_{0,\overline{1},1}^{f^{(1)}} D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\beta_1 - \delta_2)A_{0,\overline{1},1}^{f^{(1)}} p_{0,\overline{1},1}^{f^{(1)^{\beta_1}}} - \frac{\delta_2 \lambda I_1}{\rho+\lambda}\right] \tag{A–8}$$

$$B_{0,0}^{f^{(1)}} = \frac{p_{0,\overline{1},1}^{f^{(1)^{-\delta_2}}}}{\delta_1 - \delta_2}\left[\frac{\lambda(1 - \delta_1)p_{0,\overline{1},1}^{f^{(1)}} D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\delta_1 - \beta_1)A_{0,\overline{1},1}^{f^{(1)}} p_{0,\overline{1},1}^{f^{(1)^{\beta_1}}} + \frac{\delta_1 \lambda I_1}{\rho+\lambda}\right] \tag{A–9}$$

Leader

The endogenous constant, $A_{0,\underline{1},1}^{\ell^{(1)}}$, and the required investment threshold, $p_{0,\underline{1},1}^{\ell^{(1)}}$, are obtained via value–matching and smooth–pasting conditions between the two branches of (12) and are indicated in (A–10).

$$p_{0,\underline{1},1}^{\ell^{(1)}} = p_{0,\underline{1},1}^{m^{(1)}} \quad \text{and} \quad A_{0,\underline{1},1}^{\ell^{(1)}} = \left(\frac{1}{p_{0,\underline{1},1}^{\ell^{(1)}}}\right)^{\beta_1}\left[\Phi_{\underline{1}}^{m^{(1)}}\left(p_{0,\underline{1},1}^{\ell^{(1)}}\right) + A_{\underline{1},1}^{\ell^{(1)}} p_{0,\underline{1},1}^{\ell^{(1)^{\beta_1}}}\right] \tag{A–10}$$

The expression for $\Phi^{\ell^{(1)}}_{0,0}(P)$ is indicated in (A–11).

$$\Phi^{\ell^{(1)}}_{0,0}(P) = \begin{cases} A^{\ell^{(1)}}_{0,\underline{1},1} P^{\beta_1} + A^{\ell^{(1)}}_{0,0} P^{\delta_1} & , P < p^{\ell^{(1)}}_{0,\underline{1},1} \\ \frac{\lambda P D_{\underline{1}}}{(\lambda+\rho-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{\ell^{(1)}}_{\underline{1},1} P^{\beta_1} + B^{\ell^{(1)}}_{0,0} P^{\delta_1} + C^{\ell^{(1)}}_{0,0} P^{\delta_2} & , p^{\ell^{(1)}}_{0,\underline{1},1} \leq P \end{cases} \tag{A–11}$$

where $A^{\ell^{(1)}}_{0,0}$ and $C^{\ell^{(1)}}_{0,0}$ are determined via value–matching and smooth–pasting conditions between the two branches of (A–11) and are indicated in (A–12) and (A–13), respectively.

$$\begin{aligned} A^{\ell^{(1)}}_{0,0} &= \frac{p^{\ell^{(1)^{-\delta_2}}}_{0,\underline{1},1}}{\delta_2 - \delta_1} \left[\frac{(\delta_2 - 1)\lambda p^{\ell^{(1)}}_{0,\underline{1},1} D_{\underline{1}}}{(\lambda+\rho-\mu)(\rho-\mu)} - \frac{\delta_2 \lambda I_1}{\rho+\lambda} + (\delta_2 - \beta_1)\left(A^{\ell^{(1)}}_{\underline{1},1} - A^{\ell^{(1)}}_{0,\underline{1},1} \right) p^{\ell^{(1)^{\beta_1}}}_{0,\underline{1},1} \right. \\ & \qquad \left. + \ (\delta_2 - \delta_1) B^{\ell^{(1)}}_{0,0} p^{\ell^{(1)^{\delta_1}}}_{0,\underline{1},1} \right] \end{aligned} \tag{A–12}$$

$$C^{\ell^{(1)}}_{0,0} = \frac{p^{\ell^{(1)^{-\delta_1}}}_{0,\underline{1},1}}{\delta_2 - \delta_1} \left[\frac{(\delta_1 - 1)\lambda p^{\ell^{(1)}}_{0,\underline{1},1} D_1(1)}{(\lambda+\rho-\mu)(\rho-\mu)} - \frac{\delta_1 \lambda I_1}{\rho+\lambda} + (\delta_1 - \beta_1)\left(A^{\ell^{(1)}}_{\underline{1},1} - A^{\ell^{(1)}}_{0,\underline{1},1} \right) p^{\ell^{(1)^{\beta_1}}}_{0,\underline{1},1} \right] \tag{A–13}$$

■

Compulsive strategy under proprietary duopoly with N = 2

Follower

The derivation of the value function of the monopolist in each state follows the same steps as that of the follower, and, therefore, the analysis is omitted. The corresponding expression can be obtained by replacing $\overline{1}$ and $\overline{2}$ by $\underline{1}$ and $\underline{2}$ in the subscripts. In state $(\overline{1}, \overline{2}, 2)$, $A^{f^{(2)}}_{\overline{1},\overline{2},2}$ and $p^{f^{(2)}}_{\overline{1},\overline{2},2}$ are obtained via value–matching and smooth–pasting conditions between the two branches of (16) and are indicated in (A–14).

$$A^{f^{(2)}}_{\overline{1},\overline{2},2} = \frac{p^{f^{(2)^{1-\beta_1}}}_{\overline{1},\overline{2},2}}{\beta_1} \frac{D_{\overline{2}} - D_{\overline{1}}}{\rho - \mu} \quad \text{and} \quad p^{f^{(2)}}_{\overline{1},\overline{2},2} = \frac{\beta_1}{\beta_1 - 1} \frac{I_2(\rho-\mu)}{D_{\overline{2}} - D_{\overline{1}}} \tag{A–14}$$

Similarly, by applying value–matching and smooth–pasting conditions to the two branches of (17), we obtain the expressions for $A^{f^{(2)}}_{\overline{1},1}$ and $B^{f^{(2)}}_{\overline{1},1}$, that are indicated in (A–15) and (A–16) respectively.

$$A^{f^{(2)}}_{\overline{1},1} = \frac{p^{f^{(2)^{-\delta_1}}}_{\overline{1},\overline{2},2}}{(\delta_2 - \delta_1)} \left[\frac{(\delta_2 - 1)\lambda \left(D_{\overline{2}} - D_{\overline{1}} \right) p^{f^{(2)}}_{\overline{1},\overline{2},2}}{(\lambda+\rho-\mu)(\rho-\mu)} - (\delta_2 - \beta_1) A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{\overline{1},\overline{2},2} - \frac{\delta_2 \lambda I_2}{\rho+\lambda} \right] \tag{A–15}$$

$$B^{f^{(2)}}_{\overline{1},1} = \frac{p^{f^{(2)^{-\delta_2}}}_{\overline{1},\overline{2},2}}{(\delta_1 - \delta_2)} \left[\frac{(\delta_1 - 1)\lambda \left(D_{\overline{1}} - D_{\overline{2}} \right) p^{f^{(2)}}_{\overline{1},\overline{2},2}}{(\lambda+\rho-\mu)(\rho-\mu)} + (\delta_1 - \beta_1) A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{\overline{1},\overline{2},2} + \frac{\delta_2 \lambda I_2}{\rho+\lambda} \right] \tag{A–16}$$

In state $(0,\overline{1},1)$, the endogenous constant $A^{f^{(2)}}_{0,\overline{1},1}$ and investment threshold $p^{f^{(2)}}_{0,\overline{1},1}$ are obtained numerically by value matching and smooth pasting the two branches of (18), i.e:

$$A^{f^{(2)}}_{0,\overline{1},1} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} = \frac{D_{\overline{1}} p^{f^{(2)}}_{0,\overline{1},1}}{\rho-\mu} - I_1 + A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} + A^{f^{(2)}}_{\overline{1},1} p^{f^{(2)^{\delta_1}}}_{0,\overline{1},1} \tag{A–17}$$

$$\beta_1 A^{f^{(2)}}_{0,\overline{1},1} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} = \frac{D_{\overline{1}} p^{f^{(2)}}_{0,\overline{1},1}}{\rho-\mu} + \beta_1 A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} + \delta_1 A^{f^{(2)}}_{\overline{1},1} p^{f^{(2)^{\delta_1}}}_{0,\overline{1},1} \tag{A–18}$$

Finally, $A^{f^{(2)}}_{0,0}$ and $B^{f^{(2)}}_{0,0}$ are obtained via value–matching and smooth–pasting conditions between the two branches of (19) and are indicated in (A–19) and (A–20) respectively.

$$\begin{aligned} A^{f^{(2)}}_{0,0} = \frac{p^{f^{(2)^{-\delta_1}}}_{0,\overline{1},1}}{\delta_2-\delta_1} \Bigg[& \frac{(\delta_2-1)\,\lambda p^{f^{(2)}}_{0,\overline{1},1} D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho+\lambda)} - \frac{\lambda\delta_2 I_1}{\rho+\lambda} + (\delta_2-\beta_1) A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} \\ & + (\delta_2-\delta_1) A^{f^{(2)}}_{\overline{1}} p^{f^{(2)^{\delta_1}}}_{0,\overline{1},1} - (\delta_2-\beta_1) A^{f^{(2)}}_{0,\overline{1},1} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} \Bigg] \end{aligned} \tag{A–19}$$

$$\begin{aligned} B^{f^{(2)}}_{0,0} = \frac{p^{f^{(2)^{-\delta_2}}}_{0,\overline{1},1}}{\delta_1-\delta_2} \Bigg[& (\delta_1-\beta_1) A^{f^{(2)}}_{0,\overline{1},1} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} - \frac{(\delta_1-1)\,\lambda p^{f^{(2)}}_{0,\overline{1},1} D_{\overline{1}}}{(\rho+\lambda-\mu)(\rho+\lambda)} + \frac{\lambda\delta_1 I_1}{\rho+\lambda} \\ & - (\delta_1-\beta_1) A^{f^{(2)}}_{\overline{1},\overline{2},2} p^{f^{(2)^{\beta_1}}}_{0,\overline{1},1} \Bigg] \end{aligned} \tag{A–20}$$

■

Leader

In state $(\overline{1},\underline{2},2)$, $p^{\ell^{(2)}}_{\overline{1},\underline{2},2}$ and $A^{\ell^{(2)}}_{\overline{1},\underline{2},2}$ are determined via the value–matching and smooth–pasting conditions between the two branches of (21) and are indicated in (A–21).

$$p^{\ell^{(2)}}_{\overline{1},\underline{2},2} = \frac{\beta_1}{\beta_1-1}\frac{(\rho-\mu)I_2}{D_{\underline{2}}-D_{\overline{1}}} \quad \text{and} \quad A^{\ell^{(2)}}_{\overline{1},\underline{2},2} = \frac{1}{p^{\ell^{(2)^{\beta_1}}}_{\overline{1},\underline{2},2}} \left[\frac{p^{\ell^{(2)}}_{\overline{1},\underline{2},2}\left(D_{\underline{2}}-D_{\overline{1}}\right)}{\rho-\mu} - I_2 + A^{\ell^{(2)}}_{\underline{2},2}\, p^{\ell^{(2)^{\beta_1}}}_{\overline{1},\underline{2},2} \right] \tag{A–21}$$

Next, the value function in state $(\underline{1},1)$ is described in (A–22)

$$\Phi^{\ell^{(2)}}_{\underline{1},1}(P) = \frac{D_{\underline{1}}P}{\rho-\mu} - I_1 + A^{\ell^{(2)}}_{\underline{1},1} P^{\beta_1} \tag{A–22}$$

where $A^{\ell^{(2)}}_{\underline{1},1}$ is indicated in (A–23).

$$A^{\ell^{(2)}}_{\underline{1},1} = \frac{p^{f^{(2)^{1-\beta_1}}}_{0,\overline{1},1}\left(D_{\overline{1}}-D_{\underline{1}}\right)}{\rho-\mu} + A^{\ell^{(2)}}_{\overline{1},\underline{2},2} + A^{\ell^{(2)}}_{\overline{1},1} p^{f^{(2)^{\delta_1-\beta_1}}}_{0,\overline{1},1} \tag{A–23}$$

In state $(0,\underline{1},1)$, the leader holds an option to invest in the first technology with an embedded option to invest in the second. Hence, the value of the option to invest in the first technology is indicated in (A–24)

$$F^{\ell(2)}_{0,\underline{1},1}(P) \quad = \quad \begin{cases} A^{\ell(2)}_{0,\underline{1},1} P^{\beta_1}, & P < p^{\ell(2)}_{0,\underline{1},1} \\ \Phi^{\ell(2)}_{\underline{1},1}(P), & P \geq p^{\ell(2)}_{0,\underline{1},1} \end{cases} \tag{A–24}$$

where $A^{\ell(2)}_{0,\underline{1},1}$ and $p^{\ell(2)}_{0,\underline{1},1}$ are obtained via the value–matching and smooth–pasting conditions between the two branches of (A–24) and are indicated in (A–25)

$$p^{\ell(2)}_{0,\underline{1},1} = \frac{\beta_1}{\beta_1 - 1} \frac{(\rho-\mu)I_1}{D_1} \text{ and } A^{\ell(2)}_{0,\underline{1},1} = \left(\frac{1}{p^{\ell(2)}_{0,\underline{1},1}}\right)^{\beta_1} \left[\frac{D_1 p^{\ell(2)}_{0,\underline{1},1}}{\rho-\mu} - I_1 + A^{\ell(2)}_{\underline{1},1} p^{\ell(2)^{\beta_1}}_{0,\underline{1},1}\right] \tag{A–25}$$

■

Proposition 6.1: $p^{\ell(2)}_{\underline{1},\underline{2},2} > p^{f(2)}_{0,\overline{1},1}$.

Proof: From Chronopoulos and Siddiqui (2014), we know that $p^{f(2)}_{0,\overline{1},1} \leq p^{f(2)}_{0,\overline{1},2}$, where the expression for $p^{f(2)}_{0,\overline{1},2}$ is indicated in (A–26).

$$p^{f(2)}_{0,\overline{1},2} \quad = \quad \frac{\beta_1}{\beta_1 - 1} \frac{I_1(\rho-\mu)}{D_{\overline{1}}} \tag{A–26}$$

Also, the expression for $p^{\ell(2)}_{\underline{1},\underline{2},2}$ is indicated in (A–27).

$$p^{\ell(2)}_{\underline{1},\underline{2},2} \quad = \quad \frac{\beta_1}{\beta_1 - 1} \frac{I_2(\rho-\mu)}{D_{\underline{2}} - D_{\underline{1}}} \tag{A–27}$$

In order to show that $p^{\ell(2)}_{\underline{1},\underline{2},2} > p^{f(2)}_{0,\overline{1},2}$ we have:

$$\begin{aligned} p^{\ell(2)}_{\underline{1},\underline{2},2} > p^{f(2)}_{0,\overline{1},2} \quad &\Leftrightarrow \quad \frac{I_2}{D_{\underline{2}} - D_{\underline{1}}} > \frac{I_1}{D_{\overline{1}}} \\ &\Leftrightarrow \quad D_{\overline{1}} I_2 > D_{\underline{2}} I_1 - I_1 D_{\underline{1}} \\ &\Leftrightarrow \quad D_{\underline{2}} I_1 < D_{\overline{1}} I_2 + I_1 D_{\underline{1}} < D_{\underline{1}}(I_1 + I_2) \end{aligned} \tag{A–28}$$

Finally, we have $\frac{D_{\underline{2}}}{I_1+I_2} < \frac{D_{\underline{1}}}{I_1}$, which, from the assumption $\frac{D_n}{\sum_{i=1}^{n} I_i} < \frac{D_{n-1}}{\sum_{i=1}^{n-1} I_i}$, $\forall n$, holds. ■

Proposition 6.2: $p^{\ell(2)}_{0,\underline{1},1}$ *is independent of* λ.

Proof: Since $p^{\ell(2)}_{\underline{1},\underline{2},2} > p^{f(2)}_{0,\overline{1},1}$, it is not possible to invest in the second technology even it becomes available while the leader is in state $\underline{1}$. Consequently, after investing in the first technology, the leader will have to go through state $\overline{1}$ before considering investment in the second one. Hence, the value of the option to invest in state $(0, \underline{1}, 1)$ can be written as in (A–29).

$$F^{\ell(2)}_{0,\underline{1},1}(P) \quad = \quad \max_{P^{\ell(2)}_{0,\underline{1},1} > P} \left(\frac{P}{P^{\ell(2)}_{0,\underline{1},1}}\right)^{\beta_1} \left[\frac{D_{\underline{1}} P^{\ell(2)}_{0,\underline{1},1}}{\rho-\mu} - I_1 + A^{\ell(2)}_{\underline{1},1} P^{\ell(2)^{\beta_1}}_{0,\underline{1},1}\right] \tag{A–29}$$

Applying FONC to (A–29), we can write the optimal investment rule by equation the marginal benefit (MB) from delaying investment to the marginal cost (MC).

$$\frac{D_{\underline{1}}}{\rho-\mu}+\frac{\beta_1 I_1}{p_{0,\underline{1},1}^{\ell(2)}}+\beta_1 A_{\underline{1},1}^{\ell(2)}\, p_{0,\underline{1},1}^{\ell(2)^{\beta_1-1}} \quad = \quad \frac{\beta_1 D_{\underline{1}}}{\rho-\mu}+\beta_1 A_{\underline{1},1}^{\ell(2)}\, p_{0,\underline{1},1}^{\ell(2)^{\beta_1-1}} \tag{A–30}$$

The first term on the left–hand side is the MB from allowing the project to start at a higher output price, while the second term is the reduction in MC to saved investment cost. The first term on the right–hand side reflects the forgone revenues from delaying investment. Notice that the MB and MC from the embedded investment option cancel. Effectively, the MB related to this term is the higher price at which you receive the subsequent investment option, while the MC is the forgone value from not having the option sooner. These two equal each other because they both stem from the opportunity to delay. Indeed, since the threshold is chosen optimally, the MB should equal the MC. ■

References

[1] Baldursson, F and I Karatzas (1997), "Irreversible Investment and Industry Equilibrium," *Finance and Stochastics,* 1: 69–89.

[2] Bertola, G (1989), "Irreversible Investment," unpublished working paper (later published in Research in Economics in 1998), Princeton University.

[3] Bouis, R, KJM Huisman, and PM Kort (2009), "Investment in Oligopoly under Uncertainty: The Accordion Effect," *International Journal of Industrial Organisation,* 27: 320–331.

[4] Chronopoulos, M and A Siddiqui (2014), "When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology Under Uncertainty," *Discussion Paper No. 2014/26*, Norwegian School of Economics, Bergen, Norway.

[5] Dixit, AK and RS Pindyck (1994), *Investment under Uncertainty*, Princeton University Press, Princeton, NJ, USA.

[6] Doraszelski, U (2001), "The Net Present Value Method versus the Option Value of Waiting: A Note on Farzin, Huisman and Kort (1998)," *Journal of Economic Dynamics & Control,* 25: 1109–1115.

[7] Farzin, Y, KJM Huisman, and PM Kort, (1998) "Optimal Timing of Technology Adoption," *Journal of Economic Dynamics and Control,* 22: 779–799.

[8] Fudenberg, D and J Tirole (1985) "Preemption and Rent Equalisation of New Technology," *The Review of Economic Studies,* 52: 382–401.

[9] Graham, J (2011) "Strategic Real Options under Asymmetric Information," *Journal of Economic Dynamics & Control,* 35(6): 922–934.

[10] Huisman, KJM and PM Kort (1999), "Effects of Strategic Interactions on the Option Value of Waiting," working paper, Tilburg University, Tilburg, The Netherlands.

[11] Huisman, KJM (2001), *Technology Investment: A Game Theoretic Real Options Approach,* Dordrecht: Kluwer Academic Publishers.

[12] Huisman, KJM and PM Kort (2009), "Strategic Capacity Investment under Uncertainty," working paper, Tilburg University, Tilburg, The Netherlands.

[13] Lambrecht, BM and WR Perraudin (2003), "Real Options and Preemption under Incomplete Information," *Journal of Economic Dynamics and Control* 27(4): 619-643.

[14] Mason, R and H Weeds (2010), "Investment, Uncertainty and Pre-Emption," *International Journal of Industrial Organisation,* 28(3): 278–287.

[15] McDonald, RL and DS Siegel (1985), "Investment and Valuation of Firms When There is an Option to Shut Down," *International Economic Review,* 26(2): 331–349.

[16] McDonald, RL and DS Siegel (1986), "The Value of Waiting to Invest," *The Quarterly Journal of Economics,* 101(4): 707–728.

[17] Miltersen, K R and ES Schwartz, (2004). "R&D Investments with Competitive Interactions," *Review of Finance,* 8(3), 355-401.

[18] Pindyck, RS (1993), "Investment of Uncertain Cost," *Journal of Financial Economics,* 34(1): 53–76.

[19] Pindyck, RS (1988), "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review,* 79: 969–985.

[20] Paxson, D and H Pinto (2005), "Rivalry under Price and Quantity Uncertainty," *Review of Financial Economics,* 14: 209–224.

[21] Roques, F and N Savva (2009), "Investment under Uncertainty with Price Ceilings in Oligopolies," *Journal of Economic Dynamics & Control,* 33(2): 507–524.

[22] Siddiqui, A and R Takashima (2012), "Capacity Switching Options under Rivalry and Uncertainty," *European Journal of Operational Research* 222(3): 583–595.

[23] Smets, F (1993), *Essays on Foreign Direct Investment,* PhD thesis, Yale University.

[24] Spatt, CS and FP Sterbenz (1985), "Learning, Preemption and the Degree of Rivalry," *Rand Journal of Economics* 16(1): 84-92.

[25] Takashima, R, M Goto, H Kimura, and H Madarame (2008), "Entry into the Electricity Market: Uncertainty, Competition, and Mothballing Options," *Energy Economics,* 30: 1809–1830.

[26] The Financial Times (2012), "Apple iPhone sales drop as rival gains," 2 July.

[27] The Wall Street Journal (2013a), "Generic–Drug Firms Go Beyond Knockoffs," 15 June.

[28] The Wall Street Journal (2013b), "Apple Tests iPhone Screens as Large as Six Inches," 5 September.

[29] Tsekrekos, AE (2001), "Investment under Economic and Implementation Uncertainty. *R&D Management* 31(2): 127-135.

[30] Tsekrekos, AE (2003) "The Effect of First–Mover's Advantages on the Strategic Exercise of Real Options," *Real R&D Options,* D. Paxson (Ed.), 185–207.

[31] Weeds, H (1999), "Reverse Hysterisis: R& D Investment with Stochastic Innovation," *working paper,* Fitzwilliam College, University of Cambridge, UK.

[32] Weeds, H (2002), "Strategic Delay in a Real Options Model of R&D Competition," *Review of Economic Studies,* 69: 729–747.