Investment I-Game with Flexibility and Demand Uncertainty*

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Abstract

This paper embeds real options in an investment game of incomplete information in a duopolistic market, where product market competition influences the state value of the investment, and entry times are endogenously determined. The model incorporates private information over types and unveils new features of strategic interactions in imperfectly competitive markets when firms are faced with the trade-off between commitment and flexibility under demand uncertainty. The paper illustrates that type-asymmetry and/or initial demand level alone, as have been previously adopted in the literature, are insufficient criteria upon which endogenous roles under uncertainty may be determined when firms have private information over their types. Rather, the ex post market structure is determined by threshold functions whose images lie in the type-space of the firms. These functions, therefore, specify, ex ante, the firms' optimal strategies, which may involve (anti)-coordination. The model is extended to consider the plausible case where a firm is able to credibly "fool" its rival by masking its type. The threshold functions, and thus, ex post market structures obtained in equilibrium are found to be characteristically the same as with when types are truthfully revealed. Therefore, the competitive behaviour of firms remain the same whether or not there are industry regulations that make it illegal for firms to falsify, mask, or lie about their profits.

Key words: Endogenous timing, Private information, Sequential equilibrium, Real options, Imperfect markets, Anti-coordination.

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1 Introduction

Firms are often called upon to make irreversible investment decisions in the face of uncertainty about future demand, and incomplete information over the competitiveness of the rivals with whom they might be competing for market share. There are benefits to delaying investment until a future period when the uncertain elements of market fundamentals become revealed. However, there are potential costs of waiting, both in terms of foregone market activity and in losing the opportunity to preempt ones rivals. There is thus, a trade-off between commitment and flexibility.

This trade-off is central to the derivation of optimal investment decisions in strategic investment problems under uncertainty. In a new market, for instance, there may be inherent uncertainty about the scale of future demand, cost functions of potential competitors, market price of commodities etc. An optimal investment strategy must, therefore, be based on a proper consideration of the strategic value of flexibility against the opportunity cost of early commitment. By committing to an irreversible investment at an early stage, a firm may obtain a first-mover advantage in the form of a lower production cost, earning monopoly rents, or emerging as the Stackelberg leader in the subsequent stage. These benefits may however, be eroded away if market conditions become unfavourable, as the firm cannot simply recover its initial investment outlay. As a result, a firm's ability to delay making such investment until a later time, when more information arrives, that fully or partially resolves some, or all, of the uncertain elements in the market, is immensely valuable.

In most industries, firms are often able to exercise this flexibility when faced with investment opportunities under uncertainty. Which is why in practice, it is observed that firms do not invest in capital projects until price rises substantially above long-run average cost. This is in sharp contrast to the theoretical provision of the discounted cash flow (DCF) analysis or conventional net present value (NPV) approach to valuing investments. The DCF analysis specifies that an investment opportunity is viable whenever the discounted income flow is at least equal to the cost of investment (otherwise known as the *Marshallian trigger*). This trigger is, in general, less than what is observed in reality. The main shortcoming of this method is its inability to factor-in operating flexibility, i.e. the ability of management to make, revise or alter planned investment decisions as uncertainty gets resolved over time. It inherently assumes investment opportunities are "now-or-never" in nature, and hence, ignores the value of flexibility. To address this problem, an option-based valuation approach has been proposed as a tool capable of capturing managerial flexibility. It provides a dynamic decision making framework that affords firms the opportunity to delay investment decisions until such a time when more information becomes available that could influence both the timing and the level of investment.

The real options literature emphasizes the value of this sort of flexibility and derives the optimal time to make an investment, when its value is determined, in part, by an exogenous stochastic variable e.g. the market price of a commodity. The literature presents various examples of flexible investment strategy in non-strategic (monopoly) and strategic (oligopoly) investments (e.g. Dixit and Pindyck (1994), Huisman and Kort (1999), Takashima et al. (2007), Masaaki and Takashi (2005)). The general idea is that the strategic option value of waiting is lower under preemption than in a monopoly. This is because preemption erodes the option value of waiting for more information. Firms in these models, and in many others in the real options literature, are assumed to be non-atomic, and have no real influence on the macro-structure of the market. The firms are price-takers – competing in perfectly competitive markets. As such, optimal investment strategies derived in these models do not naturally generalize to industries where product market competition partly determines the value of the investment.

The reason this is important is that most markets of interest are less than perfectly competitive, particularly if they involve large sunk costs. A firm can consistently earn economic rents in such markets which are non-existent in a perfectly competitive market. The ability to sustain these economic rents over the life (finite or infinite) of a real asset has very significant impact on the value of the asset. Hence, in a perfectly competitive market, the state value of an investment opportunity with an infinite life is determined by the maximum of the expected discounted cash inflows net initial investment outlay and the deferment value; on the other hand, the state value in an imperfectly competitive market is determined by the outcome of the game that describes the market structure, i.e. Nash-Cournot, Stackelberg, or Monopoly. Therefore, as observed, for example, by Smit and Ankum (1993) and Smit and Trigeorgis (1995), and others, an options-based approach to strategic investment needs to be considered from the perspective of competitive market structure.

The manner in which the burden of uncertainty has been introduced in the literature on investment decision models in imperfect markets warrants some consideration. Most authors introduce uncertainty in terms of insufficient strategic information, for example, as private information on cost functions or some other form of idiosyncratic shock that is peculiar to individual firms. The other common form is the generic uncertainty that affects all firms in an equal way e.g. the move of nature at the start of a game of imperfect information. Unfortunately, most of these papers appear to indiscriminately introduce uncertainty in one of these forms, either in the bid to retain tractability, or just to focus on a specific problem. This presents a number of questions that beg for answers. For example, which form of uncertainty best describes reality? Which form most influences outcomes in the games, vis-$\grave{a}$-vis first and second mover advantages? Or perhaps, is there an interactive effect that may be responsible for some of the counter-intuitive outcomes in existing models? Fortunately, the options valuation approach adopted in this paper allows us to address these concerns in a very comprehensive manner that buttresses the impact of uncertainty (in either, or both forms) in investment games. Our aim is to model irreversible investment decisions by firms into imperfectly competitive markets where firms have incomplete information about their competitors' costs, and demand is uncertain.

2 Related Literature

Optimal investment strategies or role-choice in strategic investment programs under uncertainty in imperfectly competitive markets have received some following in the literature. Gal-Or (1987) demonstrates the role of strategic uncertainty in an exogenous leader-follower model with segmental private information about the level of demand. The follower is able to accurately determine the leader's private information by inverting his output function. She shows that if the leader attempts to deviate from his equilibrium output (in order to "fool" the follower into presuming that market demand is low) and produces an output whose inverse image is outside the domain of definition of his signals (if this domain is bounded or has discontinuities), the follower may then believe he has more favourable information than the leader and therefore expand his output. This first-mover disadvantage under uncertainty is sustained even when the domain of definition of the leader's signals is unbounded and continuous. In effect, with partially correlated signals and moderate uncertainty, the leader supplying more, signals high demand, and then the follower supplies more as well. She also comments on the possibility of sustaining these first-mover disadvantages in an endogenous role choice model, but only gave specifications and did not pursue it further. Mailath (1993) presents a model that allows for endogenous sequencing in an asymmetric information game, where the more informed firm has the option to enter a market in one of two periods, but the less informed firm may only enter in the second period. The less informed firm is able to gain information about market profitability by observing the more informed firm's choice. The implications of signalling distortions in this game result in an equilibrium in which the more informed firm always enters in the first period, even when he could have earned a higher ex ante payoff by moving simultaneously with the uninformed firm. The focus is on the effect private information has on the choice of roles in an endogenous setting. However, having more information does not always confer leadership rights endogenously. The option available to each firm influences competitive strategies. Normann (2002), in fact, shows that when the less informed firm in Mailath (1993) has the opportunity to invest in period 1 as well, the Stackelberg equilibrium with the uninformed firm being the leader emerges as one of the equilibria surviving the $D1$ refinement.

It is curious that simultaneous-play outcome in the second period does not feature among the equilibria in these models. This may be due, in part, to the manner in which flexibility and uncertainty are modelled. Under generic uncertainty, with equal rights to enter the market at any one of two periods, Sadanand and Sadanand (1996) show that second period Cournot outcome persists in the set of equilibria, for all levels of risk in the distribution of demand.

It is pertinent to note that flexibility in these models carries no real option value, therefore, parametrization of generic uncertainty and/or private information does not actually make it unprofitable to enter the market at any one of the entry periods, however large the level of uncertainty might be. This is not the case when initial investment outlays have to be sunk before production choices are made. For while profits may be earned (considering interior solutions alone) within the periods of output choices, the overall discounted stream of payoff less the investment outlay might not just be suboptimal, but result in an outright loss. Furthermore, investment decisions faced by firms in the business world very often require such lumpy investment outlays. Take, for instance, a pharmaceutical firm's decision to develop a new drug. The R&D phase of any drug discovery is, characteristically, capital intensive. The firm cannot simply recover sunk R&D costs in this endeavour, should it become unproductive. Or, in the event of a successful discovery, it remains uncertain if the drug will pass pre-clinical trials for approval or exactly how long it will take to get approved. Furthermore, other pharmaceuticals might be coming up with a similar drug. These, and other industry-specific forms of uncertainty bear upon investment opportunities in the real world. It often instructs decision-makers to exercise caution when making investment decisions under these circumstances (as the first-mover advantages and disadvantages are both very real).

Smit and Trigeorgis (1995)'s study quantifies the trade-off between commitment and flexibility in an investment game that incorporates real options in a strategic industrial organization framework. By developing on Fudenberg and Tirole (1984), they show how demand uncertainty influences strategic interactions in environments where the investment is propriety or shared, competitor is tough or accommodating and whether the strategic variable is quantity or price. In the contrarian (quantity competition) case, the game proceeds in two stages. In the first stage, one of the firms has the opportunity to commit to a strategic capital investment that may give him a cost, or some other form of commitment, advantage over his competition in the second stage. The nature of this capital investment may make him a tough or accommodating incumbent in the second stage. The level of demand in the second stage is unknown, but follows a simple binomial process whose initial value is known in the first stage. In the second stage, both firms have the option to either invest in the first period or defer the decision to invest until the second period, and then decide to invest, or not, having observed the favourableness of the market condition. Equilibrium payoffs are earned in each of these periods and during the entire life of the investment. The value of the investment is derived from discounted cash flows less the initial investment outlay. They show how the level of demand in the first stage provides critical thresholds that determine the market structure in the second stage in the three cases where the strategic capital investment was, a) not made, b) shared, and c) proprietary.

The market environments in these cases can be thought of as being analogous to having, a) a less efficient pioneer firm, b) symmetric firms, and c) a more efficient pioneer firm, in a single-stage multi-period investment game. This analogy allows us to think of this model as one with endogenous sequencing, and see exactly what drives the choice of roles. The critical thresholds of demand in b involves a shift to the left of those in a, i.e. with equal standing in the market (as in b), the deferment threshold is lower for the pioneer firm than in a. Similarly, a higher demand level will, in case a, be required to offset the effect of the initial sunk cost and the pioneer firm's inefficiency, before entry may become profitable. Additionally, there exists a region of indeterminacy, where either firm may emerge as the leader or the follower. It is interesting, however, to note that in case c, for all levels of demand considered, this region collapses to a null set. Therefore, for all levels of observed demand, the pioneer, more efficient, firm never defers investment when the less efficient firm invests. One of the main contributions of this paper is to posit that cost asymmetry as depicted in the analogous framework above, under exogenous uncertainty, does not always preclude a more efficient firm from deferring when firms have private information about their cost function. More succinctly, cost asymmetry alone is not enough to determine endogenous roles under uncertainty. Competitive strategies in our model are driven by a pair of continuous functions of known market parameters (initial observed level of demand and the measure of uncertainty) whose co-domain is the set of types of the firms. The images of these functions determine critical values of types that specify the optimal strategy for each firm.

Interestingly, and contrary to the stipulations in Smit and Trigeorgis (1995) and Dewit and Leahy (2001) we find a non-degenerate region of types (even for some high levels of uncertainty), where a (anti)-coordination problem materializes. Cost asymmetry gives no leverage in this region, and it is never optimal to choose the same action. The optimal strategies are for either one of the firms to choose to move early while the other defers, and vice-versa. This will ordinarily be the case if marginal costs are not private information at the start of the game. Since each firm cannot observe its rival's cost, its ex ante scheme in this region will be in mixed strategies. By modelling private information into this analogous framework, we present a baseline model that allows us to establish how private information and exogenous uncertainty individually, and interdependently, influence the choice of strategies in investment games.

The rest of the paper is structured as follows. Section 3 presents the model and assumptions, and section 4 describes equilibrium outputs, payoffs and value of the investment in each continuation game. Section 5 discusses the sequencing of actions based on the observed parameters of the model and section 6 contains extensions of the analysis that consider outcomes in a world where a firm is able to credibly lie about his type, and how the observed coordination problem might be addressed. Some concluding remarks follow. All derivations of equilibrium outputs, payoffs and expected values of the investment are collected in Appendix A. Appendix B contains all proofs.

3 The Model

The aim of this paper is to model environments in which firms have limited information about the state of demand and the competitiveness of potential rivals in a market; firms' investment decisions, whilst irreversible once made, are not 'now or never'; and competition in the market upon entry is imperfect. To capture these key features we introduce a dynamic model in which demand evolves stochastically, firms have incomplete information about each other's variable costs, and in order to enter the market firms must undertake (large) sunk cost investment but have the freedom of choice over when to undertake this investment. As such, firms face two sources of uncertainty in the model when making their investment decision: uncertainty over the level of demand; and uncertainty about the competitiveness of the (potential) rival.

Suppose there are two risk-neutral firms $\mathcal{A}$ and $\mathcal{B}$ that are considering entering a market. The sunk cost expenditure required for each firm to enter the market is $\mathcal{K}$, which is the same for each firm and common knowledge. Conversely, there is incomplete information over variable costs: each firm has constant marginal cost drawn from a distribution F with support $[\underline{c}, \bar{c}]$, which is private information.

The investment game evolves over a sequence of periods $0, 1, 2, \ldots, n$, where period 0 is a pre-play period. Firms discount future payoffs by a factor δ. To capture the key feature of demand uncertainty but retain modelling simplicity, (inverse) demand is assumed to take the following structure, which is common knowledge. Inverse demand in period t is given by $P(Q_t, \Theta(t)) = \Theta(t) - Q_t$ where Q_t is the aggregate supply from participants in the market. In period 1 the level of demand is $\Theta(1) = \theta_1$ for sure, whilst in period 2 the level of demand evolves stochastically following a binomial distribution: $\Theta(2) = u\theta_1$ with probability p and $d\theta_1$ with probability $1 - p$, where $0 < d < 1 < u$ and $0 \leq p \leq 1$. From period 3 onwards demand has the same structure as period 2 demand. u and d will be parameterized to be such that, if firms know the realization of the level of demand when making their investment decision and it is $u\theta_1$, then any type of firm would choose to invest, whilst if it is $d\theta_1$ then any type of firm would choose not to invest. At the end of each of periods $1, 2, \ldots, n$ the market clears according to the total supply from market participants. From the perspective of period 0 when investment decisions are being made, this simple demand structure captures the key idea that firms may wish to delay investing until uncertainty about demand has been resolved.

We model the investment game between the firms using the structure of an endogenous timing game with observable delay, as introduced by

Hamilton and Slutsky (1990)[1]. In period 0 firms decide whether the undertake the investment to enter the market in period 1 (I), when future demand is subject to uncertainty, or whether to defer making this decision (D) until period 2 when demand uncertainty has been resolved. If either firm enters the market they also have to decide on the level of output to supply. There are several possible scenarios to consider that are introduced now and formally analyzed in the following section.

If both firms choose to invest in period 1, (I, I), then each firm is subject to both uncertainty over period 2 demand and their rival's cost. As such, in the first period firms compete in a game of Bayesian Cournot competition with demand level θ_1. At the end of this period the market clears and output and period 1 payoffs become common knowledge so each firm can deduce the others actual marginal cost. In period 2, therefore, the firms engage in a game of Cournot competition either with demand level $u\theta_1$ with probability p, or with demand level $d\theta_1$ with probability $1-p$, which is the same from period 3 onwards.

If both firms choose to defer the investment decision until period 2, (D, D), then nothing happens in period 1 and, the level of period 2 demand is realized as either $u\theta_1$ or $d\theta_1$ before firms decide whether or not to enter the market. If the level of demand is $d\theta_1$ then both firms choose not to invest in period 2 and in all subsequent periods. If the level of demand is $u\theta_1$ then both firms will be seeking to invest in the market after which they will engage in a game of Bayesian Cournot competition, but with updated beliefs about the support of their rival's marginal cost distribution since the (observed) act of delaying reveals information about what the rival firm's marginal cost cannot be. At the end of period 2 output choices and payoffs become common knowledge which reveals the rival's marginal cost, so from period 3 onwards firms engage in each period in a game of Cournot competition with demand level $u\theta_1$.

Consider now the case where one firm chooses to invest whilst the other defers their decision, (I, D) or (D, I). In these cases the investing firm enjoys being a monopolist in period 1. Since their output and period 1 payoff

[1] The extended game with observable delay is more suited to real-world cases where there is a lag between investment decisions and actual implementation. Hamilton and Slutsky (1990) first propose this game as one of two extended games (the other being the extended game with action commitment) that endogenize the choice of roles in a duopoly with complete information. In the extended game with observable delay, firms simultaneous choose their adoption period in a pre-play stage (similar to period 0 in our model) and announces their choice before choosing an action. It is assumed that the firms are committed to whatever adoption period they choose in the preplay stage. First period Cournot competition emerges when both firms have downward sloping reaction functions. Therefore, in a quantity competition, under further restrictions to payoff functions (as in Amir (1995)), the first mover advantage is eliminated.

becomes common knowledge at the end of period 1, the firm that deferred the investment decision learns the type of their rival. After the realization of period 2 demand, this firm must decide whether to invest or not. Since the firm that invested in period 1 has installed capital, it is assumed that this firm takes the role of a Stackelberg leader whilst the other firm is relegated to making output decisions after the firm that undertook the early investment, if it chooses to invest. If the level of demand transpires to be $d\theta_1$ then it will not invest at this stage. If, on the other hand, it is $u\theta_1$ then it may. If it does so it engages in a game of Stackelberg competition as the follower. Whilst the deferring firm learned the type of its rival from its period 1 activity, the early entrant doesn't have such accurate information over its rival's cost, but the act of delaying does reveal some information so it should update its belief about the support of the deferring firm's marginal cost distribution. As such, the Stackelberg game is a game of asymmetric information in which the follower is perfectly informed. At the end of period 2 output and payoffs become common knowledge and the incumbency advantage disappears so in period 3 and all subsequent periods either the firms engage in Cournot competition if the firm that deferred its investment decision invested, or the early entrant maintains its position as the monopolist if not.

The benefits from investing early are that the firm receives profits from production in the first period and may, if its rival defers its investment decision and subsequently enters (given that demand rises), gain the advantage of being a Stackelberg leader in the second period. However, by doing so it exposes itself to losses should the level of demand fall.

Formally, we define the game as: $\mathbb{G} = (N, S, \pi)$, $N = \{\mathcal{A}, \mathcal{B}\}$ is the set of players. The inverse demand function at any period, $t = 1, 2, ...$, is given by $P(Q_t, \Theta(t)) = \Theta(t) - Q_t$, Q_t $(= Q_{t,\mathcal{A}} + Q_{t,\mathcal{B}})$ is the aggregate output, (and $Q_{t,\mathcal{A}}$ and $Q_{t,\mathcal{B}}$ are compact, convex intervals in $\Re_+ \cup \{0\}$). The demand intercept, $\Theta(t)$, follows a simple binomial process with expected value $\mathbb{E}\Theta(t)$, variance σ^2, and state space in $\Re_+ \cup \{0\}$. The evolution of $\Theta(t)$ is similar to a Markov process[2] whose absorbing state is its value in period 2. Therefore, (see Figure 1) demand remains at its period-2 level for all subsequent periods after that.

Let $\mathcal{P} = \{I, D\}$, which is the set of available actions in period 0. I corresponds to sinking the investment cost K to enter the market in period 1. D corresponds to delaying the investment decision to period 2 at which point the uncertainty about demand will be resolved, in which case (due to the parameter restrictions we impose) the firm will invest to enter the

[2]In this setup, $Pr(\Theta(3) = \delta\theta_1 | \Theta(2) = \nu\theta_1, \Theta(1) = \theta_1) = Pr(\Theta(3) = \delta\theta_1 | \Theta(2) = \nu\theta_1)$, where δ and ν take values u or d and $0 < d < 1 < u$.

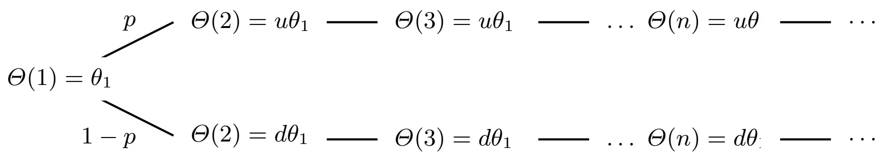

Figure 1: **Binomial Process Depicting Stage 2 Evolution of Demand**

market if and only if market demand is favourable ($\Theta(2) = u\theta_1$). The set of strategies for player $i = A, B$ is $S_i = \mathcal{P} \times \Xi_i$, where Ξ_i is the family of functions that map ζ_i into $Q_{t,i}$ for each period $t = 1, 2, \ldots$; and ζ_i is the set containing $\{(I, I), (I, D), (D, I) \times Q_{t,j}, (D, D)\}$. Define $s_i = (\sigma, \chi_{t,i}) \in S_i$, where $\sigma \in \mathcal{P}$ and $\chi_{t,i} \in \Xi_i$. Firm i's pure strategy in any period t involves the mapping $\chi_{t,i} : \zeta_i \to Q_{t,i}$, where $t = 1, 2, \ldots$. Having assumed that demand takes the same level as in period 2 from period 3 onwards, and noting that any relevant information will have been revealed by the beginning of period 3, it suffices to derive expressions of $\chi_{t,i}$ only for $t = 1, 2, 3$.

The appropriate solution concept for the extended game in this model, which ultimately determines the value of the investment, is that of sequential equilibrium introduced by Kreps and Wilson (1982). The presence of non-singleton information sets in games of incomplete information of this kind, precludes subgame perfection, as there are no proper subgames. A sequential equilibrium requires sequential rationality in the strategies, and that the beliefs held by a firm at each information set it finds itself are consistent with the strategy that got it there. Sequential rationality in our game requires that, at each information set, the output choice of firm $\mathcal{A}$ is a best response to the output choice of firm $\mathcal{B}$, given firm $\mathcal{A}$'s belief about the support of the marginal cost distribution of the firm $\mathcal{B}$. Also, the belief held by firm $\mathcal{A}$ about the support of the marginal cost distribution of firm $\mathcal{B}$, at any information set consistent with the chosen strategy, must be derived by Bayes' rule. In contrast to the perfect Bayesian equilibrium concept, sequential equilibrium specifies how a firm should form beliefs when it reaches an out-of-equilibrium information set. Because firms announce their chosen entry times in period 0, out-of-equilibrium signals may be sent in terms of outputs as well as adoption periods. Therefore, to accurately obtain the expected value of the investment for period 0 choice, we require sequential rationality and consistency in the expected output choices of the firms in all periods. Thus, a strategy profile $(\lambda_1(s_{\mathcal{A}}), \lambda_2(s_{\mathcal{B}}))$, for this game, is a sequential equilibrium if for any $s_{\mathcal{A}}$ and $s_{\mathcal{B}}$, such that for all $(a, b) \in \mathcal{P}$, there exists a probability distribution $\tilde{F}$ over $c_{(\cdot)}$ such that (a, b) is chosen to maximize expected profits (and hence, the value of the investment) given $\chi_t^{\mathcal{A}}$ and $\chi_t^{\mathcal{B}}$; also, given the choice of adoption periods and given the firms' beliefs at each information set, $\chi_t^{\mathcal{A}}$ and $\chi_t^{\mathcal{B}}$ are chosen optimally; where the beliefs about the support of the distribution of each other's marginal cost held by the firms at each information set are obtained by Bayes' rule.

4 Continuation Game Analysis

At the beginning of period 0, firm $\mathcal{A}(\mathcal{B})$ faces only one source of uncertainty regarding its decision in period 1, i.e. the marginal cost of firm $\mathcal{B}(\mathcal{A})$, $c_{\mathcal{B}(\mathcal{A})}$. In period 2, however, (looking forward from period 0), each firm faces two sources of uncertainty, i.e., $\Theta(2)$ and its rival's marginal cost. The realization of $\Theta(2)$ is common knowledge at the beginning of period 2, and marginal costs are revealed after period 1 or 2, *or not at all* (depending on period-0 choices and the realization of $\Theta(2)$). The parameters u and d governing the evolution of $\Theta(2)$ are related to the variance by: $u = \exp(\sigma\sqrt{t})$ and $d = \exp(-\sigma\sqrt{t})$ (see Cox and Rubinstein (1979)). Payoffs in each period represent cash flows generated from output competition in the product market.

The investment decision of each firm, in period 0, depends on its calculation of the expected value of the investment opportunity at each entry period $t = 1, 2, ...$, given the possible actions of the rival firm and its expectation of $\Theta(t)$. This will be the entry choice that produces, in expectation, the highest value of the investment that exceeds the initial investment outlay by an amount equal to the value of keeping the investment option alive. The investment value, $\vartheta_{(\cdot)}^{(a,b)}$, for each possible outcome in period 0, is given by the sum of the expected profits in period 1 and the expected discounted cash flows of all future periods, minus the investment outlay, $\mathcal{K}$.

Simultaneous-move Equilibria

If both firms choose to sink $\mathcal{K}$ in period 1, they play a Bayesian-Cournot game in the early production period, and the basic Cournot in the second period and all other periods after that. Let $\mathbb{E}_o(\cdot)$ denote the expected value of its argument given the information available in period 0 and let $q_{t,\mathcal{A}} \in Q_{t,\mathcal{A}} \subseteq \Re_+ \cup \{0\}$ denote the output choice for firm $\mathcal{A}$ in period t. It follows that, the firm may not find it profitable to produce outputs for all realizations of its marginal cost. As a matter of fact, we assume (as in Hurkens (2012)) that there exist some realizations of $c_{\mathcal{A}}$) for which the equilibrium output is 0. Therefore, firm $\mathcal{A}$ produces $q^*_{1,\mathcal{A}} = \frac{1}{6}(2\theta_1 - \ddot{c} - 3c_{\mathcal{A}})$ in period 1 and earns $\pi^{*(I,I)}_{1,\mathcal{A}} = \frac{1}{36}(2\theta_1 - \ddot{c} - 3c_{\mathcal{A}})^2$, where $\ddot{c} = \int_{\underline{c}}^{\alpha} c\,\mathrm{d}F(c)$. This integral is taken over an updated support of the marginal cost distribution, i.e., if $\mathcal{A}$'s rival has chosen to invest early, $\mathcal{A}$ conjectures that its rival's marginal cost must be below some threshold α above which he will rather delay if $\mathcal{A}$ invests. Therefore, $\ddot{c}$ represents firm $\mathcal{A}$'s mean belief about his rival's marginal cost. In period 2, having deduced its rival's (firm $\mathcal{B}$'s) marginal cost, firm $\mathcal{A}$ produces $q^*_{2,\mathcal{A}} = \frac{1}{3}(\nu\theta_1 - 2c_{\mathcal{A}} + c_{\mathcal{B}})$ and earns $\pi^{*(I,I)}_{2,\mathcal{A}} = \frac{1}{9}(\nu\theta_1 - 2c_{\mathcal{A}} + c_{\mathcal{B}})^2$, where ν is either u or d, given the realization of $\Theta(2)$. These are *ex post* outputs and payoffs. *Ex ante*, the expected value of period-2 payoff is $\mathbb{E}_o(\pi^{*(I,I)}_{2,\mathcal{A}}) = \frac{1}{9}(\sigma^2 + \eta^2_{\ddot{c}} + (pu\theta_1 + (1-p)d\theta_1 + \ddot{c} - 2c_{\mathcal{A}})^2)$ (see Appendix A for derivation). $\eta^2_{\ddot{c}}$ is the variance of the marginal cost derived from the updated support of its distribution. Period 3 and subsequent periods' payoffs follow accordingly. The ex ante expected value of the investment to this firm is therefore,

$$\vartheta^{(I,I)}_{\mathcal{A}} = \gamma_1 \mathbb{E}_o(\pi^{*(I,I)}_{1,\mathcal{A}}) + \gamma_2 \mathbb{E}_o(\pi^{*(I,I)}_{2,\mathcal{A}}) - \mathcal{K}, \tag{1}$$

where $\gamma_1 = 1/(1+\rho)$ and $\gamma_2 = \gamma_1/\rho$.

If both firms choose period 2, i.e. $(a, b) = (D, D)$, no production takes place in period 1. Output choices are made based upon the observed realization of $\Theta(\mathit{2})$, and only when it is $u\theta_1$. *Ex ante*, this happens with probability p, illustrating the fact that firms are not obligated to exercise their option to invest if they find it worthless. Marginal costs are still private information, but by choosing to defer, a firm, say $\mathcal{A}$, reveals information about its type. Its rival, firm $\mathcal{B}$, updates its belief about the support of the distribution of firm $\mathcal{A}$'s marginal cost, i.e. $\mathcal{B}$ believes that $\mathcal{A}$'s true marginal cost must be greater than the lower bound of the prior support of the distribution. The updated lower bound corresponds to the value of $c_{\mathcal{A}}$, say β, that makes it unprofitable for $\mathcal{A}$ to invest early if $\mathcal{B}$ defers. Bayesian updating, therefore, requires $\mathcal{B}$ to put probability zero on all types of $\mathcal{A}$ below β. Let $\hat{c}$ be each firm's updated mean belief of their marginal costs (i.e. $\hat{c} = \int_{\beta}^{\bar{c}} c\,\mathrm{d}F(c)$).

A Bayesian-Cournot game ensues in the second period, while the basic Cournot is played in subsequent periods. The ex ante expected value of the investment is given by,

$$\vartheta^{(D,D)}_{\mathcal{A}} = p\left(-\gamma_1\mathcal{K} + {\gamma_1}^2\mathbb{E}_o(\pi^{*(D,D)}_{2,\mathcal{A}}) + \gamma_1\gamma_2\mathbb{E}_o(\pi^{*(D,D)}_{3,\mathcal{A}})\right). \tag{2}$$

γ_1 and γ_2 are as previously specified. Period-2 payoff, $\mathbb{E}_o(\pi^{*(D,D)}_{2,\mathcal{A}})$, is $((2u\theta_1 + \hat{c} - 3c_{\mathcal{A}})^2)/36$, and expected payoffs for each of the subsequent periods after is $((u\theta_1 + \hat{c} - 2c_{\mathcal{A}})^2)/9$.

Sequential-move Equilibria

Choosing (I, D), as with (D, D), also reveals information about the type of each firm. Suppose $\mathcal{A}$ chooses to enter early, and $\mathcal{B}$ defers, by choosing to defer, $\mathcal{B}$ sends a signal about the lower bound of the support of its marginal cost distribution, and $\mathcal{A}$ updates its belief about its rivals marginal cost accordingly. In the asymmetric information game played in period 2, $\mathcal{A}$'s marginal cost is revealed, but $\mathcal{A}$ still has incomplete information about its rival. However, $\mathcal{A}$ believes that $\mathcal{B}$'s true marginal cost must lie in the interval $[\alpha, \bar{c}] \subset [\underline{c}, \bar{c}]$, where α is the infimum of the the set of marginal costs for which $\mathcal{B}$ finds it unprofitable to invest early if $\mathcal{A}$ invests early. Let $\check{c}$ denote $\mathcal{A}$'s mean belief about the marginal cost of $\mathcal{B}$ based upon the updated support of the marginal cost. The first-mover's expected payoff stream is as follows: $\mathbb{E}_o(\pi^{*M}_{1,\mathcal{A}}) = ((\theta_1 - c_{\mathcal{A}})^2)/4$, $\mathbb{E}_o(\pi^{*(I,D)}_{2,\mathcal{A}}) = (p(u\theta_1 + \check{c} - 2c_{\mathcal{A}})^2)/8 + ((1-p)(d\theta_1 - c_{\mathcal{A}})^2)/4$, and $\mathbb{E}_o(\pi^{*(I,D)}_{3,\mathcal{A}}) = p((u\theta_1 + \check{c} - 2c_{\mathcal{A}})^2)/9 + (1-p)(d\theta_1 - c_{\mathcal{A}})^2)/4$. The corresponding expected value for the first-mover is

$$\vartheta^{(I,D)}_{\mathcal{A}} = \Big(\gamma_1 \mathbb{E}_o(\pi^{*M}_{1,\mathcal{A}}) + \gamma_1^2 \mathbb{E}_o(\pi^{*(I,D)}_{2,\mathcal{A}}) + 9\gamma_1\gamma_2 \mathbb{E}_o(\pi^{*(I,D)}_{3,\mathcal{A}})\Big) - \mathcal{K}. \quad (3)$$

Firm $\mathcal{B}$'s expected payoff stream if, and when, it enters is $\mathbb{E}_o(\pi^{*(I,D)}_{2,\mathcal{B}}) = ((u\theta_1 - 2c_{\mathcal{B}} + 2\tilde{c} - \check{c})^2)/16$ and $\mathbb{E}_o(\pi^{*(I,D)}_{3,\mathcal{B}}) = ((u\theta_1 - 2c_{\mathcal{B}} + \tilde{c})^2)/9$, where $\tilde{c} = \int_{\underline{c}}^{\beta} c \, \mathrm{d}F(c)$ is $\mathcal{B}$'s mean belief of about $\mathcal{A}$'s marginal cost when he observes that $\mathcal{A}$ has chosen to invest early. The payoffs for all periods after period 3 are equivalent to that of period 3. Let the superscripts M indicates monopoly rent, then the follower's expected value for the investment is,

$$\vartheta^{(I,D)}_{\mathcal{B}} = p\left(-\gamma_1\mathcal{K} + {\gamma_1}^2 \mathbb{E}_o(\pi^{*(I,D)}_{2,\mathcal{B}}) + \gamma_1\gamma_2 \mathbb{E}_o(\pi^{*(I,D)}_{3,\mathcal{B}})\right). \quad (4)$$

When roles are reversed, these expected values are simply reversed as well.

It is not inconceivable that the sequential equilibria in this game may indeed be separating and perfectly revealing. In fact, with the increasing stringency on the regulations for financial and accounting reports, it might be difficult and/or illegal for a firm to misrepresent information about its costs and profits. The first part of our analyses assumes such environment. Therefore, in the sequential play outcome, for instance, the follower in period 2 observes first period (monopoly) payoff of the leader and can accurately infer his marginal cost (we have assumed the market clears after each period). Also, the follower's *ex ante* mean belief about the marginal cost of the leader uses the distribution's full support, hence $\tilde{c}$, in its best response function and in the derivation of the investment value, as seen above. In effect, it does not matter what the first mover's exact cost is, in expectation, the follower's reaction is the same. We assume that, should demand rise in period 2, the market will be shared in a Stackelberg fashion, and also, that there exists a first-mover disadvantage should demand fall in period 2. In Section 6, we illustrate, as a possible extension to this model, how the possibility of misrepresenting one's type may change or influence outcomes in this game.

5 Endogenous Timing

In this section we analyze the timing decisions of firms in the light of the analysis undertaken above. We proceed as follows. Endogenous timing in the game is based on type, i.e. marginal cost. Therefore, the marginal cost draws of each firm at the beginning of the game determines what outcomes emerge endogenously. Figure 2 represents the normal form of the extended game.

		Firm $\mathcal{B}$	
		I	D
Firm $\mathcal{A}$	I	$\vartheta_{\mathcal{A}}^{(I,I)}, \vartheta_{\mathcal{B}}^{(I,I)}$	$\vartheta_{\mathcal{A}}^{(I,D)}, \vartheta_{\mathcal{B}}^{(I,D)}$
	D	$\vartheta_{\mathcal{A}}^{(D,I)}, \vartheta_{\mathcal{B}}^{(D,I)}$	$\vartheta_{\mathcal{A}}^{(D,D)}, \vartheta_{\mathcal{B}}^{(D,D)}$

Figure 2: **Normal-Form Representation of the Game.**

Lemma 1. *For moderate levels of uncertainty, u:*

(i) $\exists\ c_1 \in [\underline{c}, \bar{c}] \ni \vartheta^{(I,I)} = \vartheta^{(D,I)}$, *and*

(ii) $\exists\ c_2 \in [\underline{c}, \bar{c}] \ni \vartheta^{(I,D)} = \vartheta^{(D,D)}$

The variance of demand provides a measure of the level of uncertainty investors face. In binomial games of this kind, we are able to represent the variance in terms of the model parameter u, i.e. $\sigma^2 = (\ln(u))^2$. It therefore follows, that very high values of u indicates high levels of uncertainty, and so, high cost firms are much more wary of committing early. Very low values of u, on the other hand, diminishes the option value of waiting, in this case, some high cost firms might find it optimal to enter early. To determine optimal choices for values of u that fall within these extremes, we study the behaviour of the investment value functions. Firstly, it is easy to see that $\vartheta^{(I,I)}$, $\vartheta^{(D,I)}$, $\vartheta^{(I,D)}$ and $\vartheta^{(D,D)}$ are strictly convex, monotone decreasing functions of c on the interval $\mathbb{I} := [\underline{c}, \bar{c}]$. This is because these value functions are monotone transformations of the individual equilibrium quantities derived within each period, which are themselves strictly decreasing and convex in the marginal cost. Secondly, and they satisfy the following conditions:

a) $\vartheta^{(I,I)}(0) > \vartheta^{(D,I)}(0)$, $\vartheta^{(I,D)}(0) > \vartheta^{(D,D)}(0)$ and

b) $\left|\dfrac{\partial\vartheta^{(I,I)}}{\partial c}\right| > \left|\dfrac{\partial\vartheta^{(D,I)}}{\partial c}\right|$ and $\left|\dfrac{\partial\vartheta^{(I,D)}}{\partial c}\right| > \left|\dfrac{\partial\vartheta^{(D,D)}}{\partial c}\right|$ on $\mathbb{I}$.

Let $f_1 = |\partial\vartheta^{(I,I)}/\partial c| - |\partial\vartheta^{(D,I)}/\partial c|$ and $f_2 = |\partial\vartheta^{(I,D)}/\partial c| - |\partial\vartheta^{(D,D)}/\partial c|$. f_1 and f_2 are simple linear monotonic decreasing function of c, and are positive for all values of c on $\mathbb{I}$ such that $\vartheta^{(I,I)}$ and $\vartheta^{(I,D)}$ are non-negative (see Appendix B for details);

c) if ξ_0, ξ_1, ε_0 and ε_1 are respectively the "*zeros*" of $\vartheta^{(I,I)}$, $\vartheta^{(D,I)}$, $\vartheta^{(I,D)}$ and $\vartheta^{(D,D)}$ on $\mathbb{I}$, then $\xi_0 < \xi_1$, $\varepsilon_0 < \varepsilon_1$ and $\xi_0 < \varepsilon_0$ *a.s.*

The roots ξ_0, ξ_1, ε_0 and ε_1 of the value functions are themselves functions of the beliefs held by the firms during the course of the game, i.e. α and β (see Appendix B for details). Now, let $g_1(\alpha, \beta) = \xi_0 - \xi_1$ and $g_2(\alpha, \beta) = \varepsilon_0 - \varepsilon_1$; we show in Appendix B that g_1 and g_2 are negative everywhere on $\mathbb{I}$ for all values of α and β. Furthermore, $\xi_0 < \varepsilon_0$ on $\mathbb{I}$, and so, $\vartheta^{(I,I)}$ will always be less than $\vartheta^{(I,D)}$.

Now, since the functions $\vartheta^{(I,I)}$, $\vartheta^{(D,I)}$, $\vartheta^{(I,D)}$ and $\vartheta^{(D,D)}$ satisfy conditions (a) and (b), along with strict convexity and monotonicity, then, there must be two distinct points of intersection in $\mathbb{I}$ where $\vartheta^{(I,I)} = \vartheta^{(D,I)}$ and $\vartheta^{(I,D)} = \vartheta^{(D,D)}$ respectively.

Lemma 2. *The interval* $[c_1, c_2]$ *is non-degenerate.*

It is easily observed that if $\xi_0 < \varepsilon_0$ as in **L**emma 1, the points c_1 and c_2, corresponding to the intersections of the pair of value functions, do not coincide, moreover, $c_1 < c_2$.

Figure 3 illustrates the critical regions on the interval $\mathbb{I}$, within which one or more of the outcomes in period 0 is dominant. These outcomes are summarized in Table 1, and presented formally, in the propositions that follow:

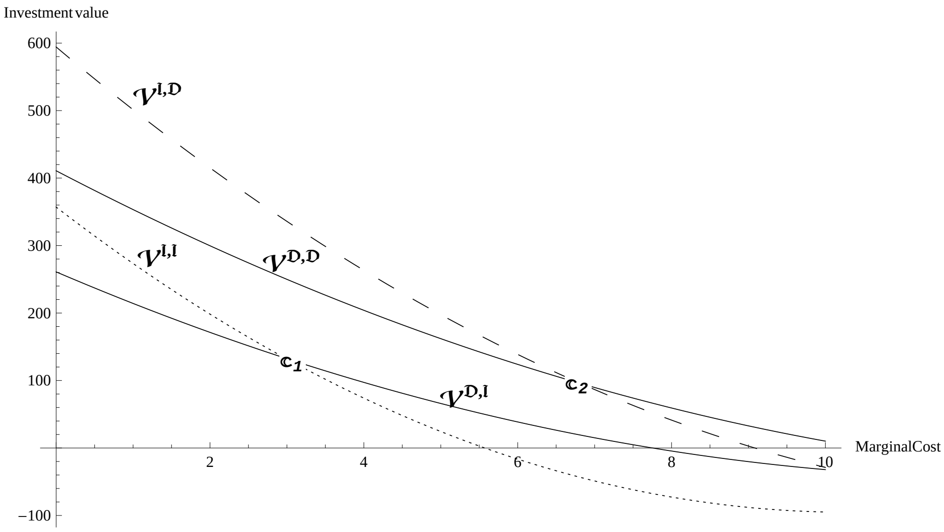

Figure 3: **Investment Values for Period-0 Outcomes**

Firm $\mathcal{A}$/Firm $\mathcal{B}$	$c_{\mathcal{B}} < c_1$	$c_1 < c_{\mathcal{B}} < c_2$	$c_{\mathcal{B}} > c_2$
$c_{\mathcal{A}} < c_1$	$\mathcal{I}, \mathcal{I}$	$\mathcal{I}, \mathcal{D}$	$\mathcal{I}, \mathcal{D}$
$c_1 < c_{\mathcal{A}} < c_2$	$\mathcal{D}, \mathcal{I}$	$\mathcal{I}, \mathcal{D}; \mathcal{D}, \mathcal{I}$	$\mathcal{I}, \mathcal{D}$
$c_{\mathcal{A}} > c_2$	$\mathcal{D}, \mathcal{I}$	$\mathcal{D}, \mathcal{I}$	$\mathcal{D}, \mathcal{D}$

Table 1:**Belief-based Equilibria**

Proposition 1. *(Simultaneous-play Equilibrium). If both firms independently draw marginal costs in the intervals* $[\underline{c}, c_1)$ *and* $(c_2, \bar{c}]$*, the dominant strategies are first- and second-period simultaneous play equilibria respectively.*

Proposition 2. *(Sequential-play Equilibrium). If the firms' marginal cost draws lie in separate regions delineated by* c_1 *and* c_2*, a sequential-play equilibrium emerges as the dominant strategy; moreover, the more efficient firm emerges endogenously as the first-mover.*

Proposition 3. *When both firms independently draw marginal costs in the interval* (c_1, c_2)*, ex ante, there is a mixed strategy in which firms randomize over* $\mathcal{I}$ *and* $\mathcal{D}$*, and an (anti)-coordination problem ensues when they use the same mixed strategy.*

Proofs for **Proposition** 1 and 2 follow directly from **Lemma** 1 and **Lemma** 2, and the summary in Table 1. See Appendix B for proof of **Proposition** 3.

6 Discussion

As is evidenced from Table 1, the choice of roles in this game is governed by the firms' types through the critical thresholds c_1 and c_2. These thresholds are parametrized by the level of demand uncertainty and the beliefs of the firms. Quite unlike Smit and Trigeorgis (1995), investment timing is not solely determined by the level of demand, and more so, a more efficient firm may not necessarily emerge endogenously as the first mover (as in case "c^*" of the analogous framework of Smit and Trigeorgis (1995) , where the strategic investment is proprietary). The belief-based equilibria we have derived in this paper show that what outcomes emerge endogenously depends primarily on the side of the critical thresholds c_1 and c_2, the firms' marginal costs lie, the values of which are estimatable at the start of the game. Simple cost(type) asymmetry is insufficient, therefore, to describe outcomes when there is private information about types under demand uncertainty. The import of private information in investment games of this nature is clearly non-trivia. For example, if both firms draw marginal costs in the intervals described in **Proposition** 1, then symmetric or not, the optimal outcome is simultaneous investment. On the other hand, no matter how close $c_{\mathcal{B}}$ might be to $c_{\mathcal{A}}$, if $c_{\mathcal{A}} < c_1 < c_{\mathcal{B}}$, then firm $\mathcal{B}$'s dominant strategy is to delay. A leader-follower equilibrium emerges endogenously only when their types are sufficiently asymmetric as in **Proposition** 2. **Proposition** 3 shows an outcome that differ from that of Smit and Trigeorgis (1995) where a more efficient firm never chooses to defer if its rival invests. As we have shown, this interval is non-degenerate and does not collapse into a null set as their model specifies. Ex post, we may, therefore, find a more efficient firm emerging as the second-mover. The intuition behind this is that when a firm draws a type that falls in this interval, it realizes that its dominant strategy is to defer if its rival invests, and to invest, if its rival defers. Unlike in the intervals $[\underline{c}, c_1)$ and $(c_2, \bar{c}]$ where a firm's dominant strategy is to invest and defer respectively, irrespective of what its rival chooses; in the interval (c_1, c_2), each firm's optimal strategy is conditional on its rival choosing the exact opposite. But firm in this interval has not knowledge of its rival's marginal cost, and therefore, must use a mixed strategy. We show in the Appendix A that if both firms use the same mixed strategy the (anti)-coordination problem will still persists.

A firm's dominant strategies evolve across the type space, and the value of its option to defer investment increasing with the type it draws. For a firm with sufficiently large marginal cost, deferring commitment decision until the second period, at which time some, or all, of the uncertainty elements of the game are resolved, becomes increasingly preferred. As the option value increases, the first-mover advantage decreases. Also, $\partial c_1/\partial u < 0$ and $\partial c_2/\partial u < 0$ for any given level of demand, meaning that the sub-spaces of the type-space where a first-mover advantage exists shrink with uncertainty for the a more efficient firm.

7 Concluding Remarks

This paper presents a baseline investment game of incomplete information under uncertainty, where product market competition influences the state value of the investment, and timing is endogenously determined. We have shown that cost asymmetry is insufficient criterion upon which outcomes may be determined when there is private information over types (see Dewit and Leahy (2001), where cost asymmetry is used to determine market structure in an investment game with observable delay under demand uncertainty). Consequently, a first-mover advantage may not exist for sufficiently large draw of types, and when it does, it diminishes with type and with the level of uncertainty. The sequential equilibrium concept employed ensures that even when a firm defies the requirement of the game with observable delay (i.e. does not commit to its period 0 choice), the rival firm, finding itself on an off-equilibrium path is able to form beliefs consistent with how he may have arrived at this information set and update its belief about the defecting firm's type appropriately.

An important assumption that drives the results in this paper is that the market clears in each period of production, and each period's payoffs are observed before the next period's output choices are made. However, an immediate extension to our model is to consider how the game might evolve if the first mover is able to credibly mask his type. In order to pursue this ideal, the firms may only be able to observe outputs and not payoffs in this particular world. This may be considered under two categories. One, is where the first mover can mask his type and the follower believes it. Two, is where the follower knows that the first mover may mask his type, and then modify his actions accordingly. Interestingly, the outcomes in both cases are similar. In the first case, we find that if the first mover shades his cost by the factor ϱ, where $0 < \varrho < 1$, then the benefit he derives from "fooling", as it were, his rival into considering him more efficient in the Cournot game in period 3 outweighs the temporary loss in revenue he would experience in period 1 and 2. The net present value of this benefit is concave in the amount, ϱ, with which he shades his cost. In the second case, even though the potential follower realizes that the first mover may be lying about his type, as a Stackelberg follower in period 2, his optimal action is to best respond to the leaders output, whether it be a lie or not. Having deduced the cost relevant to the leaders output in the second period, the third period Cournot game proceeds accordingly at which point the true marginal cost of the first mover is then revealed. The Cournot competition in subsequent periods progress as with when both firms have been truthful all along.

Our conjecture is that, this additional incentive to move first (being able to benefit from lying) may not qualitatively alter the specifications of our game. The quantitative implication may be that it reduces the values of the critical thresholds that determine the outcomes of the game. We leave the determination of the value of the investment and the specifics of the ex post market structures that emerge endogenously when masking ones type is possible as the subject of future research.

Appendix A

Equilibrium Outputs, Payoffs and Investment Value

Simultaneous-move Equilibria

If at the decision period (period-0), both firms choose to sink the investment outlay, $\mathcal{K}$, in period 1, i.e. $(a, b) = (I, I)$, then a Bayesian-Cournot game ensues in period 1 since marginal costs are still private information. Furthermore, as we have not made the common assumption that $q_t(c) > 0$ for all realizations of c, therefore, there exists some realizations of c for which the equilibrium output is 0 (see Hurkens (2012)). The firms, being ex ante symmetric, and having independently drawn their marginal costs, will be maximizing expected profits over an adjusted support of the marginal cost distribution as follows (we show this for firm $\mathcal{A}$):

$$\max_{q_{1,\mathcal{A}}(c_{\mathcal{A}})} \mathbb{E}(\pi_{1,\mathcal{A}}^{(I,I)}) = \max_{q_{1,\mathcal{A}}(c_{\mathcal{A}})} \left(q_{1,\mathcal{A}}\left(\theta_1 - q_{1,\mathcal{A}} - \mathbb{E}(q_{1,\mathcal{B}}(c_{\mathcal{B}})) - c_{\mathcal{A}}\right)\right) \tag{A1}$$

For notational convenience, we will be using $q_{1,\mathcal{A}}$ for $q_{1,\mathcal{A}}(c_{\mathcal{A}})$ with the understanding that output decisions are functions of drawn marginal cost values. Firm $\mathcal{A}$'s expectation of $\mathcal{B}$'s output in (A1) is based on the belief that there is a marginal cost threshold value, α, above which $\mathcal{A}$, itself, will find it unprofitable to enter the market in period 1 (and hence produce zero output). As the firms are ex ante symmetric, firm $\mathcal{A}$ believes this to be true of its rival, firm $\mathcal{B}$. It therefore follows from (A1) that

$$\hat{q}_{1,\mathcal{A}} = \frac{1}{2}\left(\theta_1 - \mathbb{E}(q_{1,\mathcal{B}}(c_{\mathcal{B}})) - c_{\mathcal{A}}\right) \quad \text{and} \quad \hat{q}_{1,\mathcal{B}} = \frac{1}{2}\left(\theta_1 - \mathbb{E}(q_{1,\mathcal{A}}(c_{\mathcal{A}})) - c_{\mathcal{B}}\right) \tag{A2}$$

Ex ante, $\mathbb{E}(\hat{q}_{1,\mathcal{A}}) = \mathbb{E}(\hat{q}_{1,\mathcal{B}})$. Hence, $\mathbb{E}(\hat{q}_{1,\mathcal{B}}) = \frac{1}{2}\left(\theta_1 - \mathbb{E}(q_{1,\mathcal{A}}(c_{\mathcal{A}})) - \mathbb{E}(c_{\mathcal{B}})\right)$. Note that $\mathbb{E}(c_{\mathcal{B}}) = \int_{\underline{c}}^{\alpha} c_{\mathcal{B}} \mathrm{d}F(c_{\mathcal{B}}) := \ddot{c}$, where $\alpha \in (\underline{c}, \bar{c}]$. Expected outputs for independent draws of marginal costs in this interval is

$$\mathbb{E}(q_{1,\mathcal{B}}) = \tfrac{1}{2}\left(\theta_1 - \mathbb{E}(q_{1,\mathcal{A}}(c_{\mathcal{A}})) - \ddot{c}\right). \tag{A3}$$

Since expected outputs for both firms are taken over the same support, firm $\mathcal{B}$ should therefore expect that $\mathbb{E}(q_{1,\mathcal{B}}) = \mathbb{E}(q_{1,\mathcal{A}})$. Using this in (A3), we have

$$\mathbb{E}(q_{1,\mathcal{B}}) = \frac{\theta_1 - \ddot{c}}{3} = \mathbb{E}(q_{1,\mathcal{A}}) \tag{A4}$$

Substituting (A4) for the expectations in (A2), the equilibrium outputs for the firms are hence,

$$q^*_{1,\mathcal{A}} = \frac{1}{6}\left(2\theta_1 - \ddot{c} - c_{\mathcal{A}}\right) \qquad \text{and} \qquad q^*_{1,\mathcal{B}} = \frac{1}{6}\left(2\theta_1 - \ddot{c} - c_{\mathcal{B}}\right) \tag{A5}$$

The corresponding expected equilibrium payoffs are,

$$\pi^{*(I,I)}_{1,\mathcal{A}} = \tfrac{1}{36}\left(2\theta_1 - \ddot{c} - c_{\mathcal{A}}\right)^2 \quad \text{and} \quad \pi^{*(I,I)}_{1,\mathcal{B}} = \tfrac{1}{36}\left(2\theta_1 - \ddot{c} - c_{\mathcal{B}}\right)^2. \tag{A6}$$

At the beginning of period 2, outputs and payoffs from period 1 would have been observed, therefore, the true marginal costs of each firm can be deduced. The demand level for period 2 is also observed at the start of the period. Having chosen (I, I) in period 0, the firms compete *a là* Cournot from period 2 onwards, having full information about the market parameters. Equilibrium outputs and payoffs in period 2 are

$$q^*_{2,\mathcal{A}} = \tfrac{1}{3}\left(\theta_2 - 2c_{\mathcal{A}} + c_{\mathcal{B}}\right) \qquad \text{and} \qquad \pi^{*(I,I)}_{1,\mathcal{A}} = \tfrac{1}{9}\left(\theta_2 - 2c_{\mathcal{A}} + c_{\mathcal{B}}\right)^2. \tag{A7}$$

Note, however, that these are ex post outputs and payoffs, so θ_2 in (A7) is either $u\theta_1$ or $d\theta_1$ (because Θ follows a binomial process from period 2). In order to derive the expected net present value of the investment, we require ex ante expectations of the payoffs as follows

$$
\begin{aligned}
\mathbb{E}_o(\pi_{2,\mathcal{A}}^{*(I,I)}) &= \mathbb{E}_o\left[\frac{1}{9}\left(\Theta^{(2)} - 2c_{\mathcal{A}} + c_{\mathcal{B}}\right)^2\right] \\
&= \frac{1}{9}\left(\mathrm{Var}(\Theta^{(2)} + (c_{\mathcal{B}} - 2c_{\mathcal{A}})) + \left(\mathbb{E}_o\left(\Theta^{(2)} - 2c_{\mathcal{A}} + c_{\mathcal{B}}\right)\right)^2\right) \qquad \text{(A8)} \\
&= \frac{1}{9}\left[\sigma^2_{(\Theta^{(2)})} + \eta_{\ddot{c}} + (pu\theta_1 + (1-p)d\theta_1 + \ddot{c} - 2c_{\mathcal{A}})^2\right].
\end{aligned}
$$

Where $\eta_{\ddot{c}}$ is the variance of the marginal cost distribution over the adjusted support. At period 0, the level of demand in period 2 is not known with certainty, and the marginal cost of the firm's rival is still private information, hence the expectations in (A8). Period 3, and subsequent periods' payoffs follow (A8), therefore, the expected net present value of the investment at the time of decision is

$$
\begin{aligned}
\vartheta_{\mathcal{A}}^{(I,I)} &= \frac{1}{1+\rho}\left(\frac{1}{36}\left(2\theta_1 - \ddot{c} - 3c_{\mathcal{A}}\right)^2\right) \\
&\quad + \frac{1}{9\rho(1+\rho)}\left(\sigma^2_{(\Theta^{(2)})} + \eta_{\ddot{c}} + (pu\theta_1 + (1-p)d\theta_1 + \ddot{c} - 2c_{\mathcal{A}})^2\right) - \mathcal{K}.
\end{aligned} \qquad \text{(A9)}
$$

Putting $\gamma_1 = 1/(1+\rho)$ and $\gamma_2 = \gamma_1/\rho$ in (A9) gives the expression in (1).

When both firms keep the option to delay alive until period 2, (i.e. $(a,b) = (D,D)$), then, given that demand rises in period 2, (i.e. $\theta_2 = u\theta_1$ with probability p), they simultaneously enter the market. However, while marginal costs remain private information, each firm conjectures that the choice to delay implies that its rival's true marginal cost must lie in some interval $(\beta, \bar{c}]$, where β is the upper bound of the marginal cost distribution support below which first period entry is profitable. The firms, therefore, put zero probabilities on all types within this interval.

Although second period demand level has now been observed, a Bayesian-Cournot game is played again in period 2 since marginal costs are still private information. The basic Cournot game is then played in all other periods after 2. Ex ante expected output and payoff in period 2 are

$$
q_{2,\mathcal{A}}^{*(D,D)} = \tfrac{1}{6}\left(2u\theta_1 + \hat{c} - 3c_{\mathcal{A}}\right) \qquad \text{and} \qquad \pi_{2,\mathcal{A}}^{*(D,D)} = \tfrac{1}{36}\left(2u\theta_1 + \hat{c} - 3c_{\mathcal{A}}\right)^2, \qquad \text{(A10)}
$$

where $\hat{c} = \int_{\beta}^{\bar{c}} c_{\mathcal{A}} \mathrm{d}F(c_{\mathcal{A}})$. Firm $\mathcal{B}$'s output and payoff are similarly determined.

Expected output and payoff in period 3 and all other periods after that are

$$q_{3,\mathcal{A}}^{*(D,D)} = \tfrac{1}{3}\left(u\theta_1 + \hat{c} - 2c_{\mathcal{A}}\right) \qquad \text{and} \qquad \pi_{3,\mathcal{A}}^{*(D,D)} = \tfrac{1}{9}\left(u\theta_1 + \hat{c} - 2c_{\mathcal{A}}\right)^2, \tag{A11}$$

and the value of the investment when the option is kept alive until period 2 is given by

$$\vartheta_{\mathcal{A}}^{(D,D)} = p(-k_1\mathcal{K} + \tfrac{\gamma_1^2}{36}\left(2u\theta_1 + \hat{c} - 3c_{\mathcal{A}}\right)^2 + \tfrac{\gamma_1\gamma_2}{9}\left(u\theta_1 + \hat{c} - 2c_{\mathcal{A}}\right)^2). \tag{A12}$$

γ_1 and γ_2 are as previously defined.

Sequential-move Equilibria

A number of scenarios play out when the firms choose to enter the market at different times. It suffices to consider the case for firm $\mathcal{A}$ entering early and firm $\mathcal{B}$ differing until period 2 before deciding to enter or not. After these choices are made and observed, firm $\mathcal{A}$ invests in period 1 and acts as a monopolist. His equilibrium output and payoff are

$$q_{1,\mathcal{A}}^{*(I,D)} = \tfrac{1}{2}\left(\theta_1 - c_{\mathcal{A}}\right) \qquad \text{and} \qquad \pi_{1,\mathcal{A}}^{*(I,D)} = \tfrac{1}{4}\left(\theta_1 - c_{\mathcal{A}}\right)^2. \tag{A13}$$

Firm $\mathcal{B}$ incurs no sunk cost and earns nothing in this period. Should demand rise in period 2, firm $\mathcal{B}$ exercises its right to enter the market and acts as a Stackelberg follower. Firm $\mathcal{A}$, having observed that $\mathcal{B}$ has chosen to defer , conjectures that $\mathcal{B}$'s marginal cost must lie in the interval $[\alpha, \bar{c}] \subset [\underline{c}, \bar{c}]$. $\mathcal{A}$ maximizes its Stackelberg leader payoff given the expected reaction function of $\mathcal{B}$ as follows,

$$\begin{aligned}\max_{q_{2,\mathcal{A}}} \mathbb{E}(\pi_{2,\mathcal{A}}^{(I,D)}) &= \max_{q_{2,\mathcal{A}}}\{q_{2,\mathcal{A}}\left(u\theta_1 - q_{2,\mathcal{A}} - \mathbb{E}(\hat{q}_{2,\mathcal{B}}(c_{\mathcal{B}})) - c_{\mathcal{A}}\right)\} \\ &= \max_{q_{2,\mathcal{A}}}\{q_{2,\mathcal{A}}\left(u\theta_1 - q_{2,\mathcal{A}} - \left[\frac{u\theta_1 - q_{2,\mathcal{A}} - \check{c}}{2}\right] - c_{\mathcal{A}}\right)\}.\end{aligned} \tag{A14}$$

Where $\check{c} = \int_{\alpha}^{\bar{c}} c_{\mathcal{B}} \mathrm{d}F(c_{\mathcal{B}})$. Solving (A14) and substituting into the follower's optimization problem yield the expected equilibrium outputs:

$$q_{2,\mathcal{A}}^{*(I,D)} = \tfrac{1}{2}\left(u\theta_1 + \check{c} - 2c_{\mathcal{A}}\right) \qquad \text{and} \qquad q_{2,\mathcal{B}}^{*(I,D)} = \tfrac{1}{4}\left(u\theta_1 - 2c_{\mathcal{B}} + 2\tilde{c} - \check{c}\right). \tag{A15}$$

Their respective corresponding expected payoffs are

$$\pi_{2,\mathcal{A}}^{*(I,D)} = \tfrac{1}{8}(u\theta_1 + \breve{c} - 2c_{\mathcal{A}})^2 \qquad \text{and} \quad \pi_{2,\mathcal{B}}^{*(I,D)} = \tfrac{1}{16}(u\theta_1 - 2c_{\mathcal{B}} + 2\tilde{c} - \breve{c})^2. \tag{A16}$$

Marginal costs and demand level is now revealed and the basic Cournot game is played from period 3 onwards. The expected outputs and payoffs produced and earned respectively in each of these periods are:

$$q_{3,\mathcal{A}}^{*(I,D)} = \tfrac{1}{3}\left(u\theta_1 + \breve{c} - 2c_{\mathcal{A}}\right) \qquad \text{and} \quad q_{3,\mathcal{B}}^{*(I,D)} = \tfrac{1}{3}\left(u\theta_1 + \tilde{c} - 2c_{\mathcal{B}}\right), \tag{A17}$$

and

$$\pi_{3,\mathcal{A}}^{*(I,D)} = \tfrac{1}{9}(u\theta_1 + \breve{c} - 2c_{\mathcal{A}})^2 \qquad \text{and} \quad \pi_{3,\mathcal{B}}^{*(I,D)} = \tfrac{1}{9}(u\theta_1 + \tilde{c} - 2c_{\mathcal{B}})^2. \tag{A18}$$

$\tilde{c}$ in the expressions above is $\int_{\underline{c}}^{\bar{c}} c_{\mathcal{A}} \mathrm{d}F(c_{\mathcal{A}})$ (i.e. the expected value of firm $\mathcal{A}$'s marginal cost over the full support of its distribution). The expected value of the investment for each firm is given by,

$$\begin{aligned}\vartheta_{\mathcal{A}}^{(I,D)} = \gamma_1 \frac{\theta_1 - c_{\mathcal{A}}}{4} &+ \gamma_1^2 \left(\frac{p(u\theta_1 + \breve{c} - 2c_{\mathcal{A}})^2}{8} + \frac{(1-p)(d\theta_1 - c_{\mathcal{A}})^2}{4} \right) \\ &+ 9\gamma_1\gamma_2 \left(\frac{p(u\theta_1 + \breve{c} - 2c_{\mathcal{A}})^2}{9} + \frac{(1-p)(d\theta_1 - c_{\mathcal{A}})^2}{4} \right) - \mathcal{K}\end{aligned} \tag{A19}$$

and

$$\vartheta_{\mathcal{B}}^{(I,D)} = p\left(-k_1\mathcal{K} + k_1^2 \frac{(u\theta_1 - 2c_{\mathcal{B}} + 2\tilde{c} - \breve{c})^2}{16} + k_1 k_2 \frac{(u\theta_1 + \tilde{c} - 2c_{\mathcal{B}})^2}{9} \right). \tag{A20}$$

Appendix B

Given that the value functions themselves are monotone decreasing and convex in over the support of the marginal cost distribution, it suffices, therefore, that along with conditions (a), (b) and (c), the claims in **Lemma** 1 hold. First, we show that f_1 and f_2 are positive everywhere on the support of the marginal cost distribution as follows.

(A9) and (A20) give the expected value of the investment when period 0 choices are respectively (I, I) and (D, I). Note that (A20) refers to the value of the investment to a firm who choses to defer, when its rival enters early. If $p = 1/2$ and the marginal cost expectations are evaluated over a continuous uniform distribution we obtain

$$\vartheta^{(I,I)} = \frac{\left(\frac{\alpha}{2} - 3c + 2\theta_1\right)^2}{36(1+\rho)} + \frac{\frac{\alpha^2}{12} + \left(\frac{\alpha}{2} - 2c + \frac{u\theta_1}{2} + \frac{\theta}{2u}\right)^2 + \ln(u)^2}{9\rho(1+\rho)} - \mathcal{K} \tag{B1}$$

and

$$\vartheta^{(D,I)} = \frac{1}{2}\left(\frac{\left(\frac{1}{2}(-\alpha - 10) + \beta - 2c + u\theta_1\right)^2}{16(1+\rho)^2} + \frac{\left(\frac{\beta}{2} - 2c + u\theta_1\right)^2}{9\rho(1+\rho)^2} - \frac{\mathcal{K}}{1+\rho}\right). \tag{B2}$$

Without loss of generality, let $[\underline{c}, \bar{c}] = [0, 1]$. This implies that $\alpha, \beta \in [0, 1]$.

$$\begin{aligned} f_1 &= \left|\frac{\partial \vartheta^{(I,I)}}{\partial c}\right| - \left|\frac{\partial \vartheta^{(D,I)}}{\partial c}\right| \\ &= \frac{1}{144u\rho(1+\rho)^2}\big(32\theta_1 + (32\theta_1 + u(90 + 53\alpha - 18\beta + 48\theta_1 + 14u\theta_1 \\ &\quad - 164c))\rho + 12u(\alpha + 4\theta_1 - 6c)\rho^2 - 16u(2\alpha + \beta - 4c)\big). \end{aligned} \tag{B3}$$

It is easy to see that f_1 is linear in c and differentiable on $[0, 1]$, Also, $f_1(0) > f_1(1) \geq 0$. Furthermore,

$$\frac{\partial f_1}{\partial c} = -\frac{(1+2\rho)(16+9\rho)}{36\rho(1+\rho)^2}. \tag{B4}$$

Notice that (B4) is independent of c. Also, $0 < \rho < 1$, therefore, given (B4), and the fact that $f_1(0) > f_1(1) \geq 0$, we see that f_1 is monotonic decreasing and non-negative for all c in $\mathbb{I}$, where $\vartheta^{(I,I)} \geq 0$.

Similarly, $f_2 = |\partial\vartheta^{(I,D)}/\partial c| - |\partial\vartheta^{(D,D)}/\partial c| > 0$ for all c in $\mathbb{I}$ where $\vartheta^{(I,D)} \geq 0$. f_2 is differentiable on $[0, 1]$ and $f_2(0) > f_2(1) \geq 0$. Moreover, it is linear and monotonic decreasing as shown in (B5).

$$\frac{\partial f_2}{\partial c} = -\frac{1 + 2\rho(2+\rho)}{4\rho(1+\rho)^2}. \tag{B5}$$

This verifies condition (b) of **L**emma 1.

For condition (b), we begin by noting that $g_1(\alpha, \beta) = (\vartheta^{(I,I)})^{-1}(0) -$

$(\vartheta^{(D,I)})^{-1}(0)$ and $g_2(\alpha,\beta) = (\vartheta^{(I,D)})^{-1}(0) - (\vartheta^{(D,D)})^{-1}(0)$. Again, WLOG, we let $[\underline{c}, \bar{c}] = [0,1]$. Solving for the roots of $\vartheta^{(I,I)}$ and $\vartheta^{(D,I)}$, and taking the difference, we have

$$\begin{aligned} g_1(\alpha,\beta) &= \frac{-3245 - 534\beta - 8\sqrt{15}\sqrt{49977 + 36\alpha - 86\alpha^2 - 1014\ln\left[\frac{3}{2}\right]^2}}{2028} \\ &\quad + \frac{525\alpha + 36\sqrt{10}\sqrt{3718 - (1+\alpha-\beta)^2}}{2028} \\ &\quad (\text{ set } \alpha = 1 \text{ and } \beta = 0 \text{ in the numerator of the above expression }) \\ &< -\frac{10156.58 + 7466.56}{2028} \\ &= -\frac{2690.02}{2028} \\ &< 0. \end{aligned} \tag{B6}$$

Using $\alpha = 1$ and $\beta = 0$ in the numerator of the first line in (B6) minimizes the absolute value of the negative part of it, whilst maximizing the absolute value of positive part. This allows us to obtain the inequality in the line that followed. In the same vein, we have it that,

$$\begin{aligned} g_2(\alpha,\beta) &= \frac{17989 + 445\alpha - \sqrt{55}\sqrt{2278765 - 267\alpha(34+9\alpha)}}{2968} \\ &\quad + \frac{-2147 - 83\beta + 2\sqrt{10}\sqrt{32933 - (-46+\beta)\beta}}{338} \\ &\quad (\text{ set } \alpha = 1 \text{ and } \beta = 0 \text{ in the numerators of the above expression }) \\ &< -\frac{11166.94}{2968} + \frac{18434}{2968} - \frac{2147}{338} + \frac{1147.75}{338} \\ &\approx 2.44 - 2.97 \\ &< 0. \end{aligned} \tag{B7}$$

Again, putting $\alpha = 1$ and $\beta = 0$ minimizes the absolute values of the negative terms of the first line of expression in (B7), whilst maximizing the positive terms of it.

To conclude the verification of condition (c) of **Lemma** 1, we let $h = \vartheta^{(I,I)})^{-1}(0) - \vartheta^{(I,D)})^{-1}(0)$, and show that $h < 0$ on $[0,1]$.

We have

$$h = \frac{-2519591 - 5936\sqrt{15}\sqrt{49977 + 36\alpha - 86\alpha^2 - 1014\ln\left[\frac{3}{2}\right]^2}}{1504776} + \frac{143901\alpha + 507\sqrt{55}\sqrt{2278765 - 267\alpha(34 + 9\alpha)}}{1504776}. \tag{B8}$$

Recall that α represents the believe a firm holds about its rival's marginal cost, given his period-0 choice, and it lies in the interval $[0, 1]$. Moreover, h is linear in α, therefore, it suffices to show that if $h(\alpha = 0) < 0$ and $h(\alpha = 1) < 0$, then h is negative everywhere on the interval $[0, 1]$. From (B8), $h(0) \approx -1974599.68$ and $h(1) \approx -1842439.30$.

Lastly, whenever θ takes on a value that produce a non-negative price, and α and β are chosen to minimize $\vartheta^{(I,I)}(0) - \vartheta^{(D,I)}(0)$ and $\vartheta^{(I,D)}(0) - \vartheta^{(D,D)}(0)$ as shown above, these expressions respectively yield $7482895/313632 + 100/99\ln\left(\frac{3}{2}\right)^2$ and $5067665/17424$, which proves condition (a). $\square$

Proposition 3 (Mixed Strategy Equilibrium)

Having established **Lemma** 1 and 2, and using **Table** 1, a firm, say $\mathcal{A}$, is able to determine ex ante that

a. If $c_{\mathcal{A}} < c_1$, then regardless of $c_{\mathcal{B}}$ it is a dominant strategy to play I. Since $\vartheta_{\mathcal{A}}^{I,I} > \vartheta_{\mathcal{A}}^{D,I}$ and $\vartheta_{\mathcal{A}}^{I,D} > \vartheta_{\mathcal{A}}^{D,D}$.

b. If $c_{\mathcal{A}} > c_2$, then regardless of $c_{\mathcal{B}}$ it is a dominant strategy to play D. Since $\vartheta_{\mathcal{A}}^{D,I} > \vartheta_{\mathcal{A}}^{I,I}$ and $\vartheta_{\mathcal{A}}^{D,D} > \vartheta_{\mathcal{A}}^{I,D}$.

Question is: how should the firm behave if $c_1 < c_{\mathcal{A}} < c_2$? Now, if $c_{\mathcal{B}} < c_1$ then $\mathcal{B}$ will play I and therefore, since $\vartheta_{\mathcal{A}}^{D,I} > \vartheta_{\mathcal{A}}^{I,I}$, firm $\mathcal{A}$ should play D. On the other hand, if $c_{\mathcal{B}} > c_1$ then $\mathcal{B}$ will play D and therefore, since $\vartheta_{\mathcal{A}}^{I,D} > \vartheta_{\mathcal{A}}^{D,D}$, firm $\mathcal{A}$ should play I. If $c_1 < c_{\mathcal{B}} < c_2$ then $\vartheta_{\mathcal{A}}^{D,I} > \vartheta_{\mathcal{A}}^{I,I}$ and $\vartheta_{\mathcal{A}}^{I,D} > \vartheta_{\mathcal{A}}^{D,D}$ (and similarly for $\mathcal{B}$, so the firms will (anti)-coordinate, and there will be an equilibrium in mixed strategies).

But when making its decision, firm $\mathcal{A}$ does not observe firm $\mathcal{B}$'s draw from the cost distribution, so it must base its assessment of what is the best strategy on its prior belief (knowing that, for cost draws in the same region, $\mathcal{B}$ will be doing the same thing). So, when $c_1 < c_{\mathcal{A}} < c_2$ and without observing its rival's cost draw, $\mathcal{A}$ will have to formulate its strategies in a manner consistent with its beliefs as follows.

Suppose $c_{\mathcal{B}} < c_1$ with probability p_1, $c_{\mathcal{B}} > c_2$ with probability p_2 and $c_1 < c_{\mathcal{B}} < c_2$ with probability $1 - p_1 - p_2$. Then, $\mathcal{A}$ can find a certain probability ϕ for which its dominant strategy is I when $c_{\mathcal{B}} < c_1$ and $c_{\mathcal{B}} > c_2$. However, when $c_1 < c_{\mathcal{B}} < c_2$, firm $\mathcal{B}$ formulates a similar strategy with a certain probability ψ. Therefore, when $c_1 < c_{\mathcal{A}} < c_2$ and $c_1 < c_{\mathcal{B}} < c_2$, firm $\mathcal{A}$ must expect (I, I) with probability $\phi\psi$, (I, D) with $\phi(1-\psi)$, (D, I) with $(1-\phi)\psi)$, and (D, D) with $(1-\phi)(1-\psi)$. It is clear that whenever $\phi = \psi$, the firms will (anti)-coordinate.

To find ϕ and ψ, we maximize the expected value of the investment with respect to ϕ and ψ in the following way,

$$\begin{aligned}\max_{0\leq\phi\leq 1} \mathbb{E}_{\mathcal{A}}\Pi = \max_{0\leq\phi\leq 1} \{&\vartheta_{\mathcal{A}}^{I,I}(p_1\phi + \phi\psi(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{A}}^{I,D}(p_2\phi + \phi(1-\psi)(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{A}}^{D,I}((1-\phi)p_1 + (1-\phi)\psi(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{A}}^{D,D}((1-\phi)p_2 + (1-\phi)(1-\psi)(1-p_1-p_2))\}\end{aligned} \tag{B9}$$

and

$$\begin{aligned}\max_{0\leq\psi\leq 1} \mathbb{E}_{\mathcal{B}}\Pi = \max_{0\leq\psi\leq 1} \{&\vartheta_{\mathcal{B}}^{I,I}(p_1\psi + \phi\psi(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{B}}^{I,D}(p_2\psi + \psi(1-\phi)(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{B}}^{D,I}((1-\psi)p_1 + (1-\psi)\phi(1-p_1-p_2)) \\ &+ \vartheta_{\mathcal{B}}^{D,D}((1-\psi)p_2 + (1-\psi)(1-\phi)(1-p_1-p_2))\}\end{aligned} \tag{B10}$$

Deriving the FOCs for B9 and B10 and solving for ψ and ϕ respectively yields,

$$\psi^* = \frac{(1-p_1-p_2)\left(\vartheta_{\mathcal{A}}^{I,I} - \vartheta_{\mathcal{A}}^{I,D} - \vartheta_{\mathcal{A}}^{D,I} + \vartheta_{\mathcal{A}}^{D,D}\right)}{p_1(\vartheta_{\mathcal{A}}^{D,I} - \vartheta_{\mathcal{A}}^{I,I}) + p_2(\vartheta_{\mathcal{A}}^{D,D} - \vartheta_{\mathcal{A}}^{I,D}) + (1-p_1-p_2)(\vartheta_{\mathcal{A}}^{D,D} - \vartheta_{\mathcal{A}}^{I,D})} \tag{B11}$$

and

$$\phi^* = \frac{(1-p_1-p_2)\left(\vartheta_{\mathcal{B}}^{I,I} - \vartheta_{\mathcal{B}}^{I,D} - \vartheta_{\mathcal{B}}^{D,I} + \vartheta_{\mathcal{B}}^{D,D}\right)}{p_1(\vartheta_{\mathcal{B}}^{D,I} - \vartheta_{\mathcal{B}}^{I,I}) + p_2(\vartheta_{\mathcal{B}}^{D,D} - \vartheta_{\mathcal{B}}^{I,D}) + (1-p_1-p_2)(\vartheta_{\mathcal{B}}^{D,D} - \vartheta_{\mathcal{B}}^{I,D})} \tag{B12}$$

Therefore, in the interval (c_1, c_2), $\mathcal{A}$ and $\mathcal{B}$'s mixed strategies are respectively $(\phi^*, (1-\phi^*))$ and $(\psi^*, (1-\psi^*))$.

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